Theoretical support of experiments at high energy colliders

Renat Sadykov

DLNP JINR, Dubna, 26.09.2018

Outline

- MCSANC for Drell-Yan (DY) processes at hadron colliders
  - Motivation
  - Higher order electroweak (EW) corrections to DY
  - Tuned comparison between different codes for DY (NC and CC)
  - Conclusions and plans
- ARIeL – new project for precision study of e^+e^- collisions with polarized beams
  - Motivation and goals
  - Description of the project
  - Polarized Bhabha scattering $e^+e^- \to e^+e^-$ at NLO EW
  - Preliminary results at NLO EW for other processes with polarized e^+e^- beams
  - Conclusions and plans

Part I. MCSANC for Drell-Yan processes at hadron colliders

DY processes. Motivation

- The DY processes have leptonic final states with clean signatures
- Measurements are used for PDF constraints and SM verification
- Current fixed order precision is NNLO QCD + NLO EW (FEWZ, DYNNLO, POWHEG, RADY, HORACE, MCSANC)
- LHC gives access to energies where additional corrections have to be included:
  - photon-induced contributions
  - higher order radiative corrections
  - "missed" EW radiative corrections (IFI + ISR)

DY processes. Leading two-loop corrections

In MCSANC v1.20 we follow the recipe introduced in [Phys.Lett. B319 (1993) 249-256].

The ρ parameter is defined as the ratio of the neutral current to charged current amplitudes at zero momentum transfer:

$$\rho = \frac{G_{NC}(0)}{G_{CC}(0)} = \frac{1}{1-\Delta\rho}\,,$$

where $G_{CC}(0) = G_\mu$ is the Fermi constant defined from the μ-decay width, perturbatively

$$\Delta\rho = \Delta\rho^{(1)} + \Delta\rho^{(2)} + \ldots$$

We try to estimate the effect due to the two-loop EW corrections $\Delta\rho^{(2)}$ by basic replacements in our form factors (FF). Expanding ρ to quadratic terms $\Delta\rho^2$, we have

$$\rho = 1 + \Delta\rho + \Delta\rho^2\,.$$

DY processes. Leading two-loop corrections

The leading in $G_\mu m_t^2$ NLO EW contribution to $\Delta\rho$ is explicitly given by

$$\Delta\rho^{(1)}\Big|^{G_\mu} = 3x_t = \frac{3\sqrt{2}G_\mu m_t^2}{16\pi^2}\,.$$

At the two-loop level, quantity $\Delta\rho$ contains two contributions:

$$\Delta\rho = 3x_t\,\left[1+\rho^{(2)}\,\left(M_H^2/m_t^2\right)\,x_t\right]\,\left[1-\frac{2\alpha_s(M_Z^2)}{9\pi}(\pi^2+3)\right].$$

They consist of the following:

- two-loop EW part at $\mathcal{O}(G_\mu^2)$.
- mixed two-loop EW$\otimes$QCD at $\mathcal{O}(G_\mu\alpha_s)$

DY processes. Leading two-loop corrections

Using intermediate vector boson propagators $\sim 1/(Q^2 + M_V^2)$, we derive:

$$\rho = \frac{M_w^2}{\bar{c}_w^2 M_z^2}\,,$$

where we introduced a new parameter $\bar{c}_w^2$ to distinguish from the usual c_w^2 for which we maintain the meaning $c_w^2 = M_w^2/M_z^2$ to be valid to all perturbative orders. At the lowest order (LO)

$$\rho^{(0)} = \frac{M_w^2}{c_w^2 M_z^2} = 1\,.$$

Then:

$$\bar{c}_w^2 = \frac{M_w^2}{\rho M_z^2} = (1 - \Delta\rho)\, c_w^2\,.$$

DY processes. Leading two-loop corrections

The leading in $G_\mu m_t^2$ universal higher order (h.o.) corrections may be taken into account. via the replacements, [JHEP 1001 (2010) 060]:

$$\alpha_{G_\mu} \to \alpha_{G_\mu} \frac{\bar{s}_w^2}{s_w^2}\,,$$
$$s_w^2 \to \bar{s}_w^2 \equiv s_w^2 + \Delta\rho\, c_w^2\,,$$
$$c_w^2 \to \bar{c}_w^2 \equiv 1 - \bar{s}_w^2 = (1 - \Delta\rho)\, c_w^2$$

in the LO expression for NC DY cross section.
This approach correctly reproduces terms up to $\mathcal{O}(\Delta\rho^2)$.

Given these replacements, we get the contributions of h.o. corrections to the scalar form factors of the invariant amplitude

DY processes. Leading two-loop corrections

In the LQ basis the Z exchange amplitude has the following Born-like. structure in terms of four (LL, QL, LQ and QQ) form factors:

$$\mathcal{A}_Z^{\rm IBA} = -\,i\,\frac{4\pi\alpha}{s}\left[\frac{1}{4s_W^2 c_W^2}\,\frac{s}{s-M_Z^2+i\dfrac{\Gamma_Z}{M_Z}s}\right]$$

$$\cdot\Big\{\gamma_\mu\gamma_+\otimes\gamma_\mu\gamma_+ I_e^{(3)} I_t^{(3)} F_{LL}(s,t),$$

$$\cdot + \gamma_\mu\otimes\gamma_\mu\gamma_+(-2Q_e s_W^2) I_t^{(3)} F_{QL}(s,t)$$

$$+ \gamma_\mu\gamma_+\otimes\gamma_\mu I_e^{(3)}(-2Q_f s_W^2) F_{LQ}(s,t)$$

$$+ \gamma_\mu\otimes\gamma_\mu(-2Q_e s_W^2)(-2Q_f s_W^2) F_{QQ}(s,t)\Big\},$$

DY processes. Leading two-loop corrections

$$\mathcal{A}_Z^{\rm IBA}(\textit{with h.o.}) = i\frac{4\pi\bar{\alpha}_{G_\mu}}{s}\left[\frac{1}{4}\frac{1}{s_W^2}\left[\frac{s_W^2}{\cancel{s_W^2}}\right]\frac{s}{s-M_Z^2+i\dfrac{\Gamma_Z}{M_Z}s}\right]$$

$$\left\{\gamma_\mu\gamma_+\otimes\gamma_\mu\gamma_+ I_e^{(3)} I_t^{(3)}\left[\frac{1}{\cancel{c_W^2}}\right]F_{LL}(s,t),\right.$$

$$+\gamma_\mu\otimes\gamma_\mu\gamma_+(-2Q_e)s_W^2\left[\frac{\cancel{s_W^2}}{s_W^2}\frac{1}{\cancel{c_W^2}}\right]I_t^{(3)}F_{QL}(s,t)$$

$$+\gamma_\mu\gamma_+\otimes\gamma_\mu I_e^{(3)}(-2Q_f)s_W^2\left[\frac{\cancel{s_W^2}}{s_W^2}\frac{1}{\cancel{c_W^2}}\right]F_{LQ}(s,t)$$

$$\left.+\gamma_\mu\otimes\gamma_\mu(-2Q_e)(-2Q_f)s_W^4\left[\frac{\cancel{s_W^4}}{s_W^4}\frac{1}{\cancel{c_W^2}}\right]F_{QQ}(s,t)\right\},$$

where for convenience, we multiply and divide $\cancel{s_W^2}$ on the expression s_W^2/s_W^2

DY processes. Leading two-loop corrections

The object $\frac{1}{\cancel{c}_w^2}$ will go into:

$$\frac{1}{\cancel{c}_w^2} = \frac{1}{(1-\Delta\rho)c_w^2} = (1+\Delta\rho+\Delta\rho^2)\frac{1}{c_w^2},$$

and the object $\frac{\cancel{s}_w^2}{s_w^2}$ will go into:

$$\frac{\cancel{s}_w^2}{s_w^2} = (1+\frac{c_w^2}{s_w^2}\Delta\rho).$$

.

DY processes. Leading two-loop corrections

Therefore, amplitude in NNLO order is:

$$\mathcal{A}_Z^{\rm IBA}(\textit{with h.o.}) =$$

$$\cdot - i\frac{4\pi\alpha_{G_\mu}}{s}\left[\frac{1}{4s_W^2c_W^2}\frac{s}{s-M_Z^2+i\dfrac{\Gamma_Z}{M_Z}s}\right]$$

$$\Bigg\{\gamma_\mu\gamma_+\otimes\gamma_\mu\gamma_+ I_e^{(3)}I_t^{(3)}\left[1+\Delta\rho+\Delta\rho^2\right]F_{LL}(s,t)$$

$$+\gamma_\mu\otimes\gamma_\mu\gamma_+(-2Q_e)s_W^2\left[(1+\Delta\rho+\Delta\rho^2)(1+\frac{c_W^2}{s_W^2}\Delta\rho)\right]I_t^{(3)}F_{QL}(s,t)$$

$$+\gamma_\mu\gamma_+\otimes\gamma_\mu I_e^{(3)}(-2Q_f)s_W^2\left[(1+\Delta\rho+\Delta\rho^2)(1+\frac{c_W^2}{s_W^2}\Delta\rho)\right]F_{LQ}(s,t)$$

$$+\gamma_\mu\otimes\gamma_\mu(-2Q_e)(-2Q_f)s_W^4\left[(1+\Delta\rho+\Delta\rho^2)(1+\frac{c_W^2}{s_W^2}\Delta\rho)^2\right]F_{QQ}(s,t)\Bigg\},$$

DY processes. Leading two-loop corrections

To avoid a double counting one should remove the leading NLO EW contribution from the linear in $\Delta\rho$ terms: $\Delta\rho \longrightarrow \left(\Delta\rho - \Delta\rho^{(1)}\Big|^{G_\mu}\right)$.

We showed analytically that the results obtained in this way agree with the corresponding expressions from paper [JHEP 1001 (2010) 060].

A large group of dominant radiative corrections can be absorbed into the shift of the ρ parameter from its lowest order value $\rho_{Born} = 1$.
These groups of radiative corrections are:

$$\begin{aligned}\Delta\rho \;\; = \;\; & \Delta\rho_{x_t} + \Delta\rho_{\alpha\alpha_s} + \Delta\rho_{x_t\alpha_s^2} + \left(\Delta\rho_{x_t(zt)\alpha_s^2} + \Delta\rho_{x_t(alp)_f^2}\right) \\ & + \Delta\rho_{x_t\alpha_s^3} + \Delta\rho_{x_t^2\alpha_s} + \Delta\rho_{x_t^2(bos)} + \Delta\rho_{x_t^3}\,.\end{aligned}$$

DY processes. Leading two-loop corrections

Instead of complete calculation of all perturbative orders, the dominant radiative corrections can be absorbed into the shift of the ρ parameter from it's lowest order value $\rho_{Born} = 1$:

$$\begin{aligned}\Delta\rho &= \Delta\rho_{x_t} + \Delta\rho_{\alpha\alpha_s} + \Delta\rho_{x_t\alpha_s^2} + \left(\Delta\rho_{x_t(zt)\alpha_s^2} + \Delta\rho_{x_t(alp)_f^2}\right) \\ &\quad + \Delta\rho_{x_t\alpha_s^3} + \Delta\rho_{x_t^2\alpha_s} + \Delta\rho_{x_t^2(bos)} + \Delta\rho_{x_t^3}\end{aligned}$$

For simplicity denote the corrections with indices:

notation	order of	DIZET v6.42	MCSANC
$\Delta\rho_1$	$O(\alpha\alpha_s)$	+	+
$\Delta\rho_2$	$O(\alpha^2)$	+	+
$\Delta\rho_3$	$O(\alpha_t\alpha_s^2)$	+	+
$\Delta\rho_4$	$O(\alpha_t^2\alpha_s)$	-	+
$\Delta\rho_5$	$O(\alpha_t\alpha_s^3)$	-	+

DY processes. Tuned comparison

- We paricipated in tuned comparison of different codes for Drell-Yan processes at hadron colliders in the context of *LPCC Electroweak Precision Measurements at the LHC WG*.
  The results were published in Eur.Phys.J. C77 (2017) no.5, 280
  The relevant codes can be found at:
  https://twiki.cern.ch/twiki/bin/view/Main/DrellYanComparison

- Now we invited to participate in new comparison within *W-mass 2018 worksop*. For this comparison we will use our MC generator.

