

Formation of a Quark-Gluon-Plasma : understanding the energy and system size dependence

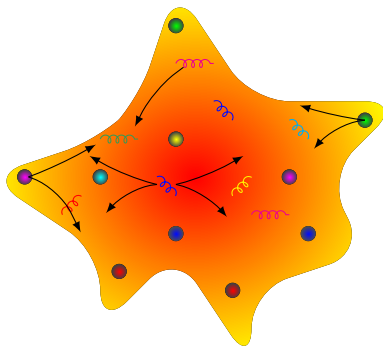
Sophys Gabriel

Supervisor : Klaus Werner at Subatech

Seminar Plenary at JINR - Dubna - 03 September

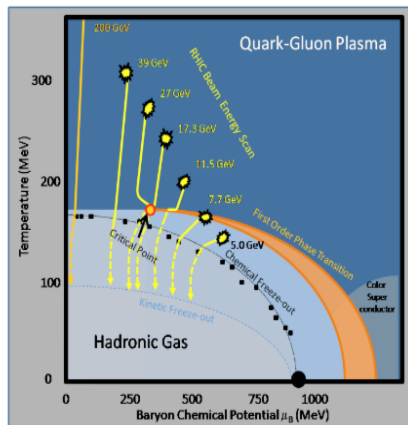
Ph.D in Theory Group at Subatech
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Context : QGP



- Quark-Gluon Plasma (QGP)
- **Partons are deconfined**
- QGP life-time : 10^{-21} s, size : 10^{-15} m \Rightarrow cannot study directly the QGP
- QGP formed in Heavy Ion collisions at accelerators
- Can we see an energy or system size dependence to create a QGP?

BES program: Dependence on Energy



STAR collaboration : [arXiv:1007.2613](https://arxiv.org/abs/1007.2613)

BES program

- At RHIC in Brookhaven National Laboratory
- Gold-Gold Collisions

Three Goals

- Find evidence of a phase transition ?
- Find critical point ?
- Evolution with $\sqrt{s_{NN}}$ of the medium ?

Small system: Dependence on system size

A new problematic of small system? A recent aspect of heavy ion physics

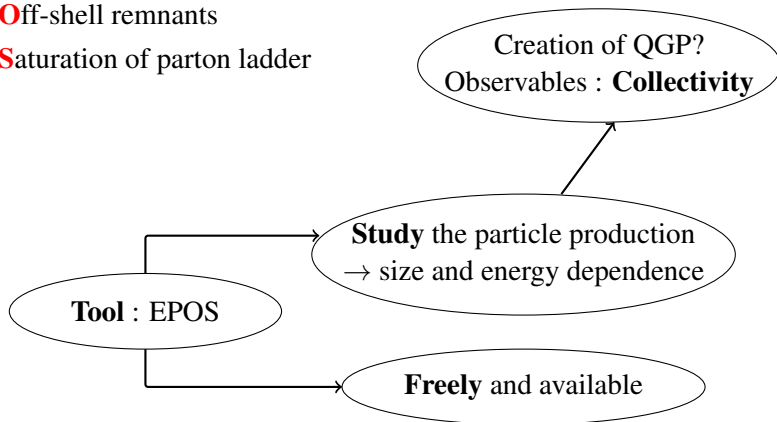
- The name "small systems" appeared at LHC Run I, it is now a **session at Quark Matter**
- A nonofficial translation could be "*system a priori too small to show characteristics of heavy ion physics and however in which we observe them, at least some*". **Caveats** "*a priori too small*" is not defined ..
- "A priori small" refers to system size: protons in initial stage with sometimes a **final state looking like a large system**, at least for charged particle multiplicity

pp/pA/AA at the same multiplicity have same behavior, role of geometry?
How is done the transition from small to large?

Introduction

Event generator : **EPOS**

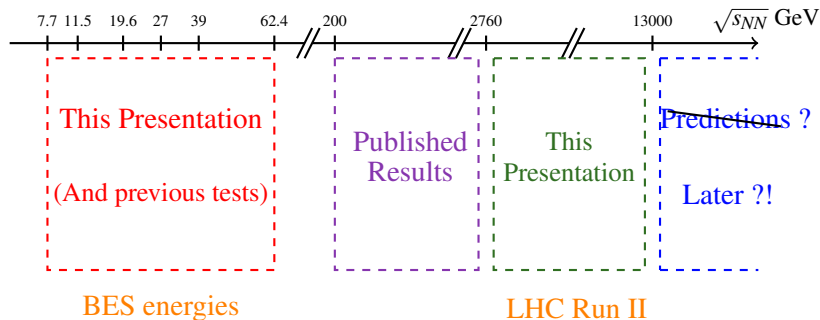
- **E**nergy conserving quantum mechanical multiple scattering approach
- based on **P**artons, partons ladders, strings
- **O**ff-shell remnants
- **S**aturation of parton ladder



Introduction

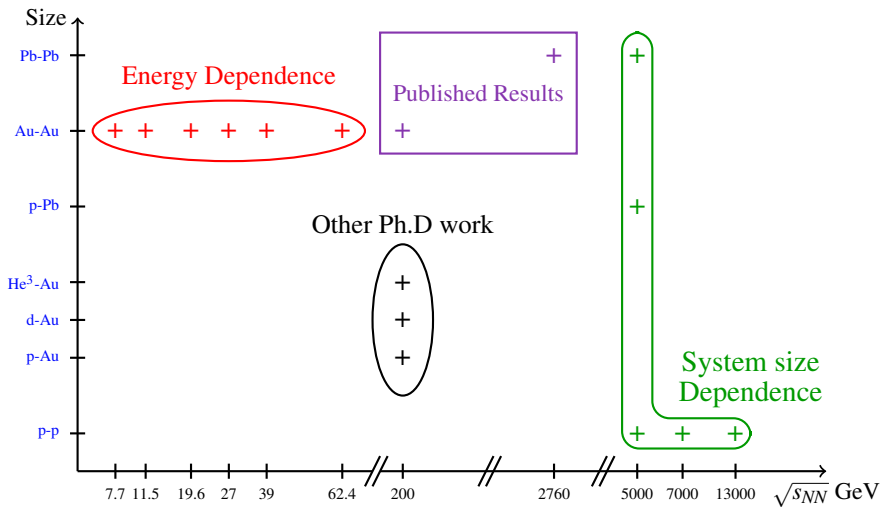
When do we use EPOS ?

