Equation of state of nuclear matter, phase transitions and radiation of particles

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Plan

- Interaction potentials, EoS, fluctuations, mean fields, phase transitions: what are these beasts.
- Phase diagram of nuclear matter (quark-gluon and hadron regions)

in application to nuclei, neutron stars, HIC.

N. Bohr: completeness of explanation and clarity, as physical picture and its mathematical description, momentum and coordinate, etc are complementary.

Concepts and principles of nonrelativistic mechanics:

Two entities: particles and fields.

Elementary particle in nonrelativistic mechanics is a small size body.

Degrees of freedom: 3 generalized spatial coordinates +time.

Reference frame. Inertial reference frames.

Interaction -- any exchange of information.

Fields--external time-space dependent potentials.

Vacuum in nonrelativistic mechanics is empty continues space in absence of external fields and particles.

Action describing trajectories of particles between any two time -moments is presented as $S = \int_{t_1}^{t_2} L dt$ where *L* is the **Lagrangian**.

Principles: **Galilei invariance, least of action, Ockham razor (simplicity) principle, causality.**

Galilei transformation, **long-range action:** $t = t'$, $\vec{r} = \vec{r}' + \vec{V}t$, $\vec{v} = \vec{v}' + \vec{V}$. +experimental check

$S = \int_{t_1}^{t_2} L(t, \{q\}, \{\dot{q}\})dt.$ *L* does not depend on second derivatives. Least of action $\delta S = \int_{t_1}^{t_2} \delta L(t, q, \dot{q})dt = 0$

Symmetry of vacuum: uniform in time and space and in all directions + simplicity → $L_{\text{kin}} = m\vec{v}^2/2$,

+external field (potential): $L = m\vec{v}^2/2 - U(t, \vec{r})$. $m_a d^2\vec{r}_a/dt^2 = \vec{F}_a$, $\vec{F}_a = -\nabla_a U$. *m* has meaning of mass, *F*-force.

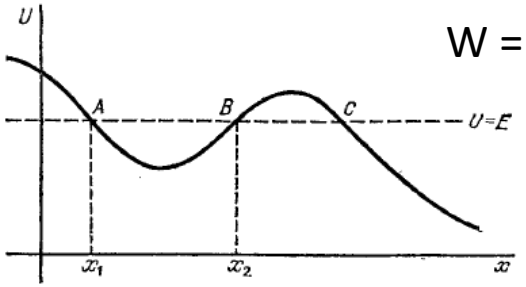
+experimental thorough check. Eq. is time reversible.

Noether theorem--conservations: uniformity of time--for *t*-independent *U*-- conservation *of E=T+U* -energy.

Uniformity of space-- for space-independent *U*-- conservation of momentum ***mv*** = $\vec{p} = \dfrac{\partial L}{\partial \vec{v}}$.

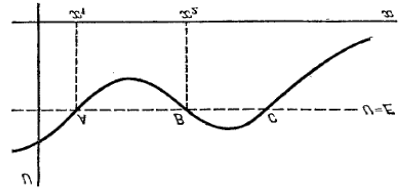
Isotropy, e.g. in central fields—conservation of angular momentum $\vec{L}_a = [\vec{r}_a, \vec{p}_a]$,

* **Semi-classics. The probability of the transition through the barrier:** motion in imaginary time.



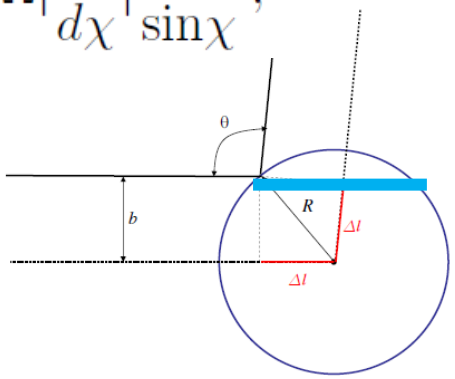
W = ψ*ψ → probability of tunneling, $W = |e^{iS(x_B, x_C)/\hbar}|^2 = \exp[-2\mathrm{Im}S(x_B, x_C)/\hbar]$.

$md^2\vec{r}_a/d\tau^2 = -\nabla_a U_E$. $U_E = -U$, $\tau = it$



Scattering of particles: effective cross section $d\sigma_{\rm ef} = dN_{\rm scat}/n_{\rm fal} = 2\pi b db = d\Omega \left|\frac{db}{d\chi}\right| \frac{b}{\sin\chi}$,

$\frac{d\sigma}{d\Omega} = \left|f(\theta,\phi)\right|^2$, *f*-scattering amplitude



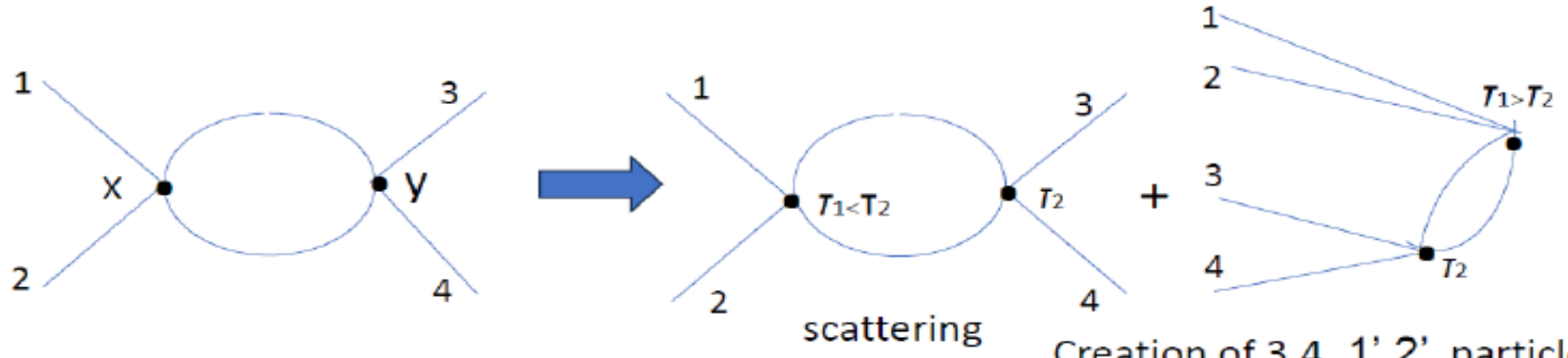
Nonrel. particle stattered in χ direction on hard sphere of radius R gets time advancement compared scattering on point center

$$\delta t_{\rm W}^{\rm cl} = -\frac{2R\cos\theta/2}{v_\infty} = -\frac{2R}{v_\infty}\sin\frac{\chi}{2} \geq -2R/v_\infty\,, \qquad \chi = \pi - \theta$$

1dim. case-attraction (a retardation), repulsion (advance)

In QFT off-shell processes include information about future

Example:



Creation of 3,4, 1',2' particles at $\tau_2 < \tau_1$ and at $\tau_1 > \tau_2$ annihilation of 1',2' with 1,2

Oscillator & Feynman graphs:

Pendulum under elastic force– oscillator--solution-- **wave**

$$\text{potential } U(x) = m\omega_0^2 x^2/2 \text{ at } \omega_0 = const > 0. \quad m\ddot{x}(t) = -m\omega_0^2 x(t)\,. \qquad x(t) = C_1 \cos(\omega_0 t - \delta_0)\,,$$

For harmonic oscillator in medium with absorption (viscosity): $\ddot{x} + \omega_R^2 x + \gamma\dot{x} = 0\,,$ *t*-irreversible, causal.

$$F_{\text{fric}} = -m\gamma\dot{x}, \text{ where } \gamma = const > 0 \text{ is the friction coefficient.} \quad x(t) = Ce^{-\gamma t/2}\cos(\omega_R t - \delta)\,.$$ δ- phase

$$\ddot{G}_0(t-t') + \omega_0^2 G_0(t-t') = -\delta(t-t')\,, \quad G_0^R(\omega) = \frac{1}{\omega^2 - \omega_0^2 + i0\omega}\,, \quad \ddot{G}(t,t') + \omega_R^2 G(t,t') + \gamma\dot{G}(t,t') = -\delta(t-t')\,, \qquad G^R(\omega) = \frac{1}{\omega^2 - \omega_R^2 + i\gamma\omega}\,.$$

Green function obeysDyson equation

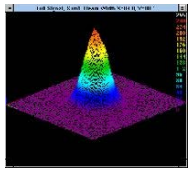
$$iG^R(\omega) = iG_0^R(\omega) + iG_0^R(\omega)(-i\Sigma^R(\omega))iG^R(\omega)\,.$$



$$\Sigma^R(\omega) = \omega_R^2 - \omega_0^2 - i\gamma\omega\,,$$

self-energy=polarization function, γ-width

Spectral function and width of resonance $A = -2\text{Im}G^R = \dfrac{\tilde{\gamma}}{(\omega^2 - \omega_0^2 - \text{Re}\Sigma^R)^2 + \tilde{\gamma}^2/4}\,, \quad \tilde{\gamma} = -2\omega\text{Im}\Sigma^R = 2\gamma\omega\,,$ as for rel. bosons!



Friction-width-damping of **resonances**.

Sum-rule—unitarity $\displaystyle\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}2\omega A = 1\,,$

Quasiparticle limit $A \to 2\pi\delta(\omega^2 - \omega_R^2)\,, \quad \widetilde{A} \to 2\pi\delta(\omega - \omega_R)\,, \quad \text{for} \quad \gamma \to 0\,,$

$$\text{for } \omega \simeq \omega_0 > 0, \quad \widetilde{A} \simeq 2\omega_R A \simeq \frac{\gamma}{(\omega - \omega_R)^2 + \gamma^2/4}$$

Similar to nonrelativistic limit

$$\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\widetilde{A} = 1\,,$$

Harmonic oscillator in matter with absorption under action of external force

$$\ddot{x} + \omega_R^2 x + \gamma\dot{x} = f(t)/m. \qquad x(t) = x_{\rm hom}(t) - \int G^R(\omega)\frac{f(\omega)}{m}e^{-i\omega t}\frac{d\omega}{2\pi}. \qquad x_{\rm inh}(\omega) = iG^R(\omega)\frac{if(\omega)}{m}$$

acquires form of Dyson eq. for field



For monochromatic force $f(t) = \mathrm{Re}(f_0(\omega)e^{-i\omega t})$ and $x = \mathrm{Re}(be^{i\delta - i\omega t})$, $\quad \delta t_{\rm gr} = \dfrac{\partial\delta}{\partial\omega} = \dfrac{\gamma}{2[(\omega-\omega_R)^2+\gamma^2/4]} = \dfrac{\widetilde{A}}{2}$

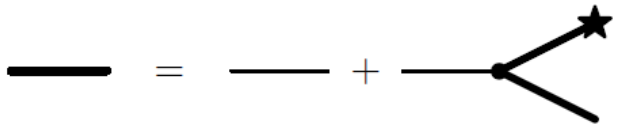
cf. resonance scat. in QM.

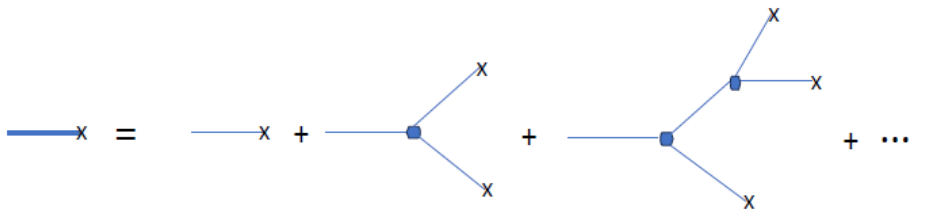
sum-rule⟶ **minimal *t-E* uncertainty condition** $\int_{-\infty}^{\infty}\delta t_{\rm gr}\frac{dE}{2\pi} = \frac{\hbar}{2}.$ $\quad E = \hbar\omega \quad$ $\delta E\delta\mathcal{T} \geq \frac{\hbar}{2}|\overline{\dot{\mathcal{T}}}|,$

→ Phase shift—density of states--time delay.

Example of solution in terms of Feynman diagrams for unharmonic oscillator $\quad \delta U = m\Lambda x^3/3.$

$$\ddot{x}(t) + \omega_R^2 x(t) + \gamma\dot{x}(t) + \Lambda x^2(t) = f(t)/m\,.$$





$$i\tilde{G}^R(\omega) = iG^R(\omega) + iG^R(\omega)(-i\Lambda)\int x(\omega')i\tilde{G}^R(\omega-\omega')d\omega'/(2\pi)\,,$$

Cf. vibration of ions in solid (pendulum) ←→ re-scatterings of phonon quasiparticles (interacting waves), **baryons + light meson excitations**

Hamilton formalism:

generalized coordinates $q_1, ..., q_s$ and $p_1, ..., p_s$ instead of $\dot{q}_1, ..., \dot{q}_s$. $\frac{\partial H(t,q,p)}{\partial p} = \dot{q}\,, \quad \frac{\partial H(t,q,p)}{\partial q} = -\dot{p}\,.$

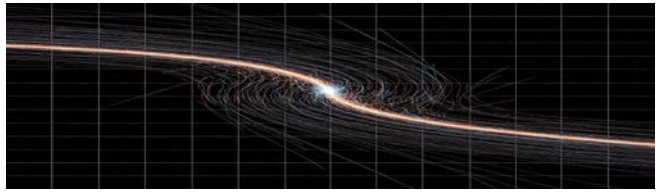
(*p*,*q*) are independent, canonical, variables.

Phase-space volume

$$d\Gamma = dp_1 ... dp_s dq_1 ... dq_s = dP_1 ... dP_s dQ_1 ... dQ_s\,.$$

Liouville theorem:

states that phase-space-volume remains constant, if particles move along their trajectories.



Statistical ensemble-- the large number of copies of the system, being under the same macroscopic conditions and ***N*-particle distribution function** showing probability density of the distribution of *N* particles in the phase space

$$dW = f_N(p, q, t)d\Gamma\,, \quad \text{where} \quad d\Gamma = \frac{dpdq}{N!(2\pi\hbar)^{3N}}; \qquad \int f_N(p, q, t)d\Gamma = 1\,.$$

Each particle moves following its mechanical trajectory in the phase space.

Liouville theorem → $f_N(p, q, t)dpdq = f_N(p', q', t')dp'dq'\,,$ $\qquad \frac{\partial f_N}{\partial t} + \{H, f_N\} = 0\,,$

Concept of entropy

$$\langle A \rangle = \int A(p,q) f_N(p,q,t) d\Gamma \, .$$

One introduces the entropy function $\eta = -\ln f_N(p,q,t)$

N-particle entropy

$$S_{\mathrm{G}} = -\int f_N(\vec{p}_1, ..., \vec{p}_N; \vec{r}_1, ...\vec{r}_N; t) \ln f_N(\vec{p}_1, ..., \vec{p}_N; \vec{r}_1, ...\vec{r}_N; t) \times \frac{d^3p_1 ... d^3p_N \, d^3x_1 ... d^3x_N}{N!(2\pi\hbar)^{3N}}$$

Properties of classical entropy

For the system of particles moving following their mechanical trajectories the Gibbs entropy following Liouville theorem does not change with time.

$$S_{\rm G}(t) = -\int f_N(p_0, q_0, 0) \ln f_N(p_0, q_0, 0) \frac{dp_0 dq_0}{N!(2\pi\hbar)^{3N}} = const\,.$$

In experiments we always deal with **coarse-grained** distributions and entropy $\tilde{f}(p,q,t) = \frac{1}{\Delta\Gamma_{\rm c.gr.}} \int_{\Delta\Gamma_{\rm c.gr.}} f(p,q,t) d\Gamma\,,$

$S_{\rm G}^{\rm c.gr.} = -\int \tilde{f} \ln \tilde{f} d\Gamma\,.$ $S_{\rm G}^{\rm c.gr.} \geq S_{\rm G} = S_{\rm G}(0)\,.$ Each time moment –approx. → **growth of coarse-grained entropy**

For discrete events--information (Shenon) entropy $S_{\rm inf} = -\sum_{k=1}^{n} p_k \ln p_k \quad \text{at} \quad \sum_{k=1}^{n} p_k = 1\,.$

If one of $p_k = 1$ and others are zero, we have $S_{\rm inf} = 0$, i.e. uncertainty in information is absent, and for all $p_k = 1/n$ the quantity $S_{\rm inf}$ gets maximum.

→ for the continues distributions $S_{\rm inf}^{\rm cl} = -\int f(p,q) \ln f(p,q) d\Gamma = S_G\,, \quad \text{at} \quad \int f d\Gamma = 1$

→ **maximum of entropy for chaos**

Coarse-graining each time moment—growth of entropy in dynamics.

Equilibrium distributions (generalization to local equilibrium is straightforward, hydro)

Necessary minimal information is encoded in conservation laws of the additive integrals of motion.

Concept of micro-canonical distribution for closed system $f^{\rm micro} = f(\mathcal{E}, \vec{P}, \vec{L}, N)\,,$

Equal probability: $f^{\rm micro}(p,q) = 1/W(\mathcal{E},N,V) \quad \text{for} \quad \mathcal{E} < H(p,q) < \mathcal{E} + \Delta\mathcal{E}$ $\qquad W(\mathcal{E},N,V) = \int_{\mathcal{E}<H(p,q)<\mathcal{E}+\Delta\mathcal{E}} d\Gamma\,.$

$S_G^{\rm micro}(\mathcal{E},V,N) = \ln W(\mathcal{E},V,N)\,.$ Additivity of entropy for independent events *S*~*N*.

→ continues distributions describe rather well even systems with not a large number of particles *N*.

木 tree 木木木 forest 木木木木 jungle

Since the statistical weight (number of states in a tiny phase space) increases with growth of the disorder, the entropy can be considered as a measure of the disorder.

Canonical and grand-canonical distributions

Particles of subsystem are assumed to exchange energies with big system-- thermostat.

Canonical distribution --for subsystem with fixed *N*. Contact of subsystem I with thermostat (II) suggests conservation of **mean** energy (and *N*) .

I+II --microcanonical $f(p,q;p',q') = 1/W(\mathcal{E},V,N)\,, \quad \mathcal{E} < H < \mathcal{E}+\Delta\mathcal{E}$

→**Canonical distribution:** $f_{\rm I}(p,q) = e^{S_{\rm II}(\mathcal{E}-H_{\rm I}(p,q))-S(\mathcal{E})}$. Expand $S_{\rm II}(\mathcal{E}-H_{\rm I}) = S_{\rm II}(\mathcal{E}) - \dfrac{\partial S_{\rm II}(\mathcal{E})}{\partial\mathcal{E}}H_{\rm I} + \dots,$

Minimizing in E_I: $S(\mathcal{E}_{\rm I}) = S_{\rm I}(\mathcal{E}_{\rm I}) + S_{\rm II}(\mathcal{E}-\mathcal{E}_{\rm I})$ $\dfrac{\partial S_{\rm II}(\mathcal{E})}{\partial\mathcal{E}} = \dfrac{\partial S_{\rm I}(\mathcal{E}_{\rm I})}{\partial\mathcal{E}_{\rm I}} = \dfrac{1}{T}.$

T is the parameter of thermostat, being **same for subsystem being in equilibrium with thermostat --** speed of change of energy with change of entropy (measure of chaoticity).

$f_{\rm I}(p,q) = Z^{-1}(V,T,N)e^{-H_{\rm I}(p,q)/T}$, $Z(T,V,N) = \int e^{-H_{\rm I}(p,q)/T}d\Gamma_{\rm I}$, $d\Gamma_{\rm I} = dp_{\rm I}dq_{\rm I}/[N_{\rm I}!(2\pi\hbar)^{3N_{\rm I}}]\,.$

$$\langle H\rangle = T^2\frac{\partial \ln Z(T,V,N)}{\partial T} = -T^2\frac{\partial}{\partial T}\left(\frac{F(T,V,N)}{T}\right)_{V,N},$$

In mechanics pressure *P* is averaged force per element of surface directed perpendicular to this element $\overline{\vec{F}} = P\delta\vec{s}.$

$\vec{F} = -\nabla U = -\partial\mathcal{E}(\vec{p},\vec{r},t)/\partial\vec{r}.$ → $P = -(\partial\mathcal{E}(S,V,N)/\partial V)_S\,.$

Grand-canonical distribution—not fixed *N*

$$f_{\rm I}(p,q,N) = Z^{-1}(V,T,\mu)e^{-(H_{\rm I}(p,q)-\mu N)/T}\,,$$

$$\Omega(V,T,\mu) = -T\ln Z(V,T,\mu) = -PV\,,$$

$$\mathcal{F}(V,T,N) = \langle H(S,V,N)\rangle - TS, \quad \langle N\rangle = -T\left(\frac{\partial\Omega(V,T,\mu)}{\partial\mu}\right)_{T,V}, \quad \mu = (\partial F(T,V)/\partial N)_{T,V}. \quad S = -\left(\frac{\partial\mathcal{F}(V,T,N)}{\partial T}\right)_V, \quad C_V = T\left(\frac{\partial S}{\partial T}\right)_V$$

Thermodynamical consistency relation: $$TS = T\langle\eta\rangle = \langle H\rangle - \Omega(V,T,\mu) - \mu\langle N\rangle = -T\left(\frac{\partial\Omega(V,T,\mu)}{\partial T}\right)_{V,\mu},$$

At equilibrium we need to know one quantity, e.g. free energy density *F(n,T)*.

Relation between pressure and energy density of thermally equilibrated system is called the equation of state.

Fluctuations:

$$\langle H'^2\rangle - \langle H'\rangle^2 = T^2\partial\langle H'\rangle/\partial T\,,$$

where $H' = H - \mu N$, $\langle H\rangle = \mathcal{E}$. Thus $T^2C_V = \langle\mathcal{E}^2\rangle - \langle\mathcal{E}\rangle^2$ and $\sqrt{|\langle N^2\rangle - \langle N\rangle^2|}/\langle N\rangle \sim 1/\sqrt{\langle N\rangle}\,,$

Smallness of fluctuations indicates that for large systems grand-canonical description differs only little from canonical and micro-canonical.

Ideal gas of N identical nonrelativistic particles in box with hot walls

$$\Omega_0(\mathcal{E},N,V)=\frac{V^N}{(2\pi\hbar)^{3N}N!}\int_{H<\mathcal{E}}d^3p_1...d^3p_N=\frac{V^N(2\pi m\mathcal{E})^{3N/2}}{(2\pi\hbar)^{3N}N!\Gamma(1+3N/2)}\,,\qquad S\simeq N\ln\frac{V}{N}+\frac{3N}{2}\ln\frac{2\mathcal{E}}{3N}+const\,,$$

$$\mathcal{E}=3NT/2\,,\quad PV=NT\,,\quad C_V=(\partial\mathcal{E}/\partial T)_{V,N}=3N/2\,,\qquad P=\tfrac{2}{3}\mathcal{E}/V\,.$$

EoS for ideal gas

→ meaning of *T* for ideal gas--mean kinetic energy.

Presence of the potential *U(q)* produces in the distribution function the pre-factor $e^{-U/T}$.

For a rare gas -- pair potential $\sum U(r_{ij})$. Pandora--Abell 2744 –collision of 4 Galaxy clusters



Three-particle binding, Efimov states. **Borromean rings**



Presenting phase space as **sum of elementary cells** with 1-particle in each cell– fermions, arbitrary *N*-bosons

$$\bar{f}^{\rm g.can}_{\rm id.}(\vec{p})=\frac{1}{e^{(\vec{p}^{\,2}/(2m)-\mu)/T}\pm1}\,,\qquad S^{\rm F/B,free}_{\rm G1}=\int[\mp(1\mp f_1^{\rm free})\ln(1\pm f_1^{\rm free})-f_1^{\rm free}\ln f_1^{\rm free}]d^3p_1d^3q_1/(2\pi\hbar)^3\,.$$

Gentile statistics, quantum anyon statistics (e.g. of quadrupolar nuclei in some liquid crystals, m.b. quarks), Tsalis statistics (m.b. for small size systems in HIC).

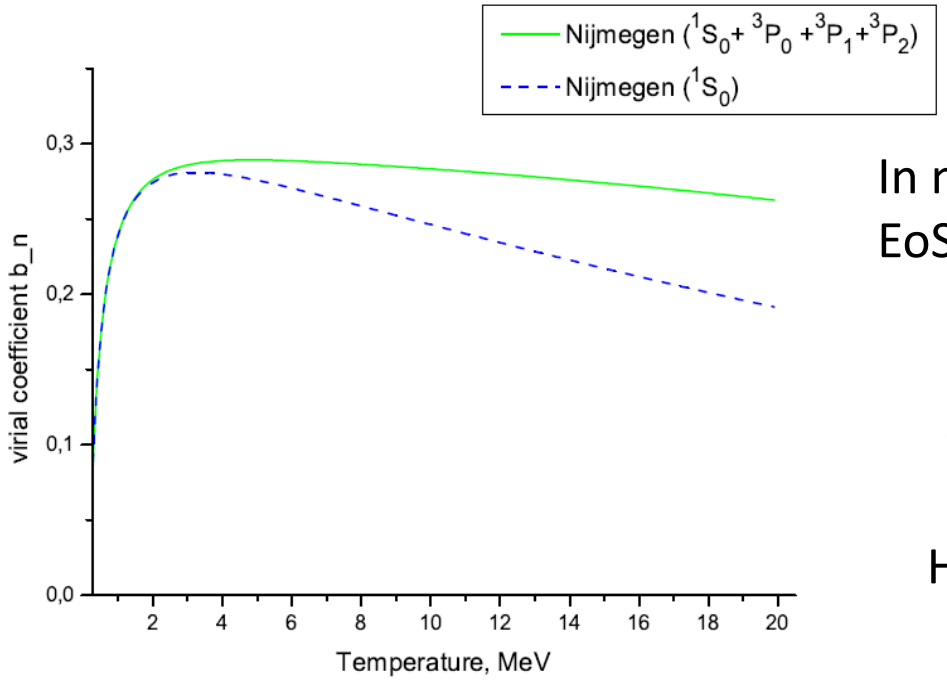
Virial equation of state presents expansion in the virial gas parameter $\lambda^3 n \ll 1, \quad \lambda = \sqrt{2\pi/(mT)}$, n is the density.

$$P \simeq \rho_n T(1 - b_n \lambda^3 \rho_n/2). \qquad b_n(T) = \frac{1}{2^{1/2}\,\pi T} \int_0^\infty dE\, e^{-E/2T}\, \delta^{\rm tot}(E) - 2^{-5/2},$$

Experimental EoS!

$$\delta^{\rm tot}(E) = \sum_{S,L,J} (2J+1)\, \delta_{\,^{2S+1}{\rm L}_J}(E)$$

$$= \delta_{^1{\rm S}_0} + \delta_{^3{\rm P}_0} + 3\,\delta_{^3{\rm P}_1} + 5\,\delta_{^3{\rm P}_2} + 5\,\delta_{^1{\rm D}_2} + \ldots$$



In neutron rare gas b_2>0, for large T, *nn* attraction decreases P(n), EoS softens, in low T limit– Pauli principle stiffens EoS

$$f_{\rm B}(\theta) = -\frac{m}{2\pi\hbar^2} \int e^{i\vec{q}\vec{r}\,'} U(\vec{r}\,')d^3r'\,, \qquad f_l = \frac{e^{2i\delta_l} - 1}{2ik} = e^{i\delta_l} \sin\delta_l/k\,.$$

Higher n short-range correlations stiffen EoS.

In case of the strong interaction the equation of state essentially differs from that for the ideal gas. As an example mention the Van der Waals equation of state describing first-order vapor-liquid phase transition.

For distributions determined within the same phase space layer and energy interval: **micro-canonical, canonical and grand-canonical distributions correspond to the maximum of the entropy.**

One particle distribution function $f_1(\vec{p}_1,\vec{q}_1,t)=\int f_N(\vec{p}_1,...,\vec{p}_N,\vec{q}_1,...,\vec{q}_N;t)\frac{d^3p_2...d^3p_N d^3q_2...d^3q_N}{(N-1)!(2\pi\hbar)^{3N-3}}\cdot$ obeys the Boltzmann eq.

N.N. Bogolyubov derived Boltzmann irreversible kinetic equation for the one-particle distribution function using reversible Liouville eq., provided particles interact by means of a weak **pair potential**, and using principle of complete **weakening of so called initial correlations**, cf. 25-th shot in cinema.

In general, *N*-particle entropy should grow after dropping of initial short (t,r) range correlations (not shown).

One-particle Boltzmann entropy $$S_{G1}=-\int f_1(\vec{p}_1,\vec{q}_1,t)\ln(f_1(\vec{p}_1,\vec{q}_1,t)/e)d^3p_1d^3q_1/(2\pi\hbar)^3\,.$$

obeys *H* theorem. $$-H=dS_{\rm Boltz}/dt=\frac{1}{4}\sum_{ab}\int\frac{d^3p_a}{(2\pi\hbar)^3}\frac{d^3p_b}{(2\pi\hbar)^3}|\vec{v}_a-\vec{v}_b|d\sigma_{ab}(f_a{}'f_b{}'-f_af_b)\ln\frac{f_a'f_b'}{f_af_b}\,.\qquad (x-y)\ln(x/y)\geq 0.$$

Few words about local equilibrium. Nonideal hydro

$$\eta = \frac{1}{15}\mathrm{Tr}\int \frac{d^4p}{(2\pi)^4}\frac{A^2\Gamma}{2}\tau_{\rm rel}\frac{\vec{p}^4 f(1\mp f)}{Tmm^*},$$

A—spectral function, cf. oscillator

$$\begin{aligned}\delta P_{\rm n.eq.} &= \frac{1}{3}\left(\Theta^{11}+\Theta^{22}+\Theta^{33}\right)_{\rm n.eq.} - \frac{1}{3}\left(\Theta^{11}+\Theta^{22}+\Theta^{33}\right)_{\rm l.eq.}\\ &= -\frac{1}{3}(\Pi^{(2)}_{ii}+\delta\Pi_{ii}) = -(\zeta+\delta\zeta)\mathrm{div}\,\vec{u}\end{aligned}$$

On stage of expansion of fireball formed in the heavy ion collisions, when $\mathrm{div}\,\vec{u} > 0,$ the nonequilibrium pressure is smaller than the local-equilibrium one. This means that realistic equilibrium equation of state might be stiffer than that allows to fit experimental data within simulations performed in the framework of the ideal hydrodynamics, where the bulk viscosity effect on the pressure is absent.

Concepts and principles of relativistic mechanics and classical field theory

Principle of locality of interaction: Signal–transfer of interaction (information) has maximum finite speed *c*.
Checked by Michelson-Morley.

Lorentz invariance, *c*=const. Interval between two events $s_{12} = \sqrt{c^2 t_{12}^2 - x_{12}^2 - y_{12}^2 - z_{12}^2}\,,$ is invariant.

Lorentz transformation: $ct' = \dfrac{ct - (V/c)x}{\sqrt{1 - V^2/c^2}}\,, \quad x' = \dfrac{-(V/c)ct + x}{\sqrt{1 - V^2/c^2}}\,, \quad y = y'\,, \quad z = z'\,.$

Vacuum is ground state without particles but **not all fields are zero in vacuum!**

Elementary particles are now excitations over the vacuum of the corresponding field.

Elementary particle now should be **point-like!**

$x^\mu = (ct, \vec{r})\,, \quad x_\mu = (ct, -\vec{r})$ 4-vector. **World is 4-dimensional Minkowski space-time. Phase space is** (t,**r**, ω,**p**) !!!
Nonequilibrium dynamics of resonances (i.e. not free particles, not quasiparticles), spectral functions ω≠ f(**p**).

Action for free particle $S_{\rm part} = \alpha \int_a^b ds \qquad \alpha = -mc$ to get in nonrel. limit $L_{\rm kin} = mv^2/2$

Phys. meaning of least of action: Elementary particles choose the trajectories of minimal path in Minkowski space.

Field is now extra degree of freedom, as q! Trajectories of particles and fields are found from least of action.

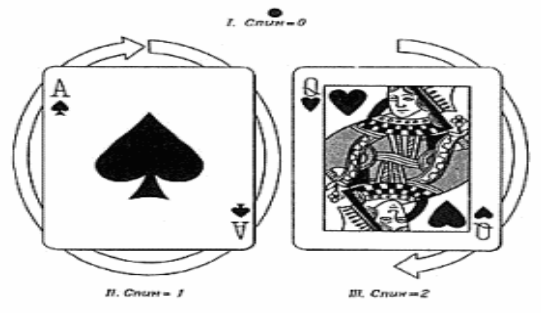
Which fields?

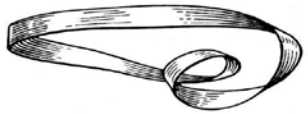
Internal degrees of freedom: isotropy and spin

2s+1 components $\Psi' = \exp\{is\phi\}\,\psi$, $\exp\{is\phi\} = \cos\{s\phi\} + i\sin\{s\phi\}$:

- scalar field s=0

σ,π





4-vector field: s=1, φ= 2π,
ϰ, ω, ρ, g

tensor: s=2, π
graviton

spinor: s=1/2, 4π
e, *μ*, *τ*, *q*, *n*, *p*, hyperons

Particles with integer spin—bosons (form boson condensate), half-integer—fermions (obey Pauli principle)







$$S_{\rm tot} = S_{\rm part} + S_{\rm int} + S_{\rm field}\,. \qquad \delta S = -\int U\,dt$$ is not Lorentz scalar!

4-vector field: electromagnetic field A^μ, ω^μ –omega meson field

Simplest Lorentz scalar $S^A_{\rm int} = -\frac{e}{c}\int A_\mu(x)dx^\mu = \frac{1}{c^2}\int j^\mu(x)A_\mu(x)d^4x\,$, + $S_A = -\frac{1}{16\pi c}\int d^4x F_{\mu\nu}F^{\mu\nu}\,.$

Variation in *x* produces Newton eq. $\frac{d\vec{p}_{kinem}}{dt} = e\vec{E} + \frac{e}{c}[\vec{v},\vec{H}]\,,\ \vec{E} = -\frac{1}{c}\frac{\partial\vec{A}}{\partial t} - \nabla\Phi\,,\quad \vec{H} = {\rm curl}\vec{A}.$ Gauge inv. $A^\mu = A'^\mu + \partial^\mu f\,,$

Variation in *A* yields Maxwell eq. $\partial_\nu\partial^\nu A^\mu_{\rm cl} = \frac{4\pi}{c}j^\mu_{\rm cl}\,,\quad \partial_\nu A^\nu_{\rm cl} = 0.$ **Vacuum (*j*=0) is trivial, *A*=0.** $T^{00}_A = \int\frac{\vec{E}^2+\vec{H}^2}{8\pi}d^3x$

The CED is applicable for $N_{\vec{k}\lambda} \gg 1.$ $\quad 2\pi c^3 E^2_{\rm cl}/(\hbar\omega^4_{max}) = 2\pi c^3 H^2_{\rm cl}/(\hbar\omega^4_{max}) \gg 1.$

→**Any static field is classical. Coulomb field=condensate of virtual photons. Rapid fields are quantum ones.**

4-vector field ω meson field, (massive photon) *m*=783 MeV, gauge non-invariant

$$S^\Omega_{\rm int} = -\sum_a \frac{e_a}{c}\int\Omega_\mu(x_a)dx^\mu_a = -\frac{1}{c^2}\int j^\mu(x)\Omega_\mu(x)d^4x\,, \qquad S_\Omega = -\frac{1}{16\pi c}\int d^4x\Omega_{\mu\nu}\Omega^{\mu\nu} + \int\frac{m^2_\Omega\Omega_\mu\Omega^\mu}{8\pi c}d^4x\,.$$

e now is baryon charge.

