$\phi\phi$ production at SPD NICA: QCD at partonic level and recursive fragmentation

SPD first stage physics meeting, May 5, 2026

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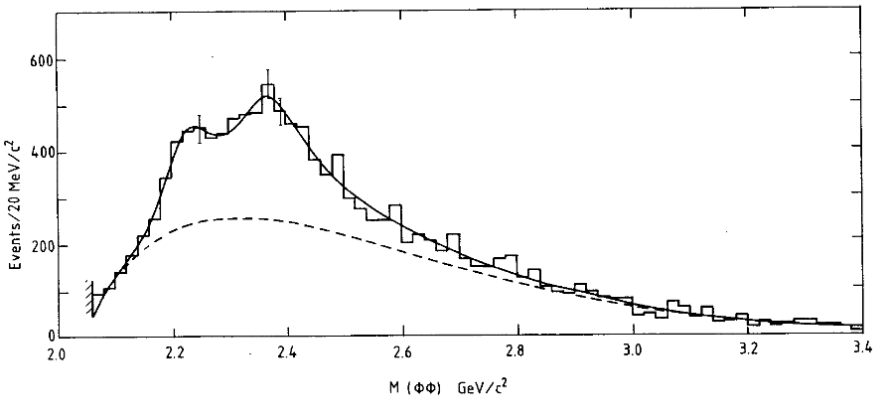
MISIS and SINP MSU

General information

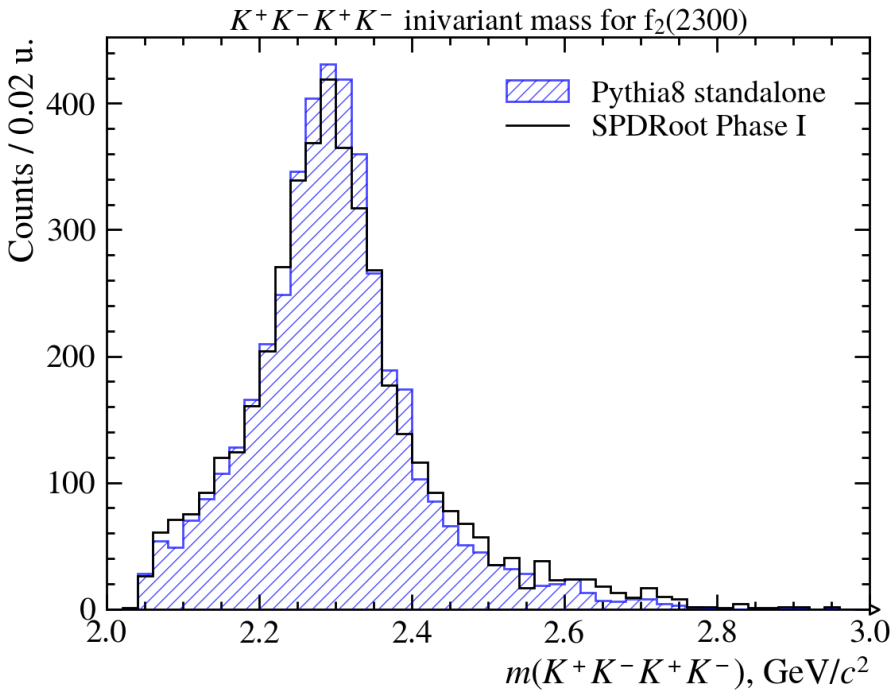
- The idea of the study – analysis of $s\bar{s}$ tensor mesons ($f_2(2150), f_2(2300), f_2(2340)$), nature of which is not exactly known. These are possible tetraquarks or glueball candidates.
- Since the library of meson cross sections and decays of PYTHIA 8 does not contain f_2 tensor mesons, to mimic processes of interest, $\chi_{c2}(1P)$ with the same quantum numbers and modified mass is used. Such substitution probably allows one to correctly capture the kinematics of the $pp \to \phi\phi \to 4K$ process, but the full cross-section cannot be meaningfully calculated. SpdRoot level study was successfully performed, which we reported in November (link).
- To estimate cross sections for $pp \to \phi\phi$ process partonic level calculations with recursive fragmentation are performed.

Event generation

- $\sqrt{s} = 10$ GeV pp collisions (PYTHIA8) with reconstruction using SPD phase I (SpdRoot)
- $f_2 \to \phi\phi \to K^+K^-K^+K^-$ on-shell decay
- Rough estimation of signal efficiency in SPD conditions $\varepsilon \sim 25 - 35\%$



Booth et al., A high statistics study of the mass spectrum, Nucl.Phys.B, Vol. 273, 1986,



Simulation of $f_2(2300)$ with Pythia8 and SpdRoot (10k events)

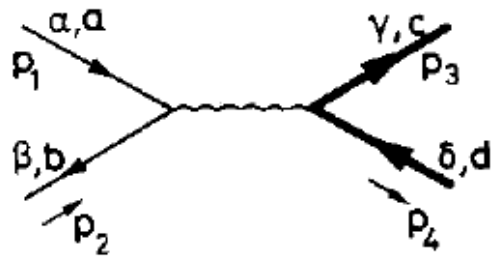
Cross section calculation

- In order to validate the SPD cross sections «from the first principles» not using any alien «black box» generators, lowest order quantum chromodynamics (QCD) processes (such as $q\bar{q} \to s\bar{s}$, $gg \to s\bar{s}$) followed by recursive fragmentation in the Field-Feynman model (FF) are used to meaningfully calculate the total cross section and distributions.
- Calculations are currently being carried out within the framework of QCD + FF using «old PDF's» with Q^2 dependence explicitly evaluated, and LHAPDF database.

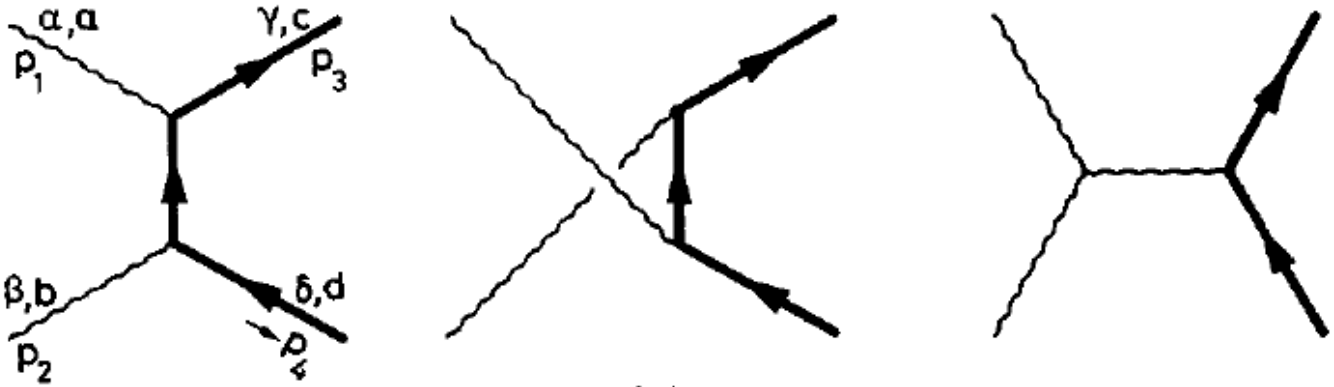
Theoretical approaches

QCD perturbative calculations

H. Georgi, S. Glashow, M. Machacek, D.V. Nanopoulos, Ann. Phys. 114 (1978) 273
M. Dubinin, D. Slavnov, Yad.Fiz. 35 (1982) 736



