Annex 4.
Project report form

APPROVED

Director of Laboratory

_____/ _____/

"____" _____ 202_

REPORT ON PROJECT
IN FIELD OF RESEARCH WITHIN
THE TOPICAL PLAN FOR JINR RESEARCH

1. General information on the Project
1.1. Scientific field
06 – Information technologies

1.2. Name of the Project
Methods of computational physics for the study of complex systems

1.3. Project code
06-6-1119-2-2024/2026

1.4. Theme code
06-6-1119-2014

1.5. Actual duration of the Project

1.6. Project Leaders
Zemlyanaya E.V., Chuluunbaatar O.

1.7 Project Deputy Leaders
Yu.L. Kalinovsky, A. Khvedelidze

2. Scientific report
2.1. Annotation
The project aims at the development and use of mathematical and computational methods for modeling complex physical systems studied within the JINR Topical Plan and described by systems of dynamic nonlinear, spatially multidimensional integral, integro-differential or differential equations driven by the parameters of the models. The evolution of the solutions of such systems may be characterized by the occurrence of critical regimes, bifurcations and phase transitions. Mathematical modeling is an indelible part of the modern scientific research. It needs adequate mathematical formulations of the problems within the models under study, the adaptation of known numerical approaches or the development of new ones to effectively take into account the features of the studied physical processes, the elaboration of algorithms and software packages for high-performance simulation on modern computer systems, primarily the MICC MLIT resources. Research within the project is conducted along the following main directions:

I. Development of methods, algorithms and software packages for carrying out numerical studies of interactions of various types in complex systems of nuclear physics, atomic physics, molecular physics, and quantum mechanics.

II. Studies of intricate processes in condensed matter and materials under external actions.

III. Solutions of modeling problems in the design of experimental facilities and optimization of their operating modes.

IV. Modeling of complex processes in dense nuclear matter, including astrophysical models and processes in experimental facilities.

V. Modeling of quantum systems using methods of quantum information theory and hybrid quantum-classical programming methods.

Research conducted in 2024–2025 yielded new results both on the development of computational methods and programs for the study of complex physical systems and on the numerical study of these systems. New information about the systems under study, which is important for the further development of mathematical models, theoretical and experimental research, was obtained.

2.2. A detailed scientific report

2.2.1. Description of the mode of operation and functioning of the main systems and equipment
2.2.2. Description of the experiments carried out (for pilot projects).
2.2.3. Description of the scientific work undertaken and the results obtained.

The results within the project are presented in more than 120 publications, including three monographs and two textbooks. The monograph [1] is devoted to the developed iterative methods, algorithms and programs for solving systems of equations of various types using the finite difference method and the high-order finite element method (FEM). The monograph [2] is devoted to the developed iterative methods and algorithms for solving systems of nonlinear equations based on the Continuous Analogue of Newton's method. The monograph [3] provides an overview of the main results that lead to an understanding of the dual nature of the chemical bond in the diatomic beryllium molecule in the ground state $X^1\Sigma_g^+$. Essentially, it is shown that, at low vibrational levels, the atoms are covalently bound, while at higher levels, they act via van der Waals forces, shifting between potential energy shapes.

The scientific review [4], published in the Natural Science Review journal, on the development of modified iterative schemes based on the Continuous Analogue of Newton's method – an important tool for solving a number of the project's problems.

To describe the collective quadrupole model of the atomic nucleus and calculate rotational-vibrational spectra and quadrupole transition probabilities, the GCMFEM program for solving boundary-value problems (BVPs) for a system of two-dimensional elliptic differential equations with mixed derivatives was developed [5,6]. The obtained results for the isotopes ^{190}Os and ^{154}Gd, are consistent with experimental data.

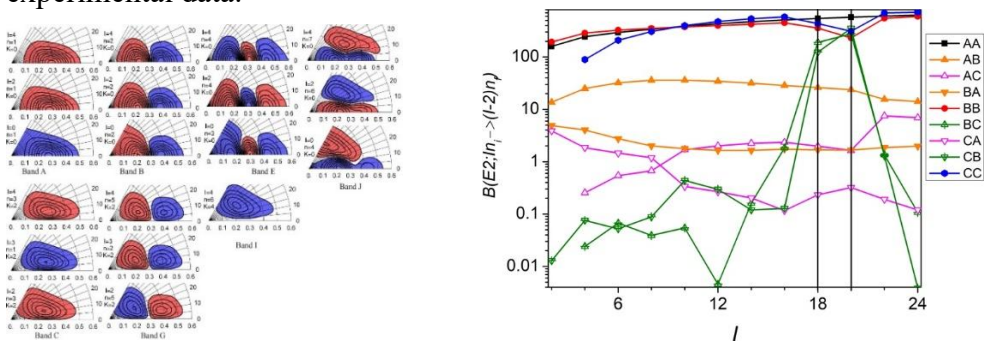


Fig.1. Components of the eigenfunctions of the collective model of the ^{154}Gd atomic nucleus obtained in [6] (left) and calculated quadrupole transitions B(E2) between the A, B and C bands in ^{154}Gd (right).

To solve equations within the coupled-channel method with complex potentials describing nuclear reactions with massive nuclei, software packages implementing the high-order FEM were developed. It was shown that the FEM and the R-matrix method were more stable than the frequently used modified Numerov method and enabled the use of significantly larger numbers of vibrational and rotational couplings. The calculations of the reactions ^{48}Ti + ^{208}Pb and ^{51}V + ^{248}Cm (Fig. 2) demonstrated that the multiphonon and high-spin states significantly smoothed the barrier distribution and resulted in better agreement with the experimental data [7].

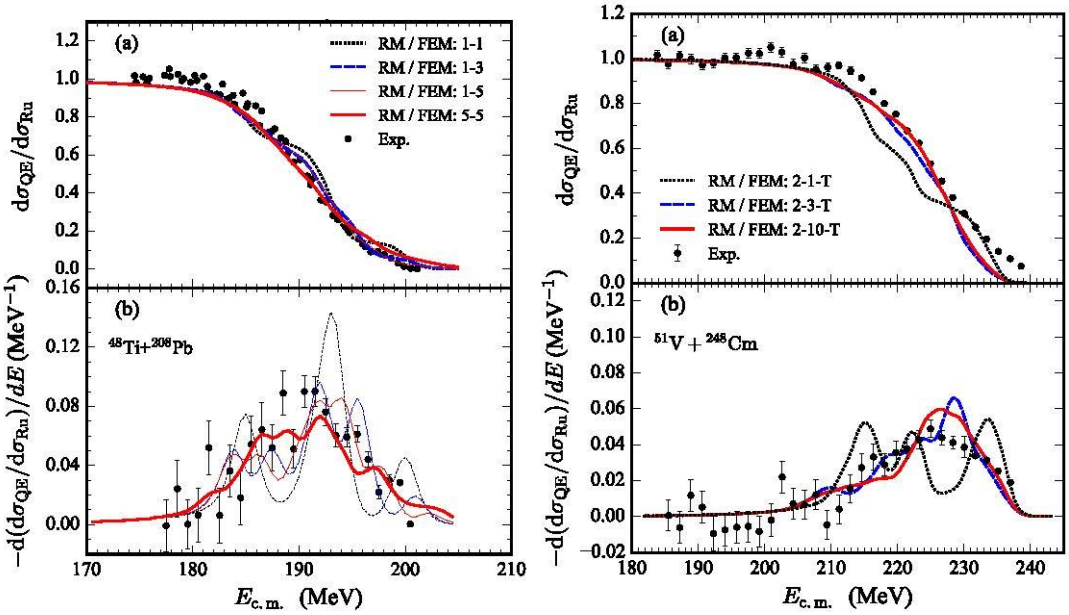


Fig. 2. (a) Quasielastic scattering cross sections at the back angle relative to the Rutherford cross section calculated in [7] for the reactions ^{48}Ti + ^{208}Pb (left) and ^{51}V + ^{248}Cm (right). (b) Corresponding barrier distributions. Experimental data [S. Mitsuoka, et al., Phys. Rev. Lett. 99 (2007) 182701 (left), and M. Tanaka, et al., J. Phys. Soc. Jpn. 91 (2022) 084201 (right)] are shown as circles.

