



Using the Wolfram Mathematica package on the HybriLIT platform to modeling of the properties of heavy mesons in effective QCD.

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Bethe–Salpeter equation:

$$\Gamma_H(q,P) = -\frac{4}{3}\int \frac{d^4p}{(2\pi)^4} D(q-p)\{\gamma_\alpha S(p_1)\Gamma_H(p,P)S(p_2)\gamma_\alpha\}. \quad (1)$$

The interaction kernel is chosen in separable form:

$$D(q-p) = D_0\varphi(q^2)\varphi(p^2), \quad (2)$$

with D_0 - interaction constant, $\varphi(p^2) = e^{-\frac{p^2}{\Lambda^2}}$, Λ - size parameter . The Green's functions in the separable model have the form:

$$S^{-1}(p) = \gamma_\mu p^\mu - m(p), \quad (3)$$

where the dynamical quark mass is determined self-consistently:

$$\boxed{m(|\bar{p}|) = m_0 + \varphi(|\bar{p}|)\Delta m} \quad (4)$$

with m_0 - starting mass.

After various transformations, the mass gap equation is obtained:

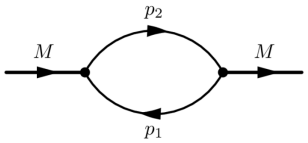
$$\boxed{\Delta m = \frac{8}{9\pi^2} D_0 N_c N_f \int_0^\infty d|\bar{p}|\,|\bar{p}|^2\,\varphi(|\bar{p}|)\,\frac{m(|\bar{p}|)}{E(|\bar{p}|)}}. \tag{5}$$

In the separable model, the vertex function factorizes as $\Gamma_H(q,P) = \varphi(q)\Gamma_H(P)$. Then the Bethe–Salpeter equation reduces to the algebraic equation:

$$\Gamma^H(P) = J^{HH'}\Gamma^{H'} \tag{6}$$

with the polarization operator

$$J^{HH'} = -\frac{8}{3} D_0 N_c N_f i \int \frac{dp}{(2\pi)^4}\varphi^2(p)\,Tr\left[\gamma^H S\left(p+\frac{P}{2}\right)\gamma^{H'} S\left(p-\frac{P}{2}\right)\right] \tag{7}$$



Next, we go to the rest frame and assume instantaneous interaction: $p = |\bar{p}|$. After transformations, the polarization operator is reduced to the calculational form:

$$\boxed{J^{VV} = \frac{8}{9}\frac{D_0 N_c N_f}{\pi^2}\int_0^\infty dp p^2 \frac{\varphi^2(p)}{E(p)}\frac{2E^2(p)+m^2(p)}{E^2-(P_0/2)^2}} \tag{8}$$

The mass of vector mesons is found from the condition for the existence of a nontrivial solution of the Bethe–Salpeter equation (6). The vector channel does not mix with any other channels, and the condition takes the form:

$$\boxed{1 - J^{VV}(M_V) = 0} \tag{9}$$

An important observable characteristic of the meson structure is its decay constant f_V, which is defined as:

$$f_V M_V \epsilon_\mu = \langle 0|\bar{q}_a \gamma_\mu q_b|V(p)\rangle \tag{10}$$

and represents a calculation on a loop with one current vertex γ_μ nd the other meson vertex, which in the separable model is respectively $\Gamma_V(p,P) = N_V \varphi(p)\gamma_\nu$. After all transformations, the equation for the decay constant is obtained:

$$\boxed{f_V M_V = \frac{\mathcal{N}_V N_c N_f}{3\pi^2} \int_0^\infty dp p^2 \frac{\varphi(p)}{E(p)} \frac{2E^2(p) + m^2(p)}{E^2 - (M_V/2)^2}} \tag{11}$$

where N_V is the normalization constant.