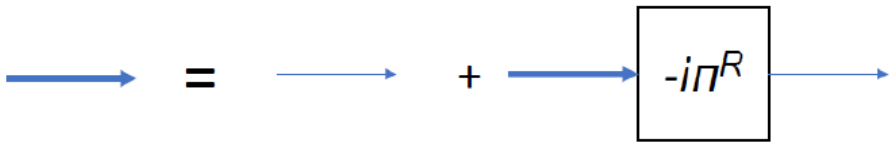
Let $j^0 = cQ_B\delta(\vec{r})$, Q_B is the baryon charge of a particle. $\quad q\Omega_0 = \frac{qQ_B}{r}e^{-\mu r}$

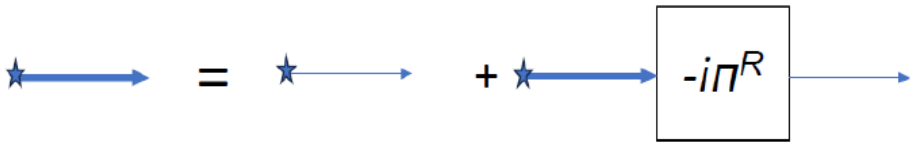
Two same sign baryon charges repulse each other

* Maxwell equations in medium :

$$\partial_x^\rho \partial_{x,\rho} A^\mu(x) = \frac{4\pi}{c} j^\mu \,, \qquad j^\mu = j_0^\mu(x) + j_{\rm ind}^\mu(x) \,,$$

$$D^{-1}_{0,\mu\nu}(x) = \frac{c}{4\pi}\delta_{\mu\nu}\partial_x^\rho \partial_{x,\rho} \,, \qquad A^\mu_{(0)}(x) = \int d^4x' D_0^{\mu\nu}(x-x') j_\nu^0(x') \,, \qquad j_{\rm ind}^\mu(x) = \int \Pi^{\mu\nu}(x,x') A_\nu(x') d^4x' \,,$$





For field *A* as were for *x* in case of pendulum

Radiation of soft photons by charged body (scattering of plane waves by oscillator)

$$\vec{E}_{\rm inc}(t,\vec{r}) = E_0\,\vec{e}_x\,\cos\left(\vec{k}\vec{r}-\omega\,t\right), \quad \vec{H}_{\rm inc}(t,\vec{r}) = \left[\vec{e}_y \times \vec{E}_{\rm inc}\right], \quad \vec{k} = |\vec{k}|\,\vec{e}_y, \quad \vec{\mathcal{P}}_{\rm inc(out)} = \frac{c}{4\pi}\left[\vec{E}_{\rm inc(out)} \times \vec{H}_{\rm inc(out)}\right]$$

Dipole radiation from piece of matter by direct reactions $|d_{\max}/e| \ll \lambda \ll r,$

$$\vec{d}(t) = ex(t) \simeq \frac{e^2\,E_0}{m}\,\frac{\cos\left(\omega\,t-\delta(\omega)\right)}{\sqrt{(\omega_R^2-\omega^2)^2+\gamma_{\rm tot}^2\,\omega^2}}\,\vec{e}_x\,, \gamma_{\rm tot} = \gamma + \gamma_{\rm rad}\cdot \quad \gamma_{\rm rad} = 2e^2\,\omega^2/(3\,m\,c^3)\,.$$

$$d\sigma = \frac{1}{|\vec{\mathcal{P}}_{\rm inc}|}\overline{dI}\,, \quad \frac{d\sigma}{d\Omega} = \left(\frac{e^2}{m\,c^2}\right)^2 \frac{\omega^4}{(\omega_R^2-\omega^2)^2+\gamma_{\rm tot}^2\,\omega^2}\,[\vec{e}_x \times \vec{n}_r]^2\,. \quad \frac{d\sigma}{d\Omega} = |f(\theta,\phi)|^2\,,$$

the scattering amplitude has only terms with $l = 1$

$$B_{\rm rad} = \gamma_{\rm rad}/\gamma_{\rm tot}, \quad \tan\delta_s(\omega) = \gamma_{\rm tot}\omega/(\omega_R^2-\omega^2)\,.$$

$$2|\vec{k}|f_1 = \frac{B_{\rm rad}\,\gamma_{\rm tot}\,\omega}{\omega_R^2-\omega^2-i\,\gamma_{\rm tot}\,\omega} = \frac{B_{\rm rad}}{\cot\delta_s(\omega) - i}$$

optical theorem is fulfilled.

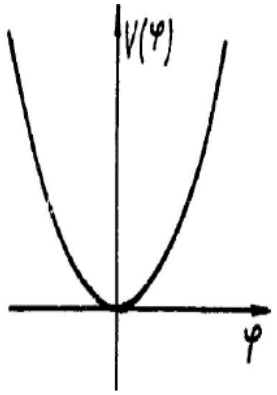
For $\omega \gg \omega_R$ $\quad \sigma_{\rm Tomp} = \frac{8}{3}\frac{e^4}{m^2c^4}$, For $\omega = \omega_R$ there is a resonance,

for $\omega \to 0$ we have $\sigma \propto \omega^4$, Rayleigh scattering (blue sky, yellow Sun, red on sunset)

Action and eq. of motion for self-interacting one-component scalar field $S_\phi = S_\phi[\partial_\mu\phi, \phi].$

q→ϕ, symmetry, simplicity+ locality, Lorentz scalar → $S_\phi = \int d^4x\mathcal{L}/c = \frac{1}{c}\int d^4x \left[\hbar^2\frac{\partial_\mu\phi\partial^\mu\phi}{2} - V(\phi(x^\mu))\right]\frac{c}{\hbar^3},$

Here Planck constant only determines convenient units of measurement of the new quantity, the scalar field



Potential $V(\phi)$ for the case $\mu^2 > 0$.

Power-law potentials--expansion $V(\phi) = \rho_{sc}\phi + \frac{c^2\mu^2\phi^2}{2} + \frac{\lambda_3\phi^3}{3} + \frac{\lambda_4\phi^4}{4} + ...$

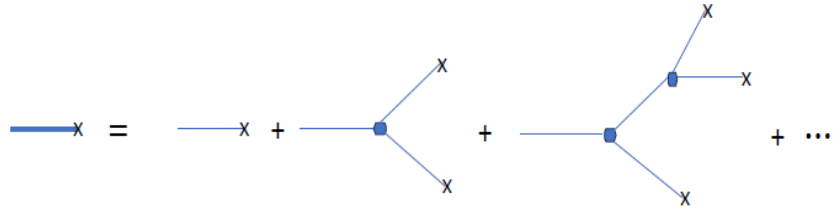
Vacuum $\phi = 0.$ In linear approx. excitations--waves $\left(\hat{\omega}^2 - \hat{\vec{k}}^2 - \mu^2\right)\phi = 0\,.$

$\omega_\pm^2(\vec{k}^2) - \vec{k}^2 = \mu^2\,,$ 2 solutions—wave (particle) and backward wave (antiparticle):

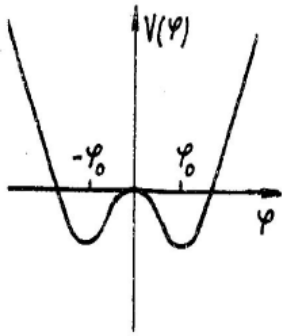
Interaction with scalar charge: $\Delta\phi - \mu^2\phi = Q_s\delta(\vec{r})\,.\; V_{Qq} = q_s\phi = -\frac{q_sQ_s}{4\pi r}e^{-\mu r}\,,$

Two same sign electric &baryon charges repulse each other. **Scalar charges of same sign attract each other.**

$\mathcal{L} = \frac{\partial_\mu\phi\partial^\mu\phi}{2} - \frac{\mu^2\phi^2}{2} - \frac{\Lambda_3\phi^3}{3} - \rho_s\phi\,,\quad \partial_\mu\partial^\mu\phi + \mu^2\phi + \lambda_3\phi^2 + \rho_s = 0\,.$



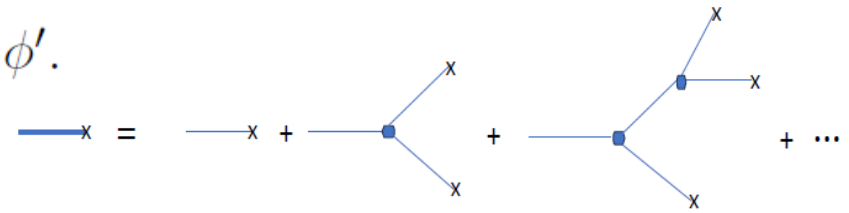
For pions, isospin *2T+1=3*, *T*-isospin, $\mathcal{L} = \frac{1}{2}\partial_\mu\vec{\phi}\,\partial^\mu\vec{\phi} - \frac{m_\pi^2}{2}\vec{\phi}^2 - \frac{\lambda_4}{4}(\vec{\phi}^2)^2\,.$



If one additionally assumes the symmetry respectively $\phi \to -\phi$,

$V = \mu^2\phi^2/2 + \lambda_4\phi^4/4$ Vacuum $\phi_0 = +\sqrt{-\mu^2/\lambda_4} > 0.$ $\mathcal{E} = -V_3\mu^4/(4\lambda_4) < 0\,,$ $V_3 = \int d^3x.$

Particles (waves) –excitations over vacuum $\phi = \phi_0 + \phi'.$



Let we deal with ordinary pions or sigma mesons (*m*>0) put in a scalar potential well (baryon matter U= *U(n)).*

$$\mathcal{L} = \frac{\partial_\mu\phi\partial^\mu\phi - (m^2+U)\phi^2}{2} - \frac{\lambda_4\phi^4}{4}\,,$$

For square potential well $m_{\rm ef}^2 = m^2 - U_0$ $\phi = \pm\phi_0\Theta(U_0 - m^2)$

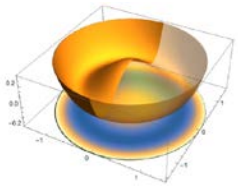
for $U_0 > m^2.$ $\phi_0 = (|m_{\rm ef}^2|/\lambda_4)^{1/2}\,.$ $\mathcal{E} = \int E d^3x = -\frac{m_{\rm ef}^4}{4\lambda_4}V_3\Theta(-m_{\rm ef}^2) + O(R^2)\,,$ $V_3 = \frac{4\pi R^3}{3}\,.$

$U_0 = U_{0c}$ can be treated as the critical point of the second-order phase transition $d^2\mathcal{E}/dU_0^2 \neq 0.$

Ehrenfest classification of phase transitions: labeled by the lowest derivative of the free energy which is discontinuous at the transition. Thus first-order phase transition exhibits a discontinuity in the first derivative of the free energy with respect to some thermodynamic variable, second-order transition -- second derivative of the free energy, etc.

→ **π condensate appears for $n>n_c>n_0$**

Phenomenological mean-field theory of II-order phase transitions



Expansion of free-energy *F(V,T)* in order parameter and coefficients in closeness to crit. point.

Superfluidity (order parameter-complex macro. wave funct.) $F[\psi,T]=\frac{|\nabla\psi|^2}{2m^*(T_c)}+\alpha(T_c)(T-T_c)|\psi|^2+\beta(T_c)|\psi|^4/2+\ldots,$

$\alpha=\alpha(T_c)(T-T_c)=m_{\rm ef}^2\,,\quad \alpha(T_c)>0,\quad |\psi|^2=\alpha(T_c)\theta(T_c-T)/\beta(T_c)\,.$ Specific heat jumps in crit. point.

Superconductivity (order parameter is pairing gap) $F[\psi,\vec{A}]=\frac{|(\nabla-iq\vec{A})\psi|^2}{2m^*}+\alpha(T_c)(T-T_c)|\psi|^2+\beta(T_c)|\psi|^4/2+\frac{(\mathrm{curl}\vec{A})^2}{8\pi}\,.$

Strong fluctuations near crit. point. Ginzburg criterion result in divergent term in specific heat.

$W\sim e^{-\delta F(V_{\rm fl})/T}$ becomes ~ 1 for $\delta F(V_{\rm fl})\sim T_{\rm fl}$ where $\delta F(V_{\rm fl})$ is the work necessary to prepare the fluctuation in the minimal volume $V_{\rm fl}=4\pi l_0^3/3$ for fluctuation,

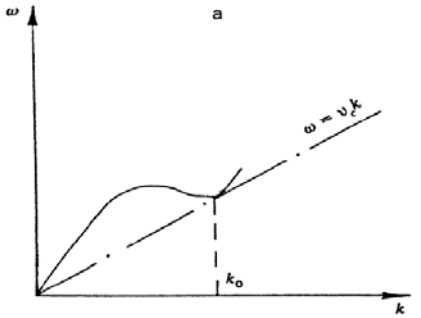
$l_0=\sqrt{2}/\sqrt{\alpha(T_c-T)}$ Ginzburg number $Gi=|T_{\rm fl}-T_c|/T_c.$ $\alpha|T_c-T_{\rm fl}|\simeq 9T_c^2\lambda^2/(8\pi^2)\,.$

With inclusion of fluctuations in 4He $F[\psi,T]=\frac{|\nabla\psi|^2}{2m^*(T_c)}+\alpha(T_c)|T-T_c|^{\mu}|\psi|^2+\beta(T_c)|T-T_c|^{\nu}|\psi|^4/2+\ldots,$

μ=4/3, ν=2/3

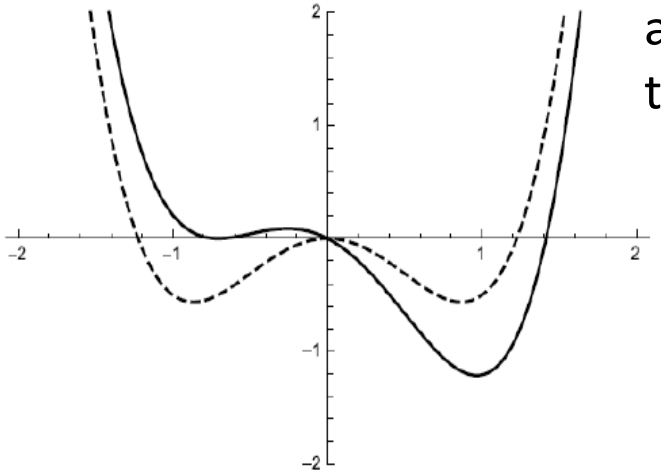
Condensate of rotons with k≠0 in moving 4He with $V>V_c$

$\phi'=\phi'_0e^{-i\omega(\vec{k}_0)t+i\vec{k}_0\vec{r}}$ $V_{\rm in}>V_c^{\rm L}=\omega(k_0)/k_0$



$\delta E^{\rm kin}=-\frac{k_0^2(V_{\rm in}-V_c^{\rm L})^2}{2\tilde{\lambda}}\theta(V_{\rm in}-V_c^{\rm L})\,.$

First-order phase transitions



Solid line–potential $V(\phi)$ for $\mu^2 < 0$ in absence of the symmetry $\phi \to -\phi$, dashed line–symmetrical potential.

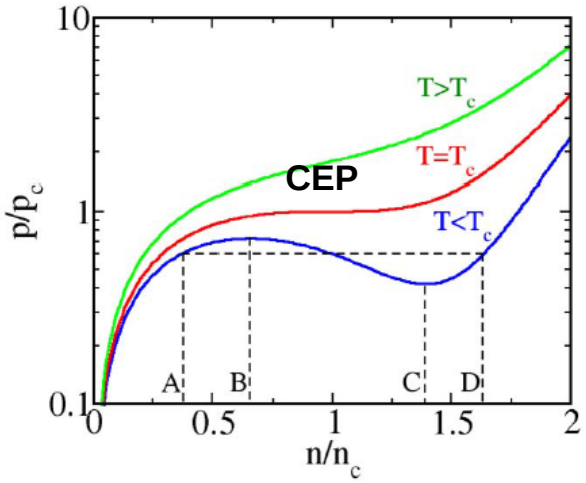
m.b. Multiverse ``sleeps'' in metastable state respectively a scalar field. If a seed has an under-critical size it melts, if it has overcritical size, it grows to stable state. The critical size of the seed, R_{cr}, can be found from relation

$$(\epsilon_{\rm st} - \epsilon_{\rm meta})V_{\rm seed}(R) + \sigma S_{\rm seed}(R) = 0,$$

The energy gain is released in heating of stable phase and the radiation. Picture is analogous to what occurs at the first-order liquid-gas phase transition.

$$E = \dot{\phi}\frac{\partial \mathcal{L}}{\partial \dot{\phi}} - \mathcal{L} = \frac{\dot{\phi}^2}{2} + \frac{(\nabla\phi)^2}{2} + \frac{\mu^2\phi^2}{2} + \frac{\lambda_4\phi^4}{4} + \rho_s\phi\,.$$

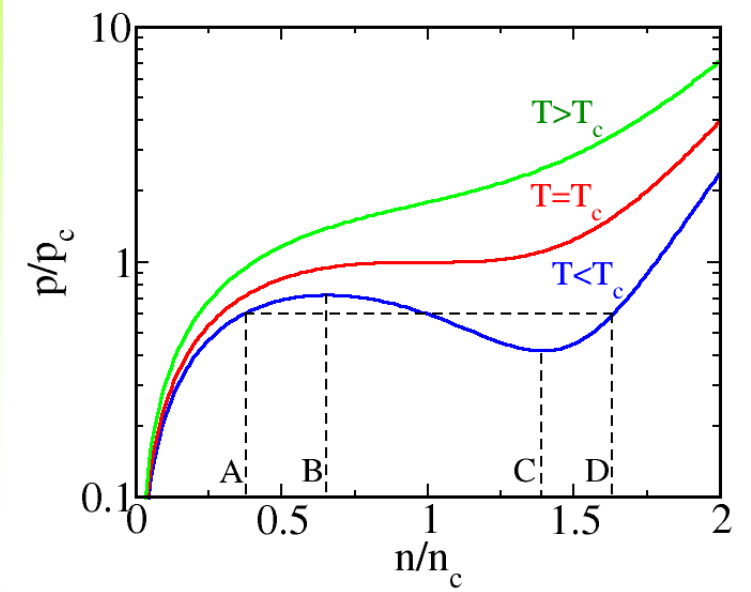
I-order vapor-liquid phase transition (order-parameter is density)



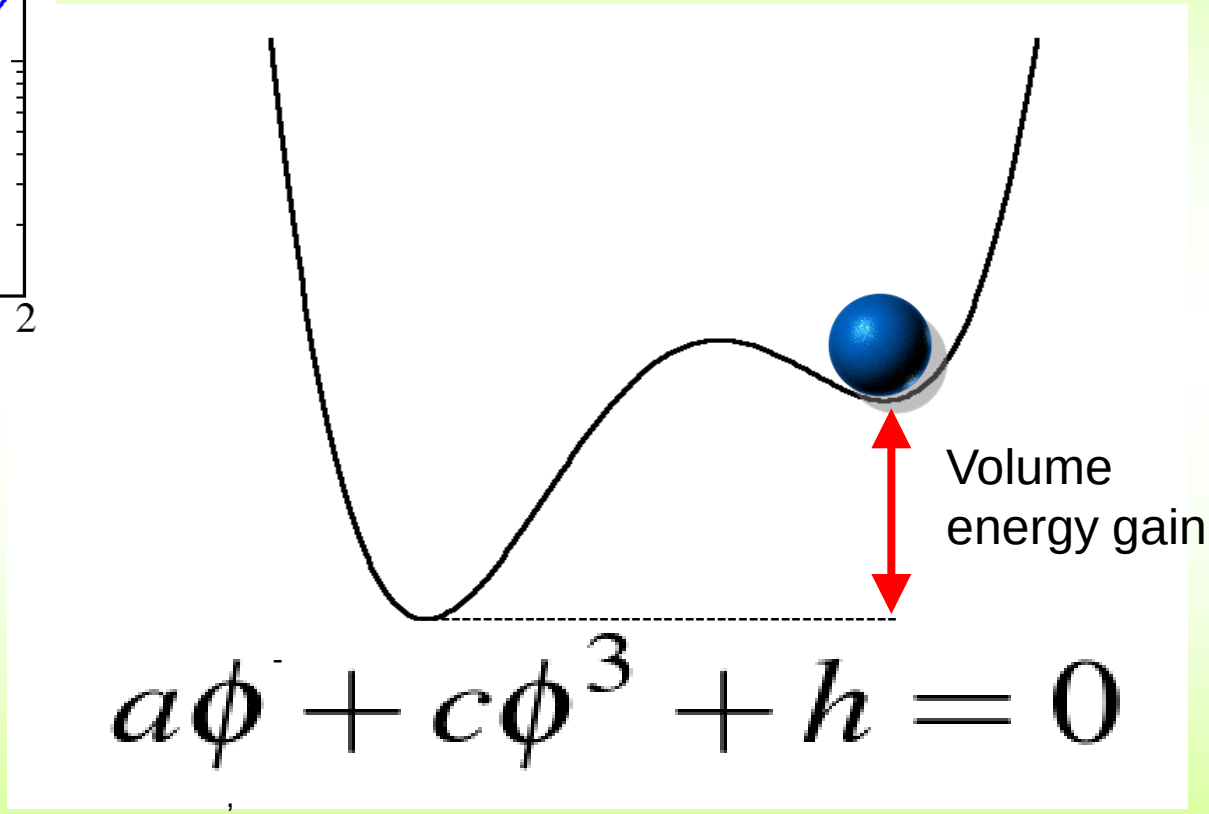
$$\delta\mathcal{F}_{\rm L} = \int \frac{d^3x}{\rho_{\rm cr}}\left\{\frac{c[\nabla(\delta\rho)]^2}{2} + \frac{\lambda[(\delta\rho)^2 - v^2]^2}{4} - \epsilon(\delta\rho + v)\right\},$$

Schematic pressure isotherms as functions of the density at a liquid--vapor-like phase transition. Horizontal dashed line shows the equilibrium Maxwell construction.

I order phase transition



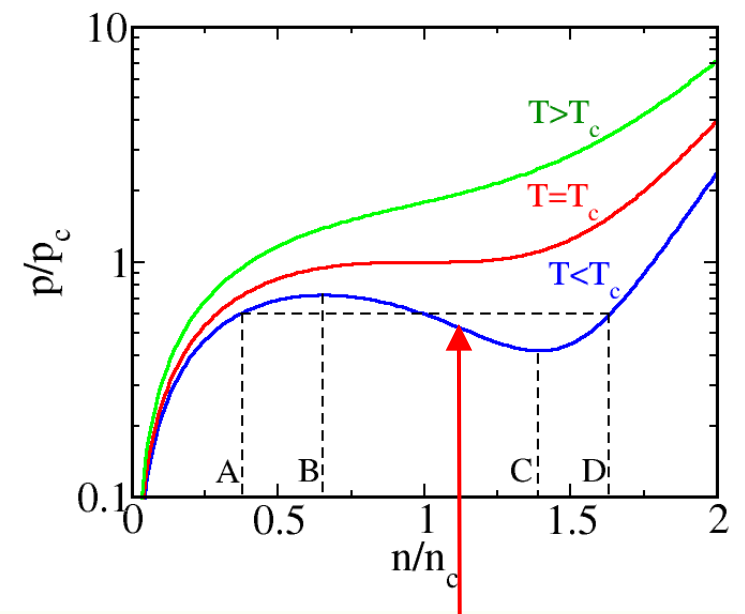
F (φ)



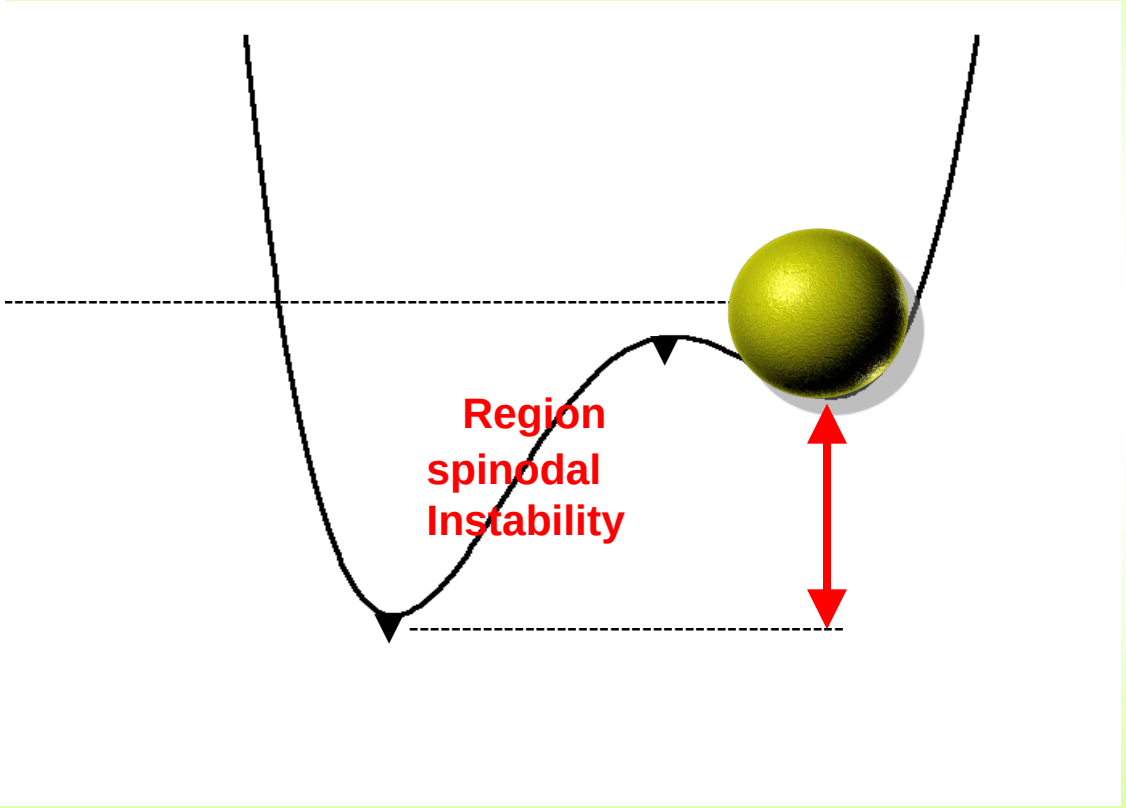
$$a\phi + c\phi^3 + h = 0$$

Surface energy loss

I order phase transition



F(φ)





In spinodal region perturbations of any size grow exponentially

Fluctuations. Normalized variance of the baryon number

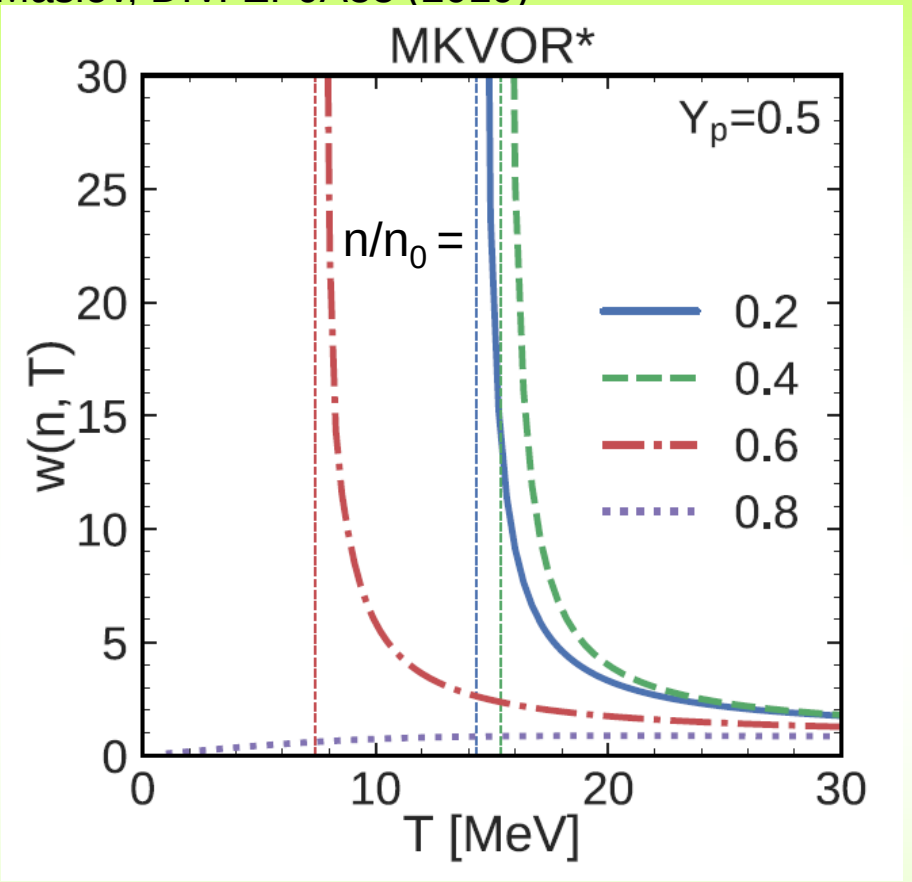
Maslov, D.V. EPJA55 (2019)

In equilibrium matter

$$w = \frac{\overline{(\Delta N)^2}}{N} = \frac{T}{n}\left(\frac{\partial n}{\partial \mu_B}\right)_T \qquad K(T,n) = \frac{1}{n\,\kappa_{\rm iso}} = n\frac{\partial \mu}{\partial n}\bigg|_T$$

diverges on ITS line

Higher fluctuation momenta – skewness and kurtosis are stronger diverging than variance at crossing of isothermal spinodal line



In HIC effects of crossing of ITS line m.b. easier to observe via increased fluctuations of conserved charges and quasiperiodic structures and anomaly in production of light nuclei (nuclear rain) rather than via effects of the closeness to the critical point due to a critical slowing down in latter case,

What one may observe if system enters spinodal region

is a structured phase with fluctuations at k ≠0

Looks like a curved spine

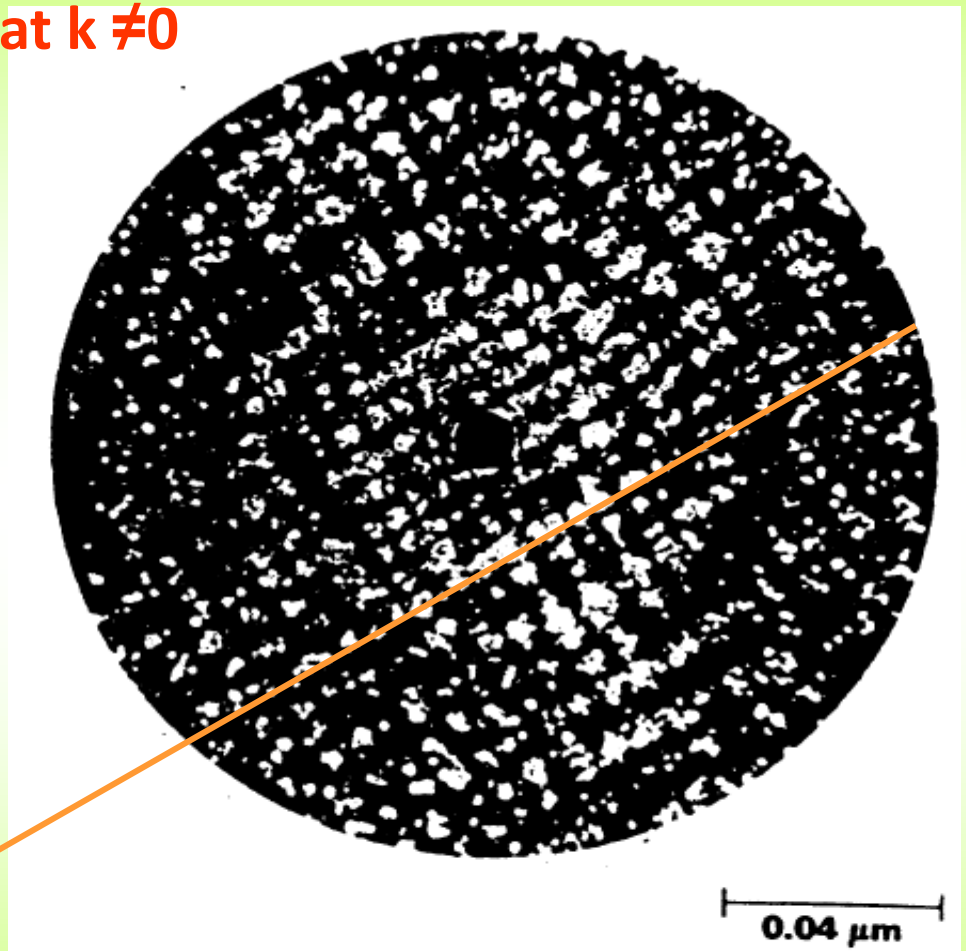


Fig. 9 Field-ion micrograph of spinodally decomposed Fe-25Be (at.%) alloy aged 20 min at 400 °C (750 °F). The axis of the needle-like specimen is [001]. The iron-rich phase images brightly because of the different contrast mechanism operating in the field-ion microscope. 375 000×. (M.K. Miller)

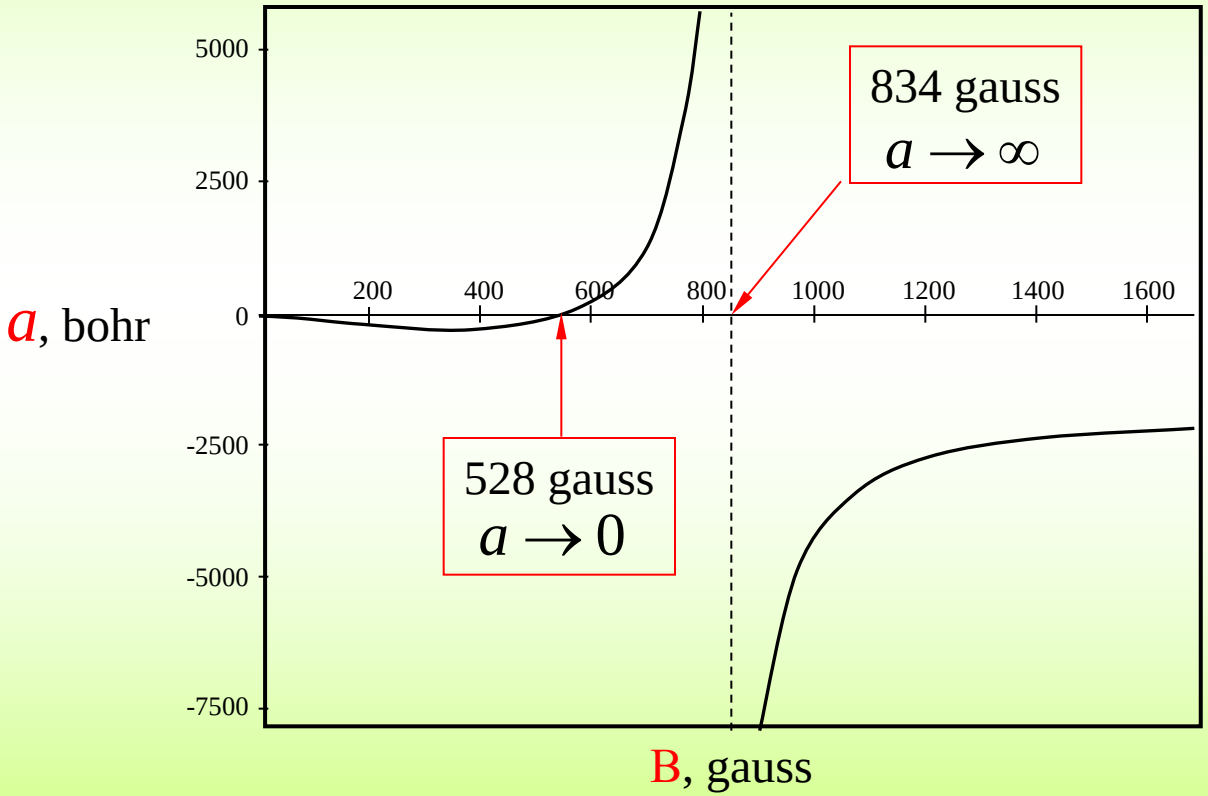
Ultracold atoms typically below several tens of microkelvin

as in nucl. systems: elliptic flow, superfl.,vortex structures, viscosity

In nuclear matter strength of interaction is changed by varying *n, T*

Scattering length a, as function of magnetic field B

$$\lim_{k\to 0} \sigma_e = 4\pi a^2 .$$



Varying mag. field, one changes strength and sign of interaction of atoms

Microscopic and phenomenological EoSs of nuclear matter

Bruekner gas expansion, Paris, Bohn etc potentials fitted from (Nijmegen) exp. phase shifts, variational APR theory,

chiral perturbation theory works up to ~n_0

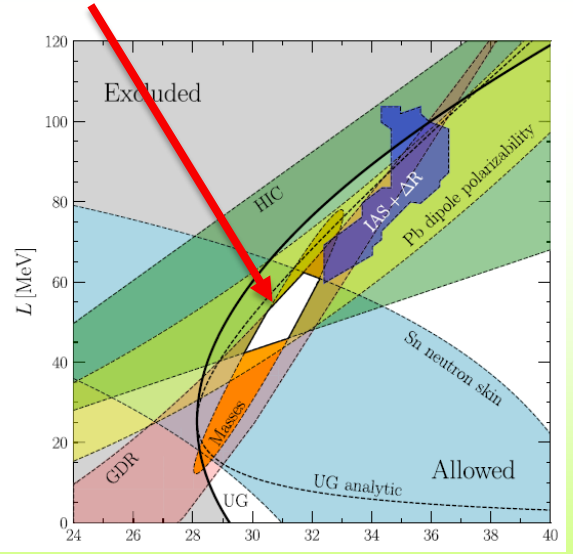
$$E = a_V + \frac{K}{18}\epsilon^2 - \frac{K'}{162}\epsilon^3 + \ldots + \beta^2\left(J + \frac{L}{3}\epsilon + \ldots\right)$$

$\epsilon = (n - n_s)/n_s$ $\beta = 1\text{–}2\,x$ $x = n_p/n$ $n = n_n + n_p.$

Dirac-Bruekner-Hartree-Fock model, RMF models.

