Standard Model Effective Field Theory

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Introduction

The Standard Model does not describe *all* physics up to *infinitely high* energies (or down to *infinitely small* distances).

At least, quantum gravity becomes important at the Planck scale.

SM does not explain dark matter and baryon asymmetry of the Universe.

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At least, quantum gravity becomes important at the Planck scale.
SM does not explain dark matter and baryon asymmetry of the Universe.

What can appear at some very short distance scale?

- Supersymmetry
- Compositeness
- Extra dimensions
- Strings / supersyrings
- Something we cannot imagine at the moment

Introduction

We can construct scenarios for new physics searches based on some known variants of the next theory (more fundamental than SM).
I. e. we can investigate some finite number of directions of departure from SM, but an infinite number of directions which we cannot now imagine remain unexplored — this is a set of measure 0.
What we need is a systematic model-independent approach for searching of some *absolutely unknown* new physics at small distances. It produces new local interactions of the SM fields.

SMEFT

$$L = L_{\mathrm{SM}}^{(4)} + \frac{1}{\Lambda}\sum_i C_i^{(5)} Q_i^{(5)} + \frac{1}{\Lambda^2}\sum_i C_i^{(6)} Q_i^{(6)} + \cdots$$

SM works well at the energy scale being investigated now
$\Rightarrow$ the new physics scale $\Lambda \gg$ the SM emergy scale.

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A theory at scales $\geq \Lambda$

- $SU(3) \times SU(2) \times U(1)$ is a subgroup of its gauge group
- Contains all SM degrees of freedom (either as elementary or as composite particles)
- Reduces to SM at scales $Q \ll \Lambda$ (unless couplings of some fields to SM ones becomes very small)

Standard Model

Matter fields

	$l = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	e_R	$q = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	u_R	d_R	$\varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$
Y	$-\frac{1}{2}$	-1	$\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{1}{2}$

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Indices:

- Weak isospin I: fundamental i, j; adjoint I, J
- Color: fundamental α, β; adjoint A, B
- Generation: not written

$$D_\mu = \partial_\mu + igt^A G_\mu^A + ig_2 t^I W_\mu^I + ig_1 Y B_\mu$$

$$\varphi_i^+ = \varphi^{i*} \qquad \tilde{\varphi}^i = \varepsilon^{ij} \varphi_j^+ \qquad \varphi^+ \overleftarrow{D}_\mu \varphi = (D_\mu \varphi)^+ \varphi$$

$$\varphi^+ \overleftrightarrow{D}_\mu \varphi = \varphi^+ (D_\mu - \overleftarrow{D}_\mu) \varphi \qquad \varphi^+ \overleftrightarrow{D}_\mu^I \varphi = \varphi^+ (t^I D_\mu - \overleftarrow{D}_\mu t^I) \varphi$$

Standard Model

$$
\begin{aligned}
L_{\mathrm{SM}}^{(4)} = & -\frac{1}{4}G_{\mu\nu}^{A}G^{A\mu\nu} - \frac{1}{4}W_{\mu\nu}^{I}W^{I\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \\
& + (D_\mu\varphi)^{+}(D^\mu\varphi) + m^2\varphi^{+}\varphi - \frac{1}{2}\lambda(\varphi^{+}\varphi)^2 \\
& + i\left(\bar{l}\not{D}l + \overline{e}\not{D}e + \overline{u}\not{D}u + \overline{d}\not{D}d\right) \\
& - \left(\bar{l}\Gamma_e e\varphi + \overline{q}\Gamma_u u\varphi + \overline{q}\Gamma_d d\varphi + \mathrm{h.c.}\right)
\end{aligned}
$$

All but 1 operators have dimension 4.

Standard Model

$$
\begin{aligned}
L^{(4)}_{\mathrm{SM}} &= -\frac{1}{4} G^A_{\mu\nu} G^{A\mu\nu} - \frac{1}{4} W^I_{\mu\nu} W^{I\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \\
&+ (D_\mu \varphi)^+ (D^\mu \varphi) + m^2 \varphi^+ \varphi - \frac{1}{2} \lambda (\varphi^+ \varphi)^2 \\
&+ i\big(\bar{l} \not{D} l + \overline{e} \not{D} e + \overline{u} \not{D} u + \overline{d} \not{D} d\big) \\
&- \big(\bar{l} \Gamma_e e \varphi + \overline{q} \Gamma_u u \varphi + \overline{q} \Gamma_d d \varphi + \text{h.c.}\big)
\end{aligned}
$$

All but 1 operators have dimension 4.

We can add ν_R and Γ_ν — neutrino (dirac) masses and mixings.

Standard Model

Full derivatives of gauge-invariant operators are irrelevant. Gauge-invariant operators = full derivatives of gauge-variant ones

$$\tilde{G}^A_{\mu\nu} G^{A\mu\nu} = 4\varepsilon^{\mu\nu\rho\sigma} \left(G^A_\nu \partial_\rho G^A_\sigma - \tfrac{1}{3} g f^{ABC} G^A_\nu G^B_\rho G^C_\sigma \right)$$

and $\tilde{W}^I_{\mu\nu} W^{I\mu\nu}$ don't affect Feynman rules and EOMs, but lead to topological nonperturbative effects.

Dimension 5

Only 1 operator (Weinberg)

$$Q_{\nu\nu}^{(5)} = (\tilde{\varphi}^{+} l)^{T} C (\tilde{\varphi}^{+} l) = (\varepsilon_{ij} \varphi^{i} l^{j})^{T} C (\varepsilon_{mn} \varphi^{m} l^{n})$$

T — transposition of the Dirac bispinor, C — the charge conjugation matrix, 2 generation indices. After spontaneous symmetry breaking gives newtrino masses and mixings.

Dimension 6

B. Grzadkowski, M. Iskrzyński, M. Misiak and J. Rosiek (2010): Warsaw basis (M. Iskrzyński M. Sc. thesis) > 900 citations

Matter fields, $X_{\mu\nu} \in \{G_{\mu\nu}, W_{\mu\nu}, B_\mu\nu\}$, D_μ

EOM: 0 contribution to S-matrix elements (eliminated by field redefinitions). EOMs at the leading order in $1/\Lambda$

$$D^2\varphi^i = m^2\varphi^i - \lambda(\varphi^+\varphi)\varphi^i - \overline{e}\Gamma_e^+ l^i + \varepsilon^{ij}\overline{q}_j\Gamma_u u - \overline{d}\Gamma_d^+ q^i$$

$$D^\nu G^A_{\nu\mu} = g\left(\overline{q}\gamma_\mu t^A q + \overline{u}\gamma_\mu t^A u + \overline{d}\gamma_\mu t^A d\right)$$

$$D^\nu W^I_{\nu\mu} = g_2\left(\varphi^+ i\overleftrightarrow{D}^I_\mu\varphi + \overline{l}\gamma_\mu t^I l + \overline{q}\gamma_\mu t^I q\right)$$

$$D^\nu B_{\nu\mu} = g_1\left(Y_\varphi\varphi^+ i\overleftrightarrow{D}_\mu\varphi + \sum Y_\psi\overline{\psi}\gamma_\mu\psi\right)$$

Bosonic

- $SU(2)$: even number of φ
- Lorentz: even number of D

$$
\begin{array}{lll}
X^3 & & \\
X^2\varphi^2 & X^2D^2 & \\
X\varphi^4 & XD^4 & X\varphi^2D^2 \\
\varphi^6 & \varphi^4D^2 & \varphi^2D^4
\end{array}
$$

With 1 $\tilde{X}$ — CP odd
$X\varphi^4$ — no scalar operators
XD^4 — always $[D_\mu, D_\nu] \to X_{\mu\nu}$

$\varphi^2 D^4$

- all D's act on one φ
- $\varepsilon_{\mu\nu\rho\sigma}$ leads to $[D_\mu, D_\nu]$
- The ordering of D_μ is irrelevant $\Rightarrow D^2\varphi$ — EOM $\rightarrow$ lower classes

