Neutrino Theory

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Lecture 2

Outline

- ways to generate ν masses
  - add RH neutrinos
  - Weinberg operator
  - seesaw mechanism
  - other ways
- leptogenesis
- neutrinoless double β decay
- towards theory of (lepton) flavor

Add RH neutrinos

(see e.g. Maggiore, QFT)

- we introduce RH neutrinos ν_R

$$\nu_R \sim (\mathbf{1}, \mathbf{1}, 0)$$

so that we can write down Yukawa interactions of the form

$$y^{\nu}_{\alpha\beta}\, \overline{L_{\alpha\, L}}\, \tilde{h}\, \nu_{\beta\, R} + \text{h.c.}$$

with $\quad L_L \sim (\mathbf{1}, \mathbf{2}, -1/2)$ and $\tilde{h} \sim (\mathbf{1}, \mathbf{2}, -1/2)$
and
where $\alpha = e, \mu, \tau$ and $\beta = 1, ..., N$ with N being number of ν_R

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- if we assign lepton number L +1 to ν_R, lepton number symmetry $U(1)_L$ remains preserved

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- if we assign lepton number L +1 to ν_R, lepton number symmetry $U(1)_L$ remains preserved
- neutrinos are then

Dirac particles

like quarks and charged leptons,
meaning we use a Dirac spinor Ψ of the form *(Maggiore, QFT)*

$$\Psi = \begin{pmatrix} \Psi_L \\ \Psi_R \end{pmatrix}$$

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where $\alpha = e, \mu, \tau$ and $\beta = 1, ..., N$ with N being number of ν_R

- we have to decide how many ν_R we want to introduce

Add RH neutrinos

- we introduce RH neutrinos ν_R

$$\nu_R \sim (\mathbf{1}, \mathbf{1}, 0)$$

- we can decide how many ν_R we want to introduce
  - two ν_R – minimal choice

  because in this case we can generate 2 non-vanishing neutrino masses

Add RH neutrinos

- we introduce RH neutrinos ν_R

$$\nu_R \sim (\mathbf{1}, \mathbf{1}, 0)$$

- we can decide how many ν_R we want to introduce
  - one per fermion generation – most conventional choice

because then ν_R are treated like all other fermion fields. This is also motivated by ideas of unification, where fermion fields e_R are put together with $\nu_{e\,R}$ in one multiplet

$$L_{e\,R} = \begin{pmatrix} \nu_{e\,R} \\ e_R \end{pmatrix} \sim \mathbf{2} \text{ under } SU(2)_R$$

like $L_{e\,L} \sim \mathbf{2}$ under $SU(2)_L$. ***(Mohapatra, arXiv:hep-ph/0211252v1)***

Add RH neutrinos

- we introduce RH neutrinos ν_R

$$\nu_R \sim (\mathbf{1}, \mathbf{1}, 0)$$

- we can decide how many ν_R we want to introduce
  - more than 3 ν_R

  because in some more fundamental theories, such as string theory, many ν_R are predicted

(see e.g. Buchmüller et al., arXiv:hep-ph/0703078v2)

Add RH neutrinos

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where $\alpha = e, \mu, \tau$ and $\beta = 1, ..., N$ with N being number of ν_R

- upon electroweak symmetry breaking, i.e. h acquires vacuum expectation value $\langle h \rangle$, we have

$$M^{\nu}_{\alpha\beta} = y^{\nu}_{\alpha\beta}\, v$$

Add RH neutrinos

- we introduce RH neutrinos ν_R

$$\nu_R \sim (\mathbf{1}, \mathbf{1}, 0)$$

- we can estimate the size of the Yukawa couplings $y^\nu_{\alpha\beta}$

$$|M^\nu_{\alpha\beta}| = |y^\nu_{\alpha\beta}|\, v \lesssim 0.1\,\mathrm{eV}\,,$$

meaning that for $v = 174\,\mathrm{GeV}$ we find

$$|y^\nu_{\alpha\beta}| \lesssim 6\ \times\ 10^{-13}$$

this is really tiny, as expected!

- such small couplings are fine from view point of quantum field theory, but not very appealing ... why so small?

Weinberg operator

- we can consider operators with mass dimension higher than 4 which are invariant under the SM gauge group
- we thus take into account non-renormalizable operators
- such operators are suppressed by a mass scale M

See Andrej Grozin's lectures on effective field theories.

Weinberg operator

(Weinberg, PRL43:1566, 1979)

- we can consider operators with mass dimension higher than 4 which are invariant under the SM gauge group
- we thus take into account non-renormalizable operators
- such operators are suppressed by a mass scale M
- the only type of operator with mass dimension 5 is the

  Weinberg operator

  which is composed of the fields L_L and h, since

$$c_{\alpha\beta} \frac{1}{M} \overline{L^c_{\alpha\,L}} \, h \, L_{\beta\,L} \, h + \mathrm{h.c.}$$

Weinberg operator

$$c_{\alpha\beta} \, \frac{1}{M} \, \overline{L^c_{\alpha\, L}} \, h \, L_{\beta\, L} \, h + \mathrm{h.c.}$$

- its mass dimension is

$$5 = 2 \times \frac{3}{2} \, (L \, \mathrm{twice}) + 2 \times 1 \, (h \, \mathrm{twice})$$

- it is invariant under the SM gauge group, since

$$L_L \sim (\mathbf{1}, \mathbf{2}, -1/2) \quad \text{and} \quad h \sim (\mathbf{1}, \mathbf{2}, 1/2)$$

- it is invariant under Lorentz symmetry

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- it is invariant under Lorentz symmetry
- but, it violates lepton number L by 2 units!

Weinberg operator

$$c_{\alpha\beta}\,\frac{1}{M}\,\overline{L^c_{\alpha\,L}}\,h\,L_{\beta\,L}\,h + \text{h.c.}$$

- upon electroweak symmetry breaking, i.e. h acquires vacuum expectation value $\langle h \rangle$, we have

$$c_{\alpha\beta}\,\frac{1}{M}\,\overline{\nu^c_{\alpha\,L}}\,\nu_{\beta\,L}\,v^2 + \text{h.c.}$$

- which corresponds to neutrino masses

$$M^{\nu}_{\alpha\beta} = c_{\alpha\beta}\,\frac{v^2}{M}$$

- since this mass term breaks lepton number L, it is called

Majorana mass term

- this type of mass term "connects" a fermion field with itself

Weinberg operator

$$c_{\alpha\beta} \, \frac{1}{M} \, \overline{L^c_{\alpha\,L}} \, h \, L_{\beta\,L} \, h + \text{h.c.}$$

- upon electroweak symmetry breaking, i.e. h acquires vacuum expectation value $\langle h \rangle$, we have

$$c_{\alpha\beta} \, \frac{1}{M} \, \overline{\nu^c_{\alpha\,L}} \, \nu_{\beta\,L} \, v^2 + \text{h.c.}$$

- which corresponds to neutrino masses

$$M^\nu_{\alpha\beta} = c_{\alpha\beta} \, \frac{v^2}{M}$$

- neutrinos with such a mass term are called

Majorana particles

- such neutrinos are their own antiparticles!

