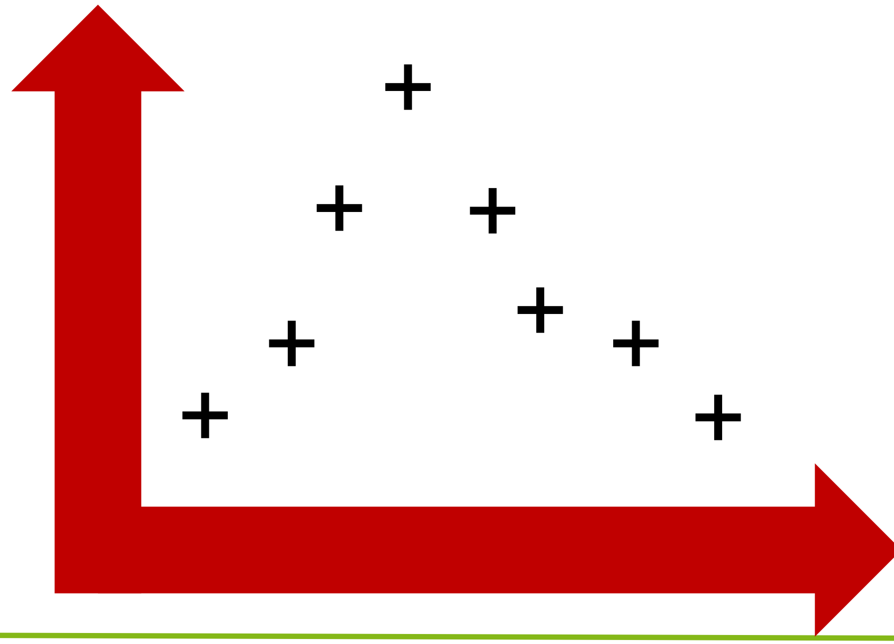
technische universität dortmund

lehrstuhl physik e5



Deconvolution in IceCube

Tim Ruhe

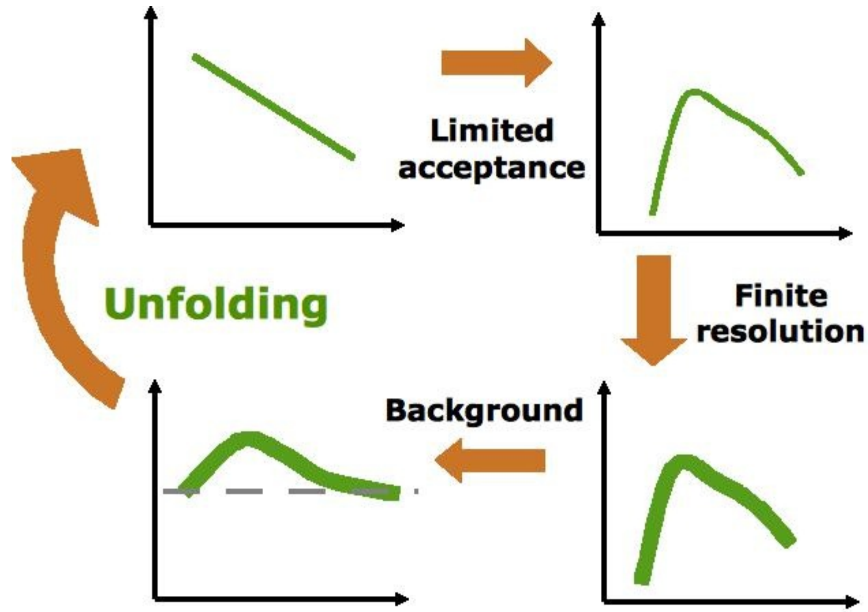
October 4th 2018

Outline

- Challenges in Deconvolution
- Selected Deconvolution Algorithms
- Some results on atmospheric neutrino spectra
- Dortmund Spectrum Estimation Algorithm

Deconvolution in a Nutshell

$$\underbrace{\frac{dN_\mu}{dE_\mu}}_{\text{Muon energy spectrum}} = \int_{E_\mu}^{\infty} \underbrace{\left(\frac{dN_\nu}{dE_\nu}\right)}_{\text{Neutrino energy spectrum}} \underbrace{\left(\frac{dP(E_\nu)}{dE_\mu}\right)}_{\text{Physics of neutrino interaction}} dE_\nu$$



- Production of charged lepton in neutrino interaction is governed by stochastical processes
- Additional smearing, due to several detector effects

Mathematically: Fredholm integral equation of the first kind:

$$g(y) = \int_{E_{min}}^{E_{max}} A(E, y) f(E) dE$$

Deconvolution in a Nutshell

Physicists generally happy with a discrete result

$$g(y) = \int_{E_{min}}^{E_{max}} A(E,y) f(E) dE$$

Integral equation

$A(E,y)$ generally not known analytically.

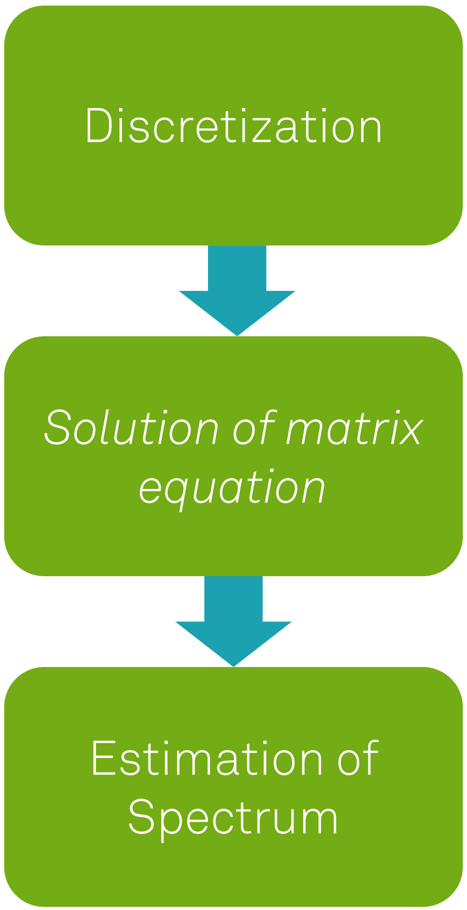
Discretizing operation

$$\vec{g}(y) = A(E,y)\vec{f}(E)dE$$

Matrix equation

Replaced my probability matrix, obtained from Monte Carlo simulations.