DY processes. Tuned comparison. Setup

Input parameters:

$$\sqrt{s} = 8\ \mathrm{GeV},$$

$$G_\mu = 1.1663787 \times 10^{-5}\ \mathrm{GeV}^{-2}, \quad \alpha = 1/137.035999074, \quad \alpha_s \equiv \alpha_s(M_Z^2) = 0.12018,$$

$$M_Z = 91.1876\ \mathrm{GeV}, \quad \Gamma_Z = 2.4952\ \mathrm{GeV},$$

$$M_W = 80.385\ \mathrm{GeV}, \quad \Gamma_W = 2.085\ \mathrm{GeV},$$

$$M_H = 125\ \mathrm{GeV},$$

$$m_e = 0.510998928\ \mathrm{MeV}, \quad m_\mu = 0.1056583715\ \mathrm{GeV}, \quad m_\tau = 1.77682\ \mathrm{GeV},$$

$$m_u = 0.06983\ \mathrm{GeV}, \quad m_c = 1.2\ \mathrm{GeV}, \quad m_t = 173.5\ \mathrm{GeV},$$

$$m_d = 0.06984\ \mathrm{GeV}, \quad m_s = 0.15\ \mathrm{GeV}, \quad m_b = 4.6\ \mathrm{GeV},$$

$$|V_{ud}| = 0.975, \quad |V_{us}| = 0.222,$$

$$|V_{cd}| = 0.222, \quad |V_{cs}| = 0.975,$$

$$|V_{cb}| = |V_{ts}| = |V_{ub}| = |V_{td}| = |V_{tb}| = 0.$$

DY processes. Tuned comparison. Setup

The detector acceptance is simulated by imposing the following transverse momentum ($p_\perp$) and pseudo-rapidity (η) cuts:

$$\text{LHC}: \qquad p_\perp^\ell > 25\ \text{GeV}, \qquad |\eta(\ell)| < 2.5,\ p_\perp^\nu > 25\ \text{GeV}, \qquad \ell = e,\ \mu,$$

where $p_\perp^\nu$ is the missing transverse momentum originating from the neutrino. These cuts approximately model the acceptance of the ATLAS and CMS detectors at the LHC. In addition we apply a cut on the invariant mass of the final-state lepton pair of $M_{l^+l^-} > 50$ GeV and $M(l\nu) > 1$ GeV in the case of γ/Z production and W production respectively.

DY processes. Tuned comparison for W^+ production

code	LO	NLO QCD	NLO EW μ	NLO EW e
HORACE	2897.38(8)	$\times$	2988.2(1)	2915.3(1)
WZGRAD	2897.33(2)	$\times$	2987.94(5)	2915.39(6)
RADY	2897.35(2)	2899.2(4)	2988.01(4)	2915.38(3)
SANC	2897.30(2)	2899.9(3)	2987.77(3)	2915.00(3)
DYNNLO	2897.32(5)	2899(1)	$\times$	$\times$
FEWZ	2897.2(1)	2899.4(3)	$\times$	$\times$
POWHEG-w	2897.34(4)	2899.41(9)	$\times$	$\times$
POWHEG_BMNNP	2897.36(5)	2899.0(1)	2988.4(2)	2915.7(1)
POWHEG_BW	2897.4(1)	2899.2(3)	2987.7(4)	($\times$)

DY processes. Tuned comparison for W^- production

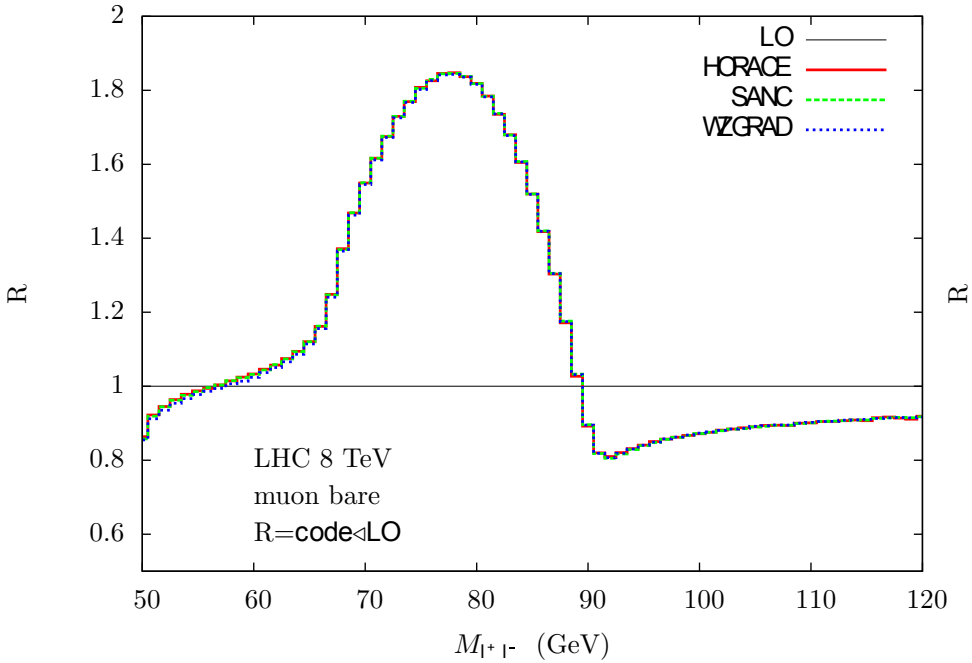
code	LO	NLO QCD	NLO EW μ	NLO EW e
HORACE	2008.84(5)	$\times$	2076.48(9)	2029.15(8)
WZGRAD	2008.95(1)	$\times$	2076.51(3)	2029.26(3)
RADY	2008.93(1)	2050.5(2)	2076.62(2)	2029.29(2)
SANC	2008.926(8)	2050.3(3)	2076.56(2)	2029.19(3)
DYNNLO	2008.89(3)	2050.2(9)	$\times$	$\times$
FEWZ	2008.9(1)	2049.97(8)	$\times$	$\times$
POWHEG-w	2008.93(3)	2050.14(5)	$\times$	$\times$
POWHEG_BMNNP	2008.94(3)	2049.9(1)	2076.9(1)	2029.71(6)
POWHEG_BW	2009.2(4)	2050.2(4)	2076.0(3)	($\times$)

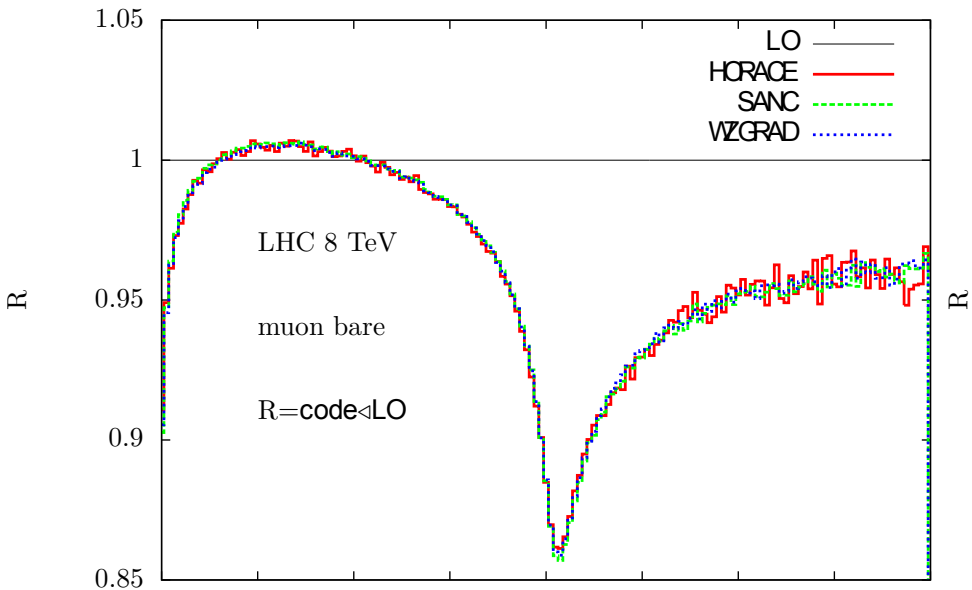
DY processes. Tuned comparison for Z production

code	LO	NLO QCD	NLO EW μ	NLO EW e
HORACE	431.033(9)	$\times$	438.74(2)	422.08(2)
WZGRAD	431.048(7)	$\times$	439.166(6)	422.78(1)
RADY	431.047(4)	458.16(3)	438.963(4)	422.536(5)
SANC	431.050(2)	458.27(3)	439.004(5)	422.56(1)
DYNNLO	431.043(8)	458.2(2)	$\times$	$\times$
FEWZ	431.00(1)	458.13(2)	($\times$)	($\times$)
POWHEG-z	431.08(4)	458.19(8)	$\times$	$\times$
POWHEG_BMNNPV	431.046(9)	458.16(7)	438.9(1)	422.2(2)