New third- and fourth-order FEM schemes with multidimensional Hermite interpolation polynomials of a *d*-dimensional hypercube for solving BVPs for an elliptic equation with mixed partial derivatives on hyperparallelepiped meshes were developed. Test calculations of the BVPs confirm the presumed order of accuracy of the developed FEM schemes [8]. Based on a combination of the FEM and the secant method, new algorithms have been developed for solving the discrete and continuous spectrum of the Coulomb two-center problem in prolate spheroidal coordinates, finding the energy eigenvalues, phase shifts, separation constants, and the corresponding eigenfunctions [9].

A numerical study [10], conducted for quasielastic collisions of hydrogen atoms with electrons carrying kinetic energies in the keV range, has established the need of extending the theoretical framework up to the second Born approximation (SBA). Methods for calculating the multidimensional singular integrals arising in the SBA was developed. The proposed methods enable further numerical studies of collisions of several keV electrons with helium atoms, for which experimental data are already available.

Within the microscopic model of the optical potential, an analysis of proton–nucleus scattering data at energies of 200–1000 MeV for various target nuclei was carried out. Based on this, estimates of the influence of the nuclear medium on the scattering amplitude as a function of the incident proton energy and the atomic mass of the target nucleus were made [11]. In [12], the developed approach was generalized to the case of lower incident proton energies and to the case of scattering of light nuclei.

In collaboration with FLNR and BLTP, a study on the scattering reaction ^{7}Li+^{10}B and the transfer reaction ^{7}Li+^{10}B→^{6}Li+^{11}B at a beam energy E_{LAB} = 58 MeV was conducted. The analysis employed a microscopic optical potential model and the distorted wave Born approximation, and the results were compared with experimental data obtained at the FLNR U-400 cyclotron. The analysis of the

experimental data pointed to the presence of a proton–neutron halo in the structure of the state $^{6}Li_{0+}$ at 3.56 MeV [13].

A quantum-chemical cluster method for the quantitative justification of effective spin models for various crystal structures of transition metal magnetic oxides were developed [14]. The method is applicable to a wide range of new magnetic materials with complex chemical compositions. The TANGELO software package for simulating quantum computations was extended to include relativistic corrections to Coulomb interactions in the calculations of chemical compounds involving superheavy elements. The computational algorithm is based on the hybrid quantum-classical adapt-VQE scheme and is implemented for the quantum-chemical calculation of wave functions and ground-state energies of heavy lead and superheavy flerovium monoxides.

A software package for simulating quantum computations based on tensor networks was developed and applied to the nonequilibrium quantum dynamics of low-dimensional spin models. Using a one-dimensional XXZ spin (s=1/2) model as an example, the influence of stochastic environmental fields that disrupt the unitary nature of the spin system's evolution was investigated. Varying the model parameters of exchange anisotropy revealed a crossover in the dynamic behavior of the spin system, consistent with the dynamic phase transition hypothesis [15].

A method for extrapolating perturbative expansions in powers of asymptotically small parameters to the domain of finite or infinite variables was proposed [16,17]. The extrapolation was accomplished using self-similar factor approximants. In a number of cases, the method allows for the exact reconstruction of the desired functions from their weak-coupling asymptotic expansions.

A study of a model of atomic optics with cold bosons was conducted [18]. It was shown that when bosons in traps were cooled to temperatures at which the atomic cloud transformed into a Bose-Einstein condensate, interference patterns, interference current, Rabi oscillations, harmonic generation, parametric conversions, and other effects could be observed.

Regularizing correlation functions were constructed for quantum statistical systems consisting of atoms or molecules and interacting with each other via singular nonintegrable potentials [19]. The method is based on solving the scattering equation in the form of asymptotic series at small distances, which can be extrapolated to arbitrary spatial variables using the theory of self-similar approximations. In [20], the method of self-similar approximations was applied to extrapolate the summation of divergent virial expansions for hard disk-type fluids, hard sphere-type fluids, and systems with power-law potentials. It was shown that the proposed method yielded more accurate results in comparison with other approaches.

A modernization of the fragmentation reaction simulation procedure based on the transport-statistical approach in the BNV-SMM model was carried out. This enabled the decrease of computation time and the improvement of the agreement of the fragmentation reactions calculations with the experimental data as compared to other known approaches. Based on the transport-statistical approach, the ratio of the isotope yields in reactions with an incident ^{18}O ion at an energy of 35 MeV with heavy ^{181}Ta and light ^{9}Be targets was studied in collaboration with FLNR. It was shown that the impact parameter of nuclear collisions plays a significant role in determining the isotopic distributions of the fragments similar to the incident nucleus. A method for accelerating the transport-statistical calculations was proposed, and a modified approach was implemented, improving the agreement of calculations with experimental data [21, 22].

A high-precision numerical method for the search of new solutions to the three-body problem with equal masses under collision-free conditions and with central symmetry was developed in collaboration with Sofia University. The method is based on a Newtonian iteration supplied with a procedure for calculating initial approximations using a grid-search method [23].

A numerical study on the influence of frequency and radius parameters on the structure and properties of standing waves in a ball of finite radius, approximating 3D oscillons in the φ^4 model, was conducted in collaboration with the University of Cape Town [24,25]. For the 3D case, coexisting types of standing waves were found, their mutual relationship was investigated, and the dependence of the energy and stability intervals of the waves on the radius and frequency was studied. The frequency range

where the variational method provides an accurate description of the (1+1)-dimensional oscillon was determined [26].