Deconvolution in a Nutshell



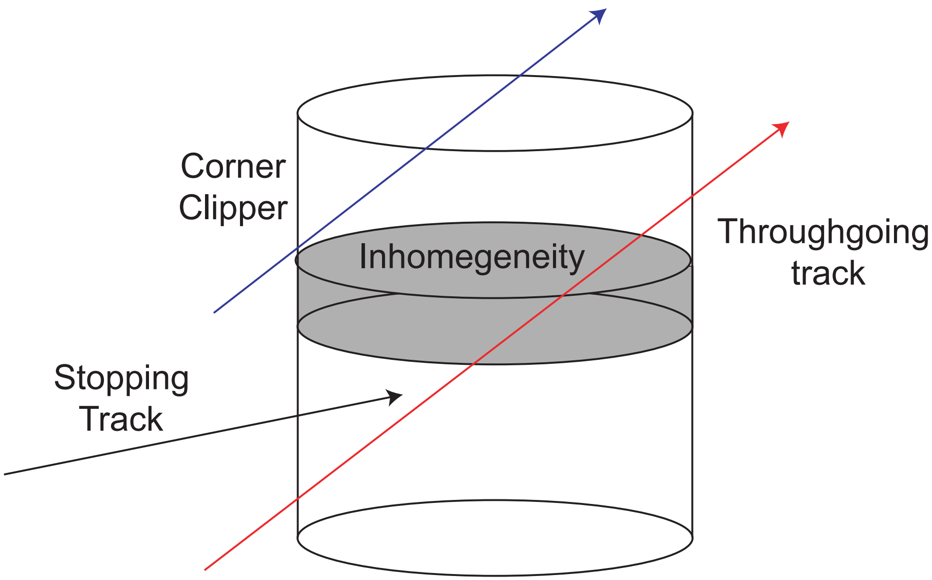
Direct inversion of A not feasible or leads to oscillating solutions.

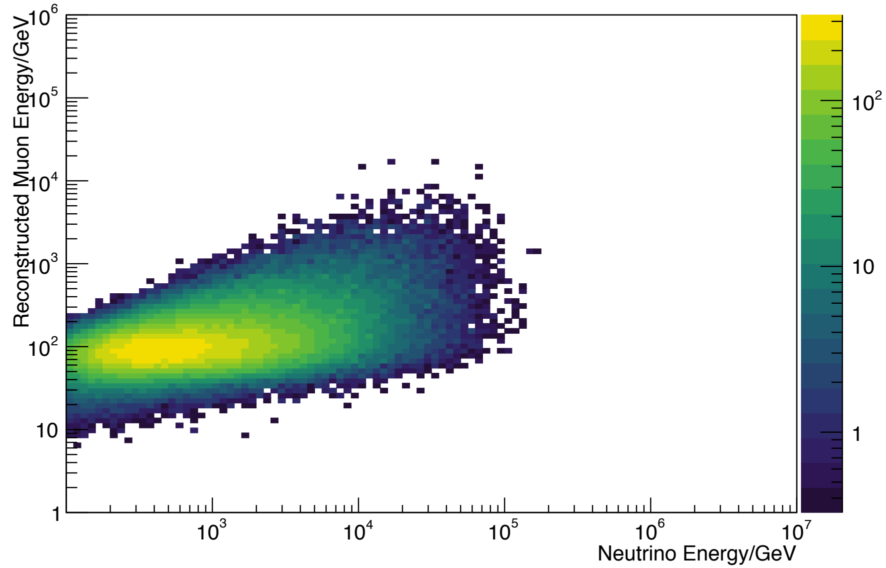
Regularisation is required,

- Assumes smoothness of f
- Adds certain amount of bias
- In most cases small second derivative

Additional Challenges

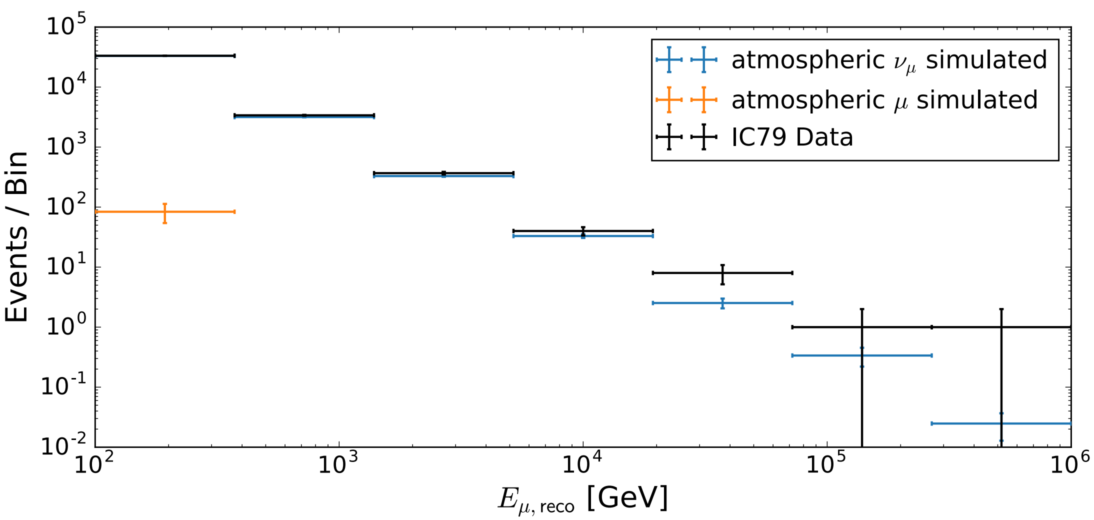
- Amount of smearing
- Small statistics at high energies
- Muon energy only an estimate
- Estimation of systematic uncertainties





Neutrinos with exactly the same energy can create very different signatures in the detector.

Before Unfoling: Purity Requirements



Purity generally above 99%.

Even more important: Make sure there are no muons in bins with small statistics.

Selected Deconvolution Algorithms

- Forward Folding
- Iterative Bayesian Unfolding D'Agostini, 2010
- Dortmund Spectrum Estimation Algorithm (DSEA), machine learning-based Ruhe et al., 2016
- Singular Value Decomposition (SVD)
- TRUEE

Based on RUN-Algorithm, uses B-splines.

Minimizes a log-likelihood.