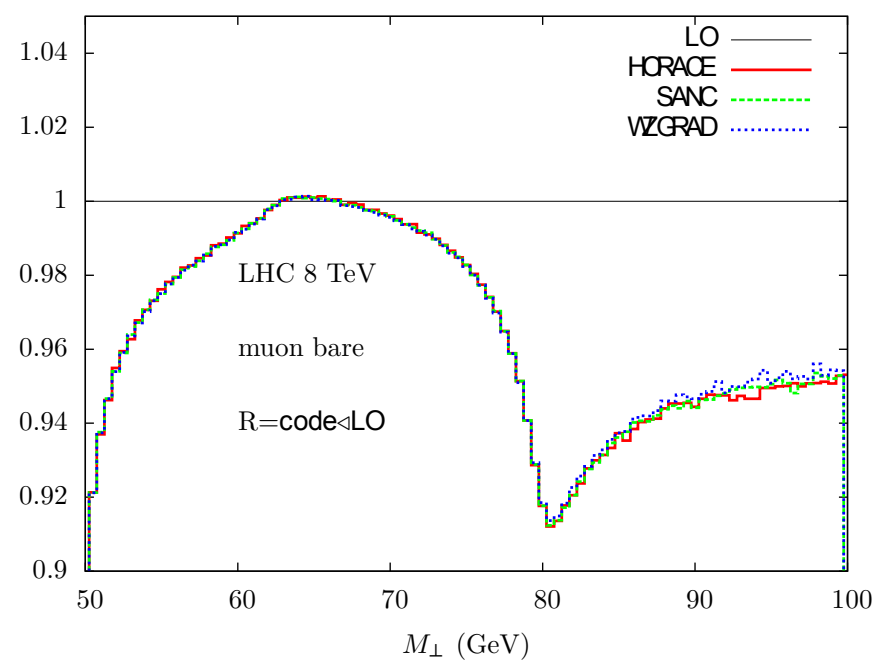
DY processes. Tuned comparison

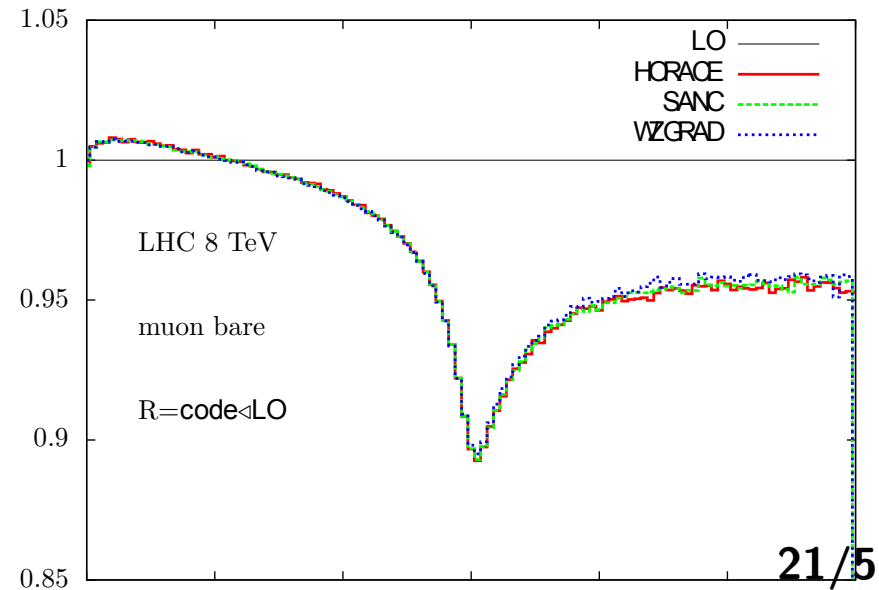
Z-production



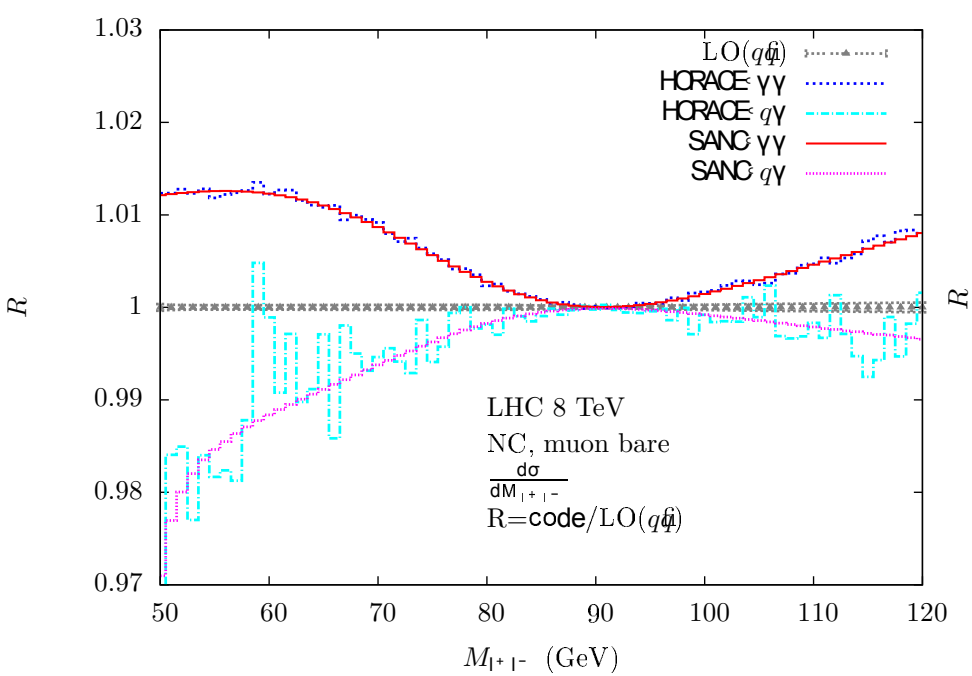


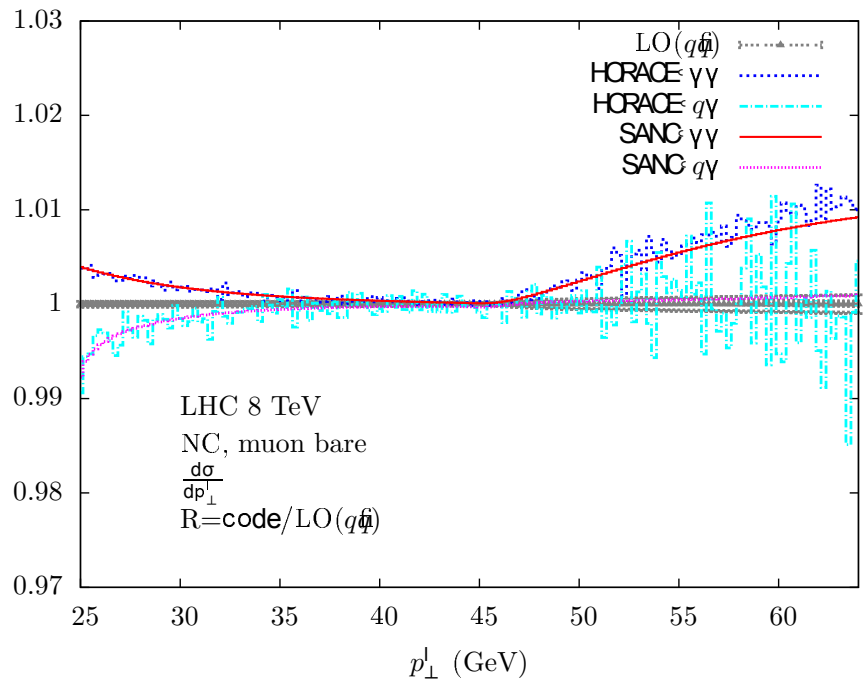
W^+-production





DY processes. Tuned comparison for photon induced contributions





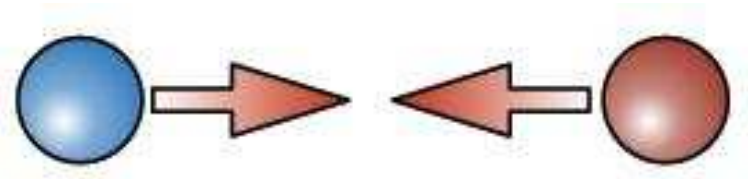
Conclusions and plans

- We participated in tuned comparison of different codes for DY processes at LHC
- New tuned comparison 2018-2019
- A set of HO leading corrections are introduced to the MCSANC generator
- Working on new tool DYTURBO: SANC EW library

Part II. ARIeL - new project for future e^+e^- colliders

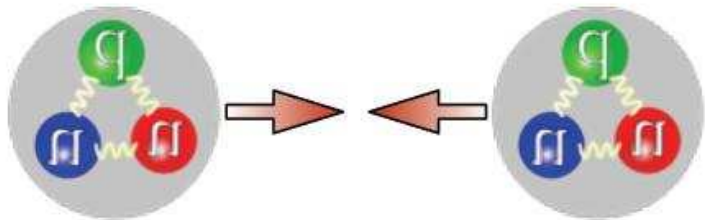
Motivation: choice of collider types

e^+e^- collisions



- Point-like particles
- Total annihilation: initial state known
- Decent background
- Limited in energy, but — precision!

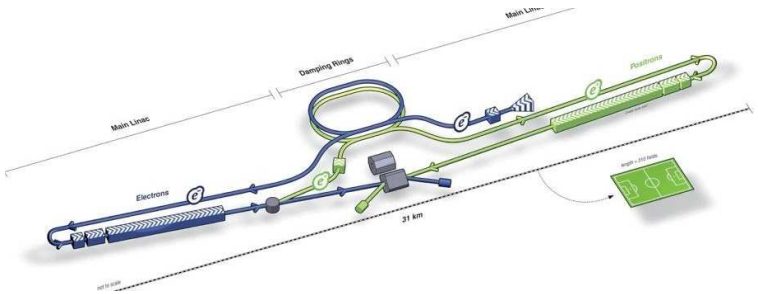
$pp(\bar{p})$ collisions



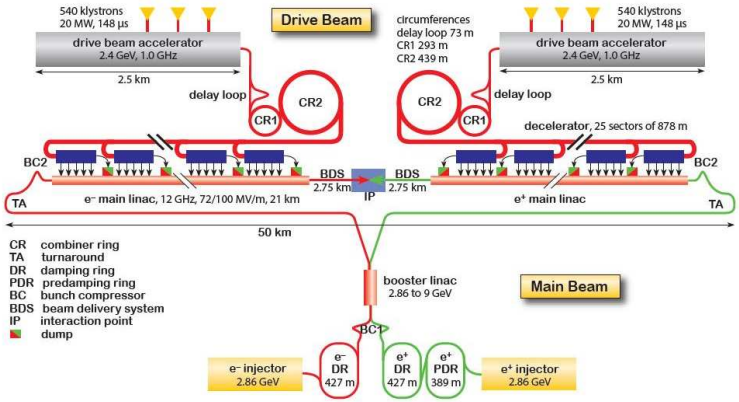
- Composite particles
- Random energy of the hard interaction
- High background
- High energy — discovery!