Model for very high energy.



Do we really have a “collectively expanding plasma” in all systems, big (PbPb) and small (pp), at high energies and low energies?

Introduction



Contents

1 Introduction

2 The event generator

EPOS, one event

EPOS : Parton Based Gribov Regge Theory

Core-Corona Separation

3 Energy Dependence

4 System Size Dependence

5 Conclusion

Event generator : EPOS

How do we construct one event ?

Universal Model for all collisions

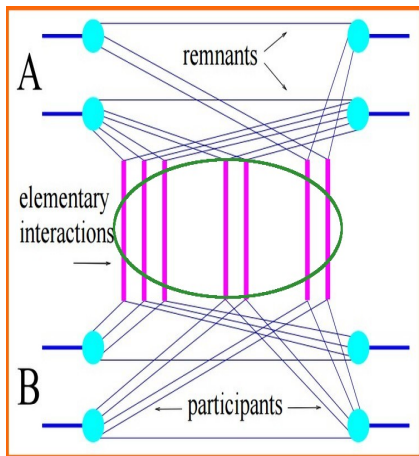
Same procedure applies, based on several stages :

- 1 Initial Conditions
- 2 Core-Corona Approach
- 3 Viscous hydrodynamic expansion
- 4 Statistical hadronization
- 5 Final state hadronic cascade

Event Generator : EPOS

Parton-Based-Gribov-Regge-Theory (PBGRT)

- ① Initial Conditions
- ② Core-Corona Approach



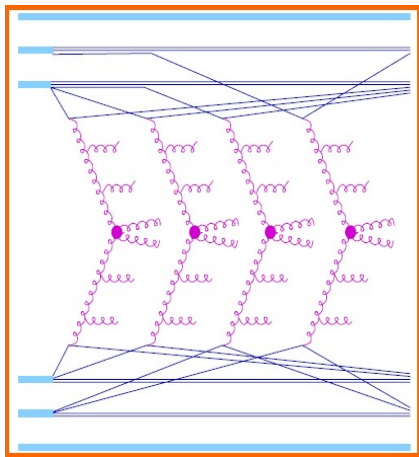
- Interaction between partons is : **Pomeron** : treated by Quantum Field Theory
- Energy conserved by partonic participants and remnants

Pomeron

H. J. Drescher, M. Hladik, S. Ostapchenko, T. Pierog and K. Werner, Phys. Rept. **350**, 93 (2001)

Event Generator : EPOS

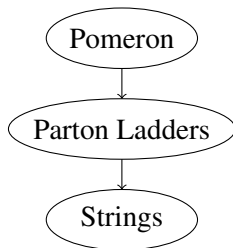
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- 2 Core-Corona Approach

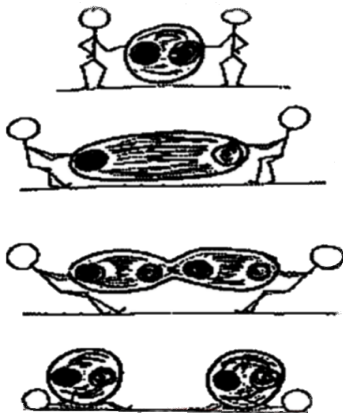
- Interaction between partons is : **Pomeron** : treated by Quantum Field Theory
- Energy conserved by partonic participants and remnants



Strings ?

String Model : A phenomenological model of hadronization

- 1 Initial Conditions
- 2 Core-Corona Approach



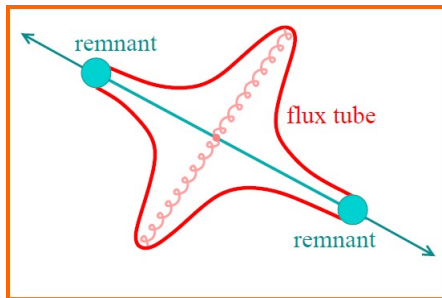
- String without mass and without color between two partons
- **Potential proportional to length**
- **When the potential is sufficient**
→ **one pair of quark-antiquark is created : Schwinger Mechanism**

Core-Corona Evolution

- 1 Initial Conditions
- 2 Core-Corona Approach

One Lund string for one scattering

Few scatterings \rightarrow we *can* treat **independently** each string



GDRE2012, Nantes, Jul 2012, Klaus WERNER, Subatech, Nantes

More scatterings \Rightarrow

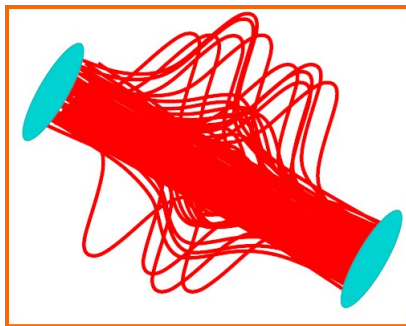
Core-Corona Evolution

- 1 Initial Conditions
- 2 Core-Corona Approach

One Lund string for one scattering

A lot of scatterings \rightarrow we **cannot** treat *independently* each string

We can observe a different string densities



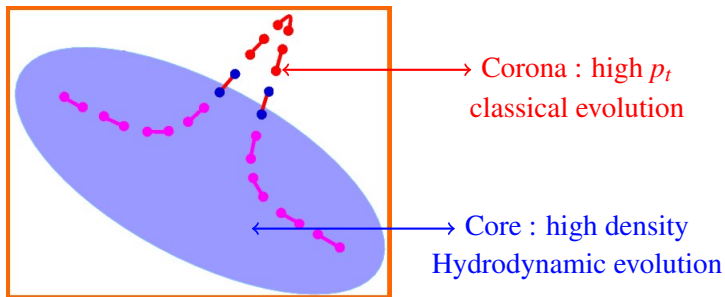
B. Guiot and K. Werner, J. Phys. Conf. Ser. **589** (2015) no.1

Core-Corona Evolution

- 1 Initial Conditions
- 2 Core-Corona Approach

High density : we use hydrodynamics → the Core is treated as **fluid**.

Low density : we do nothing → Corona becomes **hadrons** !



B. Guiot and K. Werner, J. Phys. Conf. Ser. **589** (2015) no.1

Unified Approach

- ① Initial Conditions
- ② Core-Corona Approach
- ③ Hydrodynamical expansion

Core-Corona Approach

Using hydrodynamic \rightarrow the **Core is treated as fluid**.

