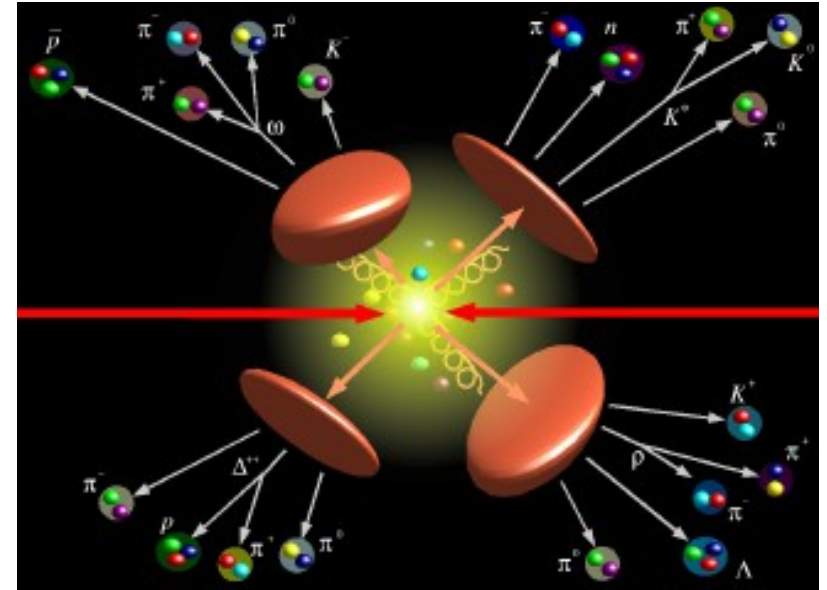
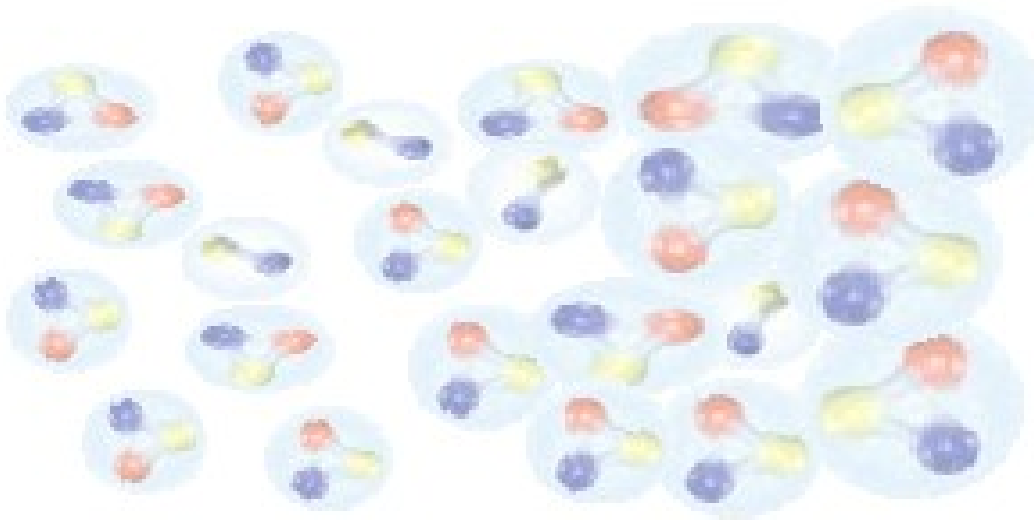


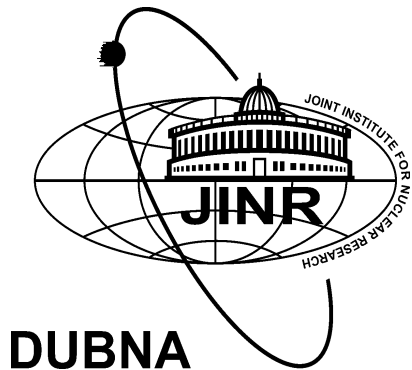
# Boiling dense QCD – Theory and event simulation for collisions at NICA and FAIR energies

**David Blaschke**

University of Wrocław, Poland & JINR Dubna & MEPhI Moscow, Russia



Association of Young Scientists and Specialists, JINR Dubna, 15.03.2016



# Boiling dense QCD – Theory and event simulation for collisions at NICA and FAIR energies

**David Blaschke**

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## 1. Introduction: Theory of Selfconsistent Quark-Hadron EoS

## 2. THESCoN: Event Simulation for NICA & FAIR

A project with FIAS, MEPhI and JINR

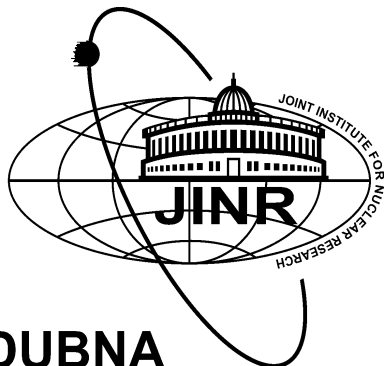
## 3. First results: Baryon stopping signal for a 1<sup>st</sup> order PT

Particization and all that ...

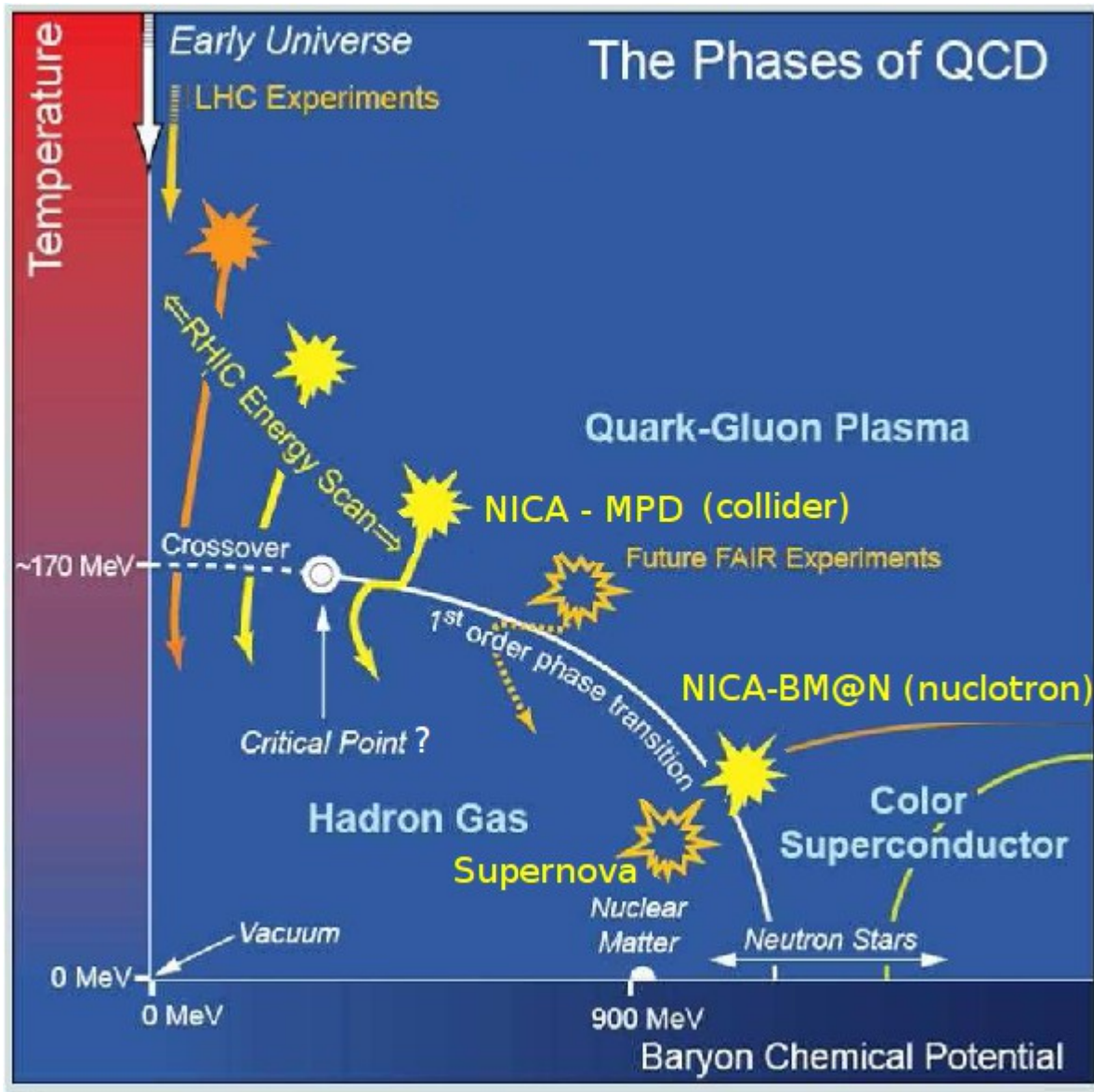
## 4. Further developments:

Flow, Detector response, new class of EoS ...

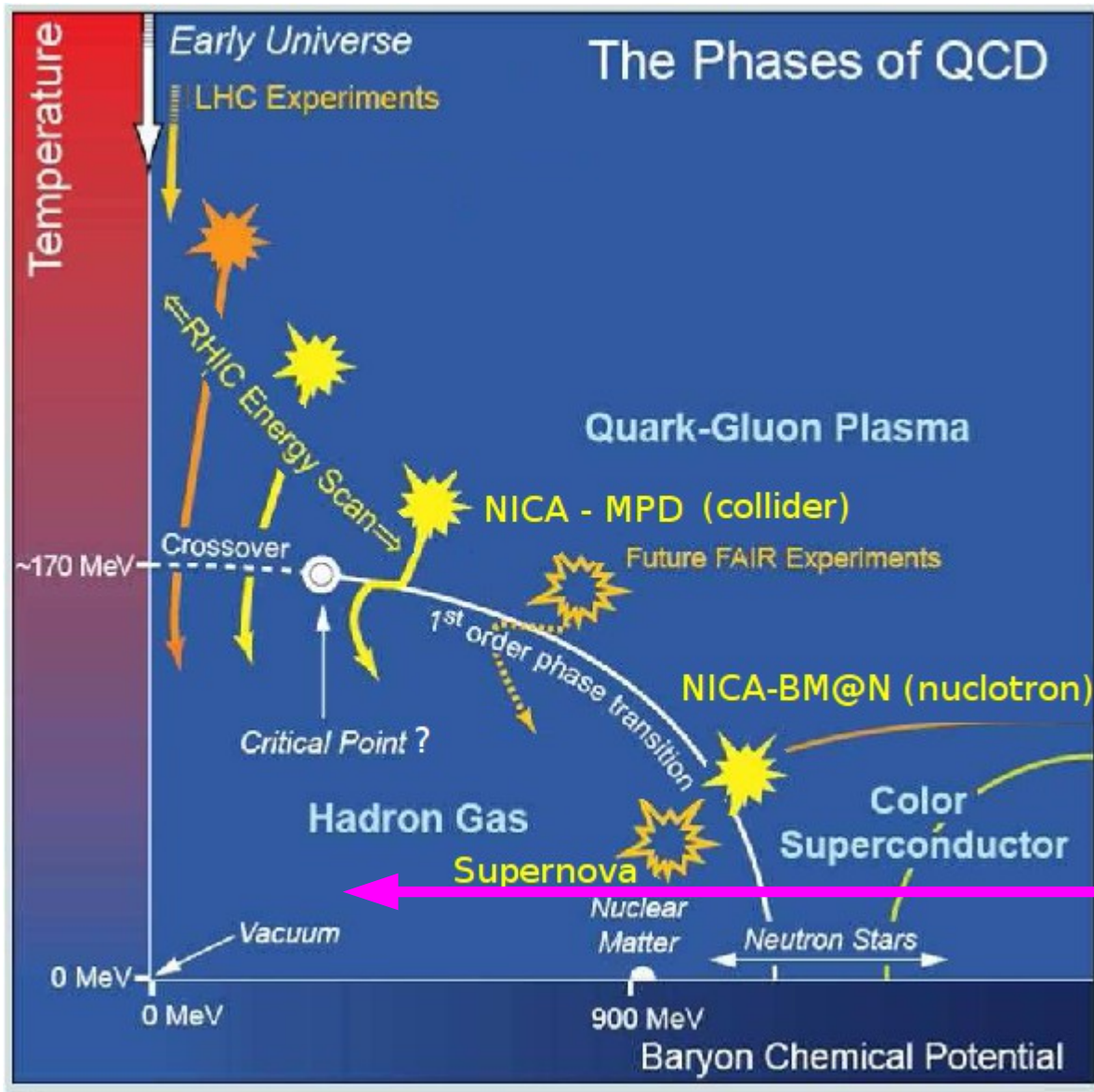
Association of Young Scientists and Specialists, JINR Dubna, 15.03.2016



# The Goal: Theory of the QCD Phase Diagram



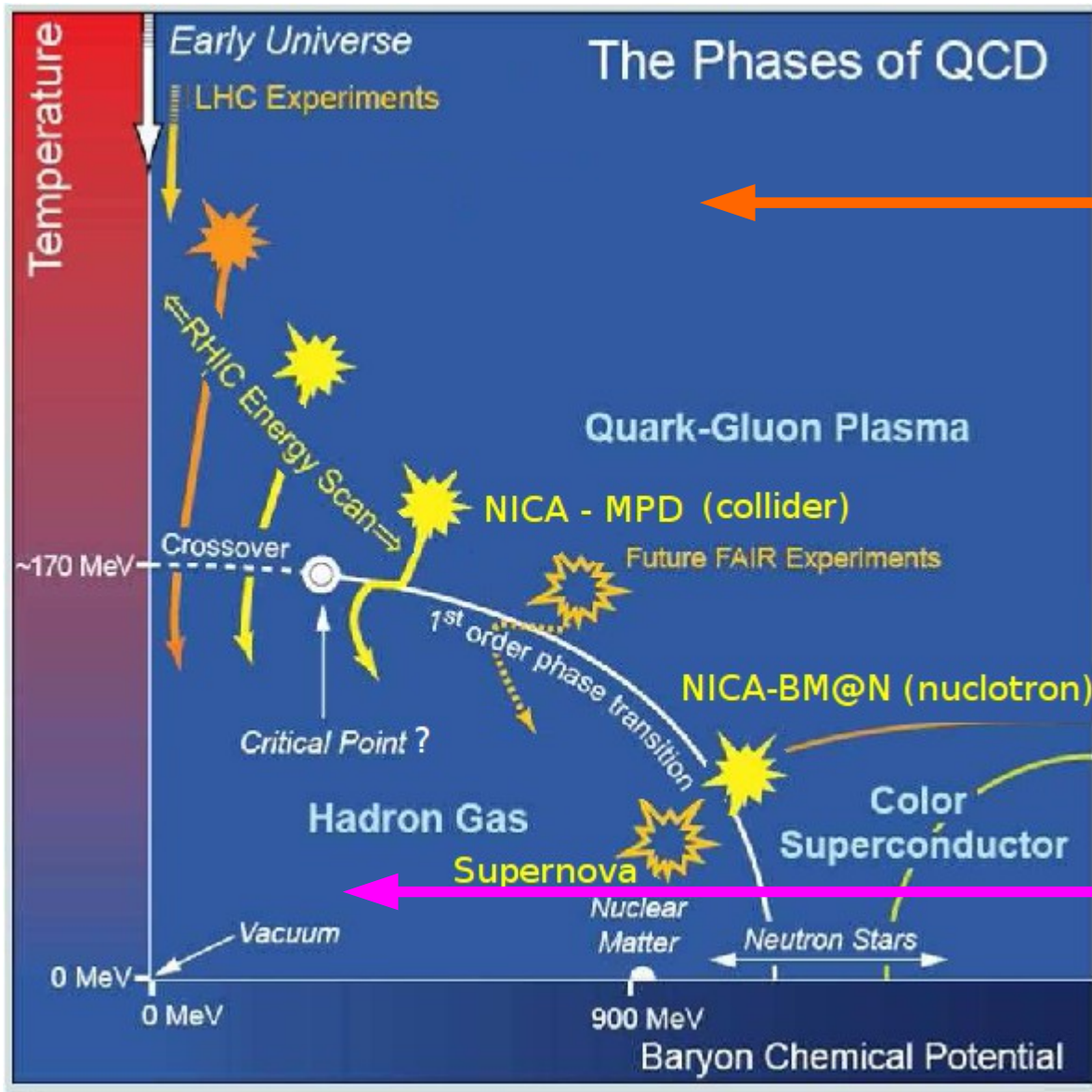
# The Goal: Theory of the QCD Phase Diagram



