Influence of rotation and acceleration on the QCD phase diagram

Lattice study

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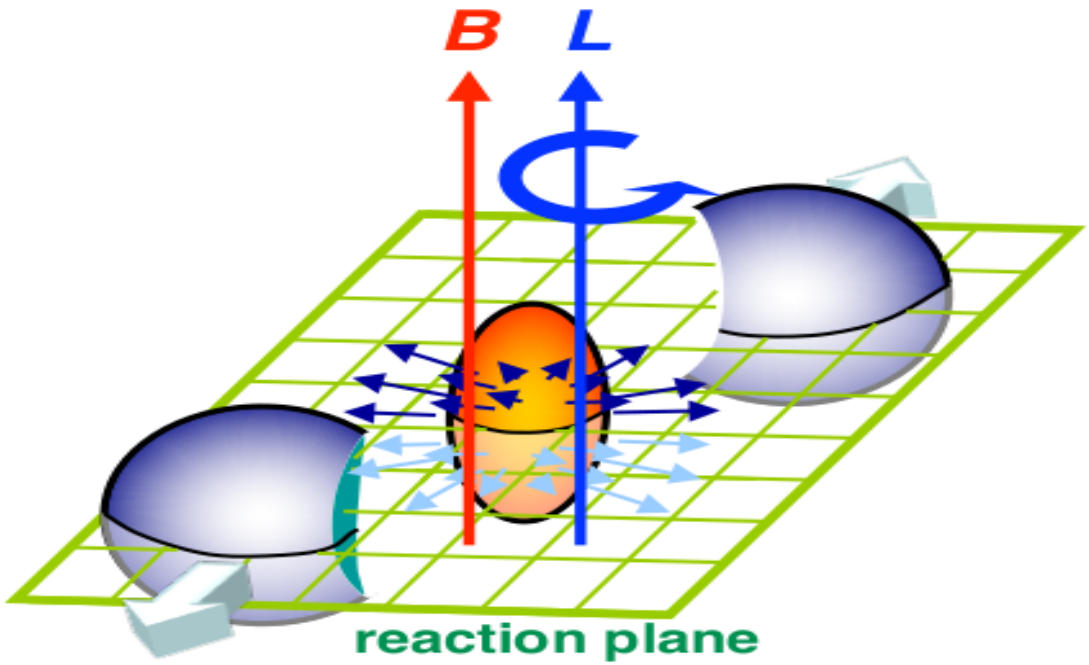
Outline:

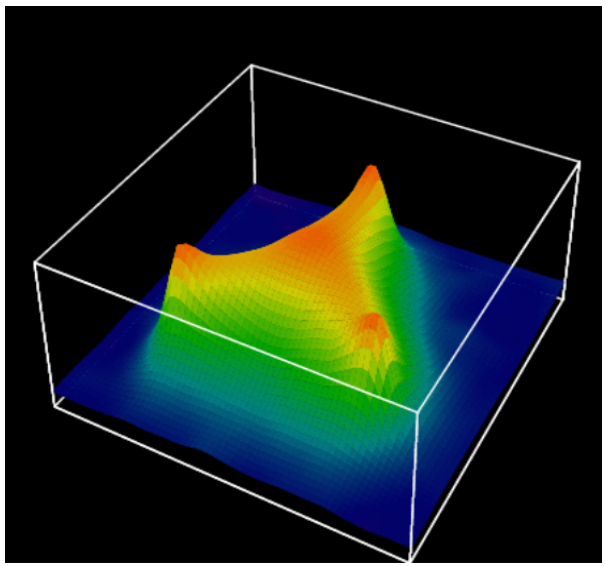
- Introduction
- Spatial confinement/deconfinement transition
- Equation of state and the moment of inertia
- Free energy of rotating static quark
- Spin potential
- Conclusion

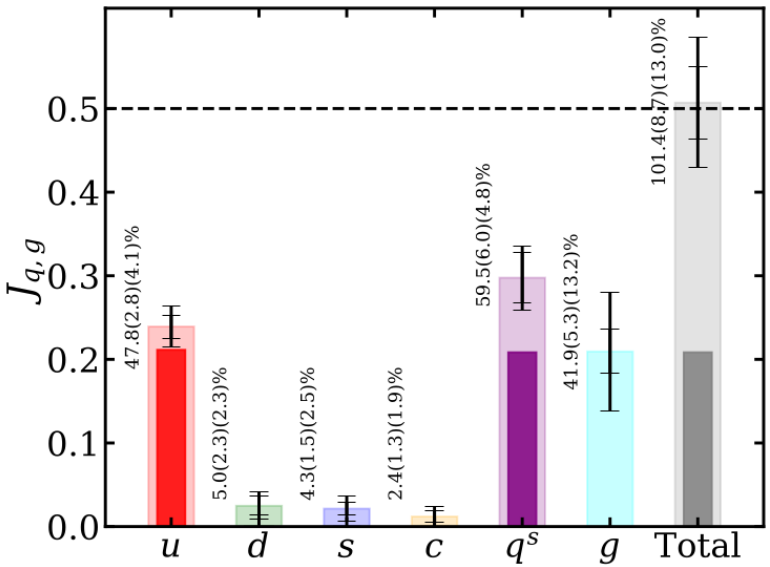
QCD under extreme conditions



- High temperature
- Intense magnetic field
- Large baryon density
- Relativistic rotation
  $\Omega \sim 10$ MeV $(v \sim c$ at $r \sim 10 - 20$ fm, $\sim 10^{22}\, s^{-1})$ [STAR'17]
- Huge acceleration
  - $\alpha \sim 1$ GeV [D. Kharzeev'2005]
  - $\alpha \sim 0.1 - 1$ GeV [G.Prokhorov'2025]

Experience from the Proton Spin Puzzle





- Jaffe-Monohar sum rules

$$\tfrac{1}{2}\Delta\Sigma + L_q + \Delta G + L_g = \tfrac{\hbar}{2}$$

- Gluon spin contribution is $\Delta G \simeq 50\%$
- Proton spin results from many-body processes with large contribution of the gluon spin
- How rotation is redistributed over different degrees of freedom?

Acceleration in HIC

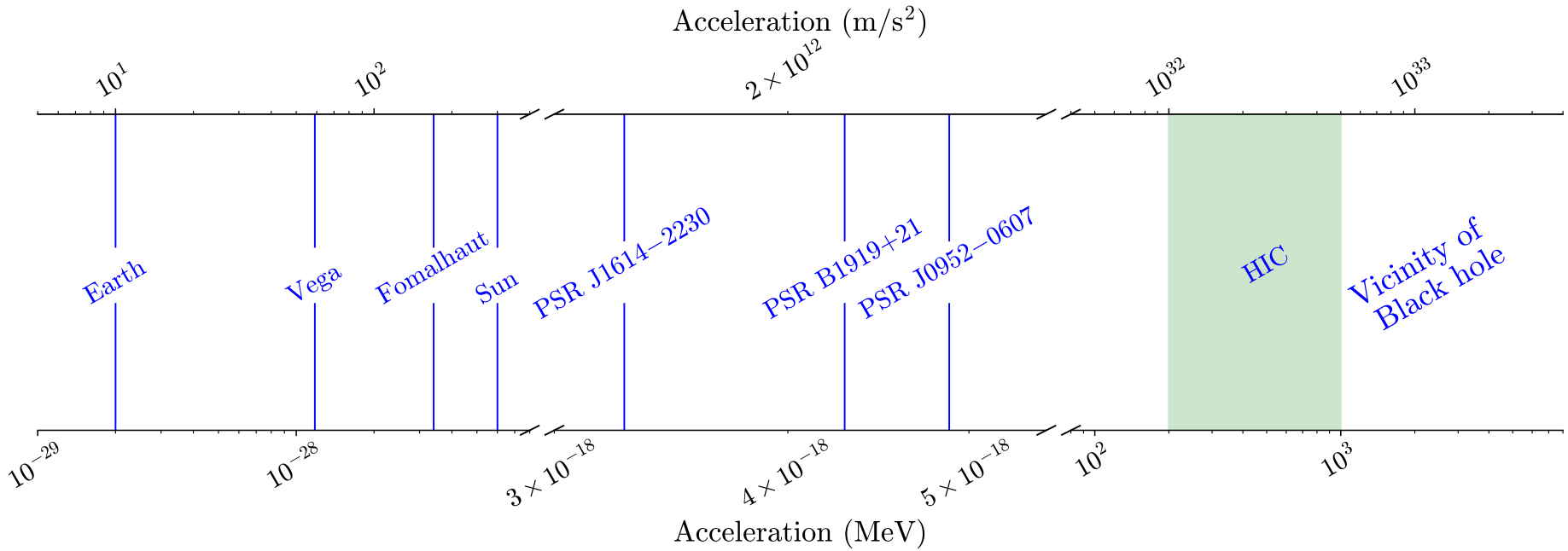


figure from [V. Braguta'2026]

- HIC accelerations can be met in the vicinity of the black hole horizon
- HIC might allow to study quantum effects in gravity?

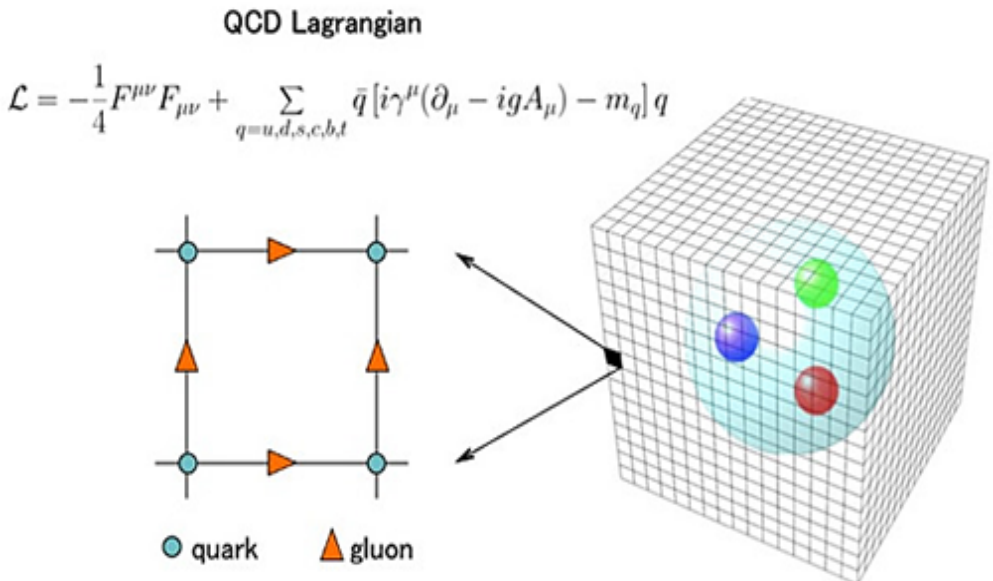
Quantum Chromodynamics(QCD)

- Degrees of freedom
  - Quarks q
  - Gluons A
- The QCD Lagrangian is well known

$$L = -\frac{1}{4}\sum_{a=1}^{8} F_a^{\mu\nu} F_{\mu\nu}^a + \sum_{f=u,d,s,\ldots} \bar{q}_f (i\gamma^\mu \partial_\mu - m) q_f + g \sum_{f=1}^{N_f} \bar{q}_f \gamma^\mu \hat{A}_\mu q_f$$

- Non-linear equations of motion with $g \sim 1$
- The main problem: calculation of observables based on the QCD Lagrangian
- Theoretical approaches contain assumptions with systematic errors which are difficult to estimate

Building lattice QCD



- Introduce regular cubic four dimensional lattice $N_s \times N_s \times N_s \times N_t = N_s^3 \times N_t$
- Lattice spacing–a
- Degrees of freedom
  - **Gluon fields:** 3x3 matrices $U \in SU(3)$, live on links
  - **Quarks fields:** column $q, \bar{q}$, live on sites