Corona becomes Jet \Rightarrow Later Hadrons !

Hydrodynamical expansion

Core evolves with respect to the equation of relativistic viscous hydrodynamics

Local energy momentum :

$$\partial_{\mu} T^{\mu\nu} = 0 \quad \nu = 0, \dots, 3$$

and the conservation of net charges,

$$\partial_{\mu} N_k^{\mu} = 0, \quad k = B, S, Q$$

with B, S and Q referring to baryon number, strangeness and electric charge

Unified Approach

- ① Initial Conditions
- ② Core-Corona Approach
- ③ Hydrodynamical expansion
- ④ **Statistical Hadronization**
- ⑤ **Hadronic cascade**

Statistical Hadronization

Core-Matter makes hadronization

Defined by a constant temperature T_H

Procedure of Cooper-Frye

K. Werner, Iu. Karpenko, T. Pierog, M. Bleicher, K. Mikhailov, arXiv:1010.0400, Phys. Rev. C 83, 044915 (2011)

Hadronic Cascade

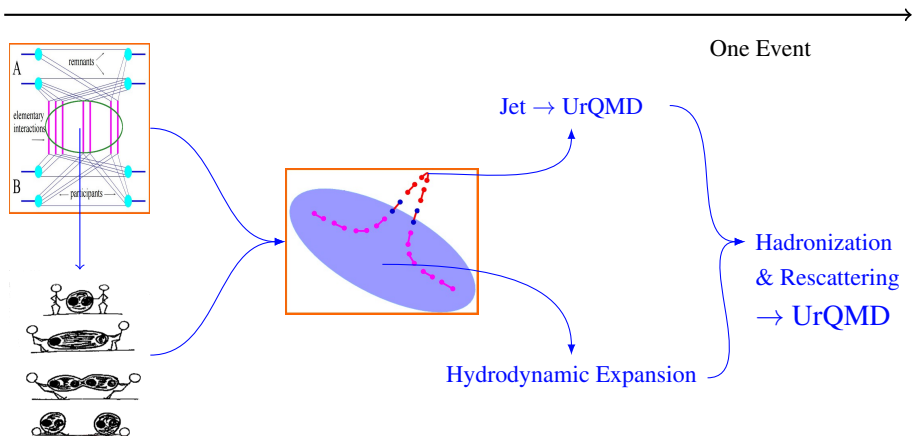
Hadron density still big \rightarrow hadron-hadron rescatterings

Use **UrQMD Model**

M. Bleicher et al., J. Phys. G25 (1999) 1859

H. Petersen, J. Steinheimer, G. Burau, M. Bleicher and H. Stoecker, Phys. Rev. C78 (2008) 044901

Unified Approach



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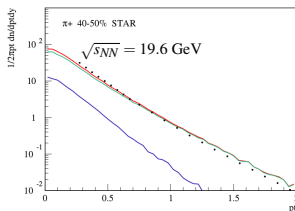
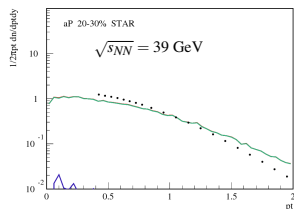
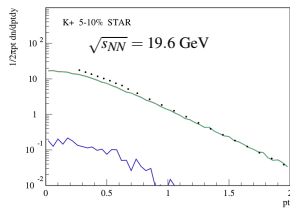
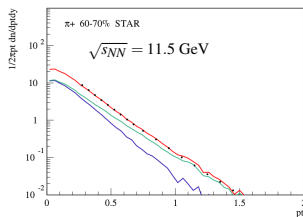
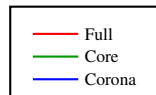
- 1 Introduction
- 2 The event generator
- 3 Energy Dependence**
 - Spectra
 - Anisotropic flow
- 4 System Size Dependence
- 5 Conclusion

pt Spectra

Au-Au collisions

$$p_T = \sqrt{p_x^2 + p_y^2}$$

$$\approx 1\text{M events}$$



- The slope is slightly different
- Big part of core (fluid) even at low energies!
- Not enough core at low p_T , and too much core at high p_T

Description of used observable

Anisotropic flow

A way of characterizing the various patterns of anisotropic flow is to use a Fourier expansion of the particle distribution function:

$$E \frac{d^3N}{d^3\mathbf{p}} = \frac{1}{2\pi} \frac{d^2}{p_t dp_t dy} \left(1 + 2 \sum_{n=1}^{\infty} v_n \cos [n(\phi - \Psi_{RP})] \right)$$

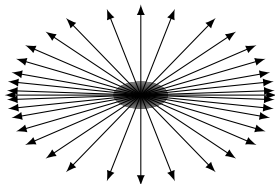
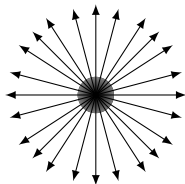
E: energy of the particle ; p: momentum ; p_t: transverse momentum ; φ: azimuthal angle ; y: rapidity ; Ψ_{RP}: reaction plane angle.

Anisotropic Flow : (n=1: Directed Flow , n=2: Elliptic Flow)

$$v_n(pt, y) = \langle \cos [n(\phi(pt, y) - \Psi_{RP})] \rangle$$

Anisotropic Flow

Anisotropy \neq Isotropy



- Elementary Collisions: **Isotropy** of particle production
 $v_2 = 0$: Elliptic Flow
- A-A Collisions: **Anisotropy** of particles production
 $v_2 > 0$
- Something more than elementary processes
- Complex observable with different methods to calculate it

Event plane method

Eta-Sub : Event Plane Method

Event Flow vector (projection of azimuthal angle):

$$Q_{n,x} = \sum_i w_i \cos(n\phi_i) = Q_n \cos(n\Psi_n)$$

$$Q_{n,y} = \sum_i w_i \sin(n\phi_i) = Q_n \sin(n\Psi_n)$$

The sum goes over all particles i used in *the event plane calculation*.
 ϕ_i and w_i are the lab azimuthal angle and weight for particle i

Where Ψ_n is **the event plane angle**:

$$\Psi_n = \frac{1}{n} \tan^{-1} \left(\frac{\sum_i w_i \sin(n\phi_i)}{\sum_i w_i \cos(n\phi_i)} \right)$$

Anisotropic flow

Eta-Sub : Event Plane Method

$$v_n^{\text{obs}}(p_T, y) = \langle \cos[n(\phi_i - \Psi_n)] \rangle$$

Average over all particles in all events with their azimuthal angles ϕ_i in a given rapidity and p_T momentum space.