Milke et al., 2013

Obtain inverse matrix A^{-1} via factorization of the form $A = U \cdot S \cdot V^T$

Challenge: Small eigenvalues in S enhance statistically insignifant contributions. Spectrum is distorted.

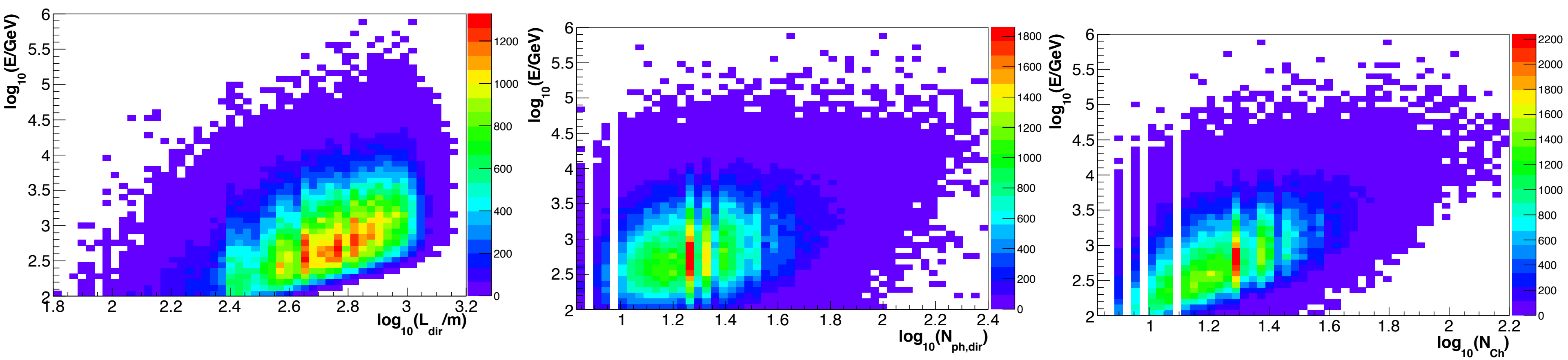
Höcker and Kartvelishvili, 1996

IC-59 NuMu Unfolding

Variables utilized:

- Length of track
- Number of unscattered photons
- Number responding DOMs

Not the beste energy estimators individually, but the combination of variables with the best unfolding result.