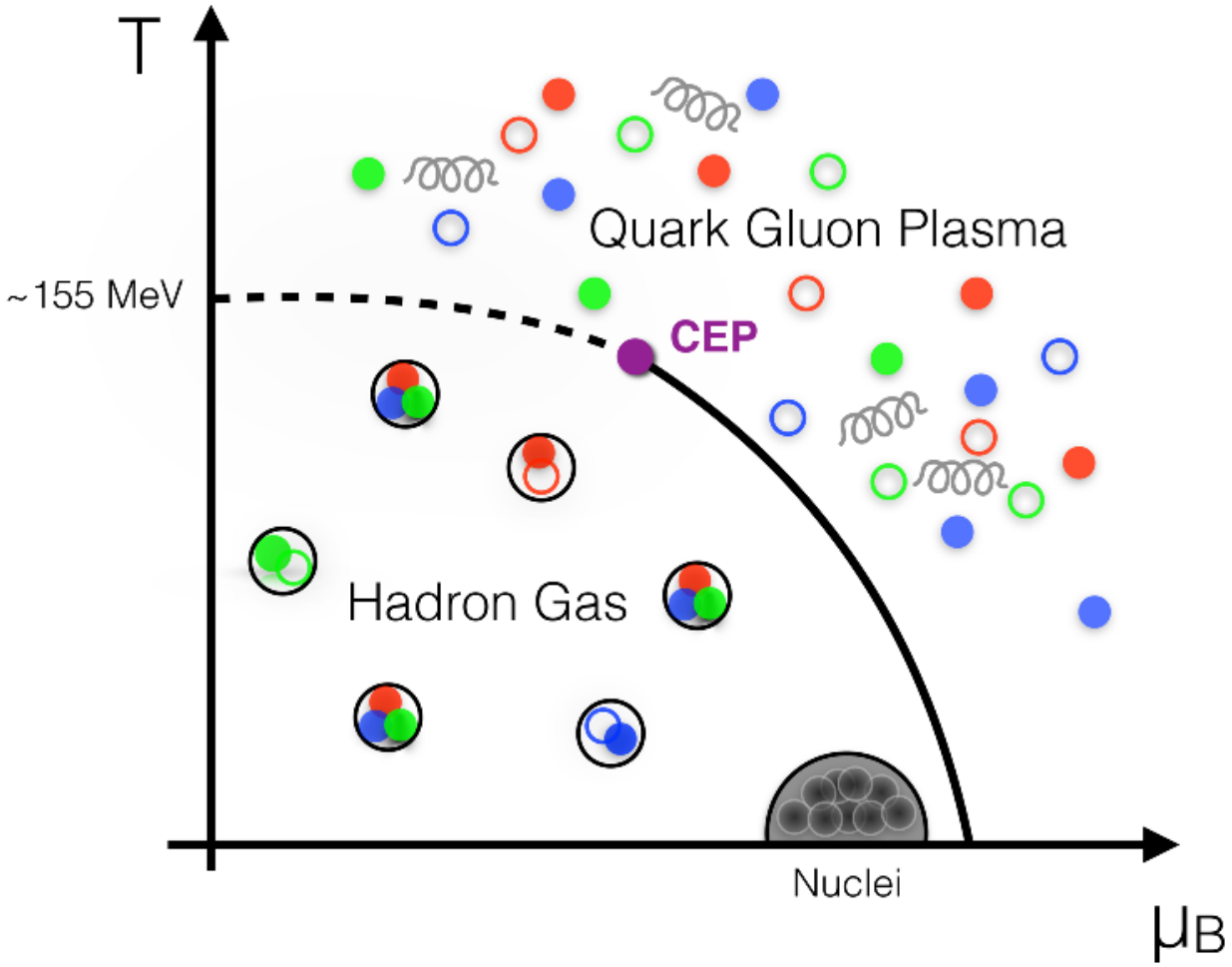
Lattice QCD

- QCD partition function (thermodynamic equilibrium!)

  $$Z = \int DU D\psi D\bar{\psi} \exp\left(-S_G - S_Q\right) = \int DU e^{-S_{eff}(U)}$$

- Hybrid Monte Carlo simulations
  (generation of gluon configurations with weight $\sim e^{-S_{eff}(U)}$)
- In continuum lattice partition function exactly reproduces QCD partition function
  - Gluon contribution: $S_G\big|_{a\to 0} = -\frac{1}{4}\sum_{a=1}^{8} F_a^{\mu\nu} F_{\mu\nu}^a$
  - Quark contribution: $S_Q\big|_{a\to 0} = \bar{q}(\gamma^\mu \partial_\mu + ig\gamma^\mu A_\mu + m)q$
- Carry out continuum extrapolation $a \to 0$
- Uncertainties (discretization and finite volume effects) can be systematically reduced
- The first principles based approach. No assumptions!
- Parameters: coupling constant $g(a)$ and masses of quarks $m_q(a)$

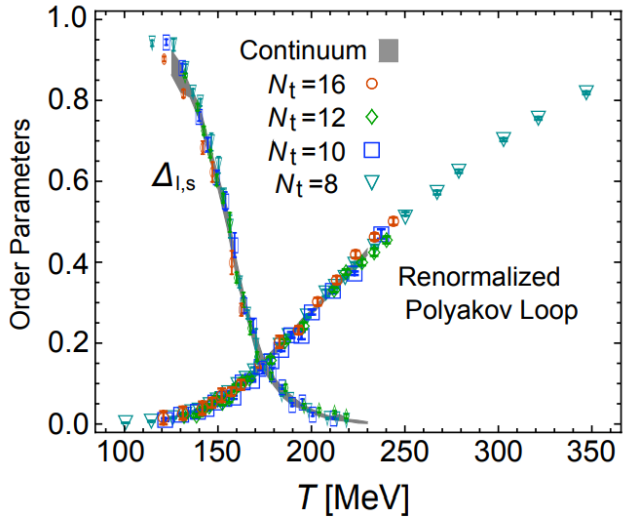
Phase transitions in QCD

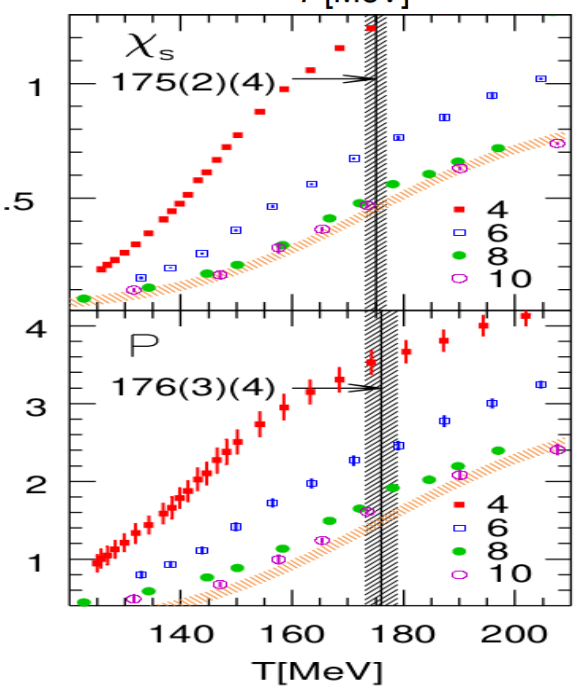


- Confinement/deconfinement transtion
- Chiral symmetry breaking/restoration transition

Confinement/deconfinement transition in QCD

- Order parameter Polyakov line:
  $P(\vec{x}) = \langle Tr P \exp\left(ig \int_0^T dx^4 \hat{A}_4(\vec{x}, x^4)\right)\rangle$
  $P = e^{-F_Q/T}$
  - Confinement: $F_Q = \infty \Rightarrow P = 0$
  - Deconfinement: $F_Q < \infty \Rightarrow P \neq 0$
- Z_3 symmetry of gluodynamics
  but $P \to e^{2\pi k/3i} P$, $k = 0, 1, 2$
- P is similar to magnetization in ferromagnetic
- First order phase transition in SU(3) gluodynamics
- Quarks violate Z_3 symmetry
- Confinement/deconfinement is crossover
  $T_c = (176 \pm 5)$ MeV
  Z. Fodor, Phys.Lett.B 643 (2006)

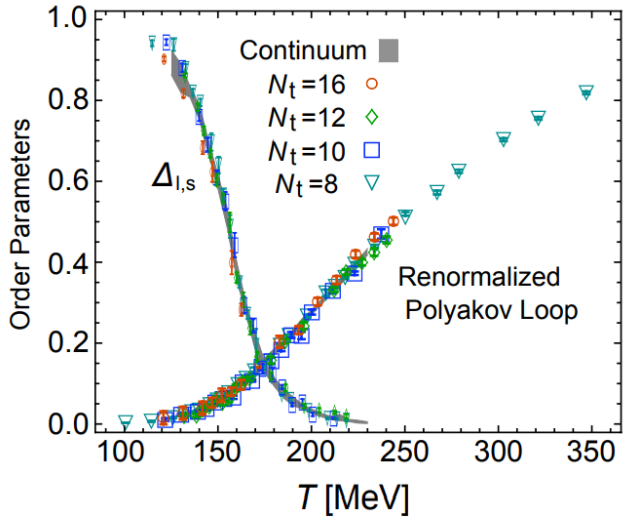


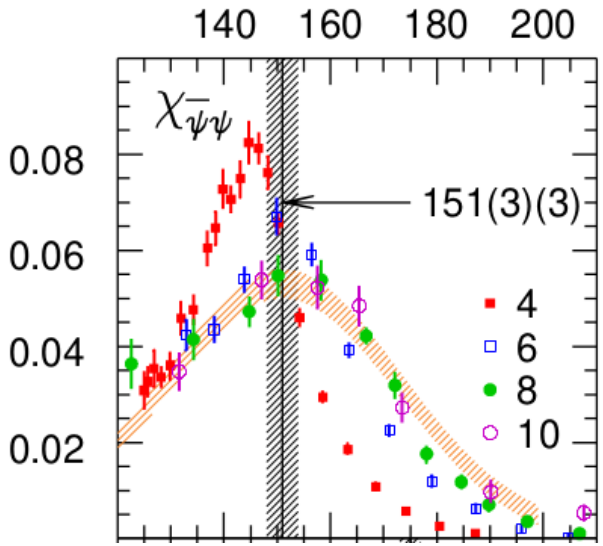


Chiral symmetry breaking in QCD

- Massless QCD Lagrangian
  $\mathcal{L} = \bar{\Psi} i\hat{D}\Psi = \bar{\Psi}_R i\hat{D}\Psi_R + \bar{\Psi}_L i\hat{D}\Psi_L$
- For N_f quarks chiral symmetry is
  $SU_L(N_f) \times SU_R(N_f) \times U_V(1) \times U_A(1)$
- Order parameter: chiral condensate
  $\langle\bar{\Psi}\Psi\rangle = \langle\bar{\Psi}_L\Psi_R\rangle + \langle\bar{\Psi}_R\Psi_L\rangle$
  - CS is broken: $\langle\bar{\Psi}\Psi\rangle \neq 0$, $M_q \neq 0$
  - CS is restored: $\langle\bar{\Psi}\Psi\rangle = 0$, $M_q = 0$
- Dynamical chiral symmetry breaking
  $SU_L(N_f) \times SU_R(N_f) \to SU_V(N_f)$
- In massless QCD the first or the second order phase transition
- But for $m_q \neq 0$ CS symmetry is explicitly broken. $M_q \sim 10 \to 300$ MeV
- CS breaking/restoration is crossover
  $T_c = (151 \pm 4)$ MeV

  Z. Fodor, Phys.Lett.B 643 (2006)





Rotating Gluodynamics/QCD

- The study is conducted in rotating reference frame (non-inertia frame)
- Influence of non-inertia reference frame can be reduced to external gravitational field
- The metric is well known $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$

$$g_{\mu\nu} = \begin{pmatrix} 1 - r^2\Omega^2 & \Omega y & -\Omega x & 0 \\ \Omega y & -1 & 0 & 0 \\ -\Omega x & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

- Aim: investigation of confinement/deconfinement phase transition in rotating gluodynamics/QCD

[V. Braguta'2024, V. Braguta'2025]

Accelerating Gluodynamics/QCD

- HIC is complicated process
- Toy model:
  uniformly accelerated observer $(\vec{a} \parallel \hat{z})$
  homogeneous magnetic field

  $\frac{d^2 z}{dt^2} = \alpha = const$
- The trajectory:

  $t(\tau) = \frac{1}{\alpha} \sinh \alpha\tau,\ z(\tau) = \frac{1}{\alpha} \cosh \alpha\tau$
- Aim: investigation of confinement/deconfinement phase transition in accelerating gluodynamics/QCD [V. Braguta'2026]
- The first lattice study of gluodynamics with acceleration was carried out without gravity using TE law [M. Chernodub'2024]



