QCD

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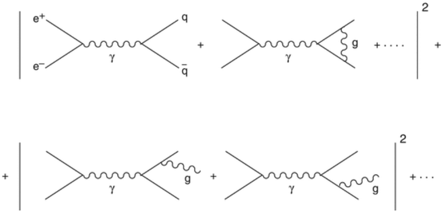
Outline

Infrared singularities.

- Higher order perturbative QCD contributions still contain divergences from infrared configurations arising in:
  - real emission of a soft or collinear partons
  - soft or collinear configurations of momenta in a virtual loops
- Infrared divergences cancel order-by-order in perturbation theory when adding real and virtual corrections:
  - Kinoshita-Lee-Nauenberg (KLN) theorem for inclusive observables, like R-ration in e^+e^- collisions, jet production.
  - for processes with hadron in initial/final states like DIS ($ep \to e' + X$), single identified hadron production in e^+e^- collisions ($e^+e^- \to h + X$) remaining collinear singularitie are universally absorbed (renormalized) in Parton Densities and Fragmentation Functions. (QCD Factorization theorem).

The KLM theorem

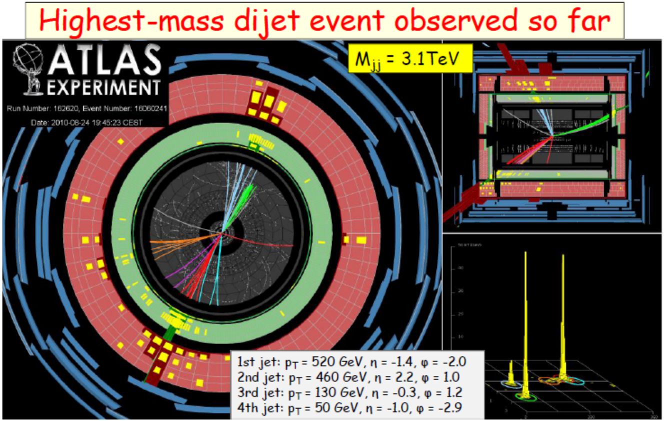
Example of $R-$ ratio in e^+e^-



- Infrared safety: definition of observable is unchanged by soft emission or collinear splitting. Thus, hard final states from virtual and real corrections are experimentally indistinguishable
- Theorem: Infrared divergences cancel in the sum of different subprocesses. Such observable is dominated by the physics of small distances and can be calculated in pQCD: $O = \sum c_n \alpha_s^n(\mu_R) + \mathcal{O}(\Lambda^2/Q^2)$

Hadron Jets production

- From high p_T di-jets LHC data we know that quarks remain elementary down to the scales $\sim \mathrm{TeV}^{-1}$



- From high p_T di-jets LHC data we know that quarks are elementary down to the scales $\sim \mathrm{TeV}^{-1}$
- Jet: clusters of particles moving in a common direction
  - Theory: jet consists of partons
  - Experiment: jet is formed by the hadrons
- Jet algorithm (the same in theory and experiment)
  - procedure to combine particles into jets, it gives some number of jets in each event
  - should be infrared safe! Do not spoil cancelation of soft/collinear singularities in the sum of virtual corrections and real gluon emissions.
    Soft: $(E_{n+1} \to 0)$: $O_{n+1}(p_1, \ldots, p_n, p_{n+1}) \to O_n(p_1, \ldots, p_n)$
    Collinear: $(p_{n+1} \parallel p_n)$:
    $O_{n+1}(p_1, \ldots, p_n, p_{n+1}) \to O_n(p_1, \ldots, p_n + p_{n+1})$

Jet algorithm

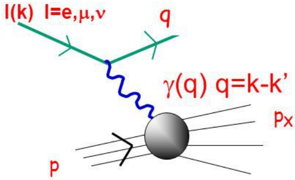
- Defines the measure of distance, y_{ij}, between the pair of particles
- Introduce the cut value, y_0, of the measure.
- In each event combine the pair (ij) with smallest y_{ij} into a pseudoparticle if $y_{ij} < y_0$ with $p_{ij} = p_i + p_j$
- Repeat until all remaining pairs have $y_{ij} > y_0$
- Remaining pseudoparticles are the jets!