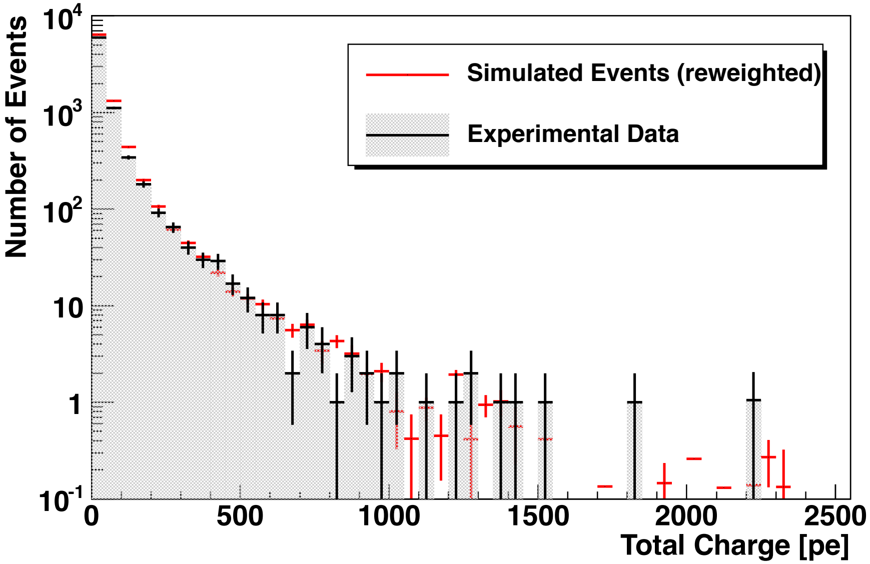
technische universität dortmund

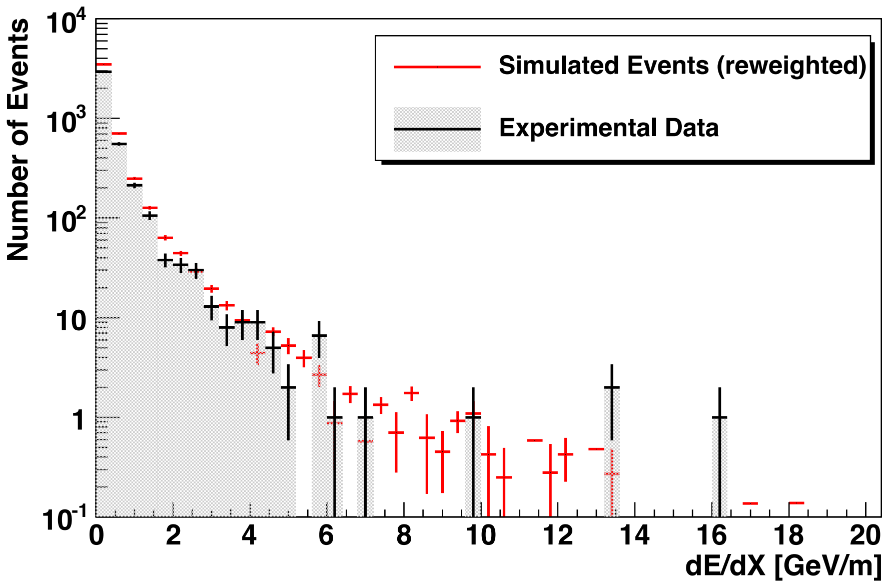
lehrstuhl physik e5

Sanity Checks in TRUEE

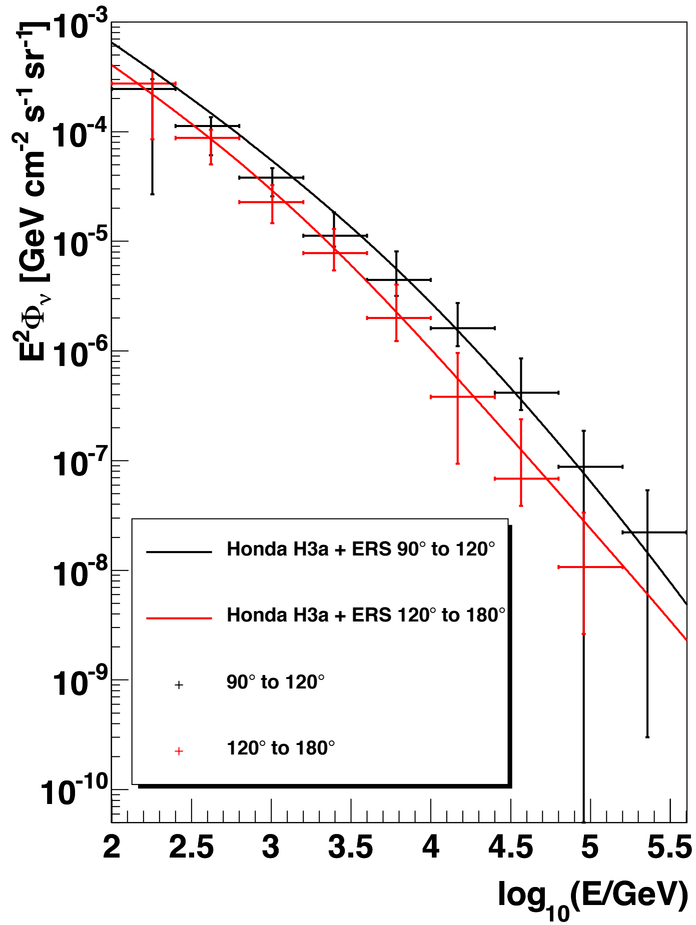
Simulations are reweighted according to obtained spectrunm and compared to experimental data.

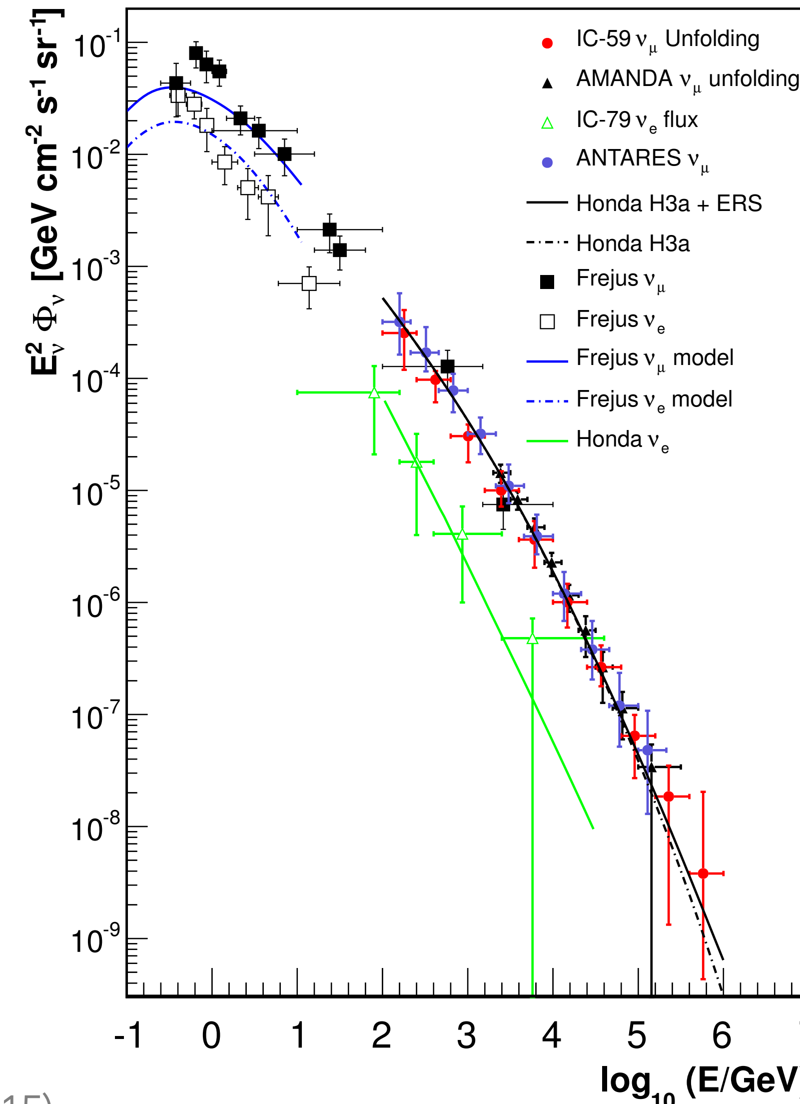
Especially useful for energy dependent quantities not used in the unfolding.