Comoving frame

- How to paramerise non-inertial frame?
- Rindler space $x^\mu = (\tilde{t}, x, y, \tilde{z})$
  $t = (1 + \alpha\tilde{z}) \sinh \alpha\tilde{t},\ z = (1 + \alpha\tilde{z}) \cosh \alpha\tilde{t}, \quad$ observer at $\tilde{z} = 0$
- Metric: $ds^2 = (1 + \alpha\tilde{z})^2 (d\tilde{t})^2 - dx^2 - dy^2 - d\tilde{z}^2$

$$g_{\mu\nu} = \begin{pmatrix} (1 + a\tilde{z})^2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

- Similar to the Schwarzschild black hole close to the horizon
  Bonus: QCD close to the black hole horizon can be studied
- The horizon: $g_{00}(\tilde{z}_h) = 0 \ \Rightarrow \ \tilde{z}_h = -\frac{1}{\alpha}$

Tolman-Ehrenfest law

- Thermal equilibrium in homogeneous system

$$T(\vec{r}) = const$$

- Thermal equilibrium in external gravity (TE law)

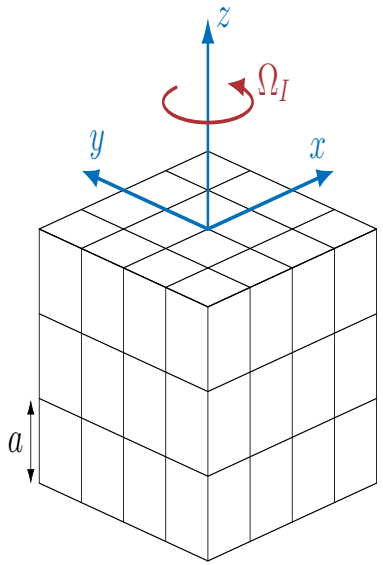
$$\sqrt{g_{00}(\vec{r})}\, T(\vec{r}) = T_0 = const$$

  - Rotation: $T(r) = \frac{T_0}{1-\Omega^2 r^2}$
  - Acceleration: $T(z) = \frac{T_0}{1+\alpha z}$

- We use the designations
  - Rotation: $T = T(r = 0)$
  - Acceleration: $T = T(z = 0)$

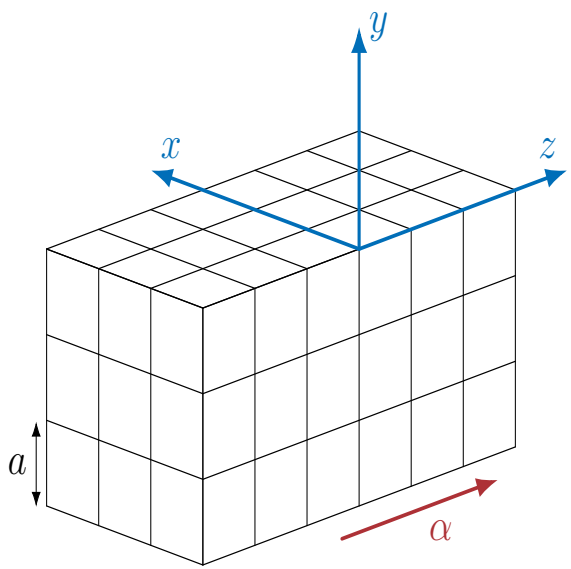
Lattice setup

- Partition function
  $Z = \int DAe^{-S}$,
  $S = \frac{1}{4g^2} \int d^4x\sqrt{g}\ g^{\mu\nu}g^{\alpha\beta}F_{\mu\alpha}F_{\nu\beta}$
- Focus on confinement/deconfinement transition
  Measure local Polyakov loop $L(\vec{r})$
- Open/Dirichle/Periodic BCs
  Don't see significant dependence on tbe BCs



Rotation

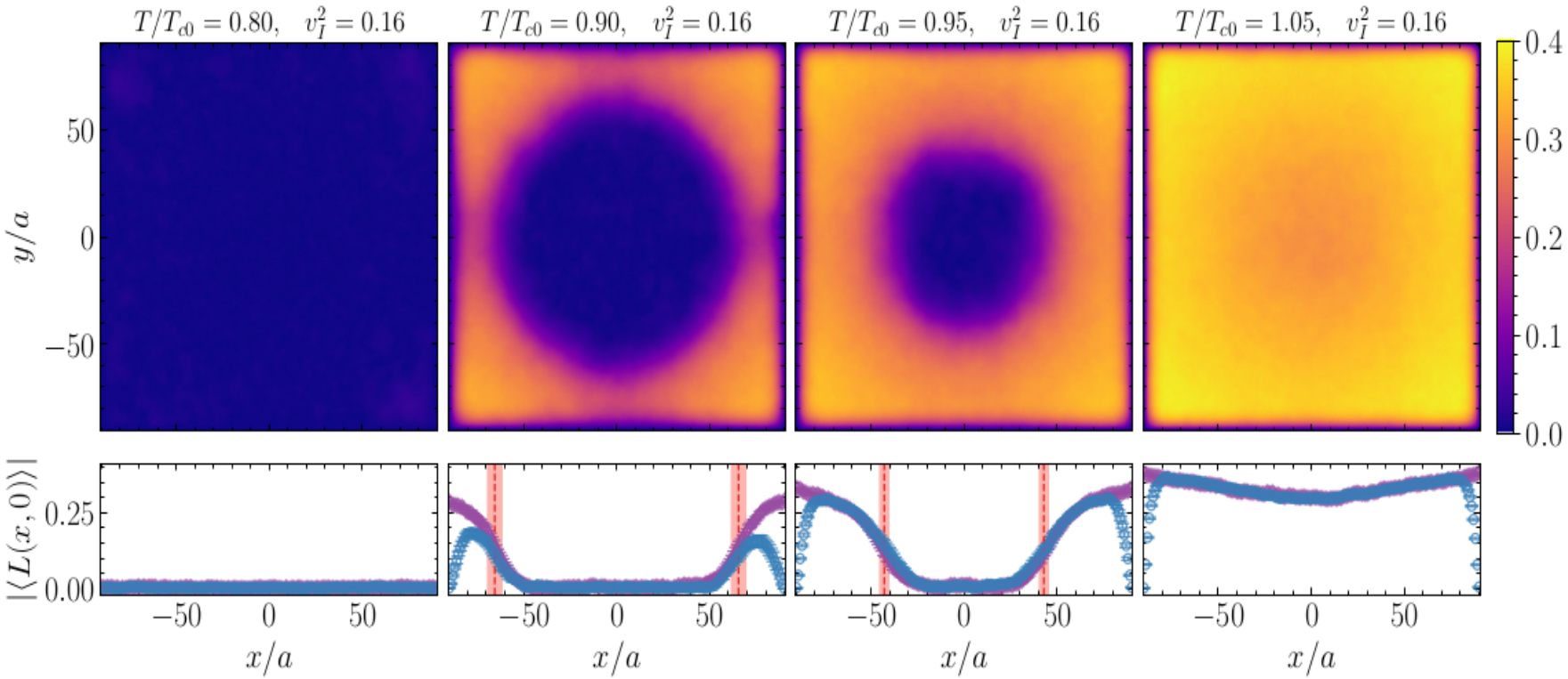
- The sign problem for $\Omega \neq 0$
  simulations are conducted as $\Omega_I = i\Omega$
- Small angular velocities:
  $\Omega \ll \Lambda_{QCD}$ and $\Omega R < 1$
- The infinite volume limit $L_z \to \infty$



Acceleration

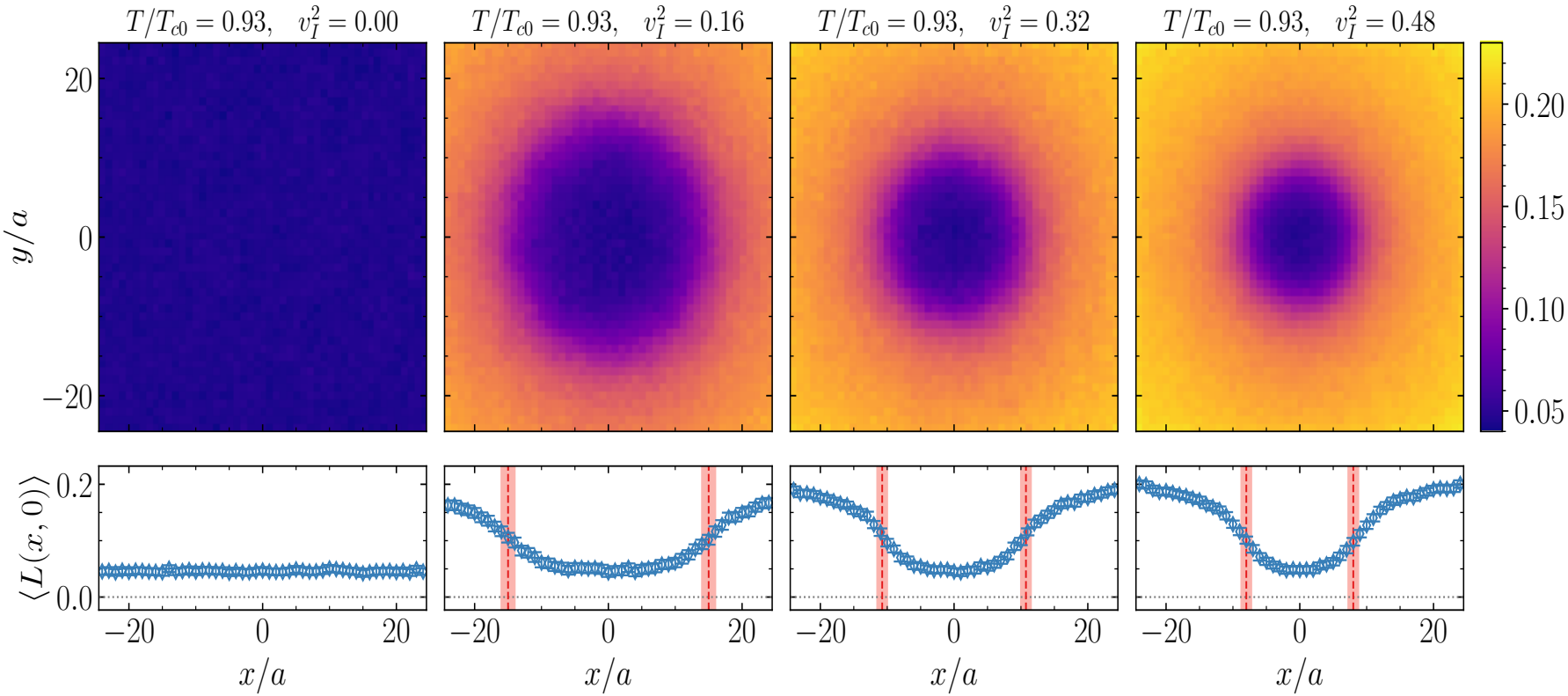
- Free from the sign problem
- Weak accelerations
  $\alpha \ll \Lambda_{QCD}$ and $\alpha L_z \ll 1$
- The infinite volume limit $L_x, L_y \to \infty$

Spatial phase transition in rotating GP



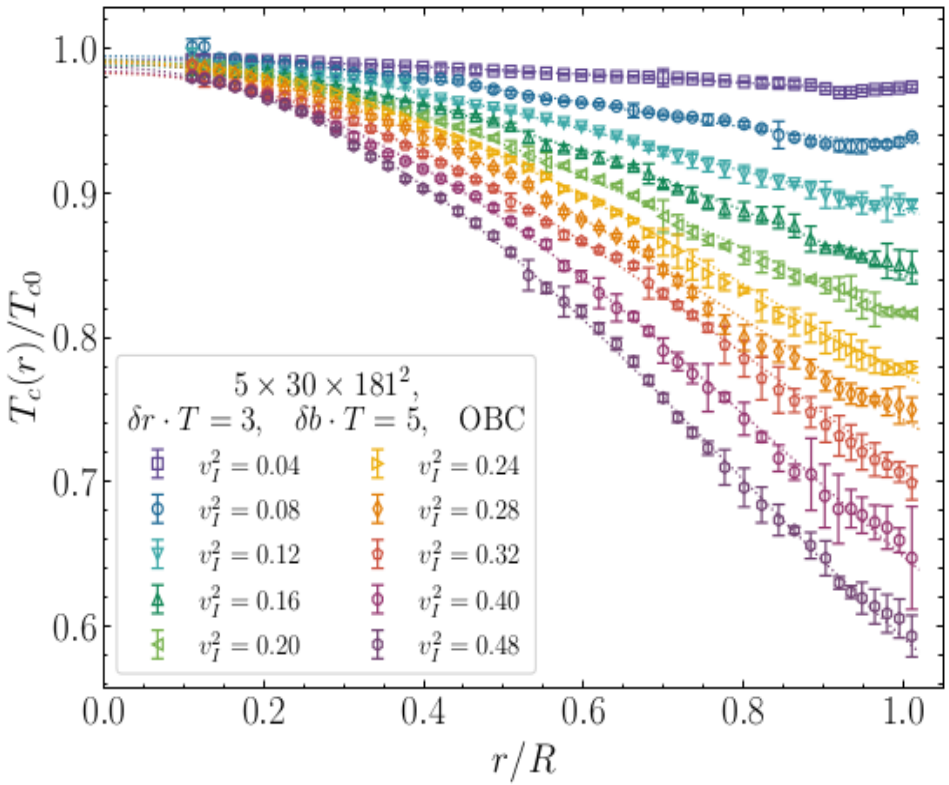
- Finite temperature confinement/deconfinement becomes spatial transition:
  For a certain parameters spatially separated confinement and deconfinement phases coexist simultaneously
- For $\Omega^2 < 0$ confinement in the center and deconfinement in the periphery
- Confinement/inhomogeneous state/deconfinement
  Inhomogeneous state takes place below T_{c0}
- Theoretical studies: [M. Chernodub'2022, S. Chen'2024]

Spatial transition in QCD



- It remains to be true for quarks (Preliminary results!)