Projects of e^+e^- collider

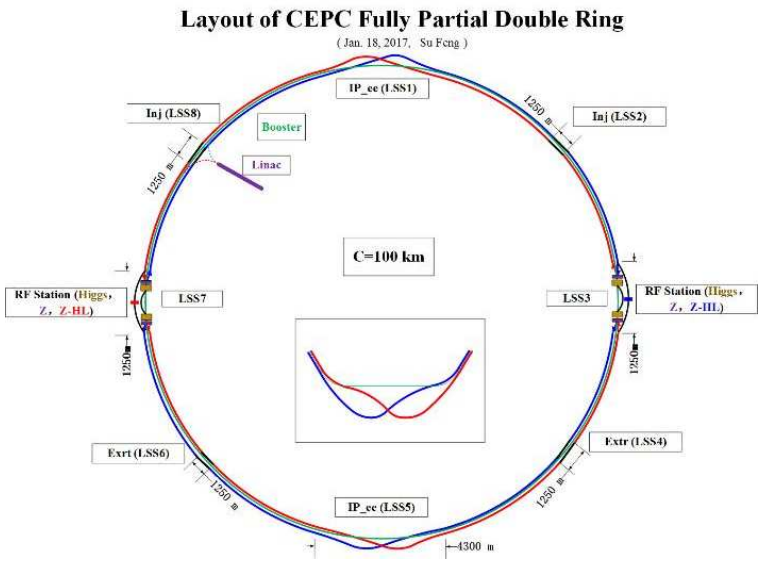
ILC (2030) 250 GeV



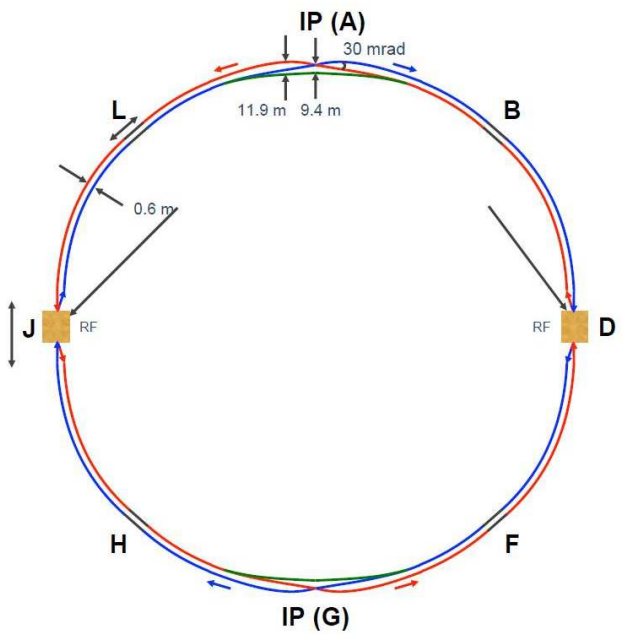
CLIC (2035) 380-3000 GeV



CEPC (2030) 240 GeV



FCC-ee (2039) 350 GeV



Why ARIeL?

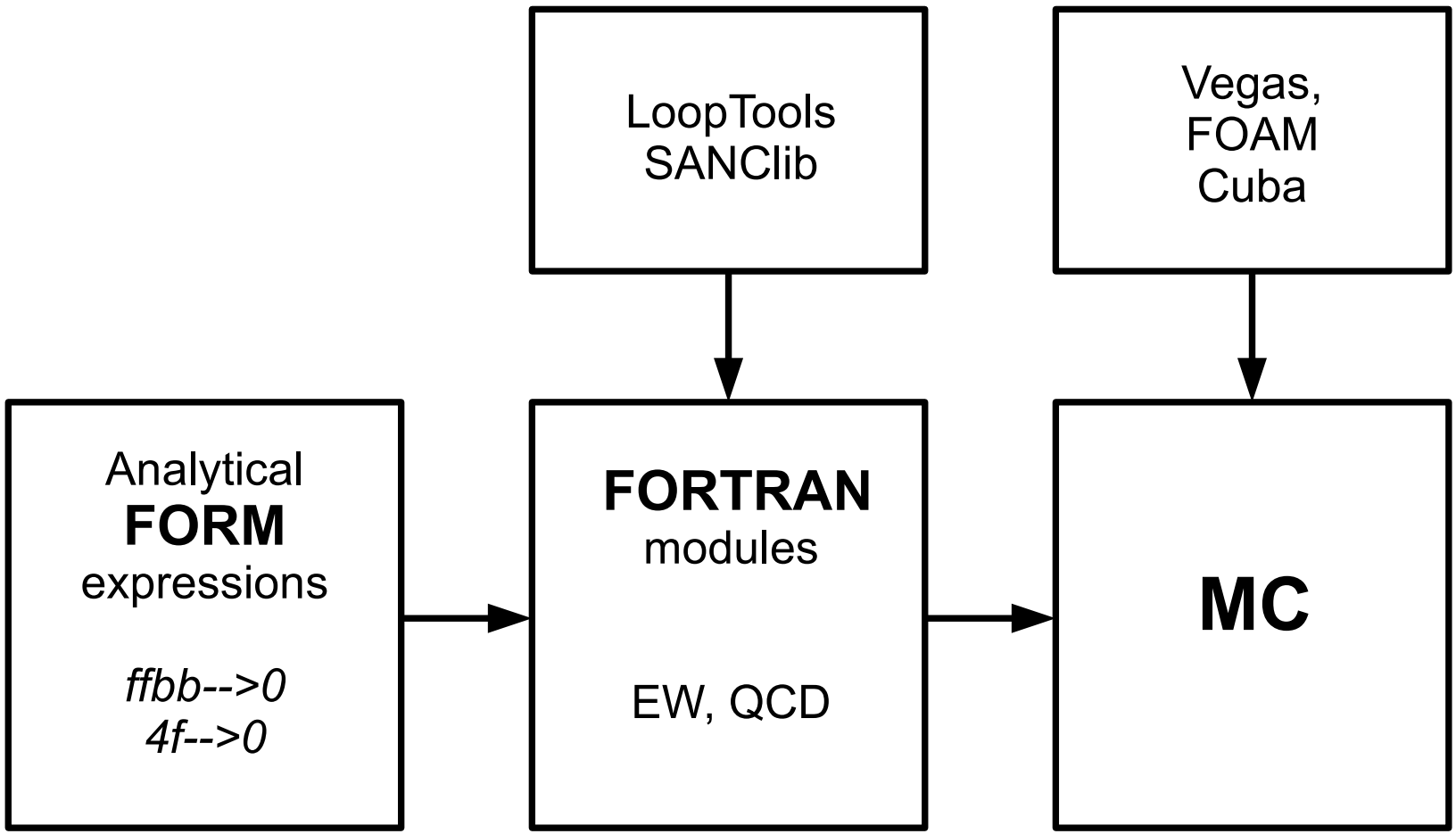
- Advanced
- Research of
- Interactions in
- e^+e^-
- coLlisions

Goals

- Preparation of CLIC research program:
  - Precision study of $e^+e^- \to \gamma\gamma$ and setting limits on the NP models
  - Precision measurement of the Higgs boson mass M_H
  - Determination of top quark polarization
  - Measurement of $\gamma\gamma \to W^+W^-$ and $\gamma\gamma \to ZZ$ and search for anomalous quarting coupling

- Theoretical support:
  - Create e^+e^- Monte Carlo generator with polarization at complete one-loop EW and leading multiloop for processes $e^+e^- \to e^+e^-$ $(\mu\mu, \tau\tau, tt, HZ, H\gamma, Z\gamma, ZZ, H\nu\nu, H\mu\mu, ff\gamma, \gamma\gamma)$
  - Interface NLO EW RC to `PYTHIA8`
  - Implement a single-resonance approach to complex processes
  - Elaborate the standard procedure for $2 \to 3, 4$ helicity amplitudes
  - Create building blocks for complete EW 2-loops and QCD 3-loops, plus leading EW 3-loops and QCD 4-loops

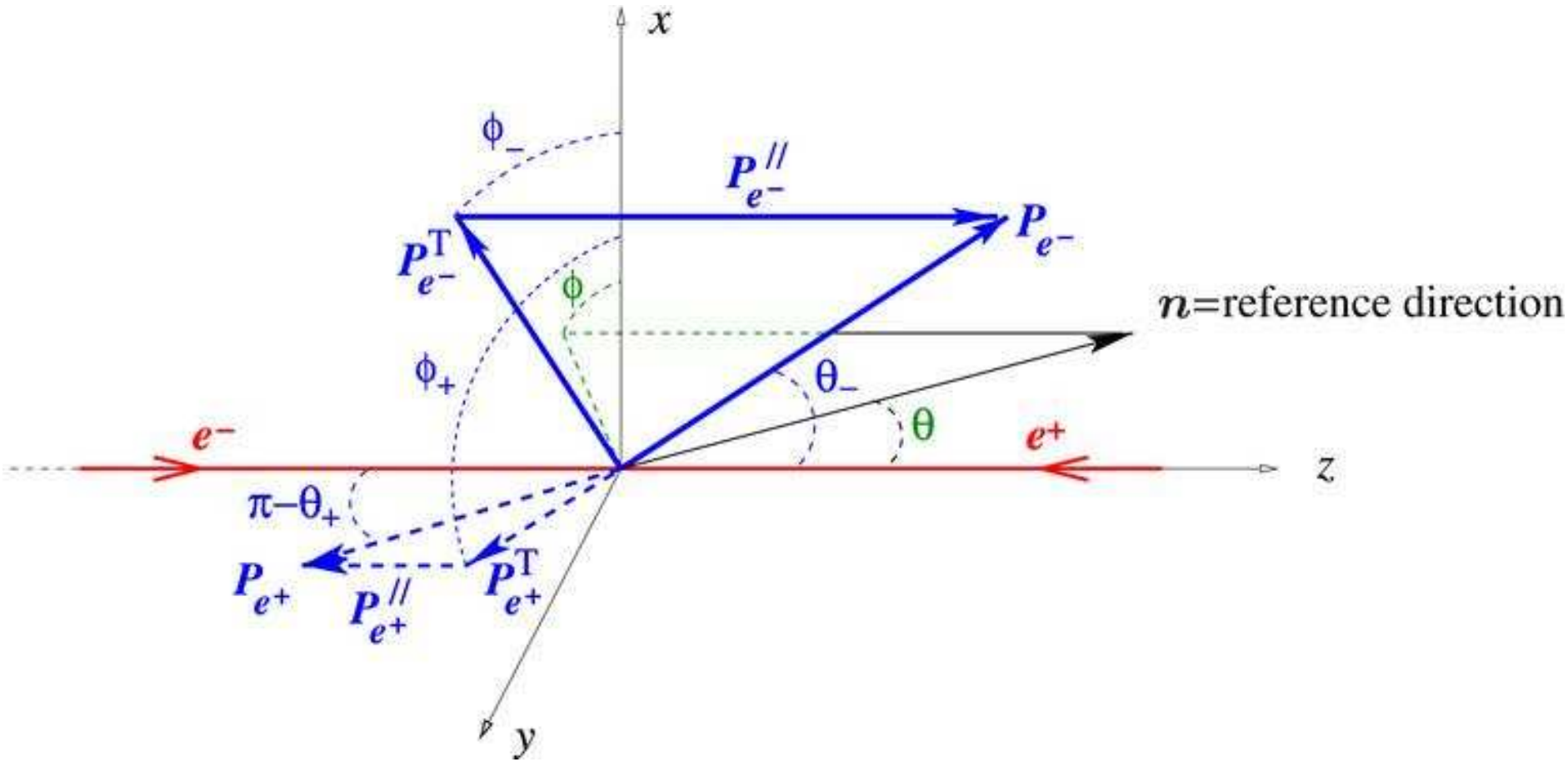
The calculation framework scheme



Processes with polarized beams in MCSANCee

- NLO EW corrections for polarized e^+e^- scattering:
  - Bhabha scattering (D.Bardin et al. Phys.Rev. D98 (2018) no.1, 013001)
  - $e^+e^- \to \mu^+\mu^-$, $e^+e^- \to \tau^+\tau^-$ (preliminary results)
  - $e^+e^- \to Z\gamma$ (preliminary results)
  - $e^+e^- \to t\bar{t}$ (in progress)
  - $e^+e^- \to ZH$ (in progress)
  - $e^+e^- \to \gamma\gamma$ (in progress)
  - $e^+e^- \to ZZ$ (in progress)
  - $e^+e^- \to f\bar{f}\gamma$ (future plans)
  - $e^+e^- \to f\bar{f}H$ (future plans)
- NLO EW corrections for polarized $\gamma\gamma$ scattering:
  - $\gamma\gamma \to \gamma\gamma$ (future plans)
  - $\gamma\gamma \to Z\gamma$ (future plans)
  - $\gamma\gamma \to ZZ$ (future plans)