IC-59 NuMu Unfolding



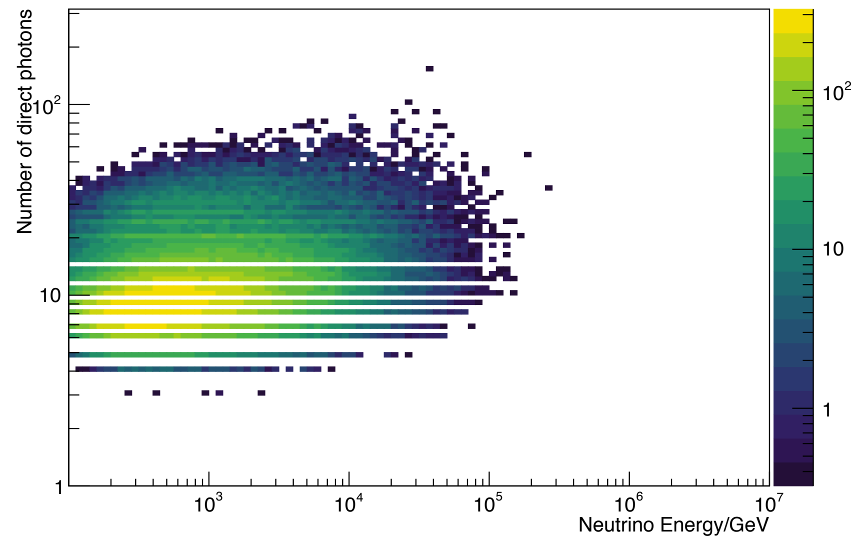


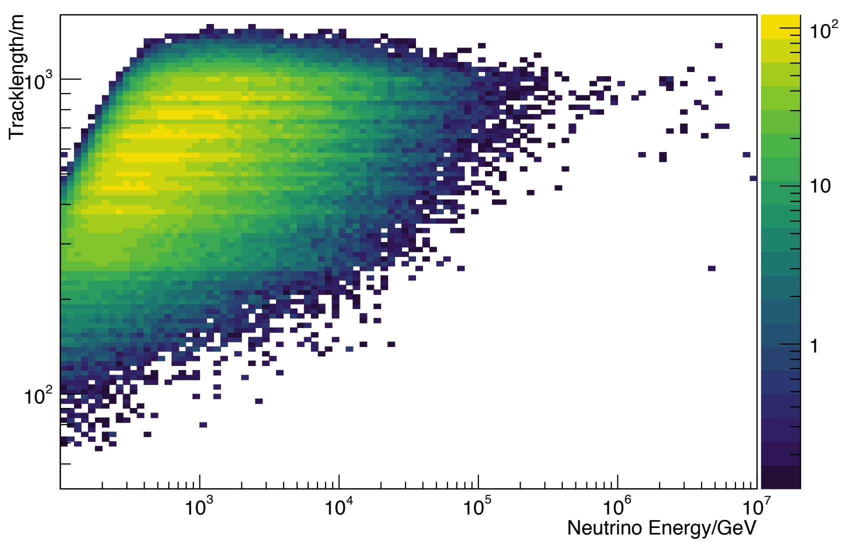
EPJC 75, 116 (2015)

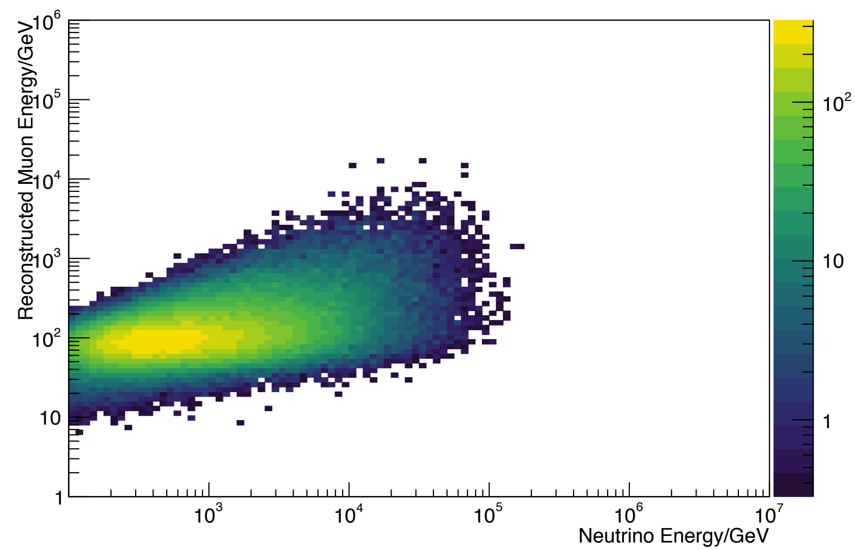
IC-79 NuMu Unfolding (TRUEE)

Three input parameters (Tracklength, Number of unscattered photons, Reconstructed Muon energy).

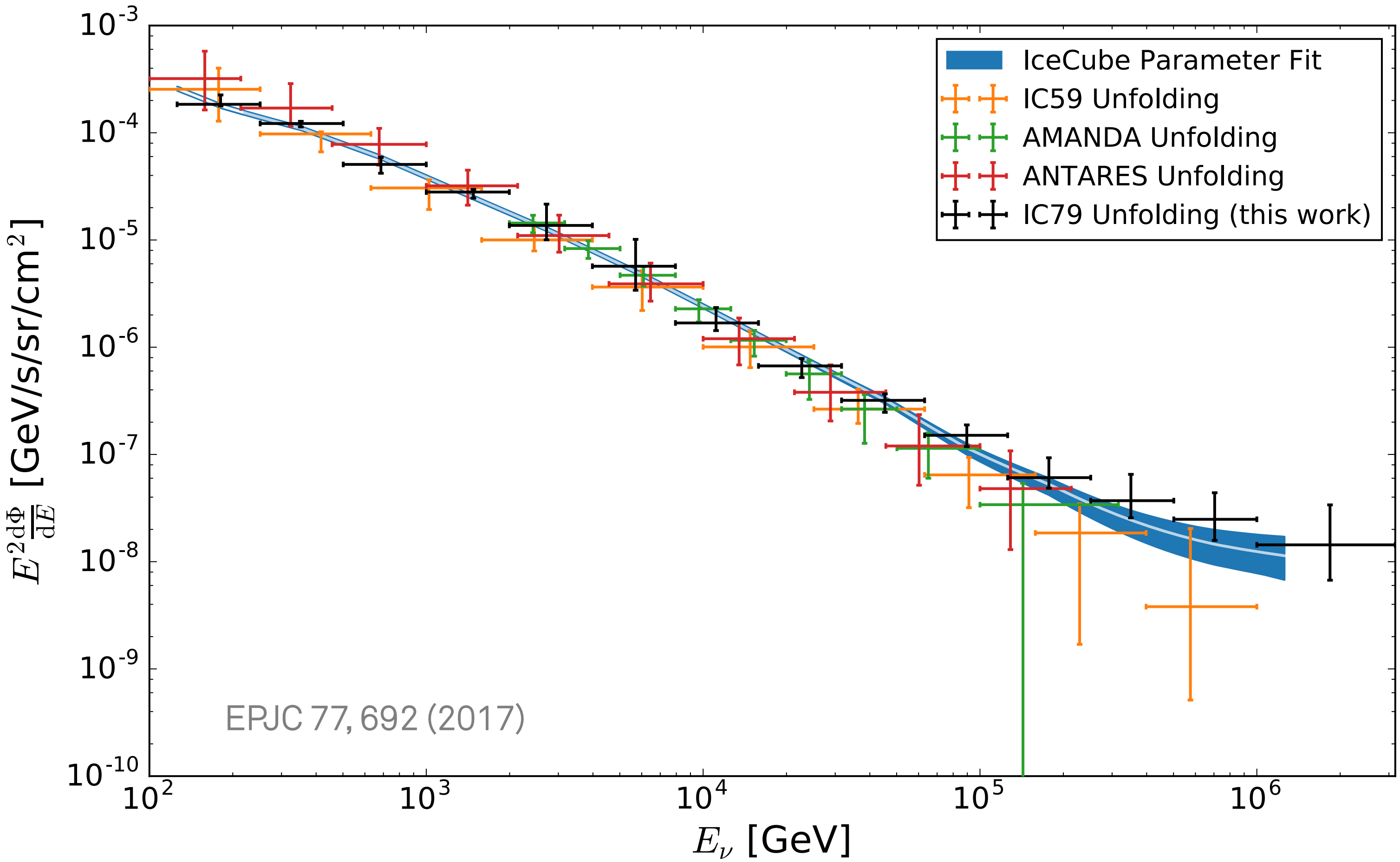
Better estimators available, but above combination gave the best unfolding result.