$\varphi^2 D^4$

- all D's act on one φ
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$X\varphi^2 D^2$

- X or $\tilde{X}$, no ε
- each D acts on its own $\varphi \rightarrow$ by parts
- both D's act on a single field: $[D_\mu, D_\nu]$
- one D acts on X and the other one on φ: EOM for X

X^2D^2

- both D's act on a single X
- X or $\tilde{X}$, no ε
- both D's are contracted with a single X: $[D_\mu, D_\nu]$
- one D is contracted with one X, the other D — with the other X: reorder $D's$, EOM $D_\mu X^{\mu\nu}$
- the D's are contracted with each other: D^2X EOM

X^3

- X, Y, Z; 1 may be dual; $X^{\lambda}{}_{\mu}Y^{\mu}{}_{\nu}Z^{\nu}{}_{\lambda}$
- $g^{\alpha\beta}X_{\alpha\mu}X_{\beta\nu}Z^{\mu\nu} = 0$, $g^{\alpha\beta}X_{\alpha\mu}\tilde{X}_{\beta\nu} = \frac{1}{4}g_{\mu\nu}X_{\alpha\beta}\tilde{X}^{\alpha\beta}$
- 3 different tensors — ε^{ABC} or ε^{IJK}

$$Q_G = f^{ABC}G^{A\lambda}{}_{\mu}G^{B\mu}{}_{\nu}G^{C\nu}{}_{\lambda} \qquad Q_{\tilde{G}} = f^{ABC}\tilde{G}^{A\lambda}{}_{\mu}G^{B\mu}{}_{\nu}G^{C\nu}{}_{\lambda}$$

$$Q_W = f^{IJK}W^{I\lambda}{}_{\mu}W^{J\mu}{}_{\nu}W^{K\nu}{}_{\lambda} \qquad Q_{\tilde{W}} = f^{IJK}\tilde{W}^{I\lambda}{}_{\mu}W^{J\mu}{}_{\nu}W^{K\nu}{}_{\lambda}$$

$X^2\varphi^2$

Singlet or triplet: $\varphi^+\varphi$ or $\varphi^+t^I\varphi$. $\varphi^+t^I\tilde{\varphi}$ has a non-zero Y.

$$Q_{\varphi G} = \varphi^+\varphi\, G^A_{\mu\nu}G^{A\mu\nu} \qquad Q_{\varphi\tilde{G}} = \varphi^+\varphi\, \tilde{G}^A_{\mu\nu}G^{A\mu\nu}$$

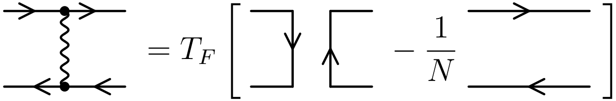
$$Q_{\varphi W} = \varphi^+\varphi\, W^I_{\mu\nu}W^{I\mu\nu} \qquad Q_{\varphi\tilde{W}} = \varphi^+\varphi\, \tilde{W}^I_{\mu\nu}W^{I\mu\nu}$$

$$Q_{\varphi B} = \varphi^+\varphi\, B_{\mu\nu}B^{\mu\nu} \qquad Q_{\varphi\tilde{B}} = \varphi^+\varphi\, \tilde{B}_{\mu\nu}B^{\mu\nu}$$

$$Q_{\varphi WB} = \varphi^+t^I\varphi\, W^I_{\mu\nu}B^{\mu\nu} \qquad Q_{\varphi\tilde{W}B} = \varphi^+t^I\varphi\, \tilde{W}^I_{\mu\nu}B^{\mu\nu}$$

φ^6

- $Y = 0 \Rightarrow (\varphi^+)^3\varphi^3$. Signlets and triplets.
- 0 triplets
- 2 triplets



$$t^{Ii}{}_j t^{Ik}{}_l = \frac{1}{2}\left[\delta^i_l \delta^k_j - \frac{1}{2}\delta^i_j \delta^k_l\right]$$

$$(\varphi^+ t^I \varphi)^2 = \frac{1}{4}(\varphi^+\varphi)^2$$

- 3 triplets

$$\varepsilon^{IJK}(\varphi^+ t^I \varphi)(\varphi^+ t^J \varphi)(\varphi^+ t^K \varphi) = 0$$

$$Q_\varphi = (\varphi^+\varphi)^3$$

$\varphi^4 D^2$

- $Y = 0 \Rightarrow (\varphi^+)^2\varphi^2$; 2 D's are contracted
- both D's act on a single $\varphi \Rightarrow$ EOM
- both D's act on both φ's or both φ^+'s $\Rightarrow$ by parts
- one D acts on φ, and the other one on φ^+: 2 isospin structures
  - (isospin Fierz)

$$(\varphi^+ t^I \varphi)\big[(D_\mu\varphi)^+ t^I (D^\mu\varphi)\big]$$
$$= \tfrac{1}{2}(\varphi^+ D_\mu\varphi)^*(\varphi^+ D^\mu\varphi) - \tfrac{1}{4}(\varphi^+\varphi)\big[(D_\mu\varphi)^+(D^\mu\varphi)\big]$$

  - EOM

$$\partial^2(\varphi^+\varphi) = 2(D_\mu\varphi)^+(D^\mu\varphi) + \varphi^+ D^2\varphi + (D^2\varphi)^+\varphi$$

$$Q_{\varphi\Box} = (\varphi^+\varphi)\partial^2(\varphi^+\varphi) \qquad Q_{\varphi D} = (\varphi^+ D_\mu\varphi)(\varphi^+ D^\mu\varphi)$$

2-fermion

Left $\psi \in \{l, e^c, q, u^c, d^c\}$

	2	3
$\overline{\psi}_1 \gamma_\mu \psi_2$	φD	$XD,\ \varphi^2 D,\ D^3$
$\psi_1^T C \psi_2$	$\varphi^2,\ D^2$	$\varphi^3,\ \varphi D^2$
$\psi_1^T C \sigma_{\mu\nu} \psi_2$	$X,\ D^2$	$X\varphi,\ \varphi D^2$

2-fermion

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$\psi_1^T C\psi_2$	$\varphi^2,\ D^2$	$\varphi^3,\ \varphi D^2$
$\psi_1^T C\sigma_{\mu\nu}\psi_2$	$X,\ D^2$	$X\varphi,\ \varphi D^2$

Dimension 5

- $\psi_1^T C\Gamma\psi_2$: $Y \neq 0$
- $\overline{\psi}_1\gamma_\mu\psi_2$: $Y \neq \pm\frac{1}{2}$

$\psi^2 X$, $\psi^2 D^2$, $\psi^2\varphi D$ — empty

$\psi^2\varphi^2$: $Y_{\varphi^2} = \pm 1$, only l^2, a single isospin structure $\Rightarrow$ Weinberg operator

Dimension 6

- Scalar and tensor currents $\overline{\psi}_1\psi_2$, $\overline{\psi}_1\sigma_{\mu\nu}\psi_2$: the number of φ is odd $\Rightarrow$ the current $I = \frac{1}{2}$
- $\overline{\psi}_1\gamma_\mu\psi_2$: the number of φ is even $\Rightarrow$ the current $I = 0$, 1. Both left or both right $\Rightarrow$ no currents with C.