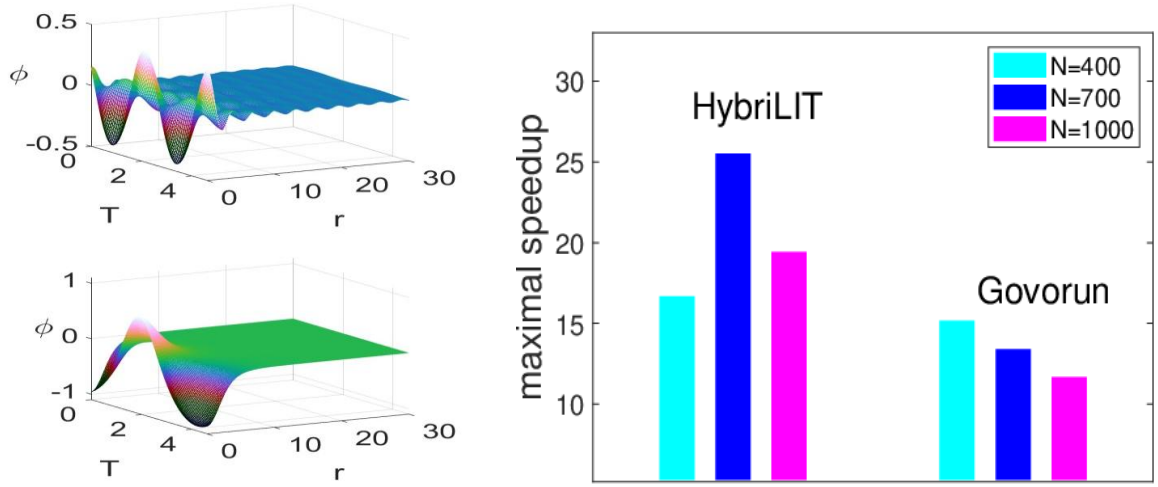


Fig. 3. Left: coexisting standing waves in the 3D φ^4 model obtained in [24]. Right: maximum speedup achieved through parallelization on the HybriLIT cluster and the "Govorun" supercomputer during MATLAB calculations of Floquet multipliers for studying the stability of standing waves in the φ^4 model. The figure from [25].

A modernization of the method for simulating the formation of hydrated electrons in a liquid was carried out, taking into account the Coulomb potential. It extends the frequency range of the scanning laser for which the calculation results agree with the experiment [27].

Based on the nonstationary heat conduction equation in a coordinate system associated with the moving evaporation front, and within the two-temperature thermal spike model, the dynamics of laser ablation of materials under the action of ultrashort laser pulses was investigated [28].

A method for the refined calculation of the spread of energy losses in crystal lattices under irradiation with heavy ions of relativistic energies was proposed. It results in a better accuracy as compared to the existing approaches in the case of irradiation with highly charged ions [29].

Molecular dynamics methods were used to model the interaction of the beta-amyloid peptide 25–35 with DMPC phospholipid membranes in the presence of a high concentration of K^+ and Cl^- ions in the solution. The results obtained provide insight into the possible processes occurring during membrane disruption in the presence of the peptide [30]. The results were included in a review[1] of works on joint research with the FLNP and LRB using methods of applied nuclear physics and molecular dynamics modeling of lipid peptide interactions, with an emphasis on the reorganization of lipid membranes caused by the Aβ (25–35) peptide under conditions simulating the preclinical stage of Alzheimer's disease.

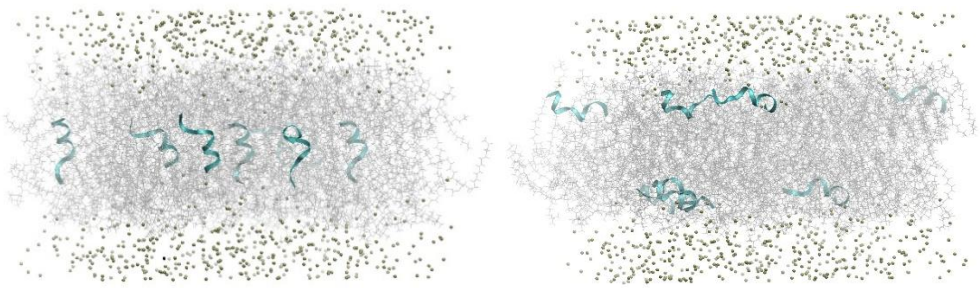


Fig. 4. Localization of peptides in the membrane center (left) and in the bilayer head groups (right) [30].

[1] Kurakin S.A., et al. Nat. Sci. Rev. 2025. V. 2. No. 2. article number 100202.

A study on the structure of phospholipid transport nanosystem vesicles and drugs based on them using small-angle neutron scattering data was conducted in collaboration with FLNP and IBMC RAS. A significant influence of a number of factors, including the presence of impurities and the solution concentration, on the vesicle size and structure was demonstrated [31]. A web interface for the online analysis of experimental small-angle scattering data was developed using the JINR cloud infrastructure, with the help of ROOT package tools to implement parallelism [32,33].

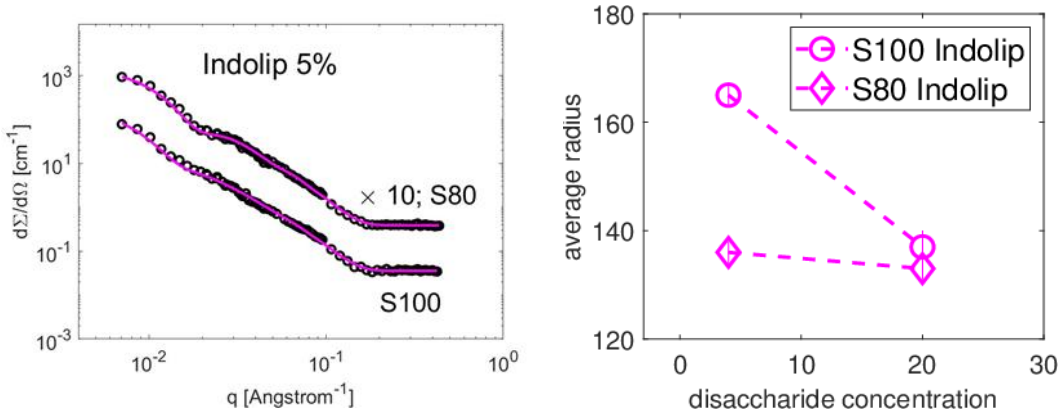


Fig. 5. Left: experimental and calculated in [31] small-angle neutron scattering cross sections for samples S80 and S100 of polydisperse vesicular systems prepared from a 5% solution of indolip in water with phosphatidylcholine content of 77% (S80) and 94% (S100). Right: dependence of the average vesicle radius in samples S80 and S100 on the concentration of indolip in water.

A FEM based combined procedure was developed [34]. It enables the reduction of the computational costs, without loss of accuracy, in the solution of 3D magnetostatics problems with complex geometries, by making use of the magnetic vector potential.

A numerical integrator of ordinary differential equations based on a combination of the Appelrot method of quadratic systems and the Kahan discretization method [35] was investigated. An analysis of reversible difference schemes (Kahan's method) was carried out, and their advantage over other approaches when applied to dynamic systems with quadratic right-hand sides was demonstrated [36].

A preconditioned conjugate gradient method for solving finite element systems with high-order approximation was developed [37]. It has the advantage of avoiding the storage of the 3D sparse finite element matrix, thereby significantly increasing the efficiency of the parallel computations.

Based on two- and three-dimensional modeling with varying the arrangement of current windings in the magnetic system, the optimization of the main characteristics of the Nuclotron superconducting dipole magnet was carried out. Joint work with physicists from VBLHEP and DLNP resulted in the development of a computer model of the MSC-230 cyclotron's current winding at 4.5 kA made from superconducting coaxial cable, taking into account a new winding option [38]. The results were reflected in a review[2] devoted to JINR's work on the development of superconducting magnets for charged particle accelerators.

A study on the possibility of using artificial intelligence to optimize and increase the speed of calculations in modeling accelerator magnetic fields was conducted [39]. The perspectives of using the computer vision and artificial intelligence to optimize the beam motion were analyzed with the aim at assessing the need of incorporating these tools into the CORD program, which performs calculations for the study of the influence of betatron oscillations and phase motion of beam particles on the magnetic field of the MSC-230 cyclotron.