IC-79 NuMu Unfolding (TRUEE)



Dortmund Spectrum Estimation Algorithm (DSEA)

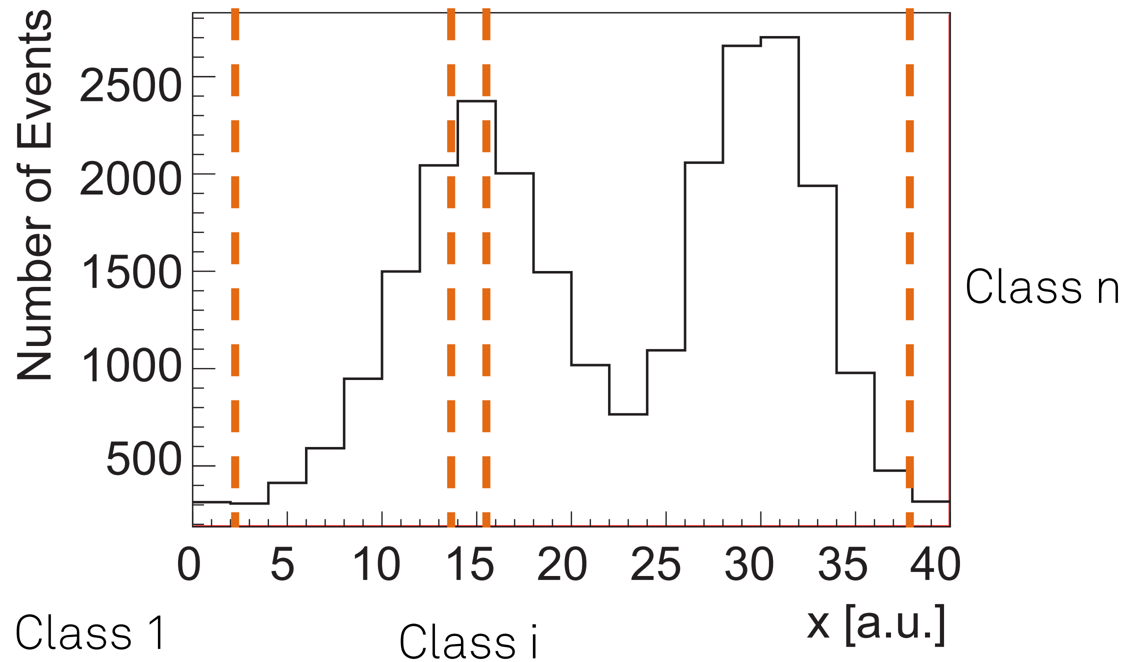
Discretization

↓

Machine Learning Algorithm

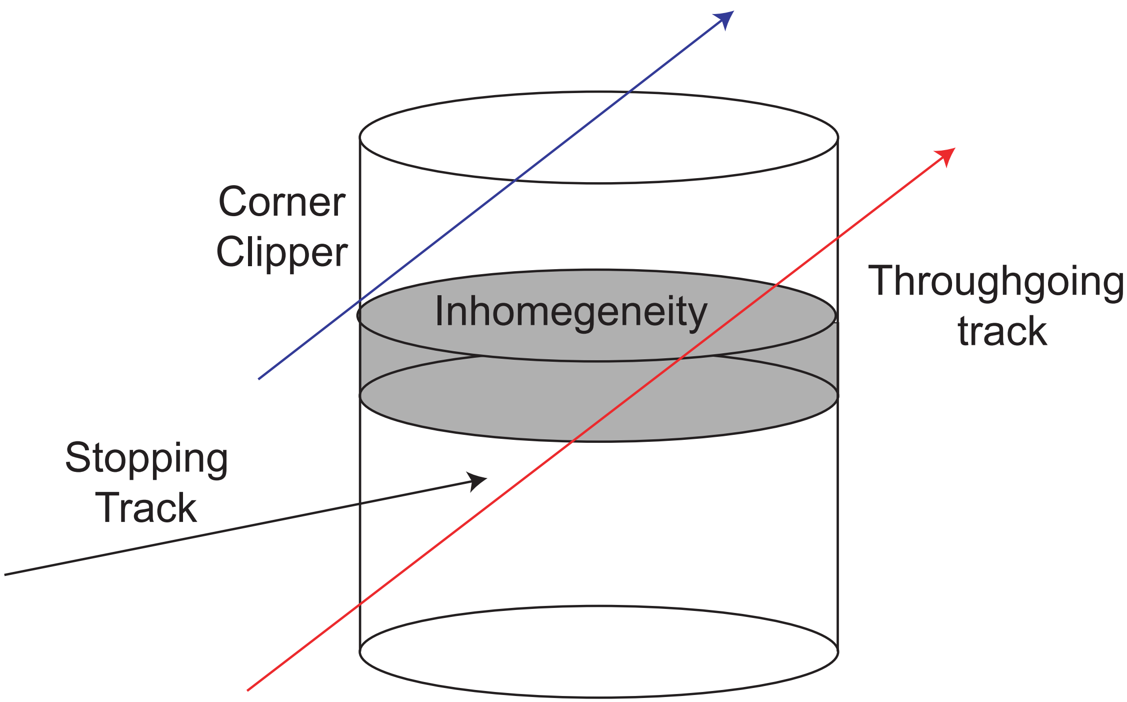
↓

Estimation of Spectrum



Inverse problem is transferred into multinominal classification problem.