Dimension 6

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$\psi^2 D^3$

- $\overline{\psi}_1\gamma_\mu\psi_2$
- D acting on $\overline{\psi}$ $\Rightarrow$ by parts
- $[D, D] \to X$
- $\overline{\psi}D^2 \not{D}\psi \Rightarrow$ EOM

$\psi^2\varphi D^2$

- $\overline{\psi}\psi$ or $\overline{\psi}\sigma_{\mu\nu}\psi$
- D acts on $\overline{\psi}$ $\Rightarrow$ by parts
- $\overline{\psi}\sigma^{\mu\nu}\psi\, D_\mu D_\nu\varphi$, $\varphi\,\overline{\psi}\sigma^{\mu\nu}D_\mu D_\nu\psi \Rightarrow \psi^2 X\varphi$
- $\overline{\psi}\psi\, D^2\varphi \Rightarrow$ EOM
- $\varphi\,\overline{\psi}D^2\psi$: $\not{D}^2 = D^2 - \frac{i}{2}\sigma^{\mu\nu}[D_\mu, D_\nu]$

$$(D_\mu\varphi)\,\overline{\psi}\sigma^{\mu\nu}D_\nu\psi = \frac{i}{2}(D_\mu\varphi)\,\overline{\psi}(\gamma^\mu\not{D} - \not{D}\gamma^\mu)\psi$$

$$(D_\mu\varphi)\,\overline{\psi}D^\mu\psi = \frac{1}{2}(D_\mu\varphi)\,\overline{\psi}(\gamma^\mu\not{D} + \not{D}\gamma^\mu)\psi$$

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$$(D_\mu\varphi)\,\overline{\psi}D^\mu\psi = \tfrac{1}{2}(D_\mu\varphi)\,\overline{\psi}(\gamma^\mu\not{D} + \not{D}\gamma^\mu)\psi$$
$$(D_\mu\varphi)\,\overline{\psi}\gamma^\mu\not{D}\psi \qquad \text{EOM}$$

$\psi^2\varphi D^2$

- $\overline{\psi}\psi$ or $\overline{\psi}\sigma_{\mu\nu}\psi$
- D acts on $\overline{\psi}$ $\Rightarrow$ by parts
- $\overline{\psi}\sigma^{\mu\nu}\psi\, D_\mu D_\nu\varphi,\ \varphi\,\overline{\psi}\sigma^{\mu\nu}D_\mu D_\nu\psi \Rightarrow \psi^2 X\varphi$
- $\overline{\psi}\psi\, D^2\varphi \Rightarrow$ EOM
- $\varphi\,\overline{\psi}D^2\psi$: $\not{D}^2 = D^2 - \frac{i}{2}\sigma^{\mu\nu}[D_\mu, D_\nu]$

$$(D_\mu\varphi)\,\overline{\psi}\sigma^{\mu\nu}D_\nu\psi = \frac{i}{2}(D_\mu\varphi)\,\overline{\psi}(\gamma^\mu\not{D} - \not{D}\gamma^\mu)\psi$$

$$(D_\mu\varphi)\,\overline{\psi}D^\mu\psi = \frac{1}{2}(D_\mu\varphi)\,\overline{\psi}(\gamma^\mu\not{D} + \not{D}\gamma^\mu)\psi$$

$$(D_\mu\varphi)\,\overline{\psi}\gamma^\mu\not{D}\psi \qquad \text{EOM}$$

$$(D_\mu\varphi)\,\overline{\psi}\not{D}\gamma^\mu\psi \to -(D_\mu\varphi)\,\overline{\psi}\overleftarrow{\not{D}}\gamma^\mu\psi - (D_\nu D_\mu\varphi)\,\overline{\psi}\gamma^\nu\gamma^\mu\psi$$

$\psi^2 \varphi D^2$

- $\overline{\psi}\psi$ or $\overline{\psi}\sigma_{\mu\nu}\psi$
- D acts on $\overline{\psi}$ $\Rightarrow$ by parts
- $\overline{\psi}\sigma^{\mu\nu}\psi\, D_\mu D_\nu \varphi$, $\varphi\, \overline{\psi}\sigma^{\mu\nu} D_\mu D_\nu \psi \Rightarrow \psi^2 X \varphi$
- $\overline{\psi}\psi\, D^2\varphi \Rightarrow$ EOM
- $\varphi\, \overline{\psi} D^2 \psi$: $\not{D}^2 = D^2 - \frac{i}{2}\sigma^{\mu\nu}[D_\mu, D_\nu]$

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$$(D_\mu\varphi)\,\overline{\psi}\gamma^\mu \not{D}\psi \qquad \text{EOM}$$

$$(D_\mu\varphi)\,\overline{\psi}\not{D}\gamma^\mu\psi \to -(D_\mu\varphi)\,\overline{\psi}\overleftarrow{\not{D}}\gamma^\mu\psi - (D_\nu D_\mu\varphi)\,\overline{\psi}\gamma^\nu\gamma^\mu\psi$$

$$(D_\mu D_\nu\varphi)\,\overline{\psi}\gamma^\mu\gamma^\nu\psi = (D^2\varphi)\,\overline{\psi}\psi - \tfrac{i}{2}([D_\mu, D_\nu]\varphi)\,\overline{\psi}\sigma^{\mu\nu}\psi$$

$\psi^2 X D$

- X may be dual, no $\varepsilon_{\mu\nu\rho\sigma}$
- $\overline{\psi}\gamma^\mu\psi \Rightarrow D$ contracted with X
- D acts on $X \Rightarrow$ EOM
- D acts on $\overline{\psi} \Rightarrow$ by parts

$$X^{\mu\nu}\,\overline{\psi}\gamma_\mu D_\nu\psi = \tfrac{1}{2}X^{\mu\nu}\,\overline{\psi}(\gamma_\mu\gamma_\nu \not{D} + \gamma_\mu \not{D}\gamma_\nu)\psi$$

$$= \tfrac{1}{2}X^{\mu\nu}\,\overline{\psi}(\gamma_\mu\gamma_\nu \not{D} - \not{D}\gamma_\mu\gamma_\nu)\psi + X^{\mu\nu}\,\overline{\psi}\gamma_\nu D_\mu\psi$$

the last term is our LHS with $-$

$$= \tfrac{1}{4}X^{\mu\nu}\,\overline{\psi}(\gamma_\mu\gamma_\nu \not{D} - \not{D}\gamma_\mu\gamma_\nu)\psi$$

$$\to \tfrac{1}{4}X^{\mu\nu}\,\overline{\psi}(\gamma_\mu\gamma_\nu \not{D} + \overleftarrow{\not{D}}\gamma_\mu\gamma_\nu)\psi + \tfrac{1}{4}(D^\rho X^{\mu\nu})\,\overline{\psi}\gamma_\rho\gamma_\mu\gamma_\nu D^\rho\psi$$

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$$\to \tfrac{1}{4}X^{\mu\nu}\,\overline{\psi}(\gamma_\mu\gamma_\nu \not{D} + \overleftarrow{\not{D}}\gamma_\mu\gamma_\nu)\psi + \tfrac{1}{4}(D^\rho X^{\mu\nu})\,\overline{\psi}\gamma_\rho\gamma_\mu\gamma_\nu D^\rho\psi$$

$$\gamma_\rho\gamma_\mu\gamma_\nu = g_{\rho\mu}\gamma_\nu - g_{\rho\nu}\gamma_\mu + g_{\mu\nu}\gamma_\rho - i\varepsilon_{\rho\mu\nu\sigma}\gamma^\sigma\gamma_5$$

$$(D^\rho X^{\mu\nu})\,\overline{\psi}\gamma_\rho\gamma_\mu\gamma_\nu D^\rho\psi = 2(D_\nu X^{\mu\nu})\,\overline{\psi}\gamma_\mu\psi - 2i(D_\nu \tilde{X}^{\mu\nu})\,\overline{\psi}\gamma_\mu\gamma_5\psi$$

$\psi^2\varphi^3$

- $\overline{\psi_1}\psi_2$ with $I = \frac{1}{2}$ — the same as in Yukawa terms in $L^{(4)}_{\text{SM}}$. The number of φ and $\tilde{\varphi}$ is given by Y.
- there are 2 ways to combine 3 isospins $\frac{1}{2}$ to $\frac{1}{2}$. One of them is 0: $\varphi^+\tilde{\varphi} = \varepsilon^{ij}\varphi_i^+\varphi_j^+ = 0$, $\varepsilon_{ij}\varphi^i\varphi^j = 0$.