Decomposition of the $e^{\pm}$ polarization vector



G. Moortgat-Pick et al. Phys. Rept. 460 (2008) 131–243

Matrix element squared

$$
\begin{aligned}
|\mathcal{M}|^2 = \quad & L^{||}_{e^-} R^{||}_{e^+} |\mathcal{H}_{-+}|^2 + R^{||}_{e^-} L^{||}_{e^+} |\mathcal{H}_{+-}|^2 + L^{||}_{e^-} L^{||}_{e^+} |\mathcal{H}_{--}|^2 + R^{||}_{e^-} R^{||}_{e^+} |\mathcal{H}_{++}|^2 \\
& -\frac{1}{2} P^T_{e^-} P^T_{e^+} Re\left[e^{i(\Phi_+ - \Phi_-)} \mathcal{H}_{++} \mathcal{H}^*_{--} + e^{i(\Phi_+ + \Phi_-)} \mathcal{H}_{+-} \mathcal{H}^*_{-+} \right] \\
& + P^T_{e^-} Re\left[e^{i\Phi_-} \left(L^{||}_{e^+} \mathcal{H}_{+-} \mathcal{H}^*_{--} + R^{||}_{e^+} \mathcal{H}_{++} \mathcal{H}^*_{-+} \right) \right] \\
& - P^T_{e^+} Re\left[e^{i\Phi_+} \left(L^{||}_{e^-} \mathcal{H}_{-+} \mathcal{H}^*_{--} + R^{||}_{e^-} \mathcal{H}_{++} \mathcal{H}^*_{+-} \right) \right],
\end{aligned}
$$

where

$$
L^{||}_{e^\pm} = \frac{1}{2}(1 - P^{||}_{e^\pm}), \quad R^{||}_{e^\pm} = \frac{1}{2}(1 + P^{||}_{e^\pm}), \quad \Phi_\pm = \phi_\pm - \phi,
$$

$\mathcal{H}_{--}$, $\mathcal{H}_{++}$,$\mathcal{H}_{-+}$,$\mathcal{H}_{+-}$ — helicity amplitudes.

Longitudinally-polarized beams

With longitudinally-polarized beams, cross-sections at an e^+e^- collider can be subdivided into four parts:

$$\sigma_{P_{e^-}P_{e^+}} = R_{e^-}R_{e^+}\sigma_{RR} + L_{e^-}L_{e^+}\sigma_{LL} + R_{e^-}L_{e^+}\sigma_{RL} + L_{e^-}R_{e^+}\sigma_{LR},$$

where σ_{RL} stands for the cross-section if the e^- beam is completely right-handed polarized ($P_{e^-} = +1$) and the e^+ beam is completely left-handed polarized ($P_{e^+} = -1$). The cross-sections σ_{LR}, σ_{RR} and σ_{LL} are defined analogously.

Polarized Bhabha scattering at one-loop

Here we present complete one-loop EW corrections to Bhabha scattering $e^+e^- \to e^+e^-$ with longitudinally polarized initilal particles.

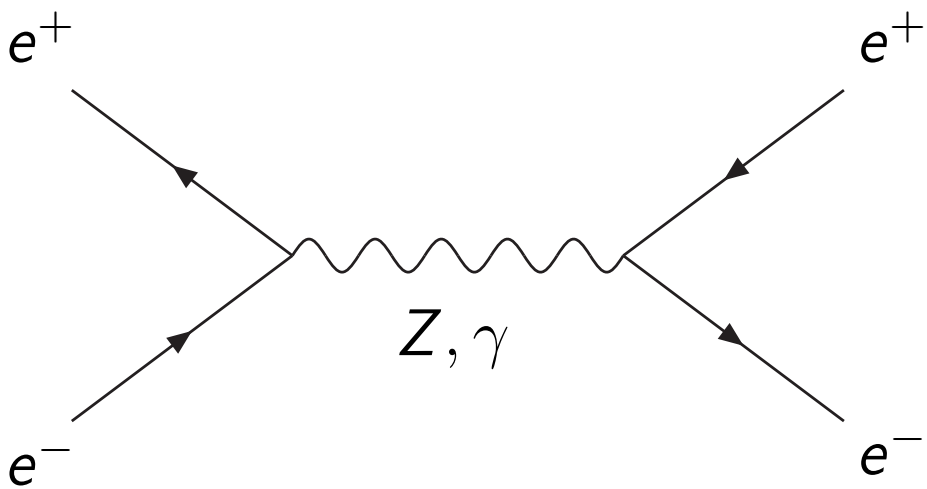
The cross-section of this process at one-loop can be divided into four parts:

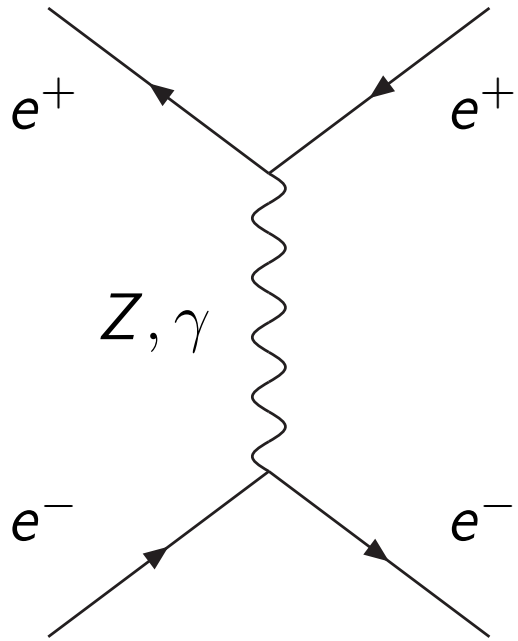
$$\sigma^{\text{1-loop}} = \sigma^{\text{Born}} + \sigma^{\text{virt}}(\lambda) + \sigma^{\text{soft}}(\lambda, \omega) + \sigma^{\text{hard}}(\omega),$$

where σ^{Born} — Born level cross-section, σ^{virt} — contribution of virtual(loop) corrections, σ^{soft} — contribution due to soft photon emission, σ^{hard} — contribution due to hard photon emission (with energy $E_\gamma > \omega \frac{\sqrt{s}}{2}$).

Auxiliary parameters λ ("photon mass") and ω cancel out after summation.

Bhabha scattering: Born-level diagrams





Polarized Bhabha scattering: HA for Born and Virtual parts

At one-loop level we have six non-zero HAs (four independent):

$$\mathcal{H}_{++++} = \mathcal{H}_{----} = -2e^2\,\frac{s}{t}\Big(\mathcal{F}_{QQ}^{(\gamma,Z)}(t,s,u) - \chi_Z^t \delta_e \mathcal{F}_{QL}^Z(t,s,u)\Big),$$

$$\mathcal{H}_{+-+-} = \mathcal{H}_{-+-+} = -2e^2\,\frac{t}{s}\Big(\mathcal{F}_{QQ}^{(\gamma,Z)}(s,t,u) - \chi_Z^s \delta_e \mathcal{F}_{QL}^Z(s,t,u)\Big),$$

$$\mathcal{H}_{+--+} = 2e^2\left(\frac{u}{s}\Big[\mathcal{F}_{QQ}^{(\gamma,Z)}(s,t,u) + \chi_Z^s\left(\mathcal{F}_{LL}^Z(s,t,u) - 2\delta_e \mathcal{F}_{QL}^Z(s,t,u)\right)\Big]\right.$$

$$\left. + \frac{u}{t}\Big[\mathcal{F}_{QQ}^{(\gamma,Z)}(t,s,u) + \chi_Z^t\left(\mathcal{F}_{LL}^Z(t,s,u) - 2\delta_e \mathcal{F}_{QL}^Z(t,s,u)\right)\Big]\right),$$

$$\mathcal{H}_{-++-} = 2e^2\left(\frac{u}{s}\mathcal{F}_{QQ}^{(\gamma,Z)}(s,t,u) + \frac{u}{t}\mathcal{F}_{QQ}^{(\gamma,Z)}(t,s,u)\right),$$

where

$$\chi_Z^s = \frac{1}{4s_W^2 c_W^2}\,\frac{s}{s - M_Z^2 + iM_Z\Gamma_Z}, \quad \chi_Z^t = \frac{1}{4s_W^2 c_W^2}\,\frac{t}{t - M_Z^2}, \quad \delta_e = v_e - a_e = 2s_W^2,$$

$$\mathcal{F}_{QQ}^{(\gamma,Z)}(a,b,c) = \mathcal{F}_{QQ}^{\gamma}(a,b,c) + \chi_Z^a \delta_e^2 \mathcal{F}_{QQ}^Z(a,b,c).$$

We get the Born level HAs by replacing $\mathcal{F}_{LL}^Z \to 1$, $\mathcal{F}_{QL}^Z \to 1$, $\mathcal{F}_{QQ}^Z \to 1$ and $\mathcal{F}_{QQ}^{\gamma} \to 1$.

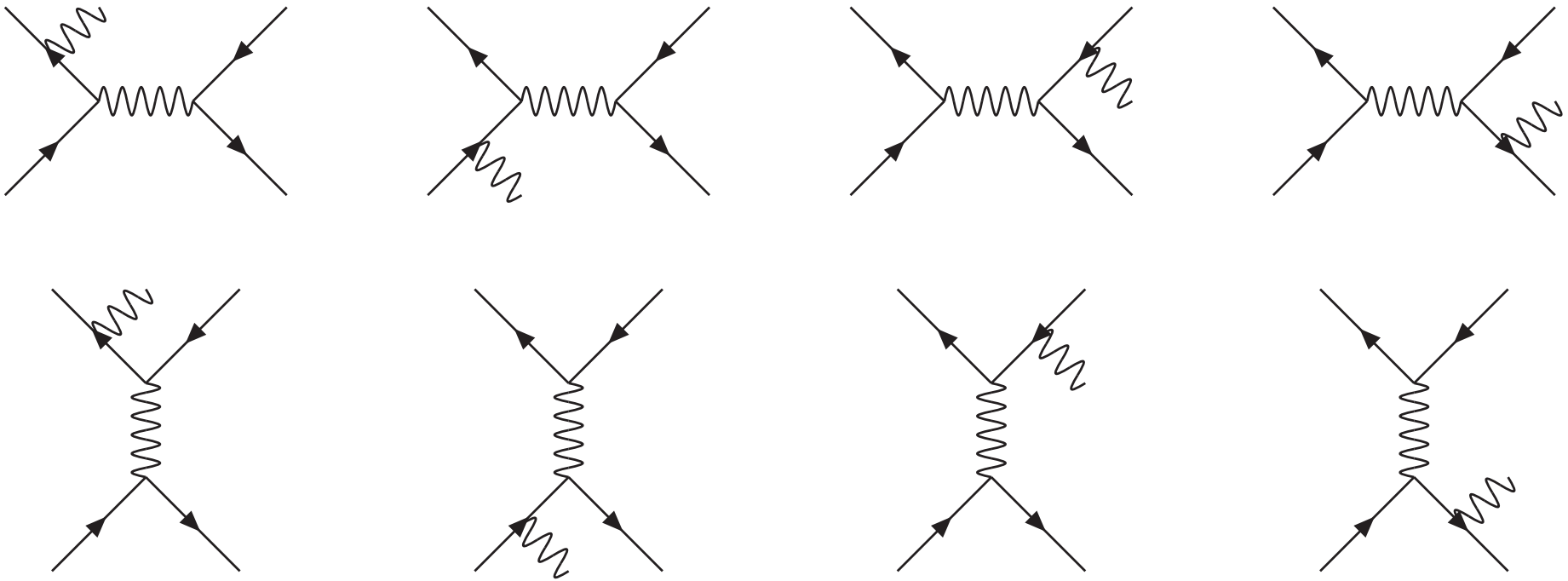
Polarized Bhabha scattering: soft photon contribution

The soft photon contribution contains the infrared divergences which compensate the infrared divergences of the one-loop QED corrections.