(a)



(b)

Рис.: Lowest order QCD diagrams: flavor creation

Theoretical approaches

Well-known, but untrivial technical issue: QCD gauge fixing condition $\omega^a = \partial_\mu A^a_\mu$ is not time-conserved like Lorentz gauge condition $\partial_\mu A_\mu = 0$ in QED. This is obvious from the equation of motion for ω^a (which appears after variation of QCD action gauge-fixing term $1/2\alpha(\partial_\mu A^a_\mu)^2$, $A^a_\mu \to A^a_\mu + \frac{1}{g_s}\nabla_\mu \alpha^a$)

$$\partial^2 \omega^a + g_s f^{abc} A^b_\mu \partial_\mu \omega^c = 0$$

which means that for elimination of ω-particles one needs to fix $\partial_\mu A^a_\mu = 0$ i.e. take only transverse gluons thus removing the unphysical contribution of $\omega^a(x)$. The matrix element of three diagrams (b), $M^{\mu\nu}_{ab} = M_1 + M_2 + M_3$

$$M_{gg\to qq} = g_s^2 \epsilon_\mu(p_1)\epsilon_\nu(p_2)\bar{u}(p_3) M^{\mu\nu}_{ab} v(p_4)$$

$$M_1 = \lambda_a\lambda_b\gamma^\mu \frac{\hat{p}_3 - \hat{p}_1 + m}{(p_3 - p_1)^2 - m^2}\gamma^\nu, \quad M_2 = \lambda_b\lambda_a\gamma^\nu \frac{\hat{p}_3 - \hat{p}_2 + m}{(p_3 - p_2)^2 - m^2}\gamma^\mu$$

$$M_3 = i f_{abc}\lambda_c \frac{g^{\mu\nu}(\hat{p}_1 - \hat{p}_2) + \gamma^\nu(2\hat{p}_2 + \hat{p}_1)^\mu - \gamma^\mu(2\hat{p}_1 + \hat{p}_2)^\nu}{(p_1 + p_2)^2}$$

does not allow «replacement of $\epsilon_\mu\gamma_\mu$ by γ_μ» for covariant summation over transverse polarisations of gluons $\partial_\mu A^a_\mu = 0$ because $p_{1\mu} M^{\mu\nu}_{ab}$ and $p_{2\mu} M^{\mu\nu}_{ab}$ are not zero. For correct calculation of $M^{\mu\nu}_{ab}$ two transverse terms $p_1^\mu \epsilon_\mu(p_1)$ and $p_2^\mu \epsilon_\mu(p_2)$ are excluded from the numerator of M_3 which is replaced by

$$M_3 = i f_{abc}\lambda_c \frac{g^{\mu\nu}(\hat{p}_1 - \hat{p}_2) + 2\gamma^\nu p_2^\mu - 2\gamma^\mu p_1^\nu}{(p_1 + p_2)^2}$$

then the redefined amplitude respects the condition $p_{1\mu} M^{\mu\nu}_{ab} = p_{2\mu} M^{\mu\nu}_{ab} = 0$. This trick is not appropriate for loop diagrams, where ghost fields need to be introduced to remove the unphysical contributions from ω.

Theoretical approaches

Calculation of QCD diagrams with the following integration in the physical region gives total partonic level cross sections

$$\sigma_{qq\to ss}(\hat{s}) = \frac{8\pi\alpha_s^2(Q^2)}{27\hat{s}}\left(1+\frac{2m_s^2}{\hat{s}}\right)\sqrt{1-\frac{4m_s^2}{\hat{s}}}, \quad Q^2 = x_1x_2s$$

$$\sigma_{gg\to ss}(\hat{s}) = \frac{\pi\alpha_s^2(Q^2)}{3\hat{s}}\left[-(7+31\frac{m_s^2}{\hat{s}})\frac{R}{4}+(1+\frac{4m_s^2}{\hat{s}}+\frac{m_s^4}{\hat{s}^2})\log\frac{1+R}{1-R}\right],$$

$$R = \sqrt{1-\frac{4m_s^2}{\hat{s}}}, Q^2 = \kappa\hat{s}, \kappa = 1/2 \text{ (t-channel)}$$

$$\frac{d^3\sigma_{pp\to ss}}{d\hat{s}dx_Ldz} = \frac{1}{x_1x_2(x_1+x_2)s}f_1^p(x_1,Q^2)f_2^p(x_2,Q^2)D_s(z)\hat{\sigma}(\hat{s}),$$

$$x_L = x_1 - x_2$$

Quark and gluon distribution functions $f_q^p(x_1,Q^2)$, $f_g^p(x_2,Q^2)$ and $\alpha_s(Q^2)$ normalized according to LHAPDF database.

PDF before HERA (ISR time period, 1971-1984, $\sqrt{s} = 20 - 62$ GeV)

A. Buras, K. Gaemers, Simple Parametrizations of Parton Distributions with q**2 Dependence Given by Asymptotic Freedom, Nucl.Phys.B 132 (1978) 249

Primary QCD fits. Parton distributions are decomposed into valence, non-charm sea and charm sea ($p, n, \lambda \to u, d, s$, two generations) with normalizing conventions

$$p(x,Q^2) = p_v(x,Q^2) + \lambda(x,Q^2),\ n(x,Q^2) = n_v(x,Q^2) + \lambda(x,Q^2)$$

$$\bar{p}(x,Q^2) = \bar{n}(x,Q^2) = \lambda(x,Q^2) = \bar{\lambda}(x,Q^2)$$

$$S(x,Q^2) = 6\lambda(x,Q^2), \quad C(x,Q^2) = c(x,Q^2) + \bar{c}(x,Q^2)$$

$$V_8(x,Q^2) = p_v(x,Q^2) + n_v(x,Q^2), \quad V_3(x,Q^2) = p_v(x,Q^2) - n_v(x,Q^2)$$

and parametrizations with $A(\bar{s})$ and $\eta(\bar{s})$ calculated in QCD by resummation of ladder diagrams