An algorithm for calculating multidimensional integrals with pole-type singularities, based on identification and bypass, typical for QCD models with singularities in Monte Carlo integration, was developed [40].

[2] Khodzhibagiyan H.G., et al. Nat. Sci. Rev. 2024. V. 1. No. 1. article number 6.

Prerequisites for the implementation of an artificial intelligence driven automatic adaptive quadrature of the one-dimensional Riemann integral were formulated. The need of this approach follows from the occurrence of critical circumstances that result in code fragility preventing the derivation of a reliable and fast output since their handling is out of the control of existing automatic adaptive quadrature algorithms [41].

An operator approach for stochastic systems, in which the fundamental kinetic equation is represented as a Schrödinger-type equation with the Hamiltonian given in terms of creation and annihilation operators, was investigated. The kernel of the evolution operator was written as a functional integral, for which an approximate calculation method was proposed on the basis of an expansion in the eigenfunctions of the Hamiltonian generating the functional integral. For the pure creation process, approximate values of the probabilities of realization of the first and second states were obtained [42].

A parallel computational scheme for integrating the three-dimensional non-stationary Schrödinger equation in a discrete variable representation was proposed and implemented. The results demonstrate a significant reduction of the computational time for problems with a large number of grid nodes [43].

A potential model with non-local interaction was developed, and a system of equations was formulated within it for the assessment of the spectrum of vector mesons: calculation of the mass spectrum, interaction constants, normalizations, and selection of suitable form factors. The phenomenological choice of the vector meson vertex used in this non-local model is sufficient to describe the properties of heavy J/ψ (Y) quarkonia, namely, the processes of quarkonia production and dissociation at heavy ion collision energies in the NICA experiment [44,45].

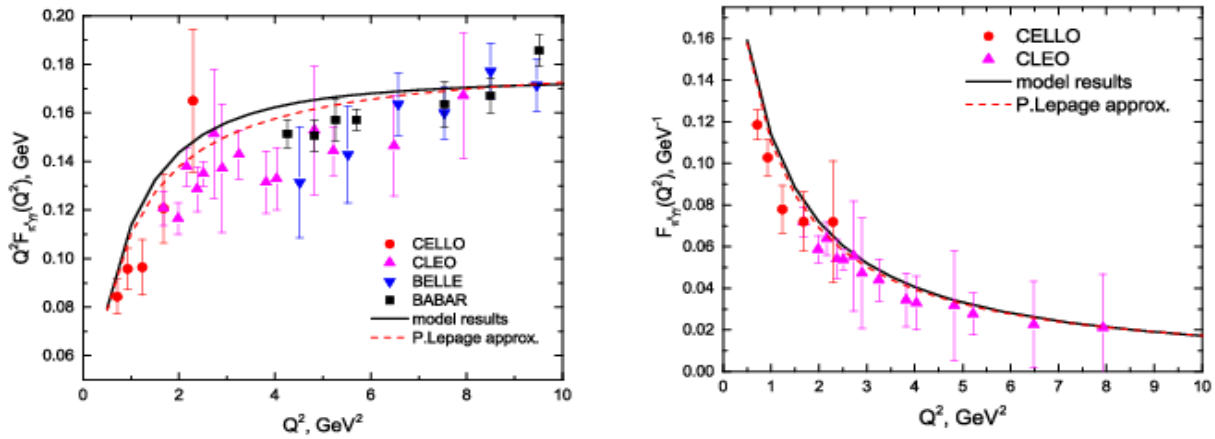


Fig. 6. Transition form factor $F_{\pi\gamma}$, calculated in [45] as a function of the space-like photon momentum Q^2, in comparison with experimental data.

Phase transitions in low-dimensional systems of charged particles, including diffusion and intertype (IT) superconductors, were studied [46,47]. Numerical modeling showed that, in diffusion multiband superconductors, the IT-area of the phase diagram expends significantly with the difference in the diffusion coefficients of the bands, which changes the nature of the vortex interaction. A numerical study of quasicrystalline superconductors revealed the absence of translational symmetry, leading to a fractal distribution of the condensate [48,49].

The ecosystem for modeling the Josephson structure, based on the *JupyterBook* platform, was supplemented with research facilities for studying the dynamics of a superconducting quantum interferometer with two Josephson junctions (DC SQUID). A computational scheme for constructing the current-voltage characteristic of the SQUID under the influence of a magnetic field was developed. The parallel implementation using the Numba library provided a significant acceleration of the calculations [50,51]. The computer simulation of a controlled magnetization flip in a chain of φ_0 junctions caused by an alternating voltage was performed. The feasibility of the controlled magnetization flip was demonstrated [52]. The capabilities provided by the Julia language in the study of a multiparameter model of a point φ_0 Josephson junction of the "superconductor–ferromagnet–superconductor" type with a direct relationship between the magnetic moment and the Josephson current

were tested. The effectiveness of this approach to the implementation of parallel computations for pinpointing the magnetic moment flip domains in the plane of model parameters was demonstrated [53]. A simulation of flip domains in a weak-dissipation "superconductor–ferromagnet–superconductor" φ_0 transition model, depending on the parameters regulating the action of an external current, was performed. The parallel implementation secured the decrease of the computation time by a factor of up to thirty [54].

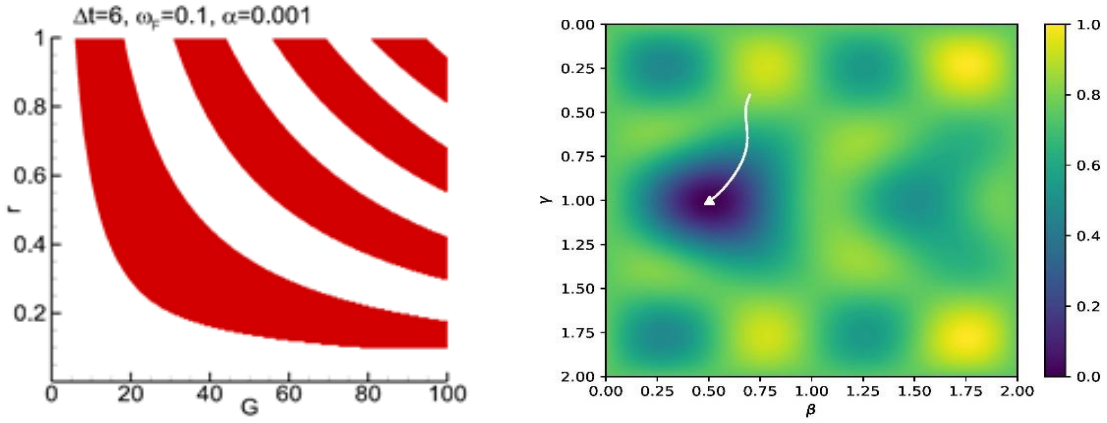


Fig. 7. Left: example of the structure of the magnetic moment flip domain arrangement calculated in [54] on the parameter plane of the "superconductor–ferromagnet–superconductor" φ_0 transition model with weak dissipation. Right: dependence of the energy on the values of the variational parameters in the Ising model with an external magnetic field, calculated on the quantum polygon [56]. The white arrow shows the gradient descent trajectory.