Local critical temperature $T_c(r, \Omega_I)$



- TE prediction:

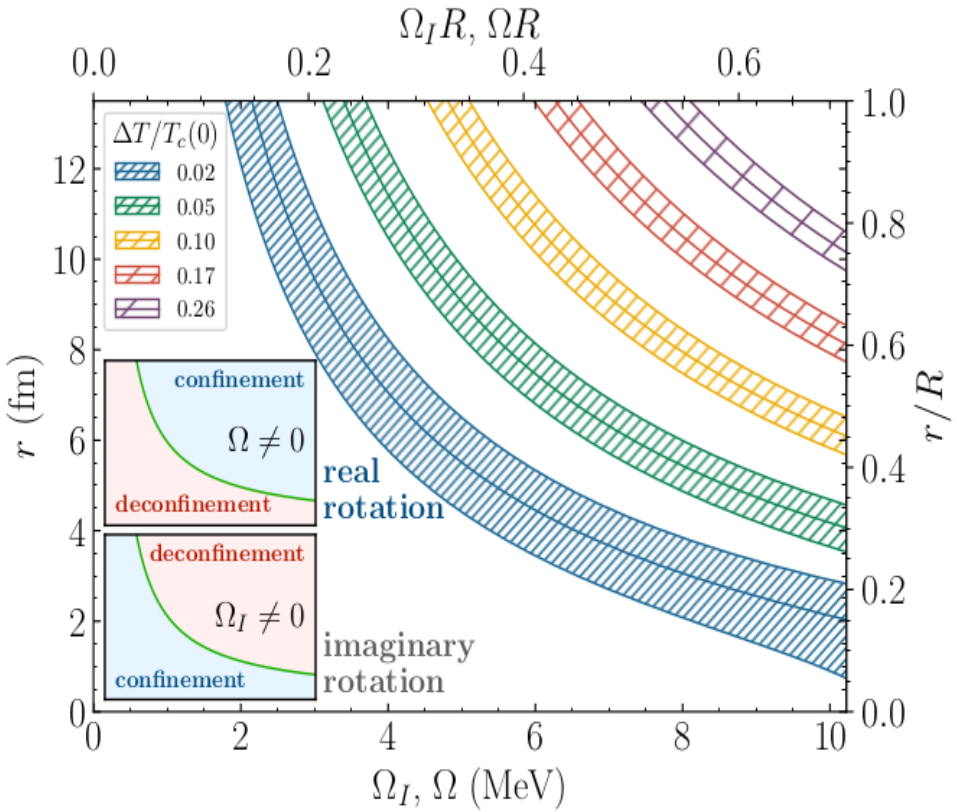
$$T(r) = \frac{T}{1+\Omega_I^2 r^2} = T_{c0} \Rightarrow T_c(r, \Omega_I) = T_{c0}(1 + \Omega_I^2 r^2)$$

- Our results can be well described by the formula

$$\frac{T_c(r,\Omega_I)}{T_{c0}} = 1 - \kappa_2 (\Omega_I r)^2, \quad \kappa_2 > 0$$

- In disagreement with TE law
- Within the uncertainty $\frac{T_c(r=0,\Omega_I)}{T_{c0}} = 1$

Analytical continuation to real rotation

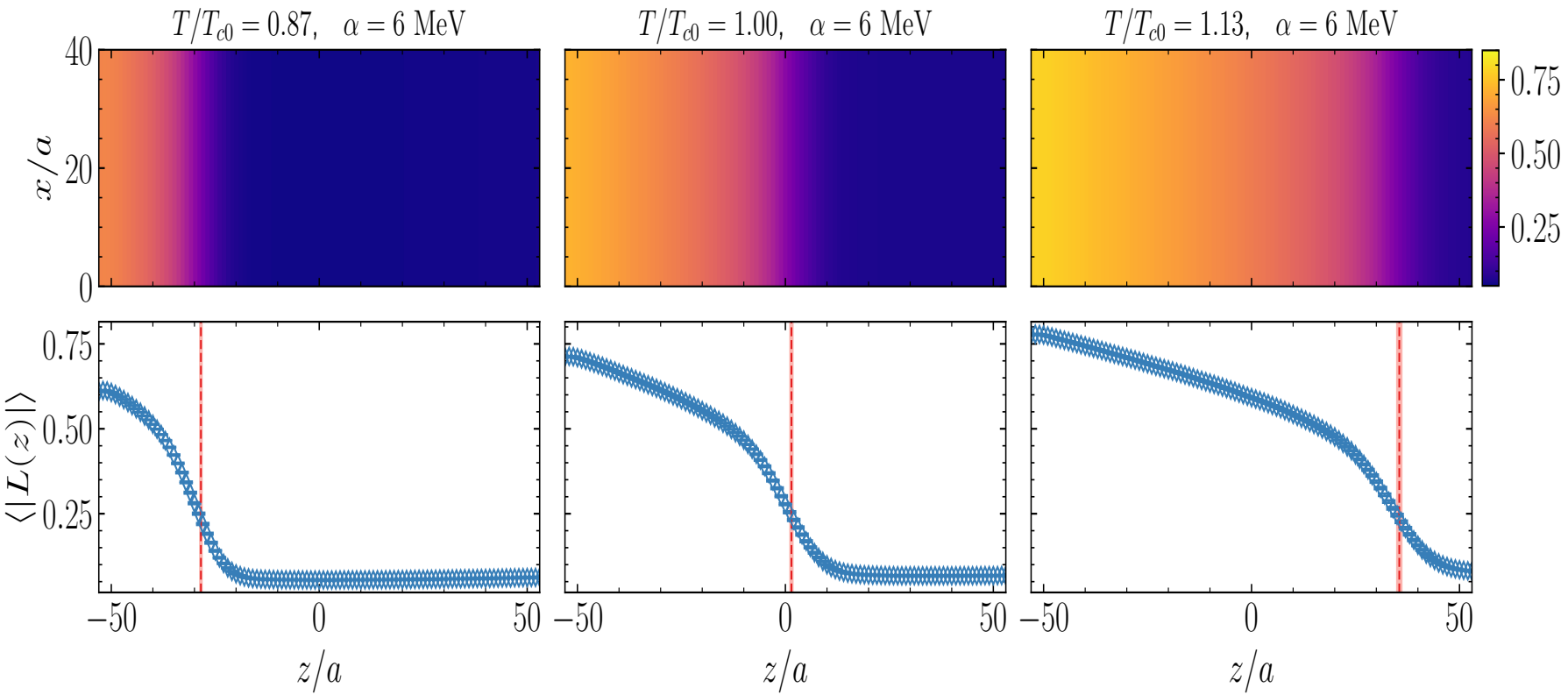


- Analytical continuation $\Omega_I^2 \to -\Omega^2$:

$$\frac{T_c(r,\Omega)}{T_{c0}} = 1 + \kappa_2 (\Omega r)^2$$

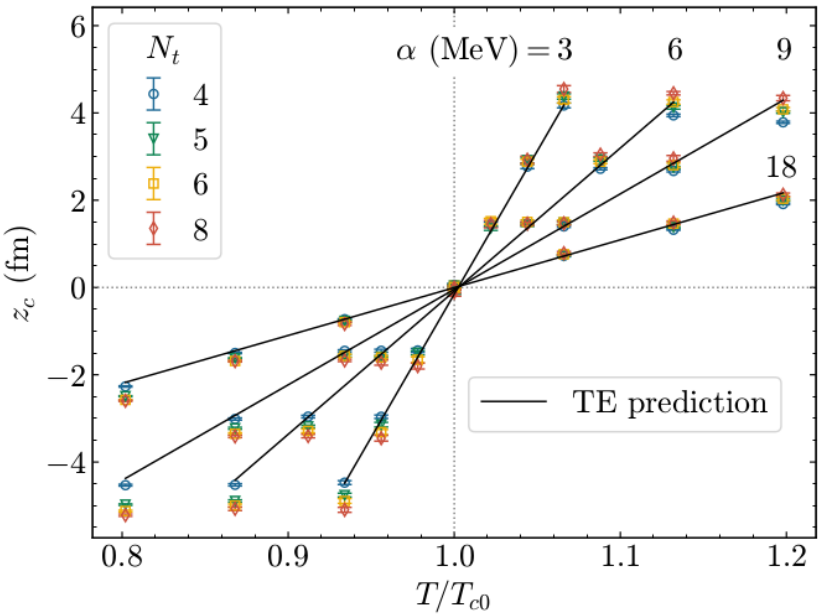
- Inhomogeneous state can be realised for $T > T_{c0}$
- Deconfinement in the center and confinement in the periphery

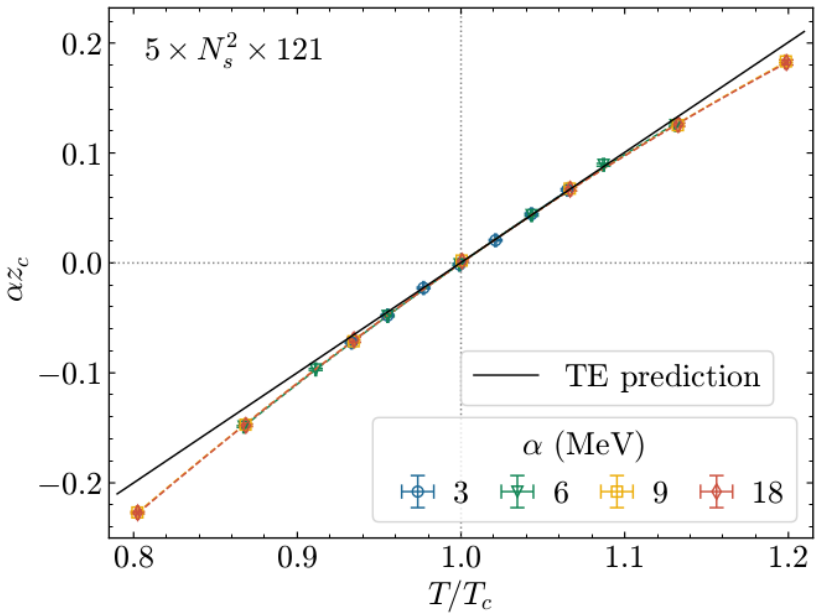
Spatial transition in accelerating GP



- Finite temperature confinement/deconfinement becomes spatial transition for $\alpha \neq 0$
- Deconfinement is close and confinement is far from the horizon
- Confinement/inhomogeneous state/deconfinement
  Inhomogeneous state takes place below and above T_{c0}