$n_0 \simeq 0.16 \pm 0.015\ \mathrm{fm}^{-3}$ $E_0 \simeq -15.6 \pm 0.6\,\mathrm{MeV}$

$K = 240 \pm 20\,\mathrm{MeV}$ J=29-32 MeV, *L* is less determined

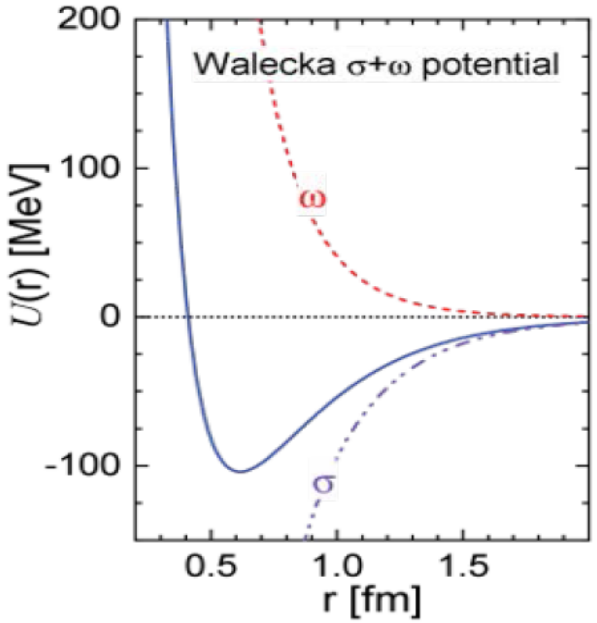


Triple role of mesons: mediators of *baryon-baryon* interaction, condensates of classical fields, collective excitations over mean fields

Relativistic mean-field models of nuclear equation of state

Scalar charges attract each other, vector field charges –repulse→ Potential between nucleons.

Response of classical scalar and vector fields on the presence of sources of scalar and baryon charges can be treated as appearance of **condensates of mean scalar and vector fields.** → RMF model of EoS of nuclear matter.



The NN potential of the Walecka $\sigma - \omega$ model.

Original Walecka model $\mathcal{L}_{N\sigma\Omega} = \frac{\partial_\mu\sigma\partial^\mu\sigma}{2} - \frac{m_\sigma^2\sigma^2}{2} + \bar{N}(i\gamma^\mu\partial_\mu - m_N)N - g_{\sigma N}\bar{N}N\sigma$

$$-\frac{\Omega_{\mu\nu}\Omega^{\mu\nu}}{4} + \frac{m_\Omega^2\Omega_\mu\Omega^\mu}{2} - g_{\Omega N}\bar{N}\gamma^\mu\Omega_\mu N\,,$$

$$m_N^* = m_N + g_{\sigma N}\sigma = m_N - \rho_{s,N}g_{\sigma N}^2/m_\sigma^2$$

Two coupling constants are fitted to describe nuclear binding and saturation density

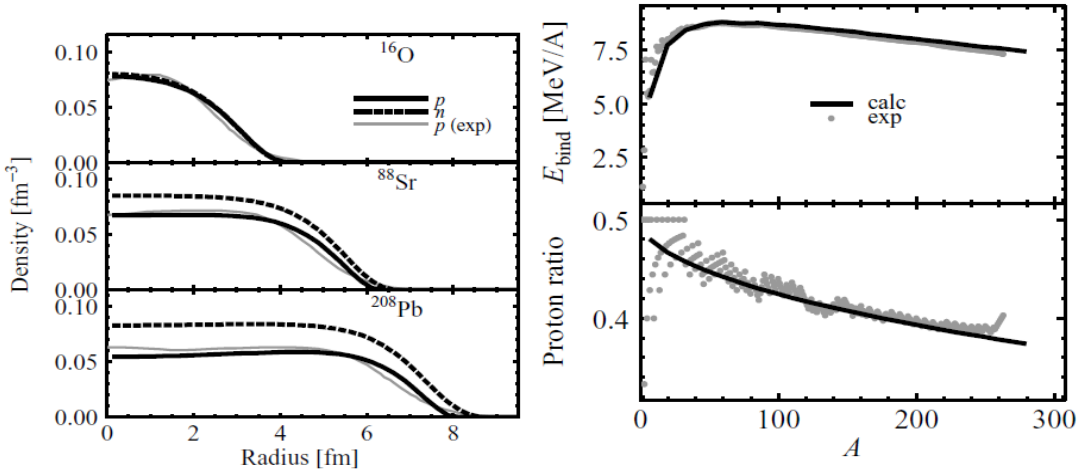
Modified Walecka model $U = b\sigma^3/3 + c\sigma^4/4$ + rho meson term

Three extra parameters are fitted to describe effective nucleon mass, compressibility and symmetry energy at nuclear saturation.

In application of a RMF to atomic nuclei, gradient terms are important

Maruyama,Tatsumi,D.V.,Tanigava,Chiba 2005,2006

+structure: Geng, Toki, Meng, Prog. Theor. Phys., 2005



RMF model with σ-field-scaled hadron masses & couplings

T=0, N>>Z Kolomeitsev, DV. 2005, T≠0, N=Z Khvorostukhin, Toneev, DV 2007, generalized to systems with arbitrary N,Z: cf. Kolomeitsev, Maslov, D.V. 2015-NPA 961 (2017), **basing on idea of partial chiral symmetry restoration**

Besides nucleons, hyperons and Δ isobars and some other resonances were included in EoS

cf. Ginzburg-Pitaevskii model of He II

$$\mathcal{L} = \mathcal{L}_{\rm bar} + \mathcal{L}_{\rm mes} + \mathcal{L}_l,$$

$$\mathcal{L}_{\rm bar} = \sum_{i=b\cup r} (\bar{\Psi}_i \left(i D_\mu^{(i)} \gamma^\mu - m_i \Phi_i(\sigma) \right) \Psi_i,$$

$$D_\mu^{(i)} = \partial_\mu + i g_{\omega i} \chi_{\omega i}(\sigma) \omega_\mu + i g_{\rho i} \chi_{\rho i}(\sigma) \vec{t} \vec{\rho}_\mu + i g_{\phi i} \chi_{\phi i}(\sigma) \phi_\mu,$$

$$\{b\} = (N, \Lambda, \Sigma^{\pm,0}, \Xi^{-,0}, \Delta^-, \Delta^0, \Delta^+, \Delta^{++})$$

$$\mathcal{L}_{\rm mes} = \frac{\partial_\mu \sigma \partial^\mu \sigma}{2} - \frac{m_\sigma^2 \Phi_\sigma^2(\sigma) \sigma^2}{2} - U(\sigma) +$$

$$+ \frac{m_\omega^2 \Phi_\omega^2(\sigma) \omega_\mu \omega^\mu}{2} - \frac{\omega_{\mu\nu}\omega^{\mu\nu}}{4} + \frac{m_\rho^2 \Phi_\rho^2(\sigma) \vec{\rho}_\mu \vec{\rho}^\mu}{2} - \frac{\rho_{\mu\nu}\rho^{\mu\nu}}{4} +$$

$$+ \frac{m_\phi^2 \Phi_\phi^2(\sigma) \phi_\mu \phi^\mu}{2} - \frac{\phi_{\mu\nu}\phi^{\mu\nu}}{4},$$

$$\omega_{\mu\nu} = \partial_\nu \omega_\mu - \partial_\mu \omega_\nu, \quad \vec{\rho}_{\mu\nu} = \partial_\nu \vec{\rho}_\mu - \partial_\mu \vec{\rho}_\nu + g_\rho \chi'_\rho [\vec{\rho}_\mu \times \vec{\rho}_\nu],$$

$$\phi_{\mu\nu} = \partial_\nu \phi_\mu - \partial_\mu \phi_\nu,$$

$$\mathcal{L}_l = \sum_l \bar{\psi}_l (i \partial_\mu \gamma^\mu - m_l) \psi_l, \quad \{l\} = (e, \mu).$$

$$\Phi_H(f) = 1 - x_{\sigma H} \frac{m_N}{m_H} \xi_{\sigma H} f,$$

where $x_{\sigma H} = g_{\sigma H}/g_{\sigma N}$ and $\xi_{\sigma H}(f) = \chi_{\sigma H}(f)/\chi_{\sigma N}(f)$.

$$f = g_{\sigma N} \chi_{\sigma N}(\sigma) \sigma / m_N$$

$$\Phi_N(f) = \Phi_m(f) = 1 - f,$$

vac. couplings follow SU(6)

For infinite matter thermodynamic quantities depend only on ratios

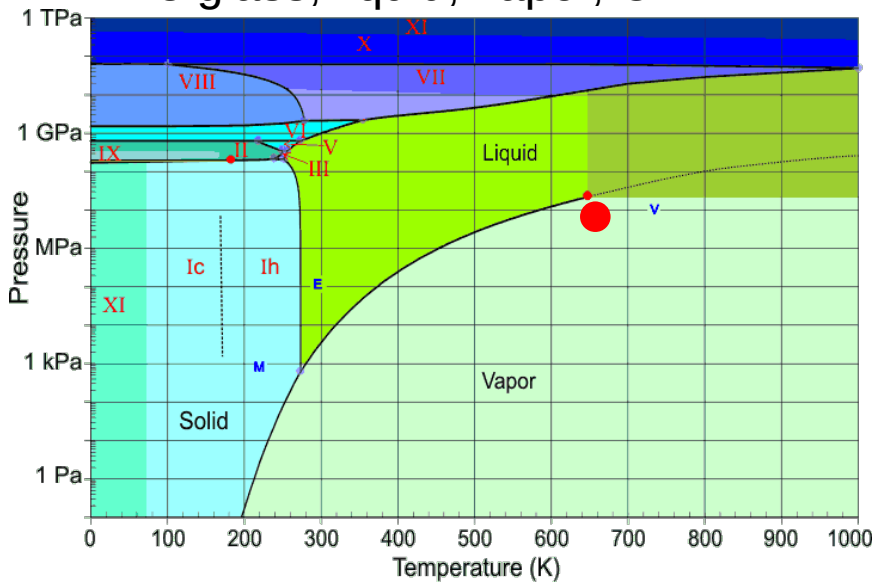
$$\eta_m(f) = \Phi_m^2(f) / \chi_{mN}^2(f)$$

Phase Diagrams

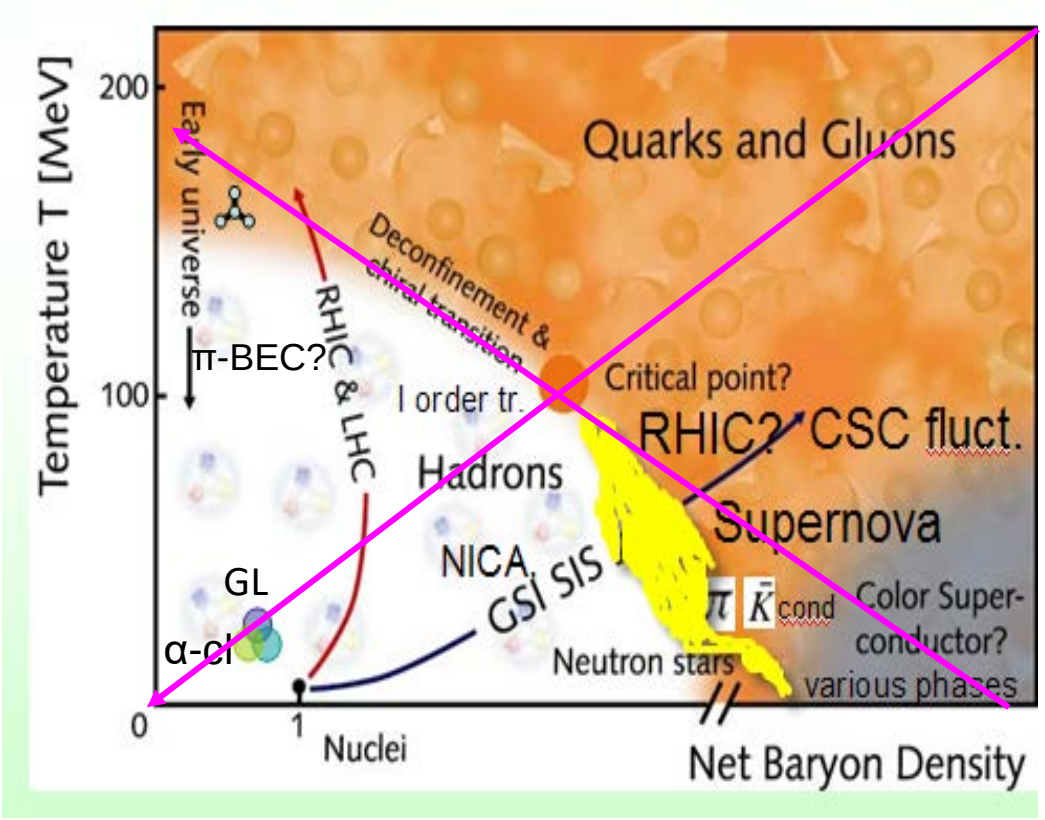
Water and Nuclear Matter

various possible phases

Variety of phases: 12 crystalline, 3 glass, liquid, vapor, CEP

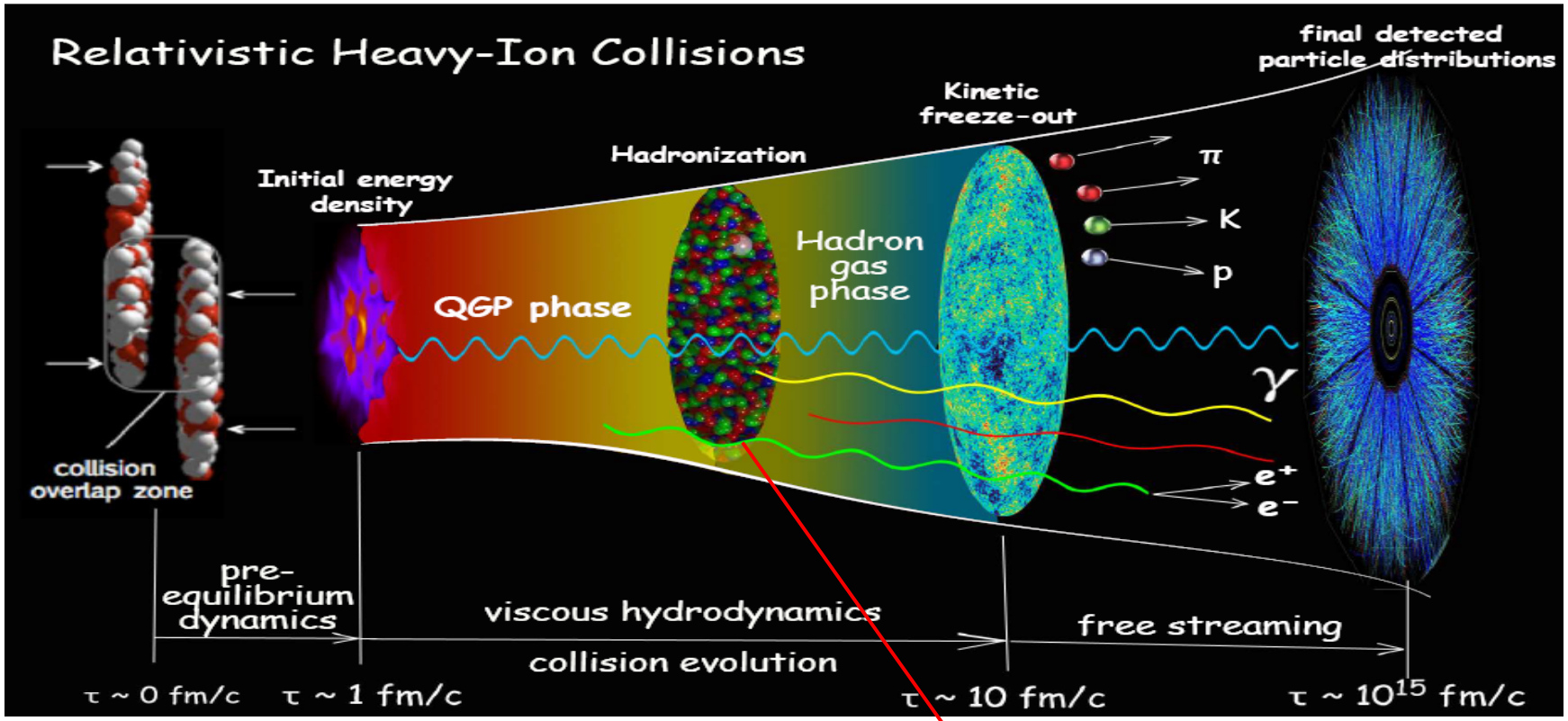


Chapline et al. (2007)



High T (T_c>150MeV), low baryon density $n < n_0 \approx 2.7 \cdot 10^{14}$ g/cm^3: deconfinement ph. tr.- crossover in *early Universe, in HIC at RHIC, LHC. W*ith growing n, decreasing *T*–CEP I order phase tr.- **challenge for NICA to seek it,** mixed QH pasta phase; Low T, low n: *HIC:* α-cl, nuclear GL ph.tr.; Low T, $n \approx n_0$: *excited nuclei:* NN pairing; *π/K atoms & hyper-nuclei*, Low T, high density*: Star mergers*,***SN,NS*:** nuclear pasta, BB-pairing, π,K,ρ condensates; *hybrid stars:* CSC, *hypothetical* metastable or stable objects: π, σ cond., strangelet drops, nuclearites, quark stars, DM objects (?)

Evolution of fireball created in RHIC



From Taranenko talk

"Tomography of fireball": various particle species measure different stages of nuclear fireball

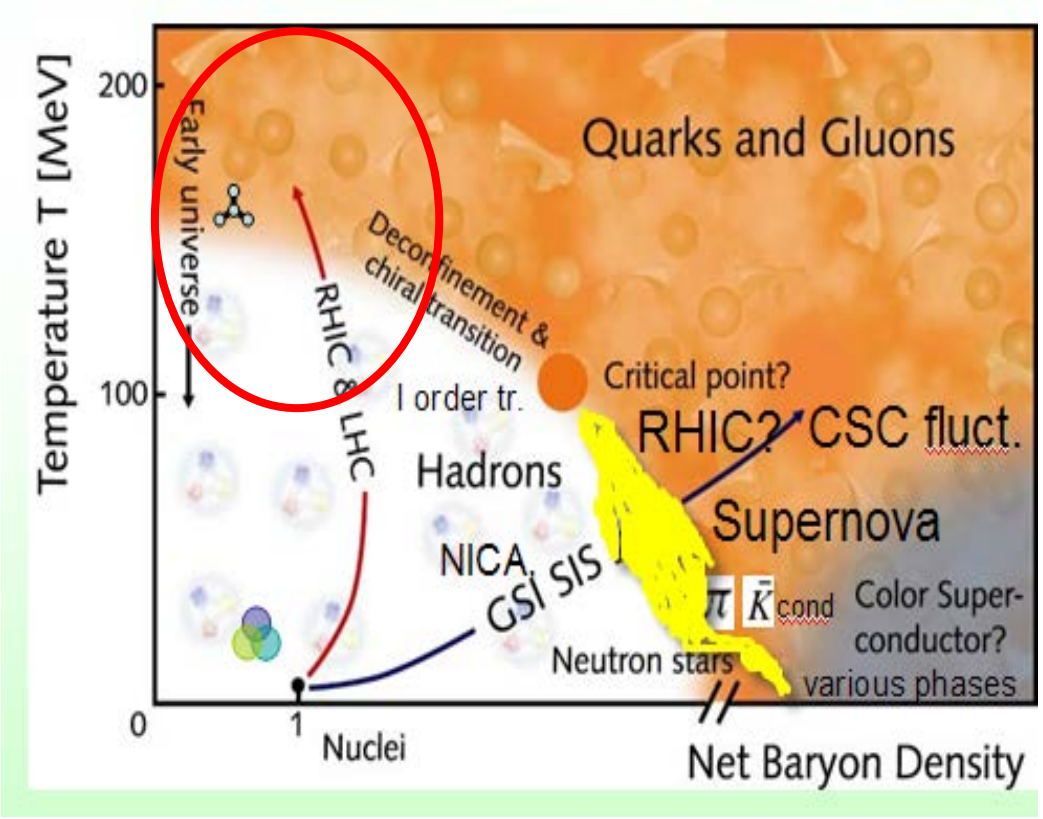
Long path-length particles (ee^+, $\mu\mu^+$, photons) fly from initial stage ,

K^+ , π with $k<1.5\ m_\pi$ fly from intermediate stage,

Short path-length particles (π with $k>1.5\ m_\pi$, N, K^-) measure late "freeze out" stage

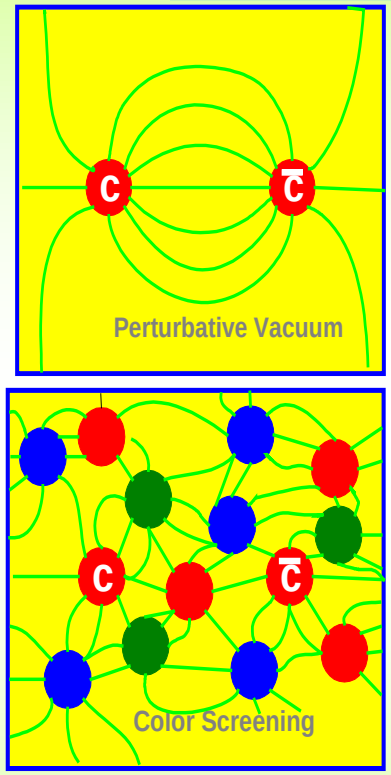
High *T*, rather low baryon density (baryon chemical potential)

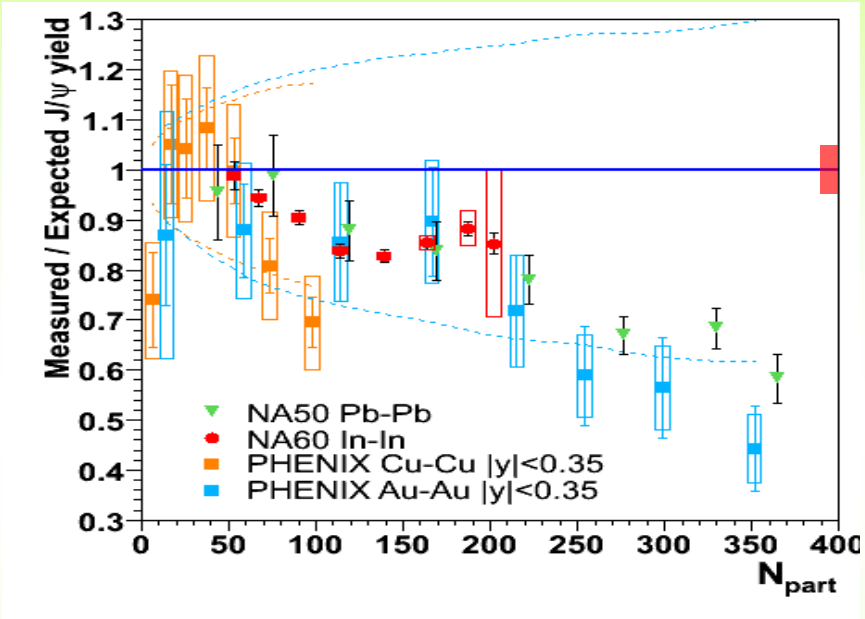
Hot QGP in early Universe and at LHC, crossover transition to hadron hot liquid



Why one says about QG state in ultrarelativistic HIC?

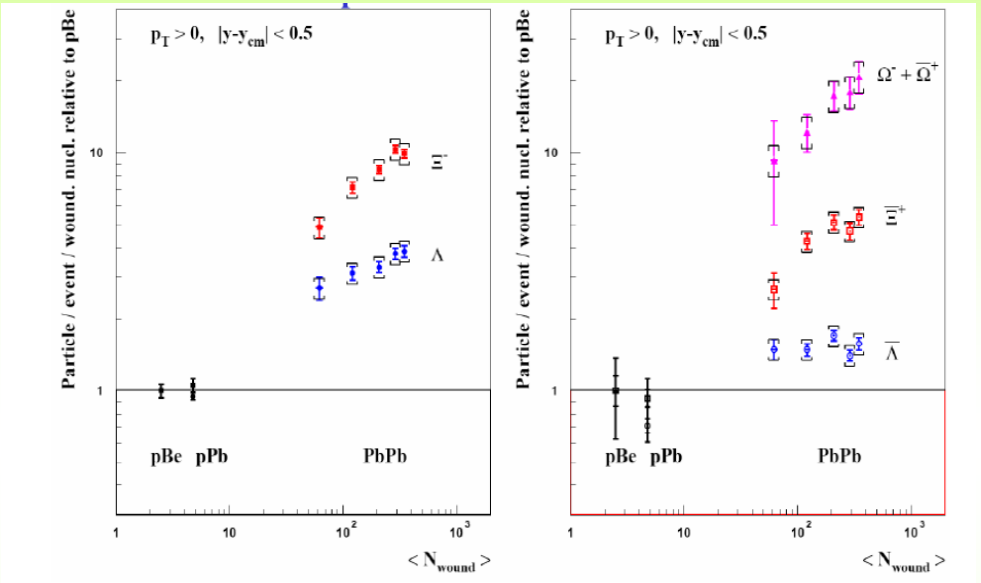
J/ψ suppression





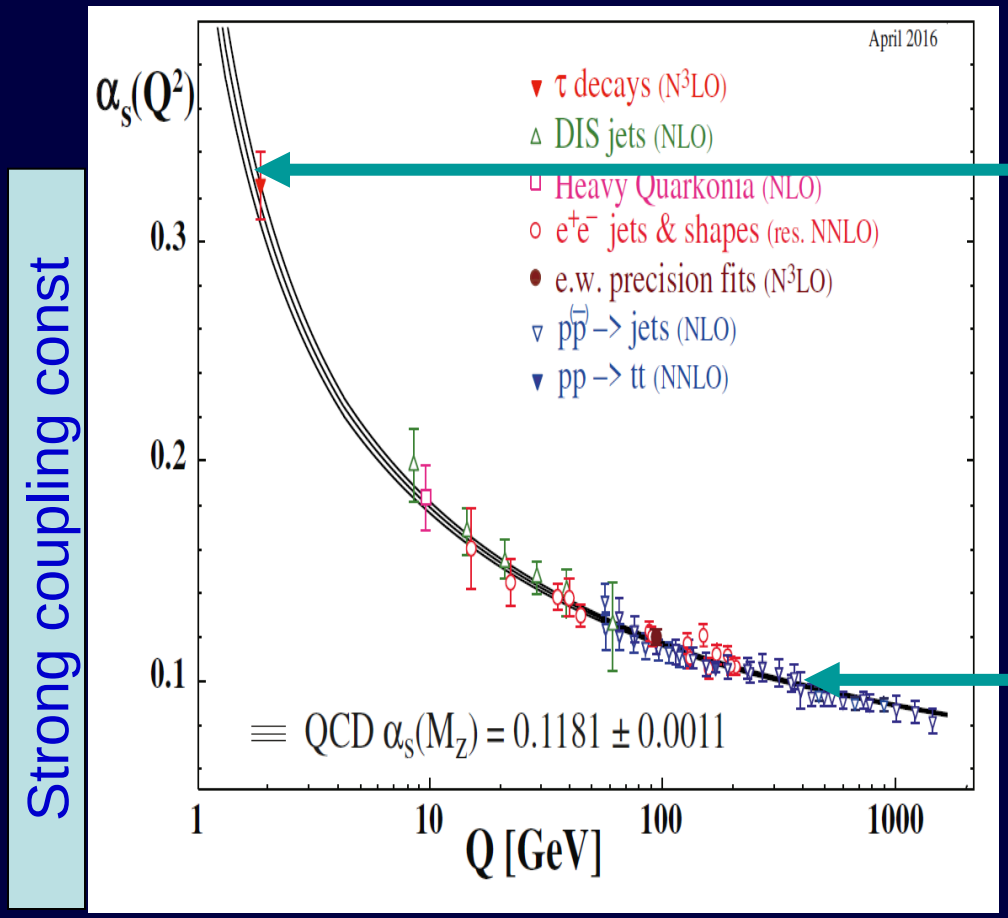
Dissociation of bounded cĉ pair in QGP

Strangeness enhancement



Interaction of energetic parton with quark medium: yield of particles at large $p_\perp$ is suppressed

Strong interaction running coupling constant



Strong coupling const

energy →

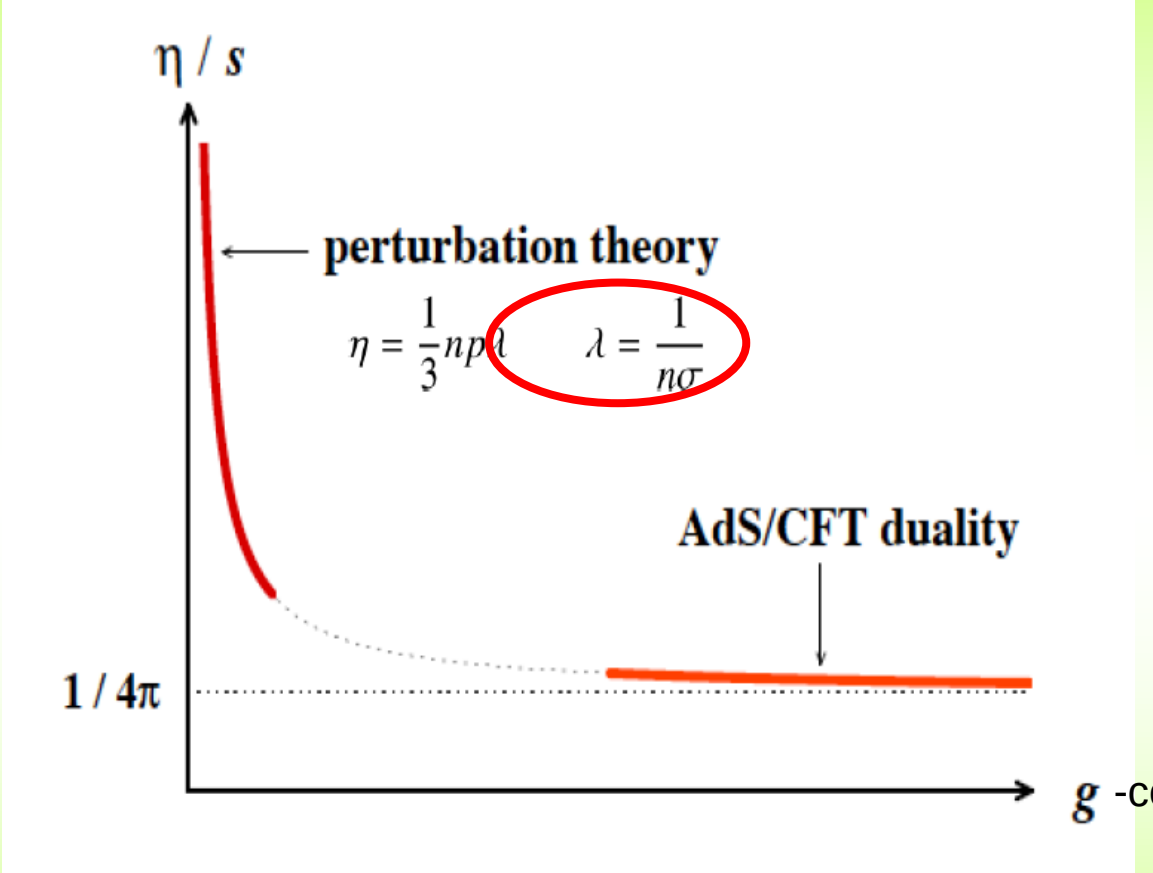
← distance

confinement
At large distance;
Quantum number – ***color.***

Asymptotic freedom
at small distance
(r<<L~0.2 fm, g →0)
Free quarks of small mass