The quantum approximation optimization algorithm (QAOA) was simulated on the Cirq quantum computing simulator of the HybriLIT quantum polygon at JINR using the example of the problem of searching for the lowest energy state of the Ising model in a magnetic field. The calculations showed that the inclusion of the cuStateVec library significantly increased the code efficiency [55,56,57]. The cases of longitudinal and transverse external fields were considered. The Hamiltonian of the model was implemented using a sequence of quantum gates. The operation of the QAOA algorithm was simulated for progressively more complex lattice configurations using the Cirq and qsim libraries with the use of a graphics processor, CUDA technology and the cuStateVec library. The obtained results indicate the preference of gradient-free methods over gradient ones.

For a finite-dimensional quantum system, the non-classicality index of a state, based on the determination of its distance from the set of "classical states", was investigated. It was shown that the set of classical states, defined by the condition of non-negativity of the Wigner function, was concentrated in the convex polyhedron of the simplex of the eigenvalues of the state density matrix [58].

A generalization of the Tate–Bryan angle method, which is a modification of the well-known Euler angle method describing the rotation of an orthogonal frame in three-dimensional Euclidean space, was proposed for spaces of arbitrary finite dimension. The developed system of elementary unitary rotations allows for the introduction of a convenient parametrization of Grassmannians and a parametrization of algebraically open subsets of conjugacy classes of Hermitian matrices [59].

A method for increasing the computational efficiency of computing the nonlocal characteristics of a pair of qubits was proposed. Within this approach, a subset of separable 2-qubit states of rank 4 was described as a semi-algebraic manifold defined by a system of third- and fourth-order polynomial inequalities on the eigenvalues of the density matrix, with the polynomial coefficients in the form of trigonometric functions defined on the direct product of two regular octahedra [60].

Within the generalized Stratonovich-Weyl formulation of quantum mechanics, the dichotomy between the elementary and composite nature of a finite-dimensional quantum system was investigated. It was shown how information about the possible realization of a virtual 2-level subsystem was encoded in the properties of the Wigner function of the full 4-level system [61].

An alternative approach to quantum mechanics in which the coordinate and momentum take values in finite Galois fields was proposed [62]. A software package that includes tools for working with Galois fields, displacement operators, Weyl–Heisenberg and Clifford groups, and Wigner functions in finite phase spaces was created [63].

With the aim of introducing the concept of entanglement based on the "aberrational" degree of freedom of photons, the classical and quantum-mechanical interpretation of the Zernike system was analyzed. Its equivalence to a Higgs oscillator on a sphere or pseudosphere was demonstrated [64].

Within the Bianchi type-I and FLRW cosmological models, the interaction of the spinor and electromagnetic fields and their influence on the evolution of the Universe were studied. For the Bianchi type-I model, it was found that the interaction of these fields softened the restrictions arising from the non-zero off-diagonal components of the stress-energy tensor [65]. In the FLRW geometry, the role of the nonlinear spinor field was investigated for different curvatures, restrictions on the field parameters were identified, and numerical solutions of the corresponding equations of state were obtained [66]. A generalized Chaplygin gas model based on a nonlinear spinor field was proposed and numerically studied, providing a unified description of dark matter and dark energy in a spherically symmetric FLRW spacetime with open geometry. The parametric identification of the model using modern data demonstrates its competitiveness in the late Universe and leads to a lower value of the Hubble constant [67].

2.2.4. A list of the main publications of JINR authors, including associated personnel on the results of the project work (list of bibliographical references).

1. Vandandoo U., Zhanlav T., Chuluunbaatar O., Gusev A., Vinitsky S., Chuluunbaatar G. High-order finite difference and finite-element methods for solving some partial differential equations // Switzerland, Springer Nature. 2024. pp. 114.
2. Zhanlav T., Chuluunbaatar O. New developments of newton-type iterations for solving nonlinear problems // Switzerland, Springer Nature. 2024. pp. 281.
3. Mitin A.V., Gusev A.A., Chuluunbaatar G., Chuluunbaatar O., Vinitsky S.I., Derbov V.L., Luong H.L. Dual nature of chemical bond and vibration-rotation spectrum of the Be_2 molecule in the ground $X_1\Sigma_g^+$ // Chemical and Materials Sciences: Research Findings V. 2. BP Intern. Publishing 2025. pp. 100–133.
4. Zemlyanaya E., Chuluunbaatar O. Computational schemes based on the continuous analogue of Newton's method in the numerical study of complex physical systems at JINR // Nat. Sci. Rev. 2026. V. 3. No. 6. pp. 200605.
5. Batgerel B., Vinitsky S.I., Chuluunbaatar O., Buša J., Jr., Blinkov Yu.A., Gusev A.A., Deveikis A., Chuluunbaatar G., Ulziibayar V. Schemes of finite element method for solving multidimensional boundary value problems // J. Math. Sci. 2024. V. 279. pp. 738–755.
6. Batgerel B., Chuluunbaatar O., Derbov V.L., Gusev A.A., Hai L.L., Deveikis A., Hess P.O., Mardyban E.V., Mardyban M.A., Vinitsky S.I., Wen P. Symbolic-numeric solving boundary value problems: Collective models of atomic nuclei // Lect. Notes Comput. Sci. 2024. V. 14938. pp. 63–81.
7. Wen P.W., Chuluunbaatar O., Descouvemont P., Gusev A.A., Lin C.J., Vinitsky S.I. Role of multi-phonon and high-spin states on the quasi-elastic barrier distributions of massive systems // Phys. Lett. B 2025. V. 863. pp. 139383.
8. Kovalev O.O., Batgerel B., Gusev A.A., Hai L.L., Derbov V.L., Chuluunbaatar O., Vinitsky S.I., Wen P.W. Symbolic-numerical algorithms for solving multidimensional boundary value problems by finite element method on hypercubes // Lect. Notes Comput. Sci. 2026. V. 16235. pp. 210–222.
9. Luong H.L., Muratova A.I., Kovalev O.O., Gusev A.A., Derbov V.L., Vinitsky S.I. Algorithms for solving two-center coulomb problem // Comput. Math. Math. Phys. 2025. V. 65. pp. 2618–2635.