Statistical Model of  
Hadron Resonance Gas

Well established for  
Description of chemical  
freezeout

# The Goal: Theory of the QCD Phase Diagram



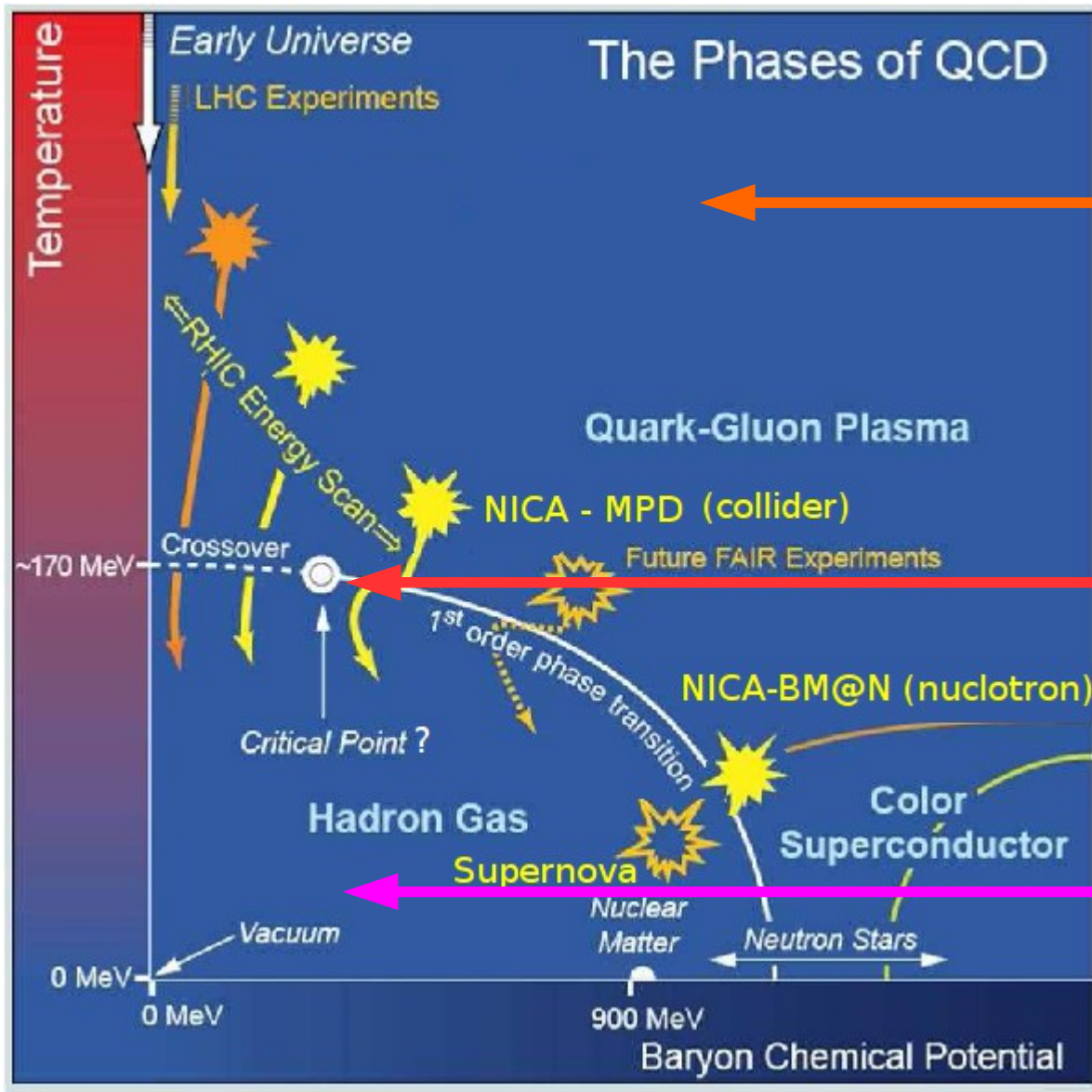
## Perturbative QCD

Approximately selfconsistent  
HTL resummation  
( $T > 2.5 T_c$  ,  $\mu > 1500$  MeV)

## Statistical Model of Hadron Resonance Gas

Well established for  
Description of chemical  
freezeout

# The Goal: Theory of the QCD Phase Diagram



## Perturbative QCD

Approximately selfconsistent HTL resummation  
( $T > 2.5 T_c$ ,  $\mu > 1500$  MeV)

## QCD Phase transition(s)

Mott dissociation of hadrons,  
Deconfinement,  $\chi$ SR

## Statistical Model of Hadron Resonance Gas

Well established for  
Description of chemical  
freezeout

# Approximately selfconsistent HTL resumm. QCD

J.-P. Blaizot, E. Iancu, A. Rebhan, Phys. Rev. D 63 (2001) 065003

## Skeleton expansion for thermodynamic potential and entropy

$$\beta\Omega[D] = -\log Z = \frac{1}{2} \text{Tr} \log D^{-1} - \frac{1}{2} \text{Tr} \Pi D + \Phi[D]$$

$$-\Phi[D] = \frac{1}{12} \text{Diagram 1} + \frac{1}{8} \text{Diagram 2} + \frac{1}{48} \text{Diagram 3} + \dots$$

↑  
Inv. Temp: 1/T

↑  
trace in conf. Space

↑  
self-energy related to D

**Dyson equation:**

$$D^{-1} = D_0^{-1} + \Pi$$

Free propagator  $D_0$  is known

Essential property of  $\Omega[D]$  is Stationarity under variation of D:  $\delta \Omega[D] / \delta D = 0$

This implies  $\delta \Phi[D] / \delta D = 1/2 \Pi$

Physical propagator and selfenergy are defined self-consistently !

**Self-consistent approximations** are defined by the **choice of  $\Phi$**

→  **$\Phi$  – derivable theories**

G. Baym, Phys. Rev. 127 (1962) 1391

# Approximately selfconsistent HTL resumm. QCD

Matsubara summation:

$$\Omega/V = \int \frac{d^4k}{(2\pi)^4} n(\omega) [\text{Im} \log(-\omega^2 + k^2 + \Pi) - \text{Im} \Pi D] + T\Phi[D]/V$$

Analytic properties:

$$D(\omega, k) = \int_{-\infty}^{\infty} \frac{dk_0}{2\pi} \frac{\rho(k_0, k)}{k_0 - \omega}, \quad \text{Im} D(\omega, k) \equiv \text{Im} D(\omega + i\epsilon, k) = \frac{\rho(\omega, k)}{2}.$$

Thermodynamics from entropy density:  $S = -\partial(\Omega/V)/\partial T$ .

$$S = - \int \frac{d^4k}{(2\pi)^4} \frac{\partial n(\omega)}{\partial T} \text{Im} \log D^{-1}(\omega, k) + \int \frac{d^4k}{(2\pi)^4} \frac{\partial n(\omega)}{\partial T} \text{Im} \Pi(\omega, k) \text{Re} D(\omega, k) + S'$$

$$S' \equiv - \left. \frac{\partial(T\Phi/V)}{\partial T} \right|_D + \int \frac{d^4k}{(2\pi)^4} \frac{\partial n(\omega)}{\partial T} \text{Re} \Pi \text{Im} D \longrightarrow 0$$

for two-loop skeletons

Loosely speaking: S' accounts for residual interactions of “independent quasiparticles”

$$d/d\omega [ \text{Im} \log D^{-1} + \text{Im} \Pi \text{Re} D ] = 2 \text{Im} [ D \text{Im}(d/d\omega D^*) \text{Im} \Pi ] = 2 \sin^2\delta d\delta/d\omega, \text{ for } D = |D|e^{i\delta}$$

D. Blaschke, in preparation (2016)



# Approximately selfconsistent HTL resumm. QCD

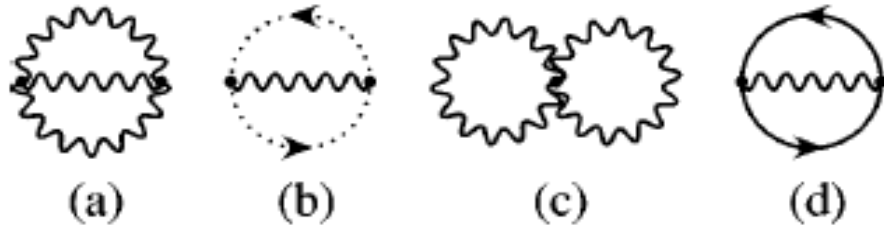


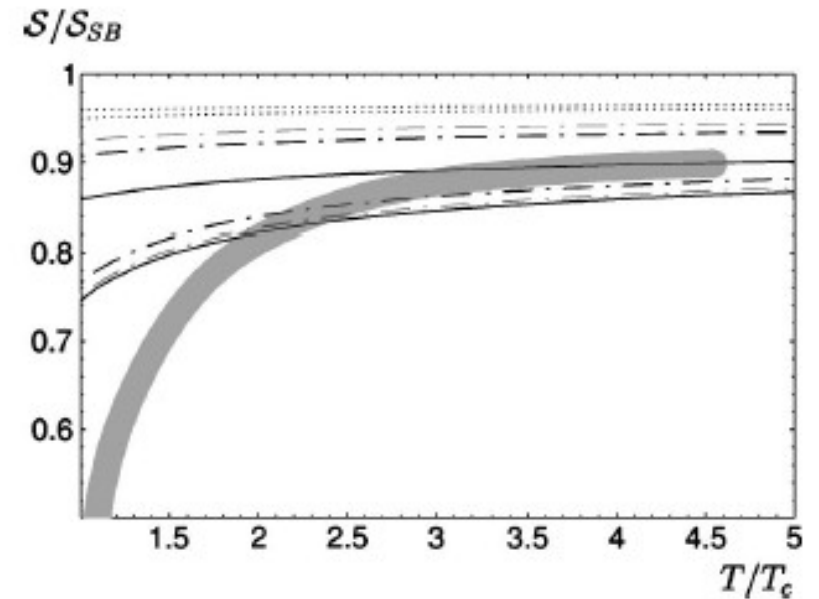
FIG. 3. Diagrams for  $\Phi$  at 2-loop order in QCD. Wiggly, plain, and dotted lines refer respectively to gluons, quarks, and ghosts.

In ghost-free gauge, HTL resummed QCD thermodyn.

$$S_2 = -\frac{g^2 N_g T}{48} \left\{ \frac{4N + 5N_f}{3} T^2 + \frac{3N_f}{\pi^2} \mu^2 \right\},$$

$$N_2 = -\frac{g^2 \mu N_g N_f}{16\pi^2} \left( T^2 + \frac{\mu^2}{\pi^2} \right),$$

$$P_2 = -\frac{g^2 N_g}{32} \left\{ \frac{4N + 5N_f}{18} T^4 + \frac{N_f}{\pi^2} \mu^2 T^2 + \frac{N_f}{2\pi^4} \mu^4 \right\}$$





# 1. Cluster expansion in the 2PI formalism

- $\Phi$ – derivable approach to the grand canonical thermodynamic potential  
[Baym, Phys. Rev. 127 (1962) 139]

$$J = -\text{Tr} \{ \ln(-G_1) \} - \text{Tr} \{ \Sigma_1 G_1 \} + \text{Tr} \{ \ln(-G_2) \} + \text{Tr} \{ \Sigma_2 G_2 \} + \Phi[G_1, G_2]$$

with full propagators:

$G_1^{-1}(1, z) = z - E_1(p_1) - \Sigma_1(1, z)$ ;  $G_2^{-1}(12, 1'2', z) = z - E_1(p_1) - E_2(p_2) - \Sigma_2(12, 1'2', z)$   
and selfenergies

$$\Sigma_1(1, 1') = \frac{\delta\Phi}{\delta G_1(1, 1')} ; \Sigma_2(12, 1'2', z) = \frac{\delta\Phi}{\delta G_2^{-1}(12, 1'2', z)}.$$

Because of stationarity equivalent to

$$n = -\frac{1}{\Omega} \frac{\partial J}{\partial \mu} = \frac{1}{\Omega} \sum_1 \int_{-\infty}^{\infty} \frac{d\omega}{\pi} f_1(\omega) S_1(1, \omega),$$

(baryon number conservation)

- Generalization to A-nucleon clusters in nuclear matter

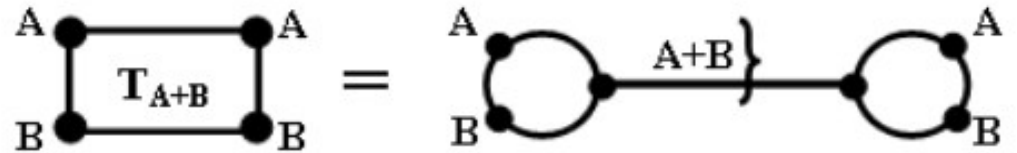
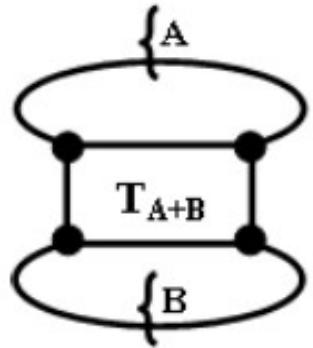
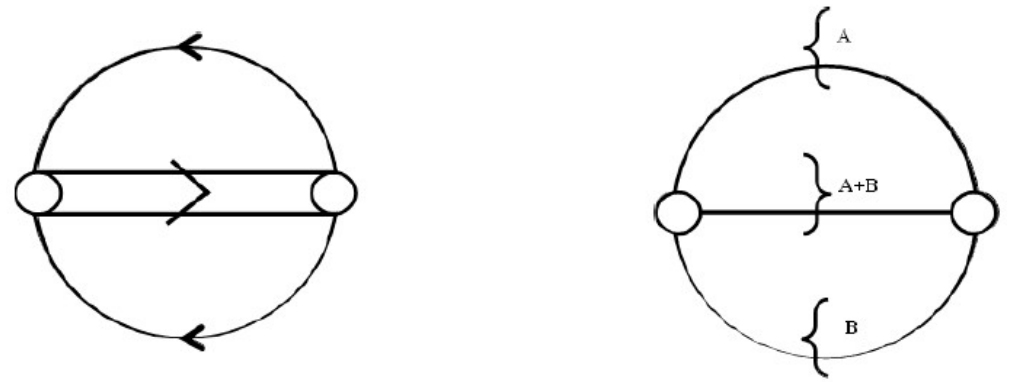
$$\Omega = \sum_A (-1)^A [\text{Tr} \ln (-G_A^{-1}) + \text{Tr} (\Sigma_A G_A)] + \Phi ,$$

$$G_A^{-1} = G_A^{(0)-1} - \Sigma_A , \quad \Sigma_A(1 \dots A, 1' \dots A', z_A) = \frac{\delta\Phi}{\delta G_A(1 \dots A, 1' \dots A', z_A)} .$$

# 1. Cluster expansion in the 2PI formalism

## A) Choice of the $\Phi$ -functional:

- 2-particle irreducible diagrams
- closed 2-loop diagram involving 3 cluster propagators (A, B, A+B) and 2 vertices
- equivalent to 1 T-matrix + 2 propagators