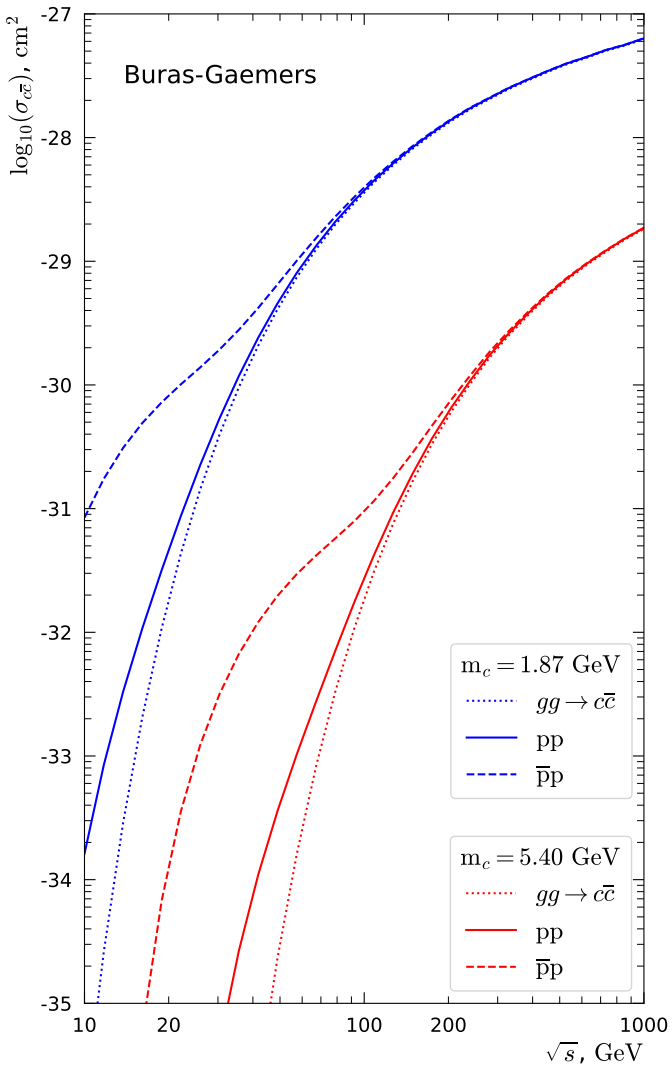
$$xV_i(x,\bar{s}) = A_i(\bar{s})x^{\eta_1^i(\bar{s})}(1-x)^{\eta_2^i(\bar{s})}$$

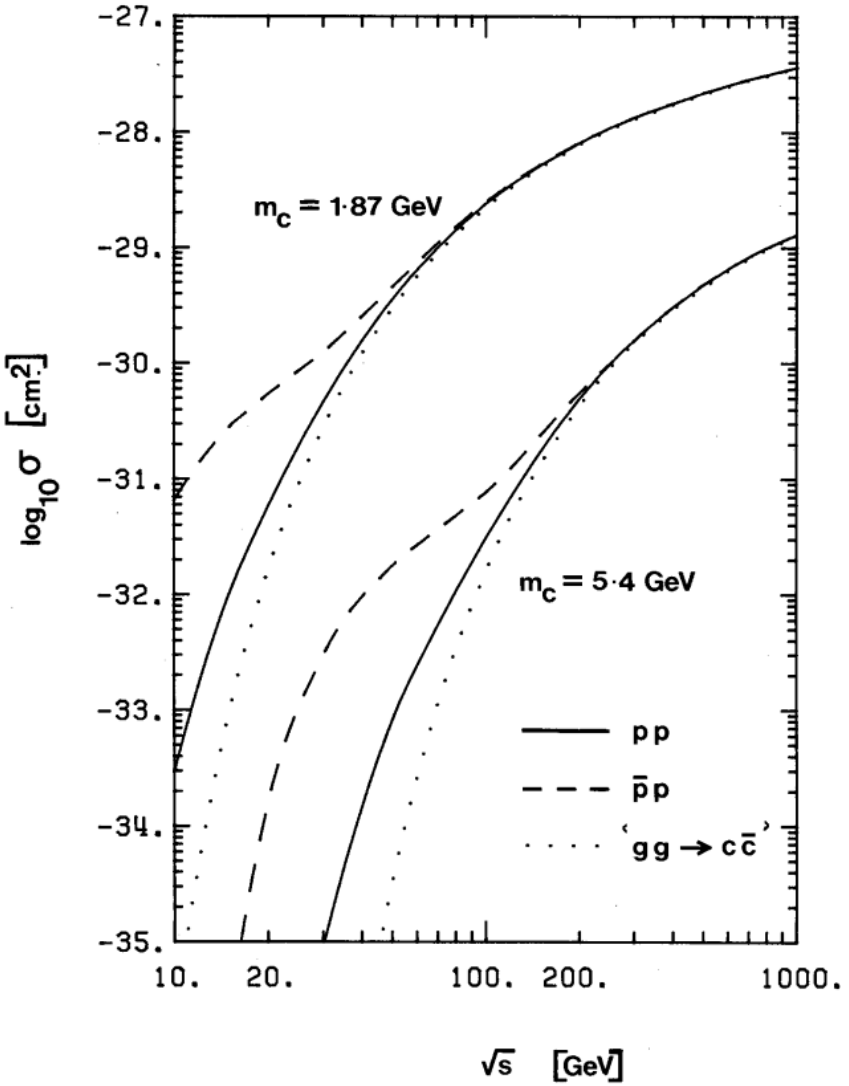
$$xS(x,\bar{s}) = A_s(\bar{s})(1-x)^{\eta_s(\bar{s})}, \quad xS(x,\bar{s}) = A_s(\bar{s})(1-x)^{\eta_s(\bar{s})}$$

$$xG(x,\bar{s}) = A_G(\bar{s})(1-x)^{\eta_G(\bar{s})}, \quad xC(x,\bar{s}) = A_C(\bar{s})(1-x)^{\eta_C(\bar{s})}$$