Anisotropic flow

Eta-Sub : Event Plane Method

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The final flow coefficients are :

$$v_n = \frac{v_n^{\text{obs}}}{\mathcal{R}_n}$$

Anisotropic flow

Eta-Sub : Event Plane Method

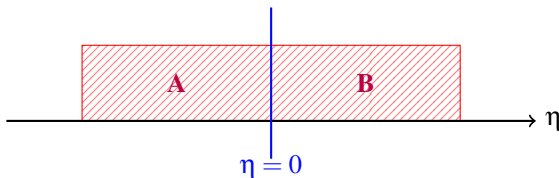
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Eta-sub method : two planes defined by negative (A) and positive (B) pseudorapidity with \approx equal multiplicity used to reduce nonflow effects:

$$\mathcal{R}_{n,\text{sub}} = \sqrt{\langle \cos[n(\Psi_n^A - \Psi_n^B)] \rangle}$$



Anisotropic flow

Eta-Sub : Event Plane Method

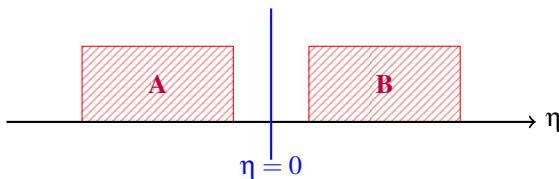
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Anisotropic flow

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$$v_n^{\text{obs}}(p_T, y) = \langle \cos[n(\phi_i - \Psi_n)] \rangle$$

The final flow coefficients are : $v_n = \frac{v_n^{\text{obs}}}{\mathcal{R}_n}$

Three planes :

$$\mathcal{R}_n = \sqrt{\frac{\langle \cos[n(\Psi_n^A - \Psi_n^B)] \rangle \times \langle \cos[n(\Psi_n^A - \Psi_n^C)] \rangle}{\langle \cos[n(\Psi_n^B - \Psi_n^C)] \rangle}}$$



Anisotropic flow

Eta-Sub : Event Plane Method

$$v_n^{\text{obs}}(p_T, y) = \langle \cos[n(\phi_i - \Psi_n)] \rangle$$

The final flow coefficients are :

$$v_n = \frac{v_n^{\text{obs}}}{\mathcal{R}_n}$$

Three planes :

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Results: Event plane

Au-Au collisions

Mesons

$$p_T = \sqrt{p_x^2 + p_y^2}$$

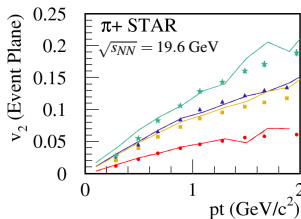
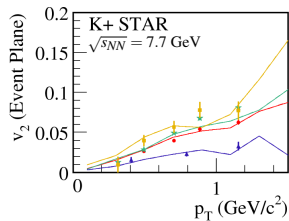
$\approx 50K$ events

● 0-80%

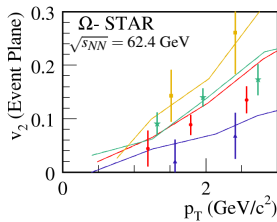
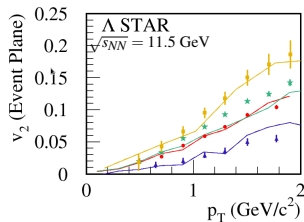
▲ 0-10%

■ 40-80%

★ 10-40%



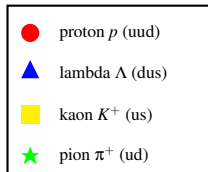
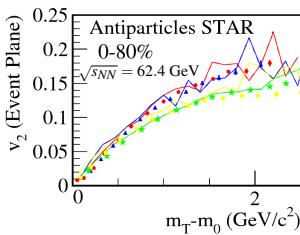
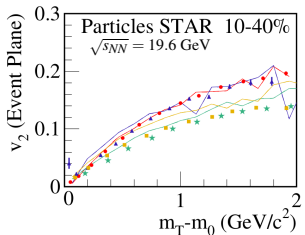
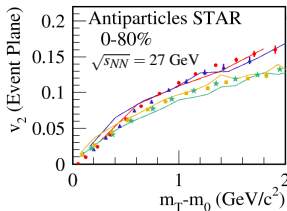
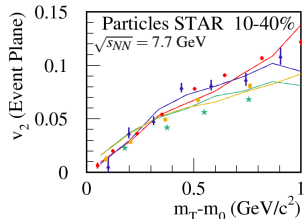
- v_2 vs p_t reveals little anisotropy for all energies
- EPOS approximatively reproduces results for all energies for several centralities



Baryons

Separation Baryons - Mesons

Au-Au collisions



- Contributions [to v_2] from particles and antiparticles reproduced for different centrality regions

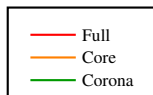
$$m_T - m_0 = \sqrt{p_T^2 + m_0^2} - m_0$$

$\approx 50K$ events

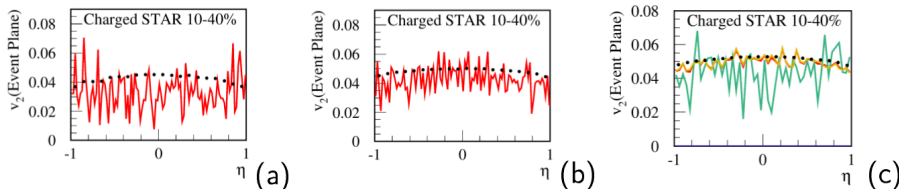
Results: Event Plane Method

Au-Au collisions

$$\eta = \frac{1}{2} \ln \left(\frac{p + p_z}{p - p_z} \right)$$



v_2 vs Pseudorapidity at EPOS 3.210



At energy collisions : $\sqrt{s_{NN}} = 7.7, 11, 39$ GeV with $\approx 30K$ events
 Seems easier to reproduce experimental data when energy increases

STAR Collaboration (Adamczyk, L. et al.) Phys. Rev. C 86, 054908 (2012)