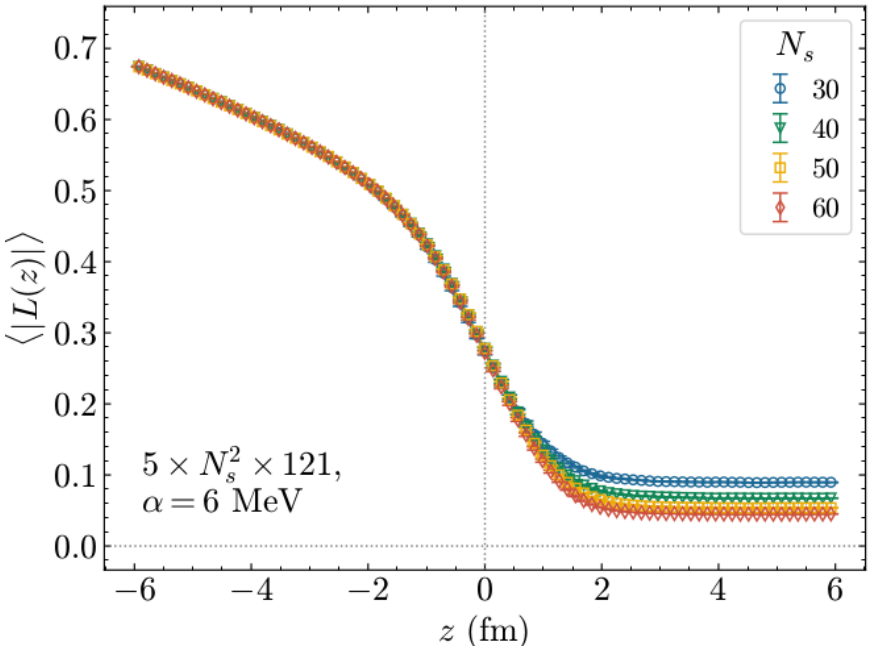
Boundary between the phases

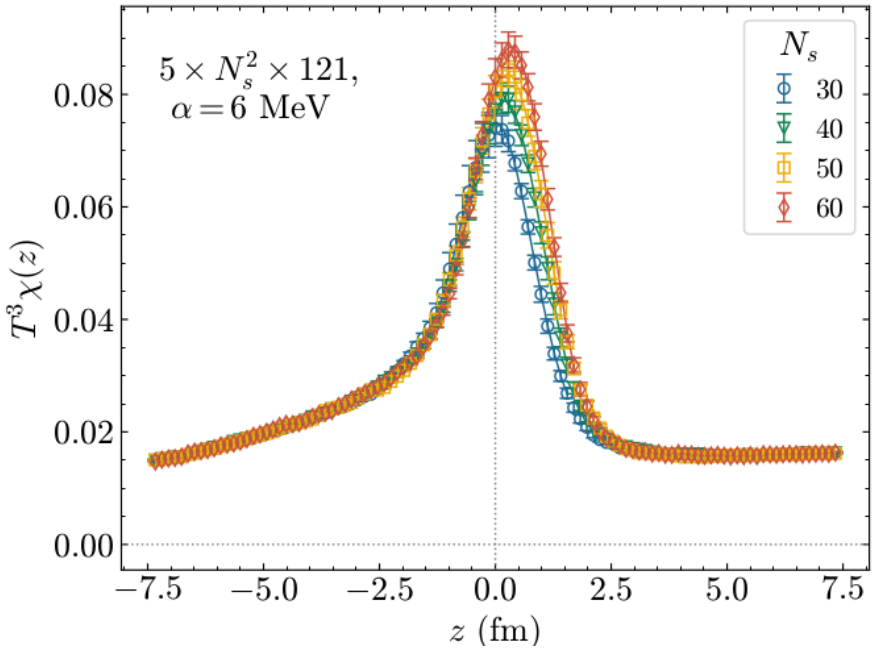




- TE prediction:
  $T_c = T(z_c) = \frac{T}{1+\alpha z_c} \Rightarrow z_c = \frac{1}{\alpha}\left(\frac{T}{T_c} - 1\right)$
- Rather good description of our data by TE law!
- Visible deviation from the TE law for larger accelerations

Properties of spatial transitions





- Type of phase transition can be studied from scaling of χ^{max} and σ
- First order phase transition: $\chi^{max} \sim V \sim N_s^2$ and $\sigma \sim 1/V \sim 1/N_s^2$
- Height: $\chi^{max}\big|_{V\to\infty} \to const$ and width $\sigma|_{V\to\infty} \to const$
- Both spatial transitions for $\Omega \neq 0$ and $\alpha \neq 0$ are crossovers
- There is the smallest width of the transition $\sigma_0 \sim 1$ fm because of finite correlation length

Local action

$$S_G = \int d^4x \left[\frac{1}{2g^2} \left((E_x^a)^2 + (E_y^a)^2 + (E_z^a)^2 + (H_y^a)^2 \right) + \right.$$
$$\left. + \frac{1}{2\tilde{g}^2} \left((H_x^a)^2 + (H_z^a)^2 \right) \right]$$

- Linear in Ω terms can be neglected
- External gravitational field leads to the asymmetric action with $\frac{g^2}{\tilde{g}^2} = 1 - (\Omega r_0)^2$
- The asymmetry $g^2/\tilde{g}^2$ is larger in the periphery region leading to the shift of the local critical temperature
- Local thermalization approximation was checked on the lattice

What about accelerating GP?

Local action for $\Omega \neq 0$

$$S_G = \int d^4x \left[\frac{1}{2g^2} \left((E_x^a)^2 + (E_y^a)^2 + (E_z^a)^2 + (H_y^a)^2 \right) + \right.$$
$$\left. + \frac{1}{2\tilde{g}^2} \left((H_x^a)^2 + (H_z^a)^2 \right) \right], \quad \frac{g^2}{\tilde{g}^2} = 1 - (\Omega r_0)^2$$

Local action for $\alpha \neq 0$

$$S_G = \int d^4x \left[\frac{1}{2g^2} \left((E_x^a)^2 + (E_y^a)^2 + (E_z^a)^2 \right) + \right.$$
$$\left. + \frac{1}{2\tilde{g}^2} \left((H_x^a)^2 + (H_y^a)^2 + (H_z^a)^2 \right) \right], \quad \frac{g^2}{\tilde{g}^2} = (1 + \alpha z_0)^2$$

EoS of rotating plasma

- Free energy of rotating QGP

  $F(T,R,\Omega) = F_0(T,R) - \frac{F_2}{2}\Omega^2 - \frac{F_4}{4}\Omega^4 + ...$

- Total momentum

  $$J_z = -\left(\frac{\partial F}{\partial \Omega}\right)_T = (F_2 + F_4\Omega^2)\cdot\Omega$$

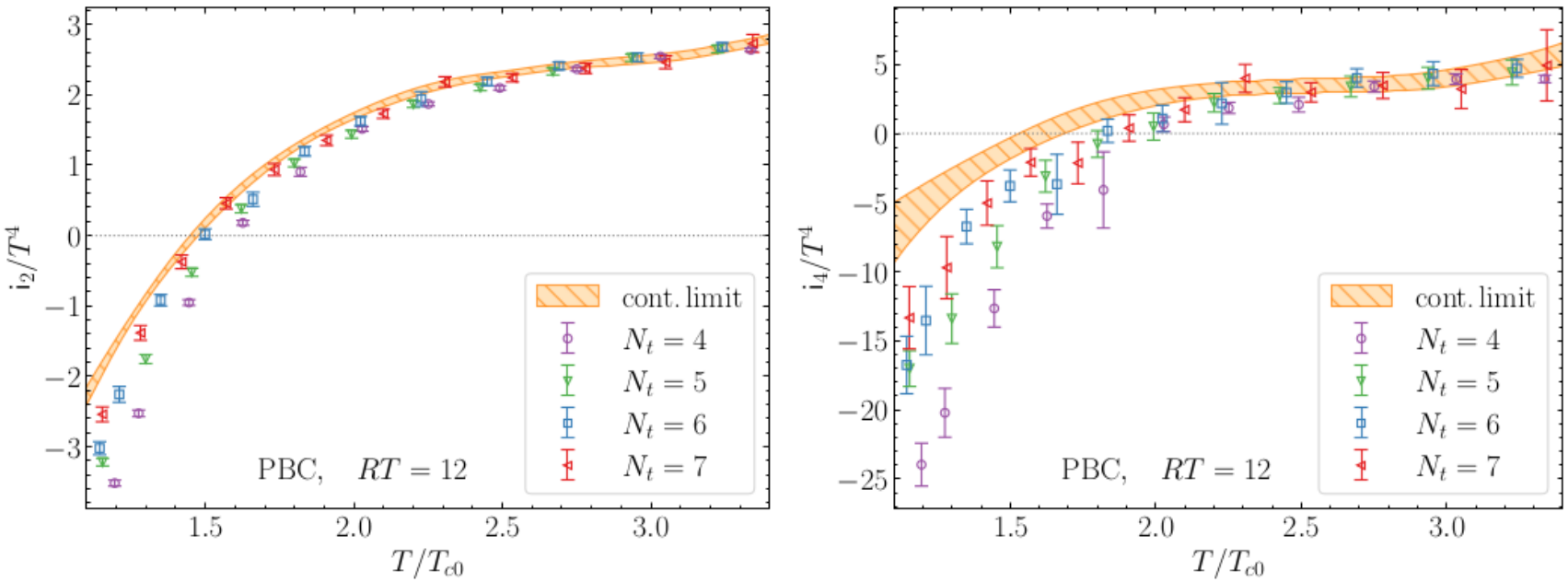
- Moment of inertia

  $$I = \frac{J_z}{\Omega} = (F_2 + F_4\Omega^2)$$

- F_2–moment of inertia at vanishing Ω
  F_4–deformation due to rotation, $F_4 > 0$
- We calculate: $i_2 = \frac{F_2}{VR^2}$, $\quad i_4 = \frac{F_4}{VR^4}$



Moment of inertia of gluon plasma

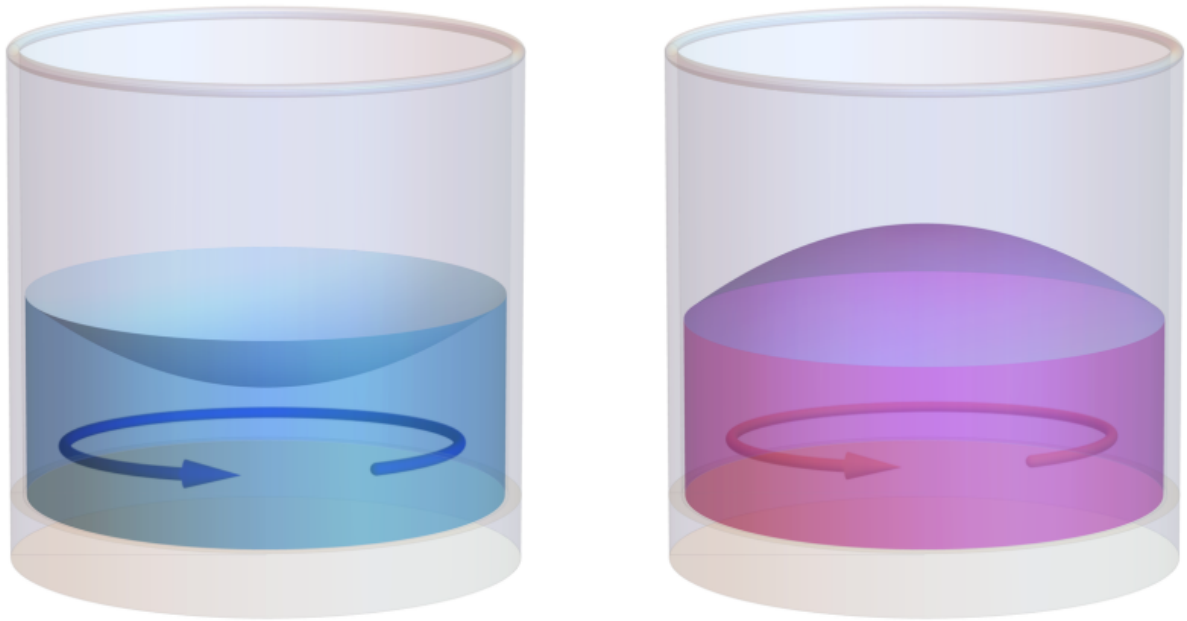


- EoS of gluon plasma

  V. Braguta et al., JETP Lett. 117 (2023) 9; 639-644, Phys.Lett.B 852 (2024) 138604; Phys.Part.Nucl. 57 (2026) 1146; to be published