$$\bar{s} = \frac{ln\frac{Q^2}{\Lambda^2}}{ln\frac{Q_0^2}{\Lambda^2}}$$

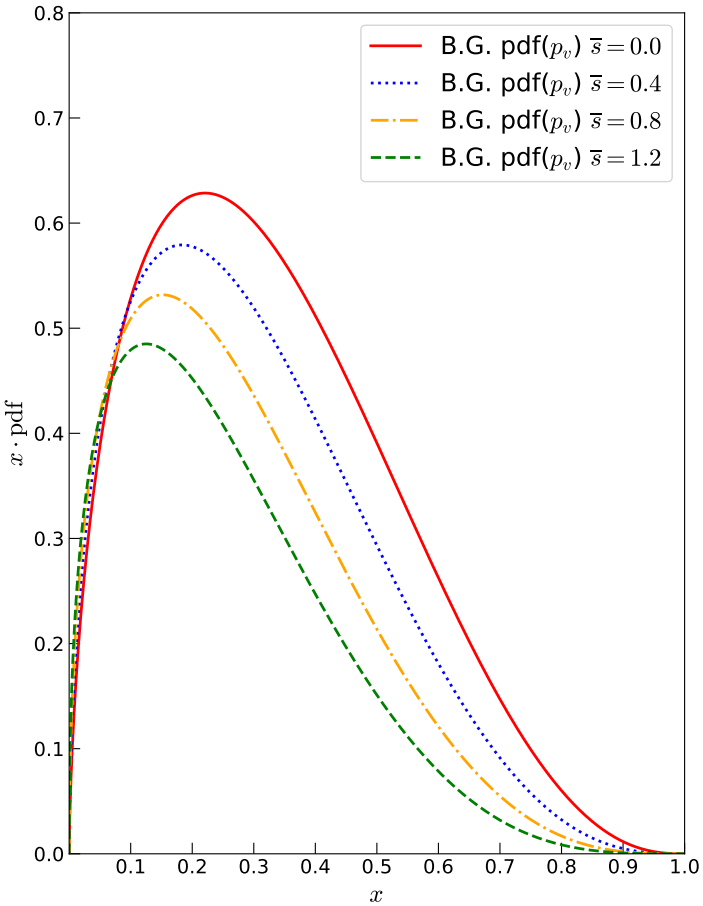
Validation with c-quark

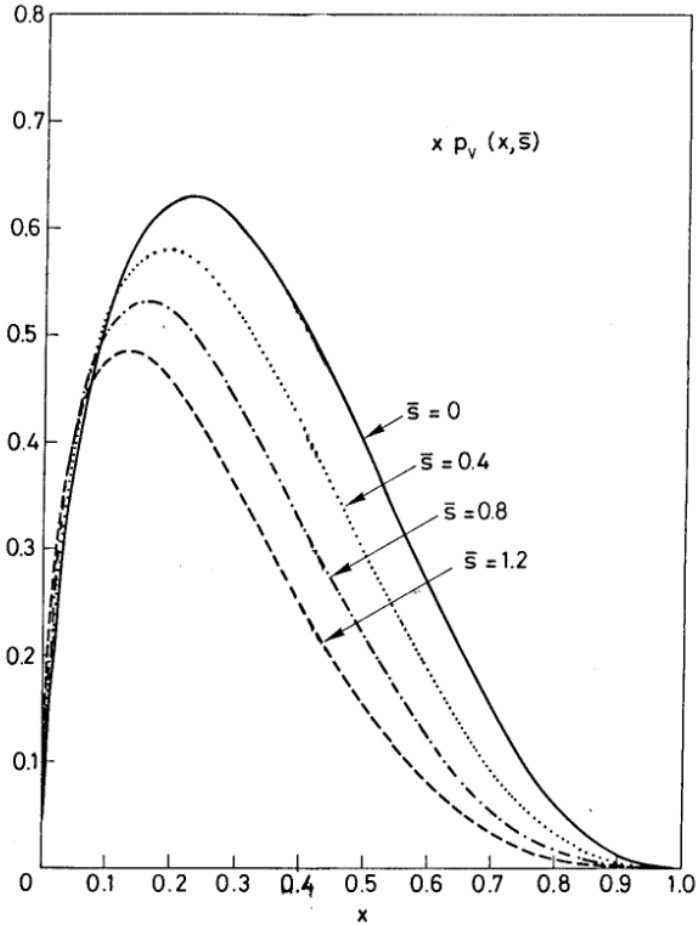




Cross-section calculated using parametrization from Buras-Gaemers and original result (Combridge, 1978)