This soft photon correction can be calculated analytically and to be factorized to Born cross section. The polarization dependence is contained in σ^{Born}.

$$\begin{aligned}\sigma^{\text{soft}}(\lambda,\omega) \quad &= -\sigma^{\text{Born}}\frac{\alpha}{\pi}\left\{\left(1+\ln\left(\frac{m_e^2}{s}\right)\right)^2+\ln\left(-\frac{u}{s}\right)^2-\ln\left(-\frac{t}{s}\right)^2\right.\\ &\quad -2\text{Li}_2\left(-\frac{u}{s}\right)+2\text{Li}_2\left(-\frac{t}{s}\right)+4\text{Li}_2(1)\\ &\quad \left.-1+2\ln\left(\frac{4\omega^2}{\lambda}\right)\left[1+\ln\left(\frac{m_e^2}{s}\right)-\ln\left(\frac{t}{u}\right)\right]\right\}.\end{aligned}$$

Polarized Bhabha scattering: hard photon contribution



Polarized Bhabha scattering: hard photon contribution

Bremsstrahlung Helicity Amplitudes:

$$\mathcal{H}^{\text{hard}} = \mathcal{H}^{\text{isr}} + \mathcal{H}^{\text{fsr}}$$

Crossing symmetry:

$$\mathcal{H}^{\text{fsr}}_{\chi_1\chi_2\chi_3\chi_4\chi_5}(p_1,p_2,p_3,p_4) = +\mathcal{H}^{\text{isr}}_{-\chi_4-\chi_3-\chi_2-\chi_1\chi_5}(-p_4,-p_3,-p_2,-p_1)$$

CP-symmetry:

$$\mathcal{H}^{\text{hard}}_{\chi_1\chi_2\chi_3\chi_4\chi_5} = -\chi_1\chi_2\chi_3\chi_4\overline{\mathcal{H}}^{\text{hard}}_{-\chi_1-\chi_2-\chi_3-\chi_4-\chi_5}$$

Polarized Bhabha scattering: Monte Carlo generator

We created Monte Carlo generator of unweighted events for the polarized Bhabha scattering $e^+e^- \to e^+e^-$ with complete one-loop EW corrections.

This generator uses adaptive algorithm mFOAM [CPC 177:441-458,2007] which is a part of ROOT [https://root.cern.ch] program.

It will be interfaced to PYTHIA8 [CPC 178 (2008) 852–867] program.

Setup for tuned comparison

We performed a tuned comparison of our results for polarized Born and hard Bremsstrahlung with the results WHIZARD [Eur.Phys.J.C71 (2011) 1742] program. The contributions of soft and virtual parts were compared with the results of aŮITALC [CPC 174 (2006) 71-82] program

Input parameters:

$$\alpha^{-1}(0) = 137.03599976,$$
$$M_W = 80.4514958 \text{ GeV}, \quad M_Z = 91.1876 \text{ GeV}, \quad \Gamma_Z = 2.49977 \text{ GeV},$$
$$m_e = 0.51099907 \text{ MeV}, \quad m_\mu = 0.105658389 \text{ GeV}, \quad m_\tau = 1.77705 \text{ GeV},$$
$$m_d = 0.083 \text{ GeV}, \quad m_s = 0.215 \text{ GeV}, \quad m_b = 4.7 \text{ GeV},$$
$$m_u = 0.062 \text{ GeV}, \quad m_c = 1.5 \text{ GeV}, \quad m_t = 173.8 \text{ GeV}.$$

Cuts:

$$|\cos\theta| < 0.9,$$
$$E_\gamma > 1 \text{ GeV} \quad \text{(for comparison of hard Bremsstrahlung).}$$

$e^+e^- \to e^+e^-$: WHIZARD vs MCSANCee (Born)

P_{e^-}, P_{e^+}	0, 0	-0.8, 0	-0.8, -0.6	-0.8, 0.6
$\sqrt{s} = 250$ GeV				
$\sigma^{\text{Born}}_{e^+e^-}$, pb	56.677(1)	57.774(1)	56.272(1)	59.276(1)
$\sigma^{\text{Born}}_{e^+e^-}$, pb	56.677(1)	57.775(1)	56.272(1)	59.275(1)
$\sqrt{s} = 500$ GeV				
$\sigma^{\text{Born}}_{e^+e^-}$, pb	14.379(1)	15.030(1)	12.706(1)	17.355(1)
$\sigma^{\text{Born}}_{e^+e^-}$, pb	14.379(1)	15.030(1)	12.706(1)	17.354(1)
$\sqrt{s} = 1000$ GeV				
$\sigma^{\text{Born}}_{e^+e^-}$, pb	3.6792(1)	3.9057(1)	3.0358(1)	4.7756(1)
$\sigma^{\text{Born}}_{e^+e^-}$, pb	3.6792(1)	3.9057(1)	3.0358(1)	4.7755(1)

$e^+e^- \to e^+e^-$: WHIZARD vs MCSANCee (hard)

P_{e^-}, P_{e^+}	0, 0	-0.8, 0	-0.8, -0.6	-0.8, 0.6
$\sqrt{s} = 250$ GeV				
$\sigma^{\text{hard}}_{e^+e^-}$, pb	48.62(1)	49.58(1)	48.74(1)	50.40(1)
$\sigma^{\text{hard}}_{e^+e^-}$, pb	48.65(1)	49.56(1)	48.78(1)	50.44(1)
$\sqrt{s} = 500$ GeV				
$\sigma^{\text{hard}}_{e^+e^-}$, pb	15.14(1)	15.81(1)	13.54(1)	18.07(1)
$\sigma^{\text{hard}}_{e^+e^-}$, pb	15.12(1)	15.79(1)	13.55(1)	18.11(2)
$\sqrt{s} = 1000$ GeV				
$\sigma^{\text{hard}}_{e^+e^-}$, pb	4.693(1)	4.976(1)	3.912(1)	6.041(1)
$\sigma^{\text{hard}}_{e^+e^-}$, pb	4.694(1)	4.975(1)	3.913(1)	6.043(1)

$e^+e^- \to e^+e^-$: aITALC vs MCSANCee (virtual+soft)

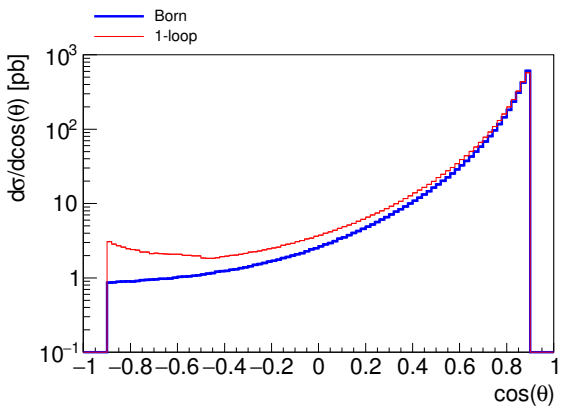
$\cos\theta$	$\sigma^{\text{Born}}_{e^+e^-}$, pb	$\sigma^{\text{Born+virt+soft}}_{e^+e^-}$, pb
-0.9	$2.16999 \cdot 10^{-1}$ $2.16999 \cdot 10^{-1}$	$1.93445 \cdot 10^{-1}$ $1.93445 \cdot 10^{-1}$
-0.5	$2.61360 \cdot 10^{-1}$ $2.61360 \cdot 10^{-1}$	$2.38707 \cdot 10^{-1}$ $2.38707 \cdot 10^{-1}$
0	$5.98142 \cdot 10^{-1}$ $5.98142 \cdot 10^{-1}$	$5.46677 \cdot 10^{-1}$ $5.46677 \cdot 10^{-1}$
$+0.5$	$4.21273 \cdot 10^{0}$ $4.21273 \cdot 10^{0}$	$3.81301 \cdot 10^{0}$ $3.81301 \cdot 10^{0}$
$+0.9$	$1.89160 \cdot 10^{2}$ $1.89160 \cdot 10^{2}$	$1.72928 \cdot 10^{2}$ $1.72928 \cdot 10^{2}$
$+0.99$	$2.06556 \cdot 10^{4}$ $2.06555 \cdot 10^{4}$	$1.90607 \cdot 10^{4}$ $1.90607 \cdot 10^{4}$
$+0.999$	$2.08236 \cdot 10^{6}$ $2.08236 \cdot 10^{6}$	$1.91624 \cdot 10^{6}$ $1.91624 \cdot 10^{6}$
$+0.9999$	$2.08429 \cdot 10^{8}$ $2.08429 \cdot 10^{8}$	$1.91402 \cdot 10^{8}$ $1.91402 \cdot 10^{8}$

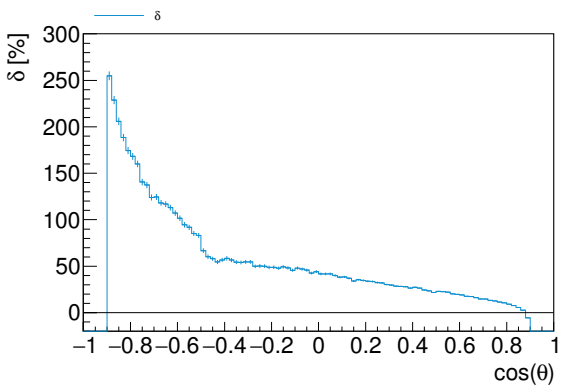
$e^+e^- \to e^+e^-$: Born vs 1-loop

P_{e^-}, P_{e^+}	0, 0	-0.8, 0	-0.8, -0.6	-0.8, 0.6
$\sqrt{s} = 250$ GeV				
$\sigma^{\text{Born}}_{e^+e^-}$, pb	56.6763(1)	57.7738(1)	56.2725(4)	59.2753(5)
$\sigma^{\text{1-loop}}_{e^+e^-}$, pb	61.731(6)	62.587(6)	61.878(6)	63.287(7)
δ, %	8.92(1)	8.33(1)	9.96(1)	6.77(1)
$\sqrt{s} = 500$ GeV				
$\sigma^{\text{Born}}_{e^+e^-}$, pb	14.3789(1)	15.0305(1)	12.7061(1)	17.3550(2)
$\sigma^{\text{1-loop}}_{e^+e^-}$, pb	15.465(2)	15.870(2)	13.861(1)	17.884(2)
δ, %	7.56(1)	5.59(1)	9.09(1)	3.05(1)
$\sqrt{s} = 1000$ GeV				
$\sigma^{\text{Born}}_{e^+e^-}$, pb	3.67921(1)	3.90568(1)	3.03577(3)	4.77562(5)
$\sigma^{\text{1-loop}}_{e^+e^-}$, pb	3.8637(4)	3.9445(4)	3.2332(3)	4.6542(7)
δ, %	5.02(1)	0.99(1)	6.50(1)	-2.54(1)