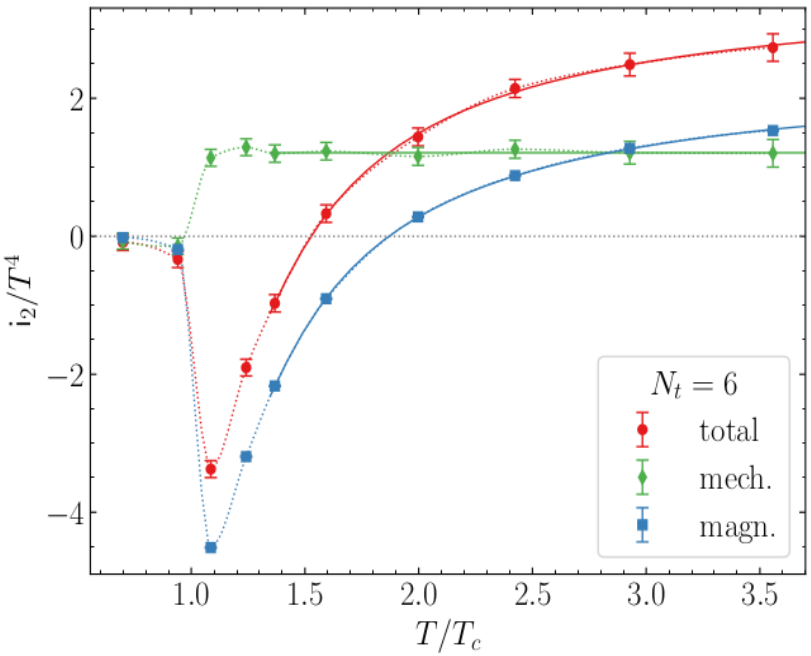
- $i_2 < 0$ for $T < 1.5T_c$ and $i_2 > 0$ for $T > 1.5T_c$
- $i_2 < 0$ is related to magnetic condensate and the scale anomaly

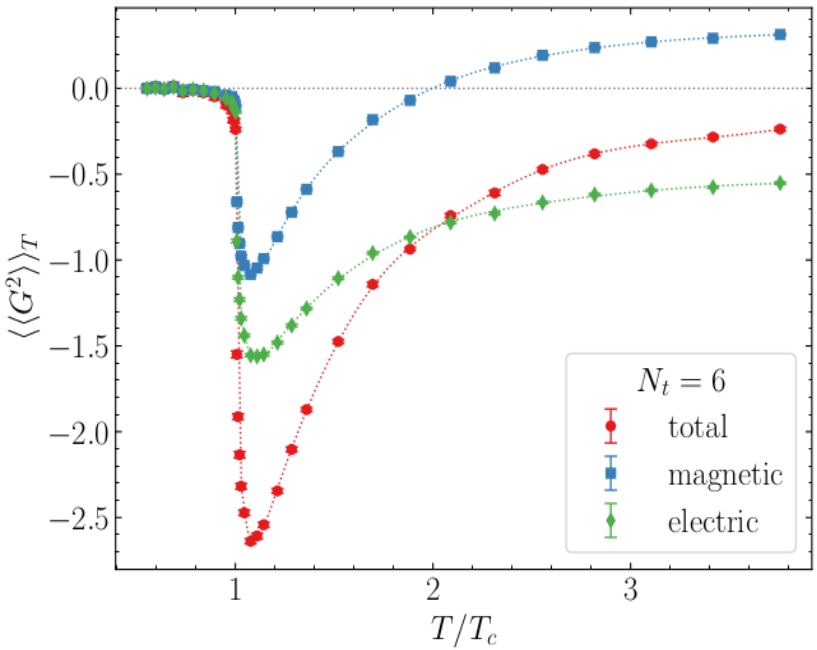
Moment of inertia of gluon plasma



- $i_4 < 0$ for $T < (1.6 - 1.8) \cdot T_c$ and $i_4 > 0$ for $T > (1.6 - 1.8) \cdot T_c$
- For $(\Omega R)^2 \sim 1/3, \quad \delta I / I_0 \sim 80\%$

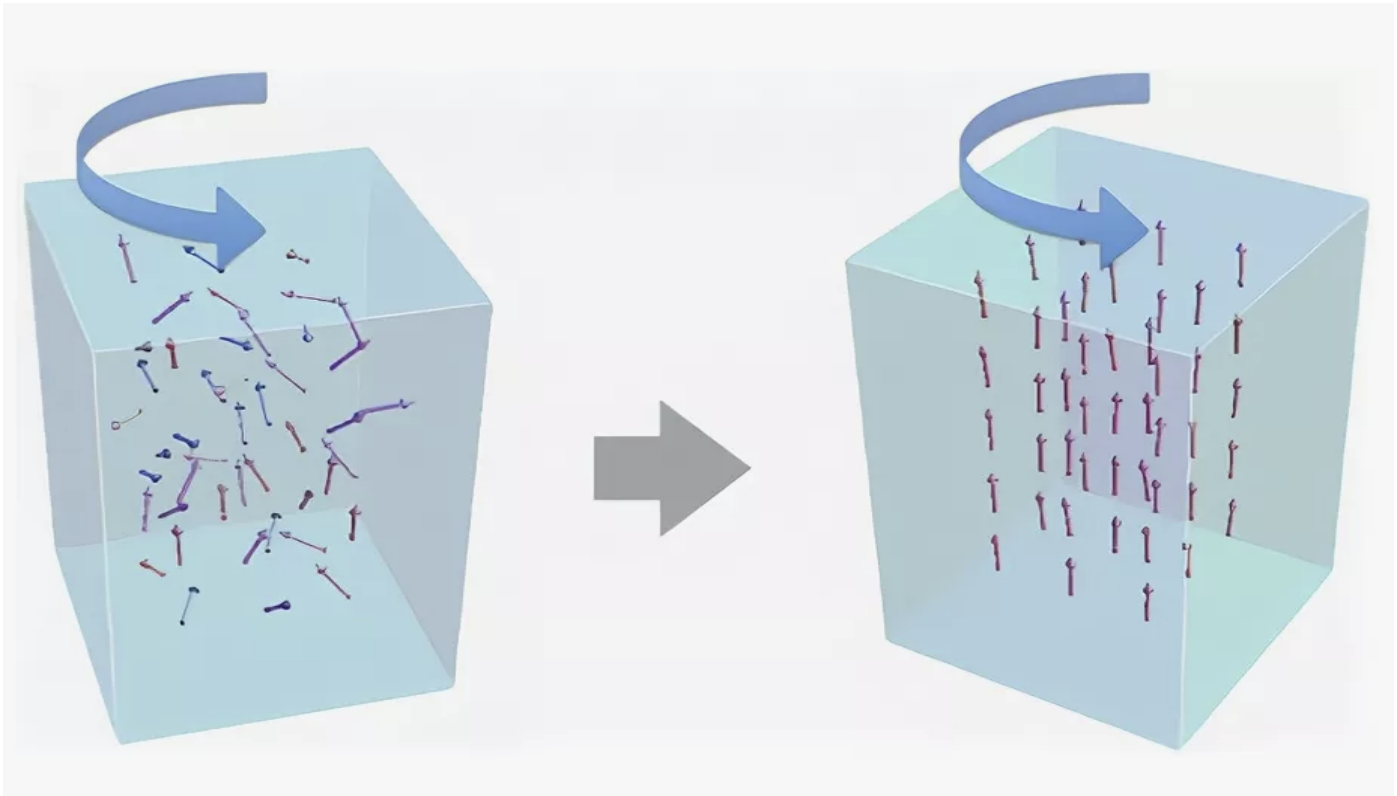
Moment of inertia of gluon plasma





- Decomposition of moment of inertia
  V. Braguta et al. Phys.Rev.D 110 (2024) 1, 014511
- $i_2 = \frac{I_0}{V R_\perp^2}, \quad I_0 = I_{mech} + I_{magn}$
  $I_{mech} = \langle J_z^2 \rangle - (\langle J_z \rangle)^2 \sim \langle {S_1}^2 \rangle$
  $I_{magn} = \frac{1}{3} \int d^3x r^2 \langle H^2 \rangle \sim \langle S_2 \rangle$
- Gluon condensate: $\langle G^2 \rangle = \langle E^2 \rangle + \langle H^2 \rangle$
- $\langle T^\mu_\mu \rangle = \epsilon - 3p = \frac{\beta_s(\alpha_s)}{4\alpha_s} \langle G^2 \rangle, \quad \beta_s(\alpha_s) = -(33 - 2n_f)\frac{\alpha_s^2}{6\pi}$

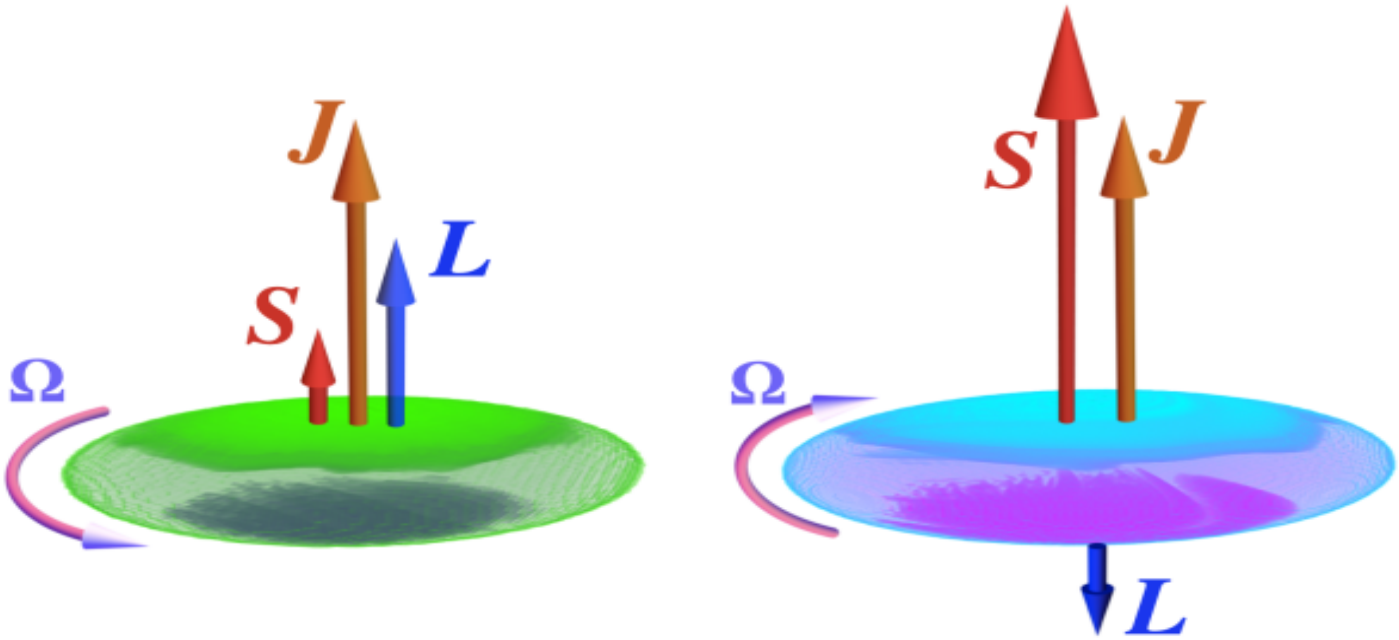
Barnett effect



- Rotation polarizes spins of ferromagnetic

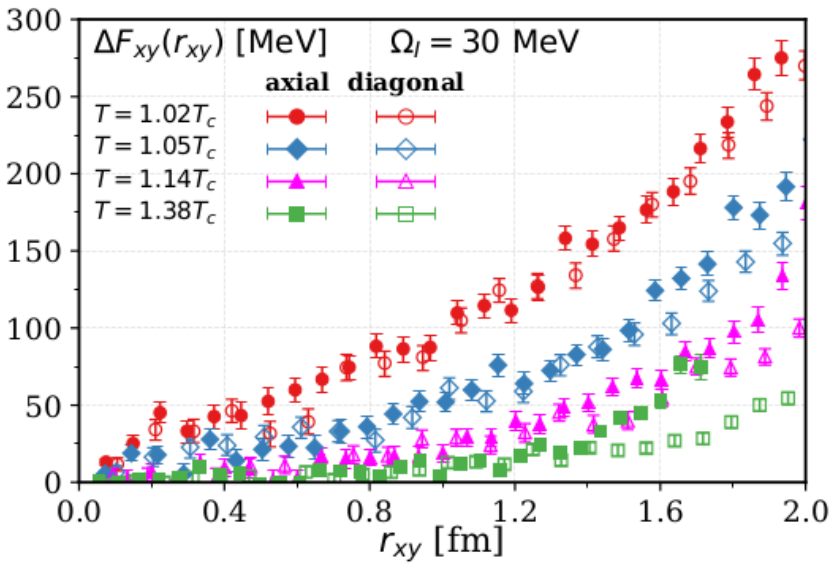
the picture from *doi: 10.3389/fphy.2015.00054*