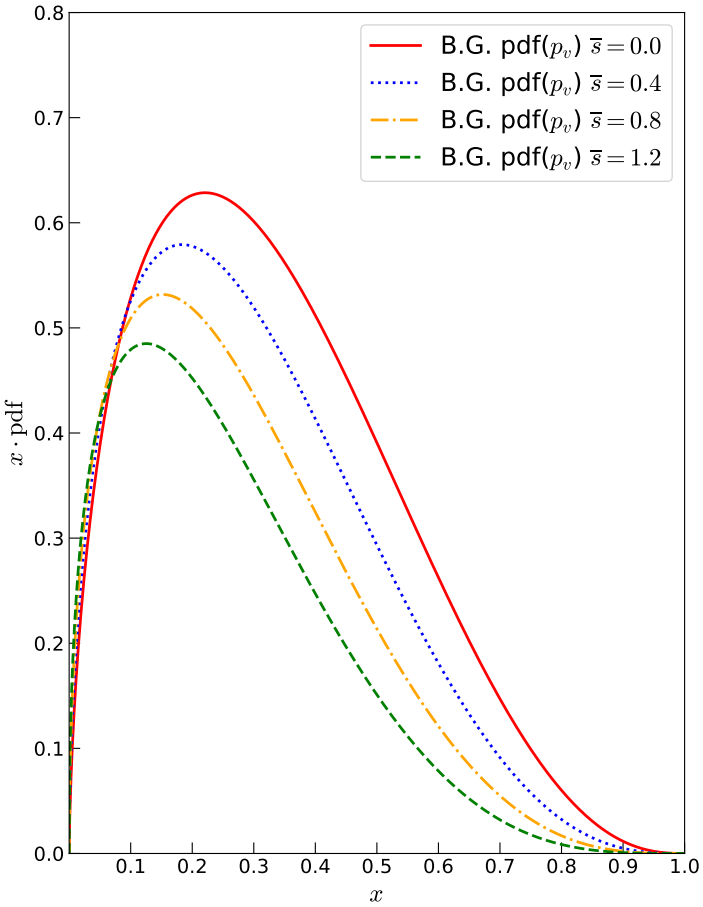
PDF investigation

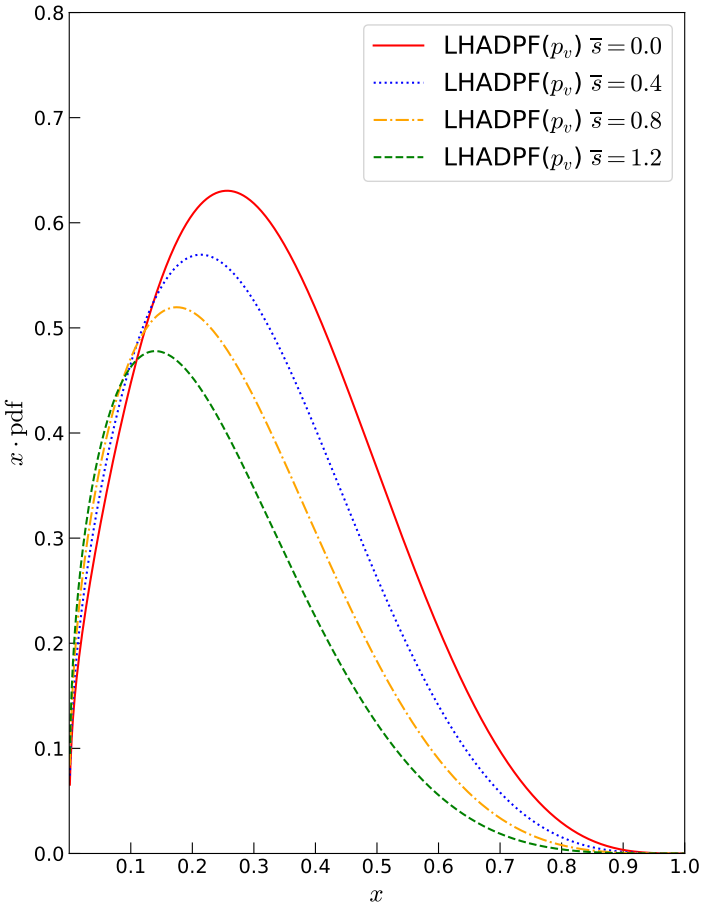




We carefully reproduced all the curves from the work of Buras, Gaemers

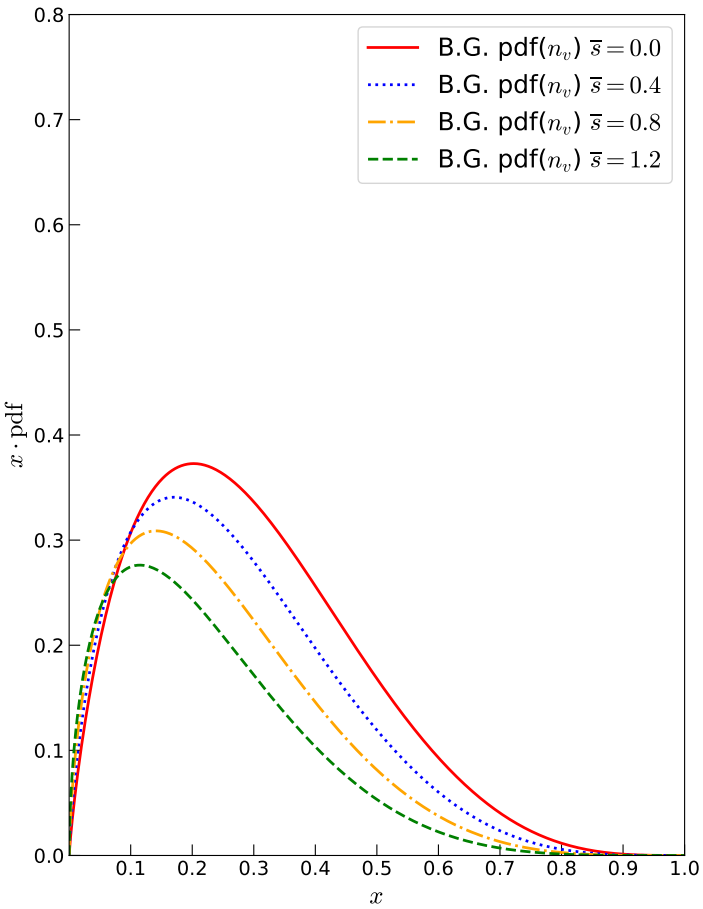
Modern PDFs

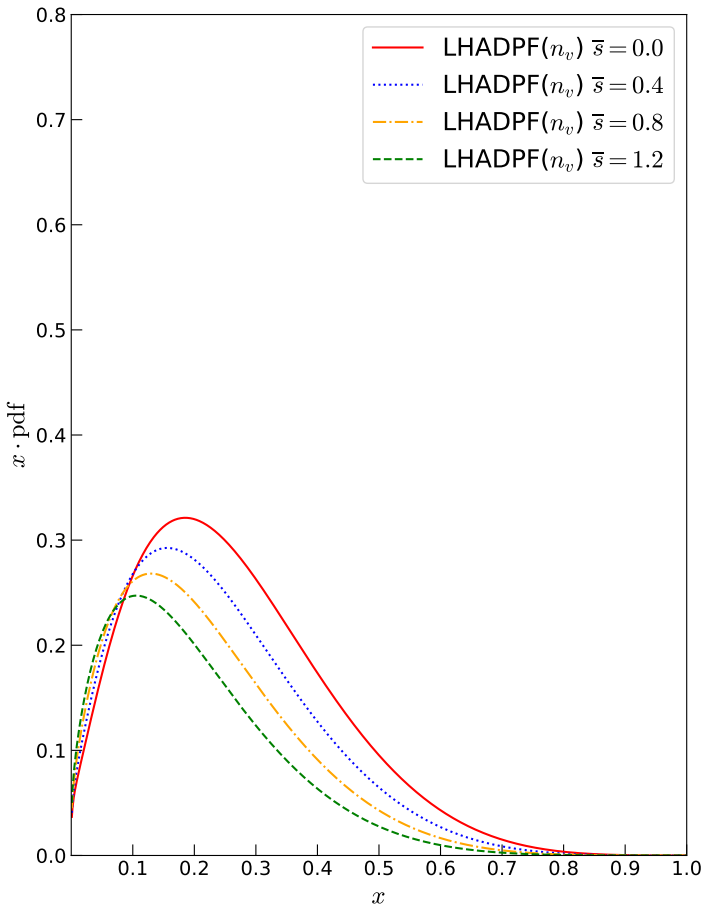




There is a difference between calculated values and modern PDFs

Modern PDFs





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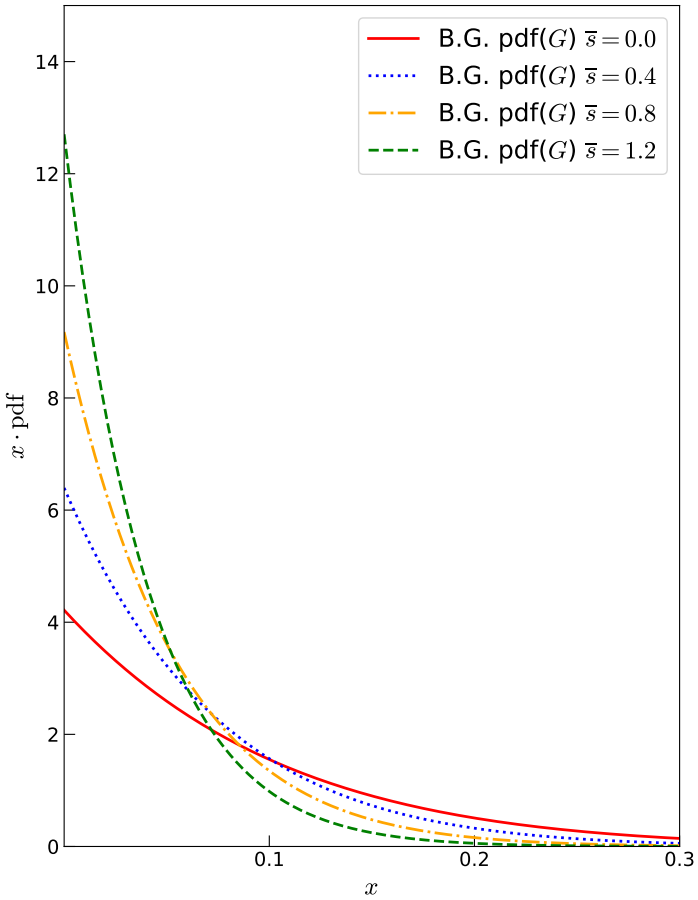