Tanabashi et al 2018

Ratio of shear viscosity to entropy density



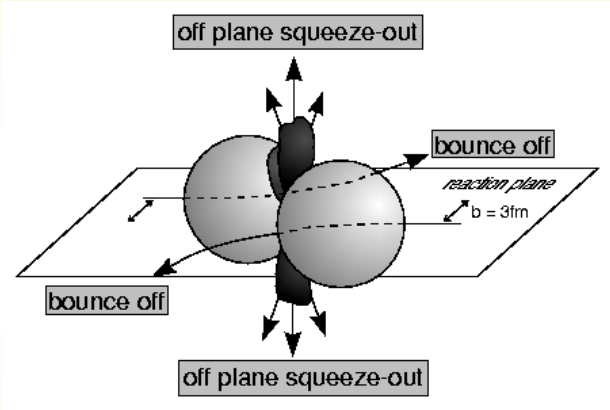
η/s [ADS/CFT]=1/4π absolute minimum

maps QGP to black hole formation in ADS_5 space, cf. arXiv 2211.16265

pQGP or sQGP? Crossover region: nonideal hydro

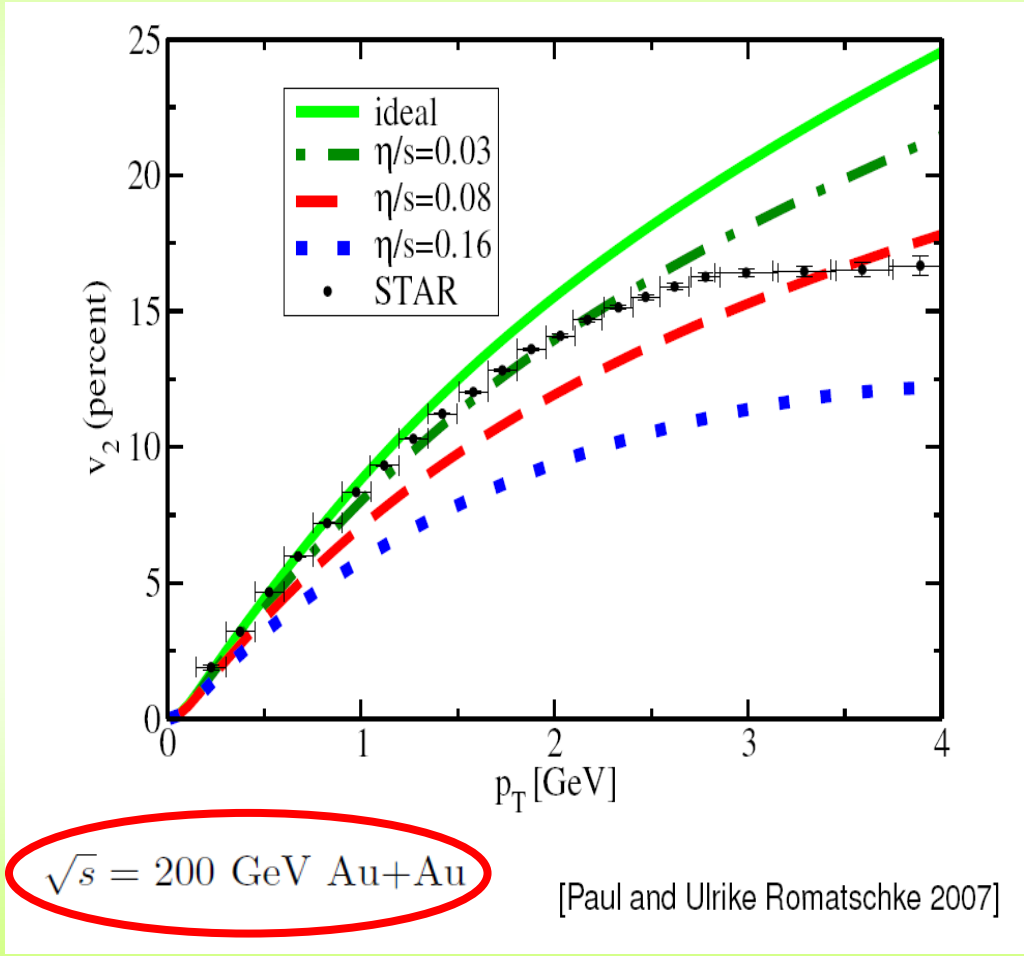
η/s – ratio of shear viscosity to entropy density

Matter is smashed in perp. direction
→ collective nucleon flow

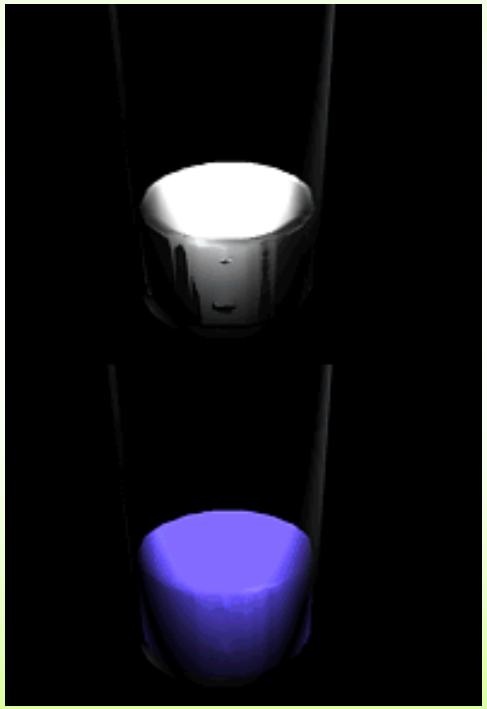


From Taranenko talk

coefficients $\mathbf{v_n}$ quantify anisotropic flow



Role of viscosity



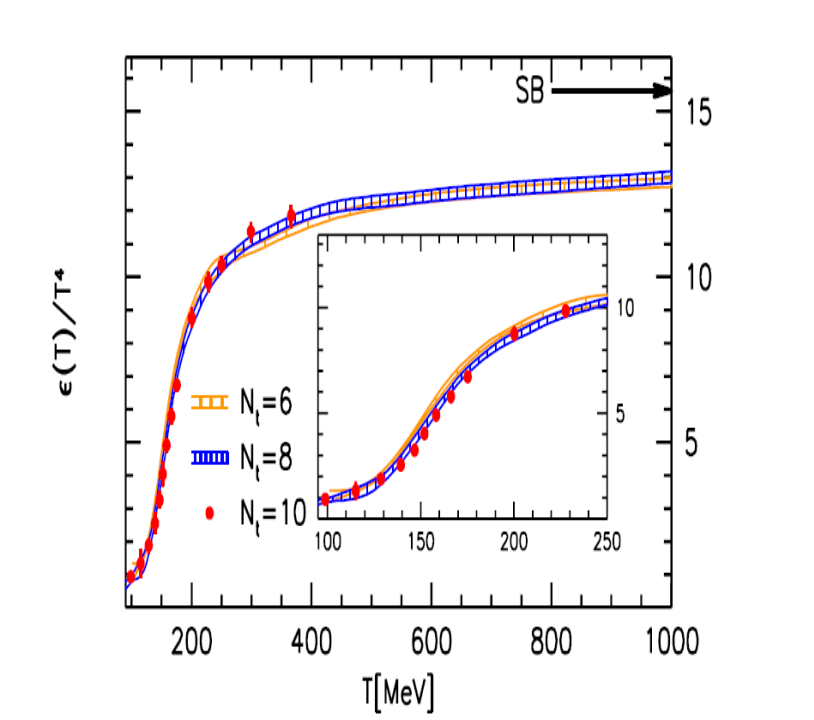
A small viscosity is required ! → **Almost ideal liquid**

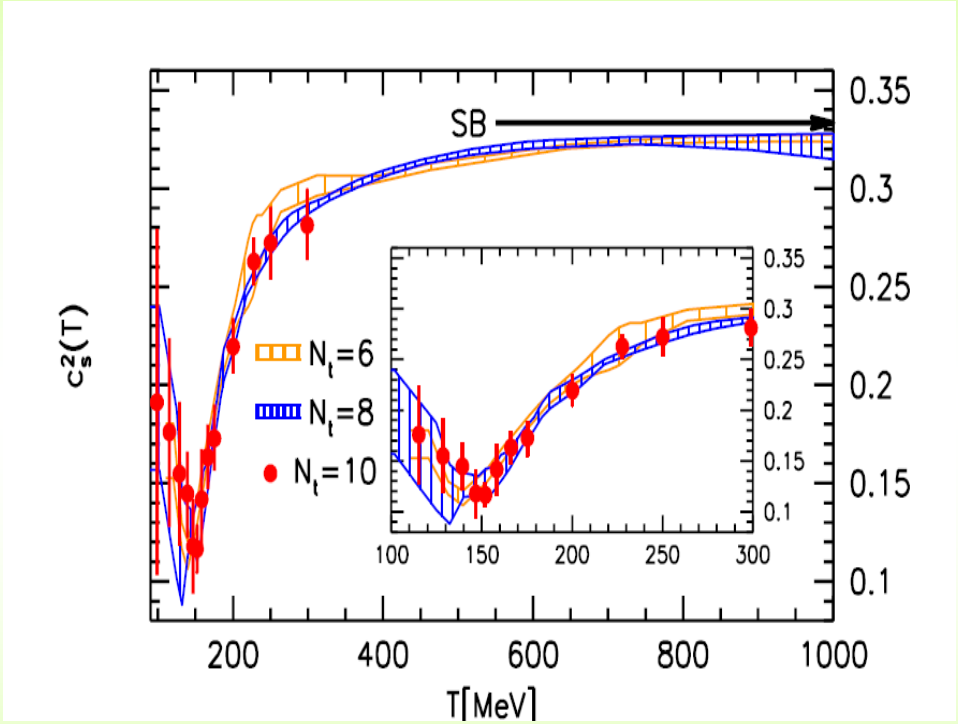
Elliptic **flow** for N, **π**, kaons, hyperons, even for photons (at LHC)

QCD on lattice

(numerical experiment on computers)

μ_B=0 - crossover





Black-body ideal gas limit is not reached ⟹ **strongly interacting QGP– liquid, not gas!**

State of pQGP is expected only for $n_B>40n_0$ or for $T> 3\ T_c$

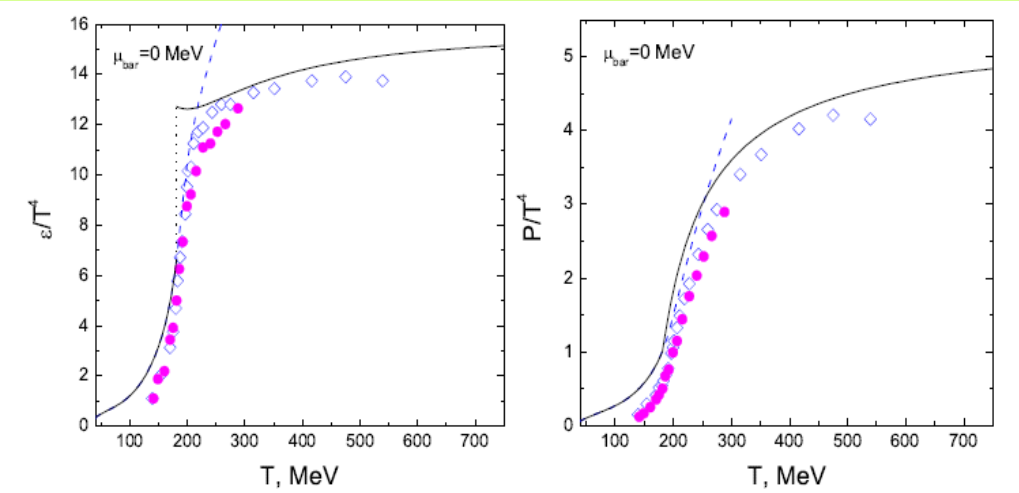
Shear and bulk viscosities are also calculated on lattice

Comparison of our RMF EoS with lattice data

energy density

pressure

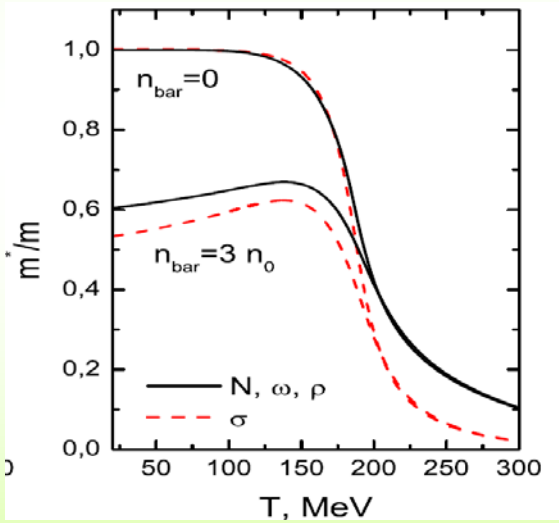
effective hadron mass



A.Bazavov et al., Phys. Rev. **D80,** 014504 (2009)

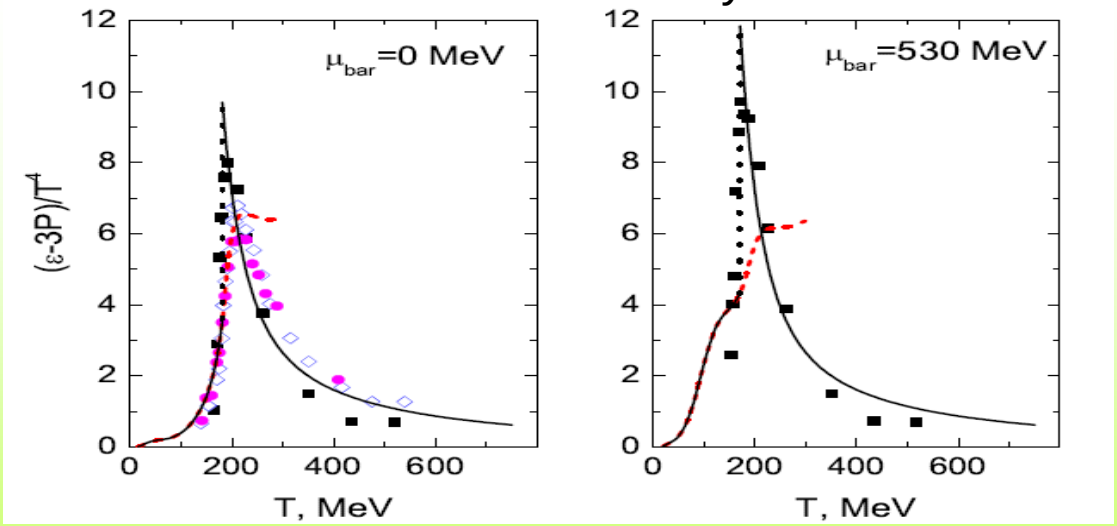
Z.Fodor et al., Phys. Lett. **B568**, 73 (2003)

p4 action
asqtad action
N_t=8
~phys. masses



Curves: our RMF σ-scaled model
Khvorostukhin, Toneev, D.V. 2007-2013

Trace anomaly

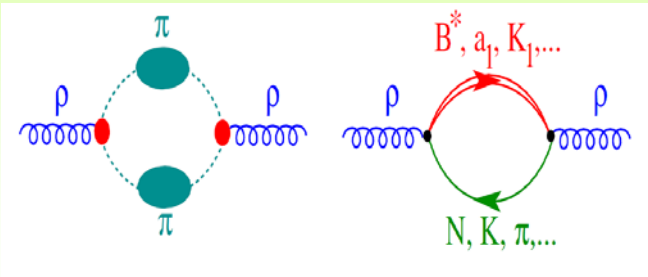


The considered baryon set is $\{b\} = N(938)$, $\Delta(1232)$, $\Lambda(1116)$, $\Sigma(1193)$, $\Xi(1318)$, $\Sigma^*(1385)$, $\Xi^*(1530)$, and $\Omega(1672)$, including antiparticles. The used σ-field dependent effective masses +couplings

Dileptons in HIC at energies SIS/HADES - RHIC-BES

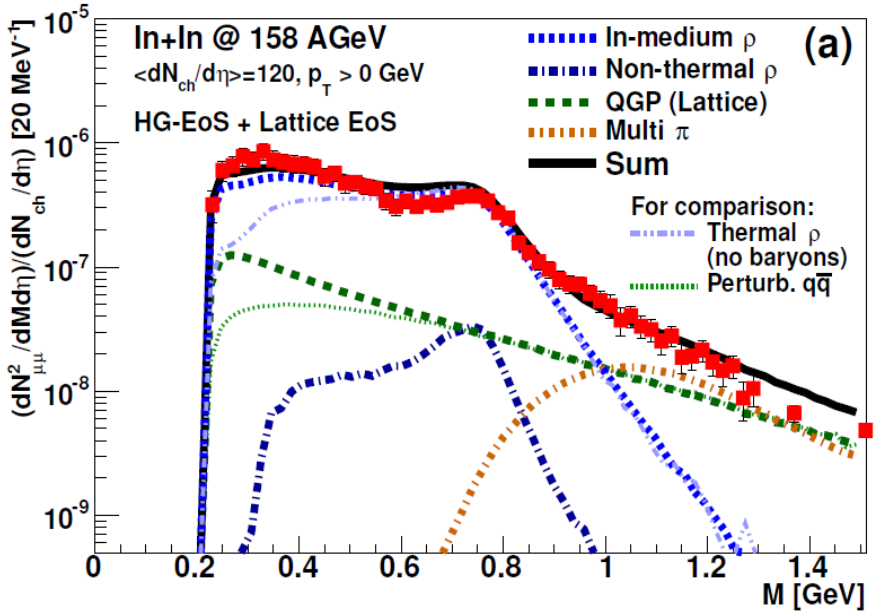
A general description (see vanHees Lect.2016) is **similar to that for neutrino emissivity from NS**, and direct soft photon radiation in HIC cf. Knoll, DV AnPh 1996

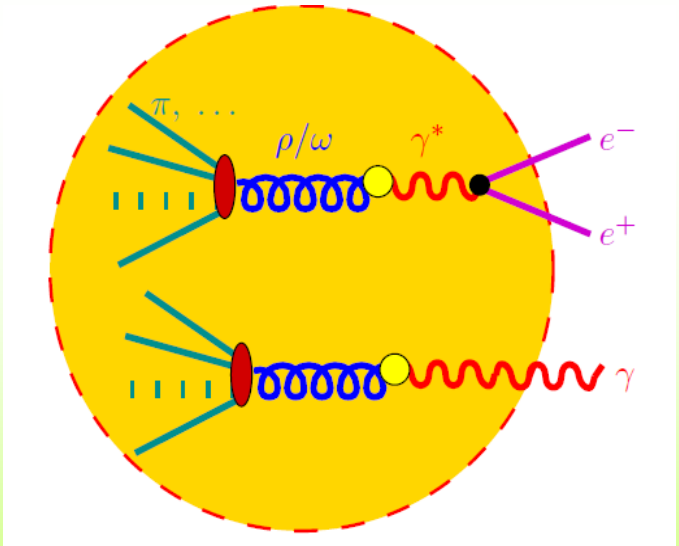
self-energies mass shift and broadening in the medium



CGUrQMD: In+In (158 *A*GeV) (SPS/NA60)

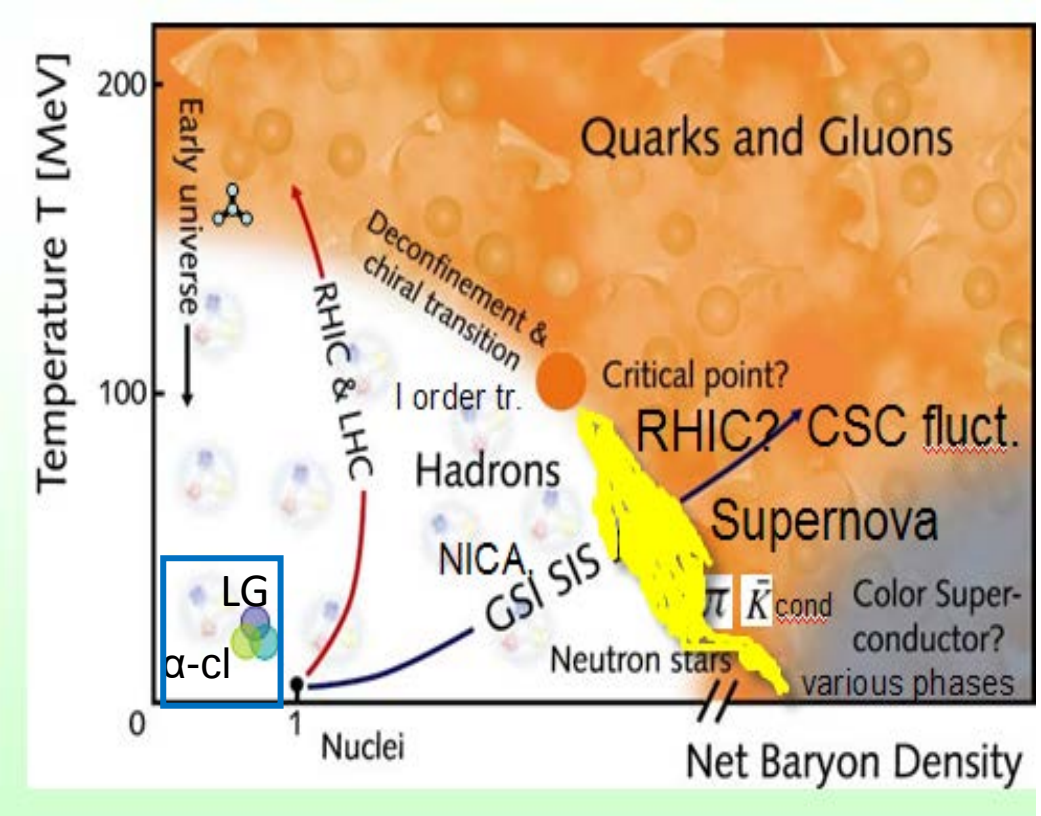
- dimuon spectra from In + In(158 *A*GeV) → $\mu^+\mu^-$ (NA60) [EHWB15]
- min-bias data ($dN_{ch}/dy = 120$)





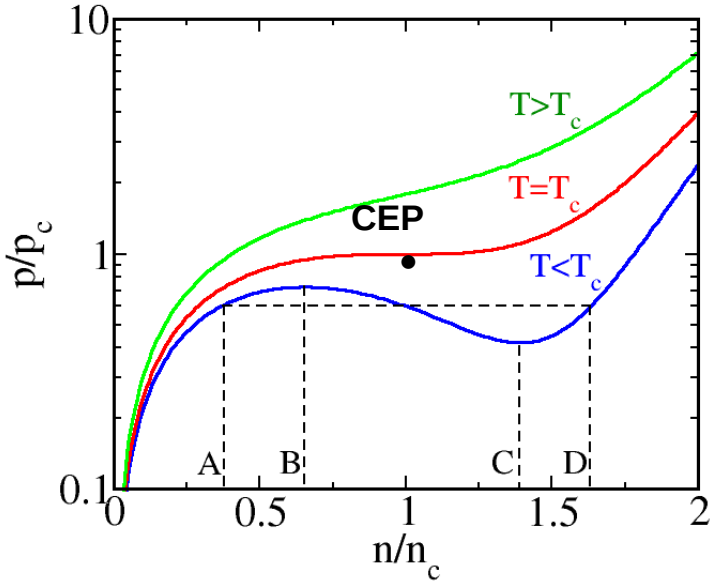
Rapp,Wambach 1999, …,

Low T, rather low baryon density (large baryon chemical potential)

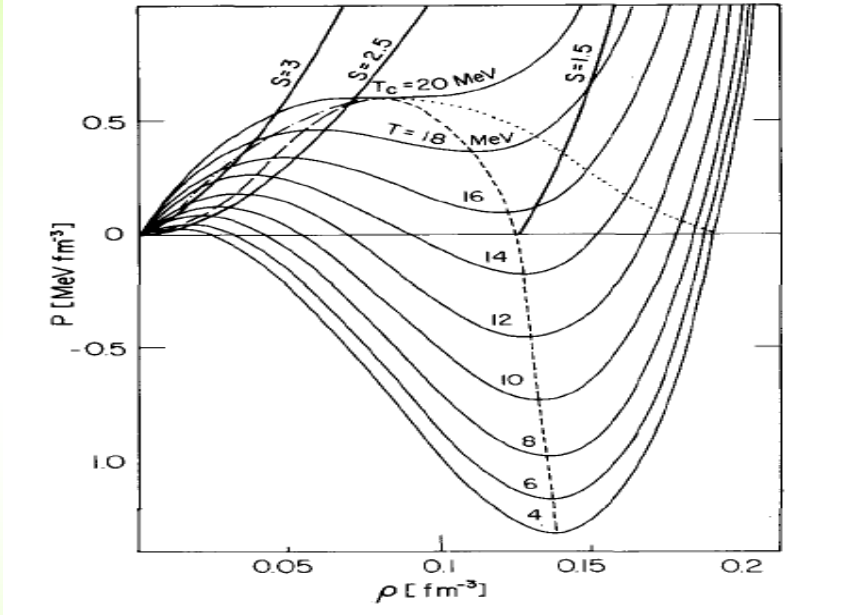


Gas (nucleons)-Liquid (nuclear clusters) I-order phase transition

EoSs for $n<n_0$ show van der Waals-like shape (expanding fireball :$S(t)$≈const, T, n decreasing till freeze out)



N=Z



In spinodal region

$$K = n\frac{\mathrm{d}^2E}{\mathrm{d}n^2} = \frac{2}{3}\epsilon_{\mathrm{F}}(1+f_0)$$ <0



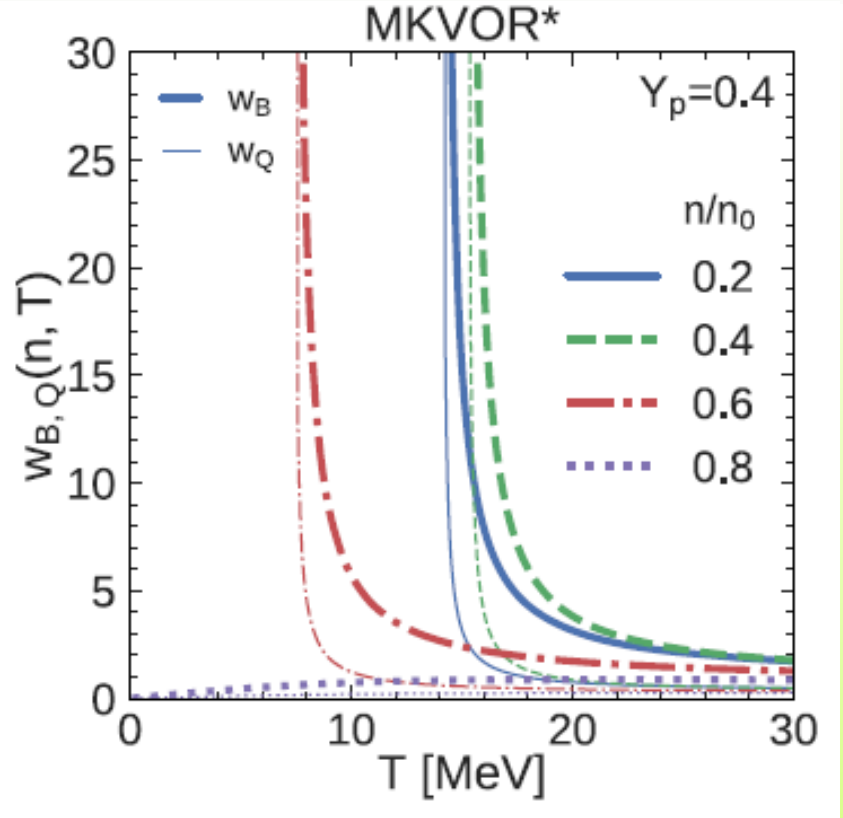
Scalar LM parameter $f_0(n)$<-1- Pomeranchuk instability In Fermi liquids

Normalized variance of the baryon number

$$w = \frac{\overline{(\Delta N)^2}}{N} = \frac{T}{n}\left(\frac{\partial n}{\partial \mu_B}\right)_T$$

diverges on ITS line

Maslov, D.V. 2019



Effects of crossing of the ITS line m.b. easier to observe than the vicinity of the critical point (due to a critical slowing down effect in latter case, since correlation time $t \sim 1/|T_c - T|^{1/2}$)

GL phase transition in HIC at low energies

Schulz, Munchow,Roepke, Schmidt 1982, see Review Borderie 2019 (ALADIN,INDRA)

N=Z

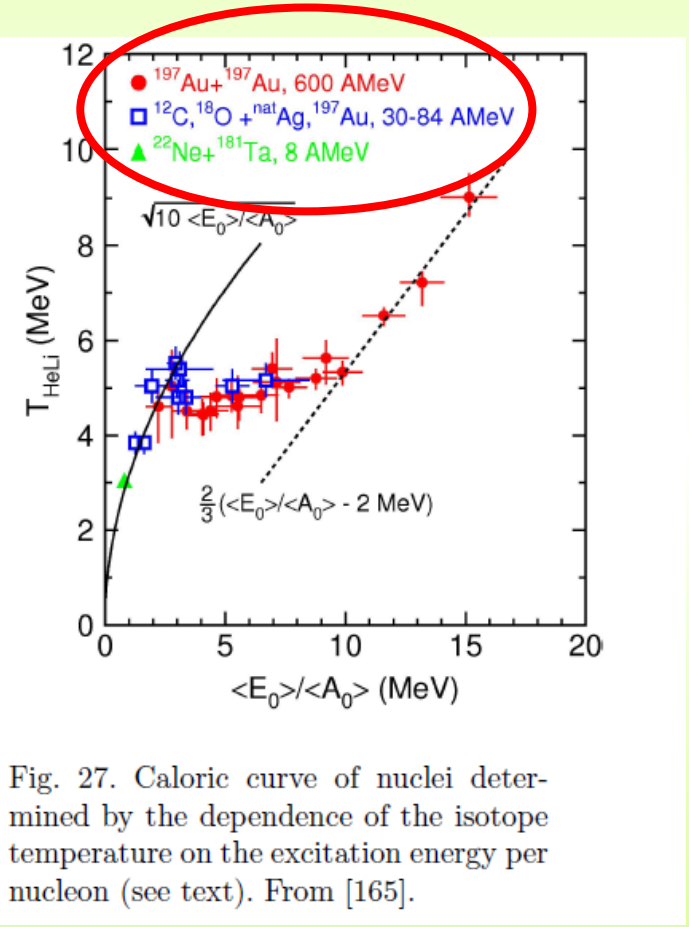
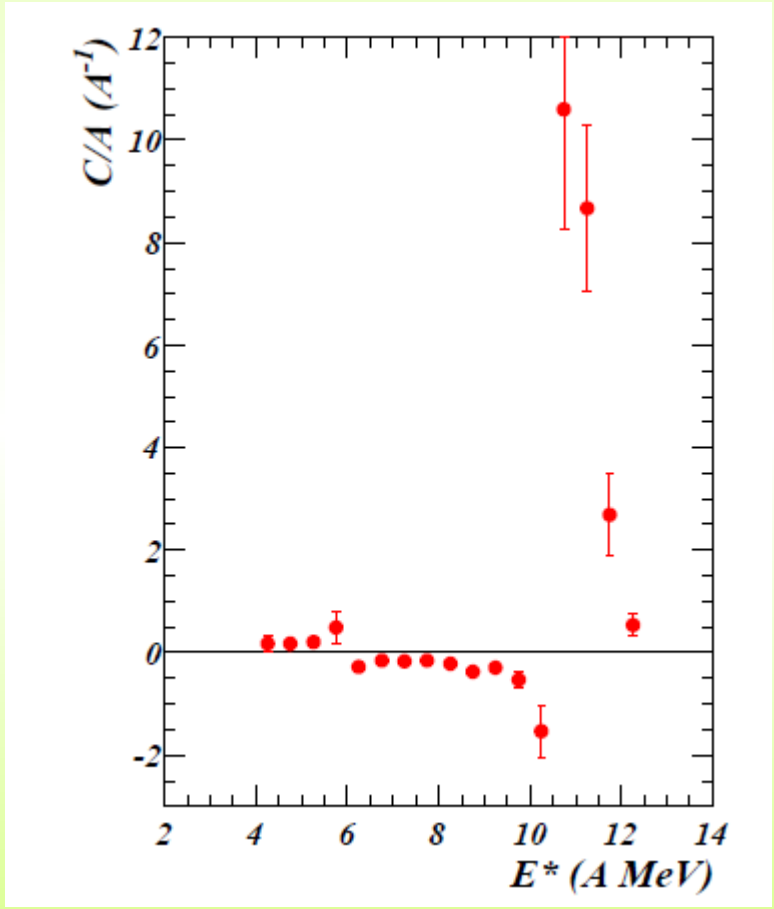


Fig. 27. Caloric curve of nuclei determined by the dependence of the isotope temperature on the excitation energy per nucleon (see text). From [165].