(Maggiore, QFT)

Weinberg operator

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Majorana particles

[Quarks and charged leptons cannot be Majorana particles]

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- which corresponds to neutrino masses

$$M^\nu_{\alpha\beta} = c_{\alpha\beta}\,\frac{v^2}{M}$$

- for a Majorana neutrino we can use a Majorana spinor Ψ_M of the form

(Maggiore, QFT; Willenbrock, arXiv:hep-ph/0410370v2)

$$\Psi_M = \begin{pmatrix} \Psi_L \\ i\,\sigma^2\,\Psi^\star_L \end{pmatrix} \quad \text{with} \quad \Psi^c_M = \Psi_M$$

Weinberg operator

$$M^{\nu}_{\alpha\beta} = c_{\alpha\beta}\,\frac{v^2}{M}$$

- this mass term is symmetric in generation/flavor space

$$M^{\nu}_{\alpha\beta} = M^{\nu}_{\beta\alpha}$$

  and for 3 generations M^{ν} has 6 complex parameters

- for neutrino masses around 0.1 eV we need

$$M \sim 3\,\times\,10^{14}\,\mathrm{GeV}$$

  for $c_{\alpha\beta} \sim 1$ and $v = 174\,\mathrm{GeV}$

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- for neutrino masses around 0.1 eV we need

$$M \sim 3\,\times\,10^{14}\,\mathrm{GeV}$$

  for $c_{\alpha\beta} \sim 1$ and $v = 174\,\mathrm{GeV}$

- however, M could also be much smaller, if $c_{\alpha\beta}$ are not of order 1, or if M is only an effective mass scale

Seesaw mechanism

There are several different variants of seesaw mechanism which are distinguished by the new particles added to the theory

- type I seesaw mechanism: we add (at least 2) fermions

$$\nu_R \sim (\mathbf{1}, \mathbf{1}, 0)$$

- type II seesaw mechanism: we add (at least 1) scalar

$$\Delta \sim (\mathbf{1}, \mathbf{3}, 1)$$

- type III seesaw mechanism: we add (at least 2) fermions

$$\Sigma \sim (\mathbf{1}, \mathbf{3}, 0)$$

- further variants – see below

Seesaw mechanism

Let's start with type I seesaw mechanism

(Minkowski, PLB67:421, 1977; Schwartz, QFT and the Standard Model)

- we add RH neutrinos ν_R

$$\nu_R \sim (\mathbf{1}, \mathbf{1}, 0)$$

- we know already that we can write down Yukawa interactions

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$$y^{\nu}_{\alpha\beta}\,\overline{L_{\alpha\,L}}\,\tilde{h}\,\nu_{\beta\,R} + \text{h.c.}$$

- since ν_R are singlets under the SM gauge group, we can also write down an explicit mass term for them

$$\frac{1}{2}\,M^R_{\alpha\beta}\,\overline{\nu^c_{\alpha\,R}}\,\nu_{\beta\,R} + \text{h.c.}$$

which is a Majorana mass term

with $M^R_{\alpha\beta} = M^R_{\beta\alpha}$

Seesaw mechanism

(Grossman, arXiv:hep-ph/0305245v1)

To better understand what happens consider 1 generation only

- the relevant terms in the Lagrangian are

$$-\mathcal{L}_\nu = M_D^\nu \, \overline{\nu_L} \, \nu_R + \frac{1}{2} \, M^R \, \overline{\nu_R^c} \, \nu_R + \text{h.c.}$$

with $\qquad M_D^\nu = y^\nu \, v$

Seesaw mechanism

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with $\qquad M_D^\nu = y^\nu \, v$

- conjugated RH fields transform as left-handed (LH) fields and thus

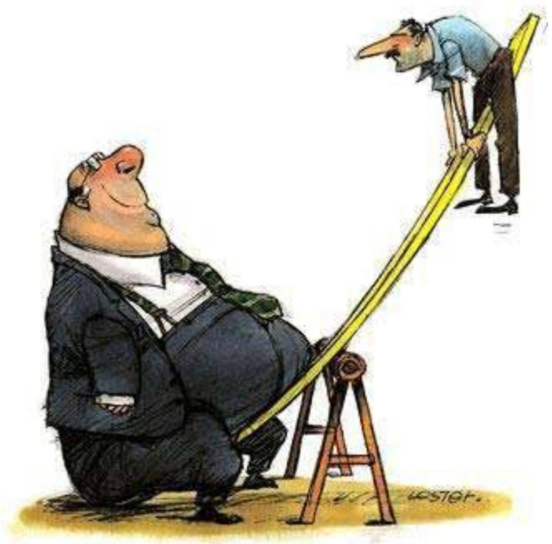
$$\left(\begin{array}{cc} \overline{\nu_L} & \overline{\nu_R^c} \end{array} \right) \mathcal{M}_\nu \left(\begin{array}{c} \nu_L^c \\ \nu_R \end{array} \right)$$

with

$$\mathcal{M}_\nu = \left(\begin{array}{cc} 0 & M_D^\nu \\ M_D^\nu & M^R \end{array} \right)$$

Seesaw mechanism

- we can diagonalize the mass matrix $\mathcal{M}_\nu$ in order to find
  - the mass eigenvalues: $m_{l(ight)}$ and $m_{h(eavy)}$
  - the mass eigenstates: $\nu_{l(ight)}$ and $\nu_{h(eavy)}$
- we assume $|M_D^\nu| \ll |M^R|$



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- we assume $|M_D^\nu| \ll |M^R|$
  and find
  $$m_l \approx \frac{(M_D^\nu)^2}{M^R} \quad \text{and} \quad m_h \approx M^R\,,$$
  meaning
  the larger $M^R \approx m_h$ (heavier ν_h), the smaller m_l (lighter ν_l)
- let's estimate $M_D^\nu = y^\nu\, v$ and M^R

  $$M_D^\nu \approx v = 174\,\mathrm{GeV} \;\; \text{and} \;\; M^R \approx 3\times 10^{14}\,\mathrm{GeV} \;\; \text{for} \;\; m_l \approx 0.1\,\mathrm{eV}$$

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  meaning
  the larger $M^R \approx m_h$ (heavier ν_h), the smaller m_l (lighter ν_l)
  as well as
  $$\nu_l \approx \nu_L - \left(\frac{M_D^\nu}{M_R}\right)\nu_R^c \;\; \text{and} \;\; \nu_h \approx \left(\frac{M_D^\nu}{M_R}\right)\nu_L + \nu_R^c$$

Seesaw mechanism

(Willenbrock, arXiv:hep-ph/0410370v2)

- since ν_R is very heavy, we can integrate ν_R out
- start with the equation of motion of ν_R

$$\frac{\partial \mathcal{L}}{\partial \nu_R} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \nu_R)} = 0$$

where $\mathcal{L}$ is composed of $\mathcal{L}_\nu$ and the kinetic term for ν_R

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where $\mathcal{L}$ is composed of $\mathcal{L}_\nu$ and the kinetic term for ν_R

- for very heavy ν_R we can neglect the kinetic term

$$\frac{\partial \mathcal{L}}{\partial(\partial_\mu \nu_R)} = 0$$

and only have to solve $\frac{\partial \mathcal{L}}{\partial \nu_R} = 0$ for the heavy field and plug the solution back into the Lagrangian

Seesaw mechanism

- we can generalize

$$m_l \approx \frac{(M_D^\nu)^2}{M^R}$$

to the case of 3 generations of ν_L and N ν_R

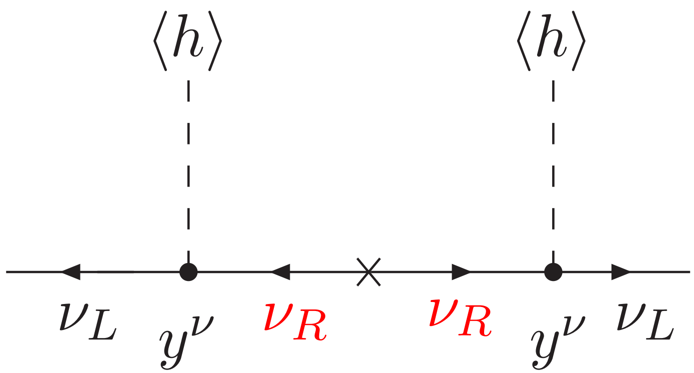
$$M^\nu \approx M_D^\nu \, (M^R)^{-1} \, (M_D^\nu)^T$$

where M_D^ν is a complex $3 \times N$-matrix
and M^R is a complex, symmetric $N \times N$-matrix