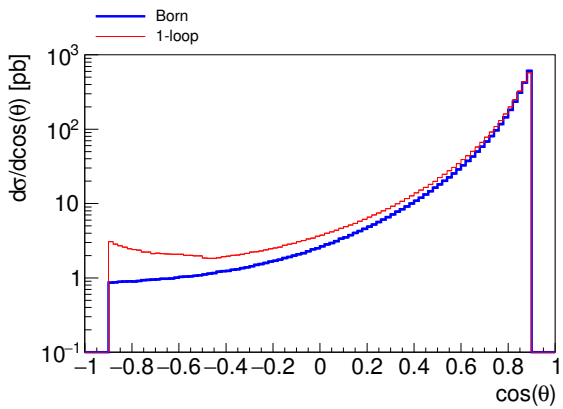
$e^+e^- \to e^+e^-$: **distributions in** $\cos\theta$

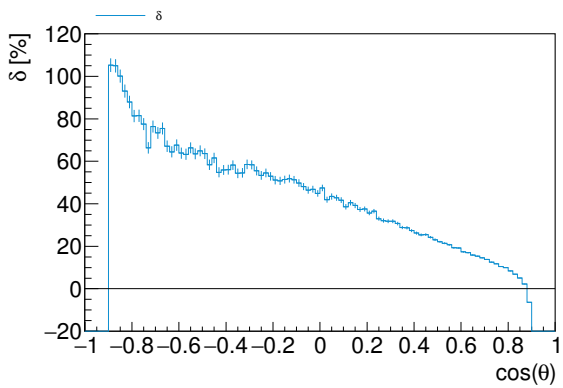
$\sqrt{s} = 250$ GeV



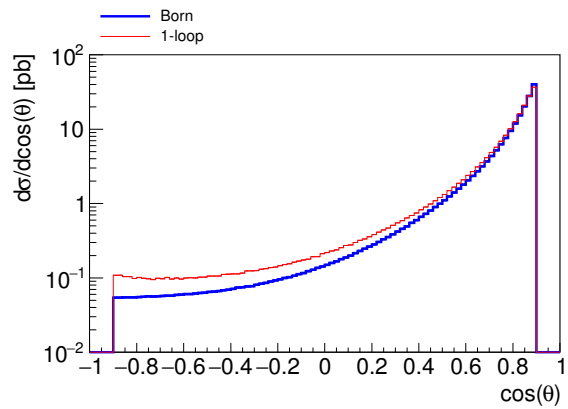


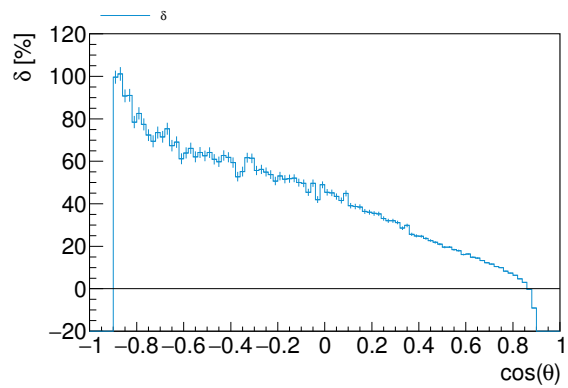
$\sqrt{s} = 500$ GeV





$\sqrt{s} = 1000$ GeV

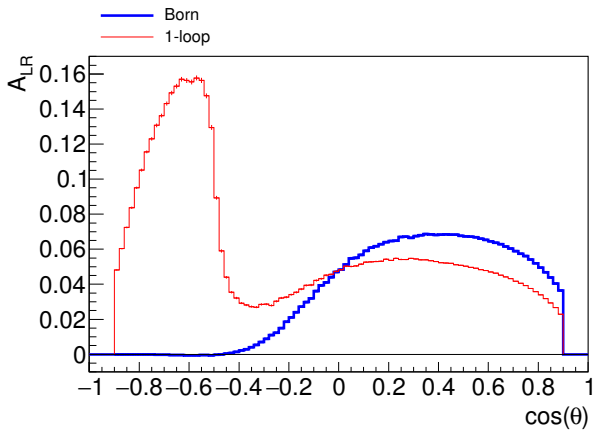




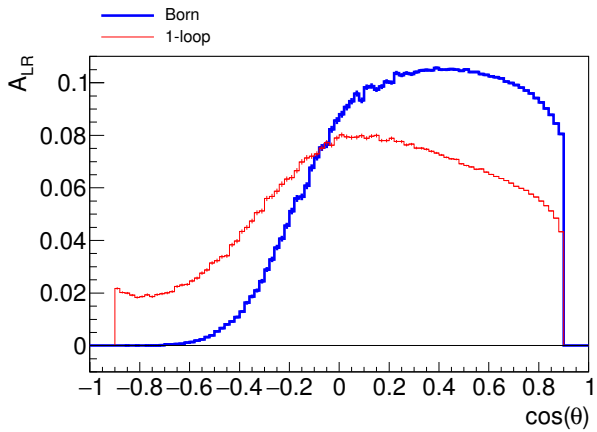
$e^+e^- \to e^+e^-$: A_{LR} distributions in $\cos\theta$

$$A_{LR} = \frac{\sigma_{LR} - \sigma_{RL}}{\sigma_{LR} + \sigma_{RL}}$$

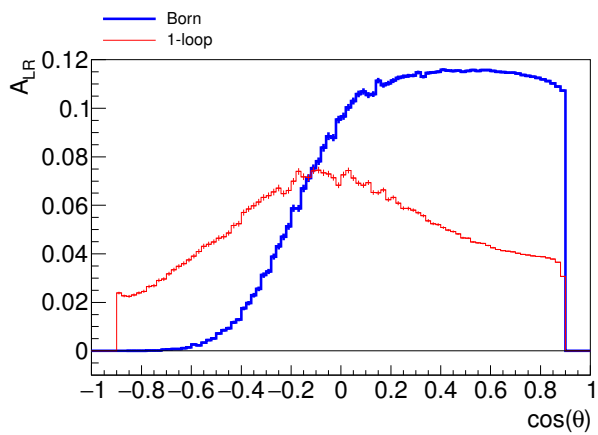
$\sqrt{s} = 250$ GeV



$\sqrt{s} = 500$ GeV



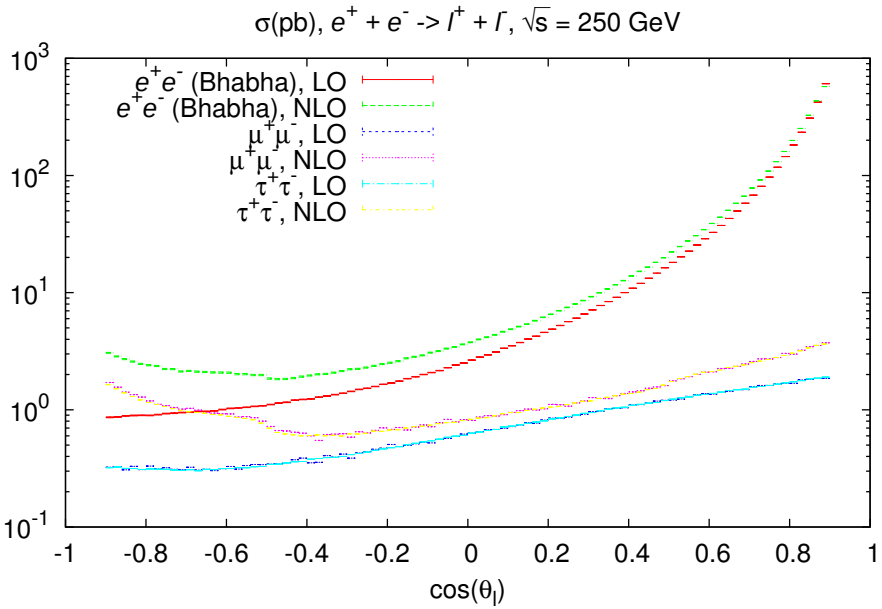
$\sqrt{s} = 1000$ GeV

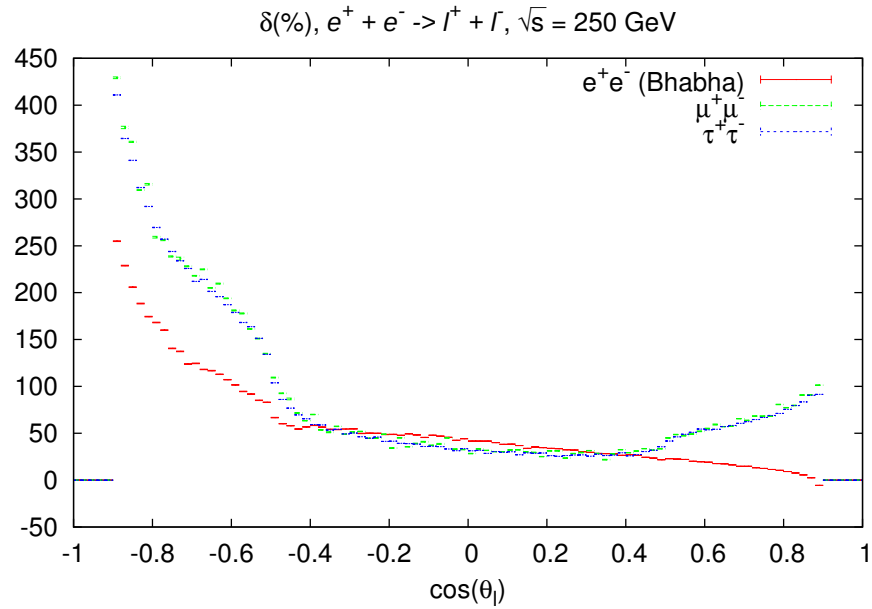


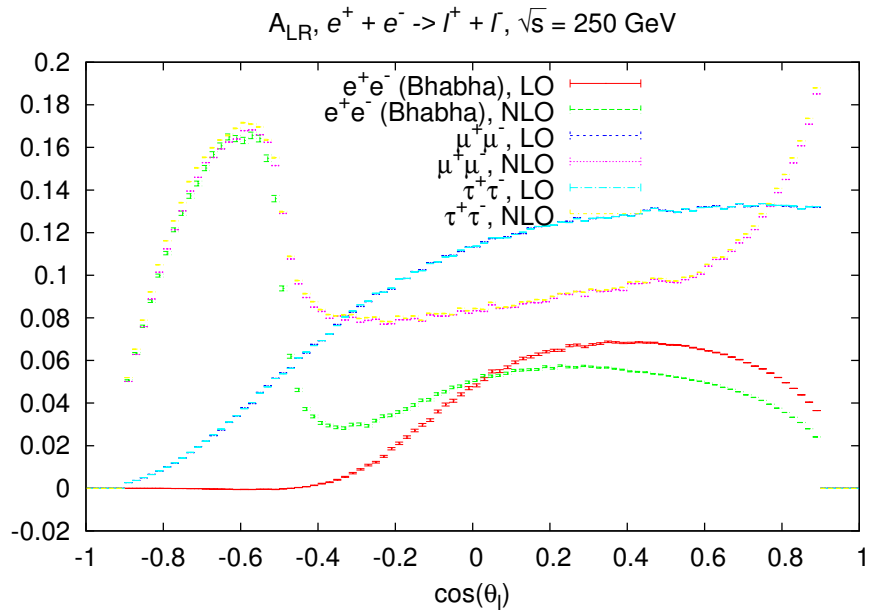
$e^+e^- \to \ell^+\ell^-$: Born vs 1-loop (preliminary)