Example of the measure (Cone Algorithm)

$$y_{ij} = (\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2$$

where ϕ azimuthal angle, $\eta = 1/2 \ln(E + p_{\parallel})/(E - p_{\parallel})$- rapidity

Kinematics



$$s = E_{c.m.}^2 = (p+k)^2$$
$$Q^2 = -q^2 = -(k-k')^2$$
$$\nu = \frac{(p \cdot q)}{M_N}$$

Bjorken dimensionless variables

$$x = \frac{Q^2}{2p \cdot q}$$
$$y = \frac{p \cdot q}{p \cdot k}$$

$$for\ p = \left(M_N, \vec{0}\right)$$
$$\nu = E - E' = E_h - M_N$$
$$Q^2 = 4EE' \sin\theta^2/2$$

$$W^2 = (p+q)^2 = M_N^2 - Q^2 + 2M_N\nu \geq 0 \rightarrow$$

$$0 \leq x \leq 1$$

$$y = \frac{p \cdot q}{p \cdot k} = \frac{1}{1 + M_N x/(2E\sin^2\theta/2)}$$
$$0 \leq y \leq \frac{1}{1 + M_N x/(2E)}$$

DIS cross section.

- The lepton-proton cross section is the contraction of a leptonic tensor $L_{\mu\nu}$ (lepton-photon coupling) and a hadronic tensor $W_{\mu\nu}$ (photon-hadron scattering)
- Integrating over the phase space of X_n and summing over all possible channels- n in the final state (inclusive cross section) we have

$$\frac{d\sigma^{lp\to l'X}}{dx dQ^2} = \sum_X dPS_X \frac{1}{4}|M|^2 L^{\mu\nu} W_{\mu\nu}$$

Hadronic tensor and DIS cross section

- Symmetries and conservation laws constrain hadronic tensor. In the case of photon exchange depends on two structure functions

$$W_{\mu\nu} = \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) F_1(x, Q^2) + \frac{\hat{p}_\mu \hat{p}_\nu}{(pq)} F_2(x, Q^2)$$

where $\hat{p}_\mu = p_\mu - \frac{(pq)}{q^2} q_\mu$

- It gives the DIS cross section for lepton-proton scattering

$$\frac{d\sigma^{lp \to l' X}}{dx dQ^2} = \frac{2\pi\alpha^2}{xQ^4} \left[\left(1 + (1 - y^2)^2 \right) F_2(x, Q^2) - y^2 F_L(x, Q^2) \right]$$

where $F_L(x, Q^2) = F_2(x, Q^2) - 2x F_1(x, Q^2)$

Parton Model

- Proton made of point-like free quarks interacting incoherently with the photon probing the proton.
- Momentum distribution of quarks in the proton described by probabilistic distribution function (parton distribution)- $f_q(\beta)$
- Cross section for lepton-proton scattering is a convolution of lepton-quark scattering cross section with quark PDF.
  Incoherent sum of lepton-quark scatterings:

$$\frac{d\sigma_{lp\to l'X}}{dxdQ^2} = \sum_q \int_0^1 d\beta f_q(\beta) \frac{d\sigma_{lq\to l'X}}{dxdQ^2}$$

- Quark distribution function $f_q(\beta)$: probability density of finding a quark with momentum βp inside proton with momentum p.
- At LO: lepton-quark scattering is elastic: $q + \gamma^* \to q'$, and therefore

$$\beta = x$$

quark momentum fraction β is equal to Bjorken variable x!

- Lepton-quark scattering cross section

$$\frac{d\sigma^{lq \to l' X}}{dx dQ^2} = \sum_q \frac{2\pi\alpha^2}{Q^4} \int_0^1 d\beta f_q(\beta) \left(1 + (1 - y^2)^2\right) \delta(x - \beta)$$

- therefore

$$F_2(x, Q^2) = \sum_q e_q^2 \, x \, f_q(x) \qquad F_L(x, Q^2) = 0$$

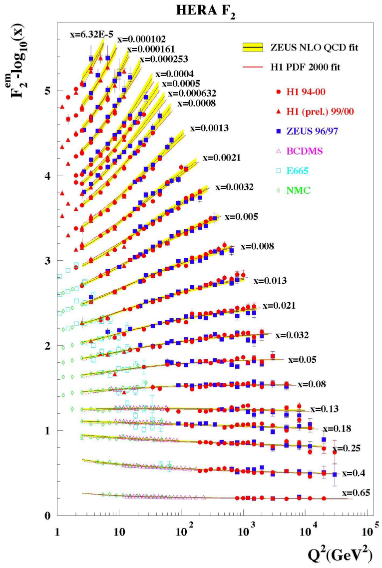
Vanishing of F_L (Callan-Gross relation): quarks spin is 1/2.
Independence on Q^2: Bjorken scaling!

Scalling

- Quark parton model prediction

$$F_2(x, Q^2) = \sum_q e_q^2 \, x \, f_q(x)$$

- Bjorken scaling due to scattering off pointlike constituents
- Small scaling violations at large x, stronger at small x
- Theoretically explained by NLO QCD!