Seesaw mechanism

The type I seesaw mechanism is given as Feynman diagram

(Romanino, arXiv:1201.6158v1 [hep-ph])



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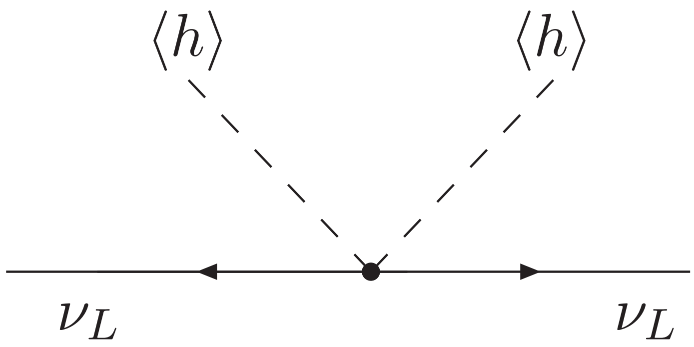
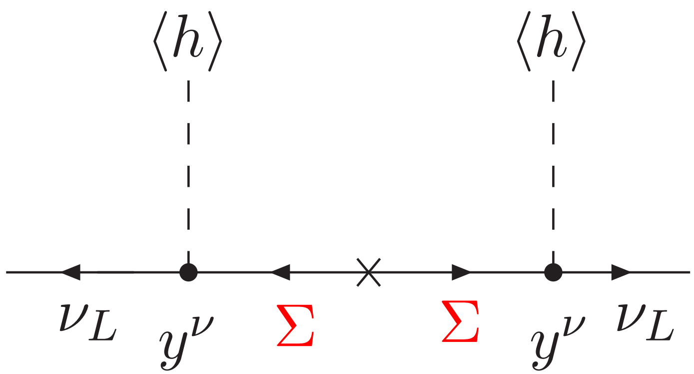


Seesaw mechanism

The type III seesaw mechanism is given by a very similar Feynman diagram **(Foot/Lew/He/Joshi, Z.Phys. C44:441, 1989)**



since $\nu_R \sim (\mathbf{1}, \mathbf{1}, 0)$ are replaced by $\Sigma \sim (\mathbf{1}, \mathbf{3}, 0)$.
Remember $\mathbf{2} \times \mathbf{2} = \mathbf{1} + \mathbf{3}$ in $SU(2)$.

Seesaw mechanism

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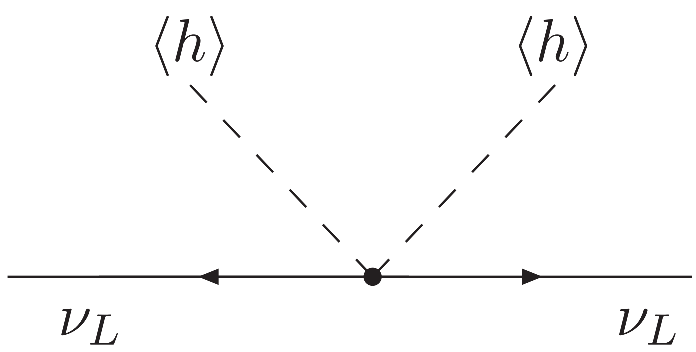

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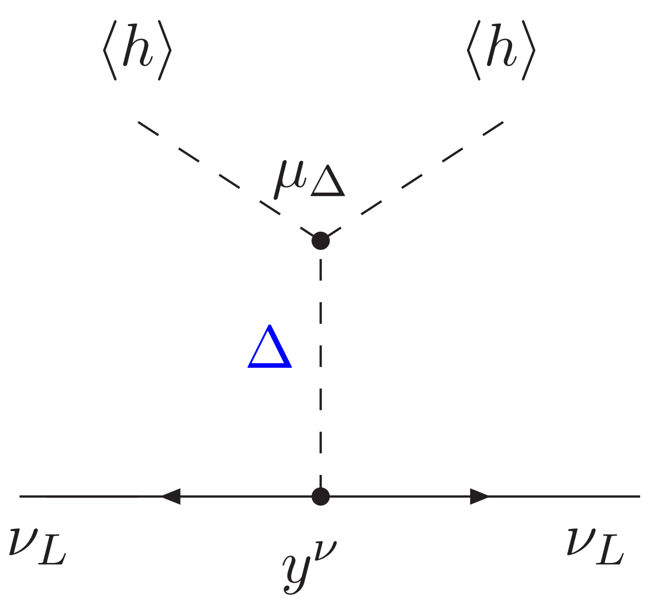
Remember $\mathbf{2} \times \mathbf{2} = \mathbf{1} + \mathbf{3}$ in $SU(2)$.

Seesaw mechanism

In order to represent the type II seesaw mechanism we have to consider a Feynman diagram of a different topology

(Magg/Wetterich, PLB94:61, 1980)



since we can consider a common vertex of the two neutrinos ν_L. But, then the new particle Δ has to be a scalar, not a fermion.

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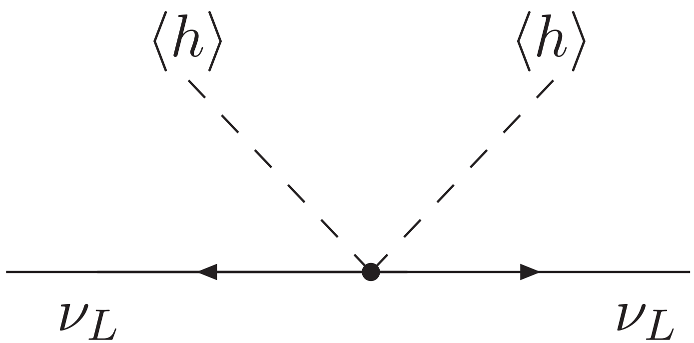

since we can consider a common vertex of the two neutrinos ν_L.
But, then the new particle Δ has to be a scalar, not a fermion.

Seesaw mechanism

Other variants of seesaw mechanism

- double seesaw mechanism
- inverse seesaw mechanism

Seesaw mechanism

Double seesaw mechanism

(Mohapatra/Valle, PRD34:1642, 1986; Mohapatra, arXiv:hep-ph/0211252v1)

- assume that you have two types of new particles

$$\nu_R \sim (\mathbf{1}, \mathbf{1}, 0) \text{ and } S \sim (\mathbf{1}, \mathbf{1}, 0)$$

- distinguishing between them seems artificial, since they transform in the same way under the SM gauge group. However, in extensions of the SM, such as $SO(10)$ grand unified theories, they can be distinguished.