Cumulants Method

A. Bilandzic, R. Snellings, and S. Voloshin Phys. Rev. C 83, 044913 – Published 26 April 2011

Q-Cumulant → Recent Method to calculate cumulants → **one loop over data**
 Faster and unbiased contrary to the previous cumulants method

$$\text{Flow vector : } Q_n = \sum_{i=1}^M e^{in\phi_i} \quad \langle 2 \rangle \equiv \langle e^{in(\phi_1 - \phi_2)} \rangle \quad \langle 4 \rangle \equiv \langle e^{in(\phi_1 + \phi_2 - \phi_3 - \phi_4)} \rangle$$

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Procedure to create cumulants by direct calculations :

- ① Decompose azimuthal correlations into expressions like $|Q_n|^2, |Q_n|^4 \dots$ in terms of $\langle 2 \rangle, \langle 4 \rangle \dots$
- ② Solve system of coupled equations for multi-particle scattering in same harmonic $\langle 2 \rangle, \langle 4 \rangle \dots$
- ③ Create $\langle\langle 2 \rangle\rangle, \langle\langle 4 \rangle\rangle$, average on all events, taking in account weights of event
- ④ Create Cumulants with terms of $\langle\langle 2 \rangle\rangle, \langle\langle 4 \rangle\rangle$ etc ...

Reduce the contribution of nonflow effects

$$\langle 2 \rangle = \frac{\text{Ex : } |Q_n|^2 - M}{M(M-1)}$$

Cumulants Method

A. Bilandzic, R. Snellings, and S. Voloshin Phys. Rev. C 83, 044913 – Published 26 April 2011

Cumulant coefficients

Cumulants for reference flow :

$$c_n\{2\} = \langle\langle 2 \rangle\rangle$$

$$c_n\{4\} = \langle\langle 4 \rangle\rangle - 2 \times \langle\langle 2 \rangle\rangle^2$$

Reference flow or integrated flow :

$$v_n\{2\} = \sqrt{c_n\{2\}}$$

$$v_n\{4\} = \sqrt[4]{-c_n\{4\}}$$

Reference Flow : v_2 vs multiplicity or vs centrality

Cumulants for differential flow :

$$d_n\{2\} = \langle\langle 2' \rangle\rangle$$

$$d_n\{4\} = \langle\langle 4' \rangle\rangle - 2 \times \langle\langle 2' \rangle\rangle \langle\langle 2 \rangle\rangle$$

Differential flow :

$$v'_n\{2\} = d_n\{2\} / \sqrt{c_n\{2\}}$$

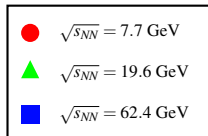
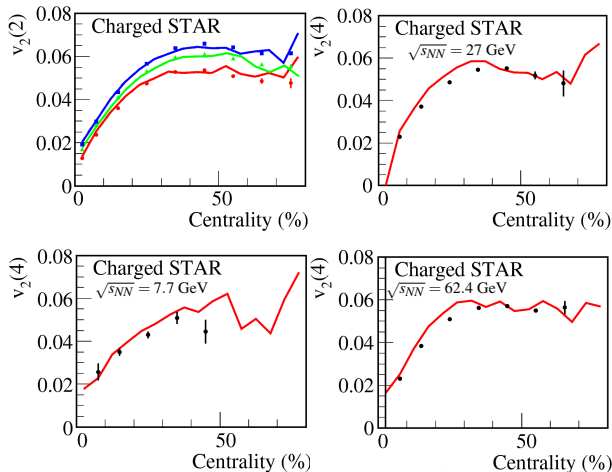
$$v'_n\{4\} = -d_n\{4\} / (-c_n\{4\})^{3/4}$$

Differential Flow : v_2 vs p_t or vs η

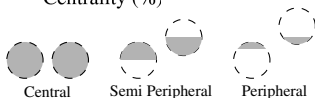
Results Cumulants Method

Au-Au collisions

$\approx 50K$
events



- Good reproduction of results for cumulants methods for $v_2\{2\}$ and $v_2\{4\}$ for each energy



Energy Dependence

Conclusion

- Do not reproduce p_t spectra at low energies
- Core contributes stronger than corona at all energies
- Implementation of event plane and cumulant methods in EPOS
- Reproduce integrated and differential flow for all energies
- No energy dependence seen at BES energies
- Same behavior for all energies

Our scenario of an hydrodynamical expansion, based on initial conditions from our Gribov-Regge approach, is supported

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1 Introduction

2 The event generator

3 Energy Dependence

4 System Size Dependence

Yields

Spectra

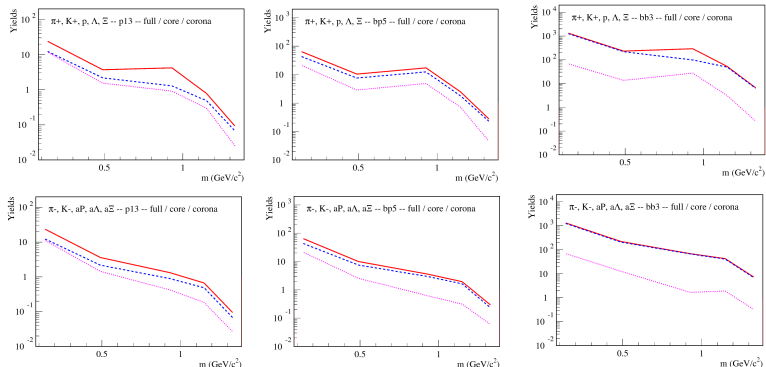
Anisotropic Flow

5 Conclusion

Yields

Theoretical test

How the fluid contributes for each type of collisions?