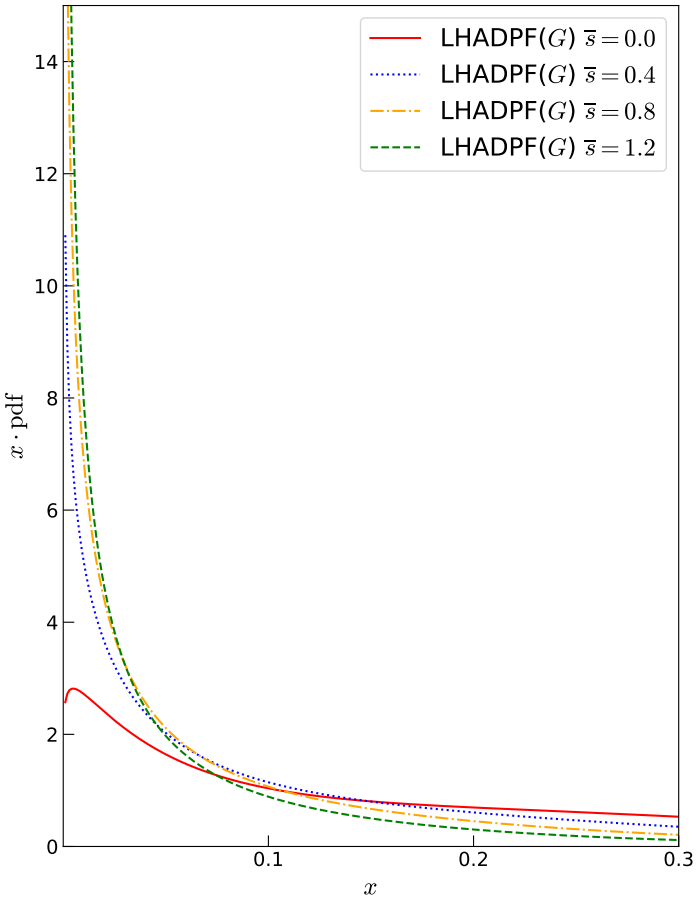

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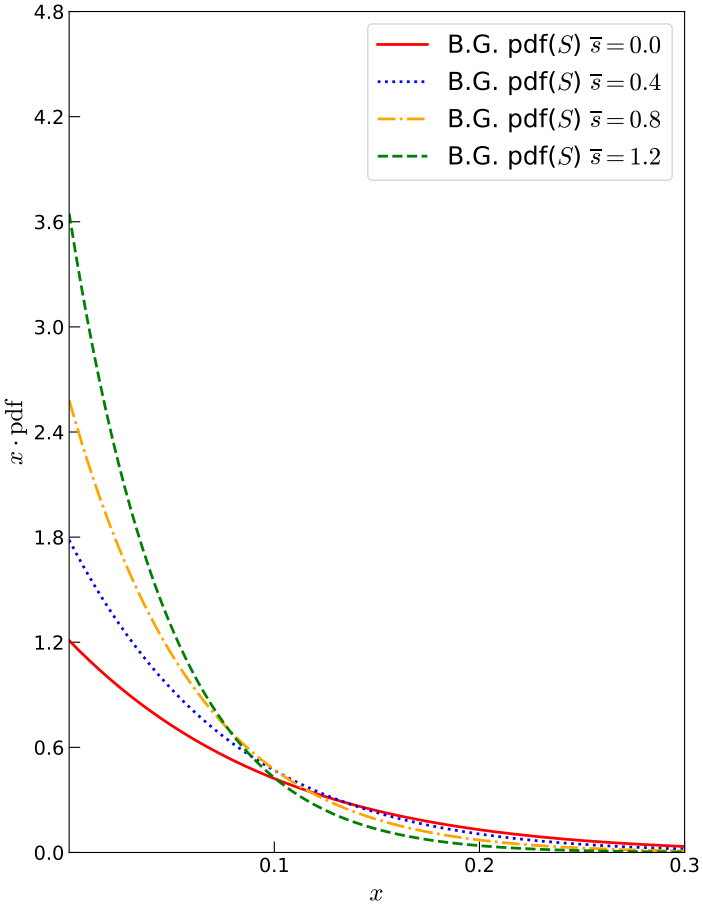