Seesaw mechanism

Double seesaw mechanism

- assume that you have two types of new particles

$$\nu_R \sim (\mathbf{1},\mathbf{1},0) \ \text{ and } \ S \sim (\mathbf{1},\mathbf{1},0)$$

- the mass matrix of ν_L, ν_R and S reads

$$\begin{pmatrix} \overline{\nu_L} & \overline{\nu_R^c} & \overline{S} \end{pmatrix} \begin{pmatrix} 0 & M_D^\nu & 0 \\ M_D^\nu & 0 & M_{\nu_R S} \\ 0 & M_{\nu_R S} & M^S \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \\ S^c \end{pmatrix}$$

- for the light neutrinos we find as mass

$$M^\nu \approx M_D^\nu \, (M_{\nu_R S} \, (M^S)^{-1} \, M_{\nu_R S}^T)^{-1} \, (M_D^\nu)^T$$

Seesaw mechanism

Double seesaw mechanism

- for the light neutrinos we find as mass

$$M^\nu \approx M_D^\nu \, (M_{\nu_R S} \, (M^S)^{-1} \, M_{\nu_R S}^T)^{-1} \, (M_D^\nu)^T$$

- let's estimate the size of M_D^ν, $M_{\nu_R S}$ and M^S
  - $M_D^\nu \sim v$, since it involves $\langle h \rangle$
  - $M_{\nu_R S}$ is of order of the grand unification scale 10^{16} GeV

Seesaw mechanism

Double seesaw mechanism

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  - $M_D^\nu \sim v$, since it involves $\langle h \rangle$
  - $M_{\nu_R S}$ is of order of the grand unification scale 10^{16} GeV
  - for $M^\nu \sim 0.1\,\mathrm{eV}$ we need

$M^S \sim 10^{18}$ GeV close to the Planck scale

Seesaw mechanism

Inverse seesaw mechanism

(Wyler/Wolfenstein, NPB218:205, 1983; Abada/Lucente, arXiv:1401.1507v2 [hep-ph])

- the crucial difference between the double seesaw mechanism and the inverse seesaw mechanism is

  scale of M^S

- M^S is considered to be very small

Inverse seesaw mechanism

- the crucial difference between the double seesaw mechanism and the inverse seesaw mechanism is

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- M^S is considered to be very small
- let's estimate the size of M_D^ν, $M_{\nu_R S}$ and M^S again
  - $M_D^\nu \sim v$, since it involves $\langle h \rangle$
  - take $M^S \sim$ keV as example

Seesaw mechanism

Inverse seesaw mechanism

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- M^S is considered to be very small
- let's estimate the size of M_D^ν, $M_{\nu_R S}$ and M^S again
  - $M_D^\nu \sim v$, since it involves $\langle h \rangle$
  - take $M^S \sim$ keV as example
  - for $M^\nu \sim 0.1$ eV we need

$$M_{\nu_R S} \sim 20\,\text{TeV}$$

Seesaw mechanism

Inverse seesaw mechanism

- the crucial difference between the double seesaw mechanism and the inverse seesaw mechanism is

  scale of M^S

- M^S is considered to be very small
- for $M^S \sim$ keV and $M_{\nu_R S} \sim 20$ TeV we have

  2 nearly degenerate mass eigenstates

$$\begin{pmatrix} 0 & M_{\nu_R S} \\ M_{\nu_R S} & M^S \end{pmatrix}$$

Seesaw mechanism

Inverse seesaw mechanism

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- for $M^S \sim$ keV and $M_{\nu_R S} \sim 20$ TeV we have

  2 nearly degenerate mass eigenstates

- if $M^S \to 0$, we can define lepton number
  - ν_L has lepton number +1
  - ν_R has lepton number +1
  - S has lepton number +1

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Inverse seesaw mechanism

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  scale of M^S

- M^S is considered to be very small
- for $M^S \sim$ keV and $M_{\nu_R S} \sim 20$ TeV we have

  2 nearly degenerate mass eigenstates

- if $M^S \to 0$, we can define lepton number
- the 2 nearly degenerate states form a

  pseudo-Dirac neutrino pair

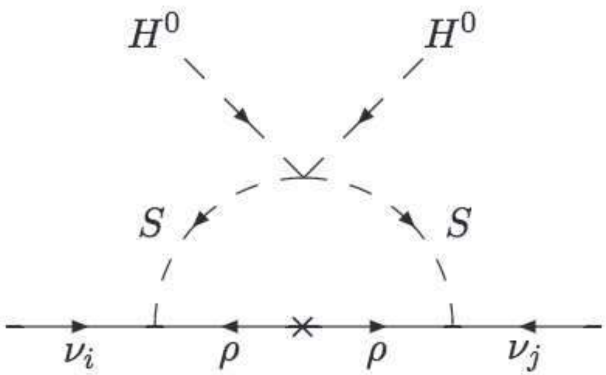
Other ways to generate ν masses

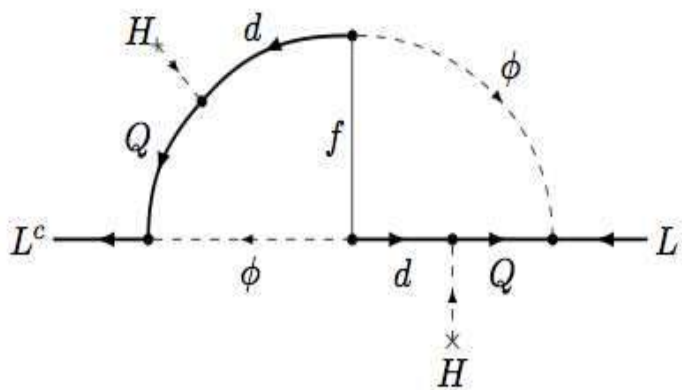
Another class of models: radiative neutrino mass models

(Boucenna/Morisi/Valle, arXiv:1404.3751v2 [hep-ph]; Cai et al., arXiv:1706.08524v3 [hep-ph])

Idea:

Neutrino masses are small due to quantum effects (loops)!



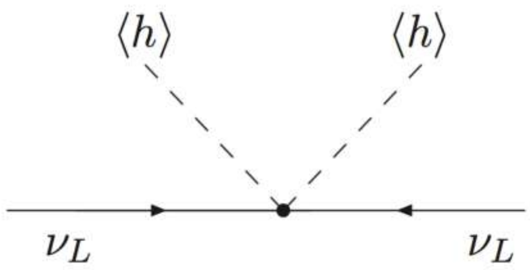


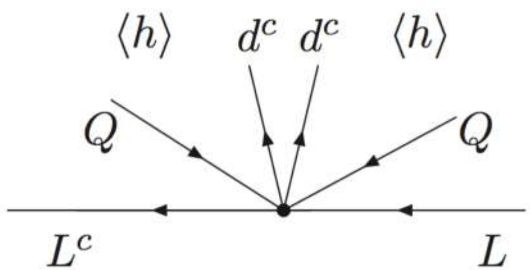
Other ways to generate ν masses

Another class of models: radiative neutrino mass models

Idea:

Neutrino masses are small due to quantum effects (loops)!





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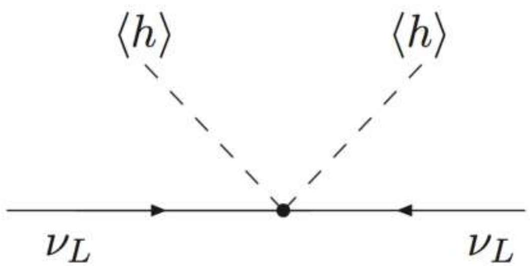

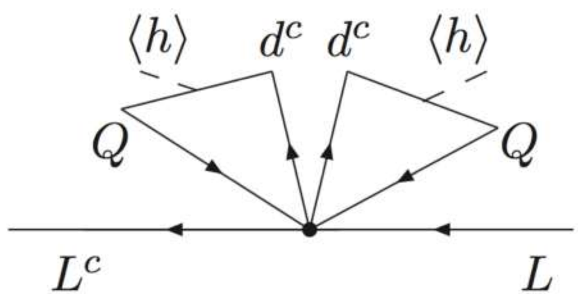

Comment on new physics scale

As we have seen, for the type I seesaw mechanism with $y^{\nu}_{\alpha\beta} \sim 1$ the mass scale of RH neutrinos is very large.
As a consequence,

- we can connect low energy with high energy physics
- this fits well with the ideas of a theory of grand unification, e.g. $SO(10)$ ***(Mohapatra, arXiv:hep-ph/0211252v1)***
- it allows for the generation of the baryon asymmetry of the Universe via leptogenesis, see below
- but it is not directly testable

Comment on new physics scale

In contrast to this, other mechanisms, like radiative neutrino mass models, *(Cai et al., arXiv:1706.08524v3 [hep-ph])*

- permit the new particles to have much smaller masses (around TeV scale)
- can be tested directly at colliders
- can be constrained through the search for rare processes
- often also have a Dark Matter candidate
- but they are usually difficult to reconcile with theories of (grand) unification
- but new mechanisms for the generation of the baryon asymmetry of the Universe have to be considered

Leptogenesis

Leptogenesis is a large category of mechanisms, where you generate the baryon asymmetry of the Universe via first generating a lepton asymmetry.