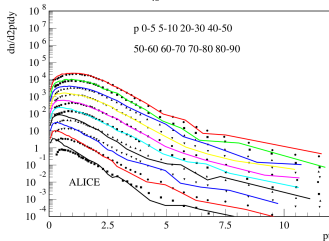
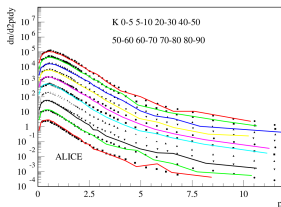
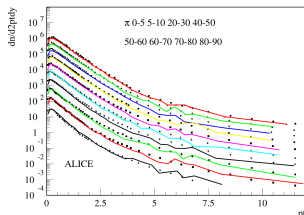


- same trend of full-core-corona contributions
- core part contributes stronger than corona part
- core part stronger for big system sizes

pt spectra

Pb-Pb Collisions at $\sqrt{s_{NN}} = 5.02$ TeV

$$p_T = \sqrt{p_x^2 + p_y^2} \approx 250\text{K events}$$



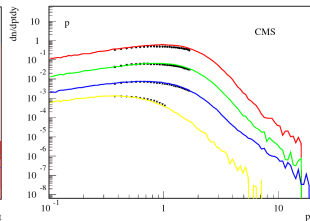
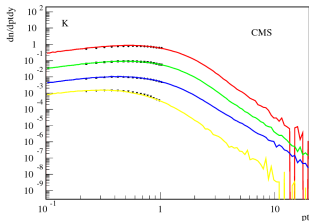
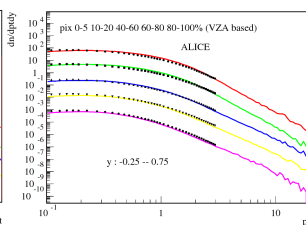
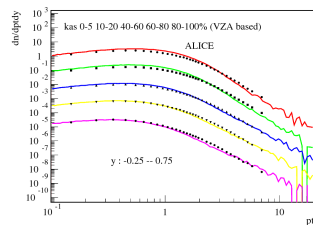
- Good reproduction for each multiplicity class



Spectra

p-Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV

$$p_T = \sqrt{p_x^2 + p_y^2} \approx 500\text{K events}$$



- EPOS approximately reproduces results for distributions in p_T

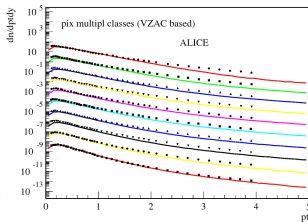
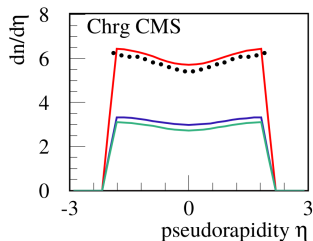
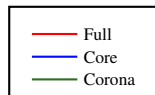
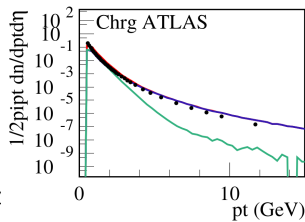
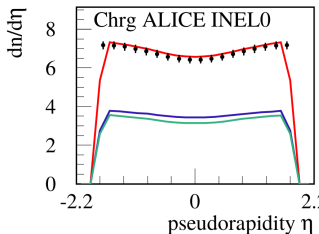


Spectra

pp collisions at $\sqrt{s_{NN}} = 13$ TeV

$$p_T = \sqrt{p_x^2 + p_y^2} \approx 1M \text{ events}$$

$$\eta = \frac{1}{2} \ln \left(\frac{|p| + p_z}{|p| - p_z} \right)$$



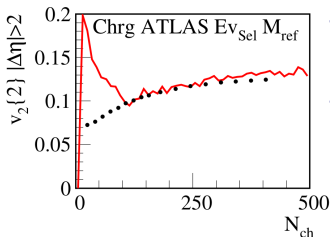
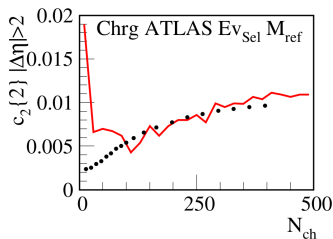
- EPOS
approximatively
reproduces results
for distributions
in p_T or η



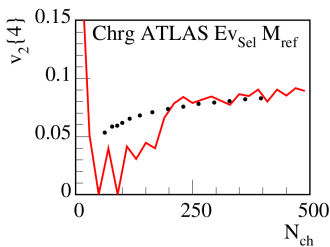
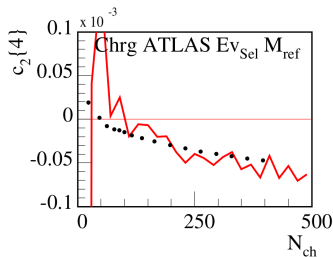
Anisotropic flow: cumulant

$\approx 240\text{K}$ events

Pb-Pb Collisions at $\sqrt{s_{NN}} = 5.02$ TeV



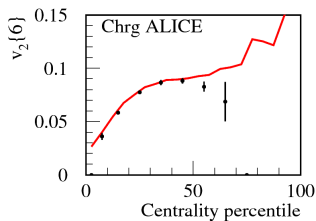
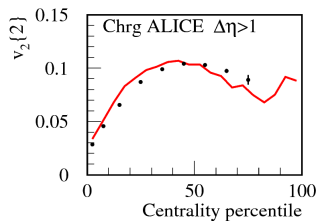
- No reproduction at low multiplicity
- Reproduction at high multiplicity (or central collisions)



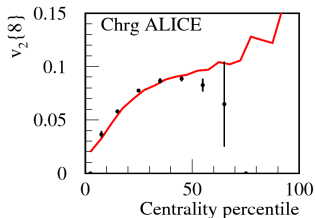
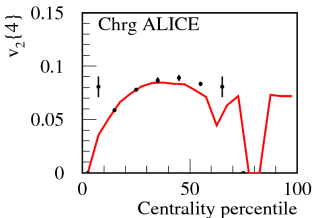
Anisotropic flow: cumulant

$\approx 240\text{K}$ events

Pb-Pb Collisions at $\sqrt{s_{NN}} = 5.02$ TeV



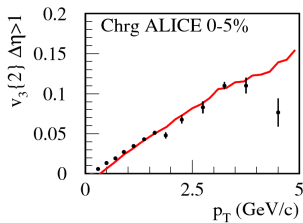
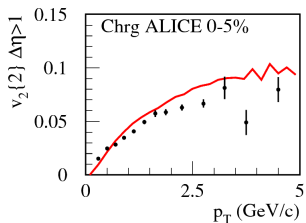
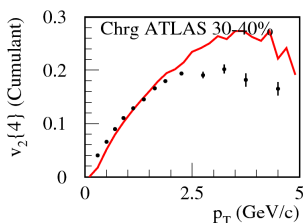
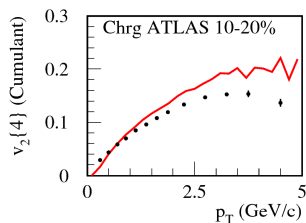
- No reproduction at low multiplicity
- Reproduction at high multiplicity (or central collisions)
- Same conclusion than v_n vs multiplicity