P_{e^-}, P_{e^+}	0, 0	-0.8, 0	-0.8, -0.6	-0.8, 0.6
$\sqrt{s} = 250$ GeV				
$e^- + e^+ \to e^- + e^+$ (Bhabha)				
$\sigma^{\text{Born}}_{e^+e^-}$, pb	56.6763(1)	57.7738(1)	56.2725(4)	59.2753(5)
$\sigma^{\text{1-loop}}_{e^+e^-}$, pb	61.731(6)	62.587(6)	61.878(6)	63.287(7)
δ, %	8.92(1)	8.33(1)	9.96(1)	6.77(1)
$e^- + e^+ \to \mu^- + \mu^+$				
$\sigma^{\text{Born}}_{e^+e^-}$, pb	1.4174(1)	1.5462(1)	0.7690(2)	2.3231(2)
$\sigma^{\text{1-loop}}_{e^+e^-}$, pb	2.3987(2)	2.616(2)	1.3015(1)	3.929(1)
δ, %	69.22(2)	69.13(1)	69.18(2)	69.12(1)
$e^- + e^+ \to \tau^- + \tau^+$				
$\sigma^{\text{Born}}_{e^+e^-}$, pb	1.4174(1)	1.5461(1)	0.7692(1)	2.3230(1)
$\sigma^{\text{1-loop}}_{e^+e^-}$, pb	2.3609(1)	2.5773(1)	1.2817(1)	3.8728(2)
δ, %	66.56(1)	66.69(1)	66.62(1)	66.71(1)

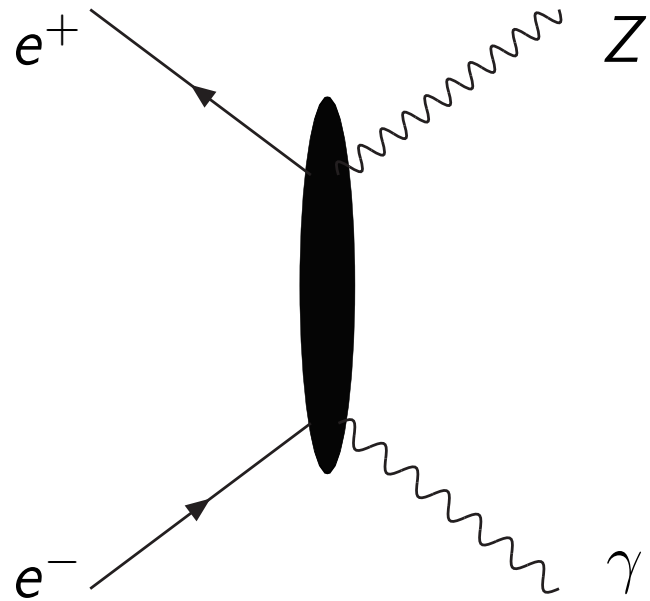
$e^+e^- \to \ell^+\ell^-$: distributions in $\cos\theta_\ell$ (preliminary)







$e^+e^- \to Z\gamma$



The expressions fo HA and the results for unpolarized case were published in Eur.Phys.J.C54:187-197,2008

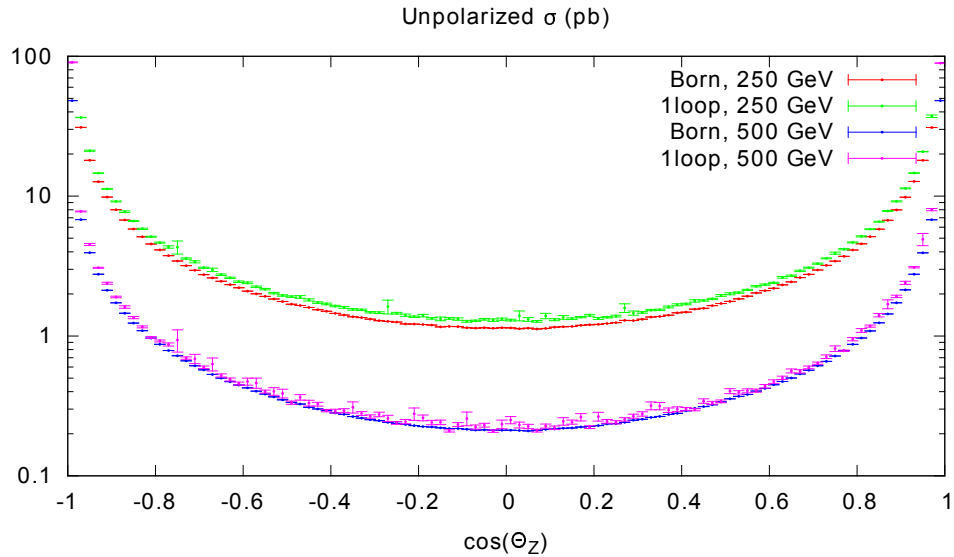
Here we present the preliminary results for 1-loop corrections taking into account the effect of polarization of e^+e^- beams.

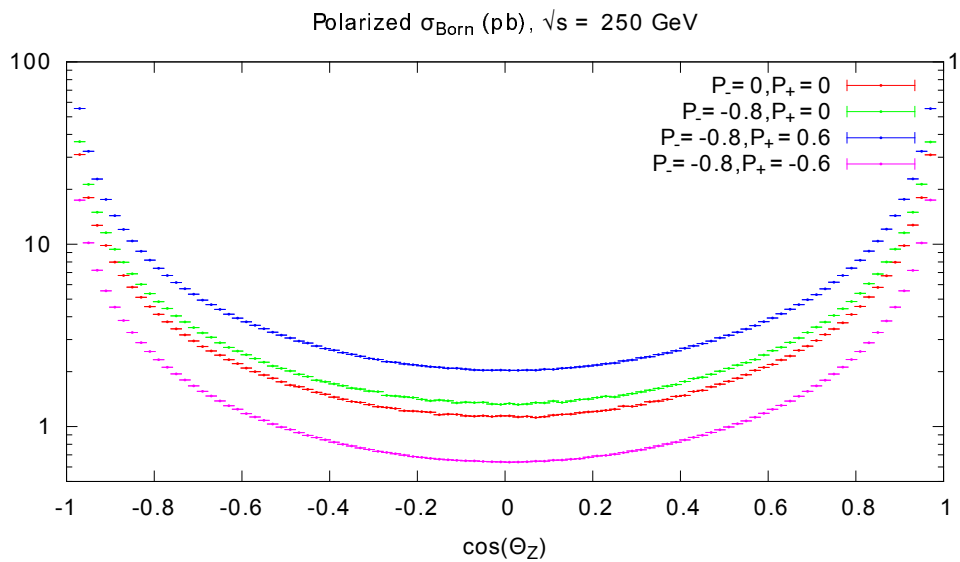
$e^+e^- \to Z\gamma$: Born vs 1-loop (preliminary)

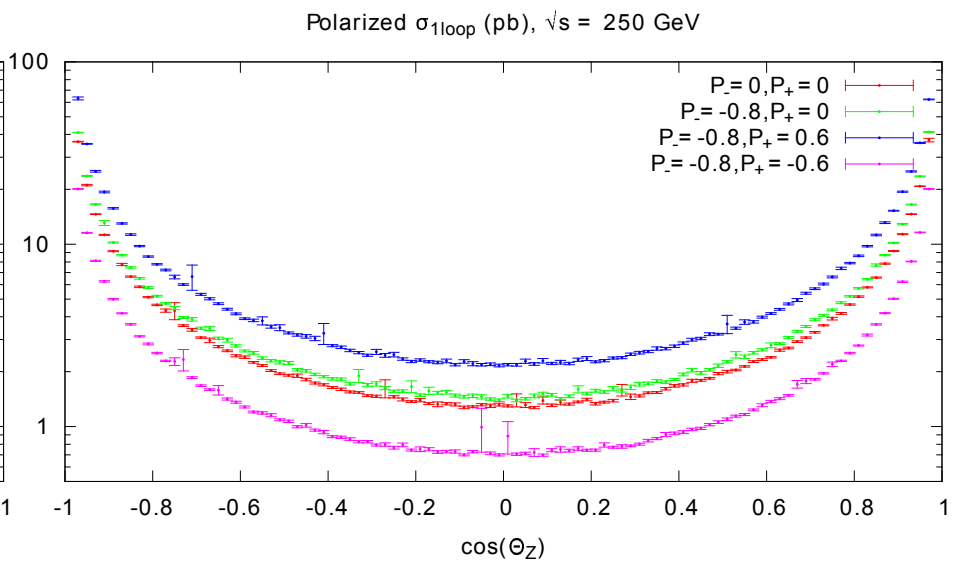
$\sqrt{s}$	250	500	1000
$\sigma_{Z\gamma}^{\text{Born}}$, pb	15.7038(6)	3.3858(3)	0.81958(3)
$\sigma_{Z\gamma}^{\text{1-loop}}$, pb	24.37(1)	5.23(6)	1.237(3)
δ, %	55.20(6)	54.4(2)	50.9(4)

P_{e^-}, P_{e^+}	0, 0	-0.8, 0	-0.8, 0.6	-0.8, -0.6
$\sqrt{s} = 250$ GeV				
$\sigma_{Z\gamma}^{\text{Born}}$, pb	15.7038(6)	18.520(4)	28.174(2)	8.870(3)
$\sigma_{Z\gamma}^{\text{1-loop}}$, pb	24.37(1)	28.00(1)	42.13(2)	13.53(1)
δ, %	55.20(6)	51.11(8)	49.55(7)	52.57(9)

$e^+e^- \to Z\gamma$: distributions in $\cos\theta_Z$ (preliminary)







Conclusions and plans

- We are building a powerful team of experimentalists and theoreticians that will prepare the physics research at the next generation e^+e^- collider
- We created the `FORTRAN` modules for polarized Bhabha scattering with complete one-loop EW corrections. Based on these modules Monte Carlo generator [Phys.Rev. D98 (2018) no.1, 013001] of unweighted events was created with possibility to produce events in Les Houches Event format
- Preliminary results for polarized $e^+e^- \to \mu^+\mu^-$, $e^+e^- \to \tau^+\tau^-$ (the paper in preparation) and $e^+e^- \to Z\gamma$ were presented. More processes will be included in MC generator in the future