(Fukugita/Yanagida, PLB174:45, 1986; Davidson/Nardi/Nir, arXiv:0802.2962v3 [hep-ph])

This lepton asymmetry is then (partially) converted to a baryon asymmetry. ***(Khlebnikov/Shaposhnikov, NPB308:885, 1988)***

Leptogenesis

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This lepton asymmetry is then (partially) converted to a baryon asymmetry.

Just to know what we are talking about

$$Y_B = \frac{n_B - n_{\bar{B}}}{s} = (8.65 \pm 0.09) \cdot 10^{-11}$$

(Planck ('15))

Leptogenesis

In order to produce a non-vanishing baryon asymmetry, we need to fulfil the three Sakharov conditions

(Sakharov, Pisma Zh. Eksp. Teor. Fiz. 5:32, 1967)

- violate baryon number
- violate C and CP
- be out of thermal equilibrium

Leptogenesis

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You can ask: Can’t we have this in the SM?

Leptogenesis

In order to produce a non-vanishing baryon asymmetry, we need to fulfil the three Sakharov conditions

- violate baryon number
- violate C and CP
- be out of thermal equilibrium

You can ask: Can't we have this in the SM?

Short answer: Yes, but not enough!

(Rubakov/Shaposhnikov, arXiv:hep-ph/9603208v2; Trodden, arXiv:hep-ph/9803479v2)

Leptogenesis

There are many variants of leptogenesis

- unflavored leptogenesis ***(Fukugita/Yanagida, PLB174:45, 1986)***
- flavored leptogenesis ***(Davidson/Nardi/Nir, arXiv:0802.2962v3 [hep-ph])***
- resonant leptogenesis ***(Pilaftsis/Underwood, arXiv:hep-ph/0309342v3)***
- Akhmedov-Rubakov-Smirnov mechanism

  (Akhmedov/Rubakov/Smirnov, arXiv:hep-ph/9803255v2)

... here we focus on

unflavored leptogenesis

Leptogenesis

Ingredients for unflavored leptogenesis

- you need heavy RH neutrinos

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$$\nu_{i\,R} \;\rightarrow\; L_{\alpha\,L} + h \text{ and } \nu_{i\,R} \;\rightarrow\; \overline{L_{\alpha\,L}} + h^c$$

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- you need C and CP violation
  we have this thanks to complex Yukawa couplings $y^{\nu}_{\alpha\beta}$
- you need to be out of thermal equilibrium
  that's also OK

Leptogenesis

What do you need to compute in practice?

(Buchmüller/Di Bari/Plümacher, arXiv:hep-ph/0401240v1; Davidson/Nardi/Nir, arXiv:0802.2962v3 [hep-ph])

- the decays of $\nu_{i\,R}$ to $L_{\alpha\,L} + h$ and $\overline{L_{\alpha\,L}} + h^c$
- or better to say the difference in the decay rates

$$\Gamma(\nu_{i\,R} \rightarrow L_{\alpha\,L} + h) - \Gamma(\nu_{i\,R} \rightarrow \overline{L_{\alpha\,L}} + h^c)$$

- then sum over α
  that's the meaning of unflavored
- lastly, normalize to the sum of the decay rates
- we get the CP asymmetry ϵ_i

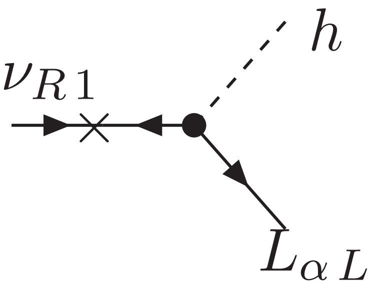
$$\epsilon_i = -\frac{\sum_\alpha [\Gamma(\nu_{i\,R} \rightarrow L_{\alpha\,L} + h) - \Gamma(\nu_{i\,R} \rightarrow \overline{L_{\alpha\,L}} + h^c)]}{\sum_\alpha [\Gamma(\nu_{i\,R} \rightarrow L_{\alpha\,L} + h) + \Gamma(\nu_{i\,R} \rightarrow \overline{L_{\alpha\,L}} + h^c)]}$$

- we need to consider up to 1-loop diagrams

Leptogenesis

Let's first look at the diagrams

- tree-level



Leptogenesis

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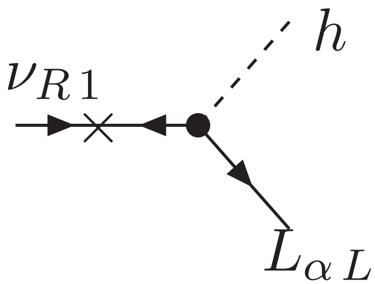
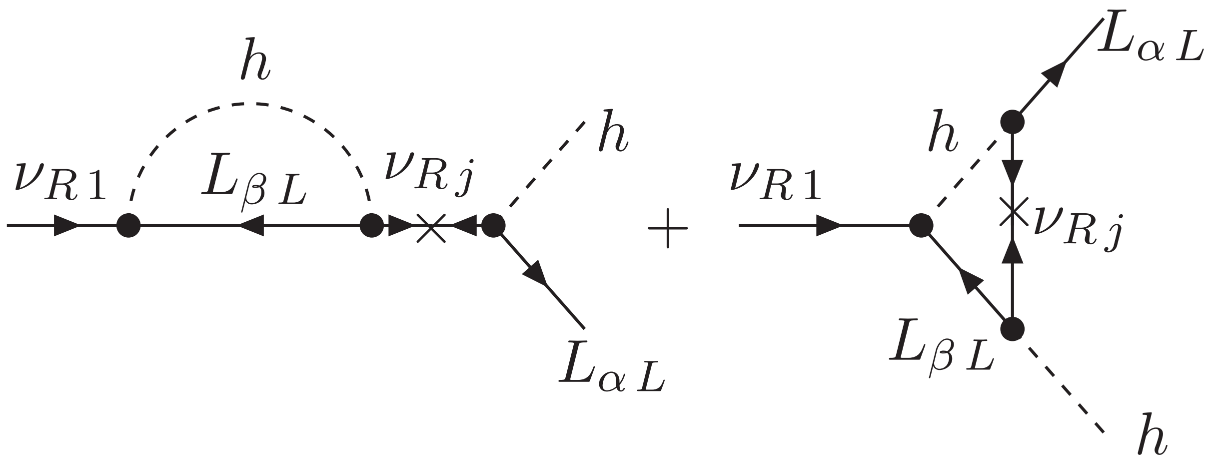


- 1-loop diagrams



- we also have to take into account processes that reduce the produced lepton asymmetry – efficiency factors η_{ij}

$$10^{-3} \lesssim \eta_{ij} \lesssim 1$$

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- and eventually we need to take into account the conversion factor for turning the lepton asymmetry to a baryon asymmetry which is around