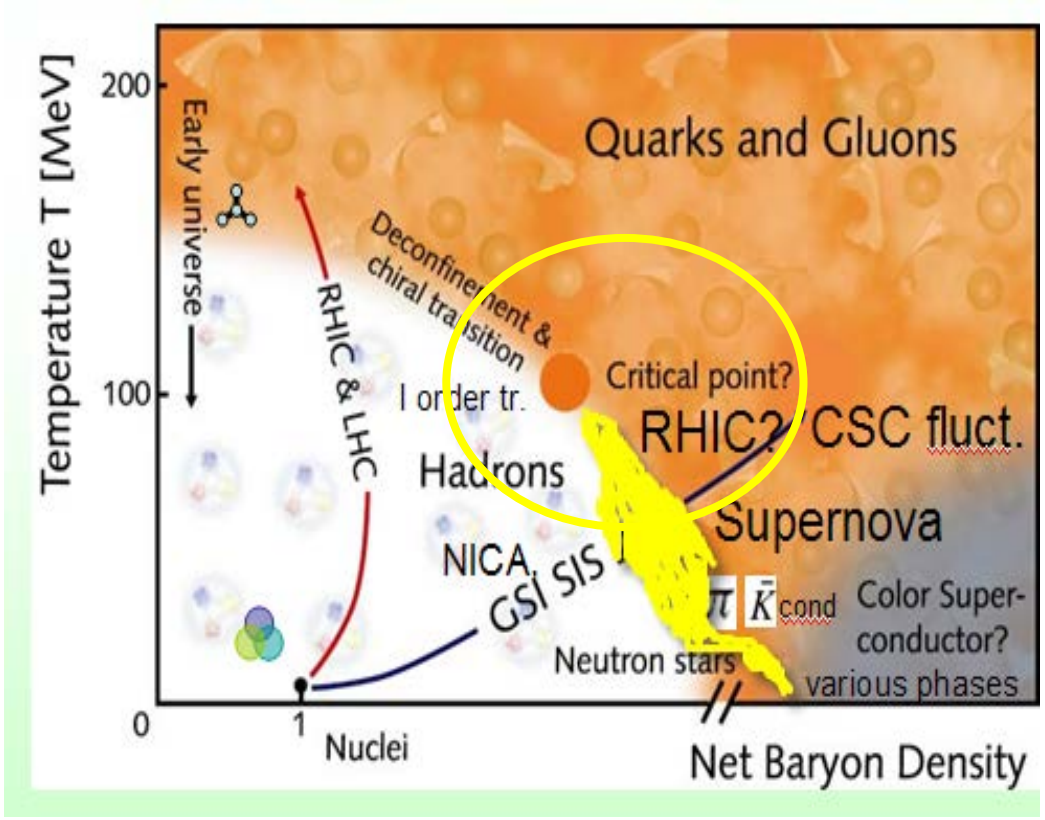
Region with C_p <0



Negative heat capacity which signs a first order phase transition for finite systems is observed

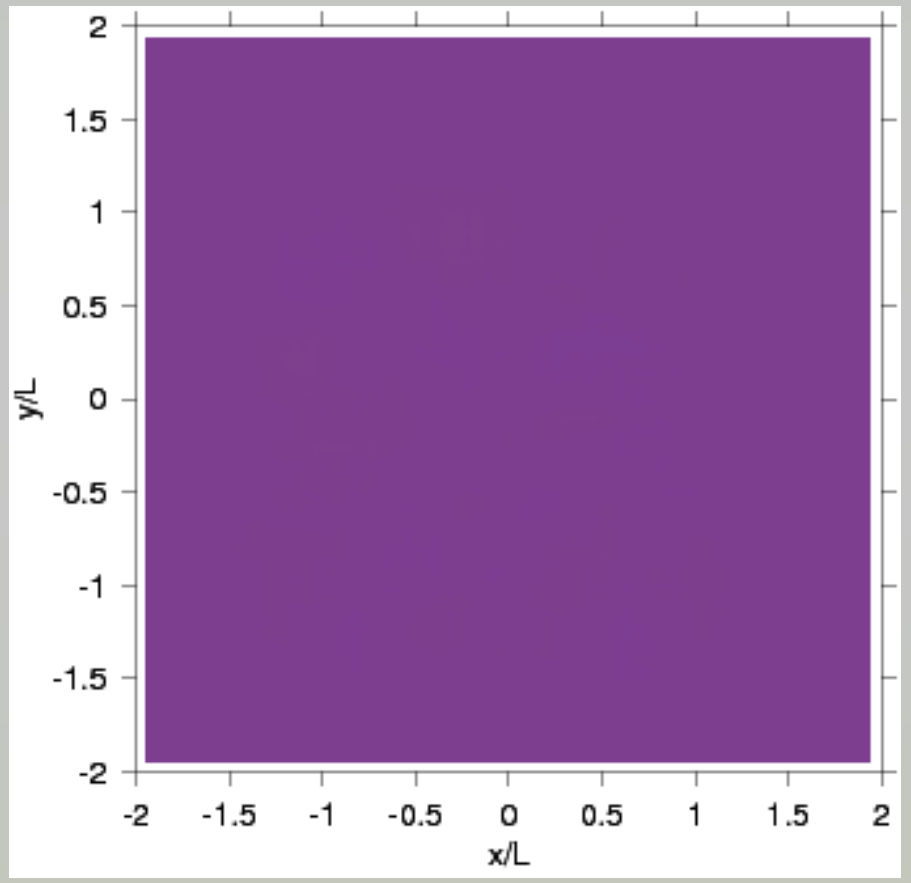
Pochodzalla et al (1995)

Rather large *T*, large baryon density (baryon chemical potential larger than *T*),
typical NICA energies



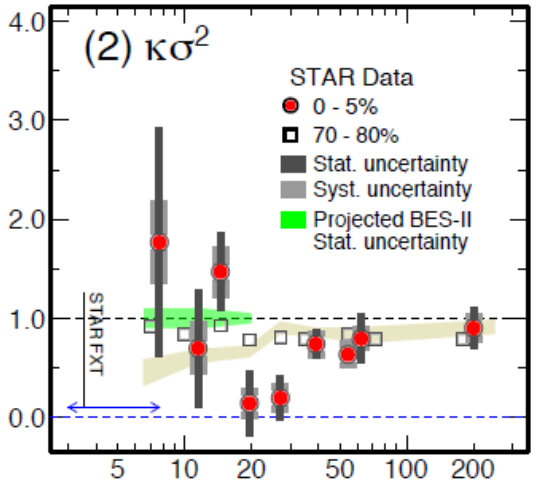
Hydrodynamics of spinodal instability at QH transition near CEP

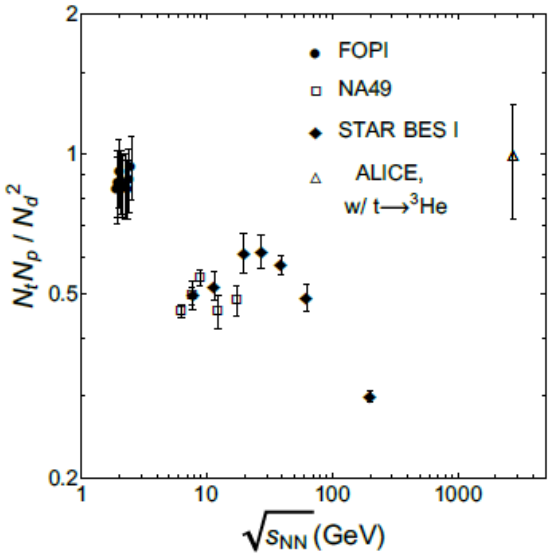
.
Blue – hadrons,
Red – quarks.



Skokov, D.V. NPA 2009,2010

Quark-hadron CEP, measurements





From Shuryak 2024

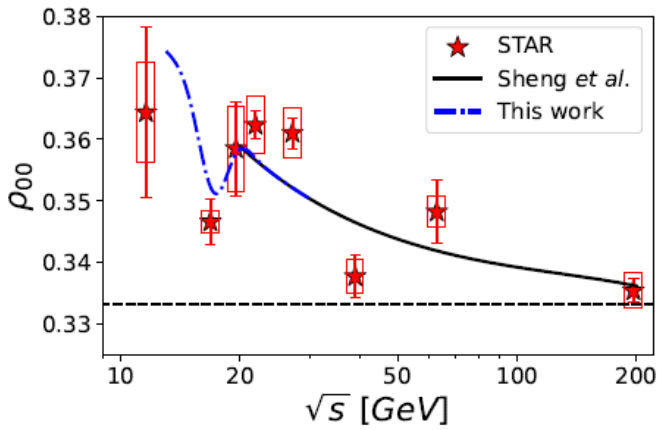
First evidence of non-monotonic variation in kurtosis of net proton number in central Au+Au collisions at RHIC -- indication to CEP (?)

In Boltzmann gas would be $N_p \propto e^{\mu/T},\ N_d \propto e^{2\mu/T},\ N_t \propto e^{3\mu/T}$

$$\mathcal{O}_{tpd} \equiv \frac{N_t N_p}{N_d^2} \simeq 0.29(1+\Delta n)$$ density fluctuations $\Delta n = \langle(\delta n)^2\rangle / \langle n \rangle^2$

Similar irregularity for $\mathcal{O}_{\alpha p^3\mathrm{He}d} \equiv \frac{N_\alpha N_p}{N_{^3\mathrm{He}} N_d}$

Variance of quark spin

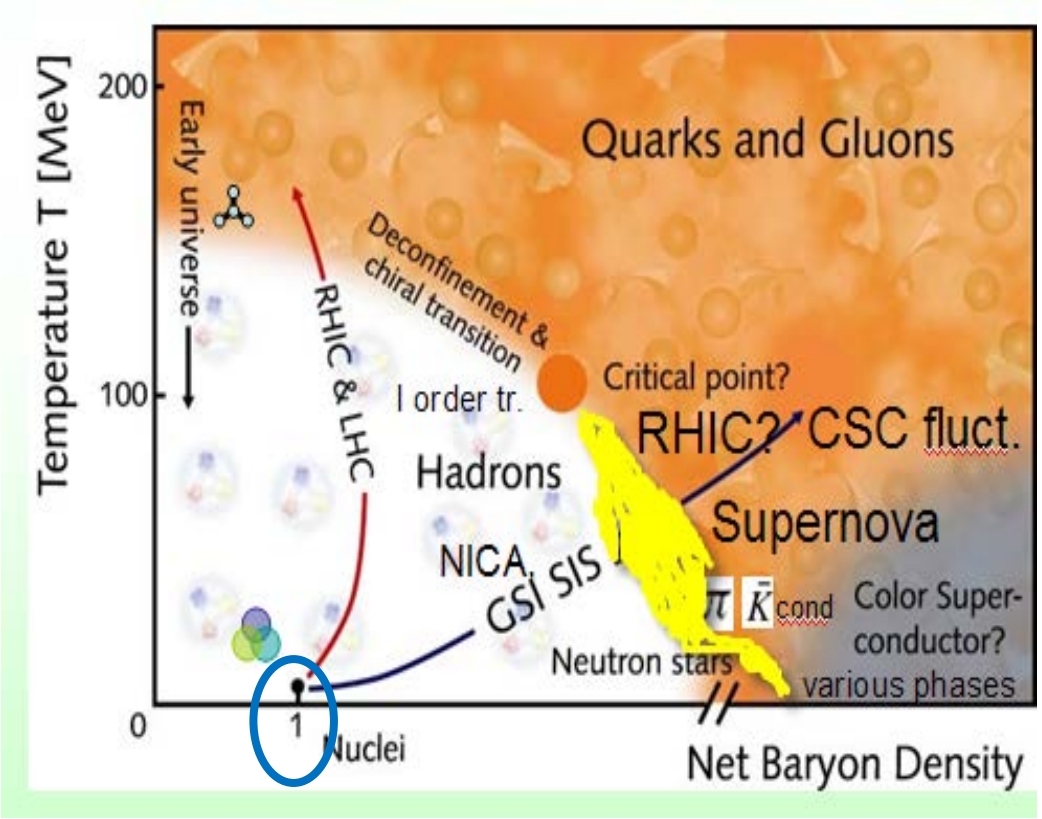


00 component of spin density matrix

Schematic figure including critical fluctuation.

Data Wilks 2024,
Lines-calcul. Chen et al. PRL2025

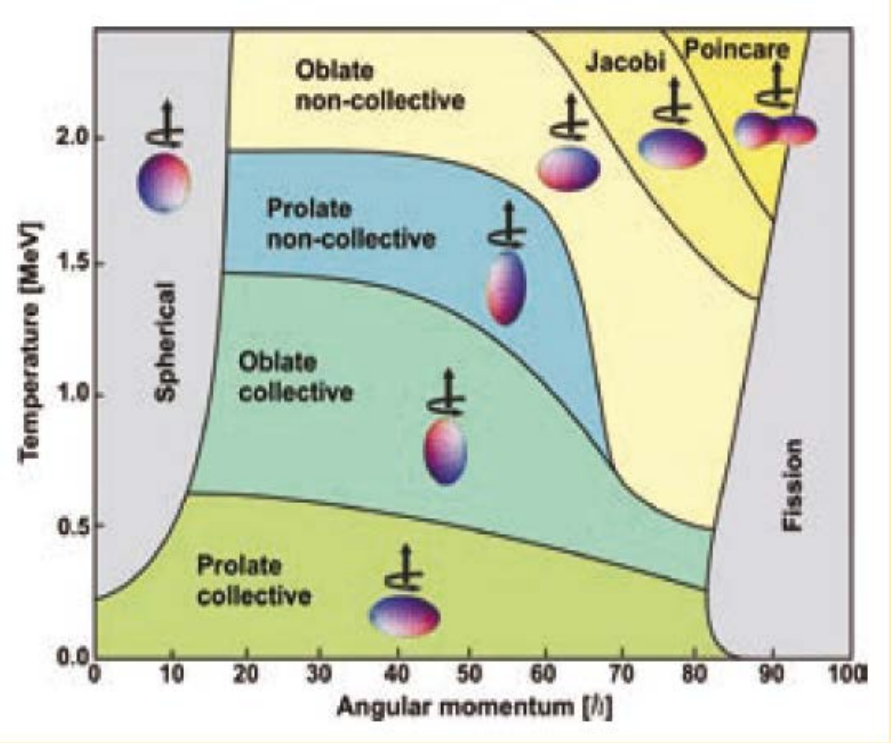
Phase transitions in excited nuclei



(Super)deformed nuclei

Nuclei with high angular momentum

phase transition with change of nucleus form



Prolate-lemon, oblate-orange, triaxial-mango, pear-like shapes arXiv 2211.06037

Ratio of nuclear ellipsoid axes 2:1:1, $\omega \sim 10^{20}$ Hz.

Pulsars: vortex structure at

$\omega_{c1} \sim 10^{-14}$ Hz $< \omega < 10^{4}$ Hz

Rapidly spinning fireballs: in peripheral HIC

$\omega \sim 10^{21}$ -10^{22} Hz

Rotation bodies with charged constituents→strong magnetic fields

In HIC $H \sim 10^{17}$ -10^{19} G.~(0.1-10) m^2_{π}, DV, Anisimov JETP1980, Toneev et al. , IntJ.ModPh2009.

In magnetars ~10^{15} -10^{16} G at surface, m.b. up to ~10^{18} G in central regions of NS

Related problem: spin polarization and vorticity in HIC

Cooper pairing transition in nuclei

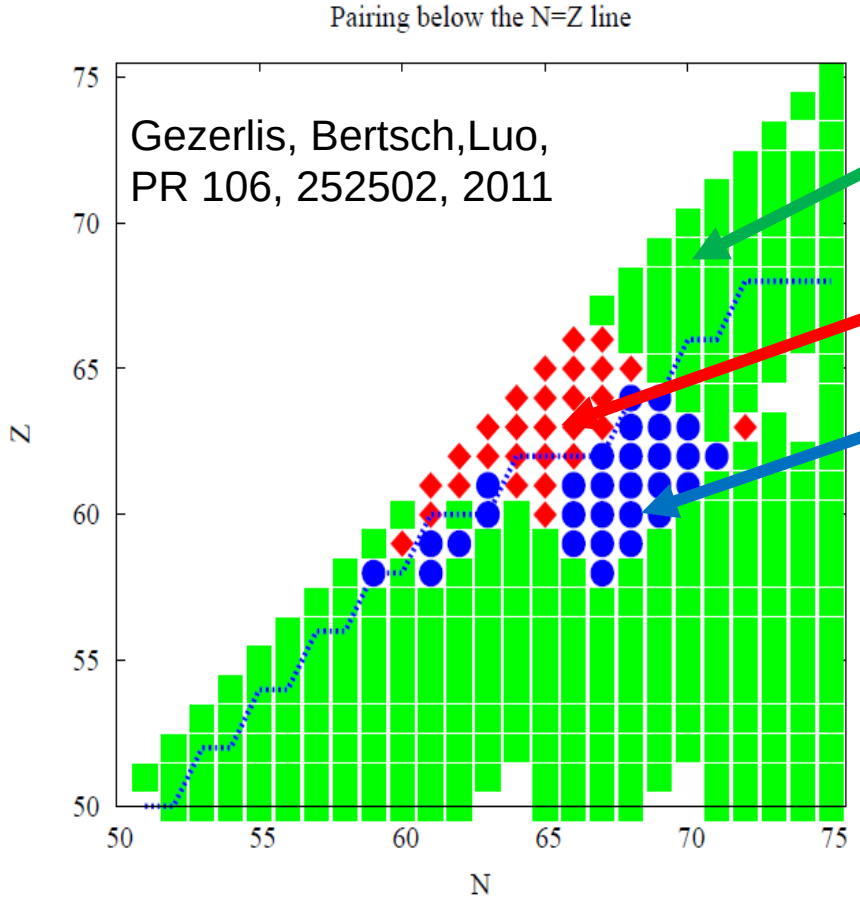


FIG. 1: (color online) Chart of nuclides with $Z \leq N$ for neutron numbers from 50 to 75. Blank squares denote nuclei that exhibit practically no pairing ($E_{corr} < 0.5$ MeV), green squares signify the case where the pairing condensate is mostly spin-singlet, red diamonds are used for the nuclei that exhibit spin-triplet pairing, while blue circles denote nuclei for which the pairing is a mixture of spin-singlet and spin-triplet. The blue dashed line is the proton-drip line from Ref. [16].

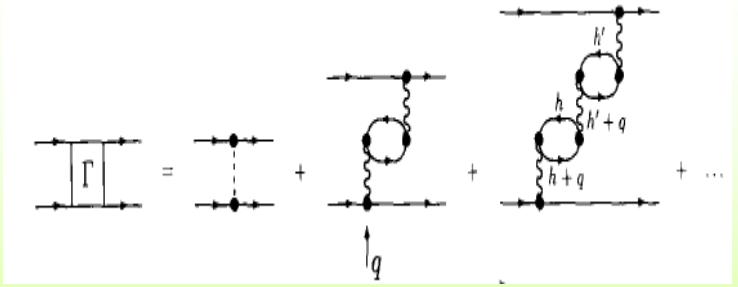
Singlet pairing of nn, pp
(np-exp. Cederwall et al, Nature 2011)

Triplet nn pairing

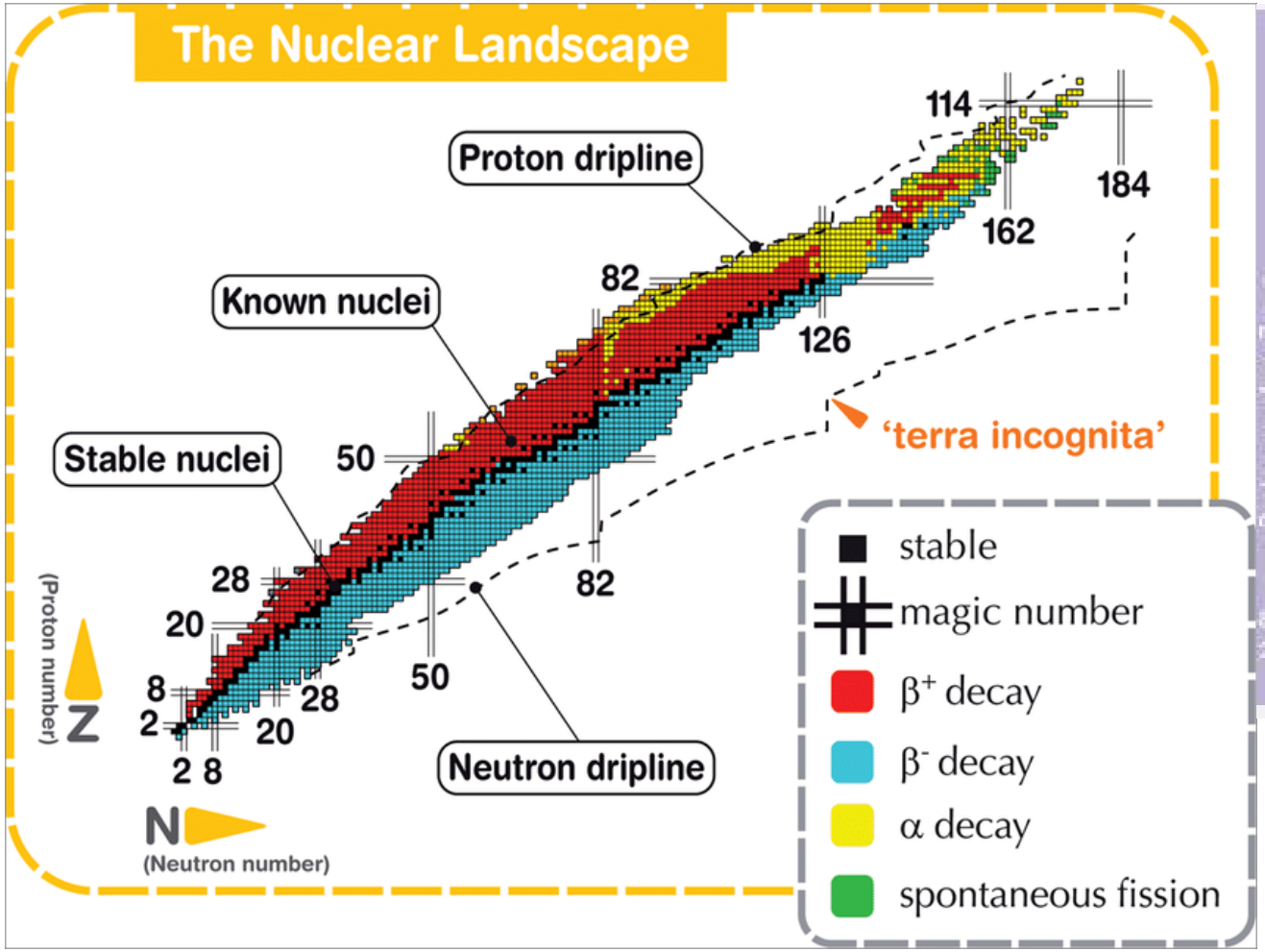
Mixture of singlet and triplet pairing

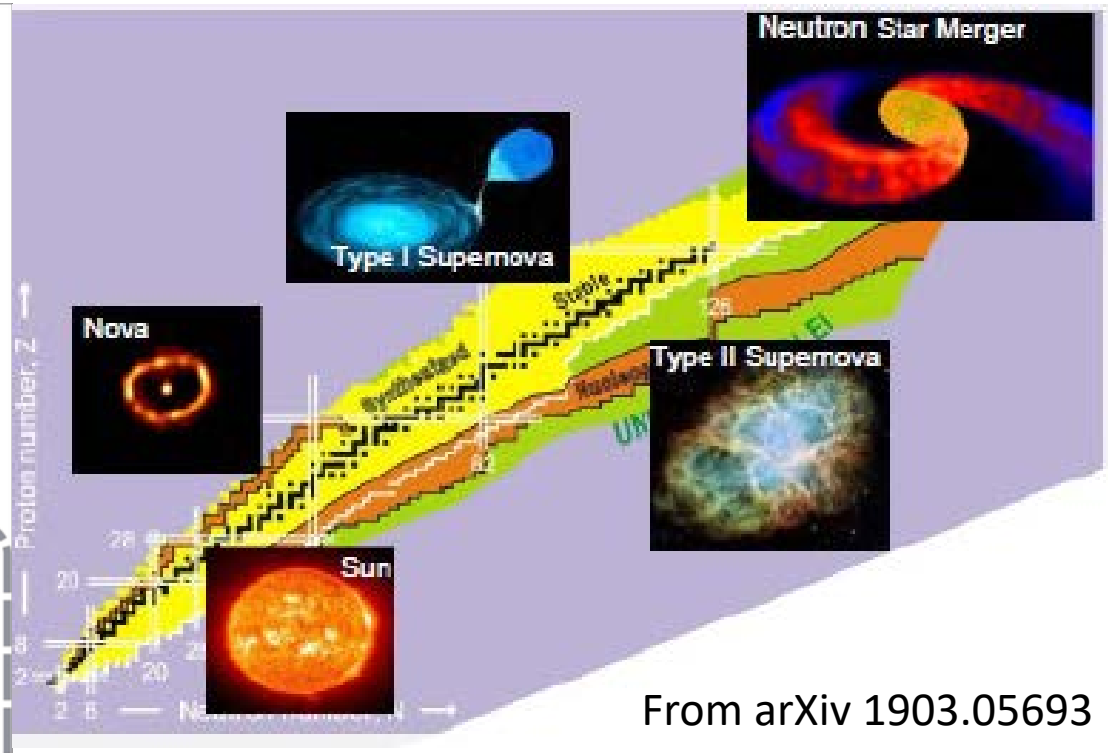
Fulde–Ferrell–Larkin–Ovchinnikov **periodic Δ(r) for N ≠Z** **(not yet found in nuclei)**

In-medium effects are important



Pairing influences moments of inertia of nuclei (data)

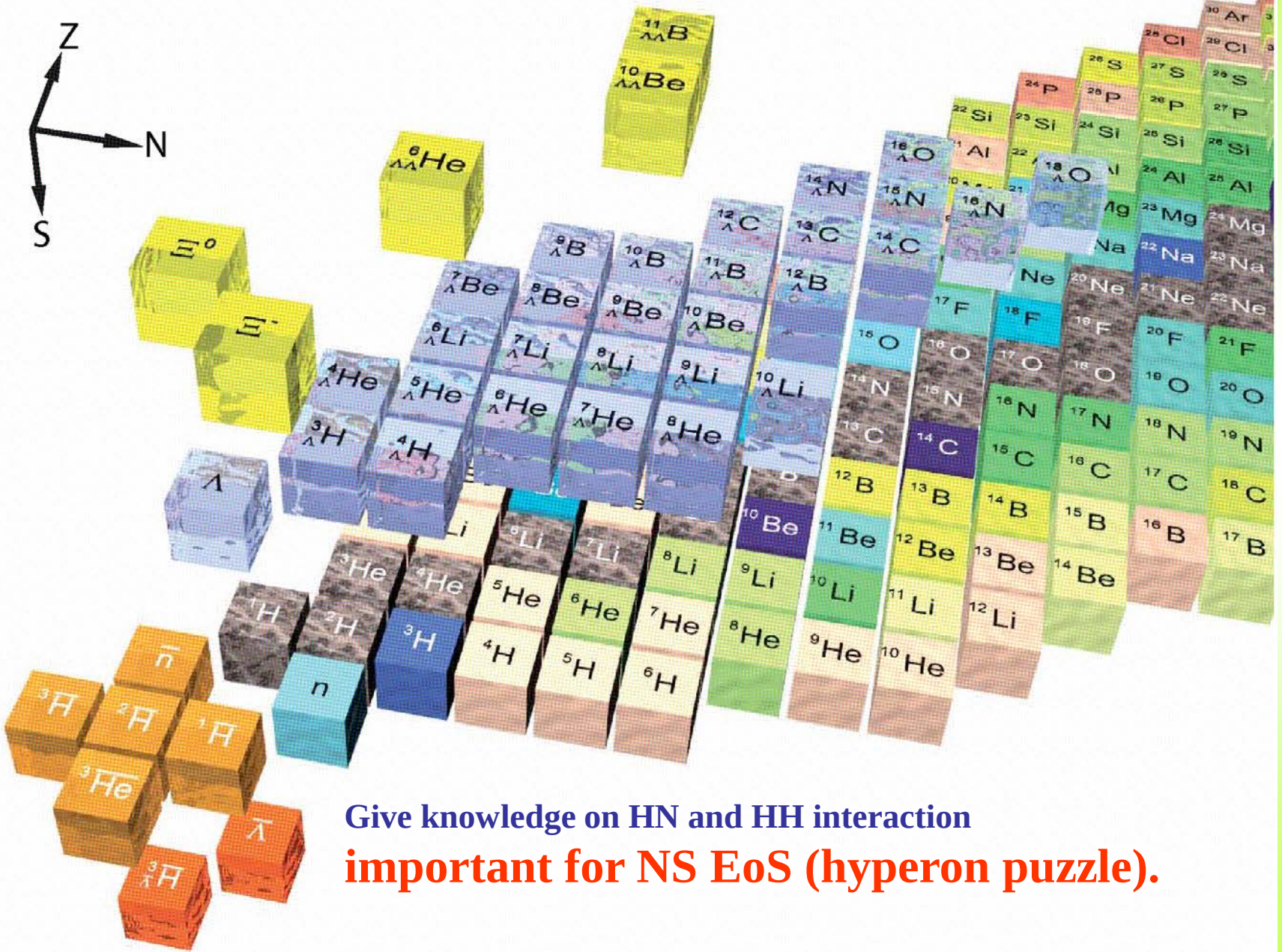




From arXiv 1903.05693

Only about ~1/3 of possible states is identified. Important Info for astrophysics and HIC.
Renaissance of low-energy nuclear physics.

Hyper-nuclei



Give knowledge on HN and HH interaction

important for NS EoS (hyperon puzzle).

Hadron fireball expansion

at low collision energies (<2 GeV/A)

with a model for EoS at hand

$$E_{\text{c.m.}} \simeq E(\rho_{\text{m}}, T_{\text{m}}) - E_{\text{bind}}, \qquad -16\ \text{MeV} \cdot A < E_{\text{bind}} < 0\,. \tag{2.1}$$

Here $E(\rho_{\text{m}}, T_{\text{m}})$ corresponds to the energy minimum as function of density. This value follows from the known equation of state. E_{bind} is the binding energy of the nucleus of a given atomic number $A = A_{\text{f}}$ at $T = 0$, where A_{f} is the fireball baryon number. Thus, the values of the energy, entropy and other thermodynamical characteristics at maximum temperature and density as well as the values T_{m} and ρ_{m} are unambiguously determined by the known initial collision energy, $E_{\text{c.m.}}$. Under the influence of internal pressure the fireball expands into the vacuum. Due to a lot of collisions the expansion velocity is not too high. For this reason the viscosity effects are not as essential and the entropy remains nearly constant during the fireball expansion. Therefore, the equality

$$S(\rho, T(\rho)) \simeq S(\rho_{\text{m}}, T_{\text{m}}) \tag{2.2}$$

determines the $T(\rho)$ dependence during the expansion process.

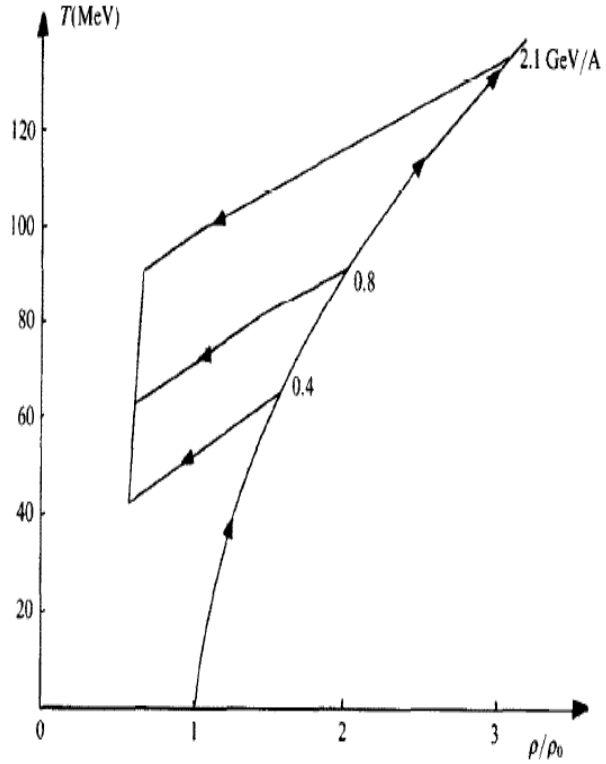
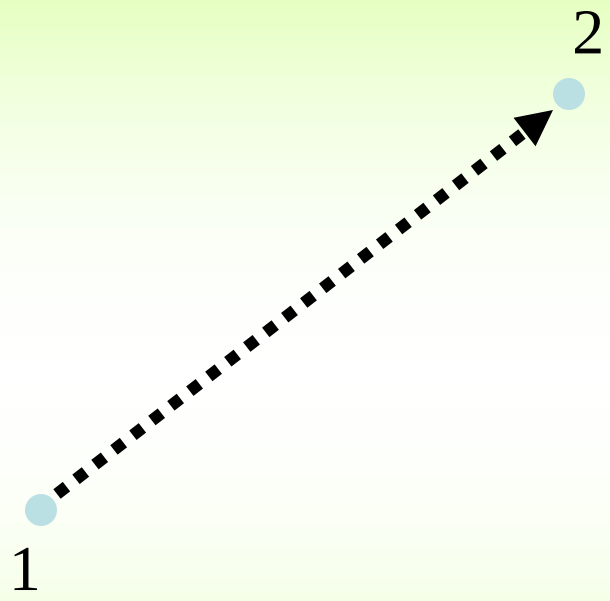


Fig. 6. $T(\rho)$ trajectories of hydrodynamical compression and expansion of nuclear matter at collisional energies $E_{\text{lab}} = 0.4$; 0.8 and 2.1 GeV/A [ref. [7]].

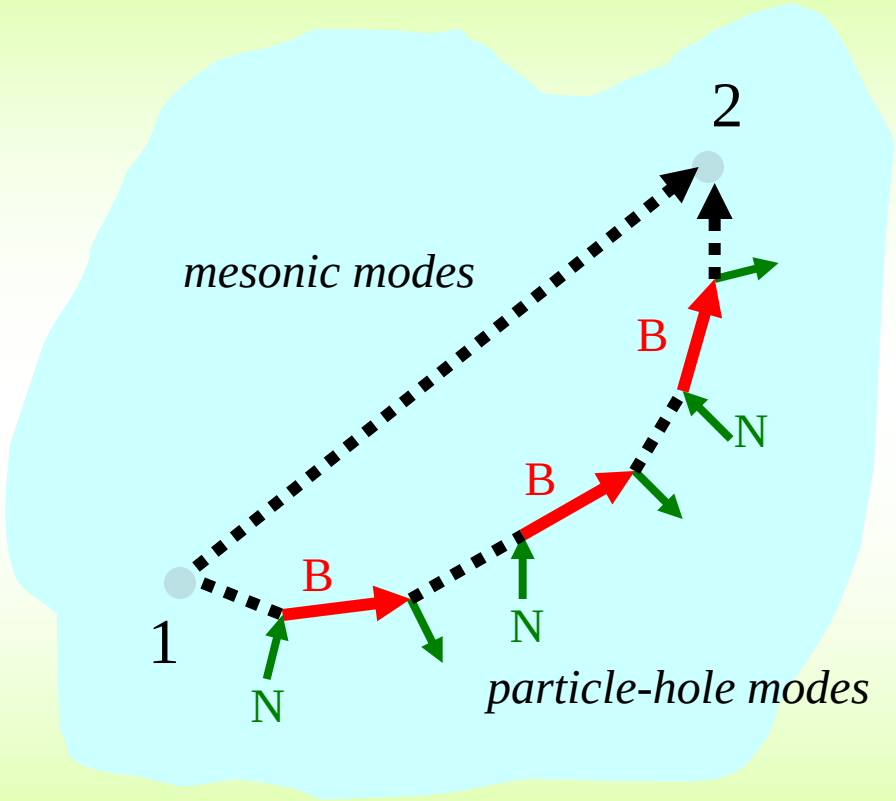
Meson propagator

probability that a state with the given quantum number being produced at point 1 is observed at point 2

vacuum



nuclear medium



$$D^{(0)}(\omega, \boldsymbol{k}) = \frac{1}{\omega^2 - \boldsymbol{k}^2 - m_\eta^2 + i\,0}$$

$$\frac{1}{D(\omega, \boldsymbol{k})} = \frac{1}{D^{(0)}(\omega, \boldsymbol{k})} - \Pi(\omega, k)$$

polarization operator

Relativistic π are incorporated explicitly with the help of the full spectral function

$$A_\pi = -2\Im D_\pi^R .$$

$$A_\pi(t,\vec{r},\omega,\vec{k}) = \frac{\Gamma_\pi(t,\vec{r},\omega,\vec{k})}{[\omega^2 - k^2 - m_\pi^2 - \Re\Sigma^R(t,\vec{r},\omega,\vec{k})]^2 + \Gamma_\pi^2(t,\vec{r},\omega,\vec{k})/4},$$

Approximations:

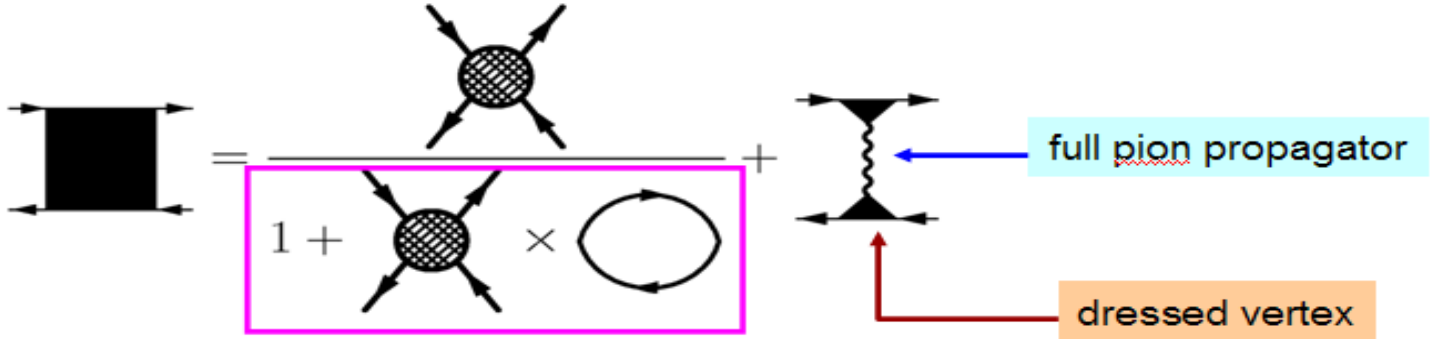
Heavy *N* (7 times heavier than π) are good quasiparticles, except vicinity of π condensation transition and except for large *T* (*T*>100 MeV),

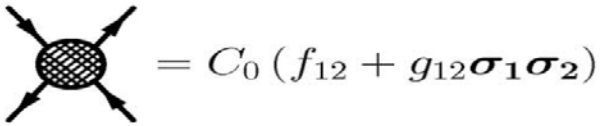
Δ (1232) isobars have width ~100 MeV in vacuum which is included by shift of energy.