10. Chuluunbaatar O., Popov Yu.V., Kanaya S., Onitsuka Y., Takahashi M. Electron-hydrogen quasi-elastic scattering at high momentum transfer: Calculations of second Born singular integrals // Phys. Part. Nucl. 2025. V. 56. No. 6. pp. 1400–1406.
11. Abdul-Magead I.A.M., Lukyanov V.K., Zemlyanaya E.V., Lukyanov K.V. Analysis of the proton amplitude of scattering on bounded nuclear nucleons based on proton-nucleus scattering data // Int. J. Mod. Phys. E. 2024. V. 33. No. 11. pp. 2441012.
12. Abdulmagead I., Lukyanov K., Lukyanov V., Zemlyanaya E. Nuclear in-medium effects on proton-nucleus and light nucleus-nucleus scattering amplitudes // Interact. 2026. V. 247. article number 86.
13. Zemlyanaya E.V., Lukyanov K.V., Lukyanov V.K., Penionzhkevich Yu.E., Sobolev Yu.G., Stukalov S.S. Microscopic analysis of elastic scattering and transfer reactions in the ^{7}Li+^{10}B collision at a beam energy of 58 MeV // Bull. Russ. Acad. Sci. Phys. 2025. V. 89. No. 8. pp. 1236–1240.
14. Yushankhai V.Yu., Siurakshina L.A. Variational quantum algorithm for computational problems of quantum chemistry: from UCCSD-VQA to ADAPT-VQA // Phys. Part. Nucl. 2025. V. 56. No. 4. pp. 1004–1008.
15. Siurakshina L.A., Yushankhai V.Yu. Nonequilibrium quantum dynamics of a low-dimensional spin system // Phys. Part. Nucl. Lett. 2025. V. 22. No. 3. pp. 536–539.
16. Yukalov V.I., Yukalova E.P. Strong-coupling limits induced by weak-coupling expansions // Ann. Phys. 2024. V. 467. pp. 169716.
17. Yukalov V.I., Yukalova E.P. Strong-coupling extrapolation of Gell-Mann-Low functions // Phys. Part. Nucl. 2024. V. 55. No. 4. pp. 1026–1032.
18. Yukalov V.I., Yukalova E.P. Atom optics with cold bosons // Bull. Russ. Acad. Sci. Phys. 2024. V. 88. No. 6. pp. 922–928.
19. Yukalov V.I., Yukalova E.P. Quantum systems of atoms with highly singular interaction potentials // Laser Phys. 2025. V. 35. No. 4. pp. 045501.
20. Yukalov V.I., Yukalova E.P. Self-similar summation of virial expansions // Mol. Phys. 2026. V. 124. No. 6. article number e256302.
21. Mikhailova T.I., Erdemchimeg B., Sereda Yu.M. Target dependence of the isotope distributions in heavy-ion reactions at Fermi energy // Bull. Russ. Acad. Sci. Phys. 2025. V. 89. No. 8. pp. 1216–1222.
22. Mikhailova T.I., Erdemchimeg B. Modified transport approach for description of fragmentation reactions in heavy-ion collisions // Phys. Part. Nucl. 2025. V. 56. No. 6. pp. 1536–1542.
23. Hristov I., Hristova R., Puzynina T., Sharipov Z., Tukhliev Z. Numerical search for three-body periodic free-fall orbits with central symmetry // Commun. Nonlinear Sci. Numer. Simul. 2025. V. 51. pp. 109066.
24. Zemlyanaya E., Bogolubskaya A., Bashashin M., Alexeeva N. Numerical study of the φ^4 standing waves in a ball of finite radius // Discrete Contin. Models Appl. Comput. Sci. 2024. V. 32. No. 1. pp. 106–111.
25. Zemlyanaya E.V., Bogolubskaya A.A., Alexeeva N.V., Bashashin M.V. φ^4 oscillons as standing waves in a ball: a numerical study // Phys. Part. Nucl. 2025. V. 56. No. 6. pp. 1655–1659.
26. Alexeeva N.V., Barashenkov I.V., Dika A., De Sousa R. The energy-frequency diagram of the (1+1)-dimensional Φ^4 oscillon // J. High Energy Phys. 2024. V. 10. article number 136.
27. Волохова А.В., Земляная Е.В., Лахно В.Д., Рихвицкий В.С., Башашин М.В. Исследование влияния кулоновского потенциала на процесс гидратации электрона в рамках динамической модели полярона // Материалы ITTMM-2024, Изд. РУДН. 2024. сс. 330–335.
28. Amirkhanov I.V., Sarkhadov I., Tukhliev Z.K., Gafurov H. Simulation of laser ablation of materials within the thermal spike model // J. Surf. Investig.: X-ray, Synchrotron Neutron Tech. 2024. V. 18. No. 2. pp. 348–353.
29. Kats P.B., Kudravets A.V., Rimashevskaya A.S., Voskresenskaya O.O. Comparative study of some rigorous and approximate methods for calculating the energy loss straggling // Radiat. Phys. Chem. 2024. V. 222. pp. 111860.
30. Kurakin S., Badreeva D., Dushanov E., Shutikov A., Efimov S., Timerova A., Mukhametzyanov T., Murugova T., Ivankov O., Mamatkulov K., Arzumanyan G., Klochkov V., Kučerka N.

Arrangement of lipid vesicles and bicelle-like structures formed in the presence of Aβ (25–35) peptide // Biochimica et Biophysica Acta (BBA)-Biomembranes. 2024. V. 1866. Iss. 1, pp. 184237.
31. Zemlyanaya E., Kiselev M., Chausova V., Tereshkina Yu., Tikhonova E. Analysis of structure of the PhTNS vesicles based on small-angle neutron scattering data obtained at the “Yellow Submarine” spectrometer // Int. J. Mod. Phys. E 2025. https://doi.org/10.1142/S021830132641003X
32. Соловьев А.Г., Соловьева Т.М., Лукьянов К.В., Земляная Е.В. Веб-интерфейс на базе облачной инфраструктуры ОИЯИ для фитирования экспериментальных данных малоуглового рассеяния с использованием инструментов пакета ROOT // Современные информационные технологии и ИТ-образование. 2024. Т. 20. вып. 3. сс. 590–601.
33. Lukyanov K., Soloviev A., Solovjeva T., Zemlyanaya E. A web application for fitting experimental data using JINR cloud infrastructure and ROOT package tools // Phys. Part. Nucl. 2025. V. 56. No. 6. pp. 1509–1513.
34. Chervyakov A.M. On finite-element modeling of large-scale magnetization problems with combined magnetic vector and scalar potentials // Phys. Part. Nucl. Lett. 2024. V. 21, No. 5. pp. 1074–1083.
35. Malykh M., Ayryan E., Lapshenkova L., Sevastianov L. Difference schemes for differential equations with a polynomial right-hand side, defining birational correspondences // Math. 2024. V. 12. article number 2725.
36. Malykh M.D., Ayryan E.A., Dulatov I.T., Lapshenkova L.O., Sevastianov L.A. Finite difference models of dynamical systems with polynomial right-hand side // Phys. Part. Nucl. 2025. V. 56. No. 6. pp. 1520–1523.
37. Yuldasheva M.B., Yuldashev O.I., Novitsky V.V. Modelling magnetic field of a compact electromagnetic device of experimental setup to study rot-effects in fission of heavy nuclei // Russ. J. Numer. Anal. Math. Model. V. 40. No. 2. pp. 153–167 (2025).
38. Akishin P.G., Vesenkov V.A., Vlasov A.I. et al. Medical superconducting cyclotron MSC-230: Project status // Phys. Part. Nucl. Lett. 2025. V. 22. No. 4. pp. 847–851.
39. Karamyshev O.V., Lyapin I.D., Karamysheva T.V., Shuravin M.A. Using artificial intelligence in cyclotron design // Phys. Part. Nucl. Lett. 2025. V. 22. No. 4. pp. 690–698.
40. Friesen A.V., Goderidze D., Kalinovsky Yu.L. Calculation of a multidimensional integral with a singularity by dividing the integration domain into subsegments // Phys. Part. Nucl. Lett. 2024. V. 21. No. 4. pp. 782–784.
41. Adam Gh., Adam S. Prerequisites for artificial intelligence driven automatic adaptive quadrature // Phys. Part. Nucl. 2025. V. 56. No. 6. pp. 1255–1263.
42. Ayryan E., Buša J., Hnatič M., Lučivjanský T., Malyutin V. Approximate calculation of functional integrals arising from the operator approach // Phys. A: Stat. Mech. Appl. 2025. V. 670. pp. 130616.
43. Ayriyan A.S., Buša J., Jr., Melezhik V.S. Parallelization of computational scheme based on two-dimensional dvr for integrating time-dependent three-dimensional Schrödinger equation // Phys. Part. Nucl. Lett. 2024. V. 22. No. 3. pp. 617–621.
44. Tawfik A.N., Ahmadov A.I., Friesen A., Kalinovsky Yu., Aparin A., Hanafy M. Derivation of meson masses in SU(3) and SU(4) extended linear sigma model at finite temperature // Particles 2025. V. 8. No. 1. article number 9.
45. Friesen A., Kalinovsky Yu., Khmelev A. Mesons in nonlocal model with four-dimensional separable kernel // Eur. Phys. J. A 2026. V. 62. article number 6.
46. Nikonov E.G., Nazmitdinov R.G., Glukhovtsev P.I. Computer studies of a dependence of equilibrium state structure on a number of particles for a two-dimensional system of charged particles confined in a disk potential // Discrete Contin. Models Appl. Comput. Sci. 2024. V. 32. No. 1. pp. 128–36.
47. Nikonov E.G., Nazmitdinov R.G. Glukhovtsev P.I. Manifestation of the hexatic phase in confined two-dimensional systems with circular symmetry // J. Surf. Investig.: X-ray, Synchrotron Neutron Tech. 2024. V. 18. No. 2. pp. 248–254.