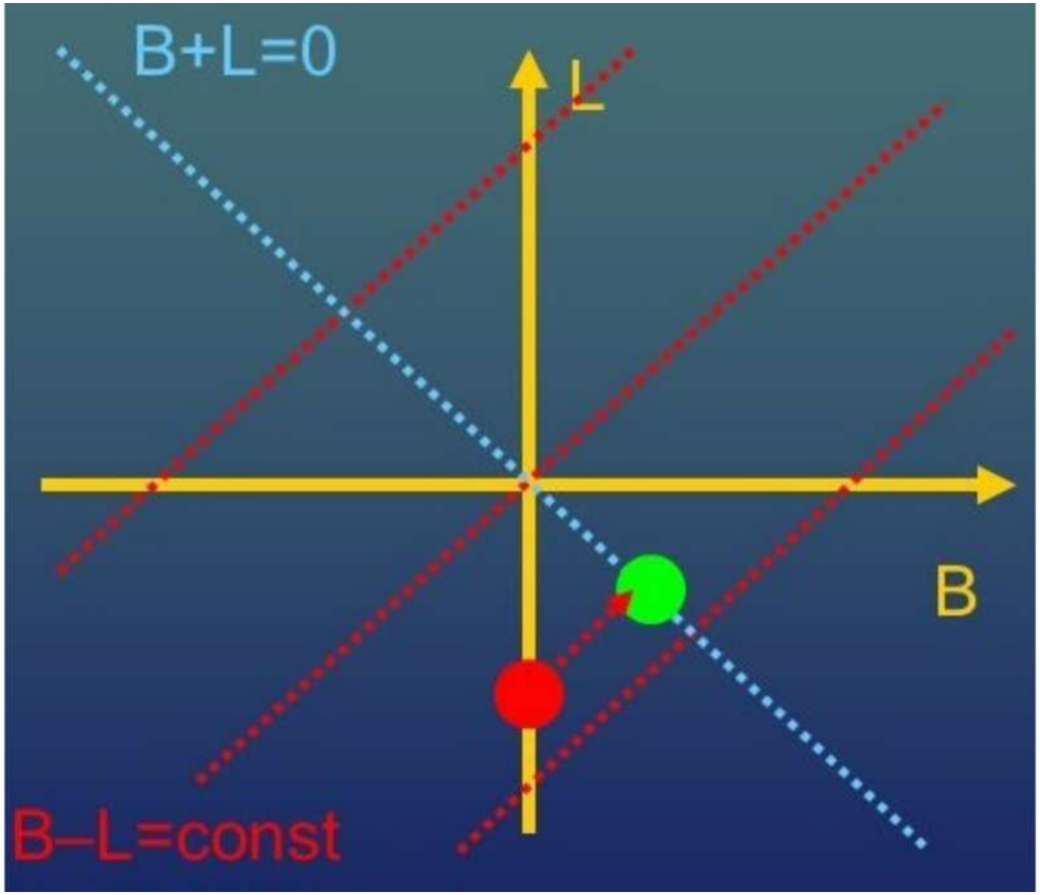
$$10^{-3} \text{ in the SM}$$

- this means ϵ_i has to be of order 10^{-7} to 10^{-4}

Leptogenesis

Crucial for the conversion are sphaleron processes

(Khlebnikov/Shaposhnikov, NPB308:885, 1988)



Neutrinoless double β decay

Depending on the mechanism of generating ν masses there can be different possibilities to test the Majorana nature of neutrinos.

The

search for neutrinoless double β decay

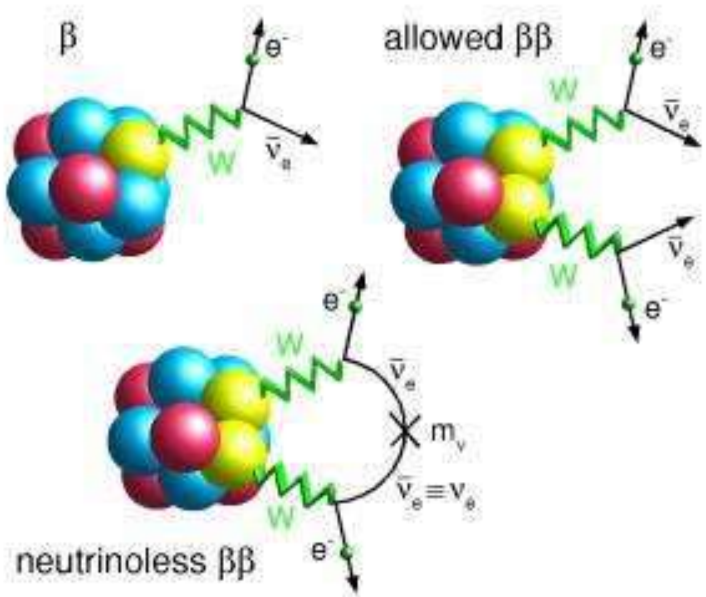
is the one most directly related to ν masses & lepton mixing parameters. ***(Dell'Oro et al., arXiv:1601.07512v2 [hep-ph])***

For details see Marcos Dracos' lectures on neutrino experiments.

Neutrinoless double β decay

Neutrinoless double β decay

$$(A, Z) \rightarrow (A, Z+2) + 2\,e^-$$



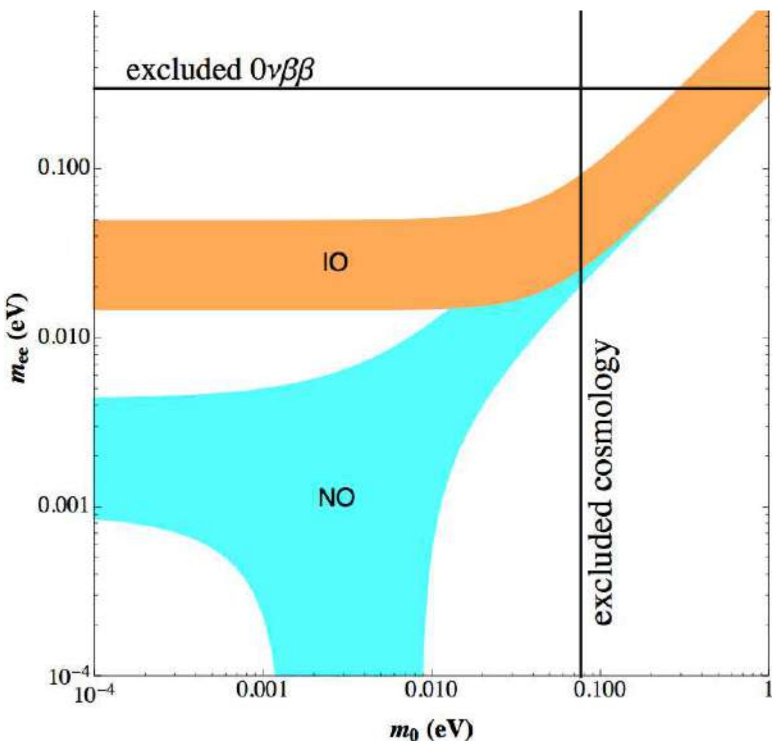
This process violates lepton number L by 2 units!