Anisotropic flow: cumulant

$\approx 240\text{K}$ events

Pb-Pb Collisions at $\sqrt{s_{NN}} = 5.02$ TeV



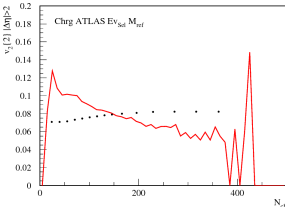
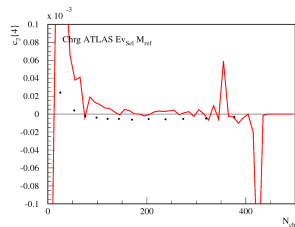
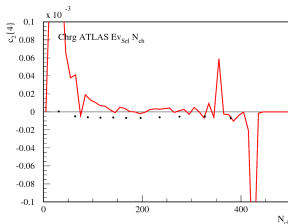
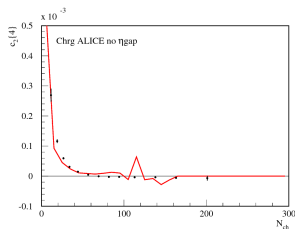
- Reproduction at high multiplicity (or central collisions)
- Seems difficult to reproduce data at high p_T : hydro model
- Same conclusion than v_n vs multiplicity



Anisotropic flow: cumulant

p-Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV

≈ 500 K events



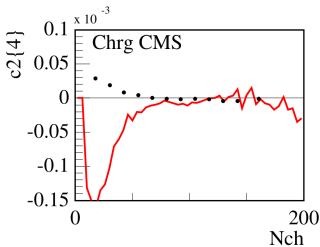
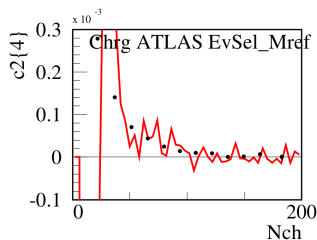
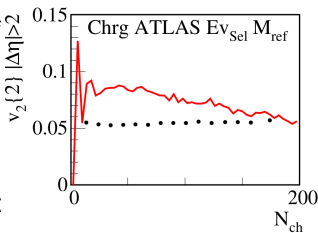
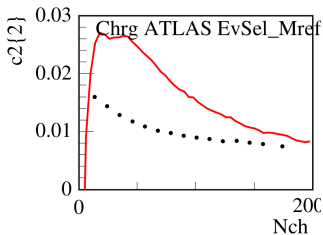
- almost agreement at high multiplicity but do not reproduce anisotropic observables
- $c_2\{4\}$ seems higher than 0, collectivity not related to the fluid???



Anisotropic flow: cumulant

pp collisions at $\sqrt{s_{NN}} = 13$ TeV

$\approx 1M$ events



- at high multiplicity, EPOS seems more adapted to reproduce data
- But we do not reproduce anisotropic observables



Conclusion

- **EPOS part**

Use EPOS at different energy ranges **to moving forward its release**
Implementation of **event plane** and the **cumulant** methods with or without pseudorapidity gap.

- **Energy dependence**

Good data reproduction for anisotropic observables: **a collectively expanding plasma seems exist at these energies.**

No Good data reproduction for pt spectra, first time that we use EPOS at these energies, *work in progress.*

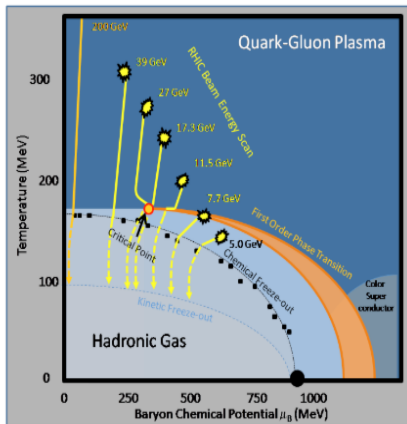
- **System Size dependence**

Confirmation of the utilisation of EPOS for **high multiplicity Pb-Pb collisions.**

Good reproduction of spectra observables for each system size with **high core contribution** but not for anisotropic observables.

Work on *size of initial* fluid to solve the problem of anisotropy for pp collisions

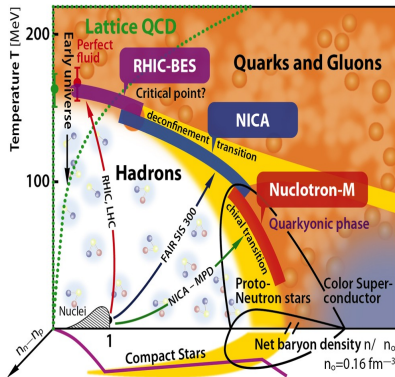
BES and NICA program



STAR collaboration: arXiv:1007.2613

Both programs study physics at $\mu \neq 0$:

- anisotropic flow
- strange particles production
- yield & spectra ..



<http://nica.jinr.ru/physics.php>

Try to work on Nica energies?

Thank you for your attention !



Core Corona separation

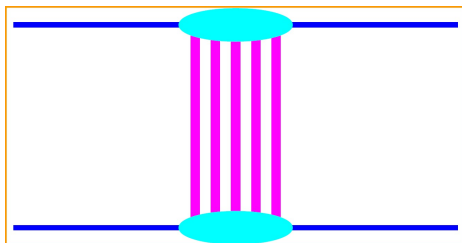
The core corona separation start by the identification of string fragments in core or corona depending of **critical density ρ_0** . However, some string fragments of bulk matter can escape the plasma depending of their transverse momenta. To know exactly which fragments escape, we compute:

$$p_{new}^t = p_t - f_{Eloss} \int_{\gamma} \rho \, dL \quad (1)$$

where γ is the trajectory of the segment and f_{Eloss} a model parameter using some interpolation. The p_t^{new} can be positive or negative, if the new p_t is positive, the string fragments can escape the bulk matter and can be identify as a corona fragment. In the other case ($p_{new}^t < 0$), the segment cannot escape the plasma, therefore it contributes to the core.