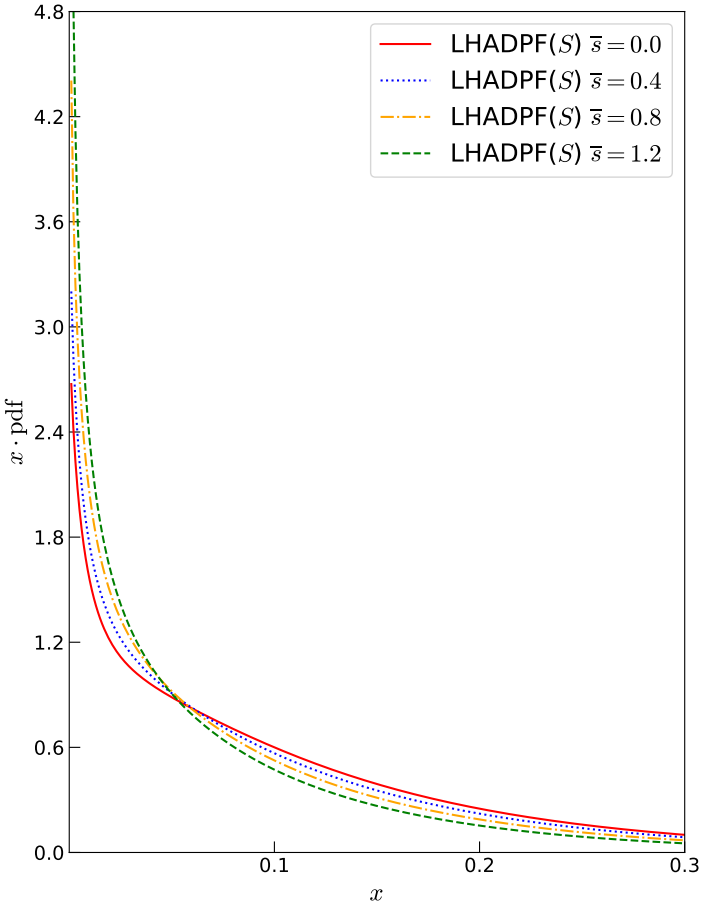

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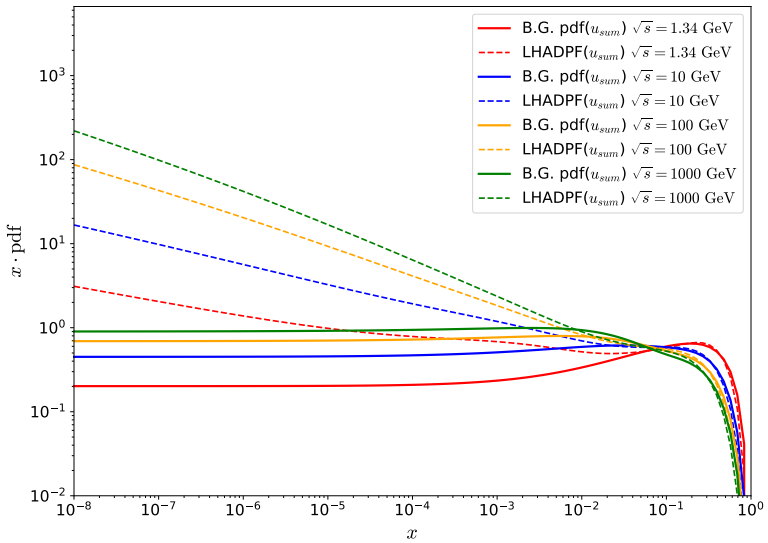
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$x \cdot \mathrm{pdf}$
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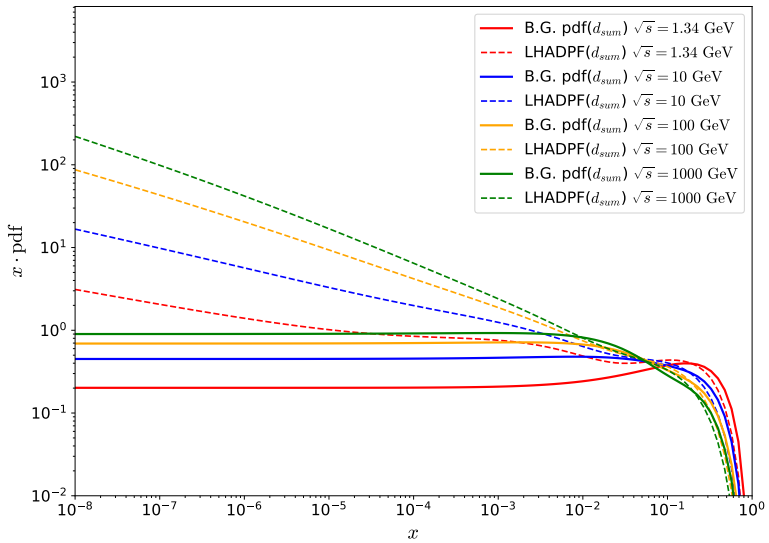
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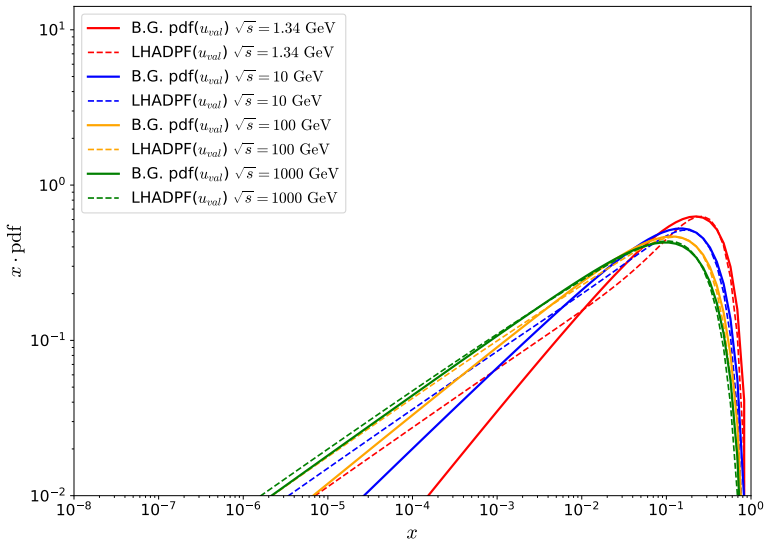
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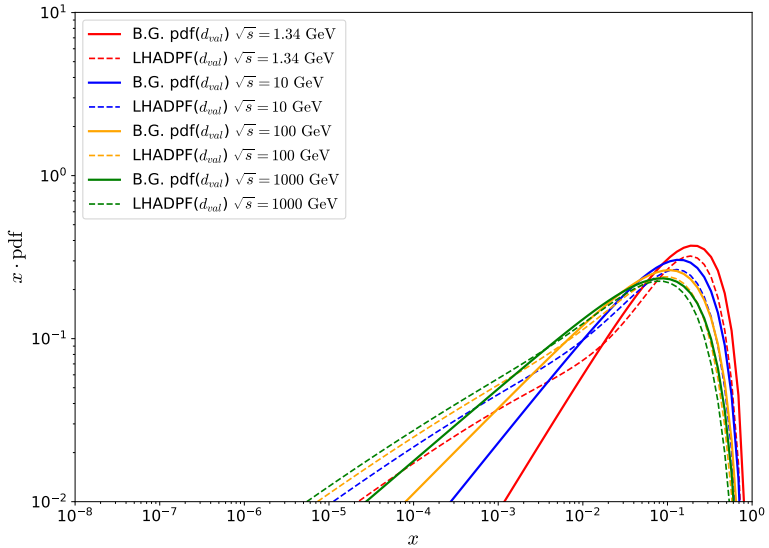
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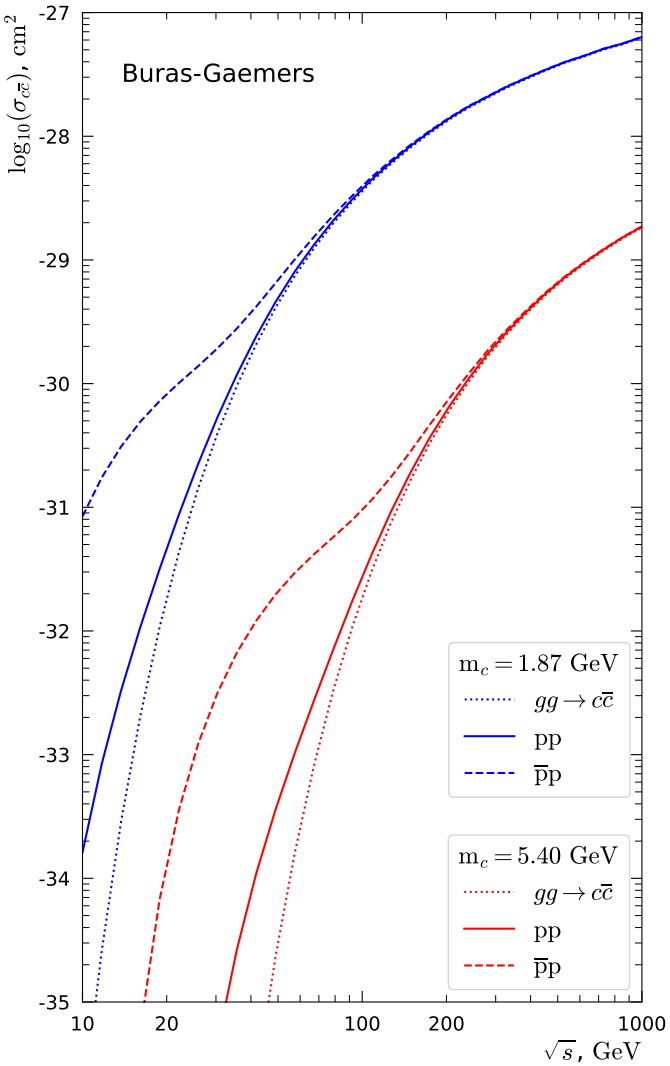
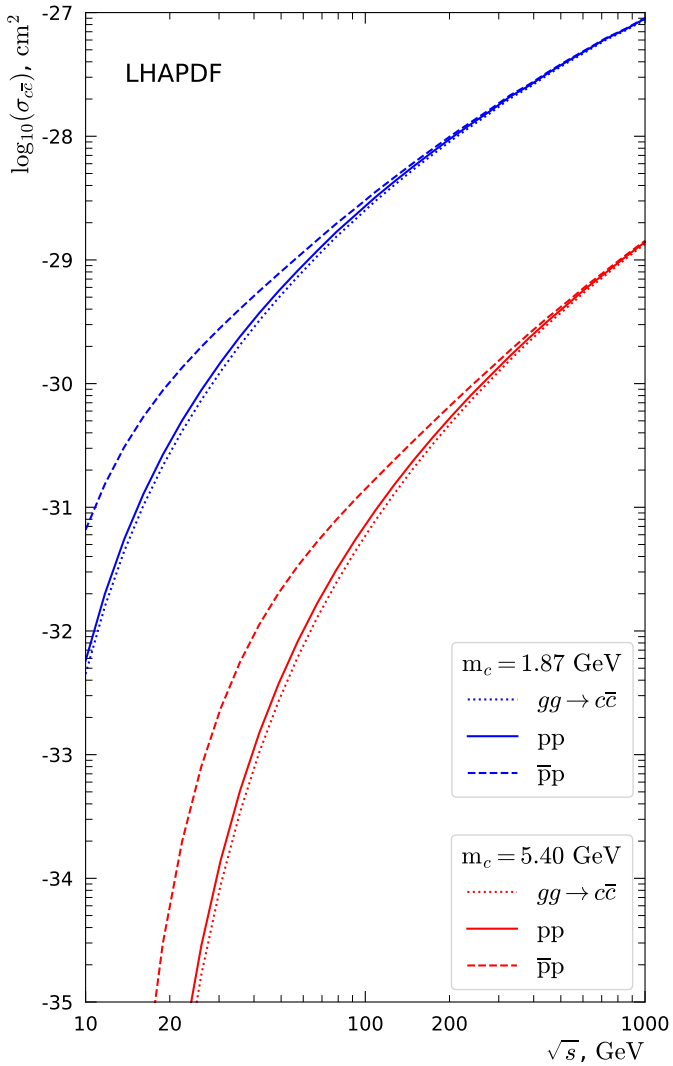
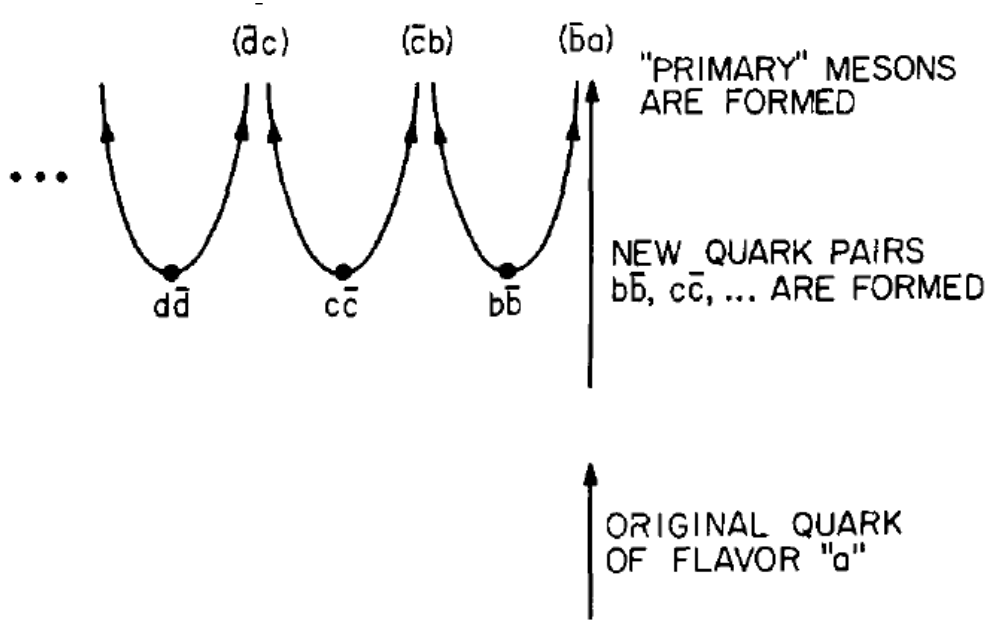
LHAPDF results