Yukawa terms in $L^{(4)}_{\text{SM}}$ times $\varphi^+\varphi$.

$$Q_{e\varphi} = (\varphi^+\varphi)(\bar{l}e\varphi)$$

$$Q_{u\varphi} = (\varphi^+\varphi)(\bar{q}u\varphi) \qquad Q_{d\varphi} = (\varphi^+\varphi)(\bar{q}d\varphi)$$

$\psi^2 X \varphi$

- $\overline{\psi_1}\sigma^{\mu\nu}\psi_2$ with $I = \frac{1}{2}$
- $Y = 0 \Rightarrow$ fermion–higgs combinations like in $L_{\text{SM}}^{(4)}$
- $W^I_{\mu\nu}$, $G^A_{\mu\nu}$ contracted with t^I, t^A — unique way
- Dual B, W, G — nothing new: $\varepsilon_{\mu\nu\rho\sigma}\sigma^{\rho\sigma} = \gamma_5\sigma_{\mu\nu}$, γ_5 absorbed into ψ

$$Q_{eW} = (\bar{l}\sigma^{\mu\nu}e)t^I\varphi W^I_{\mu\nu} \qquad Q_{eB} = (\bar{l}\sigma^{\mu\nu}e)\varphi B_{\mu\nu}$$

$$Q_{uG} = (\bar{q}\sigma^{\mu\nu}t^A u)\tilde{\varphi}G^A_{\mu\nu} \qquad Q_{uW} = (\bar{q}\sigma^{\mu\nu}u)t^I\tilde{\varphi}W^I_{\mu\nu}$$

$$Q_{uB} = (\bar{q}\sigma^{\mu\nu}u)\tilde{\varphi}B_{\mu\nu}$$

$$Q_{dG} = (\bar{q}\sigma^{\mu\nu}t^A d)\varphi G^A_{\mu\nu} \qquad Q_{dW} = (\bar{q}\sigma^{\mu\nu}d)t^I\varphi W^I_{\mu\nu}$$

$$Q_{dB} = (\bar{q}\sigma^{\mu\nu}d)\varphi B_{\mu\nu}$$

$\psi^2\varphi^2 D$

- $\overline{\psi}_1\gamma^\mu\psi_2$; D acts on $\psi \Rightarrow$ EOM
- higgs part: $I = 0, 1$; color-singlet $\Rightarrow$ fermion part too
- $Y = 0 \Rightarrow$ the number of φ, φ^+
- $\varphi^+(D_\mu + \overleftarrow{D}_\mu)\varphi$, $\varphi^+(t^I D_\mu + \overleftarrow{D}_\mu t^I)\varphi$: by parts, EOM. $\overleftrightarrow{D}_\mu$, $\overleftrightarrow{D}^I_\mu$ remain
- $\tilde{\varphi}^+(D_\mu + \overleftarrow{D}_\mu)\varphi = 0$, $\tilde{\varphi}^+\overleftrightarrow{D}_\mu\varphi = 2\tilde{\varphi}^+D_\mu\varphi$, the Hermitian conjugate operator is independent

$$Q^{(1)}_{\varphi l} = (\varphi^+ i\overleftrightarrow{D}_\mu\varphi)(\bar{l}\gamma^\mu l) \qquad Q^{(3)}_{\varphi l} = (\varphi^+ i\overleftrightarrow{D}^I_\mu\varphi)(\bar{l}\gamma^\mu t^I l)$$

$$Q_{\varphi e} = (\varphi^+ i\overleftrightarrow{D}_\mu\varphi)(\bar{e}\gamma^\mu e)$$

$$Q^{(1)}_{\varphi q} = (\varphi^+ i\overleftrightarrow{D}_\mu\varphi)(\bar{q}\gamma^\mu q) \qquad Q^{(3)}_{\varphi q} = (\varphi^+ i\overleftrightarrow{D}^I_\mu\varphi)(\bar{q}\gamma^\mu t^I q)$$

$$Q_{\varphi u} = (\varphi^+ i\overleftrightarrow{D}_\mu\varphi)(\bar{u}\gamma^\mu u) \qquad Q_{\varphi d} = (\varphi^+ i\overleftrightarrow{D}_\mu\varphi)(\bar{d}\gamma^\mu d)$$

$$Q_{\varphi ud} = (\tilde{\varphi}^+ i\overleftrightarrow{D}_\mu\varphi)(\bar{u}\gamma^\mu d)$$

4-fermion

Left $\psi \in \{l, e^c, q, u^c, d^c\}$. Scalar products of the currents $\overline{\psi}_1 \gamma_\mu \psi_2$, $\psi_1^T C \psi_2$, $\psi_1^T C \sigma_{\mu\nu} \psi_2$ never contain $\psi^3 \overline{\psi}$ or $\psi \overline{\psi}^3$.

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Search for all $Y = 0$ combinations (computer code)

- reduce to products of $Y = 0$ currents
-

$\bar{l}\bar{e}^c d^c q \qquad q u^c q d^c \qquad l e^c q u^c \qquad (B \text{ conserving})$

$qqql \qquad d^c u^c u^c e^c \qquad q q \overline{u}^c \overline{e}^c \qquad q l \overline{u}^c \overline{d}^c \qquad (B \text{ violating})$

and their Hermitian conjugates

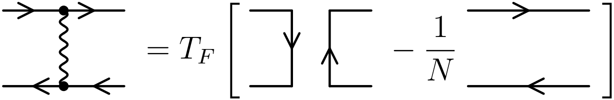
Products of $Y = 0$ currents

- Fierz

$$(\overline{L}_1\gamma_\mu L_2)(\overline{L}_3\gamma_\mu L_4) = (\overline{L}_1\gamma_\mu L_4)(\overline{L}_3\gamma_\mu L_2)$$

$\Rightarrow Y = 0$ currents

- to usual isospin singlets: $(\overline{L}L)(\overline{L}L)$, $(\overline{R}R)(\overline{R}R)$, $(\overline{L}L)(\overline{R}R)$
- isospin: singlets, triplets; color: singlets, octets



$$t^{Ii}{}_j t^{Ik}{}_l = \tfrac{1}{2}\left[\delta^i_l\delta^k_j - \tfrac{1}{2}\delta^i_j\delta^k_l\right]$$

$$t^{A\alpha}{}_\beta t^{A\gamma}{}_\delta = \tfrac{1}{2}\left[\delta^\alpha_\delta\delta^\gamma_\beta - \tfrac{1}{3}\delta^\alpha_\beta\delta^\gamma_\delta\right]$$

$(\overline{L}L)(\overline{L}L)$

$$(\bar{l}_p \gamma_\mu t^I l_q)(\bar{l}_r \gamma^\mu t^I l_s) = \frac{1}{2}(\bar{l}_p \gamma_\mu l_s)(\bar{l}_r \gamma^\mu l_q) - \frac{1}{4}(\bar{l}_p \gamma_\mu l_q)(\bar{l}_r \gamma^\mu l_s)$$

$(\overline{L}L)(\overline{L}L)$

$$(\bar{l}_p \gamma_\mu t^I l_q)(\bar{l}_r \gamma^\mu t^I l_s) = \tfrac{1}{2}(\bar{l}_p \gamma_\mu l_s)(\bar{l}_r \gamma^\mu l_q) - \tfrac{1}{4}(\bar{l}_p \gamma_\mu l_q)(\bar{l}_r \gamma^\mu l_s)$$

$$(\overline{q}_p \gamma_\mu t^A q_q)(\overline{q}_r \gamma^\mu t^A q_s) = -\tfrac{1}{6}(\overline{q}_p \gamma_\mu q_q)(\overline{q}_r \gamma^\mu q_s)$$

$$+ \tfrac{1}{2}(\overline{q}_{pi\alpha} \gamma_\mu q_q^{i\beta})(\overline{q}_{rj\beta} \gamma^\mu q_s^{j\alpha})$$