## B) Ansatz for thermodynamic potential:

$$\Omega = \sum_A (-1)^A [\text{Tr} \ln (-G_A^{-1}) + \text{Tr} (\Sigma_A G_A)] + \sum_{A,B} \Phi[G_A, G_B, G_{A+B}],$$

$$G_A^{-1} = G_A^{(0)-1} - \Sigma_A, \quad \Sigma_A(1 \dots A, 1' \dots A', z_A) = \frac{\delta \Phi}{\delta G_A(1 \dots A, 1' \dots A', z_A)}.$$

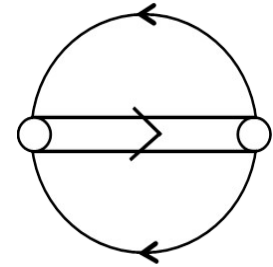
## C) Check: conservation laws, e.g.:

(correspondence to GF formalism)

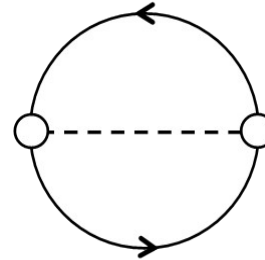
$$n = -\frac{1}{V} \frac{\partial \Omega}{\partial \mu} = \frac{1}{V} \sum_1 \int_{-\infty}^{\infty} \frac{d\omega}{\pi} f_1(\omega) A_1(1, \omega)$$

# 1. Cluster virial expansion in the 2PI formalism, Examples:

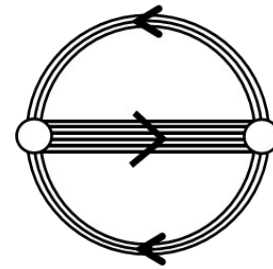
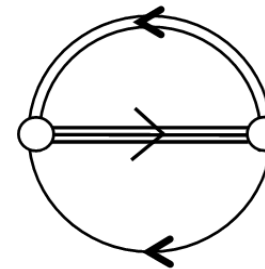
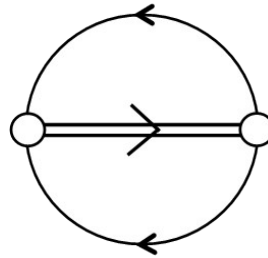
A) Deuterons in nuclear matter (check):



B) Mesons in quark matter:

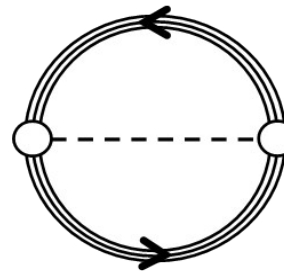
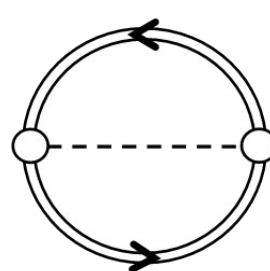


C) Nucleons in quark matter:



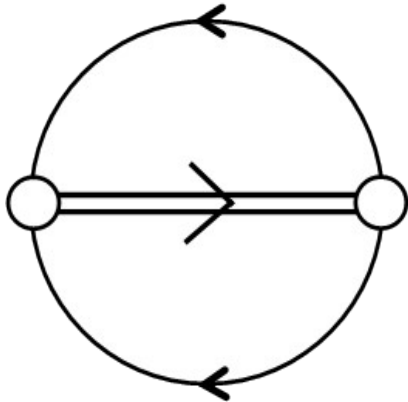
D) Nucleons and mesons (hadron resonance gas) in quark matter:

B) + C) +

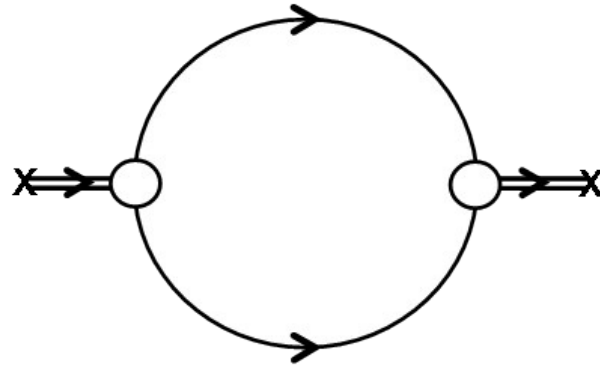


## 2. Example A: Deuterons in nuclear matter

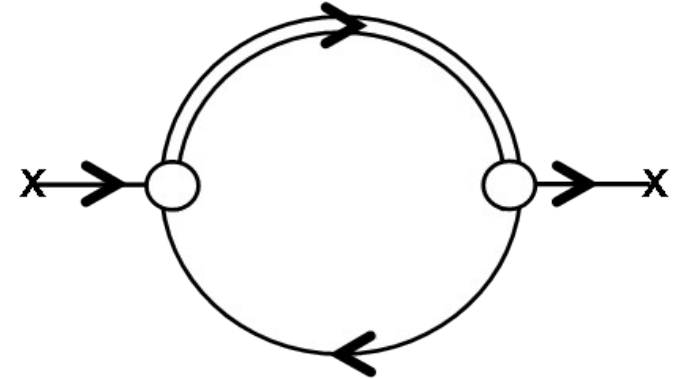
$\Phi$ -functional



deuteron selfenergy " $\Pi$ "



nucleon selfenergy



$$T^{-1} = V^{-1} - \Pi$$

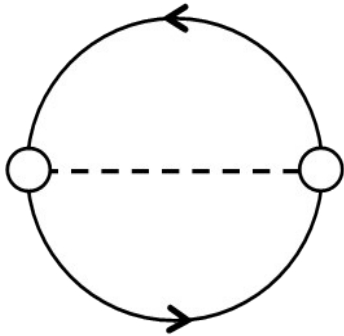
Bethe-Salpeter equation for deuteron T-matrix, ladder approximation

$$\Sigma(1, z) = \sum_2 \int \frac{d\omega}{2\pi} A(2, \omega) \left\{ f(\omega) V(12, 12) - \int \frac{dE}{\pi} \Im T(12, 12; E + i\eta) \frac{f(\omega) + g(E)}{E - z - \omega} \right\}$$

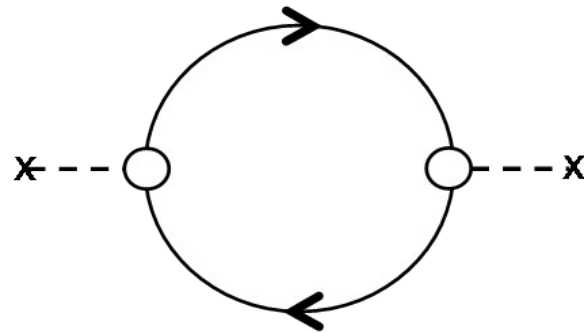
$$n(\mu, T) = n_{\text{free}}(\mu, T) + 2n_{\text{corr}}(\mu, T), \quad n_{\text{sc}} = \int \frac{dE}{2\pi} g(E) 2 \sin^2 \delta(E) \frac{d\delta(E)}{dE}.$$

### 3. Example B: Mesons in quark matter

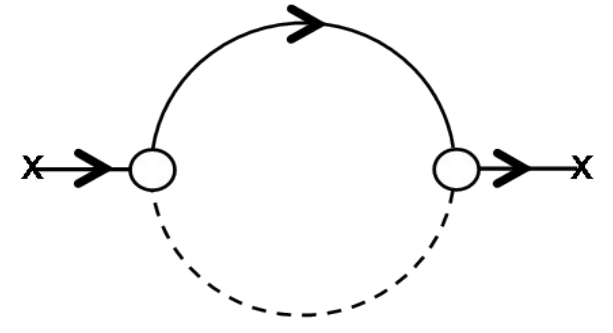
$\Phi$ -functional



Meson selfenergy (RPA)



Quark selfenergy



$$T_M^{-1}(q, \omega + i\eta) = G_S^{-1} - \Pi_M(q, \omega + i\eta) = |T_M(q, \omega)|^{-1} e^{-i\delta_M(q, \omega)}, \quad \delta_M(q, \omega) = \arctan(\Im T_M / \Re T_M)$$

$$\Omega = \Omega_{\text{MF}} + \Omega_M, \quad \Omega_{\text{MF}} = 2N_f N_c G_S \int \frac{d^3 p}{(2\pi)^3} \frac{m}{E_p} [1 - f_-(E_p) - f_+(E_p)],$$

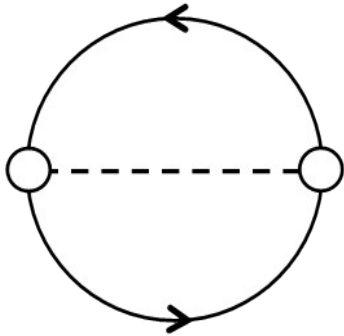
$$\Omega_{\text{MF}} = \frac{\sigma_{\text{MF}}^2}{4G_S} - 2N_c N_f \int \frac{d^3 p}{(2\pi)^3} \left[ E_p + T \ln \left( 1 + e^{-(E_p - \Sigma_+ - \mu)/T} \right) + T \ln \left( 1 + e^{-(E_p + \Sigma_- + \mu)/T} \right) \right],$$

$$\Omega_M = d_M \int \frac{d^3 k}{(2\pi)^3} \int \frac{d\omega}{2\pi} \left\{ \omega + 2T \ln \left[ 1 - e^{-\omega/T} \right] 2 \sin^2 \delta_M(k, \omega) \frac{\delta_M(k, \omega)}{d\omega} \right\},$$

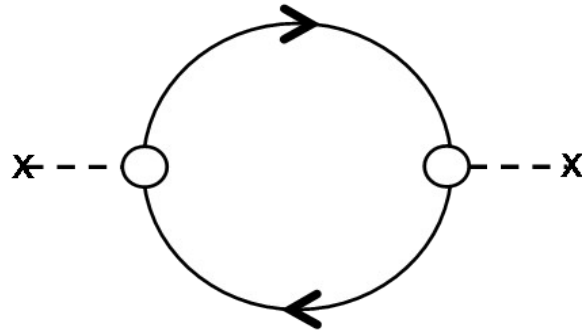
$$\Sigma_M(\mathbf{0}, p_0) = d_M \int \frac{d^4 q}{(2\pi)^4} \pi \rho_M(\mathbf{q}, q_0) \left\{ \frac{(\gamma_0 + m/E_q)[1 + g(q_0) - f_-(E_q)]}{q_0 - p_0 + E_q - \mu - i\eta} + \frac{(\gamma_0 - m/E_q)[g(q_0) + f_+(E_q)]}{q_0 - p_0 - E_q - \mu - i\eta} \right\},$$

### 3. Example B: Mesons in quark matter

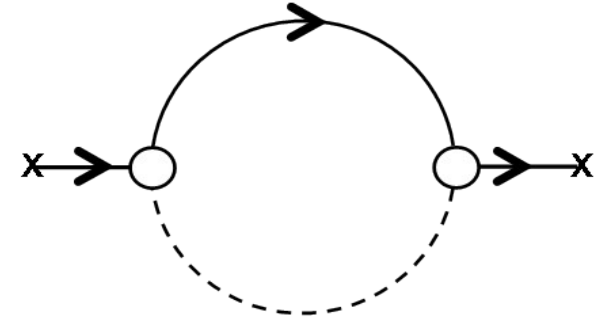
$\Phi$ -functional



Meson selfenergy (RPA)



Quark selfenergy



$$T_M^{-1}(q, \omega + i\eta) = G_S^{-1} - \Pi_M(q, \omega + i\eta) = |T_M(q, \omega)|^{-1} e^{-i\delta_M(q, \omega)}, \quad \delta_M(q, \omega) = \arctan(\Im T_M / \Re T_M)$$

$$\Omega = \Omega_{\text{MF}} + \Omega_M, \quad \sigma_{\text{MF}} = 2N_f N_c G_S \int \frac{d^3 p}{(2\pi)^3} \frac{m}{E_p} [1 - f_-(E_p) - f_+(E_p)],$$

$$\Omega_{\text{MF}} = \frac{\sigma_{\text{MF}}^2}{4G_S} - 2N_c N_f \int \frac{d^3 p}{(2\pi)^3} \left[ E_p + T \ln \left( 1 + e^{-(E_p - \Sigma_+ - \mu)/T} \right) + T \ln \left( 1 + e^{-(E_p + \Sigma_- + \mu)/T} \right) \right],$$

$$\Omega_M = d_M \int \frac{d^3 k}{(2\pi)^3} \int \frac{d\omega}{2\pi} \left\{ \omega + 2T \ln \left[ 1 - e^{-\omega/T} \right] \boxed{2 \sin^2 \delta_M(k, \omega)} \frac{\delta_M(k, \omega)}{d\omega} \right\}$$

not yet !

$$\Sigma_M(\mathbf{0}, p_0) = d_M \int \frac{d^4 q}{(2\pi)^4} \pi \rho_M(\mathbf{q}, q_0) \left\{ \frac{(\gamma_0 + m/E_q)[1 + g(q_0) - f_-(E_q)]}{q_0 - p_0 + E_q - \mu - i\eta} + \frac{(\gamma_0 - m/E_q)[g(q_0) + f_+(E_q)]}{q_0 - p_0 - E_q - \mu - i\eta} \right\}$$