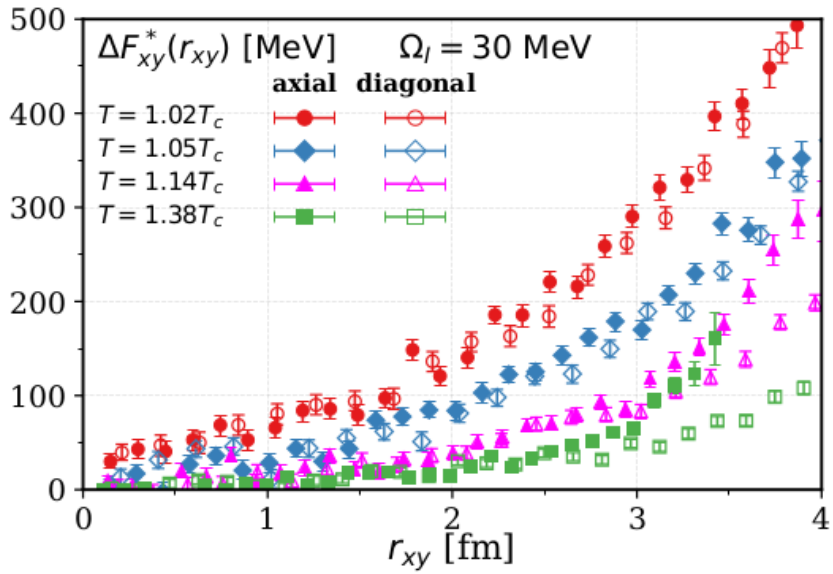
Negative Barnett effect



- $\vec{J} = I_0 \vec{\Omega} = -\left(\frac{\partial F}{\partial \vec{\Omega}}\right)_T, \quad \vec{J} = \vec{L} + \vec{S}$
- For classical motion $\vec{J} = \vec{L}$ and $I_0 > 0$
- But the spin $\vec{S}$ is quantum
- The term $\langle H^2 \rangle$ is related to spin of gluons
- $\vec{L} \uparrow\uparrow \vec{\Omega}, \quad \vec{S} \uparrow\downarrow \vec{\Omega}$ might lead to $\vec{J} \uparrow\downarrow \vec{\Omega}$ and $I_0 < 0$

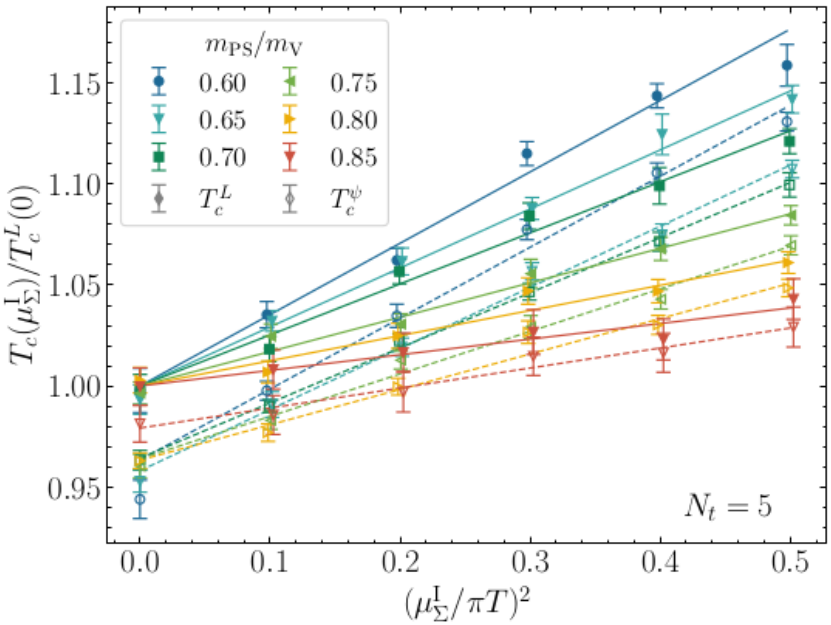
Free energy of rotating static quark

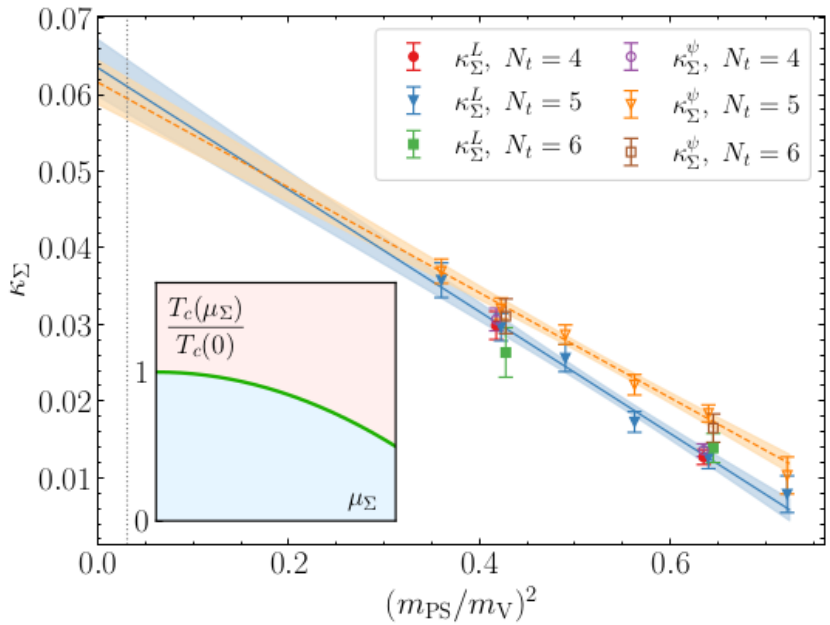




- Interaction potential of rotating static quarks
  Heng-Tong Ding et al., e-Print: 2607.26939
- $\Delta F = a\Omega^2 + b(\Omega r)^2$, $b > 0$ $\Rightarrow$ centripetal force acts on static quark
- For usual rotation $\Delta F = b(\Omega r)^2$, $b < 0$ $\Rightarrow$ centrifugal force
- This phenomenon takes place in the region $T \sim T_c$

Spin potential





- Spin potential μ_Σ: $Z = Tr[e^{-\beta\hat{H}}] \to Tr[e^{-\beta(\hat{H}-\mu_\Sigma\hat{S})}]$
- Lattice simulation at imaginary μ_Σ

  V. Braguta et al., Phys.Rev.D 111 (2025) 11, 114508
- Spin potential decreases the critical temperatures

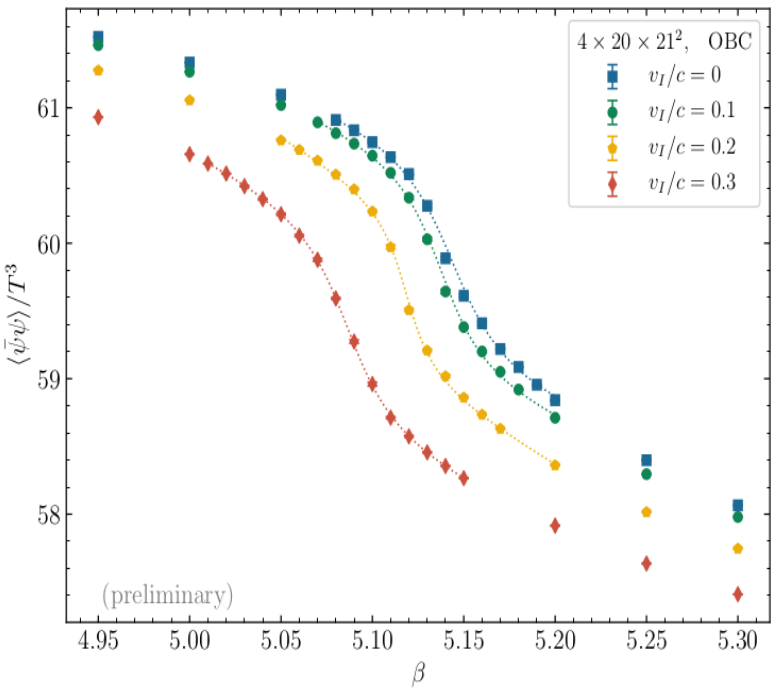
$$\frac{T_c(\mu_\Sigma)}{T_c} = 1 - \kappa_\Sigma \mu_\Sigma^2, \quad \kappa_\Sigma > 0$$

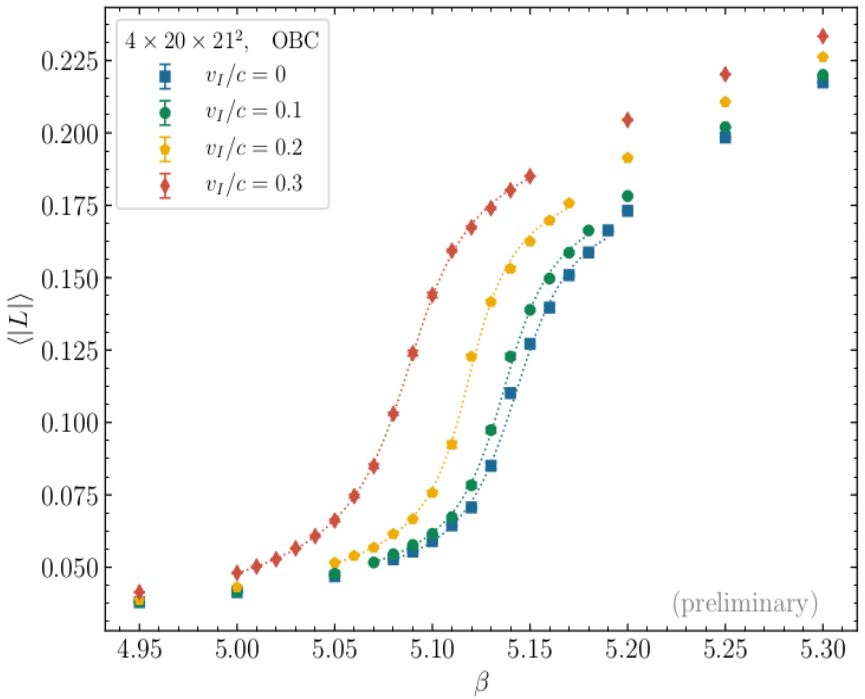
Conclusion

- Lattice studies of rotating gluodynamics and QCD have been reviewed
  - Equation of state and the moment of inertia
  - Spatial confinement/deconfinement transition
  - Free energy of rotating static quark
  - Spin potential
- One observes unusual phenomena in the region $T \sim T_c$: Negative moment of inertia, spatial deconfinement transition, centripetal force for rotating static quark
- These phenomena disappear at larger temperatures

THANK YOU!