Low energy excitations in nuclear Fermi liquid (Migdal approach)

Resummed NN interaction

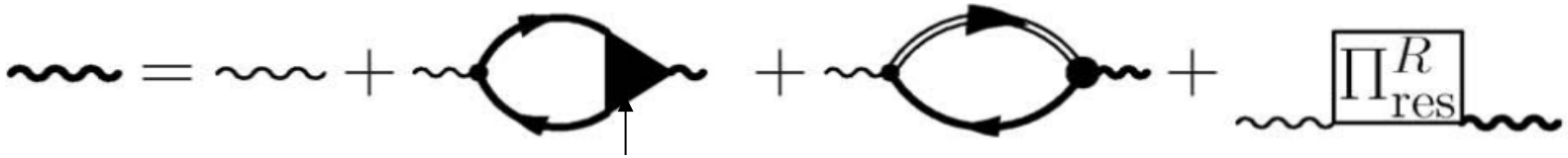
based on a separation of long and short scales





Info. on short-range interact. is extracted from atomic nuclei exp. and model calcul.

Poles yield zero-sound modes in scalar and spin channels



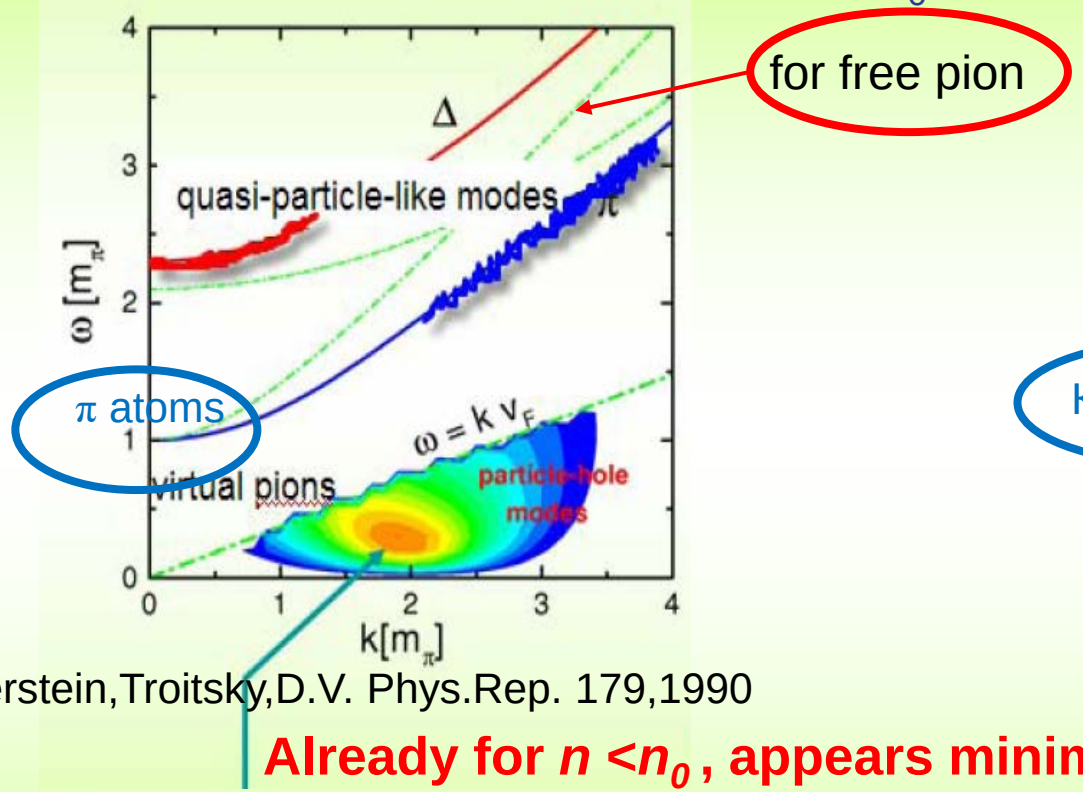
includes s-wave inter. & pion fl.

Migdal Rev.Mod.Phys. 50, 1978, Migdal,Saperstein,Troitsky,D.V. Phys.Rep.192 (1990).

Pion and kaon in nuclear matter are complicated collective Bose excitations:

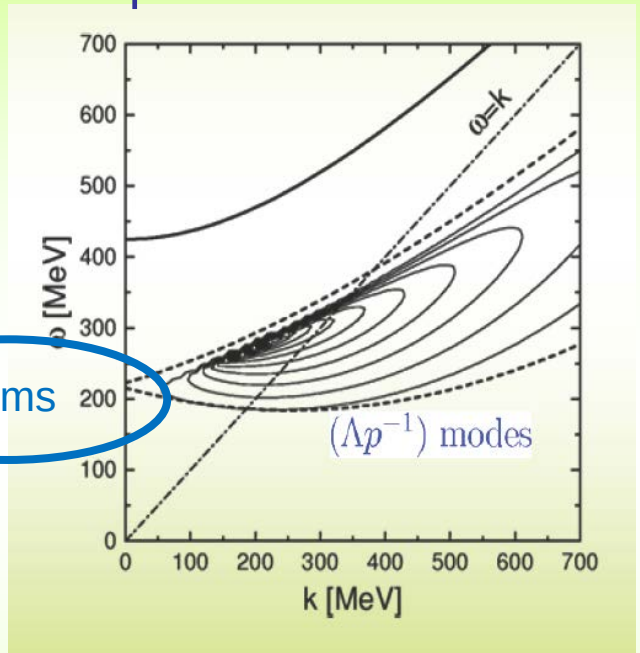
(nucleons—pendulum--ions, pions,kaons—waves--phonons)

Pion spectrum for N=Z and for π^0 at N>> Z at $n \sim n_0$



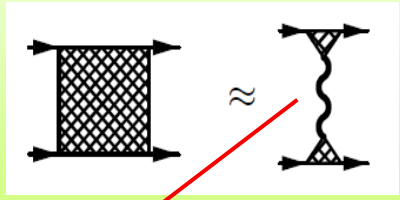
Migdal,Saperstein,Troitsky,D.V. Phys.Rep. 179,1990

Spectrum of antikaons at $n \sim n_0$



Kolomeitsev,D.V.,Kampfer 1996

Already for $n < n_0$, appears minimum in complex ω plane and it decreases with *growing n.* *NN* amplitude Is strongly enhanced with increasing n owing to in-medium pion exchange



For $n > n_c \sim (2\text{--}3) n_0$, $\omega^2 < 0$ *solution, and* occurs p-wave (k≠0) pion condensation

Pion and similarly antikaon condensation may appear in dense NS interior

Pion quasiparticle path lengths in nuclear matter

$$\lambda_\pi = \frac{\mathrm{d}\omega(k)}{\mathrm{d}k}\tau_\pi, \qquad \frac{\mathrm{d}\omega}{\mathrm{d}k} = k\left(1+\frac{\partial\,\mathrm{Re}\,\Pi^{\mathrm{A}}}{\partial k^2}\right)\left[\omega\left(1-\frac{\partial\,\mathrm{Re}\,\Pi^{\mathrm{A}}}{\partial\omega^2}\right)\right]^{-1}\Bigg|_{\omega=\omega_i(k)}. \qquad \tau_\pi^{-1} = \frac{\mathrm{Im}\,\Pi^{\mathrm{A}}}{\omega(k)[1-\partial\,\mathrm{Re}\,\Pi^{\mathrm{A}}/\partial\omega^2]}\Bigg|_{\omega=\omega_i(k)}$$

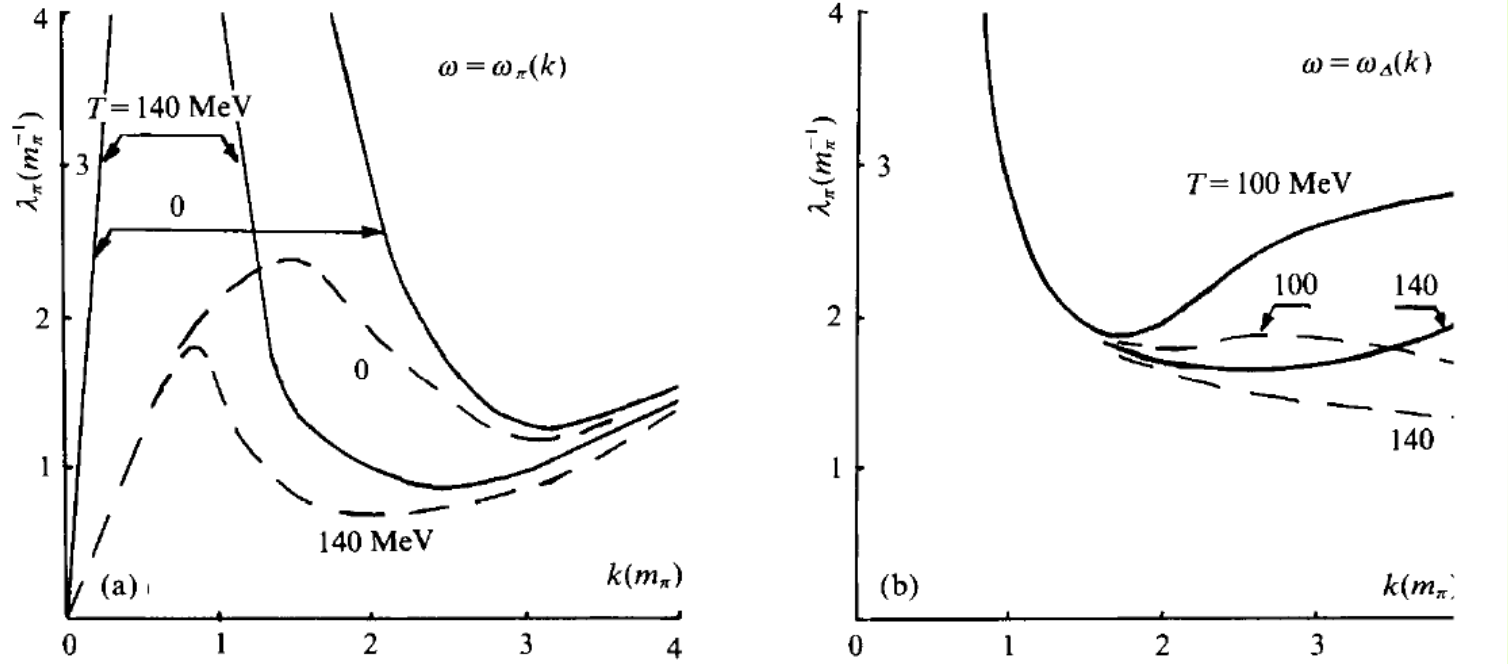


Fig. 2. Pion-quasiparticle path length at $\omega=\omega_\pi(k)$ and $\omega=\omega_\Delta(k)$. Solid curves correspond to $\rho\simeq 0.6\rho_0$ whereas dashed curves correspond to $\rho\simeq 1.5\rho_0$. Corresponding temperatures are shown in MeV.

Pion path-length is small for $k>1.5\ m_\pi$ (radiate at freeze out) and large for $k< m_\pi$ (radiate from intermediate collision stage in direct reactions)

cf. D.V. NPA555 (1993)

Sudden Decay of Nuclear Fireball

Assume the breakup happens within the time interval $t_0 - \frac{1}{2}\tau_b < t < t_0 + \frac{1}{2}\tau_b$

τ_b is a breakup time

momentum distribution of particles after breakup

$$\frac{dN^{\rm out}}{dX^3\,dk^3/(2\pi)^3} = 2\sqrt{m^2+k^2}\int {\rm d}\xi e^{ik\xi}\left\langle \Psi(t+\tau_b/2)\middle|\phi^\dagger(X+\xi)\phi(X-\xi)\middle|\Psi(t+\tau_b/2)\right\rangle$$

$$= 2\sqrt{m^2+k^2}\,i\int D^{-+}_{\rm free}\left(t_0+\frac{\tau_b}{2},\boldsymbol{X},\omega,\boldsymbol{k}\right)\frac{{\rm d}\omega}{2\pi}$$

Wigner function

fireball state-vector

If τ_b is sufficiently short then the wave function of the system **does not change**

$$\left|\Psi(t+\tau_b/2)\right\rangle \approx \left|\Psi(t-\tau_b/2)\right\rangle$$

spectral density

$$D^{-+}_{\rm free}\left(t_0+\frac{\tau_b}{2},\boldsymbol{X},\omega,\boldsymbol{k}\right) \approx D^{-+}_{\rm med}\left(t_0-\frac{\tau_b}{2},\boldsymbol{X},\omega,\boldsymbol{k}\right)$$

$$\approx 2\,i\frac{{\rm Im}\,D^R_{\rm med}\left(\omega,\boldsymbol{k};T(X,t_0),n(X,t_0)\right)}{e^{\omega/T(X,t)}\pm 1}$$

promptness condition: τ_b shorter than the quantum-mechanical transition time from an in-medium state to a vacuum state

$$\tau_b < \frac{1}{|\omega-\sqrt{m^2+k^2}|}$$

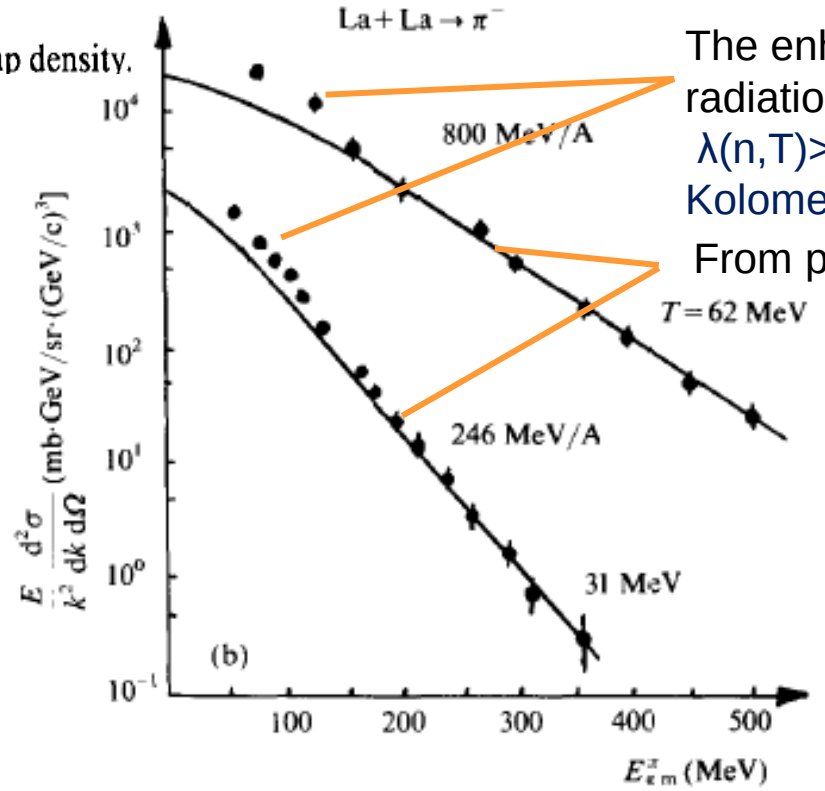
Pion production in La+La collisions

$$E\frac{d\sigma}{dk} = \sum_i \frac{A^{5/3}\omega_k \Gamma_i}{\rho_b[\exp(\omega_i/T_b)-1]}$$

$$\times 15.64\,\mathrm{mb/GeV^2}\,c^3\,\mathrm{st}\,,$$

where ρ_b is the assumed break up density.

from D.V. Nucl. Phys. A555 (1993)

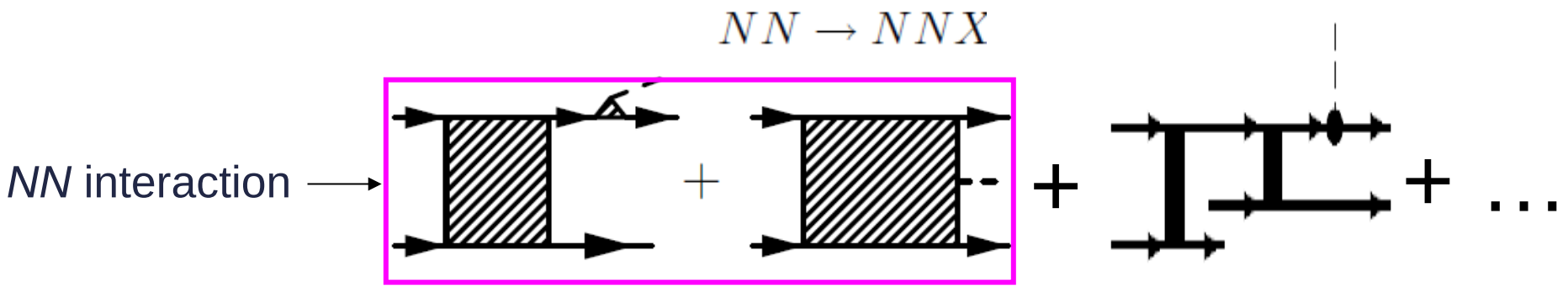


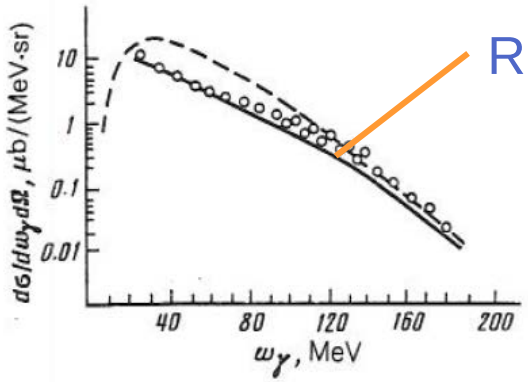
The enhancement can be described by radiation of pions in **direct reactions**
$\lambda(n,T)>R(n)$ at $n>n_b$
Kolomeitsev,D.V., Yad.Fiz.56 (1995) 92

From prompt **freeze out**

With in-medium effects included, pions with k>(150-200) MeV have short mean-free path and radiate from thermal freeze-out, $n\approx 0.6\, n_0$, pions with smaller momenta radiate from an earlier stage.

In quasiparticle approximation





Radiation in direct reactions, $e^2<<1$, in quasipart. model

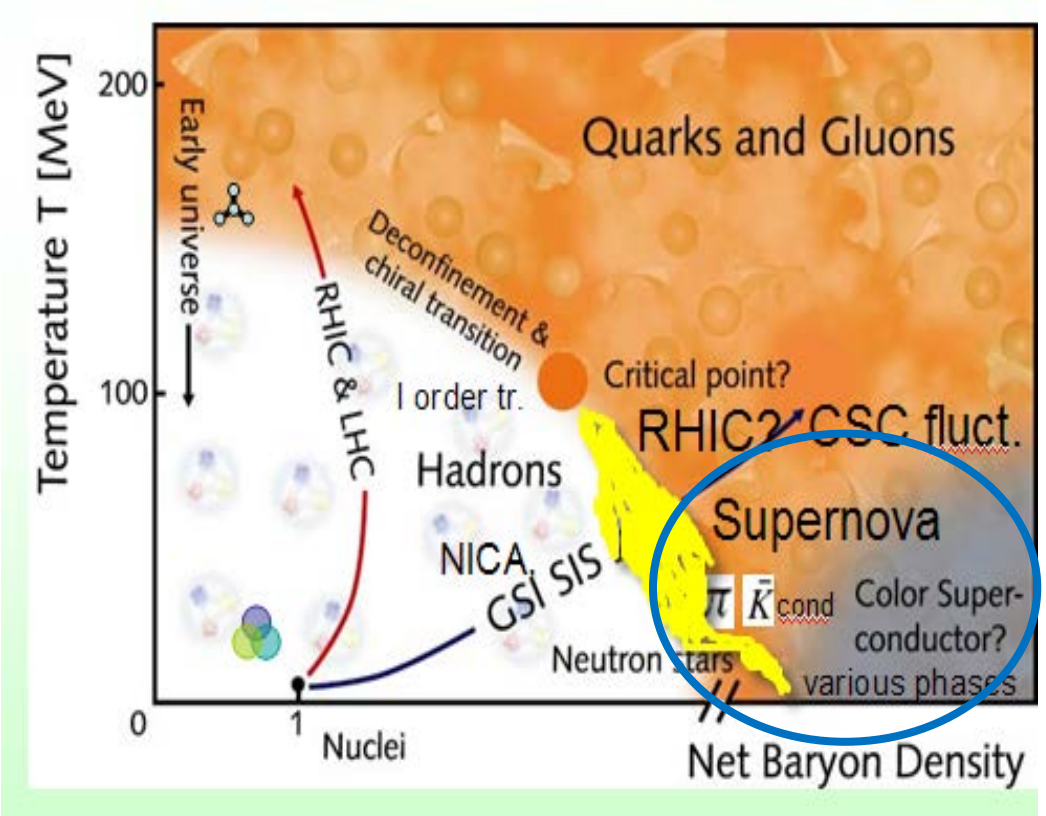
From D.V., Senatorov, Sov.Nucl.Ph.52 (1990)

FIG. 5. Invariant differential cross section for the photons in the reaction $^{36}Ar + {}^{27}Al \to X + \gamma$ at $E_{lab} = 85$ MeV/nucleon at $\theta = 150$, 90, and 60°. The solid curve is the calculation according to Eq. (26), and the dashed curve is the black-body law (37).

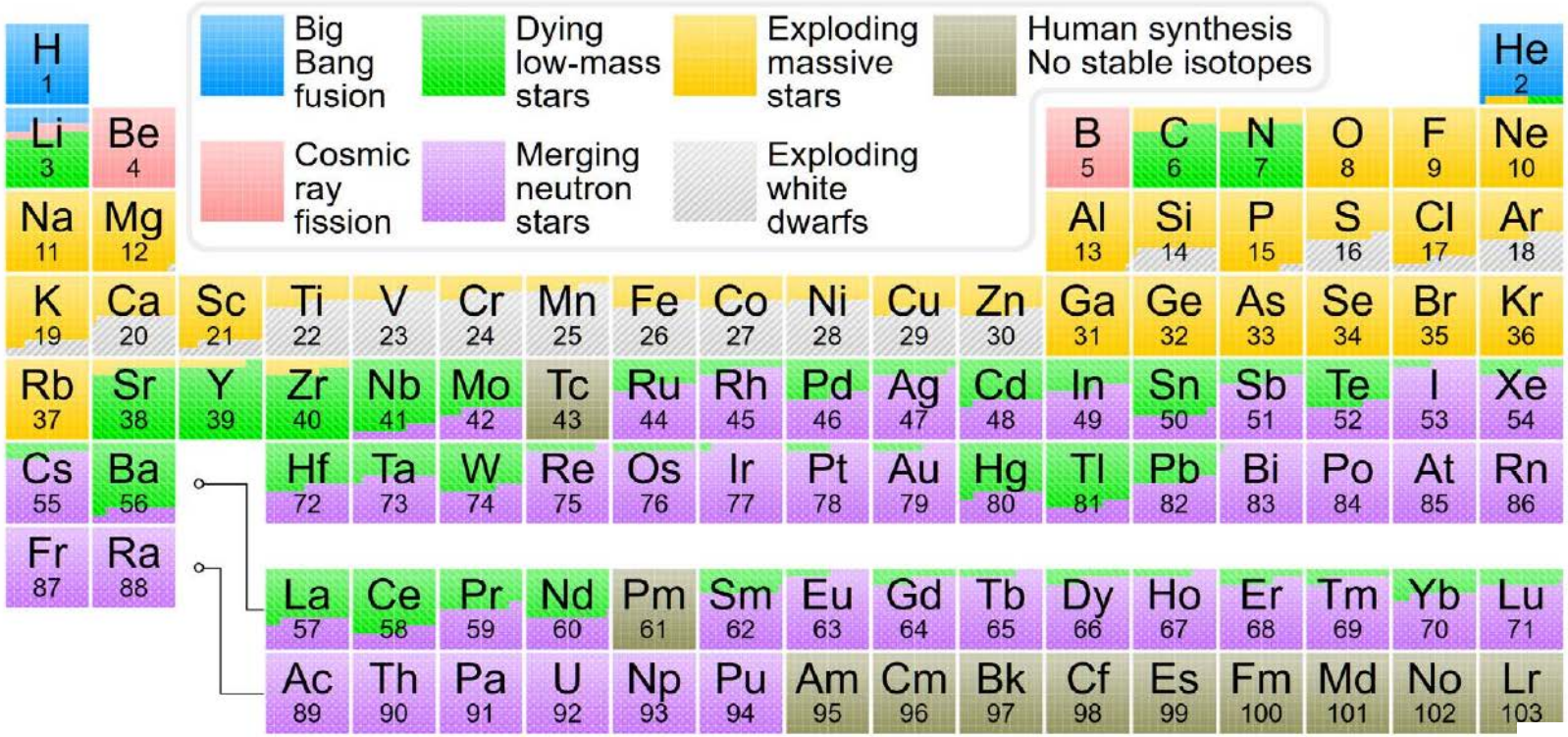
However in matter there are no asymptotic states →

at least in some cases quasiparticle picture is not sufficient

Mergers, supernovas, neutron stars, hybrid stars



Most-heavy elements are produced in mergers of NSs (our parents)

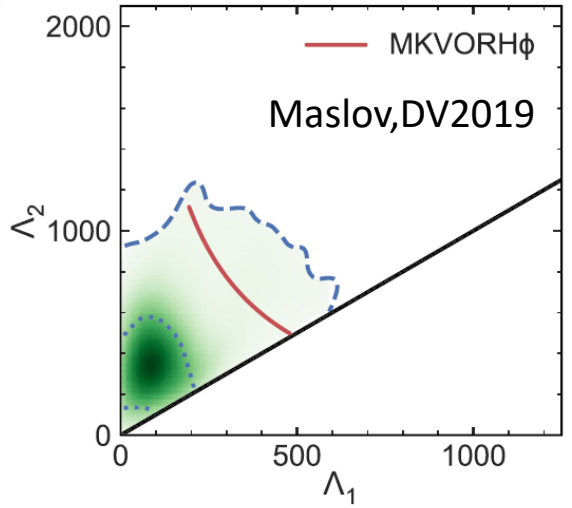


Gravitational waves from neutron star mergers (LIGO/Virgo)

GW170817



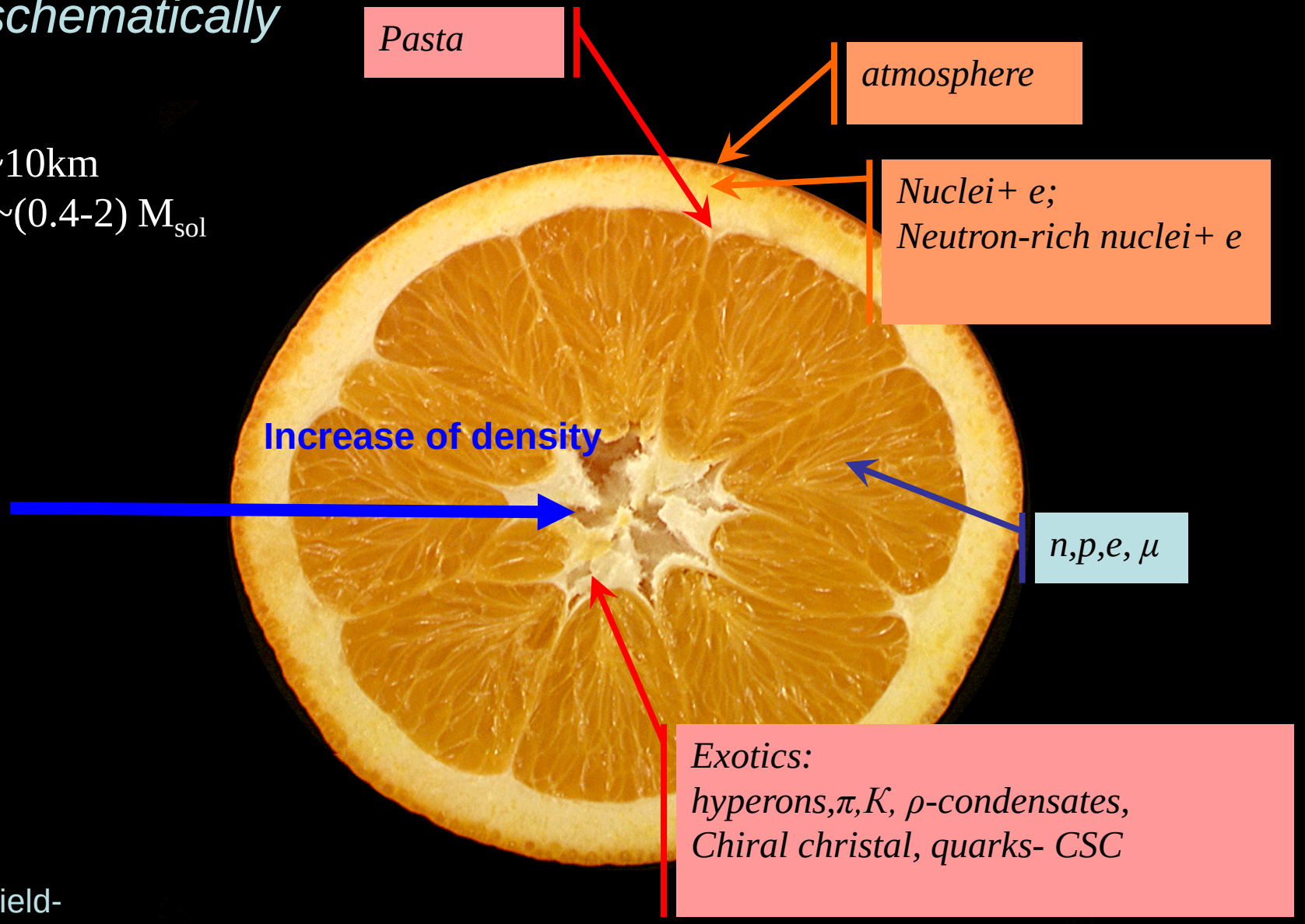
Tidal deformabilities



DataB.P. Abbott, et al 2018

Neutron star schematically

R~10km
M~(0.4-2) M_{sol}



Magnetic field-

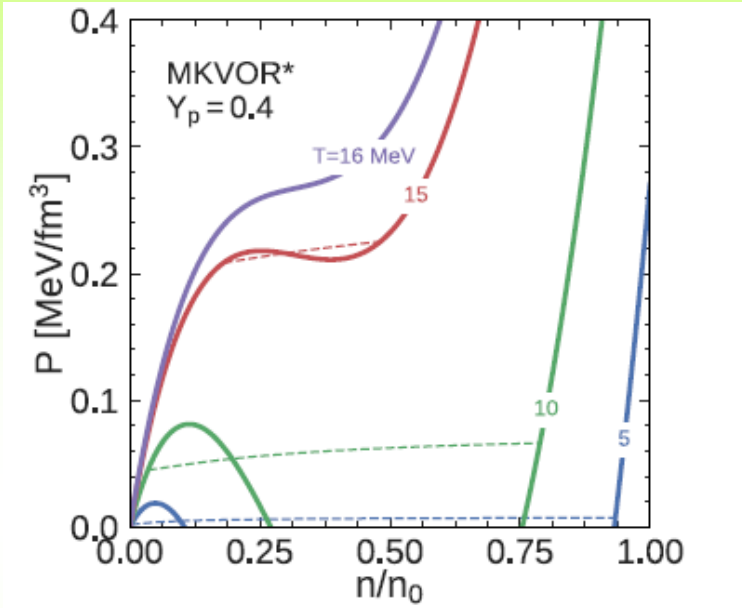
Up to ~10^{15} Гс in magnetars

Fig. E. Kolomeitsev

About peculiarity of 1-order phase transition in asymmetric nuclear matter (N≠Z)

In case of two conserved charges: P(n) has saddle

***np*-gas-nuclear liquid**

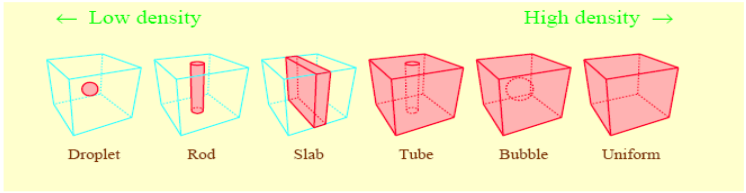


Maxwell construction horizontal line transforms to a curve since

electric charge can be conserved globally: balance of surface tension

+ screening – WZ cells

Rawenhall, Pethick, Wilson 1983, Glendening 1992

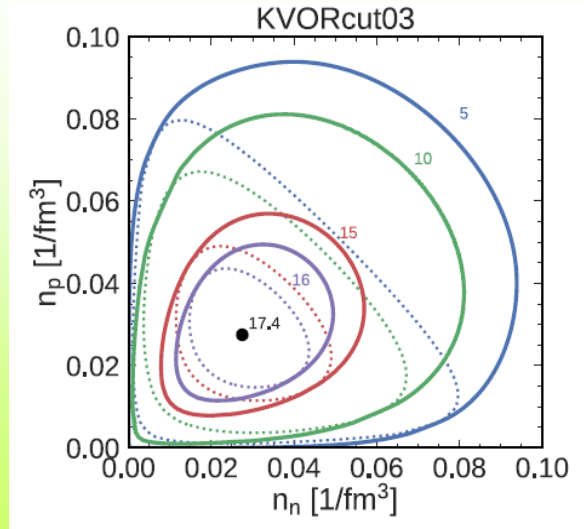


Density →

In NSs crust ($n<n_0$) structured nuclear pasta phase (*n*-gas-nuclear liquid).