$(\overline{L}L)(\overline{L}L)$

$$(\bar{l}_p\gamma_\mu t^I l_q)(\bar{l}_r\gamma^\mu t^I l_s) = \tfrac{1}{2}(\bar{l}_p\gamma_\mu l_s)(\bar{l}_r\gamma^\mu l_q) - \tfrac{1}{4}(\bar{l}_p\gamma_\mu l_q)(\bar{l}_r\gamma^\mu l_s)$$

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$$+ \tfrac{1}{2}(\overline{q}_{pi\alpha}\gamma_\mu q_s^{j\alpha})(\overline{q}_{rj\beta}\gamma^\mu q_q^{i\beta})$$

$(\overline{L}L)(\overline{L}L)$

$$(\bar{l}_p\gamma_\mu t^I l_q)(\bar{l}_r\gamma^\mu t^I l_s) = \frac{1}{2}(\bar{l}_p\gamma_\mu l_s)(\bar{l}_r\gamma^\mu l_q) - \frac{1}{4}(\bar{l}_p\gamma_\mu l_q)(\bar{l}_r\gamma^\mu l_s)$$

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$$+ (\overline{q}_p\gamma_\mu t^I q_s)(\overline{q}_r\gamma^\mu t^I q_q) + \frac{1}{4}(\overline{q}_p\gamma_\mu q_s)(\overline{q}_r\gamma^\mu q_q)$$

$(\overline{L}L)(\overline{L}L)$

$$(\bar{l}_p\gamma_\mu t^I l_q)(\bar{l}_r\gamma^\mu t^I l_s) = \tfrac{1}{2}(\bar{l}_p\gamma_\mu l_s)(\bar{l}_r\gamma^\mu l_q) - \tfrac{1}{4}(\bar{l}_p\gamma_\mu l_q)(\bar{l}_r\gamma^\mu l_s)$$

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$$+ (\overline{q}_p\gamma_\mu t^I q_s)(\overline{q}_r\gamma^\mu t^I q_q) + \tfrac{1}{4}(\overline{q}_p\gamma_\mu q_s)(\overline{q}_r\gamma^\mu q_q)$$

$$(\overline{q}_p\gamma_\mu t^I t^A q_q)(\overline{q}_r\gamma^\mu t^I t^A q_s) = -\tfrac{1}{6}(\overline{q}_p\gamma_\mu t^I q_q)(\overline{q}_r\gamma^\mu t^I q_s)$$

$$+ \tfrac{1}{2}(\overline{q}_{pi\alpha}\gamma_\mu {t^{Ii}}_j q_q^{j\beta})(\overline{q}_{rk\beta}\gamma^\mu {t^{Ik}}_l q_s^{l\alpha})$$

$(\overline{L}L)(\overline{L}L)$

$$(\bar{l}_p\gamma_\mu t^I l_q)(\bar{l}_r\gamma^\mu t^I l_s) = \tfrac{1}{2}(\bar{l}_p\gamma_\mu l_s)(\bar{l}_r\gamma^\mu l_q) - \tfrac{1}{4}(\bar{l}_p\gamma_\mu l_q)(\bar{l}_r\gamma^\mu l_s)$$

$$(\overline{q}_p\gamma_\mu t^A q_q)(\overline{q}_r\gamma^\mu t^A q_s) = -\tfrac{1}{6}(\overline{q}_p\gamma_\mu q_q)(\overline{q}_r\gamma^\mu q_s)$$

$$+ (\overline{q}_p\gamma_\mu t^I q_s)(\overline{q}_r\gamma^\mu t^I q_q) + \tfrac{1}{4}(\overline{q}_p\gamma_\mu q_s)(\overline{q}_r\gamma^\mu q_q)$$

$$(\overline{q}_p\gamma_\mu t^I t^A q_q)(\overline{q}_r\gamma^\mu t^I t^A q_s) = -\tfrac{1}{6}(\overline{q}_p\gamma_\mu t^I q_q)(\overline{q}_r\gamma^\mu t^I q_s)$$

$$- \tfrac{1}{8}(\overline{q}_{pi\alpha}\gamma_\mu q_q^{i\beta})(\overline{q}_{rj\beta}\gamma^\mu q_s^{j\alpha}) + \tfrac{1}{4}(\overline{q}_{pi\alpha}\gamma_\mu q_q^{j\beta})(\overline{q}_{rj\beta}\gamma^\mu q_s^{i\alpha})$$

$(\overline{L}L)(\overline{L}L)$

$$(\bar{l}_p\gamma_\mu t^I l_q)(\bar{l}_r\gamma^\mu t^I l_s) = \tfrac{1}{2}(\bar{l}_p\gamma_\mu l_s)(\bar{l}_r\gamma^\mu l_q) - \tfrac{1}{4}(\bar{l}_p\gamma_\mu l_q)(\bar{l}_r\gamma^\mu l_s)$$

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$$+ (\overline{q}_p\gamma_\mu t^I q_s)(\overline{q}_r\gamma^\mu t^I q_q) + \tfrac{1}{4}(\overline{q}_p\gamma_\mu q_s)(\overline{q}_r\gamma^\mu q_q)$$

$$(\overline{q}_p\gamma_\mu t^I t^A q_q)(\overline{q}_r\gamma^\mu t^I t^A q_s) = -\tfrac{1}{6}(\overline{q}_p\gamma_\mu t^I q_q)(\overline{q}_r\gamma^\mu t^I q_s)$$

$$- \tfrac{1}{8}(\overline{q}_{pi\alpha}\gamma_\mu q_s^{j\alpha})(\overline{q}_{rj\beta}\gamma^\mu q_q^{i\beta}) + \tfrac{1}{4}(\overline{q}_p\gamma_\mu q_s)(\overline{q}_r\gamma^\mu q_q)$$

$(\overline{L}L)(\overline{L}L)$

$$(\bar{l}_p\gamma_\mu t^I l_q)(\bar{l}_r\gamma^\mu t^I l_s) = \tfrac{1}{2}(\bar{l}_p\gamma_\mu l_s)(\bar{l}_r\gamma^\mu l_q) - \tfrac{1}{4}(\bar{l}_p\gamma_\mu l_q)(\bar{l}_r\gamma^\mu l_s)$$

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$$+ (\overline{q}_p\gamma_\mu t^I q_s)(\overline{q}_r\gamma^\mu t^I q_q) + \tfrac{1}{4}(\overline{q}_p\gamma_\mu q_s)(\overline{q}_r\gamma^\mu q_q)$$

$$(\overline{q}_p\gamma_\mu t^I t^A q_q)(\overline{q}_r\gamma^\mu t^I t^A q_s) = -\tfrac{1}{6}(\overline{q}_p\gamma_\mu t^I q_q)(\overline{q}_r\gamma^\mu t^I q_s)$$

$$- \tfrac{1}{4}(\overline{q}_p\gamma_\mu t^I q_s)(\overline{q}_r\gamma^\mu t^I q_q) + \tfrac{3}{16}(\overline{q}_p\gamma_\mu q_s)(\overline{q}_r\gamma^\mu q_q)$$