Neutrinoless double β decay

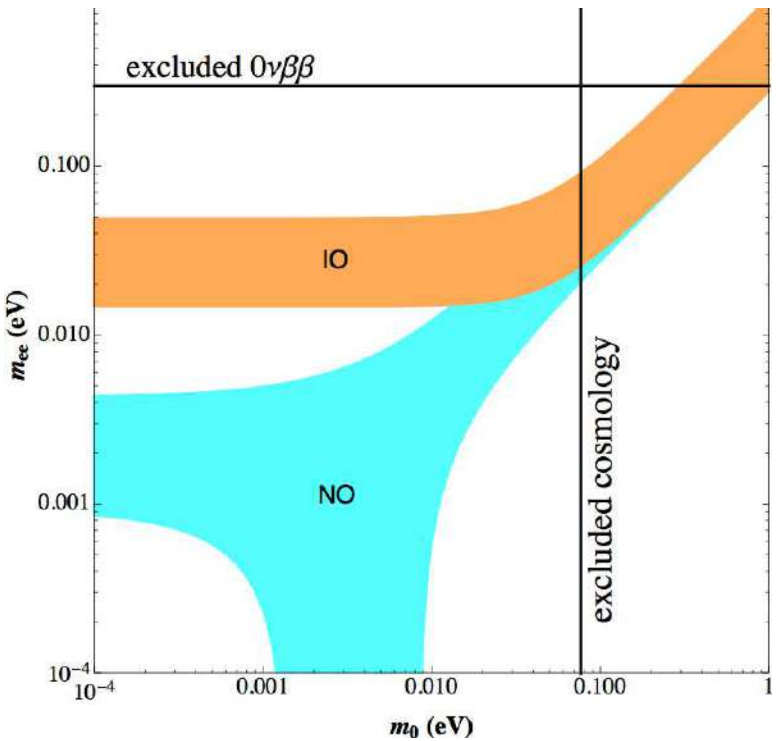
The relevant quantity for neutrinoless double β decay is

(PDG, Chapter 14 ('18); Dell'Oro et al., arXiv:1601.07512v2 [hep-ph])

$$
\begin{aligned}
m_{ee} &= \left| U_{e\,1}^2\, m_1 + U_{e\,2}^2\, m_2 + U_{e\,3}^2\, m_3 \right| \\
&= \left| c_{12}^2\, c_{13}^2\, m_1 + s_{12}^2\, c_{13}^2\, e^{i\,\alpha_{21}}\, m_2 + s_{13}^2\, e^{i\,(\alpha_{31} - 2\,\delta)}\, m_3 \right|
\end{aligned}
$$

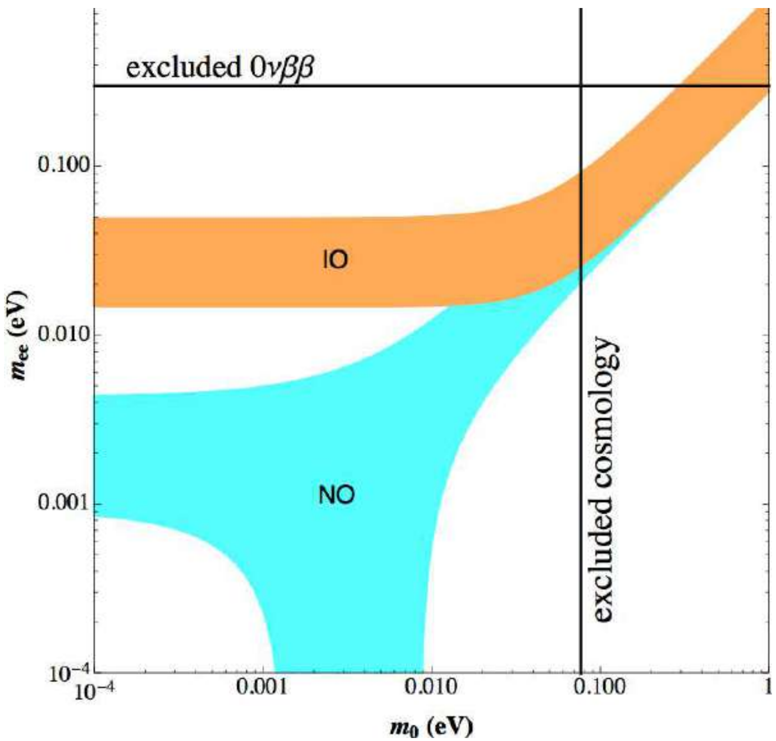


Neutrinoless double β decay



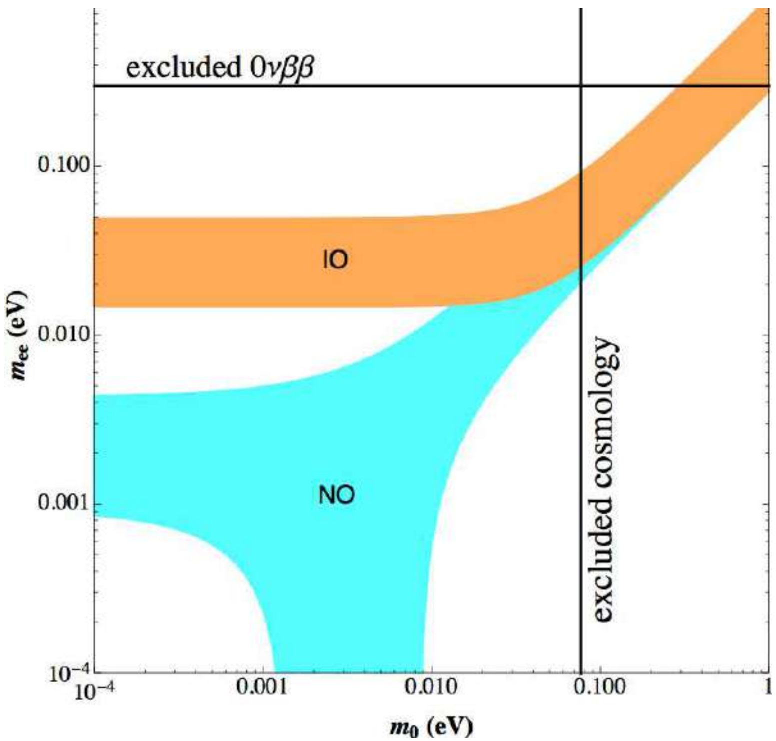
Allowed area for IO neutrino masses can be tested in the future.

Neutrinoless double β decay



If neutrino masses are NO, m_{ee} may be very small, although neutrinos are Majorana particles.

Neutrinoless double β decay

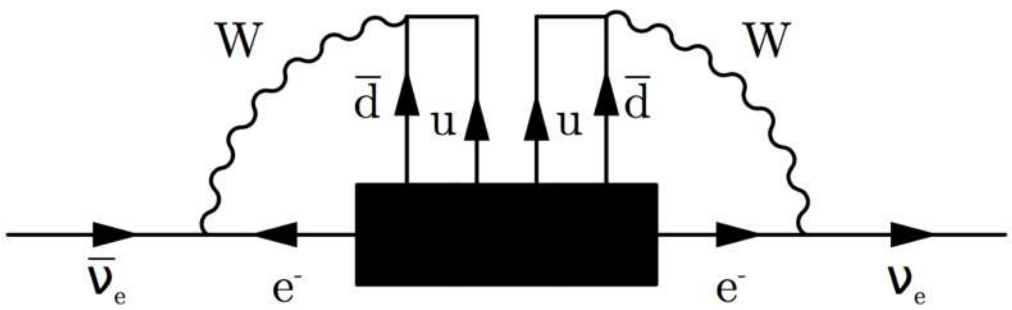


At points on boundaries of areas CP is conserved.

Neutrinoless double β decay

If neutrinoless double β decay is observed,
neutrinos have (at least) a small Majorana mass of order 10^{-24} eV.
This observation is stated in the “black box theorem".

(Schechter/Valle, PRD25:2951, 1982; Duerr/Lindner/Merle, arXiv:1105.0901v3 [hep-ph])



Towards theory of (lepton) flavor

Most of the free parameters in the SM and in theories beyond appear in the Yukawa interactions and thus in the fermion mass matrices.

Understand the values of the fermion mixing angles.