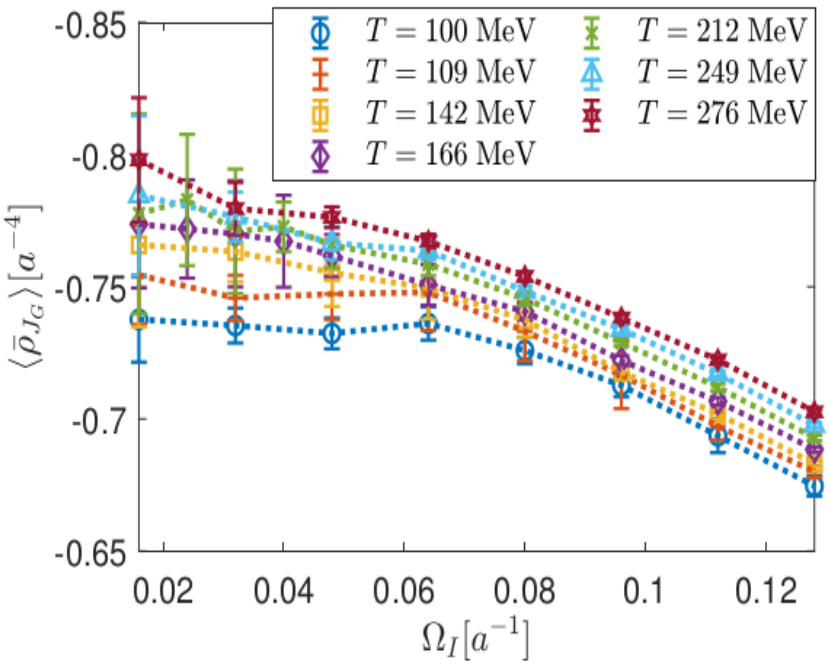
Backup slides: Simulation with fermions

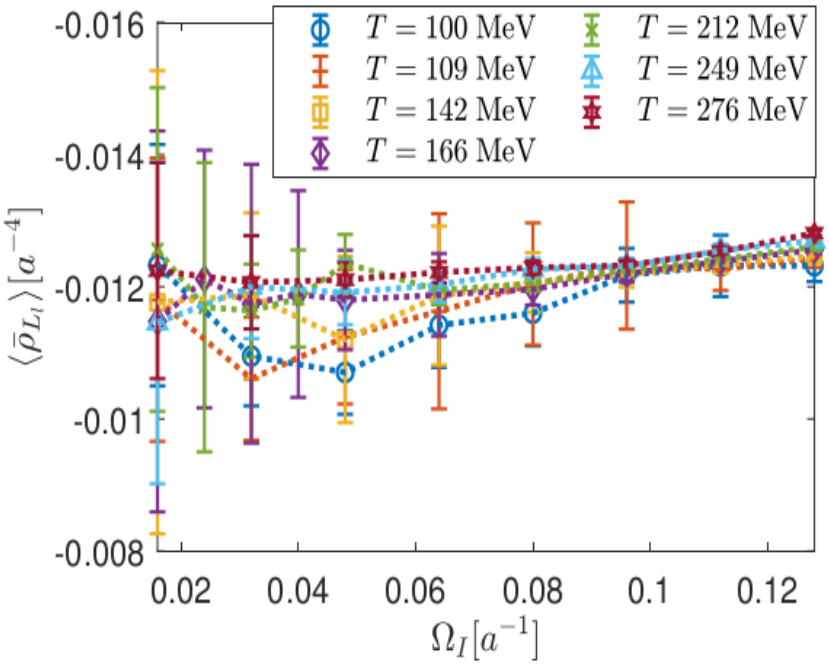




- Lattice simulation with Wilson fermions
- Critical couplings of both transitions coincide
- Critical temperatures are increased

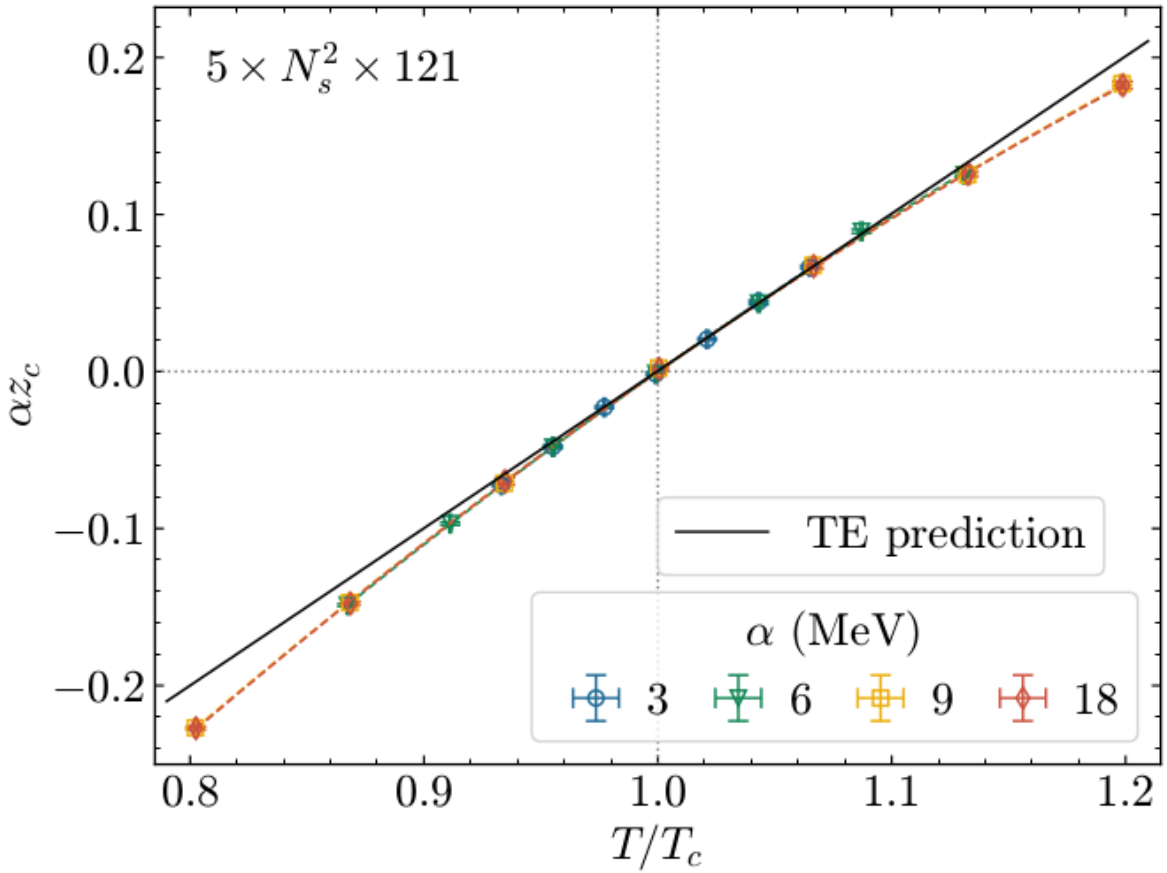
Backup slides:
Simulation with fermions (e-Print: 2307.05755)





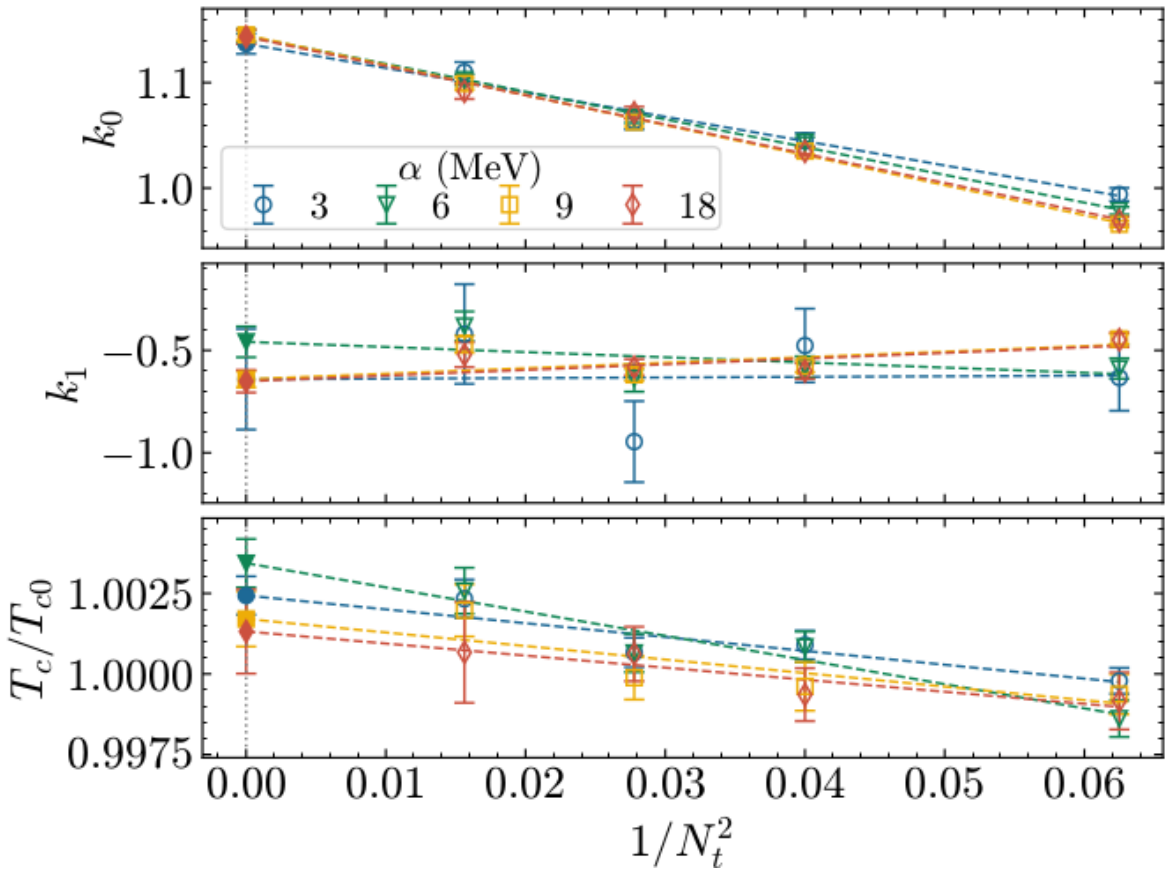
- Rotational rigidities: $\rho_{J_G} = \frac{J_G}{\Omega R^2}, \quad \rho_{L_f} = \frac{L_f}{\Omega R^2}$
- Spin susceptibility: $\zeta_f = \frac{s}{\Omega}$
- Negative moment of inertia

Backup slides: Beyond TE law



- Fitting of data
  $\alpha z_c = k_0 \left(\frac{T}{T_c} - 1\right) + k_1 \left(\frac{T}{T_c} - 1\right)^2$
- k_0, k_1 account deviation from TE law, $T_c(\alpha)$ possible shift of T_{c0}

Backup slides: Beyond TE law



- We tried a lot of options: in the continuum limit $k_0 \sim 1.10 - 1.15$
- Possible explanations: lattice renormalization of the α, crossover transition, higher order α-corrections to TE law...
- Within the uncertainty $T_c(\alpha) = T_{c0}$

Backup slides: Property of Rindler space

- The observer is located at $z = 0$

$$ds^2 = -(1+\alpha z)^2\, d\tau^2 - dx^2 - dy^2 - dz^2$$

- Is the coordinate of the observer exceptional?
- No! One can change the coordinate of the observer
- For the observer at $z = z_1$

$$ds^2 = -(1+\alpha_1(z-z_1))^2\, d\tau^2 - dx^2 - dy^2 - dz^2$$

$$\alpha_1 = \frac{\alpha}{1+\alpha(z_1-z_0)}, \quad T_1 = \frac{T_0}{1+\alpha_0(z_1-z_0)}$$

- To change the observer $z_0 \to z_1$, one has to change parameters $\alpha_0 \to \alpha_1, \quad T_0 \to T_1$

Backup slides: Gluodynamics in the Rindler space

- The observer at $z = z_0$:
  $S_G = \frac{1}{2g^2} \int d\tau_0 d^3x \left(\frac{(\vec{E}^a)^2}{1+\alpha_0(z-z_0)} + (1+\alpha_0(z-z_0))(\vec{H}^a)^2 \right) =$
  $\frac{1}{2g^2} \int d\tau_0 d^3x \left((\vec{E}^a)^2 + (\vec{H}^a)^2 \right) + O(\alpha)$
- The observer at $z = z_1$:
  $S_G = \frac{1}{2g^2} \int d\tau_1 d^3x \left(\frac{(\vec{E}^a)^2}{1+\alpha_1(z-z_1)} + (1+\alpha_1(z-z_1))(\vec{H}^a)^2 \right) =$
  $\frac{1}{2g^2} \int d\tau_0 d^3x \left((\vec{E}^a)^2 + (\vec{H}^a)^2 \right) + O(\alpha)$
- The only difference is $T_0 \neq T_1$
- For this reason TE law describes the data well
- This approximation works for $\alpha\zeta \ll 1$
  It is violated for $\alpha \sim 1/\zeta \sim \Lambda \sim 200$ MeV
- What do we have close to the black hole horizon?