Fragmentation

QCD perturbative calculations + Field-Feynman model of recursive fragmentation (Nucl.Phys.B 136 (1978)1-76)



Meson fragmentation function $F(z) = f(1-z) + \int f(\eta)F(z/\eta)d\eta/\eta$ where $f(\eta)$ is the probability that primary meson 1 leaves the fraction of momentum η to the remaining cascade, $\int f(\eta)d\eta$ =1, solutions of the integral equation are found for $f(\eta) = (a+1)\eta^a$. Double fragmentation function is defined by analogous integral equation $F_2(z_1, z_2) = f(1-z_1)F(z_2/(1-z_1))/(1-z_1) + \int f(\eta F_2(z_1/\eta, z_2/\eta)d\eta/\eta^2$.

Fragmentation functions

Jing Hua, Bo-Qiang Ma, Fragmentation functions of mesons in the Field-Feynman model, Eur.Phys.J.C30:207-212 (2003)
Qiaomu Peng, Bo-Qiang Ma, Determination of meson fragmentation functions in the Field-Feynman model, Phys. Rev. C 107, 055202 (2023)

$$D_s^{\phi}(z) = \beta_{ss}(1-2\beta_{ss})\bar{F}(z), \tag{1}$$

$$D_{\bar{s}}^{\phi}(z) = \beta_{ss} f(1-z) + \beta_{ss}(1-2\beta_{ss})\bar{F}(z), \tag{2}$$

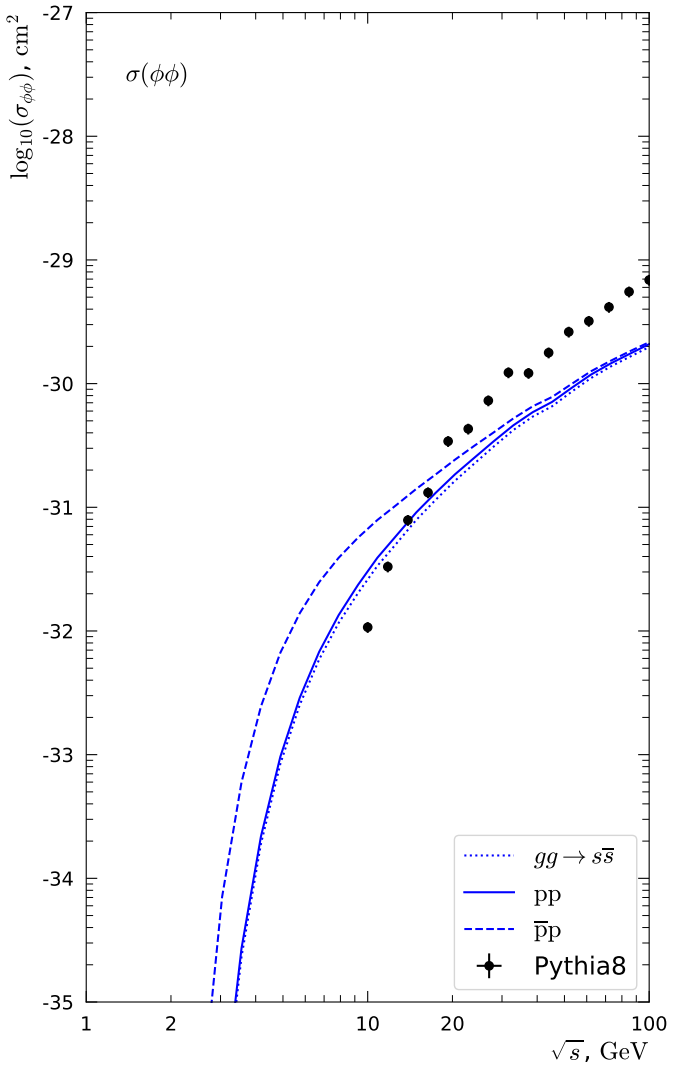
$$f(z) = (d+1)z^d, \tag{3}$$

$$\bar{F}(z) = (1/z-1)f(1-z). \tag{4}$$

Exact coefficients to use are debated.

Results for NICA energies

- Pythia8 has problems generating events near phase space threshold
- $\sim$ nanobarns at NICA energies
- We still need to validate fragmentation functions before we conclude this study



Outlook

- The primary PDF-1980 fits and Q^2 dependencies produced specifically for $\sqrt{s} \sim 50$ GeV (before MRS and CTEQ) are not the same as the fits of the MRS, CTEQ $\rightarrow$ LHAPDF database at the TeV energy scale. Validation of PDF's at the energy scale $\sqrt{s} \sim 10$ GeV is appropriate using «benchmark processes» like Drell-Yan, etc.
- It makes sense to go beyond PYTHIA 8 and reconsider the parton-level field-theoretic models and fragmentation models that were successfully used 20-25 years ago to describe the ISR data and the SPS experiments with extracted beams, taking into account the latest results on calculating radiative corrections to multiparticle final states and the results on QCD resummations for jets.
- We hope that fully validated procedure could be used to calculate cross-sections of any new process of interest



Status

Physics

- Machinery to generate events with Pythia8
- Reconstruction with SpdRoot
- Estimates of reconstruction efficiency
- Estimates of cross sections

Organisational

- Small group in MISIS (2 former PhD students)
- No external funding
- We have cluster access, but haven't tried generation there