Gribov-Regge Theory and Pomeron

Effective Field Theory
 Elementary interaction \rightarrow Pomeron exchange
 Pomeron : Quantum numbers of vacuum
 Vladimir Gribov in \approx 1960



Elastic Amplitude : $T(s, t) \approx i s^{\alpha_0 + \alpha' t}$

Collectivity in small system ?

What is collectivity ? \Rightarrow A lot of definitions !

My definition : **multiple particles are correlated across rapidity or pseudorapidity due to a common origin.**

Why do we care? : learn about : medium (transport), initial state (saturation), and microscopic processes (MPI, strings, parton ladders ...)

We want to find collectivity by measure of multiparticles correlations :

$$\underbrace{
 \begin{array}{cccc}
 v_2\{2\} & \leq v_2\{4\} & \approx v_2\{6\} & \approx v_2\{8\} \\
 c_2\{2\} > 0 & , c_2\{4\} < 0 & , c_2\{6\} > 0 & , c_2\{8\} < 0
 \end{array}
 }_{\text{measure in small system ?}}$$

- Learn about the medium (transport)
- Learn about the initial state (saturation)
- Small and dilute system: chance to learn about microscopic processes (MPI, strings, parton ladders ...)

Elliptic Flow

Differential Flow

Definitions of vectors p and q :

For particles labeled as POI :

$$p_n \equiv \sum_{i=1}^{m_p} e^{in\psi_i}$$

For particles labeled as **both**
POI and REP :

$$q_n \equiv \sum_{i=1}^{m_q} e^{in\psi_i}$$

Average of two- and four-particles azimuthal correlations :

$$\langle 2' \rangle = \frac{\mathcal{R} [p_n Q_n^*] - m_q}{m_p M - m_q} \quad \langle 4' \rangle \propto \mathcal{R} [p_n Q_n Q_n^* Q_n^*] + \mathcal{R} [q_n Q_n^*] \dots$$

Hydrodynamic equations

Based on the four-momenta of string segments, we compute the energy momentum tensor and the flavor flow vector at some position x (at $\tau = \tau_0$) as :

$$T^{\mu\nu} = \sum_i \frac{\delta p_i^\mu \delta p_i^\nu}{\delta p_i^0} g(x - x_i)$$

$$N_q^\mu(x) = \sum_i \frac{\delta p_i^\mu}{\delta p_i^0} q_i g(x - x_i)$$

where $q = u, d, s$

arXiv:1312.1233v1 [nucl-th] 4 Dec 2013

Elliptic Flow

Event Weight

Event Average :

$$\langle\langle 2 \rangle\rangle = \frac{\sum_{events} (W_{\langle 2 \rangle})_i \langle 2 \rangle_i}{\sum_{events} (W_{\langle 2 \rangle})_i} \quad \langle\langle 4 \rangle\rangle = \frac{\sum_{events} (W_{\langle 4 \rangle})_i \langle 4 \rangle_i}{\sum_{events} (W_{\langle 4 \rangle})_i}$$

$$\langle\langle 2' \rangle\rangle = \frac{\sum_{events} (w_{\langle 2' \rangle})_i \langle 2' \rangle_i}{\sum_{events} (w_{\langle 2' \rangle})_i} \quad \langle\langle 4' \rangle\rangle = \frac{\sum_{events} (w_{\langle 4' \rangle})_i \langle 4' \rangle_i}{\sum_{events} (w_{\langle 4' \rangle})_i}$$

Definition of weights :

$$W_2 = M(M-1) \quad W_4 = M(M-1)(M-2)(M-3)$$

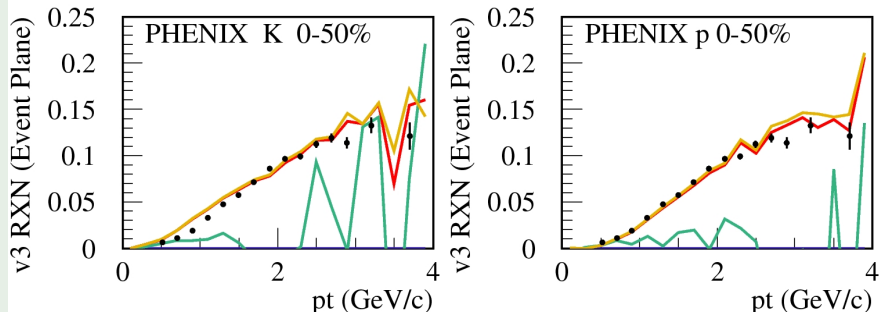
$$w_{2'} = m_p M - m_q \quad w_{4'} = (m_p M - 3m_q)(M-1)(M-2)$$

Results

Event Plane Method

$$p_t = \sqrt{p_x^2 + p_y^2}$$

v_3 vs p_t at EPOS 3.210



At energy collisions : $\sqrt{s_{NN}} = 200$ GeV with 287300 events

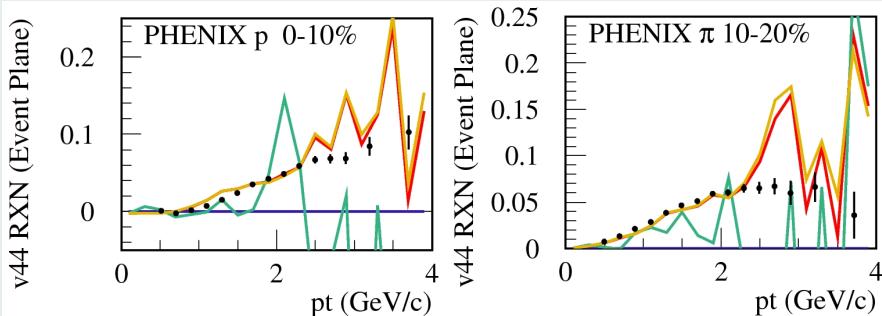
PHENIX Collaboration (A. Adare et al.) Phys. Rev. C 93, 051902 – 2016

Results

Event Plane Method

$$p_t = \sqrt{p_x^2 + p_y^2}$$

v_4 vs p_t at EPOS 3.210



At energy collisions $\sqrt{s_{NN}} = 200$ GeV with 287300 events

PHENIX Collaboration (A. Adare et al.) Phys. Rev. C 93, 051902 – 2016