QCD at NLO, Scalling violation.

- NLO correction to DIS consists of two contributions:
  - loop correction to LO suprocess $q\gamma^* \to q$. It is $\sim f_q(x)$
  - gluon radiation, subprocess $q\gamma^* \to q + G$. Its cross section $\sim \int\limits_x^1 d\beta f_q(\beta)$, initial quark should have bigger energy to produce extra gluon. But region of β that is closed to $\beta = x$ – soft limit of real gluon emission.
- Soft gluon singularity in virtual corrections and in real soft gluon production both $\sim f_q(x)$ and cancel each other.
- Collinear singularity remains! Initial states entering the collision are distinguishable (quark with or without initial state radiation have different momenta). KLM theorem does not apply.

Factorization

- Small angle emission happens at long distance/time before the hard collision, it is non-perturbative. It should be included into redefinition of $f_q(x)$, absorbing the initial state collinear divergences from the partonic process into quark PDF.

$$f_q^{\text{bare}}(x) \longrightarrow f_q^{\text{physical}}(x, \mu_F)$$

- Redefinition performed at fixed factorisation scale μ_F, making parton distributions scale-dependent.
- Essential property of perturbative QCD, ensured by Collins-Soper-Sterman factorisation theorem: process-independence of initial-state collinear singularities.

The relations between the bare and renormalized quantities in the $\overline{\mathrm{MS}}$ scheme are

$$f_q(x) = f_q(x,\mu_F) - \frac{\alpha_s}{2\pi}\left(\frac{1}{\hat{\epsilon}} + \ln\frac{\mu_F^2}{\mu^2}\right)\int_x^1 \frac{dz}{z}\left[P_{qq}(z) f_q(\frac{x}{z},\mu_F) + P_{qg}(z) f_g(\frac{x}{z},\mu_F)\right] ,$$

$$f_g(x) = f_g(x,\mu_F) - \frac{\alpha_s}{2\pi}\left(\frac{1}{\hat{\epsilon}} + \ln\frac{\mu_F^2}{\mu^2}\right)\int_x^1 \frac{dz}{z}\left[P_{gq}(z) f_q(\frac{x}{z},\mu_F) + P_{gg}(z) f_g(\frac{x}{z},\mu_F)\right] ,$$

where $\frac{1}{\hat{\epsilon}} = \frac{1}{\epsilon} + \gamma_E - \ln(4\pi) \approx \frac{\Gamma(1-\epsilon)}{\epsilon(4\pi)^\epsilon}$,
Using independence of bare distributions on μ_F,

$$\frac{\partial f_{q,g}(x)}{\partial \ln \mu_F^2} = 0$$

we obtain DGLAP evolution equations.

DGLAP evolution equations

$$\frac{\partial f_q(x,\mu_F)}{\partial \ln \mu_F^2} = \frac{\alpha_s(\mu_F)}{2\pi}\int\limits_x^1 \frac{dz}{z}\left[P_{qq}(z) f_q(\frac{x}{z},\mu_F) + P_{qg}(z) f_g(\frac{x}{z},\mu_F)\right] \;,$$

$$\frac{\partial f_g(x,\mu_F)}{\partial \ln \mu_F^2} = \frac{\alpha_s(\mu_F)}{2\pi}\int\limits_x^1 \frac{dz}{z}\left[P_{gq}(z) f_q(\frac{x}{z},\mu_F) + P_{gg}(z) f_g(\frac{x}{z},\mu_F)\right] \;,$$

They resum initial-state collinear parton radiation at $\mathcal{O}(\alpha_s^n \ln^n Q^2)$

LO DGLAP kernels

$$P_{gq}(z) = C_F \frac{1+(1-z)^2}{z} \ ,$$

$$P_{qg}(z) = T_R \left[z^2 + (1-z)^2\right] \ ,$$

$$P_{qq}(z) = C_F \left(\frac{1+z^2}{1-z}\right)_+ = C_F \left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right] \ ,$$

$$P_{gg}(z) = 2C_A \left[\frac{1}{(1-z)_+} + \frac{1}{z} - 2 + z(1-z)\right] + \left(\frac{11}{6}C_A - \frac{n_f}{3}\right)\delta(1-z) \ ,$$

with + prescription

$$\int_0^1 dx f(x)[g(x)]_+ = \int_0^1 dx g(x)[f(x) - f(0)]$$

that originate from canselation of soft gluon radiation in virtual and real emissions.

DGLAP differential equations: solution requires boundary condition at some low scale μ_0 (non-perturbative input distribution)- extracted from experiment.