The normalization constant is calculated as the derivative of the polarization operator at the point $P_0 = M_V$:

$$\mathcal{N}_V^{-2} = \frac{1}{2D_0} \left. \frac{\partial J^{VV'}(P_0)}{\partial P_0^2} \right|_{P_0^2 = M_V^2}, \tag{12}$$

After differentiation of (8), we obtain an explicit expression for the normalization:

$$\boxed{\mathcal{N}_V^{-2} = \frac{1}{9} \frac{N_c N_f}{\pi^2} \int_0^\infty dp p^2 \frac{\varphi^2(p)}{E(p)} \frac{2E^2(p) + m^2(p)}{[E^2 - (M_V/2)^2]^2}} \tag{13}$$

Subsequently, the normalization will be needed for calculations of processes involving vector mesons.

Thus, the system of equations is as follows:

$$\begin{cases} \Delta m = \dfrac{8}{9\pi^2} D_0 N_c N_f \displaystyle\int_0^\infty dp\, p^2\, \varphi(p)\, \frac{m(p)}{E(p)}, \\ 1 - J^{VV}(M_V) = 0, \\ \mathcal{N}_V^{-2} = \dfrac{1}{9} \dfrac{N_c N_f}{\pi^2} \displaystyle\int_0^\infty dp\, p^2\, \frac{\varphi^2(p)}{E(p)}\, \frac{2E^2(p) + m^2(p)}{\left[E^2(p) - (M_V/2)^2\right]^2}, \\ f_V M_V = \dfrac{\mathcal{N}_V N_c N_f}{3\pi^2} \displaystyle\int_0^\infty dp\, p^2 \frac{\varphi(p)}{E(p)}\, \frac{2E^2(p) + m^2(p)}{E^2 - (M_V/2)^2}. \end{cases} \tag{14}$$

This system of equations must be solved numerically. First, find the optimal parameters: m_0, Λ, D_0, then solve the system with them and obtain the meson mass M_V, the decay constant f_V, and the normalization constant N_V.

Results of Numerical Calculations

The decay constant $f_{J/\psi}(f_V)$ and normalization $\mathcal{N}_{J/\psi}(N_V)$ were obtained.

	m_q [GeV]	$2m_q$ [GeV]	M_V [GeV]	N_V	f_V [GeV]
Model	1.57741	3.15481	3.09072	1.67646	0.416452
Experiment	—	—	3.0970	—	0.4160

Таблица: Comparison of J/ψ parameters with experimental data.

m_0 [GeV]	Λ [GeV]	D_0 [GeV]
1.25	1.8	0.8

Таблица: Values of optimal parameters

HLIT VDI

VDI (Virtual Desktop Infrastructure) technology is a form of desktop virtualization in which all elements of the user's desktop are hosted remotely, and the user connects to their desktop from a client device. It is possible to work with applications that support a graphical user interface (GUI).

Instructions

`https://hlit.jinr.ru/newhlit-vdi/`

`https://hlit.jinr.ru/guide_hybrilit/##_14`

4 virtual machines

- 16 GB of RAM
- 4 CPU cores Intel Xeon E5-2698 v4
- Nvidia Tesla V100, 16 GB
- Lustre user file storage
- Operating system AlmaLinux 9

Mathematical packages

- Matlab
- Mathematica
- Maple
- Comsol

How can access the HLIT-VDI service?

How to launch HLIT-VDI

- Become a user of the HybriLIT platform
  `https://hlit.jinr.ru/for_users/registration/`
- in the web interface using a browser:
  `https://hlit-vdi.jinr.ru/`

Remote login to the HybriLIT platform for Linux users

- Open a terminal and enter:
  `$ ssh USERNAME@hydra.jinr.ru`
  where `USERNAME` is the login received during registration.
- Enter the password.
- Now your home directory is available to you.

Remote login to the HybriLIT platform for Windows users

- Install MobaXterm.
- Launch it and in the top menu under **Sessions** select **New session**.
- Enter the server address `hydra.jinr.ru` in the **Remote host** field and click **OK**.
- A new tab will open where you need to enter your login and password to access the platform.

Thank you for your attention