$(\overline{L}L)(\overline{L}L)$

$$(\bar{l}_p\gamma_\mu t^I l_q)(\bar{l}_r\gamma^\mu t^I l_s) = \tfrac{1}{2}(\bar{l}_p\gamma_\mu l_s)(\bar{l}_r\gamma^\mu l_q) - \tfrac{1}{4}(\bar{l}_p\gamma_\mu l_q)(\bar{l}_r\gamma^\mu l_s)$$

$$(\overline{q}_p\gamma_\mu t^A q_q)(\overline{q}_r\gamma^\mu t^A q_s) = -\tfrac{1}{6}(\overline{q}_p\gamma_\mu q_q)(\overline{q}_r\gamma^\mu q_s)$$

$$+ (\overline{q}_p\gamma_\mu t^I q_s)(\overline{q}_r\gamma^\mu t^I q_q) + \tfrac{1}{4}(\overline{q}_p\gamma_\mu q_s)(\overline{q}_r\gamma^\mu q_q)$$

$$(\overline{q}_p\gamma_\mu t^I t^A q_q)(\overline{q}_r\gamma^\mu t^I t^A q_s) = -\tfrac{1}{6}(\overline{q}_p\gamma_\mu t^I q_q)(\overline{q}_r\gamma^\mu t^I q_s)$$

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$$Q_{ll} = (\bar{l}\gamma_\mu l)(\bar{l}\gamma^\mu l)$$

$$Q_{qq}^{(1)} = (\bar{q}\gamma_\mu q)(\bar{q}\gamma^\mu q) \qquad Q_{qq}^{(3)} = (\bar{q}\gamma_\mu t^I q)(\bar{q}\gamma^\mu t^I q)$$

$$Q_{lq}^{(1)} = (\bar{l}\gamma_\mu l)(\bar{q}\gamma^\mu q) \qquad Q_{lq}^{(3)} = (\bar{l}\gamma_\mu t^I l)(\bar{q}\gamma^\mu t^I q)$$

$(\overline{R}R)(\overline{R}R)$

$$(\overline{u}_p\gamma_\mu t^A u_q)(\overline{u}_r\gamma_\mu t^A u_s) = \frac{1}{2}(\overline{u}_p\gamma_\mu u_s)(\overline{u}_r\gamma_\mu u_q) - \frac{1}{6}(\overline{u}_p\gamma_\mu u_q)(\overline{u}_r\gamma_\mu u_s)$$
$$(\overline{d}_p\gamma_\mu t^A d_q)(\overline{d}_r\gamma_\mu t^A d_s) = \frac{1}{2}(\overline{d}_p\gamma_\mu d_s)(\overline{d}_r\gamma_\mu d_q) - \frac{1}{6}(\overline{d}_p\gamma_\mu d_q)(\overline{d}_r\gamma_\mu d_s)$$

$$Q_{ee} = (\overline{e}\gamma_\mu e)(\overline{e}\gamma^\mu e)$$
$$Q_{uu} = (\overline{u}\gamma_\mu u)(\overline{u}\gamma^\mu u) \qquad Q_{dd} = (\overline{d}\gamma_\mu d)(\overline{d}\gamma^\mu d)$$
$$Q_{eu} = (\overline{e}\gamma_\mu e)(\overline{u}\gamma^\mu u) \qquad Q_{ed} = (\overline{e}\gamma_\mu e)(\overline{d}\gamma^\mu d)$$
$$Q_{ud}^{(1)} = (\overline{u}\gamma_\mu u)(\overline{d}\gamma^\mu d) \qquad Q_{ud}^{(8)} = (\overline{u}\gamma_\mu t^A u)(\overline{d}\gamma^\mu t^A d)$$

$(\overline{L}L)(\overline{R}R)$

$$Q_{le} = (\bar{l}\gamma_\mu l)(\bar{e}\gamma^\mu e)$$

$$Q_{lu} = (\bar{l}\gamma_\mu l)(\bar{u}\gamma^\mu u) \qquad Q_{ld} = (\bar{l}\gamma_\mu l)(\bar{d}\gamma^\mu d)$$

$$Q_{qe} = (\bar{q}\gamma_\mu q)(\bar{e}\gamma^\mu e)$$

$$Q_{qu}^{(1)} = (\bar{q}\gamma_\mu q)(\bar{u}\gamma^\mu u) \qquad Q_{qu}^{(8)} = (\bar{q}\gamma_\mu t^A q)(\bar{u}\gamma^\mu t^A u)$$

$$Q_{qd}^{(1)} = (\bar{q}\gamma_\mu q)(\bar{d}\gamma^\mu d) \qquad Q_{qd}^{(8)} = (\bar{q}\gamma_\mu t^A q)(\bar{d}\gamma^\mu t^A d)$$

Exceptional $\overline{L}^2 L^2$

- 1 scalar $(\overline{L}\gamma_\mu L)(\overline{L}\gamma^\mu L)$
- $SU(2)$: 2 singlets, 2 doublets $\Rightarrow$ 1 structure
- $SU(3)$: $\psi^\alpha \overline{\psi}_\beta$ (B preserving) or $\psi^\alpha \psi^\beta \psi^\gamma$ (B violating) $\Rightarrow$ 1 structure

Exceptional L^4

- 2 scalars 1×1, $\sigma_{\mu\nu} \times \sigma^{\mu\nu}$

$$(\psi_1^T C \sigma_{\mu\nu} \psi_2)(\psi_3^T C \sigma^{\mu\nu} \psi_4) =$$
$$4(\psi_1^T C \psi_2)(\psi_3^T C \psi_4) + 8(\psi_1^T C \psi_4)(\psi_3^T C \psi_2)$$

  (choose the rhs everywhere except $le^c qu^c$ where we want color-singlet currents). $qu^c qd^c$, $qqql$, $d^c u^c u^c e^c$: 2 pairings — just permulations of generation indices.
- Isospin and color: 2 structures for $qu^c qd^c$, $qqql$; 1 for $d^c u^c u^c e^c$

$(\overline{L}R)(\overline{R}L)$ and $(\overline{L}R)(\overline{L}R)$

$$Q_{ledq} = (\bar{l}e)(\bar{d}q)$$

$$Q^{(1)}_{quqd} = \varepsilon^{ij}(\overline{q}_i u)(\overline{q}_j d) \qquad Q^{(8)}_{quqd} = \varepsilon^{ij}(\overline{q}_i t^A u)(\overline{q}_j t^A d)$$

$$Q^{(1)}_{lequ} = \varepsilon^{ij}(\bar{l}_i e)(\overline{q}_j u) \qquad Q^{(3)}_{lequ} = \varepsilon^{ij}(\bar{l}_i \sigma_{\mu\nu} e)(\overline{q}_j \sigma^{\mu\nu} u)$$

B violating

$$Q_{duq} = \varepsilon_{\alpha\beta\gamma}\varepsilon_{ij}\left[\left(d^{\alpha}\right)^T C u^{\beta}\right]\left[\left(q^{i\gamma}\right)^T C l^{j}\right]$$

$$Q_{qqu} = \varepsilon_{\alpha\beta\gamma}\varepsilon_{ij}\left[\left(q^{i\alpha}\right)^T C q^{j\beta}\right]\left[\left(u^{\gamma}\right)^T C e\right]$$

$$Q_{qqq}^{(1)} = \varepsilon_{\alpha\beta\gamma}\varepsilon_{ij}\varepsilon_{kl}\left[\left(q^{i\alpha}\right)^T C q^{j\beta}\right]\left[\left(q^{k\gamma}\right)^T C l^{l}\right]$$

$$Q_{qqq}^{(3)} = \varepsilon_{\alpha\beta\gamma}t^{I}_{ij}t^{I}_{kl}\left[\left(q^{i\alpha}\right)^T C q^{j\beta}\right]\left[\left(q^{k\gamma}\right)^T C l^{l}\right]$$

$$Q_{duu} = \varepsilon_{\alpha\beta\gamma}\left[\left(d^{\alpha}\right)^T C u^{\beta}\right]\left[\left(u^{\gamma}\right)^T C e\right]$$