48. Marychev P.M., Nikonov E.G., Stolyarov V.S., Vagov A. Crossover between types I and II in diffusive superconductors: perturbative study // Phys. Rev. B 2025. V. 111. pp. 134501.
49. Nikonov E., Chen Y., Doria M., Shanenko A. Hartree-Fock interaction in superconducting condensate fractals // Beilstein J. Nanotechnol. 2025. V. 16. pp. 2177–2182.
50. Rahmonova A., Rahmonov I., Streltsova O., Zuev M. Toolkit in python for simulation of Shapiro step on the current-voltage characteristic of a Josephson junction // Phys. Part. Nucl. 2024. V. 55. No. 3. pp. 528–531.
51. Rahmonova A., Streltsova O., Rahmonov I., Zuev M. Python-реализация алгоритмов и инструментарий для моделирования динамики сверхпроводникого квантового интерферометра с двумя джозефсоновскими переходами (СКВИД постоянного тока) // Электронная публикация, 2024. http://studhub.jinr.ru:8080/jjbook/DC-SQUID.html
52. Rahmonova A., Streltsova O., Rahmonov I., Zuev M. Simulation of a controllable magnetization reversal in a chain of φ_0 junctions by an alternating voltage pulse // Phys. Part. Nucl. 2025. V. 56. No. 6. pp. 1592–1598.
53. Bashashin M.V., Rahmonov I.R., Zemlyanaya E.V. Applicability of the Julia programming language for mass solving of systems of differential equations using the example of the problem of modeling magnetic moment reversal in the φ_0-Josephson junction // Материалы ITTMM-2024, Изд. РУДН. 2024. сс. 393–398.
54. Bashashin M., Zemlyanaya E., Rahmonov I. Simulation of the magnetization reversal effect depending on the current pulse duration within the φ_0 Josephson junction model using MPI and OpenMP parallel computing technique // Phys. Part. Nucl. 2024. V. 55. No. 3. pp. 498–501.
55. Palii Yu., Bogolubskaya A. Yanovich D. Quantum approximation optimization algorithm for the Ising model in an external magnetic field // Phys. Part. Nucl. 2024. V. 55. No. 3. pp. 600–602.
56. Palii Yu.G., Bogolubskaya A.A. Yanovich D.A. Simulating QAOA operation using Cirq and qsim quantum frameworks // Discrete Contin. Models Appl. Comput. Sci. 2025. V. 33. No. 4. pp. 440–460.
57. Palii Yu., Belyakov D., Bogolubskaya A., Zuev M., Yanovich D. Simulation of the QAOA algorithm at the JINR quantum testbed // Phys. Part. Nucl. 2025. V. 56. No. 4. pp. 1661–1669.
58. Khvedelidze A., Torosyan A. The nonclassicality distance indicator of qudits // Phys. Part. Nucl. 2024. V. 55. No. 3. pp. 591–593.
59. Babich M.V., Bordag L.A., Khvedelidze A.M., Mladenov D.M. On multidimensional analogs of Euler (Tait-Bryan) angles and Grassmannians // Записки научных семинаров ПОМИ. 2025. Т. 543 сс. 43–55.
60. Khvedelidze A., Mladenov D., Torosyan A. Towards parameterizing the entanglement body of a qubit pair // Phys. Part. Nucl. 2025. V. 56. No. 6. pp. 1493–1497.
61. Khvedelidze A., Rogozhin I. On the dichotomy between quatrits and pairs of qubits in the Stratonovich-Weyl correspondence // Phys. Part. Nucl. 2025. V. 56. No. 4. pp. 1025–1029.
62. Kornyak V.V. Constructive quantum mechanics and calculations with finite groups // Program. Comput. Softw. 2025. V. 51. No 1. pp. 6–16.
63. Корняк В.В. Группы Вейля – Гейзенберга и Клиффорда и квантовая физика // Записки Научных Семинаров ПОМИ. 2025. Т. 543 сс. 126–142.
64. Abgaryan V., Nersessian A., Yeghikyan V. Zernike system revisited: Imaginary gauge and Higgs oscillator // Mod. Phys. Lett. A 2025. V. 41. No. 02n03. pp. 2550222.
65. Saha B. Interacting spinor and electromagnetic fields in cosmology // Пространство, время и фундаментальные взаимодействия. 2024. Т. 1. № 46. сс. 22–26.
66. Saha B. Maxwell-Dirac system in cosmology // Int. J. Mod. Phys. A. 2024. V. 39. No. 29. pp. 2450117.
67. Goray M., Saha B. Observational constraints on a spinor field generalized Chaplygin gas model in a spherically symmetric FLRW space-time // Eur. Phys. J. C 2025. V. 85. No. 10. article number 1146.

2.2.5. A complete list of publications (electronic annex, for journal publications with journal impact factor).
2.2.6. List of papers presented at international conferences and meetings (electronic annex).
2.2.7. Patent activity (if any)

2.3. Status and stage (TDR, CDR, ongoing project) of the project (including percentage of implementation of the declared milestones of the project *(if applicable)*

2.4. Results of related activities

2.4.1. Research and education activities. List of defended thesis.

1. On 12 September 2025, project participant M.V. Bashashin defended his thesis, "Numerical study of superconducting processes and physical characteristics in multiparameter models of Josephson structures", for the degree of candidate of physical and mathematical sciences in specialty 1.2.2 – mathematical modeling, numerical methods, and software packages.

2. Two textbooks were prepared:

- Kalinovskaya L.V., Kalinovsky Yu.L. Matrices and Determinants // Moscow, Infra-Engineering. 2024. 164 p. The textbook covers basic concepts, general techniques, and methods for working with matrices, including the calculation of determinants of various orders.
- Ayriyan A.S., Bashashin M.V., Zemlyanaya E.V., Streltsova O.I. Parallel Programming Based on MPI Technology: A Textbook. // Dubna: Publishing House of Dubna University, 2025. 86 p. The textbook is aimed at a wide range of readers studying parallel programming for accelerating numerical solutions of computational problems. It presents the fundamentals of using MPI technology for implementing parallelism and increasing computational performance.