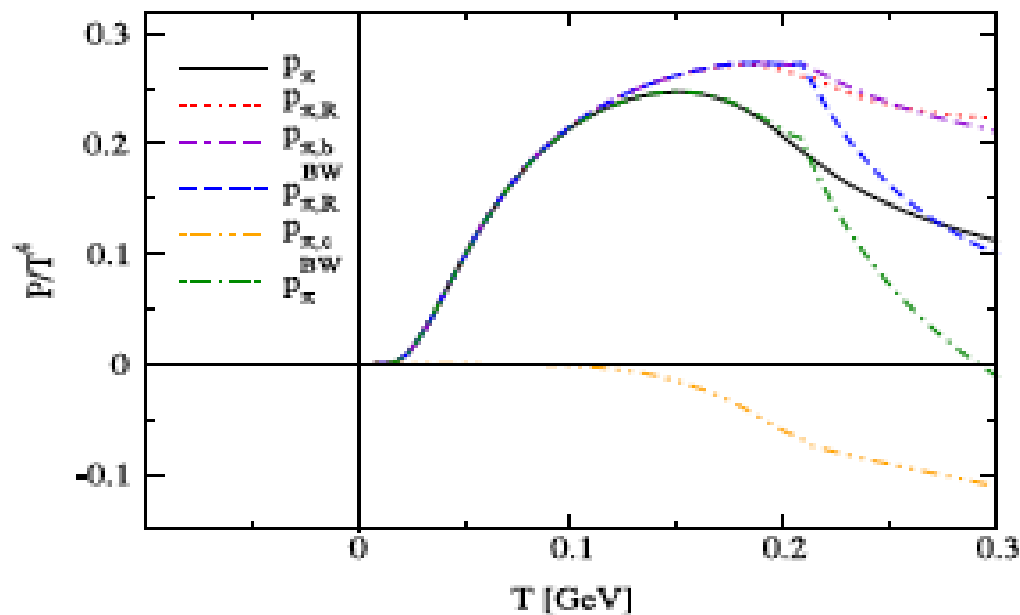


### 3. Example B: Mesons in quark matter

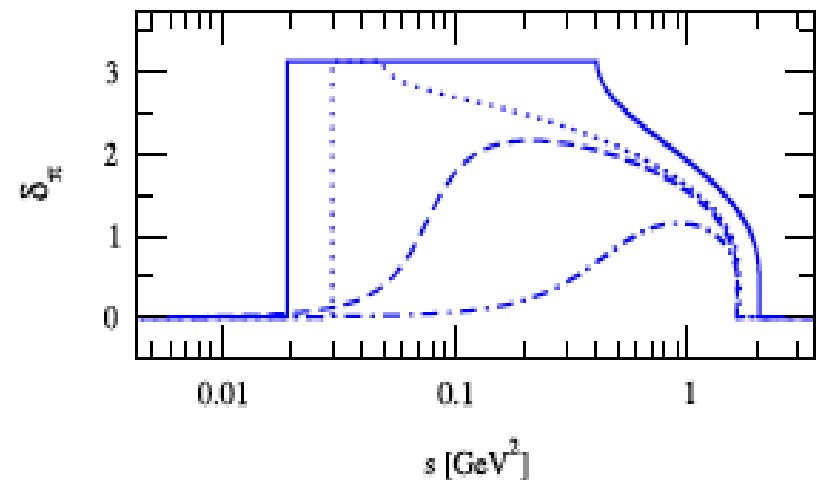
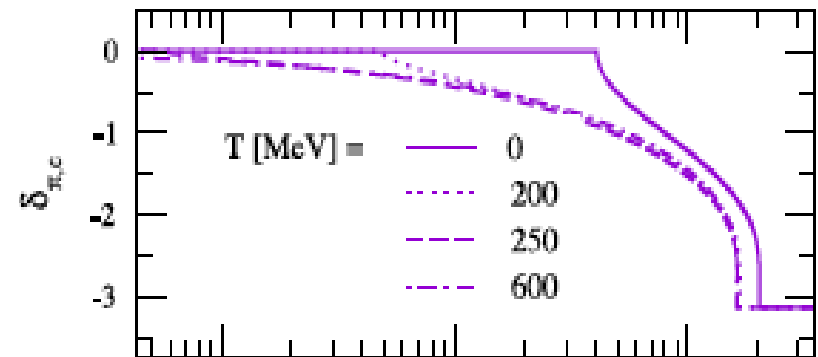
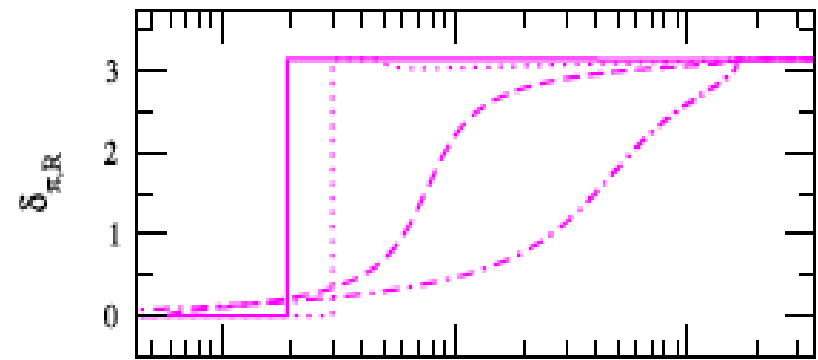
$$\Omega_X(T, \mu) = -d_X \int \frac{d^3q}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} n_X^-(\omega) \delta_X(\omega, \mathbf{q}),$$

$$\int_0^{\infty} d\omega \frac{1}{\pi} \frac{d\delta_X(\omega; T)}{d\omega} = 0 = \underbrace{\int_0^{\omega_{\text{thr}}(T)} d\omega \frac{1}{\pi} \frac{d\delta_X(\omega; T)}{d\omega}}_{n_{B,X}(T)} + \underbrace{\frac{1}{\pi} \int_{\omega_{\text{thr}}(T)}^{\infty} d\omega \frac{d\delta_X(\omega; T)}{d\omega}}_{\frac{1}{\pi} [\delta_X(\infty; T) - \delta_X(\omega_{\text{thr}}; T)]},$$

$$p_\pi(T) = -d_\pi T \int \frac{d^3q}{(2\pi)^3} \int_0^{\infty} \frac{d\omega}{\pi} \ln(1 - e^{-\omega/T}) \frac{d\delta_\pi(\omega, \mathbf{q})}{d\omega}$$



$$\delta_\pi = \delta_{\pi,c} + \delta_{\pi,R}$$



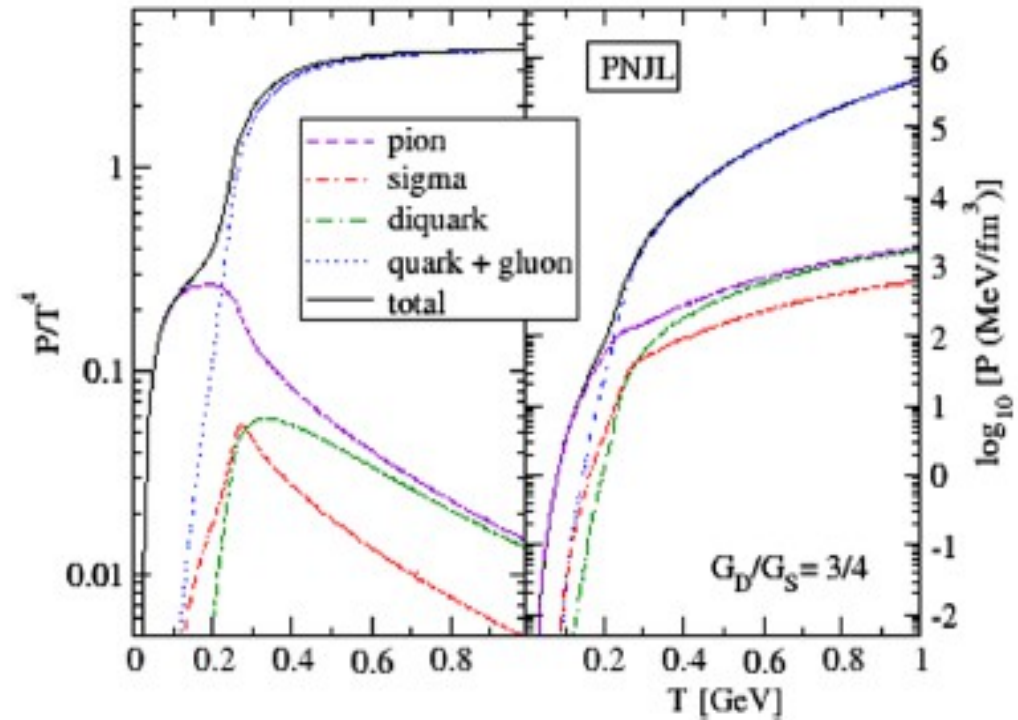
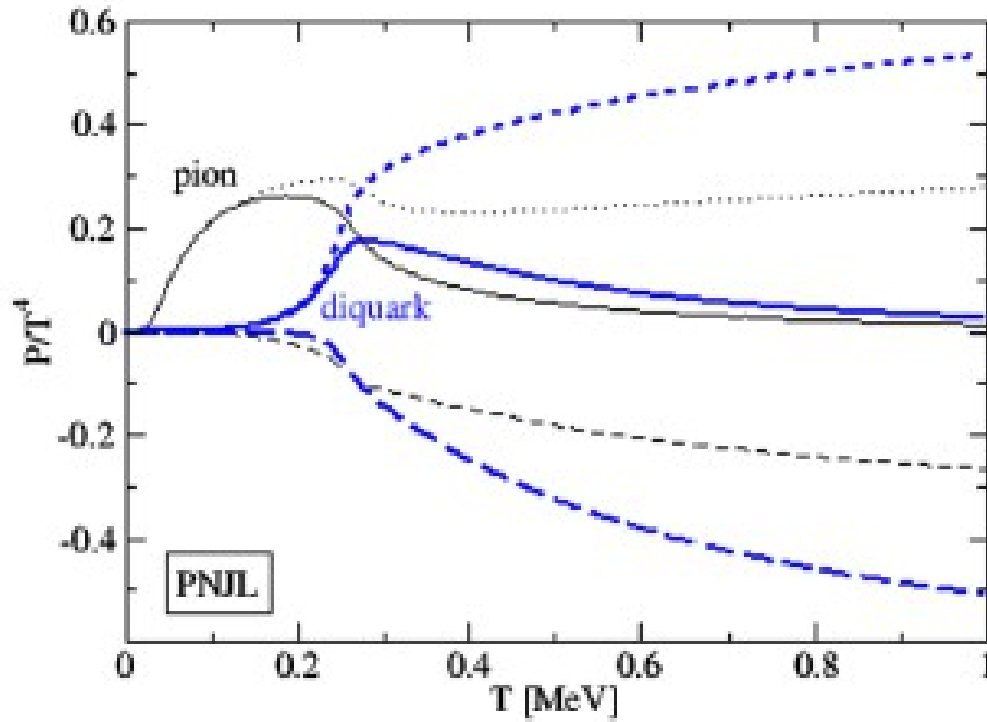
### 3. Example B\*: Mesons+diquarks in quark matter

$$\Omega_Q = -\frac{2N_c N_f}{3} \int \frac{d^3 p}{(2\pi)^3} \frac{p^4}{E_p} [f_{\bar{\Phi}}^+(E_p) + f_{\bar{\Phi}}^-(E_p)], \quad f_{\bar{\Phi}}^+(E_p) = \frac{(\bar{\Phi} + 2\Phi Y)Y + Y^3}{1 + 3(\bar{\Phi} + \Phi Y)Y + Y^3}, \quad Y = e^{-(E_p - \mu)/T}$$

$$\Omega_D = -3 \int \frac{d^3 p}{(2\pi)^3} \int \frac{d\omega}{2\pi} [g_{\bar{\Phi}}^+(\omega) + g_{\bar{\Phi}}^-(\omega)] \delta_D(\omega), \quad g_{\bar{\Phi}}^+(\omega) = \frac{(\Phi - 2\bar{\Phi} X)X + X^3}{1 - 3(\Phi - \bar{\Phi} X)X - X^3}, \quad X = e^{-(\omega - 2\mu)/T}$$

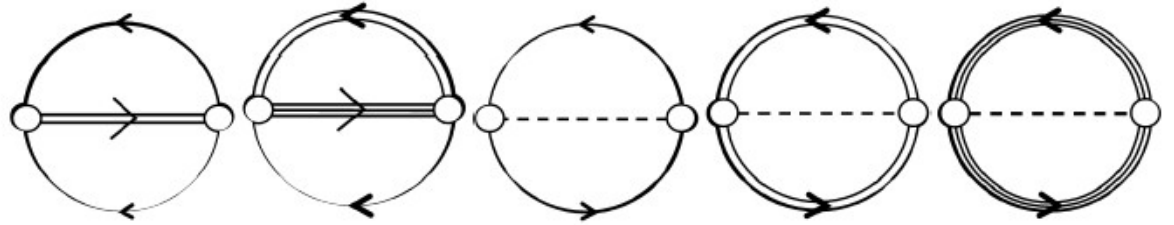
Suppression of colored states by Polyakov-loop  $\Phi$

Confinement:  $\Phi=0$

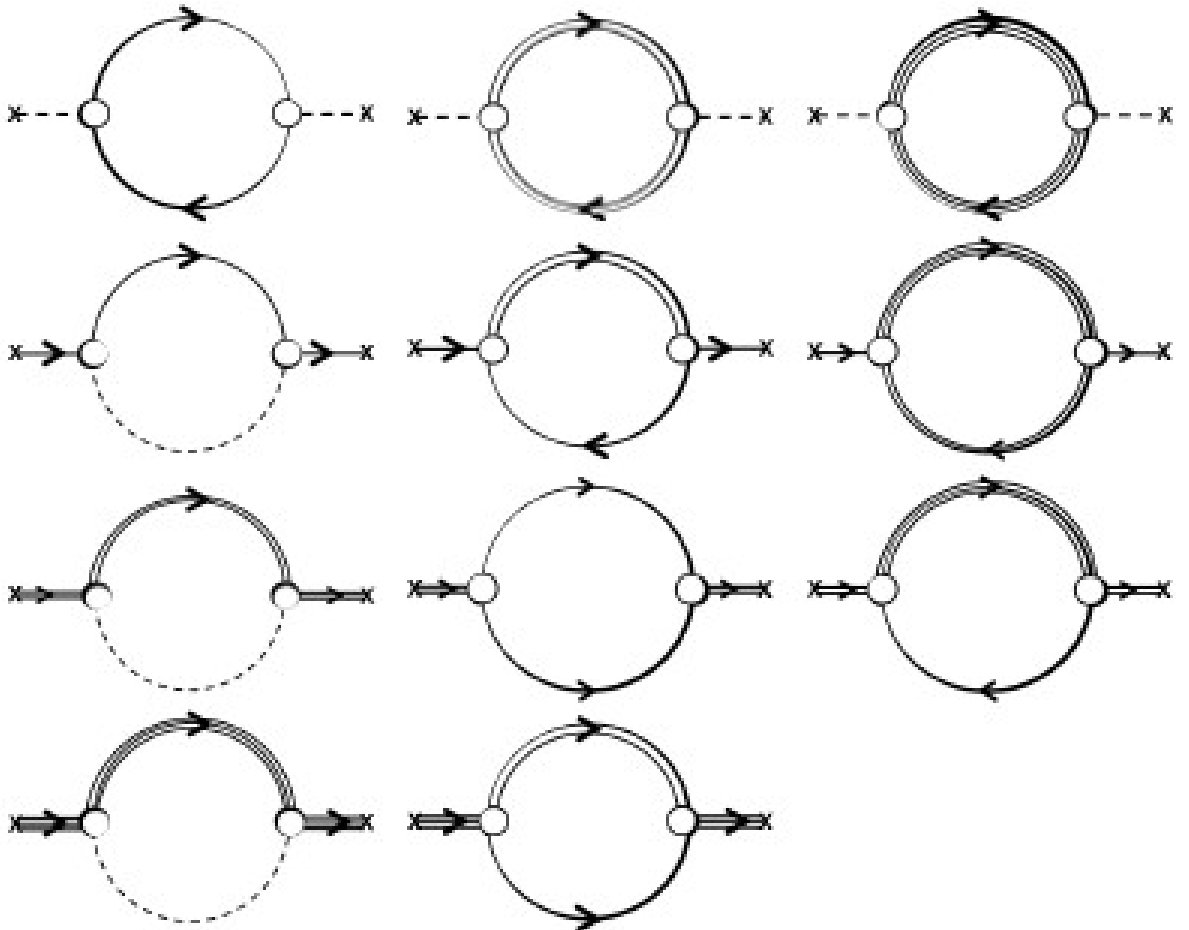


# 3. Example C: Hadron resonance gas – toy model

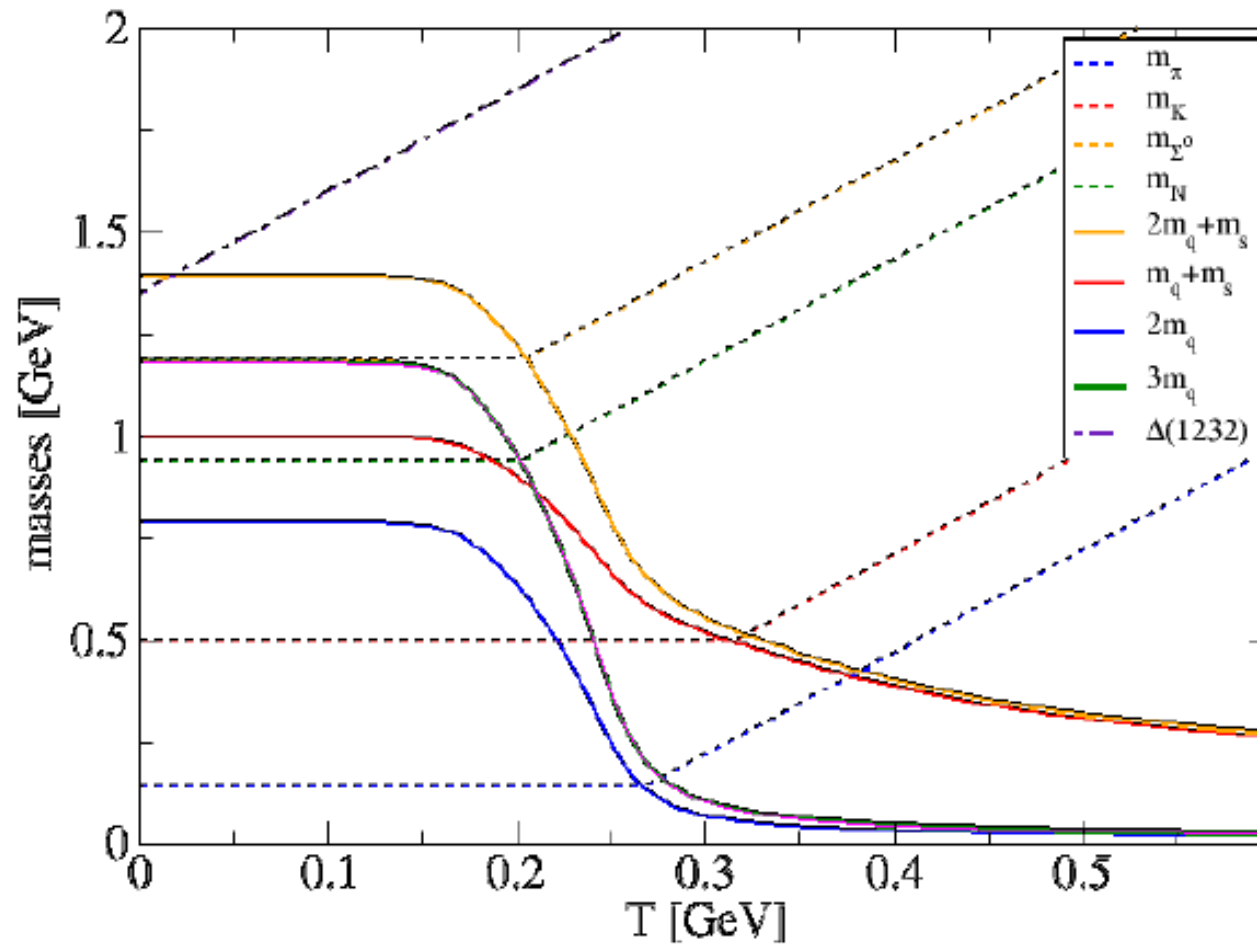
$\Phi$ -functional:



Selfenergies:



### 3. Example C: Hadron resonance gas – toy model



$$M_i(T) = M_i(0) + \Gamma_i(T) ,$$

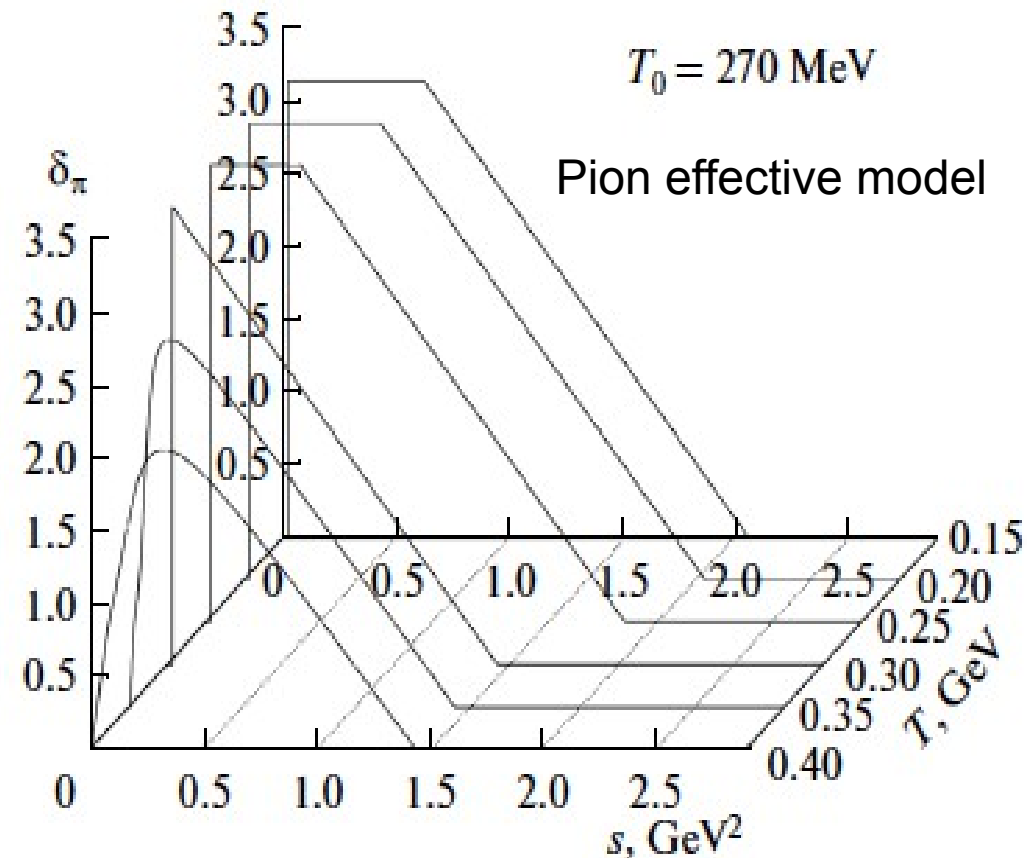
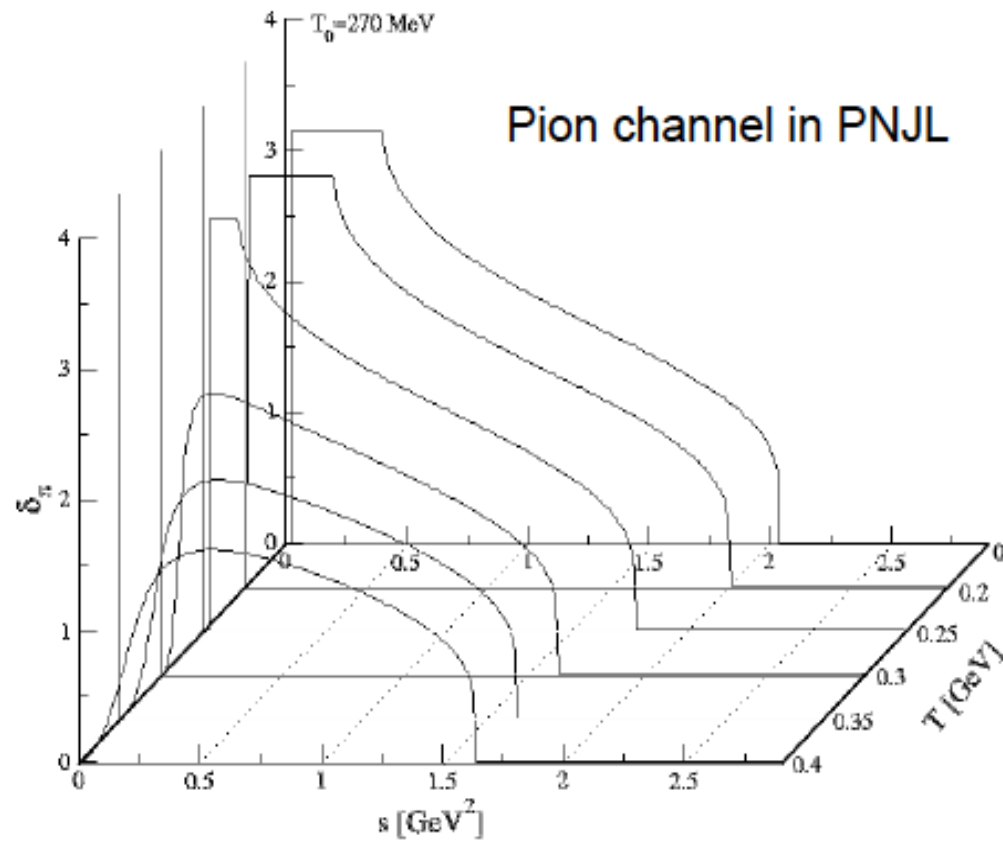
$$\Gamma_i(T) = a (T - T_{\text{Mott},i}) \Theta(T - T_{\text{Mott},i})$$

$$M_i(T_{\text{Mott},i}) = m_{\text{thr},i}(T_{\text{Mott},i}) ,$$

$$m_{\text{thr},M}(T) = (2 - N_s)m(T) + N_s m_s(T)$$

$$m_{\text{thr},B}(T) = (3 - N_s)m(T) + N_s m_s(T)$$

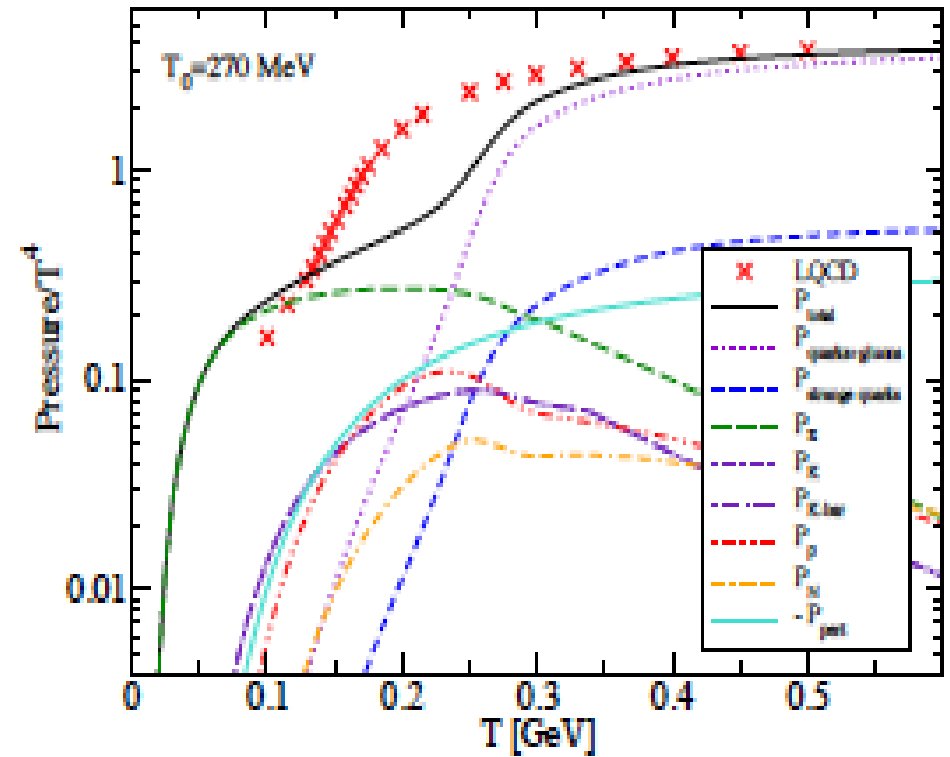
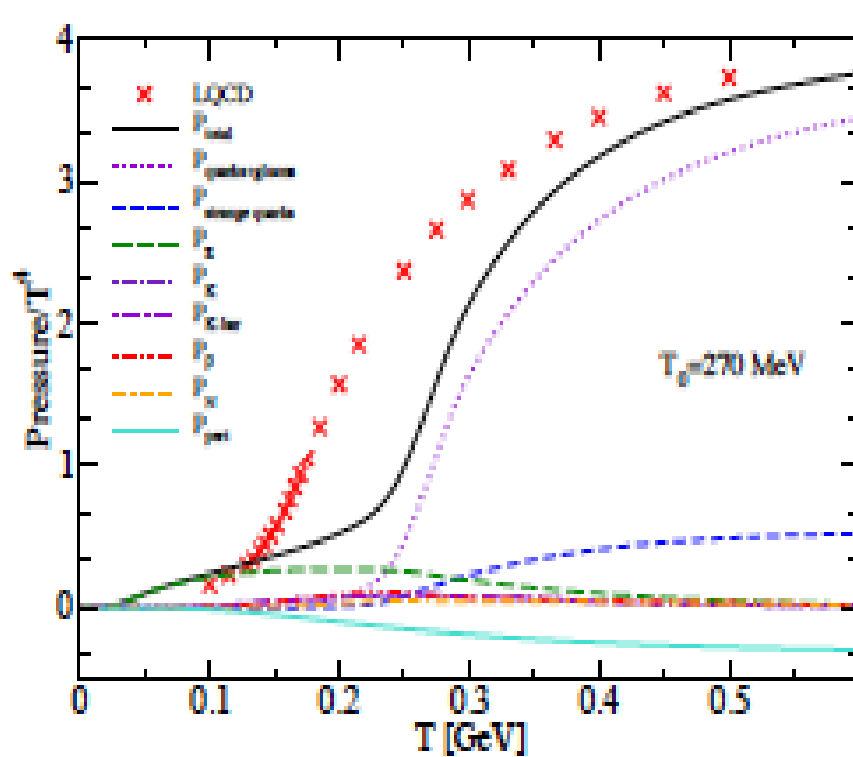
# 3. Example C: Hadron resonance gas – toy model



Effective model for in-medium hadron phase shifts

$$\delta_i(s; T) = \left[ \frac{\pi}{2} + \arctan \left( \frac{s - M_i^2(T)}{M_i(T)\Gamma_i(T)} \right) \right] \left\{ \Theta[m_{\text{thr},i}^2 - s] + \Theta[s - m_{\text{thr},i}^2] \Theta[m_{\text{thr},i}^2 + N_i^2 \Lambda^2 - s] \left[ \frac{[m_{\text{thr},i}^2 + N_i^2 \Lambda^2 - s]}{N_i^2 \Lambda^2} \right] \right\} \quad (7)$$

### 3. Example C: Hadron resonance gas – toy model

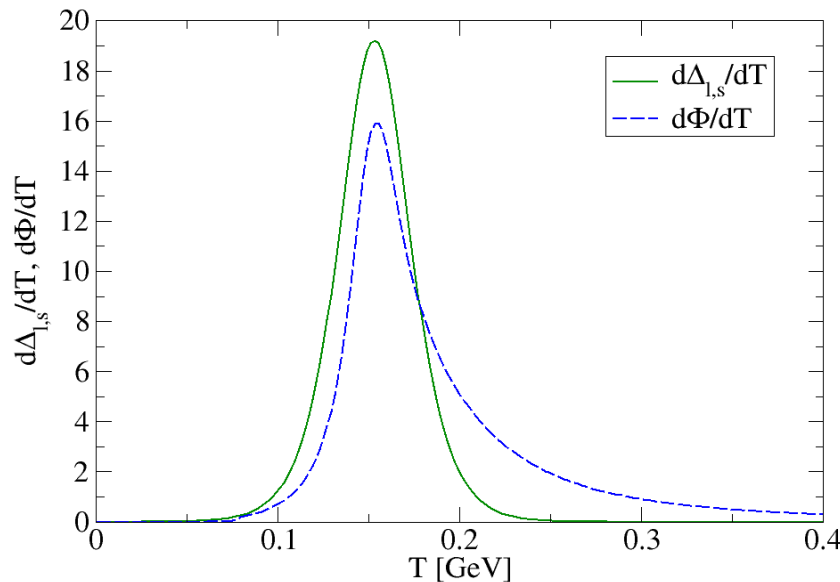
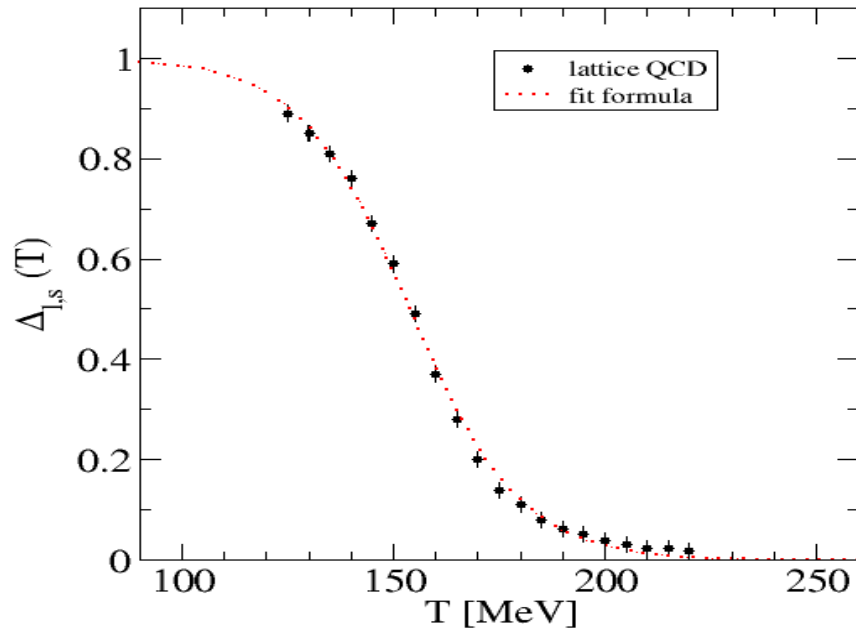


$$P_i(T) = d_i \int_0^\infty \frac{dp p^2}{2\pi^2} \int_0^\infty \frac{dM}{\pi} \frac{M}{\sqrt{p^2 + M^2}} f_i(\sqrt{p^2 + M^2}) \delta_i(M^2; T)$$

$$P_i(T) = \mp d_i \int_0^\infty \frac{dp p^2}{2\pi^2} \int_0^\infty dM T \ln(1 \mp e^{-\sqrt{p^2 + M^2}/T}) \frac{1}{\pi} \frac{d\delta_i(M^2; T)}{dM}$$

$$P(T) = \sum_{i=M,B} P_i(T) + P_{\text{PNJL}}(T) + P_{\text{pert}}(T)$$

# 3. Example C: Mott HRG – improved toy model



$$P_{\text{PNJL}}(T) = P_{\text{FG}}(T) + \mathcal{U}[\Phi; T] ,$$

$$P_{\text{FG}}(T) = 4 \sum_{\sigma=u,d,s} \int \frac{d^3p}{(2\pi)^3} T \ln [1 + 3\Phi(Y + Y^2) + Y^3]$$

$$Y(E_p) = \exp(-E_p/T)$$

$$\mathcal{U}[\Phi; T] = -\frac{a(T)}{2}\Phi^2 + b(T) \ln(1 - 6\Phi^2 + 8\Phi^3 - 3\Phi^4)$$