Summary

Not counting generations and Hermitian conjugated operators

$\#\psi$	
0	15
2	19
4	25
	59

Summary

Not counting generations and Hermitian conjugated operators

$\#\psi$	
0	15
2	19
4	$25 + 5$
	64

Higgs sector

$$L_H = (D_\mu \varphi)^+ (D^\mu \varphi) + m^2 (\varphi^+ \varphi) - \frac{\lambda}{2} (\varphi^+ \varphi)^2 + C_\varphi (\varphi^+ \varphi)^3$$
$$+ C_{\varphi \Box} (\varphi^+ \varphi) \partial^2 (\varphi^+ \varphi) + C_{\varphi D} (\varphi^+ D_\mu \varphi)^* (\varphi^+ D^\mu \varphi)$$
$$\varphi = \begin{pmatrix} \Phi^+ \\ \frac{1}{\sqrt{2}} (v + H + i \Phi^0) \end{pmatrix}$$

Higgs sector

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$$+ C_{\varphi\Box} (\varphi^+ \varphi) \partial^2 (\varphi^+ \varphi) + C_{\varphi D} (\varphi^+ D_\mu \varphi)^* (\varphi^+ D^\mu \varphi)$$

$$\varphi = \begin{pmatrix} \Phi^+ \\ \frac{1}{\sqrt{2}} (v + H + i\Phi^0) \end{pmatrix}$$

$$v = m \sqrt{\frac{2}{\lambda}} \left[1 + \frac{3}{2} \frac{m^2}{\lambda^2} c_\varphi \right]$$

Higgs sector

$$L_H = (D_\mu\varphi)^+(D^\mu\varphi) + m^2(\varphi^+\varphi) - \frac{\lambda}{2}(\varphi^+\varphi)^2 + C_\varphi(\varphi^+\varphi)^3$$
$$+ C_{\varphi\Box}(\varphi^+\varphi)\partial^2(\varphi^+\varphi) + C_{\varphi D}(\varphi^+ D_\mu\varphi)^*(\varphi^+ D^\mu\varphi)$$

$$\varphi = \begin{pmatrix} \Phi^+ \\ \frac{1}{\sqrt{2}}(v + H + i\Phi^0) \end{pmatrix}$$

$$v = m\sqrt{\frac{2}{\lambda}}\left[1 + \frac{3}{2}\frac{m^2}{\lambda^2}c_\varphi\right]$$

$$L_H^{(2)} = \frac{1}{2}\left[1 + 2v^2\left(\frac{1}{4}C_{\varphi D} - C_{\varphi\Box}\right)\right](\partial_\mu H)(\partial^\mu H)$$
$$+ \left(\frac{m^2}{2} - \frac{3}{4}\lambda v^2 + \frac{15}{8}C_\varphi v^4\right)H^2$$
$$+ \frac{1}{2}\left(1 + \frac{1}{2}C_{\varphi D}v^2\right)(\partial_\mu\Phi^0)(\partial^\mu\Phi^0) + (\partial_\mu\Phi^-)(\partial^\mu\Phi^+)$$

Higgs sector

$$h = Z_h H \qquad G^0 = Z_{G^0} \Phi^0 \qquad G^\pm = \Phi^\pm$$

$$Z_h = 1 + \left(\frac{1}{4} C_{\varphi D} - C_{\varphi \Box} \right) v^2$$

$$Z_{G^0} = 1 + \frac{1}{4} C_{\varphi D} v^2 \Phi^0$$

Higgs sector

$$h = Z_h H \qquad G^0 = Z_{G^0}\Phi^0 \qquad G^{\pm} = \Phi^{\pm}$$

$$Z_h = 1 + \left(\frac{1}{4}C_{\varphi D} - C_{\varphi\Box}\right) v^2$$

$$Z_{G^0} = 1 + \frac{1}{4}C_{\varphi D} v^2 \Phi^0$$

$$L_h^{(2)} = \frac{1}{2}(\partial_\mu h)(\partial^\mu h) - \frac{M_h^2}{2} h^2$$

$$M_h^2 = 2m^2 \left[1 - \frac{m^2}{\lambda^2}\big(3C_\varphi + \lambda(C_{\varphi D} - 4C_{\varphi\Box})\big)\right]$$

Gauge sector: QCD

$$L_{\mathrm{QCD}} = -\frac{1}{4} G^A_{\mu\nu} G^{A\mu\nu} + C_{\varphi G} (\varphi^+ \varphi) G^A_{\mu\nu} G^{A\mu\nu}$$

Gauge sector: QCD

$$L_{\mathrm{QCD}} = -\frac{1}{4} G^A_{\mu\nu} G^{A\mu\nu} + C_{\varphi G} (\varphi^+ \varphi) G^A_{\mu\nu} G^{A\mu\nu}$$

$$\bar{G}^A_\mu = Z_G G^A_\mu \qquad \bar{g} = Z_G^{-1} g$$

$$Z_G = 1 - C_{\varphi G} v^2$$

$$L^{(2)}_{\mathrm{QCD}} = -\frac{1}{4} \bar{G}^A_{\mu\nu} \bar{G}^{A\mu\nu}$$

Gauge sector: electroweak

$$L_{\mathrm{EW}} = -\frac{1}{4}W^I_{\mu\nu}W^{I\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + (D_\mu\varphi)^+(D^\mu\varphi)$$
$$+ C_{\varphi W}(\varphi^+\varphi)W^I_{\mu\nu}W^{I\mu\nu} + C_{\varphi B}(\varphi^+\varphi)B_{\mu\nu}B^{\mu\nu}$$
$$+ C_{\varphi WB}(\varphi^+ t^I\varphi)W^I_{\mu\nu}B^{\mu\nu} + C_{\varphi D}(\varphi^+ D_\mu\varphi)^*(\varphi^+ D^\mu\varphi)$$

Gauge sector: electroweak

$$
\begin{aligned}
L_{\mathrm{EW}} = & -\frac{1}{4}W^I_{\mu\nu}W^{I\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + (D_\mu\varphi)^+(D^\mu\varphi) \\
& + C_{\varphi W}(\varphi^+\varphi)W^I_{\mu\nu}W^{I\mu\nu} + C_{\varphi B}(\varphi^+\varphi)B_{\mu\nu}B^{\mu\nu} \\
& + C_{\varphi WB}(\varphi^+ t^I\varphi)W^I_{\mu\nu}B^{\mu\nu} + C_{\varphi D}(\varphi^+ D_\mu\varphi)^*(\varphi^+ D^\mu\varphi)
\end{aligned}
$$

$$\bar{W}^I_\mu = Z_W W^I_\mu \qquad \bar{g}_2 = Z_W^{-1} g_2$$

$$\bar{B}_\mu = Z_B B_\mu \qquad \bar{g}_1 = Z_B^{-1} g_1$$

$$Z_W = 1 - C_{\varphi W} v^2 \qquad Z_B = 1 - C_{\varphi B} v^2$$

Gauge sector: electroweak

$$L_{\rm EW} = -\frac{1}{4}W^I_{\mu\nu}W^{I\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + (D_\mu\varphi)^+(D^\mu\varphi)$$
$$+ C_{\varphi W}(\varphi^+\varphi)W^I_{\mu\nu}W^{I\mu\nu} + C_{\varphi B}(\varphi^+\varphi)B_{\mu\nu}B^{\mu\nu}$$
$$+ C_{\varphi WB}(\varphi^+ t^I\varphi)W^I_{\mu\nu}B^{\mu\nu} + C_{\varphi D}(\varphi^+ D_\mu\varphi)^*(\varphi^+ D^\mu\varphi)$$

$$\bar{W}^I_\mu = Z_W W^I_\mu \qquad \bar{g}_2 = Z_W^{-1} g_2$$
$$\bar{B}_\mu = Z_B B_\mu \qquad \bar{g}_1 = Z_B^{-1} g_1$$
$$Z_W = 1 - C_{\varphi W} v^2 \qquad Z_B = 1 - C_{\varphi B} v^2$$