Motivation for DSEA

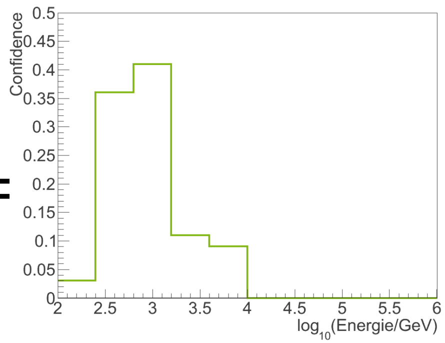


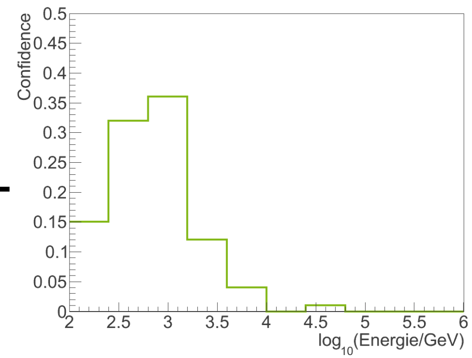
Geometric information is hard to include.

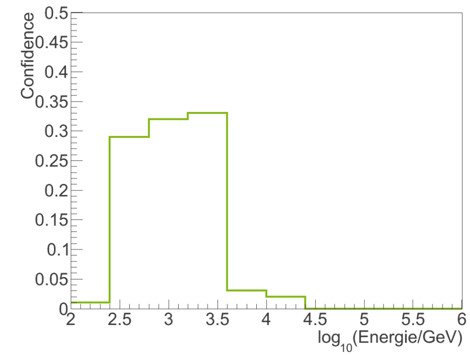
Information on individual events is lost.

Don‘t know which event contributed to which energy bin and how much...

DSEA in greater detail

$\hat{f} =$ 

$+$ 

$+ \ldots +$ 

Iterate:

1. Discretize
2. Train Model
3. Apply Model
4. Reconstruct spectrum
5. Update weights according to unfolding result

Choice of learning algorithm largely arbitrary (and probably somewhat problem dependent).

Some overlap with IBU in case Naive Bayes is used as a learner.

Variable Step Width in DSEA+

Step Width

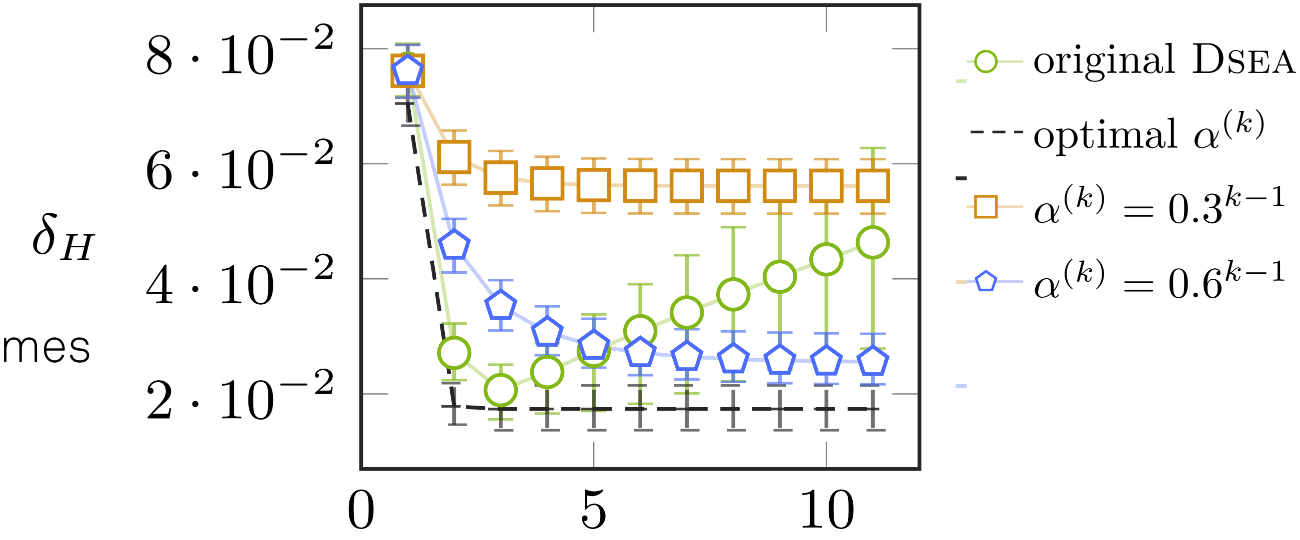
$$p_k = f_k - f_{k-1}$$

Next estimate then becomes

$$f_k = f_{k-1} + \alpha_k p_k$$

Find optimal α via:

$$\alpha = \arg\min_{\alpha \geq 0} l(f_{k-1} + \alpha_k p_k)$$



Get the software:
https://sfb876.tu-dortmund.de/deconvolution/index.html

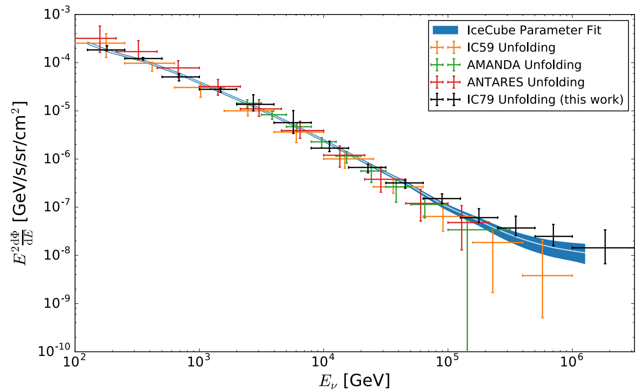
For details see: https://sfb876.tu-dortmund.de/PublicPublicationFiles/bunse_2018a.pdf