T-dependent quark masses from fit to LQCD

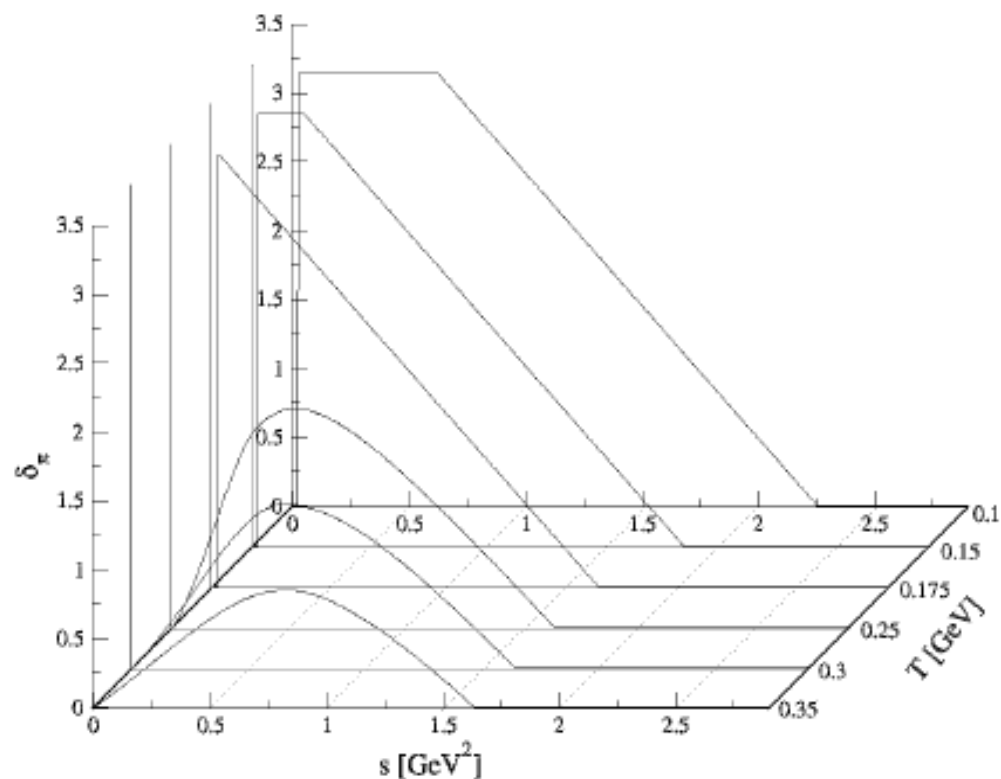
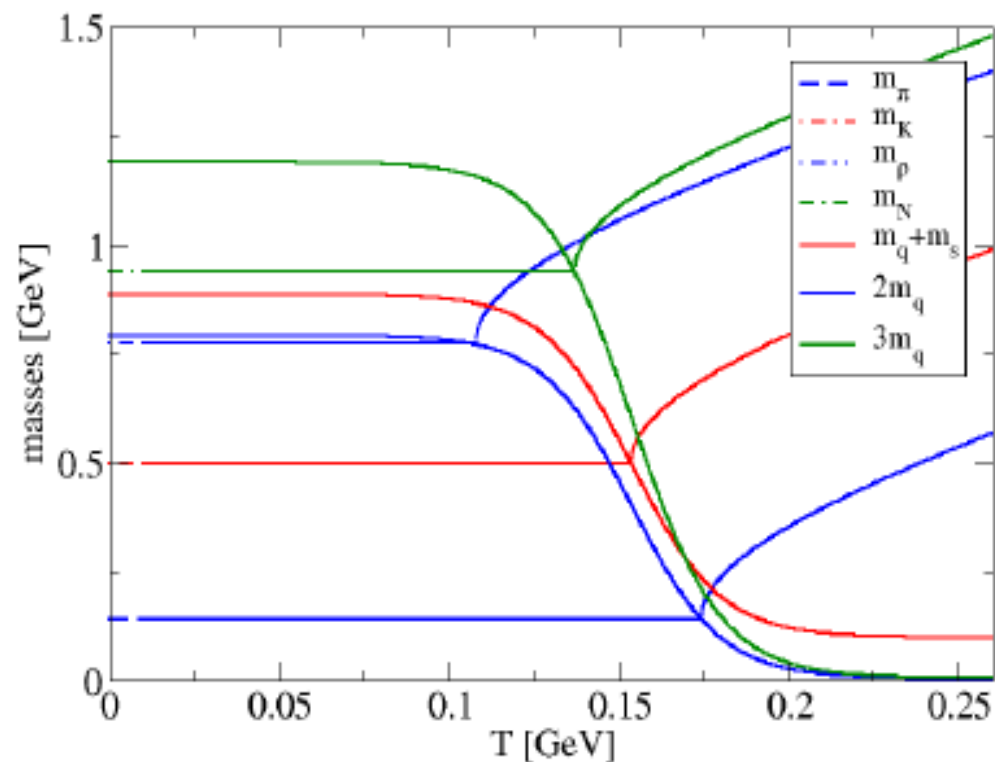
$$m(T) = [m(0) - m_0]\Delta_{l,s}(T) + m_0 ,$$

$$m_s(T) = m(T) + m_s - m_0 ,$$

$$\Delta_{l,s}(T) = \frac{1}{2} \left[ 1 - \tanh \left( \frac{T - T_c}{\delta_T} \right) \right]$$

$$T_c = 154 \text{ MeV} \quad \delta_T = 26 \text{ MeV}$$

### 3. Example C: Mott HRG – improved toy model



Hadrons + Mott effect

$$P_i(T) = \mp d_i \int_0^\infty \frac{dp p^2}{2\pi^2} \int_0^\infty dM T \ln \left( 1 \mp e^{-\sqrt{p^2+M^2}/T} \right) \frac{2}{\pi} \sin^2 \delta_i(M^2; T) \frac{d\delta_i(M^2; T)}{dM}$$

Quarks + rescattering effects

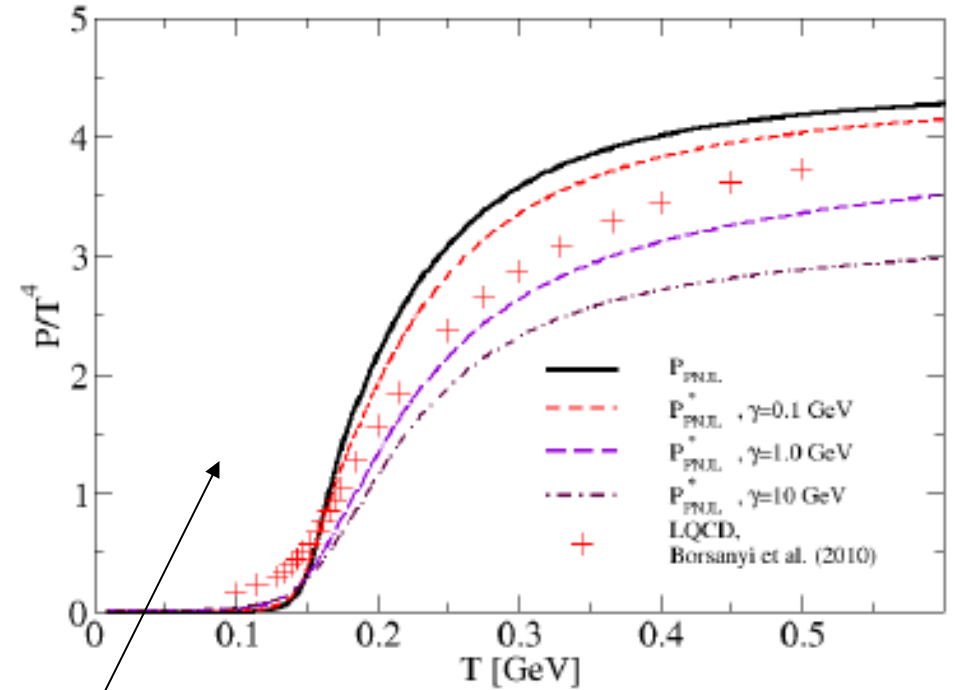
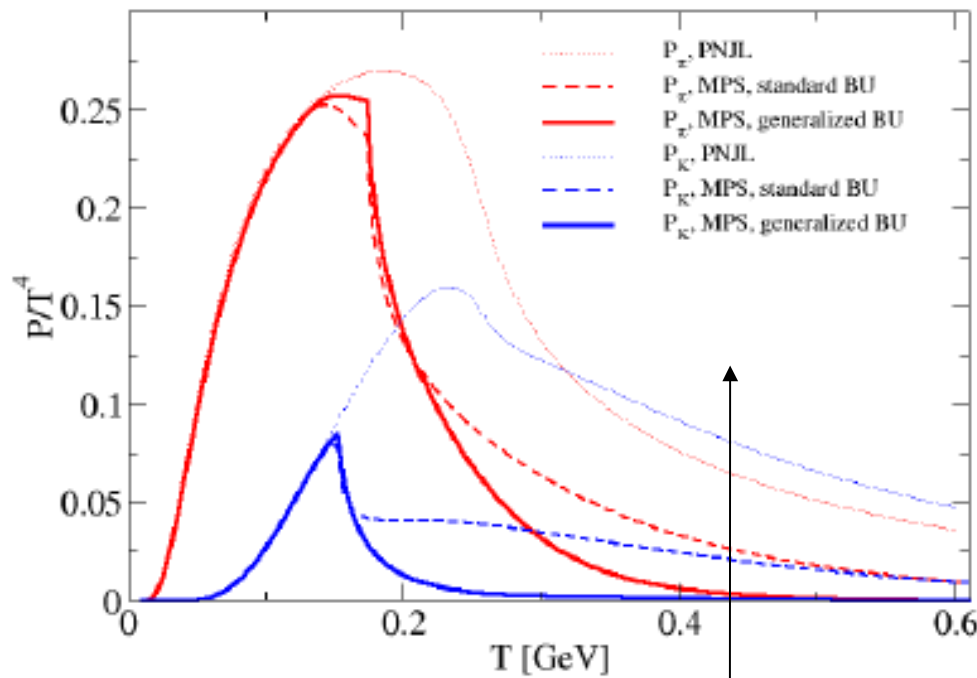
$$P_{FG}^*(T) = 4N_c \sum_{q=u,d,s} \int \frac{dp p^2}{2\pi^2} \int \frac{d\omega}{\pi} f_\Phi(\omega) \delta_q(\omega; \gamma),$$

$$f_\Phi(\omega) = \frac{\Phi(1+2Y)Y + Y^3}{1 + 3\Phi(1+Y)Y + Y^3},$$

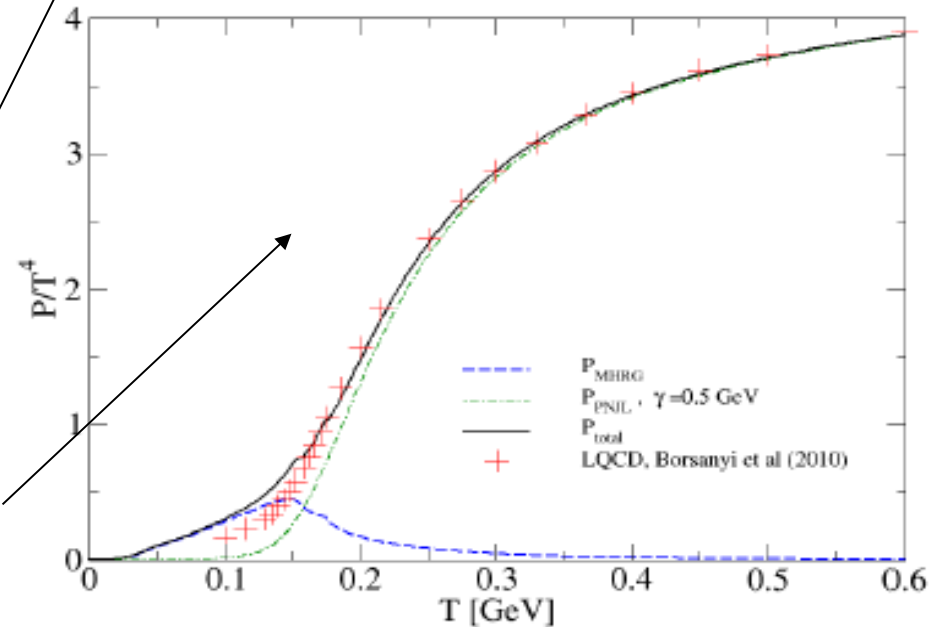
$$\delta_q(\omega; \gamma) = \frac{\pi}{2} + \arctan \left[ \frac{\omega - \sqrt{p^2 + m_q^2}}{\gamma} \right]$$



# 3. Example C: Mott HRG – improved toy model

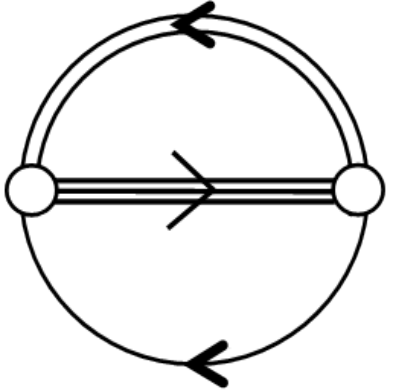


- Mott dissociation of hadrons (here pi, K) at the Chiral restoration temperature  $T_c = 153$  MeV
- Asymptotic behaviour of scaled quark-gluon Pressure can be adjusted with rescattering Parameter gamma
- Very good correspondence between lattice QCD Thermodynamics and improved MHRG model; Hadronic and partonic contributions quantified