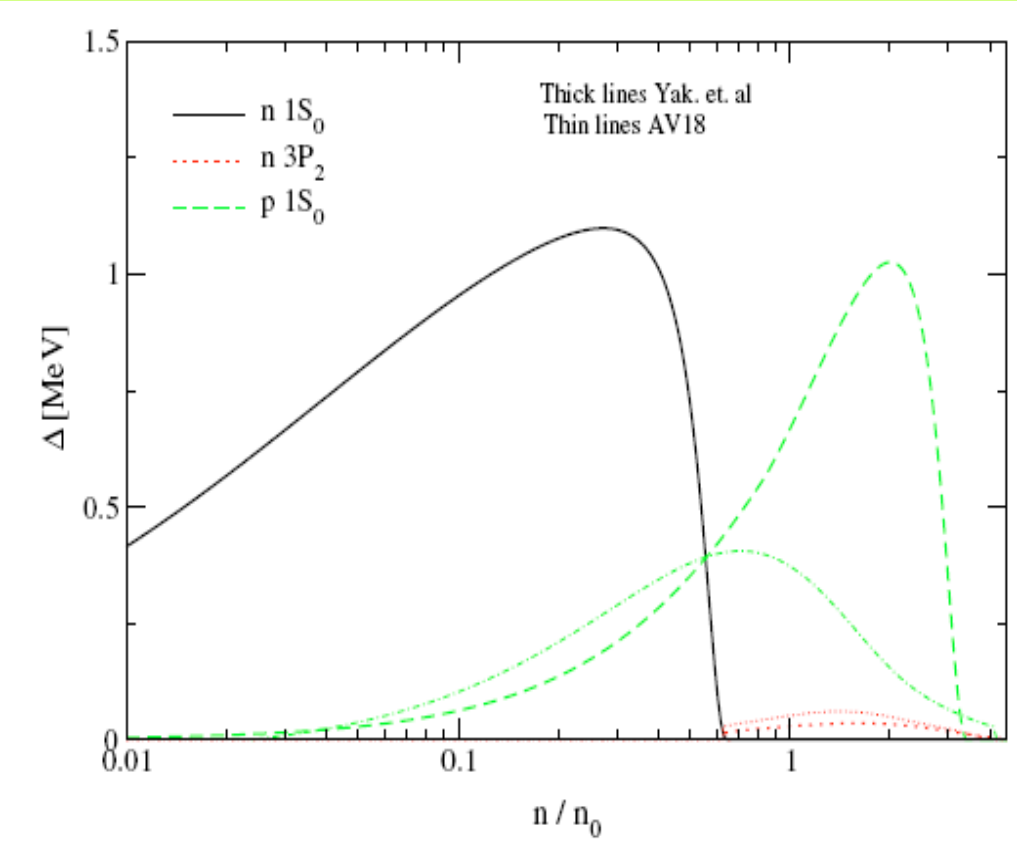
For larger density ($n>2\text{-}3\ n_0$) Nπ-cond., B-antikaon cond., HQ pasta (?)

D.V.,Yasuhira,Tatsumi 2002,T.Maruyama,…2005, Maslov, D.V. 2019



Features of pasta in HIC (?)

NN, HH pairings in NSs and *qq* in hybrid stars



Calcul.: $1S_0$-typical NN **Δ <MeV**,
For nn $3P_2$, Δ ~(0.01-0.1) MeV

In case of II-order supercond.: vortex **Abrikosov structure**,
$3P_2$:ferromag. superfluidity, D.V. PRD 2020, specific reaction on external mag. field, rotation.

Cooper BB pairing → superfluidity: glitches of pulsars and post-glitch relaxation.
Cooling of compact stars is sensitive to *$3P_2$ nn and $1S_0$ pp*,
+ hyperon-hyperon, and qq pairings in dense interiors

Constraints on EoS, masses of compact stars

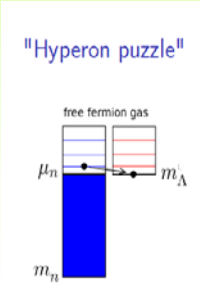
Bethe & Brown 1994: **M~1.4<1.56 M_{sol}**

Data: NS masses are changed in interval ~(1-2.1) M_{sol} PSRJ0740-6620 Cromartie et al. Nature 2020

A general analysis yields estimate $M_{\max} > 2.1 M_{\odot}.$ **M=2.07+/- 0.07 Msol** $2.08(7) M_{\odot}$ for PSR J0740+6620

Radii are also measured: 12.5+-1 km

For rotat. PSR J0952−0607 $2.35(17) M_{\odot}.$



In NSs depending on M: n_{centr} <(6-8) n_0

Strangeness appears in NS already for n>(2-4) n_0

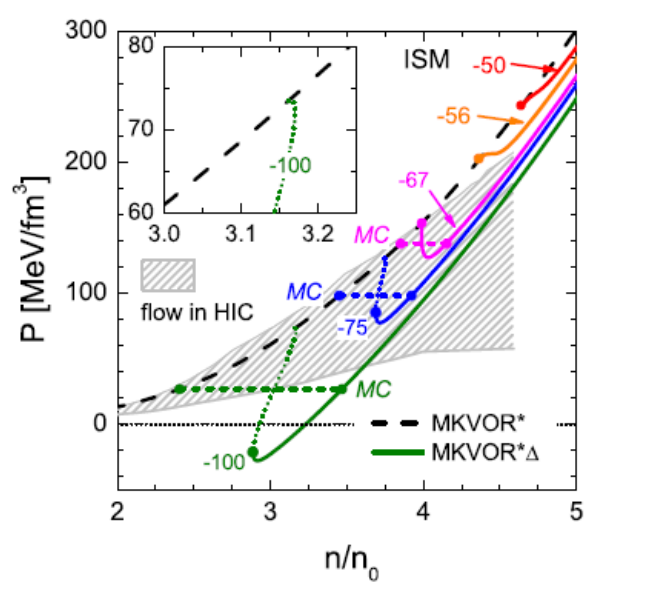
With *H* and Δ EoS softens (*P*(*n*), & max. NS *mass* decrease): Hyperon and Δ isobar puzzles.

HIC need soft EoS, NSs need stiff EoS, most of EoSs cannot fulfill all known constraints.

Delta-isobars and hyperons in cold matter

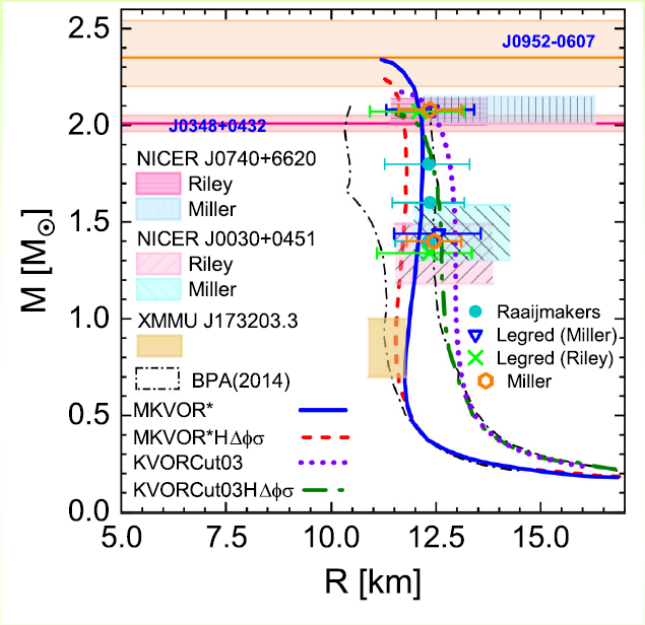
Examples of appropriate EoS:

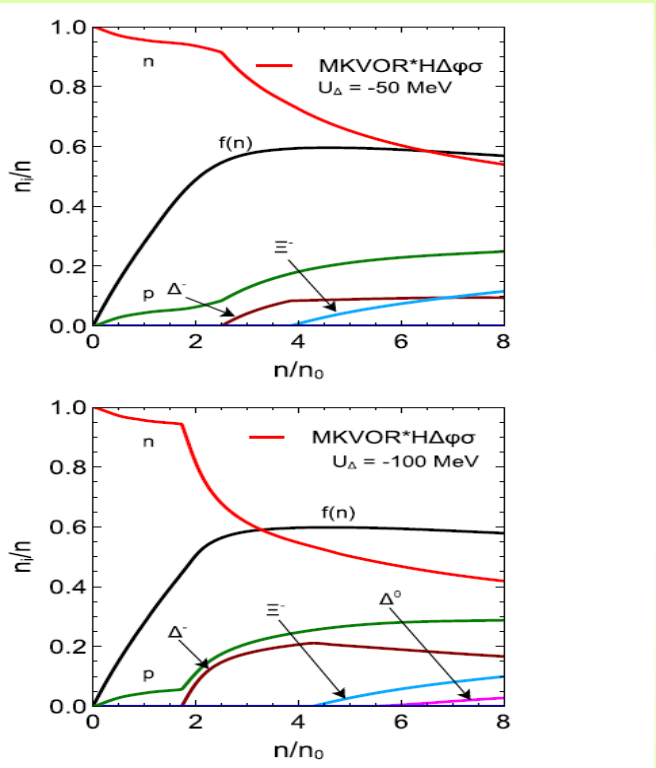
Iso-symmetric matter



$U_\Delta \sim -(50\text{–}100)$ MeV at n_0.

NS matter



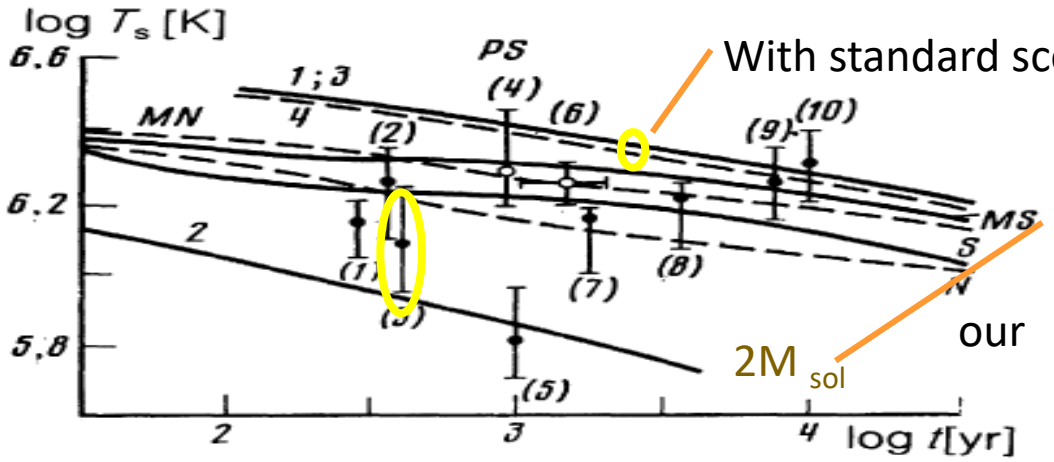


Possible Delta-resonance matter I order ph. tr. In HIC in case of a strong Δ attracting optical potential,
III-order transitions with production of hyperons and Δ in dense NS matter

Kolomeitsev, Maslov, D.V. 2016, Kolomeitsev, D.V. 2024

D.V., Senatorov JETP Lett. 1984, JETP 63,1986 all the data (**only upper limits on T_s!**) were explained by medium-modified Urca (MMU) process **assuming different masses of NS** (different mean densities)

whereas following Brown, Bethe till 2000-th most of authors thought that all NSs have masses M≈1.4 M_{sol} ,



With standard scenario

1M_{sol}

2M_{sol}

our

Now it is exp. known that

NS masses are different

Pulsar J0348-04232

M = 2.01(4) M_{sol}

PSRJ0453+1559: M=1.174(4)M_{sol}

$2.08(7)M_{\odot}$ for PSR J0740+6620

Max. M>(2-2.1) M_{sol}

FIG. 6. Comparison of theoretical calculations with data of the Einstein observatory. The abscissa is the logarithm of the time, the ordinate the logarithm of the surface temperature. The equation of state is that of Pandharipande and Smith ($M \approx 1.3\ M_{\odot}$, $R \approx 8$ km, $R^{\infty} = 11$ km). The experimental data (see Ref. 7): 1) Cas A, 2) Kepler, 3) Tycho, 4) Crab,[5] 5) SN1006, 6) RCW103, 7) RCW86, 8) W28, 9) G350, 0-18, 10) G22, 7-02. The open circles represent observed sources, the black circles are upper limits. The curves are as follows: *S* for superfluid neutron stars, *N* for normal stars, *MS* for magnetic superfluid stars, and *MN* for magnetic normal stars.[7] Curves 1 and 2 represent our calculations with mean density $\bar{\rho} = \rho_0$, $\bar{\omega}_{\pi} = m_{\pi}$, $\gamma = 0.47$ and $\bar{\rho} = 2\rho_0$, $\bar{\omega}_{\pi} = 0.5m_{\pi}$, $\gamma = 0.47$, respectively. Curves 3 and 4 are our calculations for the same values of $\bar{\rho}$ but with luminosity from Ref. 27.

If in the future central sources are discovered in supernova remnants with low values of T_s (see Fig. 6), then they could be associated with neutron stars having a denser internal region than other neutron stars with higher T_s

It was thought that no source in Cas A , now exist 20 year measurements of T_s in Cas A,+ ~20 other data points.

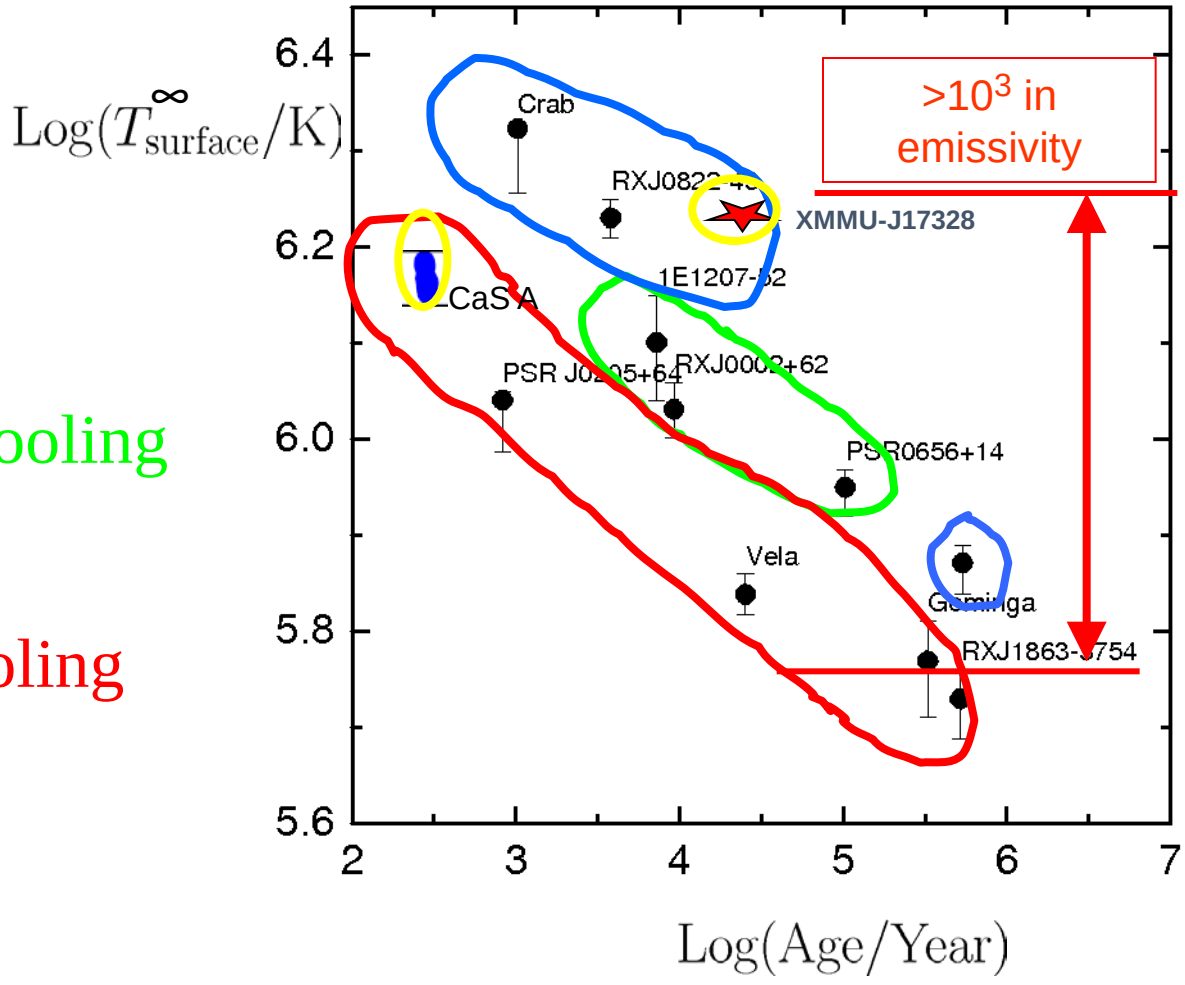
Neutron star cooling

3 groups+Cas A:

slow cooling

intermediate cooling

rapid cooling



Difficult to describe all data in MCS.

With pion softening effect included data are described within one cooling scenario.

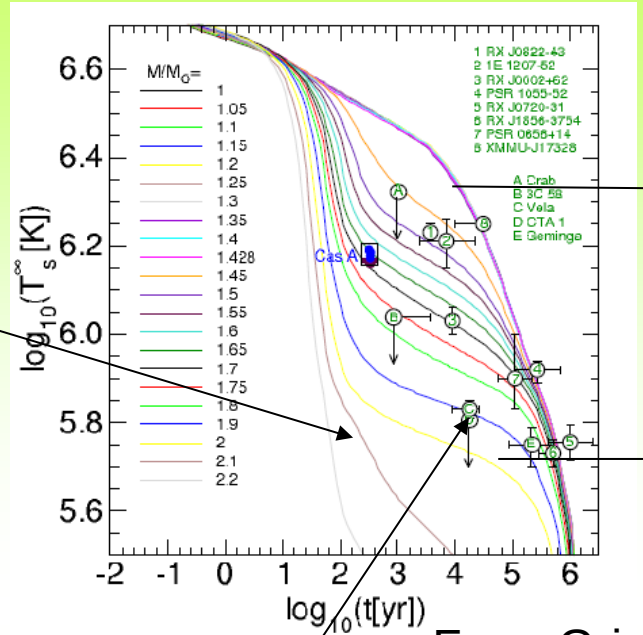
Neutrino cooling of neutron stars *(similar to di-lepton production in HIC)*

for $10^2 s<t<10^6$yr, T<1 MeV $\lambda_\nu >> R$ white-body radiation:

Direct Urca reactions $n \rightarrow pe\bar{\nu}$

yield too rapid cooling of pulsars

To occur DU needs >11-15% of *protons*

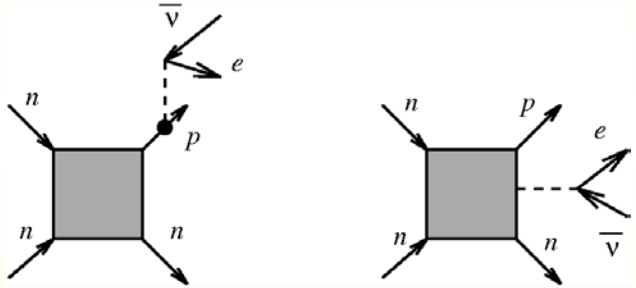


difference by 3 orders of magnitude in neutrino emissivity

From Grigorian, D.V., Maslov 2018

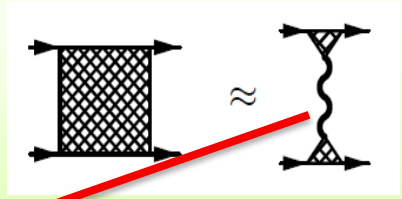
A **constraint on *n*-dependence of symmetry energy**

MMU processes $Nn \rightarrow N\, pe\bar{\nu}$



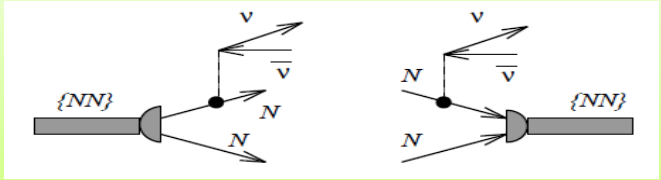
Senatorov, D.V. 1984-1987, Blaschke,Grigorian,D.V. 2004-2016

Strong enhancement of NN amplitude is needed.
It occurs due to exchange by **in-medium pion,** *cf. pion spectrum shown on slide 26*,



Strongly density/*M* dependent

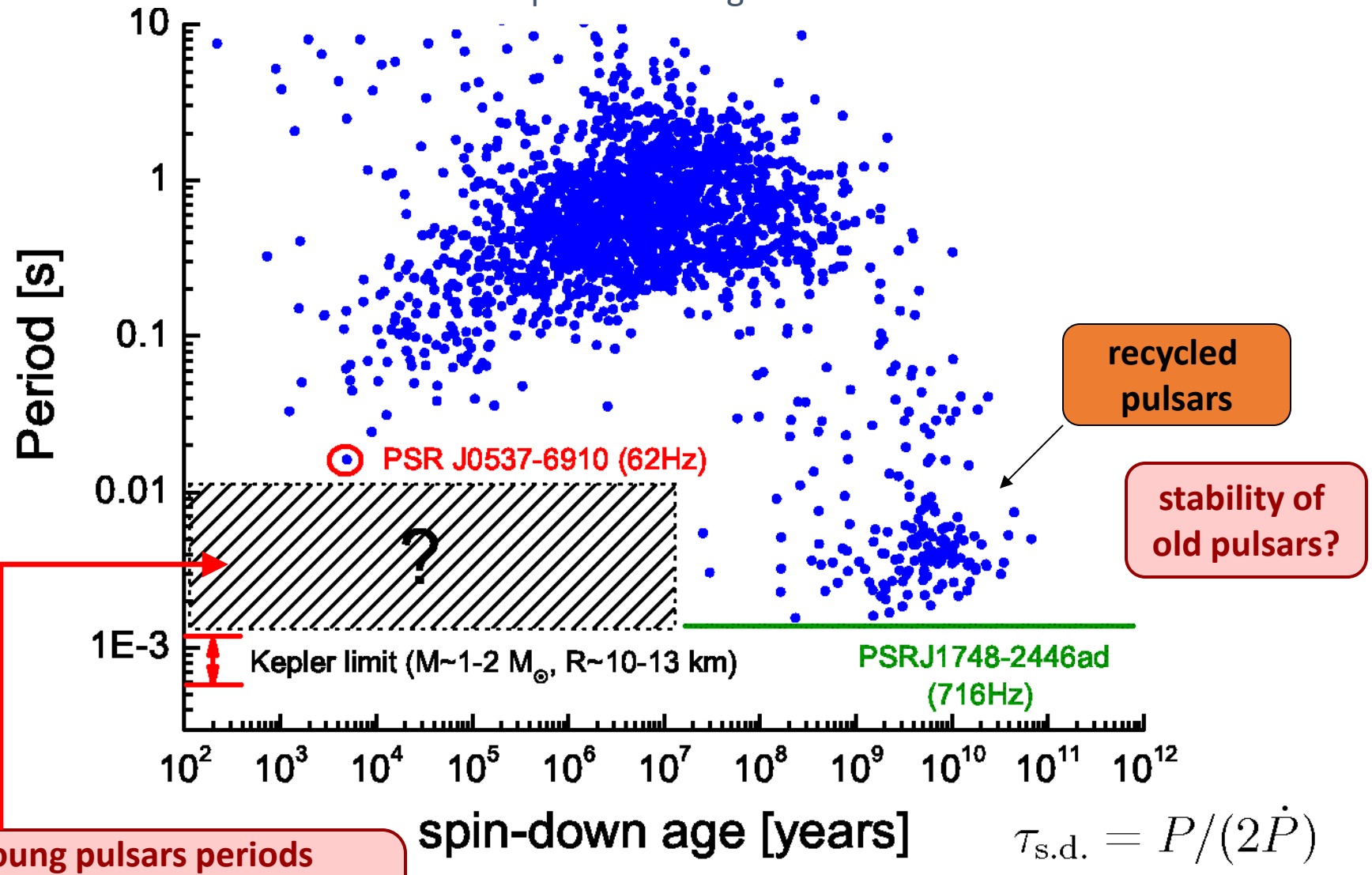
Pairing is important:
PBF processes



Data are explained with strongly *n*-dependent NN interaction +NN pairing:
low mass NSs cool slowly, massive NSs-rapidly

Pulsar period-age diagram for pulsars

- The ATNF pulsar catalogue

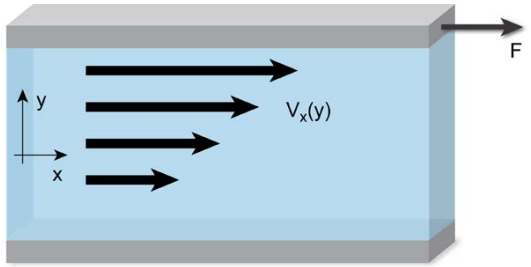


Viscosity

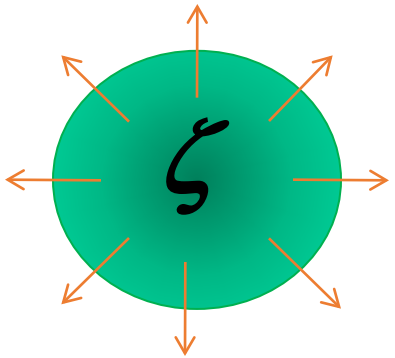
Navier-Stokes equation:

$$\rho\partial_t \boldsymbol{v}_i + \boldsymbol{v}_i \mathrm{div}\boldsymbol{v} = -\nabla_i p + \partial_k \sigma_{ki}$$

$$\sigma_{ik} = \eta\left(\nabla_k v_i + \nabla_i v_k - \frac{2}{3}\delta_{ik}\mathrm{div}\boldsymbol{v}\right) + \zeta\delta_{ik}\mathrm{div}\boldsymbol{v}$$



In HIC one considers only collisional viscosity terms



NS Shear viscosity: *lepton contributions*
nucleon contributions
phonon contributions
neutrino contributions

Inputs: *EoS,*
density profiles in NS,
pairing gaps,
NN-interaction

NS Bulk viscosity: *collisional viscosity*
"soft-mode" viscosity
radiative viscosity

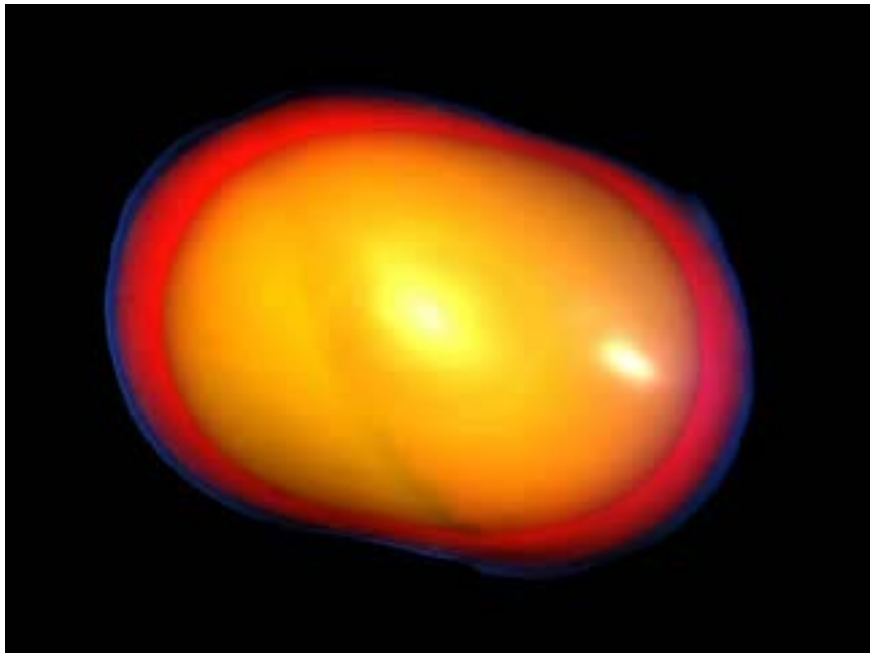
Kolomeitsev,D.V. EPJA (2014), PRC (2015)

r-mode is unstable if $\frac{1}{\tau} = -\frac{1}{\tau_G} + \frac{1}{\tau_\eta} + \frac{1}{\tau_\zeta}$ >0

Again viscosity effect!

In a dense system like a NS the Rossby waves (driven by Coriolis force) are sources of r-mode instability

Andersson, Friedman and Morsnik 1998

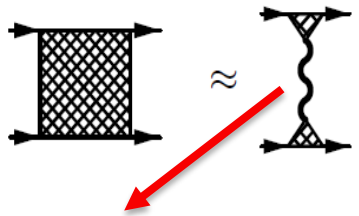


r-modes can either destroy the star
or the star stops rotating

r-mode is stable if

$$\frac{1}{\tau} = -\frac{1}{\tau_G} + \frac{1}{\tau_\eta} + \frac{1}{\tau_\zeta} \quad >0$$

In presence of "soft-mode" bulk viscosity increases

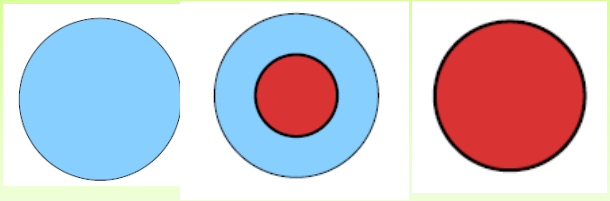


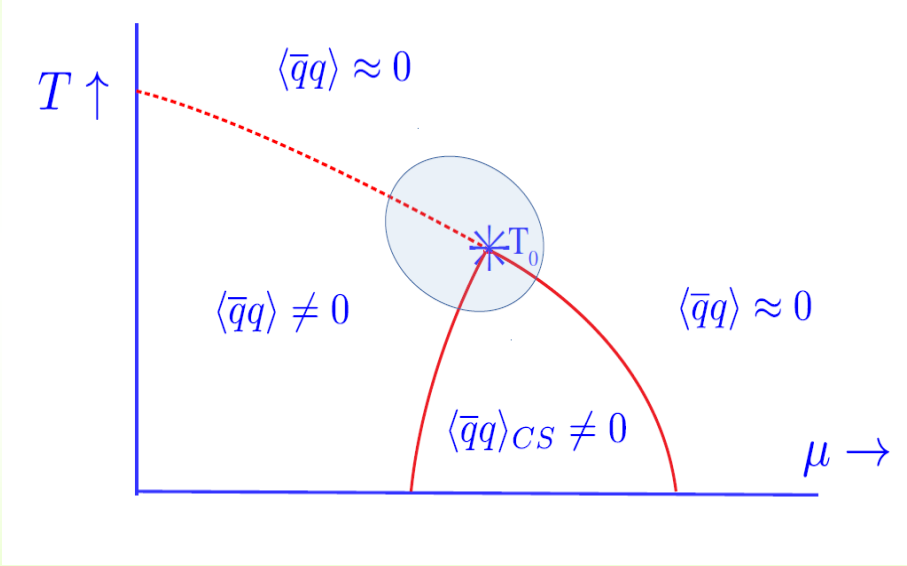
Bulk viscosity of NS matter is increased again owing to in-medium pion exchange that allows to damp r-modes and save the rotating star

Kolomeitsev, D.V. 2015, Kraav et al 2023

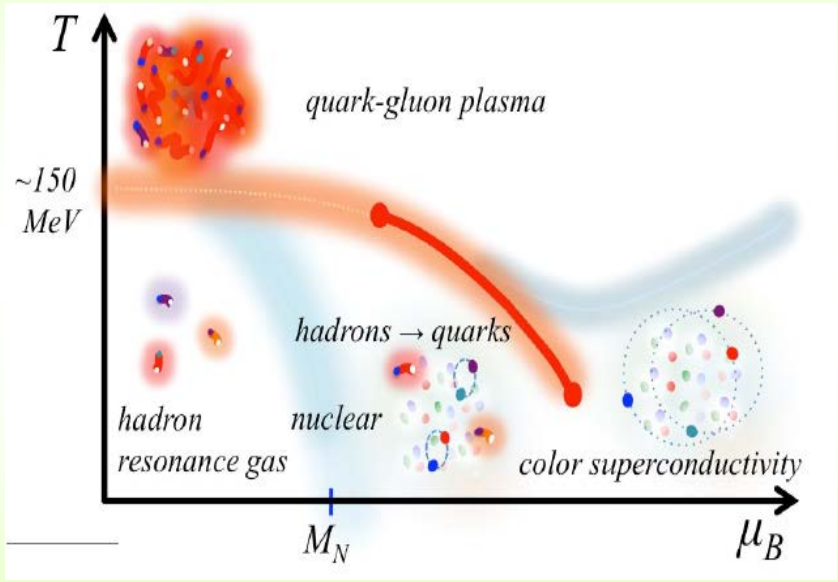
More exotics: quarkyonic crystals, similar to p-wave pion and kaon condensates and A-smectic liquid crystals

Baryon-quarkyonic-quark Fermi seas
cf. Duarte et al., 2302.04781:





Pisarski, Skokov, Tsvelik PRD 2018



Baym et al, Rep.Prog.Ph. 2017: m.b. continues tr. from nucleon to quark pairing, **two CEP points (?)**

Color superconductivity in most massive NSs (hybrid stars)

for n>~5 n_0 (T<T_c), M>~1.7 M_{sol}

Typical values of superconducting diquark gap: T_c ~**Δ~ 10-100 MeV** ~ 10^{11}-10^{12} K,

Order parameter- matrix:

Alford, Rajagopal, and Wilczek, 1998

many different phases (2SC, CFL,CSL etc) in most massive NS.

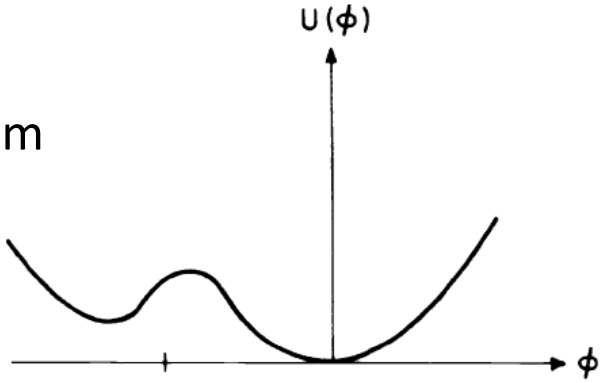
Due to strong interaction- broad fluctuation region at T~ T_c as in He-II

Precritical CSC fluctuations in HIC (?) at T<1.5 T_c ~100 -150 MeV

Kitazawa et al PRD2002,PTEP2022, DV PRC2004, Kerbikov PRD2020

M.b. more exotics (?)

If energy as function of order parameter (e.g. density, or a field) had second minimum



would exist stable/metastable nuggets of arbitrary size (without gravity) glued by condensates, strange quarks, and quark stars and, m.b., objects including DM.

Migdal 1971, Bodmer 1971, T.D. Lee & Wick 1974, Migdal, Popov, D.V. 1976, Chernoutsan, Sorokin, D.V. 1977, Witten 1984, Glashow & De Rujula 1984, …, Zhitnitsky 2025, Kolomeitsev, D.V. 2026.