3. In 2024 and 2025, project participants Bashashin M.V., Bordag L., Buša J., Jr., Chuluunbaatar O., Gusev A.A., Kalinovsky A.A., Khvedelidze A., Kokaev D.A., Lukyanov K.V., Nikonov E.G., Rakhmonova A.R., and Zemlynaya E.V. took part in Autumn IT Schools held annually at MLIT JINR. The aim of these schools is to familiarize students and postgraduates from various universities with the research conducted at JINR and involve them in scientific work within the project.

4. The project participants are engaged in teaching activities and serve as supervisors for bachelor's and master's theses at the following universities:

- at Dubna State University (Ayriyan A.S., Bashashin M.V., Chuluunbaatar G., Gusev A.A., Kalinovsky Yu.L., Lukyanov K.V., Nikonov E.G., Rakhmonova A.R., Siurakshina L.A., Streltsova O.I., Volokhova A.V., Zemlyanaya E.V.);
- at Tver State University (Ayryan E.A., Streltsova O.I.);
- at the Dubna branch of MSU (Ayriyan A.S., Bashashin M.V., Kalinovsky Yu.L., Lukyanov K.V., Rakhmonova A.R., Streltsova O.I., Zemlyanaya E.V.)
- at Peoples' Friendship University of Russia named after Patrice Lumumba (Chuluunbaatar G., Malykh M.D., Saha B.)

2.4.2. JINR grants (scholarships) received.

JINR Grants for Young Scientists and Specialists in 2024 – Badreeva D.R.
M.G. Meshcheryakov Scholarship in 2024 – Abgaryan V., Bashashin M.V., Goderidze D.
M.G. Meshcheryakov Scholarship in 2025 – Badreeva D.R., Khmelev A.V.
M.G. Meshcheryakov Scholarship in 2026 – Goderidze D., Rakhmonova A.R.

2.4.3. Awards and prizes.

The project participants are co-authors of the following works awarded JINR Prizes for methodology, research and technology papers:

JINR Third Prize for 2024: “Numerical methods and problem-oriented program complexes for solving some partial differential equations of physical processes and systems.” Authors: **Gusev A.A., Chuluunbaatar O., Chuluunbaatar G., Buša J., Jr.**, Vinitsky S.I., Zhanlav T., Batgerel B., Ulziibayar V., Hai L.L., Wen P.W.

JINR First Prize for 2025: “Development and creation of superconducting magnets for the NICA Collider.” Authors: Nikiforov D.N., **Akishin P.G.**, Bespalov Yu.G., Bychkov A.V., Zolotykh D.A., Korovkin S.A., Kuznetsov G.L., Merkuriev A.V., Petrov M.V., Svidetelev A.N.

2.4.4. Other results (expert, scientific-organisational, scientific-propaganda activities).

2024:
Gratitude from the Ministry of Education and Science of the Russian Federation – Chuluunbaatar O., Podgainy D.V.;

2025:
Gratitude from the Head of the Dubna Urban District – Streltsova O.I.;
Honorary Medal of the Government of Mongolia “Honorary Scientist of Mongolia” – Gusev A.A.;

3. International scientific and technical cooperation.
Actually participating countries, institutions and organisations

Organisation	Country	City	Participants	Type of agreement
AANL	Armenia	Yerevan	Ananikyan N. Piloyan A.	Collaborations
IP	Azerbaijan	Baku	Mirzaev M.N.	Collaborations
IM NASB	Belarus	Minsk	Egorov A.D. Malyutin V. B.	Agreement
SU	Bulgaria	Sofia	Hristov I.G., Hristova R.D. Mladenov D.M.	Collaborations
HCMUE	Vietnam	Ho Chi Minh City	Luongh L.H.	Collaborations
GTU	Georgia	Tbilisi	Giorgadze G.	Collaborations
TSU	Georgia	Tbilisi	Elashvili A. Giorgadze G.	Collaborations
UG	Georgia	Tbilisi	Gogilidze S.	Collaborations
CU	Egypt	Giza	Abdul-Magead I.	Collaborations
CIAE	China	Beijing	Lin C.J. Wen P.W.	Collaborations
UNAM	Mexico	Mexico City	Hess P.O.	Collaborations
IMDT MAS	Mongolia	Ulaanbaatar	Batgerel B.	Collaborations
MUST	Mongolia	Ulaanbaatar	Ulziibayar V.	Collaborations
Collaboration BM@N	Russia	Dubna	Kapishin M.N.	Collaborations
Collaboration MPD	Russia	Dubna	Ryabov V.G.	Collaborations
PFUR	Russia	Moscow	Bronnikov K.A. Sevastianov L.A. Rybakov Yu.P.	Collaborations
IMPB RAS	Russia	Pushchino	Lakhno V.D.	Collaborations
SGU	Russia	Saratov	Derbov V.L.	Collaborations

UCT	South Africa	Cape Town	Alexeeva N.V Dika A.	Collaborations, grant JINR-NRF
IMRAM	Japan	Sendai	Takahashi M. Kanaya S.	Collaborations

4. Plan/actual analysis of resources used: human (including associated personnel), financial, IT, infrastructure

4.1. Human resources (actual at the time of reporting, total number of participants)
(total number of participants)

№№ п/а	Employee category	Core staff Amount of FTE	Associated personnel Amount of FTE
1.	scientific staff	40.8 (51 persons)	
2.	engineers	5.4 (8 persons)	
3.	leaders	0.8 (1 person)	
	Total:	47.0	

4.2. The actual estimated cost of the project

Names of costs, resources, funding sources			Cost (thousands of dollars) resource requirements	Proposal from the laboratory for allocation of funding and resources				
				2024	2025	2026 (plan)		
		International cooperation (IC)	246,8	57,4	99,4	90,0		
		Materials						
		Equipment and third-party services	145,1	24,4	17,8	102,9*		
		Commissioning work						
		Services of research organisations						
		Acquisition of software	132,4	30,7	0	101,7*		
		Design/construction						
		Service costs (*planned in case of direct project affiliation)*						
Resources required	**Normo-hours**	Resources						
		– The amount of FTE,						
		– accelerator/installation,						
		– reactor, ...						
Sources of funding	**Budgetary resources**	JINR budget *(budget items*						

	Extrabudgetary (supplementary estimates)	Contributions by co-contractors Funds under contracts with customers Other sources of funding						

*2026 costs are given taking into account the transfer of obligations under 2025 contracts

4.3. Other resources

Computering resources	**Distribution by year**				
	1st year	**2nd year**	**3rd year**	**4th year**	**5th year**
Data storage (TB) - EOS - Tapes					
Tier 1 (core-hour)					
Tier 2 (core-hour)					
SC “Govorun” (core-hour) - CPU - GPU					
Clouds (CPU cores)					

5. Conclusion

6. Proposed reviewers

Polyakov S.V., Doctor of Physics and Mathematics Sciences, deputy director of the Keldysh Institute of Applied Mathematics, Russian Academy of Sciences

Theme Leader

_____/_____/

_____/_____/
"_____"_____ 202_

Project Leaders

_____/_____/

_____/_____/
"_____"_____ 202_

Laboratory Economist

_____/_____/
"_____"_____ 202_