Predict leptonic CP violation.

Aim:

Reduce the number of free parameters!

Make theory predictive!

Towards theory of (lepton) flavor

Aim:

Reduce the number of free parameters!

Make theory predictive!

- simple approaches are
  - set some of the elements of the mass matrix to zero (texture zeros)

(see e.g. Frampton/Glashow/Marfatia, arXiv:hep-ph/0201008v2)

$$M^\nu = \begin{pmatrix} 0 & \star & 0 \\ \star & \star & \star \\ 0 & \star & \star \end{pmatrix}$$

Towards theory of (lepton) flavor

Aim:

Reduce the number of free parameters!

Make theory predictive!

- simple approaches are
  - set some of the elements of the mass matrix to zero (texture zeros)
  - equate some of the matrix elements

    (see e.g. Grimus/Lavoura, arXiv:hep-ph/0305046v2)
  - assume that the mass matrix is hermitean or symmetric

    (see e.g. Ramond/Roberts/Ross, arXiv:hep-ph/9303320v1)

Towards theory of (lepton) flavor

Aim:

Reduce the number of free parameters!

Make theory predictive!

- simple approaches are textures
- more sophisticated approach is to use a symmetry

Towards theory of (lepton) flavor

Aim:

Reduce the number of free parameters!

Make theory predictive!

- simple approaches are textures
- more sophisticated approach is to use a symmetry
  - symmetry principle is very successful for gauge interactions (gauge symmetry)
  - apply symmetry also to flavor sector: flavor symmetry G_f
  - ... but there are many choices of G_f
  - ... and not much/enough data (masses and mixing)

Towards theory of (lepton) flavor

The symmetry G_f could be ...

- ... abelian or non-abelian

abelian	:	parity P
non-abelian	:	$SU(2)_L$

Towards theory of (lepton) flavor

The symmetry G_f could be ...

- ... abelian or non-abelian
- ... continuous or discrete

continuous	:	$SU(2)_L$
discrete	:	parity P, charge conjugation C

Towards theory of (lepton) flavor

The symmetry G_f could be ...

- ... abelian or non-abelian
- ... continuous or discrete
- ... local or global

local	:	the gauge groups, like $SU(3)_c$
global	:	lepton number $U(1)_L$

Towards theory of (lepton) flavor

The symmetry G_f could be ...

- ... abelian or non-abelian
- ... continuous or discrete
- ... local or global
- ... spontaneously broken or explicitly

spontaneously broken	:	$SU(2)_L \times U(1)_Y$ to $U(1)_{em}$
explicitly broken	:	parity P in the SM

Towards theory of (lepton) flavor

The symmetry G_f could be ...

- ... abelian or non-abelian
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- ... spontaneously broken or explicitly
- ... broken arbitrarily or to non-trivial subgroups

arbitrarily broken	:	$U(1)_X$ to nothing
broken to non-trivial subgroups	:	$SU(2)_L \times U(1)_Y$ to $U(1)_{em}$

Towards theory of (lepton) flavor

The symmetry G_f could be ...

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- ... broken at low or high energies

low scales	:	electroweak scale $\mathcal{O}(100)\,\mathrm{GeV}$
high scales	:	seesaw scale $\sim 10^{14}\,\mathrm{GeV}$,
		GUT scale $\sim 10^{16}\,\mathrm{GeV}$

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Its maximal possible size depends on the gauge group,
e.g. in the SM without ν_R: $G_f \subset U(3)^5$.
in $SO(10)$: $G_f \subset U(3)$.

Towards theory of (lepton) flavor

Possible continuous symmetries G_f

- $U(1)$ symmetry (Froggatt-Nielsen symmetry)

  (Froggatt/Nielsen, NPB147:277, 1979; Chankowski et al., arXiv:hep-ph/0501071v1)

  Example:

  3 generations of q_L have charges under $U(1)$: $(-2,-1,0)$

  3 generations of u_R have also $U(1)$ charges: $(2,1,0)$

  Higgs field h has no $U(1)$ charge

Towards theory of (lepton) flavor

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  3 generations of u_R have also $U(1)$ charges: $(2, 1, 0)$
  Higgs field h has no $U(1)$ charge
  - only the term $\overline{q_{3\,L}}\,\tilde{h}\,u_{3\,R}$ is allowed
  - other entries of the matrix M^u are only generated, if $U(1)$ is broken

Towards theory of (lepton) flavor

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  Example:
  3 generations of q_L have charges under $U(1)$: $(-2,-1,0)$
  3 generations of u_R have also $U(1)$ charges: $(2,1,0)$
  Higgs field h has no $U(1)$ charge
  - only the term $\overline{q_{3\,L}}\,\tilde{h}\,u_{3\,R}$ is allowed
  - we need a scalar field θ with $U(1)$ charge -1 to have, e.g.

$$\frac{1}{M^2}\,\overline{q_{2\,L}}\,\tilde{h}\,u_{2\,R}\,\theta^2 \;.$$

    If $\langle\theta\rangle \ll M$,
    then M^u_{22} is much smaller than M^u_{33}.

Towards theory of (lepton) flavor

Possible continuous symmetries G_f

- $U(1)$ symmetry (Froggatt-Nielsen symmetry)
  - mass hierarchies and order of magnitudes can be easily explained
  - breaking such a symmetry is simple
  - $U(1)$ can be easily implemented in different models
  - precise values, e.g. $\theta_{23} = 45^\circ$, difficult to achieve

Towards theory of (lepton) flavor

Possible continuous symmetries G_f

- $U(1)$ symmetry (Froggatt-Nielsen symmetry)
- non-abelian symmetries: $SU(2)$, $SO(3)$ and $SU(3)$

  (see e.g. King/Ross, arXiv:hep-ph/0108112v3)

  - existence of 3 generations is understood
  - precise values, e.g. $\theta_{23} = 45^\circ$, can be explained
  - breaking such a symmetry can be difficult to implement (many fields, study of scalar potential, ...)
  - not straightforward to construct such models

Towards theory of (lepton) flavor

G_f could also be a discrete symmetry.

(Ishimori et al., arXiv:1003.3552v2 [hep-th]; King/Luhn, arXiv:1301.1340v3 [hep-ph])

This choice has several advantages

- such groups have (only) small representations
- they have preferred directions
- they can be broken easily, in particular along such directions
- they are very suitable for understanding lepton mixing
- it is easy to achieve particular values for mixing parameters

However, they may not be suitable for explaining mass hierarchies and the small quark mixing.

Towards theory of (lepton) flavor

There are many possible choices for discrete G_f

(Grimus/Ludl, arXiv:1110.6376v4 [hep-ph])

- permutation symmetries: S_N and A_N with $N \in \mathbb{N}$
- dihedral symmetries: D_n and D'_n with $n \in \mathbb{N}$
- further double-valued groups: T', O', I'
- subgroups of $SU(3)$: series of $\Delta(3\,n^2)$ and $\Delta(6\,n^2)$ groups with $n \in \mathbb{N}$, as well as finite number of Σ groups
- subgroups of $U(3)$ such as $\Sigma(81)$ and subgroups of the listed groups such as $T_7 \cong Z_7 \rtimes Z_3 \subset \Delta(147)$

NB: there can be isomorphisms among the groups,

e.g. $S_3 \cong D_3 \cong \Delta(6)$.

Summary – Lecture 2

- there are indeed many possibilities to generate neutrino masses
- generation of the baryon asymmetry of the Universe can also be related to neutrino mass generation
- signal of neutrinoless double β decay would indicate lepton number L violation by 2 units
- there are many ideas for theories beyond the SM to understand fermion masses and mixing

Thank you for your attention.
Questions? Comments?