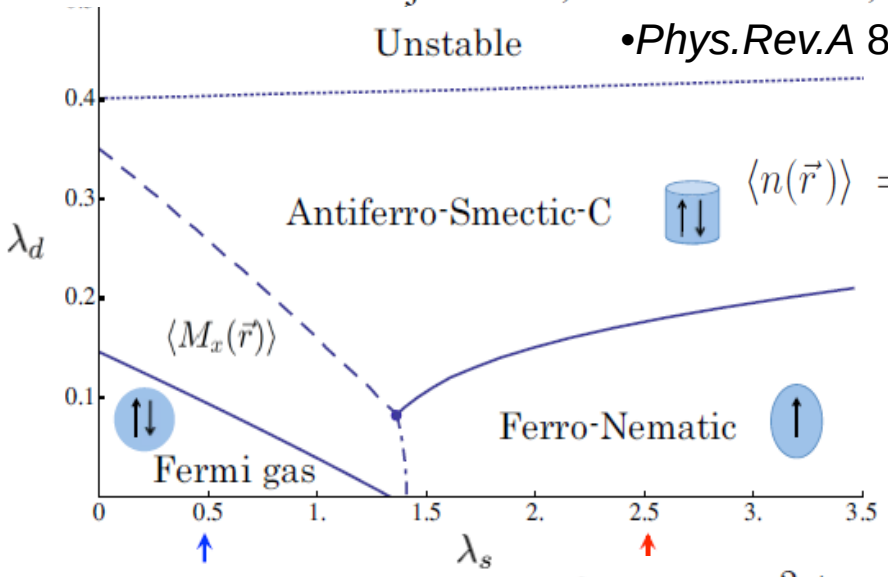
Although no abnormal states were found, presently a number of observations demonstrate anomalies that defy conventional physics interpretations e.g. 10 anomalous bursts correlated with lightnings were observed in thunderstorms occurred during 5 years. The estimated energy from individual events within the bursts is five to six orders of magnitude higher than the energy estimated by an event rate.

Some supplementary slides

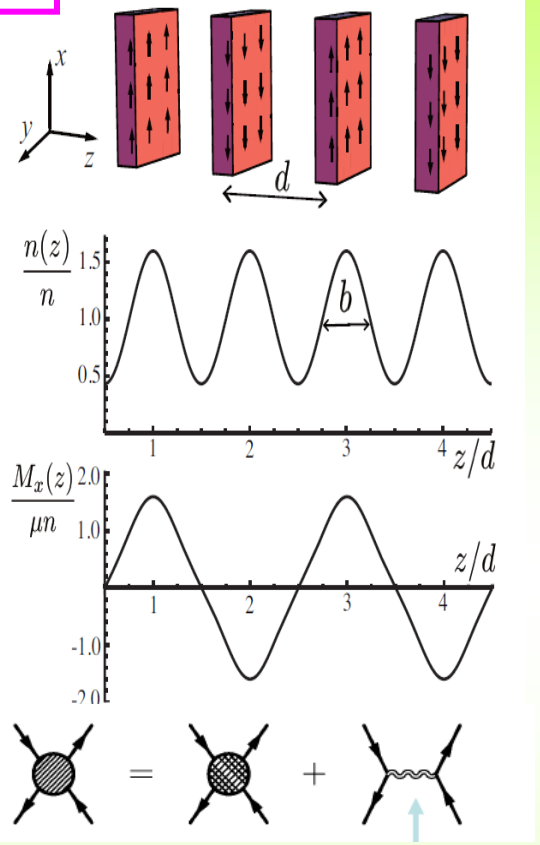
Antiferrosmectic ground state of two-component dipolar Fermi gases
– an analog of meson condensation in nuclear matter

Kenji Maeda,[1] Tetsuo Hatsuda,[2,3] and Gordon Baym[4]

•*Phys.Rev.A* 87 (2013) 2, 021604



$$\langle n(\vec{r})\rangle = \frac{nd}{b\sqrt{\pi}} \sum_{\ell=-\infty}^{\infty} e^{-(z-d\ell)^2/b^2},$$



$$\lambda_d = n\mu^2/\epsilon_F, \quad \lambda_s = ng/\epsilon_F,$$

FIG. 2: Schematic phase structure of dipolar fermions as a function of λ_s and λ_d, showing the Fermi gas phase, the ferronematic phase, the onset of spatially varying magnetization and antiferrosmectic-C phase. The dashed line shows where the AFSC phase becomes favorable compared with the uniform unmagnetized interacting Fermi gas, and the dash-dot line the transition between the uniform Fermi gas and the FN phase. Beyond the upper dotted line the system becomes unstable against collapse.

analog of tensor forces, p-wave πNN inter.

$$V(\vec{r}_1,\vec{r}_2)^{ij}_{\alpha\alpha',\beta\beta'} = \frac{\mu^2}{r^3}\{\sigma^i_{\alpha\alpha'}(\delta_{ij}-3\hat{r}_i\hat{r}_j)\sigma^j_{\beta\beta'}\} + g\delta_{\alpha\alpha'}\frac{\delta_{ij}}{3}\delta(\vec{r}_1-\vec{r}_2)\delta_{\beta\beta'},$$

analog of NN correlations

^{161}Dy and ^{163}Dy are the most magnetic fermionic atoms
novel nearly quantum degenerate dipolar Bose-Fermi gas

Similarly, for fermionic polar molecules, e.g., ^{40}K-Rb with large electric dipole moments

Other peculiarities of fluctuations in HIC

- **critical slowing down** (not enough time for critical thermal fluctuations to be developed), **dynamical effects are important!**

- **finite size effects associated with fluctuations in peripheral HIC:**

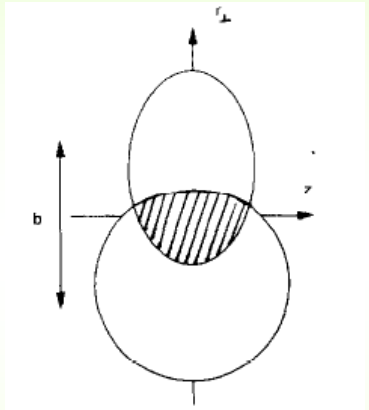
effectively reduced dimensionality near CEP :

2d geometry, if coherence length $l \sim 1/|T_{cr}-T|^{1/2}$

fulfills inequalities $R_{\perp} > l > R_{\parallel}$,

1d for $R_{\parallel} > l > R_{\perp}$,

0 d for $l > R$



Depending on impact parameter in a vicinity of T_c fluctuations may behave as in case d<3 !

Effects of different geometry for fluctuations in central and peripheral HIC (?)

About electromagnetic fields and rotation in HIC

Typical magnetic field in non-central HIC at ~ GeV A: $h=h_{VA} \sim (Ze^6)^{1/3} H_{\pi}$,
$H_{\pi}=m^2_{\pi} /e \sim 3.5 \times 10^{18}$ G, D.V., Anisimov, JETP51, 1980,

For ultrarelativistic HIC $h_{SIT} \sim \gamma h_{VA}$ Skokov,Illarionov,Toneev,Int.JMPh.A24, 2009

however quantum effects (**rapid fields are not classical**) may result in a decrease of estimate h_{SIT} resulting in that the maximum value of the magnetic field in ultrarelativistic HIC is $h_{max} \sim Ze\ h_{VA}$,D.V. PRD2024

The global polarization of Lambda (1116) hyperon observed by the STAR collaboration in noncentral Au-Au collisions indicated existence of a vorticity with rotation frequency $\Omega \sim (9 \mp 1) \times 10^{21}$ Hz$\sim 0.05 m_{\pi}$, large angular momenta are $L < 10^5$-10^6

Nuclear medium cooling scenario

with stiff EoSs without/with hyperons included

Blaschke, Grigorian, D.V. PRC 2013, EPJA 2016, HDD EoS without hyperons

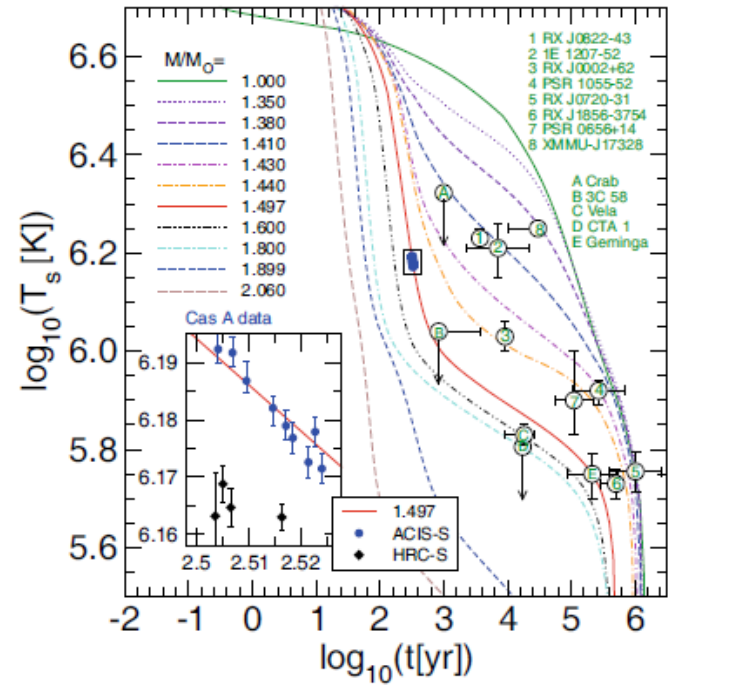
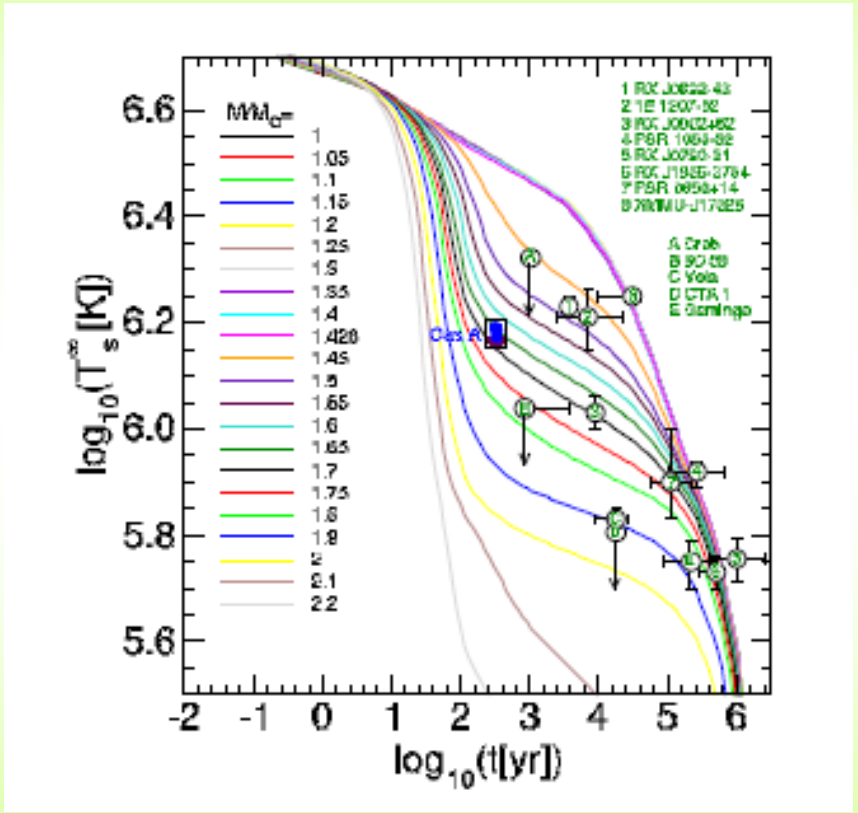


Fig. 4. Cooling curves for a NS sequence according to the hadronic HDD EoS; T_s is the redshifted surface temperature, t is the NS age. The effective pion gap is given by the solid curve 1a+1b in fig. 2, $n_c^\pi = 3n_0$. The $1S_0$ pp pairing gap corresponds to model I. The mass range is shown in the legend. Comparison with Cas A ACIS-S and HRC-S data is shown in the inset. Cooling ACIS-S data for Cas A are explained with a NS mass of $M = 1.497 M_\odot$.

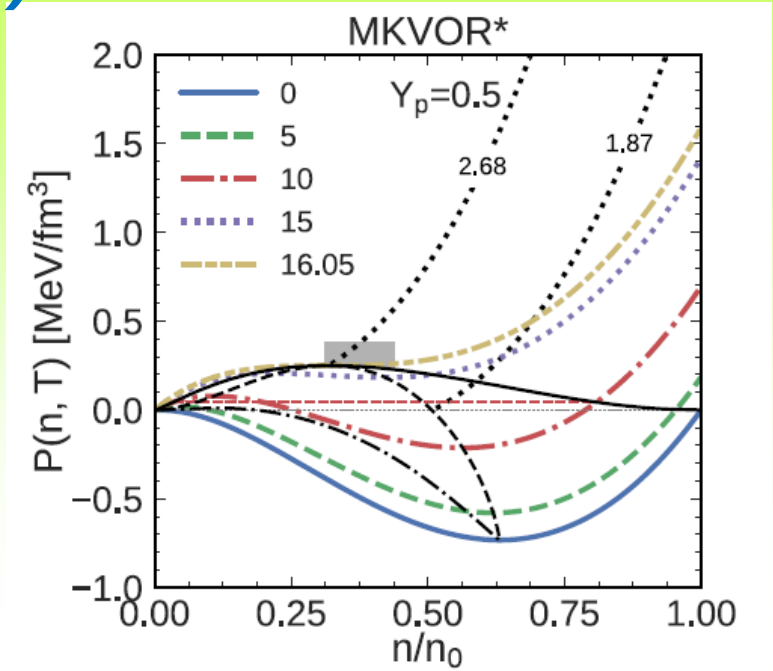
Grigorian, Maslov, D.V. 2018: Example of cooling calcul. with stiff MKVOR EoS with hyperons

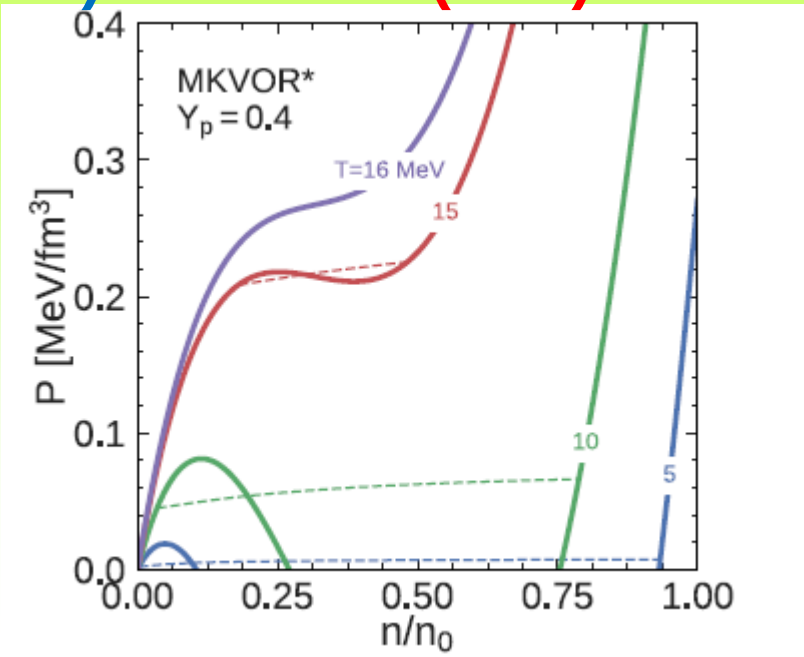


DU is allowed in our EoSs only for most massive NSs

MMU-main processes: low mass objects-slow coolers, more massive objects – more rapid coolers. Data are explained with EoSs, as with hyperons, as without

P(n) isotherms for LG tr. in ISM (*N=Z*) and IASM (*N≠Z*)





In case of two conserved charges: Maxwell construction from horizontal line transforms into curve - mixed phase, on spinodal diverge both *w*$_B$ and w$_Q$

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D.V.,Yasuhira,Tatsumi Phys.Lett.B(2002),T.Maruyama,…D.V.PRC(2005), Maslov,…D.V. PRC(2019)

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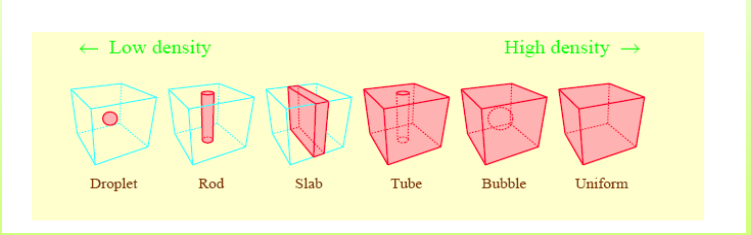
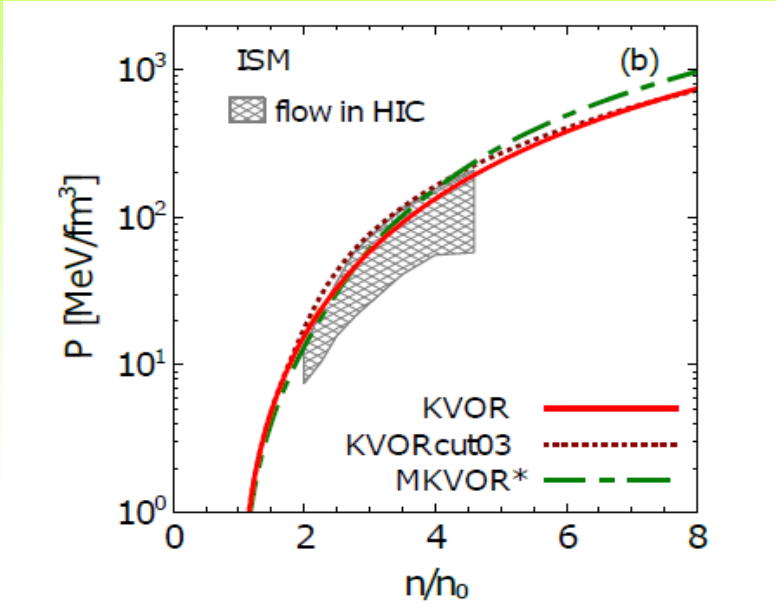


Maslov, D.V. EPJA 55(2019)

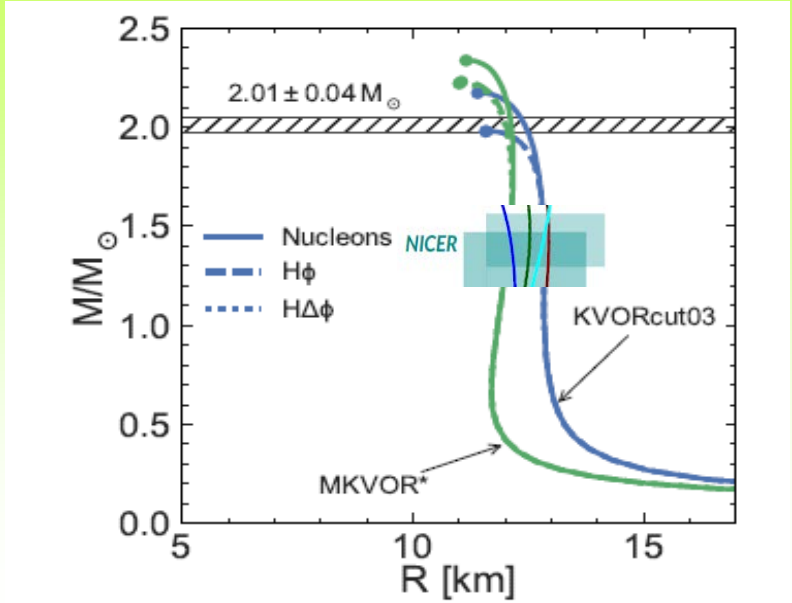
Features of pasta in HIC (?)

Scalings with quenching in vector ω (KVORcut03), and isovector ρ (MKVOR) channels

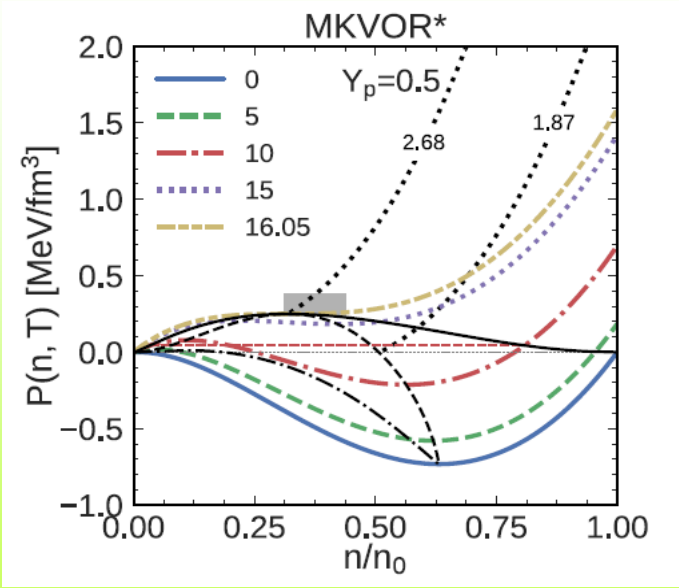
see K. Maslov, PhD thesis 09.2020



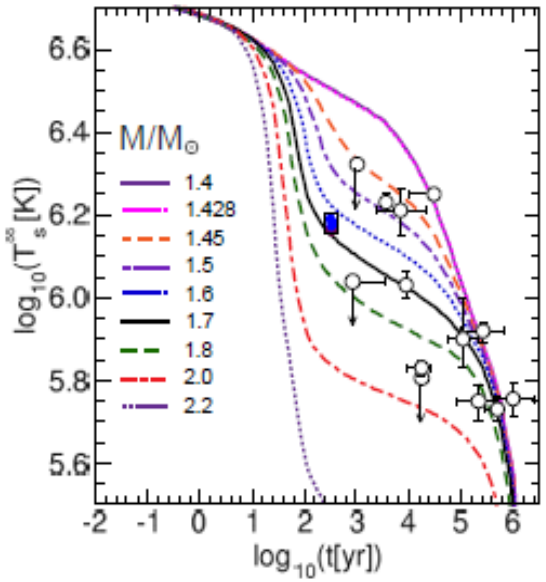
Low T,n EoS is similar to van der Waals EoS



HΔφ -with hyperons and Delta included



H. Grigorian, D.N. Voskresensky, K.A.M. Nucl.Phys A 980 (2018)



A qualitative picture for HIC at Bevalac, GSI, AGS energies:

- A short strongly nonequilibrium q-g and/or hadron stage. Formation of hadron fireball + peripheral region.
- Local equilibrium T(t,r), n(t,r) stage of fireball expansion till freezeout.
- Long-path-length particles probe dense hot interior:

dilepton and photon radiation in direct reactions, cf. neutrino radiation from NS after ~hour after formation during next 10^5 yr

- Pions with $k<m_\pi$ and $K^{+,0}$ (s=1), having larger path lengths, fly from intermediate fireball expansion s $\lambda_{K^+} \gg \lambda_{K^-} \quad \lambda_{K^-} \simeq 3$ fm for $\rho = 0.5\rho_0$.
- Nucleons, pions with $k>1.5\ m_\pi$ and K^- (s=-1) having shorter path lengths probe freezeout (breakup) stage, cf. surface radiation of photons in NS

Migdal,Saperstein,Troitsky,D.V. Phys.Rep.192 (1990), D.V. NPA555 (1993)

Picture for HIC at SPS, RHIC, LHC energies is different:

Baryon-less matter, chemical (elastic collisions) and kinetic (thermal) freezeout

Manifestation of in-medium effects in exp. pion distributions

``Free`` pion distribution in nonequilibrium matter $n_\pi(t,\mathbf{r},\mathbf{k}) = 2\sqrt{m_\pi^2+k^2}\langle\Phi(t)|\pi^+\pi|\Phi(t)\rangle$

$= 2i\sqrt{m_\pi^2+k^2}\, D_\pi^{-+}(t,\mathbf{r},t,\mathbf{k})\,.$

In thermal equilibrium matter $iD_\pi^{-+}(\omega,k) = -\dfrac{2\,\mathrm{Im}\,D_\pi^{\mathrm{ret}}(\omega,k)}{\exp\left(\dfrac{\omega}{T}\right)-1}\,.$ Senatorov, D.V. PhLett B219 (1989)

For a sudden freeze out

$D_\pi^{-+}(t_0-0,\mathbf{r},t_0-0,\mathbf{r}) = D_\pi^{-+}(t_0-0,\mathbf{r},t_0-0,\mathbf{r})\,,$ $\dfrac{d^3N_\pi}{\dfrac{d^3k}{(2\pi)^3}V(t_0)} = -2\sqrt{m_\pi^2+k^2}\int \dfrac{2\,\mathrm{Im}\,D_\pi^{\mathrm{ret}}(\omega,k,\rho(t_0),T(t_0))}{\exp\left(\dfrac{\omega}{T(t_0)}\right)-1}\dfrac{d\omega}{2\pi}\,,$

in simplified quasiparticle-based picture

quasiparticle+virtual pion terms $n_k^\pi = \sum_j \Gamma_j \dfrac{1}{\exp\left(\dfrac{\omega_j(k)}{T(t_0)}\right)-1} + \delta n_k^\pi$

dressing factors

$\Gamma_i = \dfrac{2\sqrt{m_\pi^2+k^2}}{\left(2\omega - \dfrac{\partial\,\mathrm{Re}\,\Pi^{\mathrm{ret}}}{\partial\omega}\right)\Big|_{\omega=\omega_i(k)}}\,, \quad i=\pi,\ \Delta$

$\delta n_k^\pi \approx k_0\sqrt{m_\pi^2+k_0^2}\,T^2\beta/[24\sqrt{\gamma}\tilde{\omega}^3(k_0)]$ for $\rho > \rho_{c1}=(0.5\text{-}0.8)\rho_0$

$\beta T \ll \tilde{\omega}^2(k_0)$

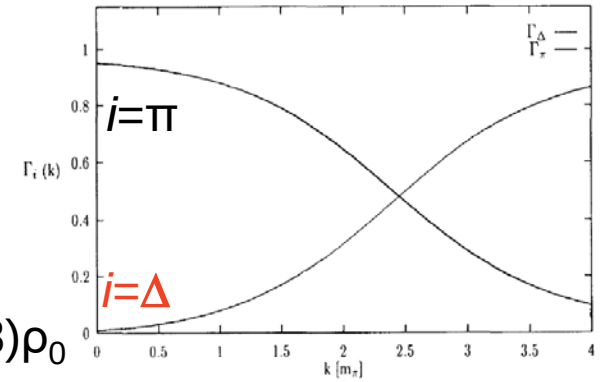
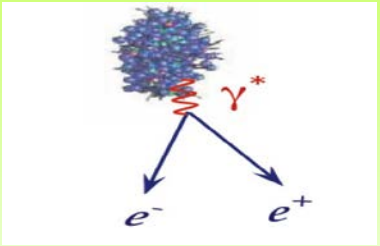


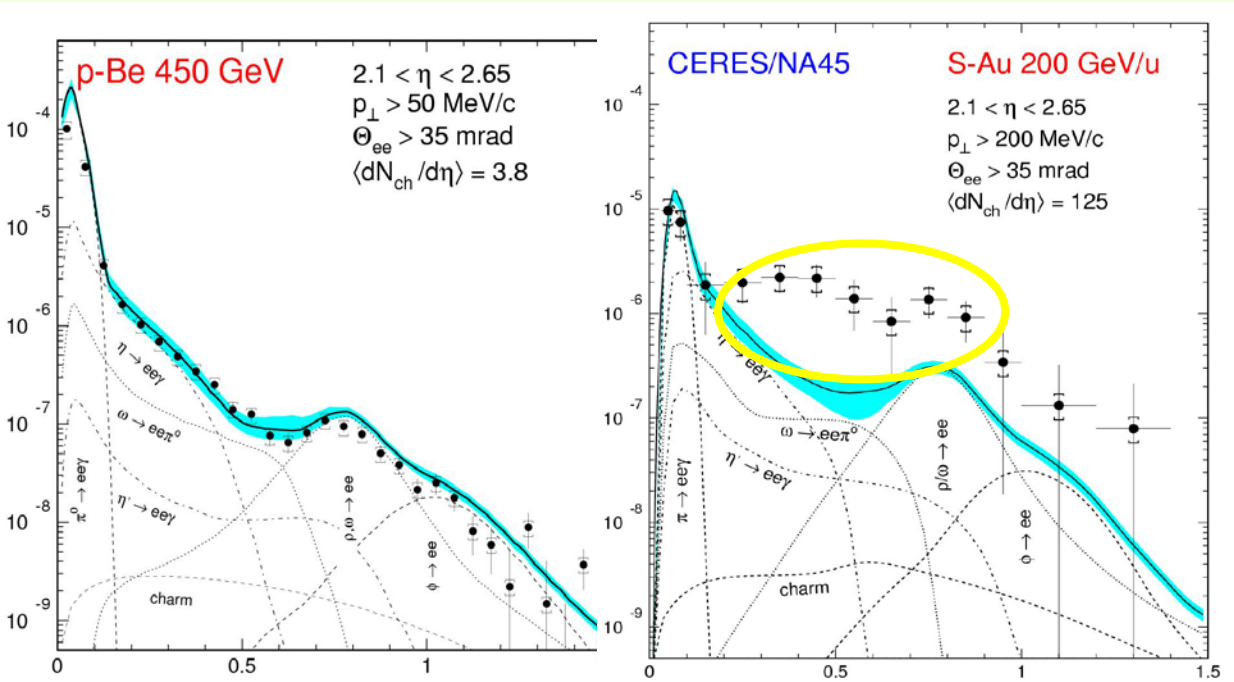
Fig. 2. Admixture factors Γ_π, Γ_Δ.

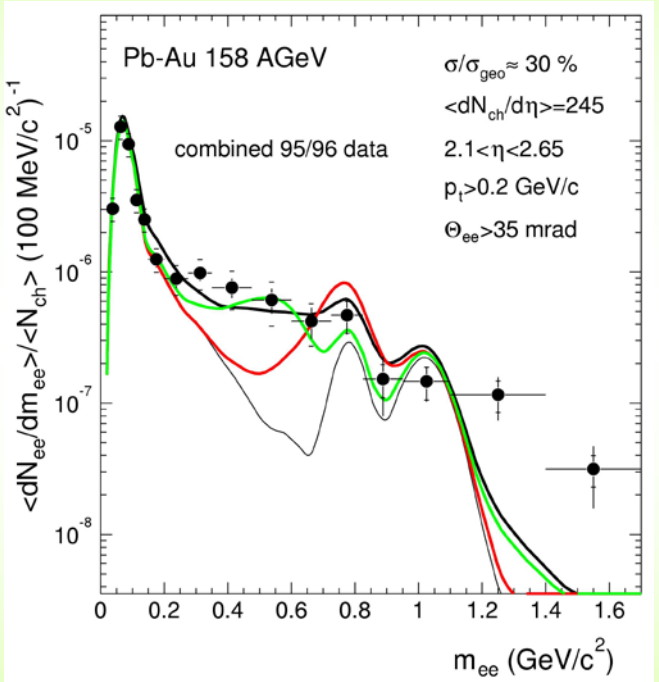
What means sudden b.up in reality: $\tau_{\mathrm{breakup}} < |\omega_\pi(k) - \sqrt{m_\pi^2+k^2}|^{-1}$ $\tau_{\mathrm{breakup}} < |\omega_\Delta(k) - \sqrt{m_\pi^2+k^2}|^{-1}$

at $\rho_b \approx 0.6\,\rho_0$ and T_b breakup is **always sudden for ω_i** (k, ρ_b, T_b) close to $\omega_{\mathrm{free}}(k)$ and adiabatic for virtual pions, and δn_k^π strongly decreases for $\rho < \rho_{c1}$

Dilepton production in p-Be, S-Au, Pb-Au







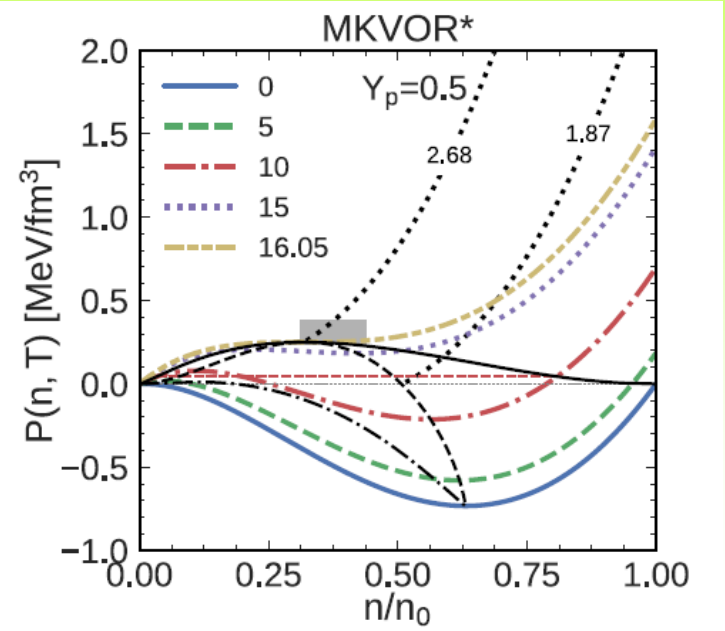
— without participation of ρ μεσονσ

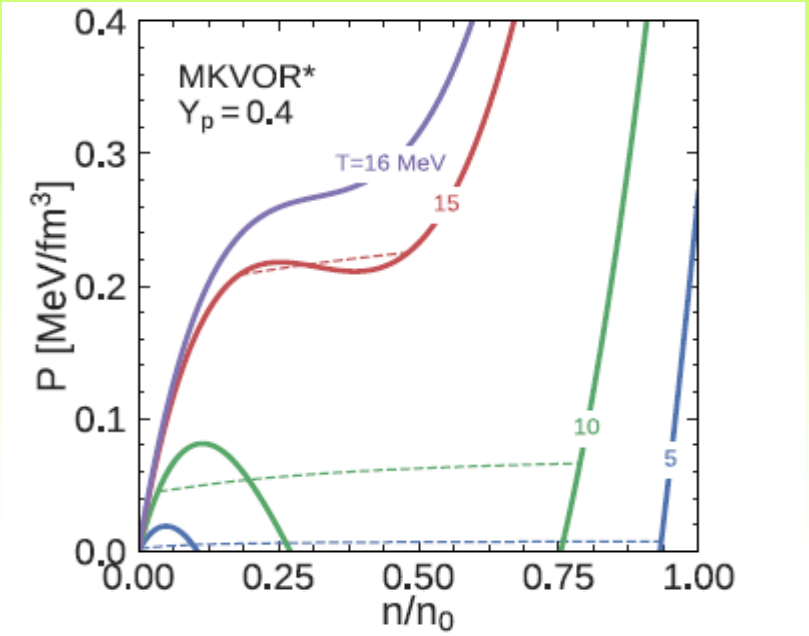
free ρ

with account of decrease of ρ mass

with account of brodening of ρ σπεχτραλ φυνχτιον ιν μεδιυμ

P(n) isotherms for LG tr. in ISM (*N=Z*) and IASM (*N≠Z*)





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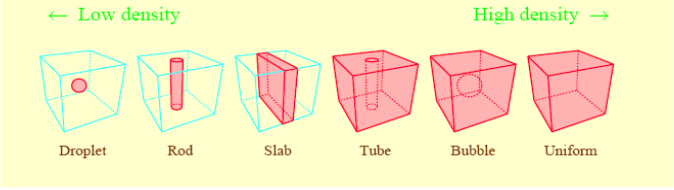

Features of pasta in HIC (?)