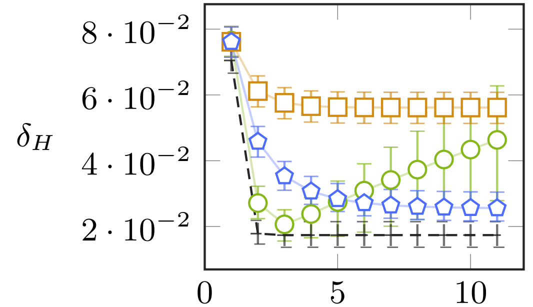
Summary and Outlook

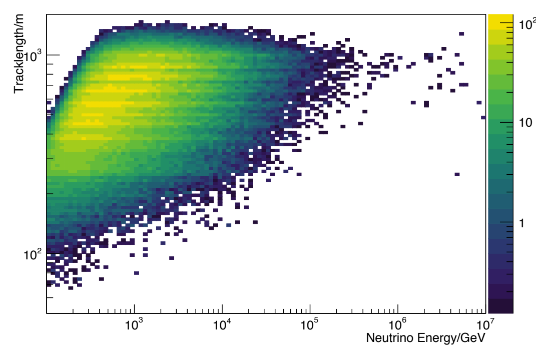
Discretization

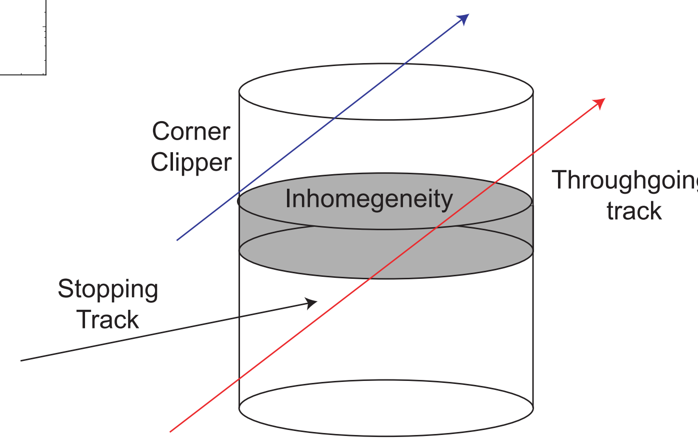
Solution of matrix equation

Estimation of Spectrum







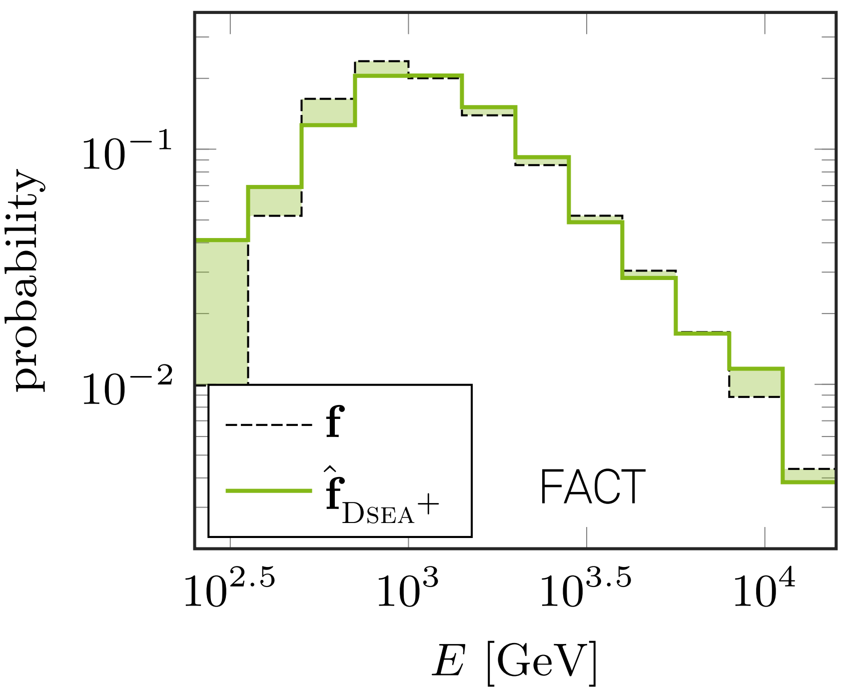


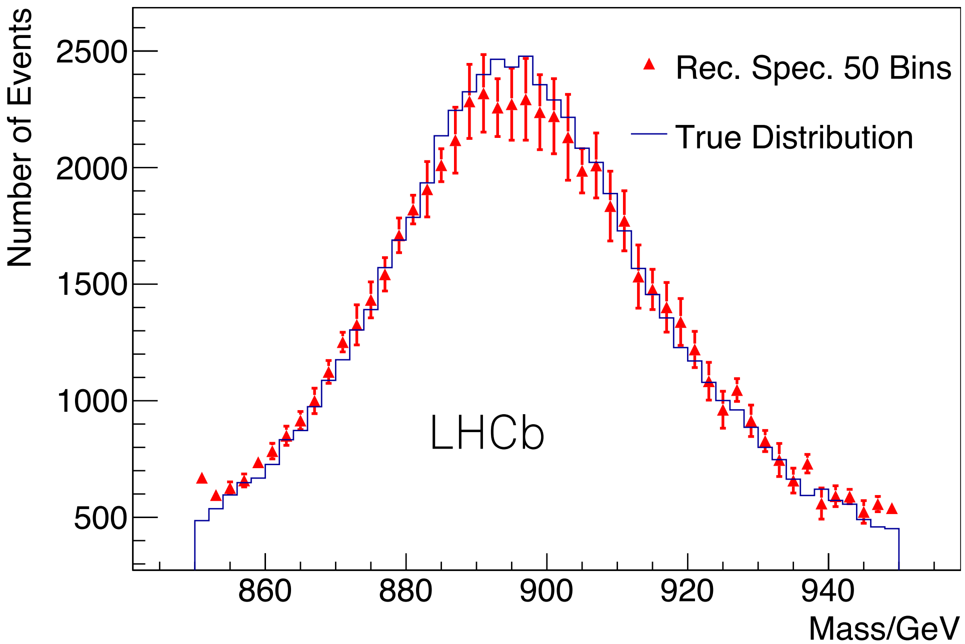
$$g(y) = \int_{E_{min}}^{E_{max}} A(E,y) f(E) dE$$

Hellinger Distance

$$H^2(f,\hat{f}) = \frac{1}{2}\sum_{j=1}^{m}\sqrt{f_j} - \sqrt{\hat{f}_j}$$

Some Preliminary Results





Strategies for choosin the step-size

strategy	parameter	step size
constant factor	$\alpha > 0$	$\alpha^{(k)} = \alpha$
multiplicative decay (slow)	$0 < \eta < 1$	$\alpha^{(k)} = k^{(\eta-1)}$
exponential decay (fast)	$0 < \eta < 1$	$\alpha^{(k)} = \eta^{(k-1)}$

Table 3.1: Some simple strategies determine the step size $\alpha^{(k)}$. η is referred to as the *decay rate*. Each parameter controls the speed of the convergence in DSEA$^+$.

IC-79 NuMu Unfolding