$$L^{(2)}_{\rm EW} = -\frac{1}{4}\sum_{I=1,2}\bar{W}^I_{\mu\nu}\bar{W}^{I\mu\nu} - \frac{1}{4}\left(\bar{W}^3_{\mu\nu}\bar{B}_{\mu\nu}\right)\begin{pmatrix}1 & \epsilon\\ \epsilon & 1\end{pmatrix}\begin{pmatrix}\bar{W}^{3\mu\nu}\\ \bar{B}^{\mu\nu}\end{pmatrix}$$
$$+ \frac{\bar{g}_2^2 v^2}{8}\sum_{I=1,2}\bar{W}^I_\mu\bar{W}^{I\mu} + \frac{v^2}{8}Z^2_{G^0}\left(\bar{W}^3_\mu\bar{B}_\mu\right)\begin{pmatrix}\bar{g}_2^2 & -\bar{g}_1\bar{g}_2\\ -\bar{g}_1\bar{g}_2 & \bar{g}_1^2\end{pmatrix}\begin{pmatrix}\bar{W}^{3\mu}\\ \bar{B}^{\mu}\end{pmatrix}$$

$$\epsilon = 2C_{\varphi WB}v^2$$

Gauge sector: electroweak

$$W_\mu^\pm = \frac{1}{\sqrt{2}}\left(\bar{W}_\mu^1 \mp i\bar{W}_\mu^2\right) \qquad M_W^2 = \frac{1}{2}\bar{g}_2 v$$

Gauge sector: electroweak

$$W_\mu^\pm = \frac{1}{\sqrt{2}}\left(\bar{W}_\mu^1 \mp i\bar{W}_\mu^2\right) \qquad M_W^2 = \frac{1}{2}\bar{g}_2 v$$

$$\begin{pmatrix} \bar{W}_\mu^3 \\ \bar{B}_\mu \end{pmatrix} = X \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}$$

$$X = \begin{pmatrix} 1 & -\frac{\epsilon}{2} \\ -\frac{\epsilon}{2} & 1 \end{pmatrix} \begin{pmatrix} \cos\bar{\theta} & \sin\bar{\theta} \\ -\sin\bar{\theta} & \cos\bar{\theta} \end{pmatrix}$$

$$\tan\bar{\theta} = \frac{\bar{g}_1}{\bar{g}_2} + \frac{\epsilon}{2}\left(1 - \frac{\bar{g}_1^2}{\bar{g}_2^2}\right)$$

$$M_Z = \frac{1}{2}\sqrt{\bar{g}_1^2 + \bar{g}_2^2}\, v \left(1 + \frac{\bar{g}_1\bar{g}_2}{\bar{g}_1^2 + \bar{g}_2^2}\epsilon\right) Z_{G^0}$$

$$M_A = 0$$

Gauge–glodstone mixing

$$
\begin{aligned}
&(D_\mu\varphi)^+(D^\mu\varphi) + C_{\varphi D}(\varphi^+ D_\mu\varphi)(\varphi^+ D^\mu\varphi) \Rightarrow \\
&- i\frac{\bar{g}_2 v}{2\sqrt{2}}\bar{W}^1_\mu\partial^\mu(\Phi^+ - \Phi^-) + \frac{\bar{g}_2 v}{2\sqrt{2}}\bar{W}^2_\mu\partial^\mu(\Phi^+ + \Phi^-) \\
&- \frac{\bar{g}_2 v}{2} Z^2_{G^0}\bar{W}^3_\mu\partial^\mu\Phi^0 + \frac{\bar{g}_1 v}{2} Z^2_{G^0}\bar{B}_\mu\partial^\mu\Phi^0
\end{aligned}
$$

Gauge–glodstone mixing

$$
\begin{aligned}
&(D_\mu\varphi)^+(D^\mu\varphi) + C_{\varphi D}(\varphi^+ D_\mu\varphi)(\varphi^+ D^\mu\varphi) \Rightarrow \\
&- i\frac{\bar{g}_2 v}{2\sqrt{2}}\bar{W}^1_\mu\partial^\mu(\Phi^+ - \Phi^-) + \frac{\bar{g}_2 v}{2\sqrt{2}}\bar{W}^2_\mu\partial^\mu(\Phi^+ + \Phi^-) \\
&- \frac{\bar{g}_2 v}{2} Z^2_{G^0}\bar{W}^3_\mu\partial^\mu\Phi^0 + \frac{\bar{g}_1 v}{2} Z^2_{G^0}\bar{B}_\mu\partial^\mu\Phi^0 \\
&= iM_W\left(W^+_\mu\partial^\mu G^- - W^-_\mu\partial^\mu G^+\right) - M_Z Z_\mu\partial^\mu G^0
\end{aligned}
$$

Fermion sector

$$
\begin{aligned}
L_f = {} & i(\bar{l}'\not{D}l' + \overline{e}'\not{D}e' + \overline{q}'\not{D}q' + \overline{u}'\not{D}u' + \overline{d}'\not{D}d') \\
& - (\bar{l}'\Gamma_e e'\varphi + \overline{q}'\Gamma_u u'\tilde{\varphi} + \overline{q}'\Gamma_d d'\varphi + \text{h.c.}) \\
& + (C_{\nu\nu}(\tilde{\varphi}^+ l')^T C(\tilde{\varphi}^+ l') + \text{h.c.}) \\
& + (\varphi^+\varphi)(\bar{l}'C_{e\varphi}e'\varphi + \overline{q}'C_{u\varphi}u'\tilde{\varphi} + \overline{q}'C_{d\varphi}d'\varphi + \text{h.c.})
\end{aligned}
$$

Fermion sector

$$L_f = i(\bar{l}' \not{D} l' + \bar{e}' \not{D} e' + \bar{q}' \not{D} q' + \bar{u}' \not{D} u' + \bar{d}' \not{D} d')$$
$$- (\bar{l}' \Gamma_e e' \varphi + \bar{q}' \Gamma_u u' \tilde{\varphi} + \bar{q}' \Gamma_d d' \varphi + \text{h.c.})$$
$$+ (C_{\nu\nu} (\tilde{\varphi}^+ l')^T C (\tilde{\varphi}^+ l') + \text{h.c.})$$
$$+ (\varphi^+ \varphi)(\bar{l}' C_{e\varphi} e' \varphi + \bar{q}' C_{u\varphi} u' \tilde{\varphi} + \bar{q}' C_{d\varphi} d' \varphi + \text{h.c.})$$
$$L_m = -\frac{1}{2} \nu_L'^T C M_\nu' - \bar{e}_L' M_e' e_R' - \bar{u}_L' M_u' u_R' - \bar{d}_L' M_d' d_R' + \text{h.c.}$$
$$M_\nu' = -C_{\nu\nu} v^2 \qquad M_e' = \frac{v}{\sqrt{2}} \left(\Gamma_e - C_{e\varphi} \frac{v^2}{2} \right)$$
$$M_u' = \frac{v}{\sqrt{2}} \left(\Gamma_u - C_{u\varphi} \frac{v^2}{2} \right) \qquad M_d' = \frac{v}{\sqrt{2}} \left(\Gamma_d - C_{d\varphi} \frac{v^2}{2} \right)$$

Fermion sector

$$\psi'_X = U_{\psi_X}\psi_X \qquad \psi = \nu,\ e,\ u,\ d \qquad X = L,\ R$$

$$U^T_{\nu_L} M'_\nu U_{\nu_L} = M_\nu = \mathrm{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3})$$

$$U^+_{e_L} M'_e U_{e_R} = M_e = \mathrm{diag}(m_e, m_\mu, m_\tau)$$

$$U^+_{u_L} M'_u U_{u_R} = M_u = \mathrm{diag}(m_u, m_c, m_t)$$

$$U^+_{d_L} M'_d U_{d_R} = M_d = \mathrm{diag}(m_d, m_s, m_b)$$