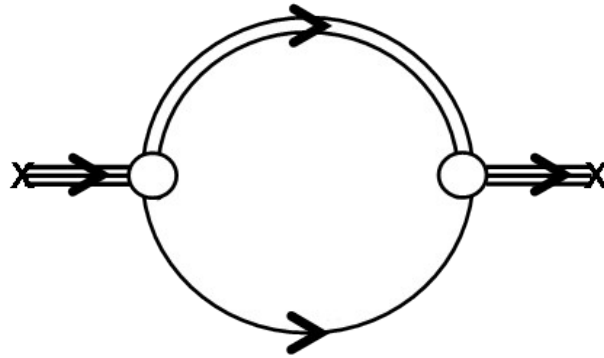


# 3. Example D: Nucleons in quark matter

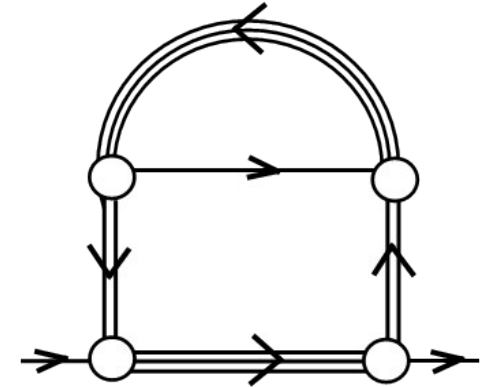
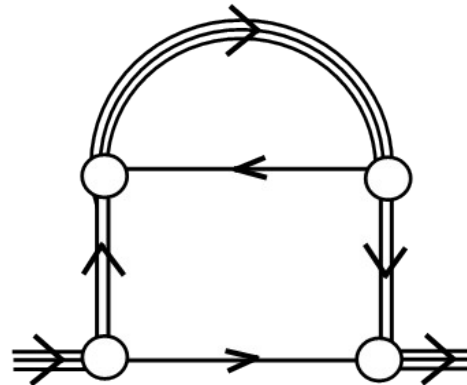
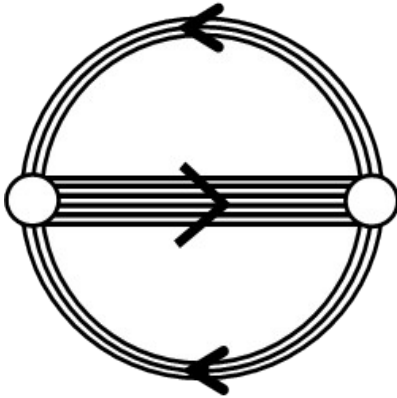
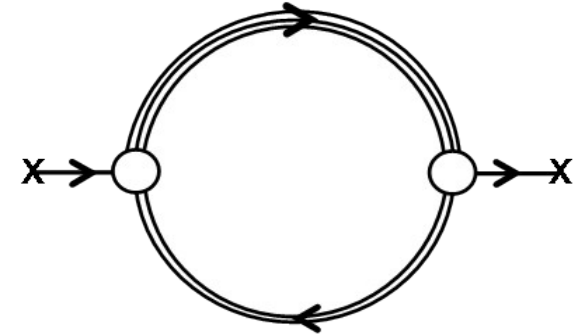
$\Phi$ -functional



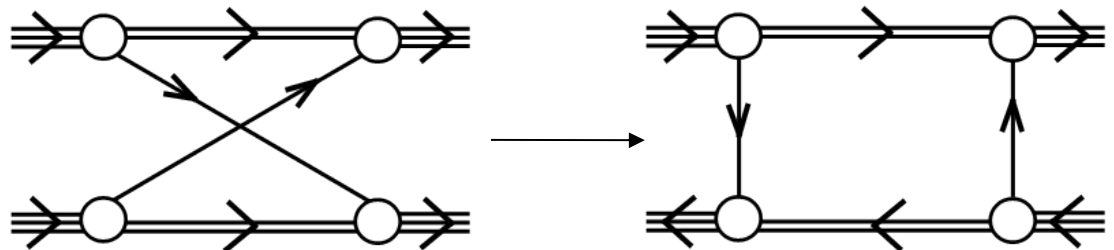
nucleon selfenergy



quark selfenergy

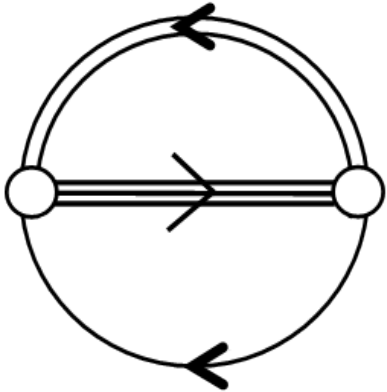


quark exchange interaction between nucleons:

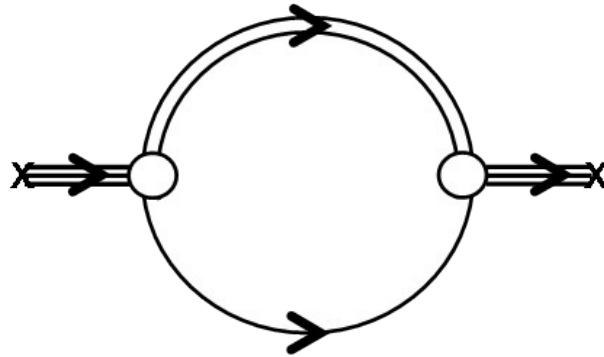


# 5. Example D: Nucleons in quark matter

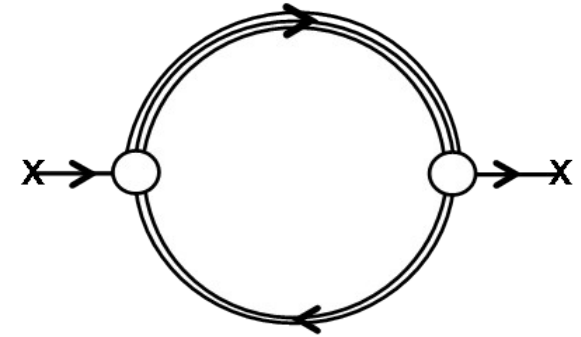
$\Phi$ -functional



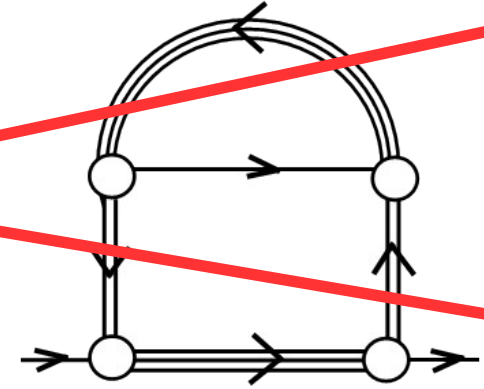
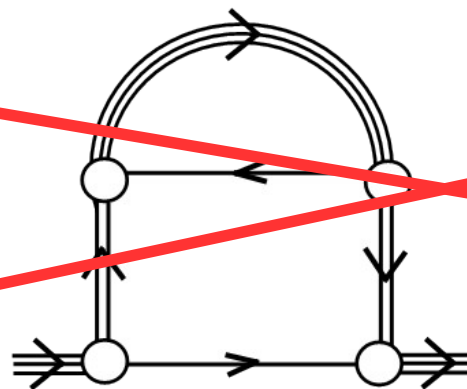
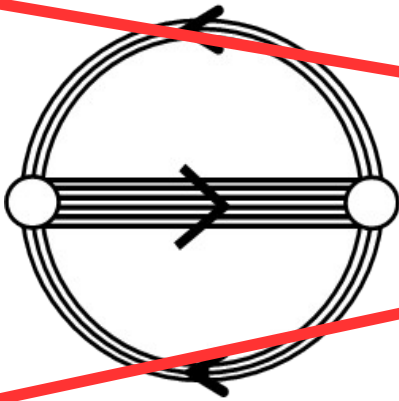
nucleon selfenergy



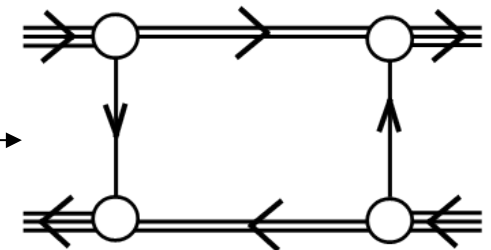
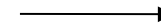
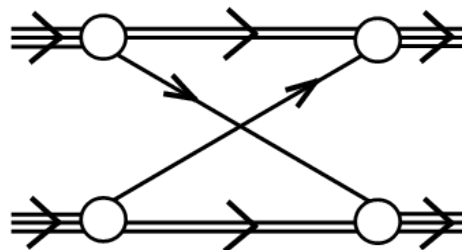
quark selfenergy



Not new! Already contained in above diagrams!

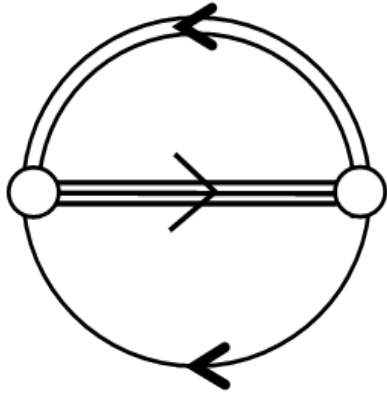


quark exchange interaction  
between nucleons:

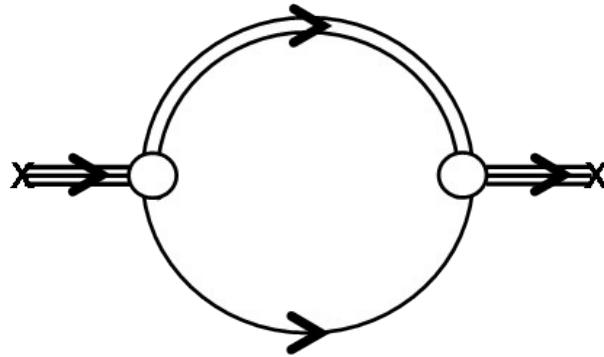


# 5. Example D: Nucleons in quark matter

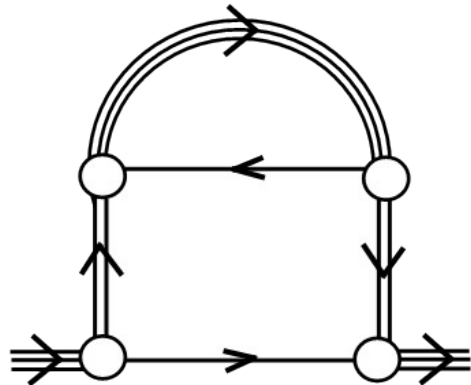
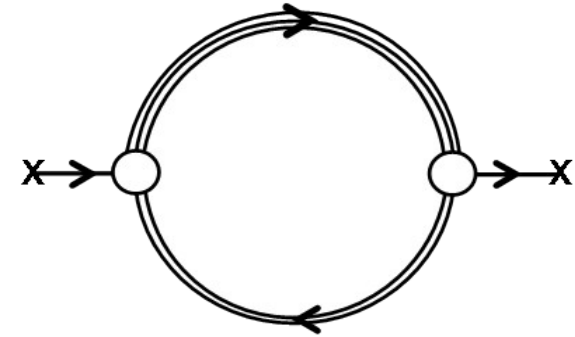
$\Phi$ -functional



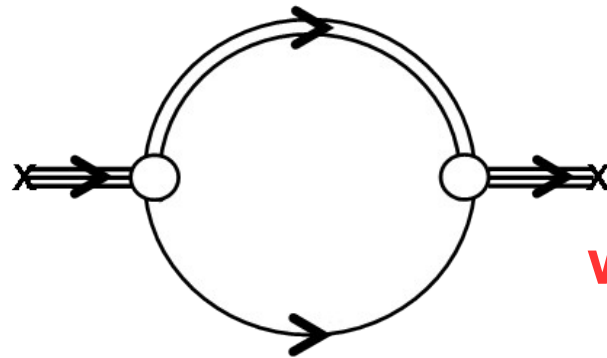
nucleon selfenergy



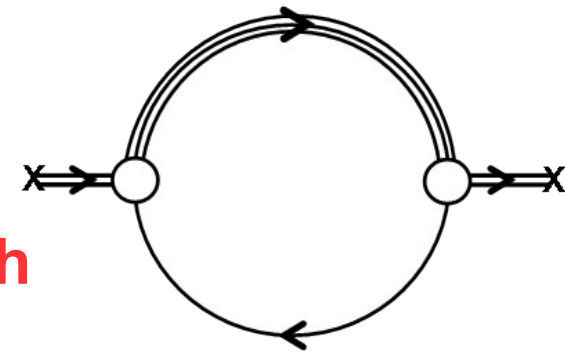
quark selfenergy



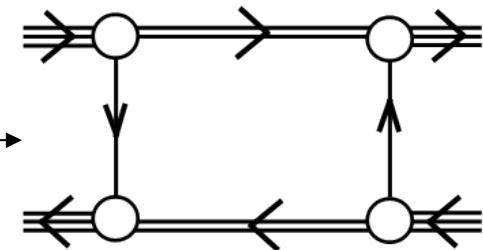
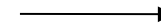
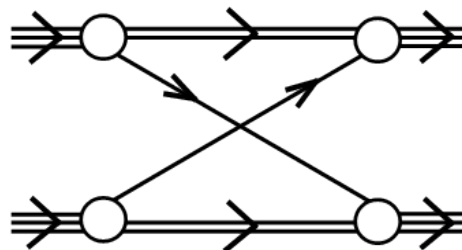
=



with



quark exchange interaction  
between nucleons:



# Intermezzo: Structure of the baryon?



12-Apostle  
Church,  
Kars

# Intermezzo: Structure of the baryon?



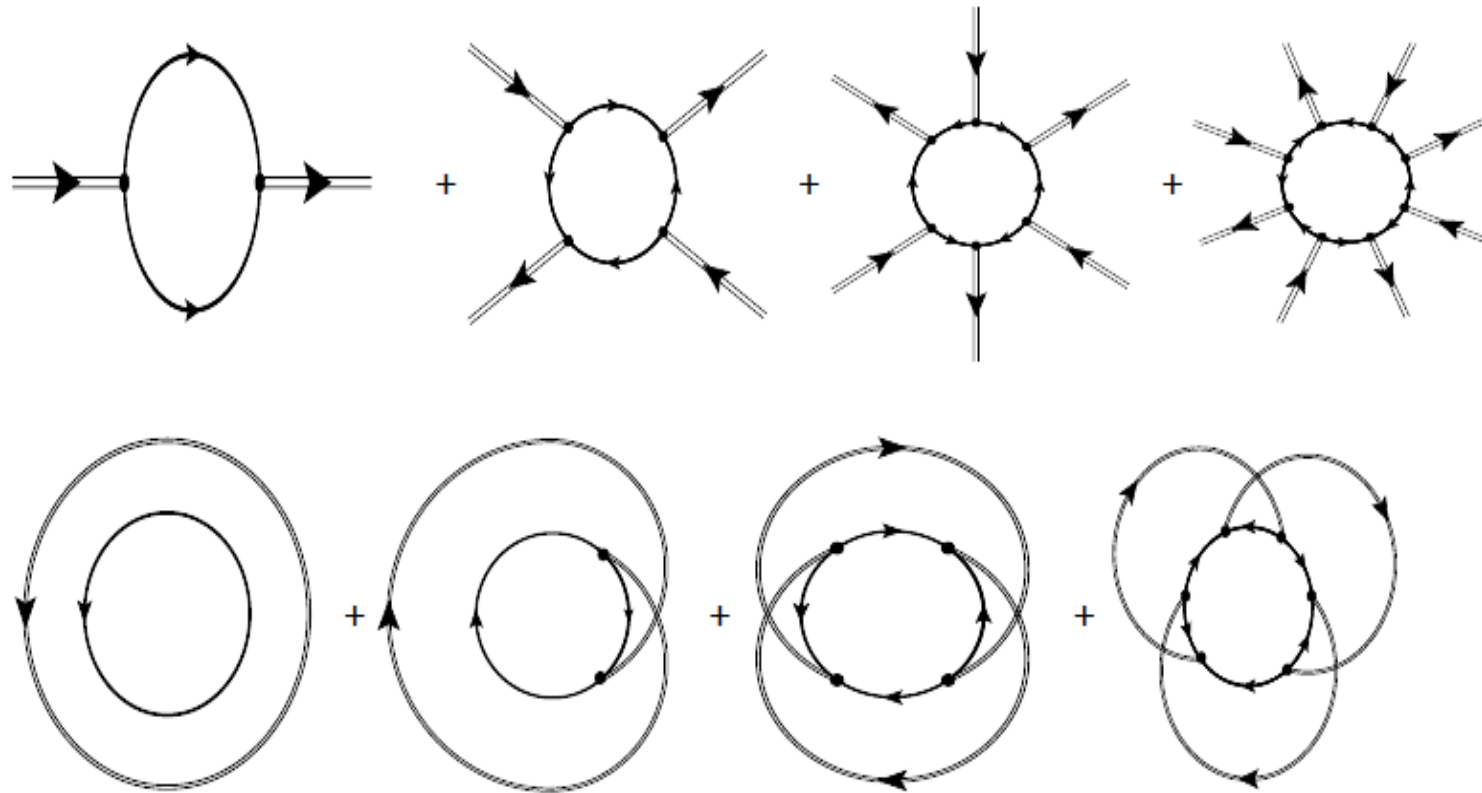
12-Apostle  
Church,  
Kars

# Intermezzo: Structure of the baryon?

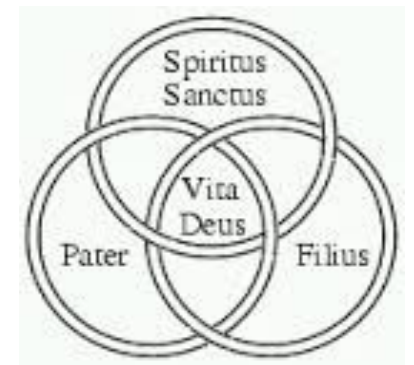
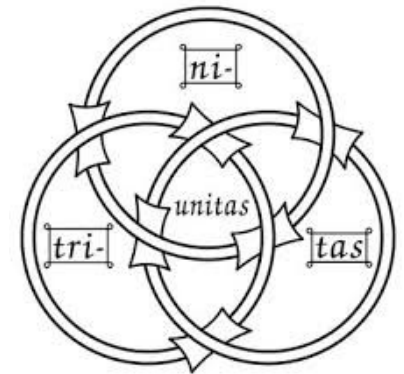
$$Z_{\text{fluct}} = \int D\Delta^\dagger D\Delta D\phi \exp\left\{-\frac{|\Delta|^2}{4G_D} - \frac{\phi^2}{4G} - \text{Tr} \ln S^{-1}[\Delta, \Delta^\dagger, \phi]\right\}$$

Cahill, Roberts, Prashifka: *Aust. J. Phys.* 42 (1989) 129, 161

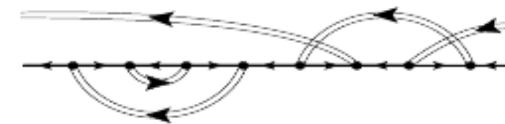
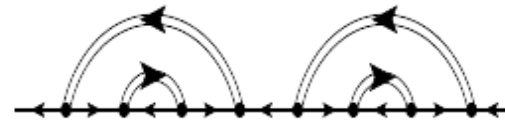
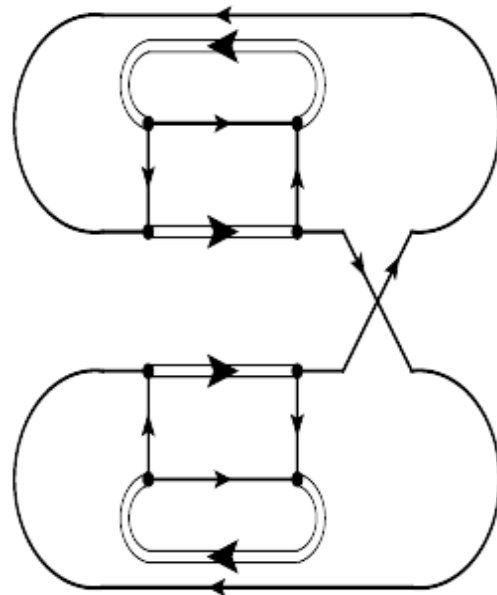
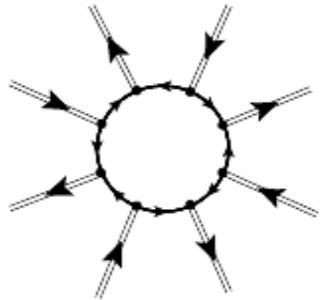
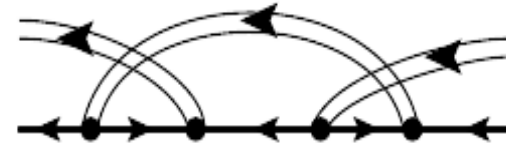
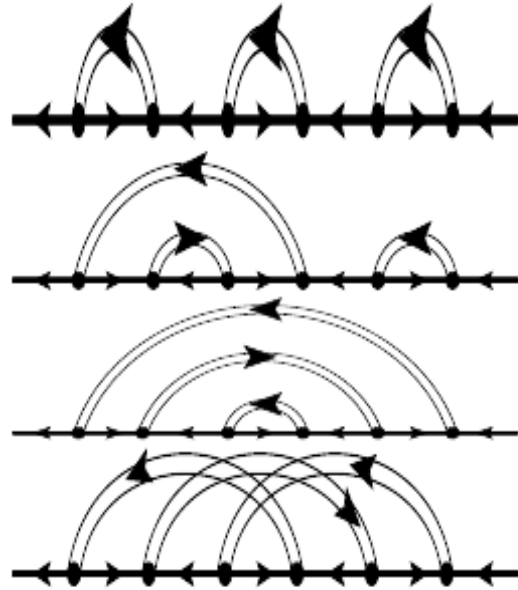
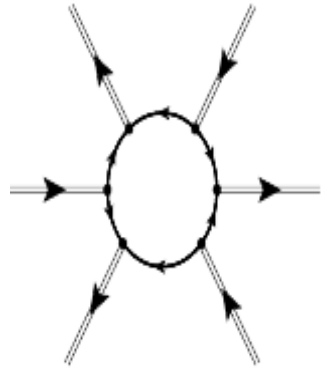
Cahill, *ibid*, 171; Reinhardt: *PLB* 244 (1990) 316; Buck, Alkofer, Reinhardt: *PLB* 286 (1992) 29



Borromean ? !!

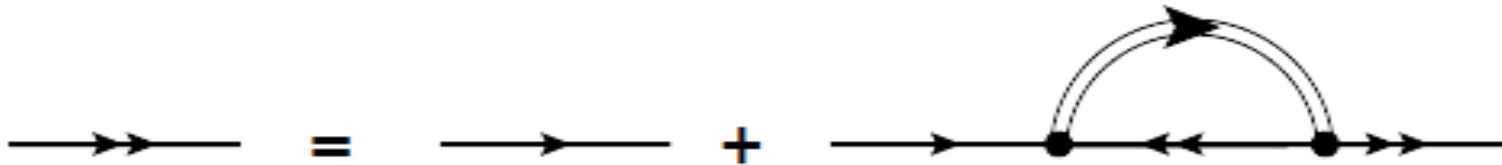
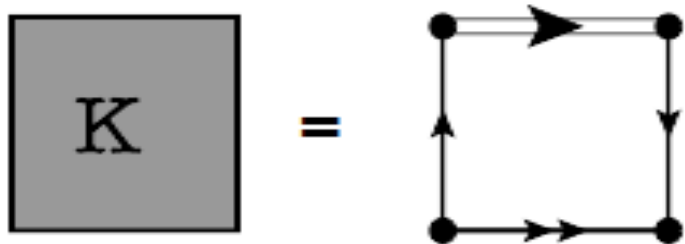
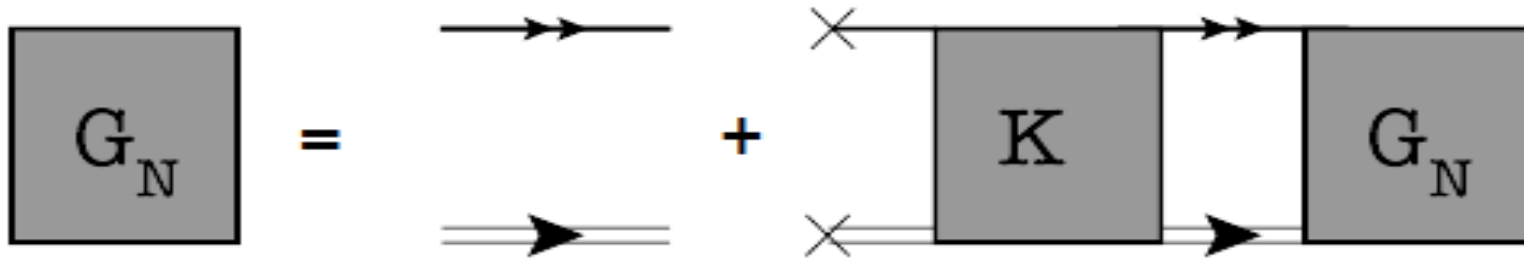


# Intermezzo: Structure of the baryon?





# Intermezzo: Structure of the baryon?



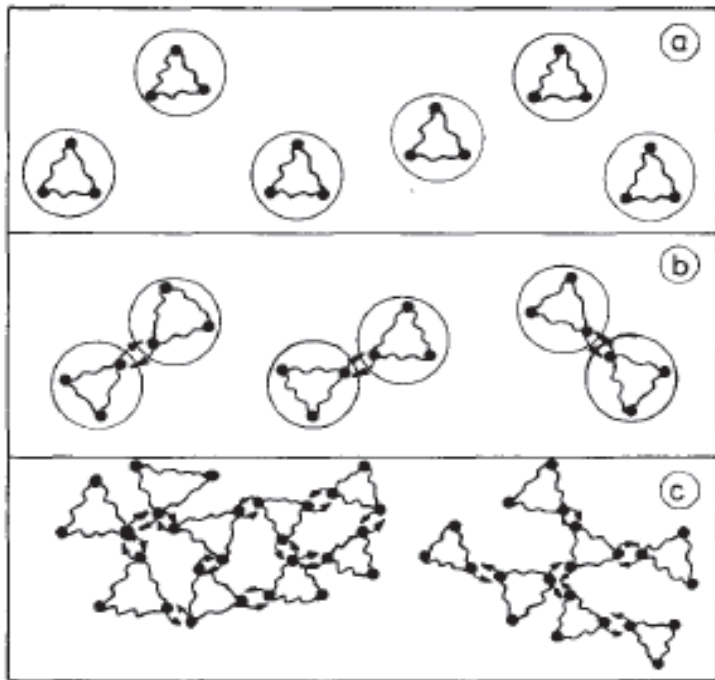
Generalized Bethe-Salpeter equation for the baryon

Dyson- Schwinger equation for the quarks ... important:

Bethe-Salpeter (RPA) equation for the diquark propagator must be **consistent** !

==> see the Phi- derivable approach !

# 4. Example C: Pauli blocking among baryons

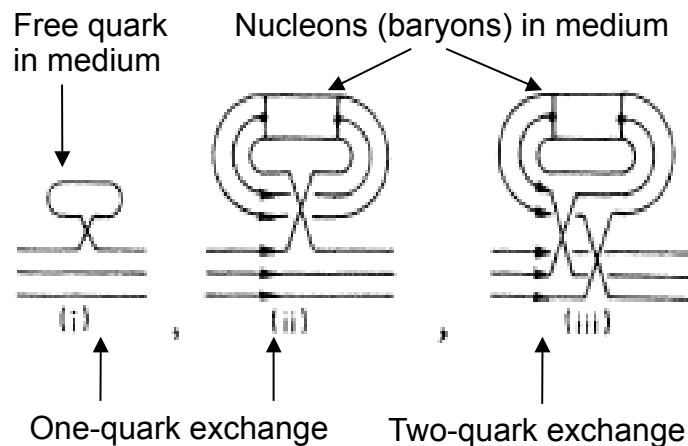


a) Low density: Fermi gas of nucleons (baryons)

b) ~ saturation: Quark exchange interaction and Pauli blocking among nucleons (baryons)

c) high density: Quark cluster matter (string-flip model ...)

Roepke & Schulz, Z. Phys. C 35, 379 (1987); Roepke, DB, Schulz, PRD 34, 3499 (1986)



Nucleon (baryon) self-energy --> Energy shift

$$\Delta E_{\nu P}^{\text{Pauli}} = \sum_{123} |\psi_{\nu P}(123)|^2 [E(1) + E(2) + E(3) - E_{\nu P}^0] [f_{\alpha_1}(1) + f_{\alpha_2}(2) + f_{\alpha_3}(3)]$$

$$+ \sum_{123} \sum_{456} \sum_{\nu P'} \psi_{\nu P}^*(123) \psi_{\nu P'}(456) f_3(E_{\nu P'}^0) \{ \delta_{36} \psi_{\nu P}(123) \psi_{\nu P'}^*(456) - \psi_{\nu P}(453) \psi_{\nu P'}^*(126) \}$$

$$\times [E(1) + E(2) + E(3) + E(4) + E(5) + E(6) - E_{\nu P}^0 - E_{\nu P'}^0]$$

$$= \Delta E_{\nu P}^{\text{Pauli, free}} + \Delta E_{\nu P}^{\text{Pauli, bound}}$$



PHYSICAL REVIEW D

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## Pauli quenching effects in a simple string model of quark/nuclear matter

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(Received 16 December 1985)

# 4. Example C: Pauli blocking in NM - details

$$\Sigma_\nu(p, p_{Fn}, p_{Fp}) = \sum_{\nu'=\{n,p\}} \sum_{\alpha=1,2} C_{\nu\nu'}^{(\alpha)} W_\alpha(p_{F\nu'}, p)$$

$$W_\alpha(p_{F\nu'}, p) = \frac{\Omega}{2\pi^2} \int_0^{p_{F\nu'}} p'^2 \bar{V}^{(\alpha)}(p, p') dp';$$

$$\bar{V}^{(\alpha)}(p, p') = \frac{1}{2} \int_{-1}^1 V^{(\alpha)}(\vec{p}, \vec{p}') dz;$$

$$V^{(\alpha)}(\vec{p}, \vec{p}') = \frac{V_0 b}{\Omega m} \left( \frac{15}{2} - \lambda_\alpha^2 (\vec{p} - \vec{p}')^2 \right) \exp(-\lambda_\alpha^2 (\vec{p} - \vec{p}')^2).$$

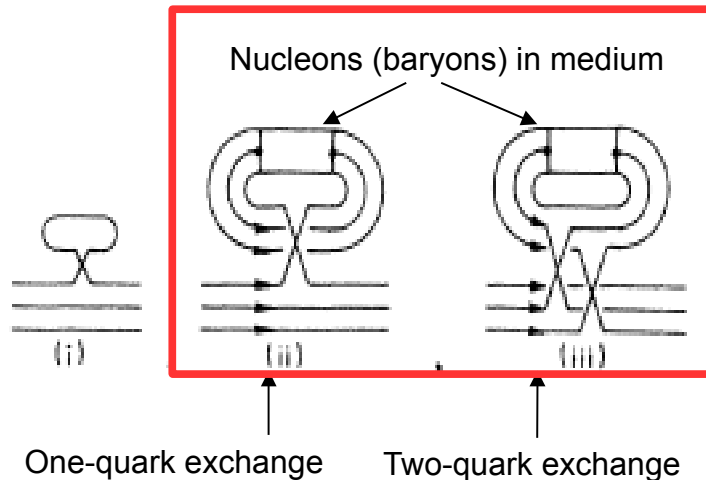
	$C_{n\nu}^{(1)}$	$C_{n\nu}^{(2)}$
neutron	$-\frac{96}{243}$	$\frac{10}{27}$
proton	$-\frac{28}{81}$	$\frac{8}{27}$

$$b^{-2} = \sqrt{3} m \omega$$

$$\omega = 178.425 \text{ MeV}$$

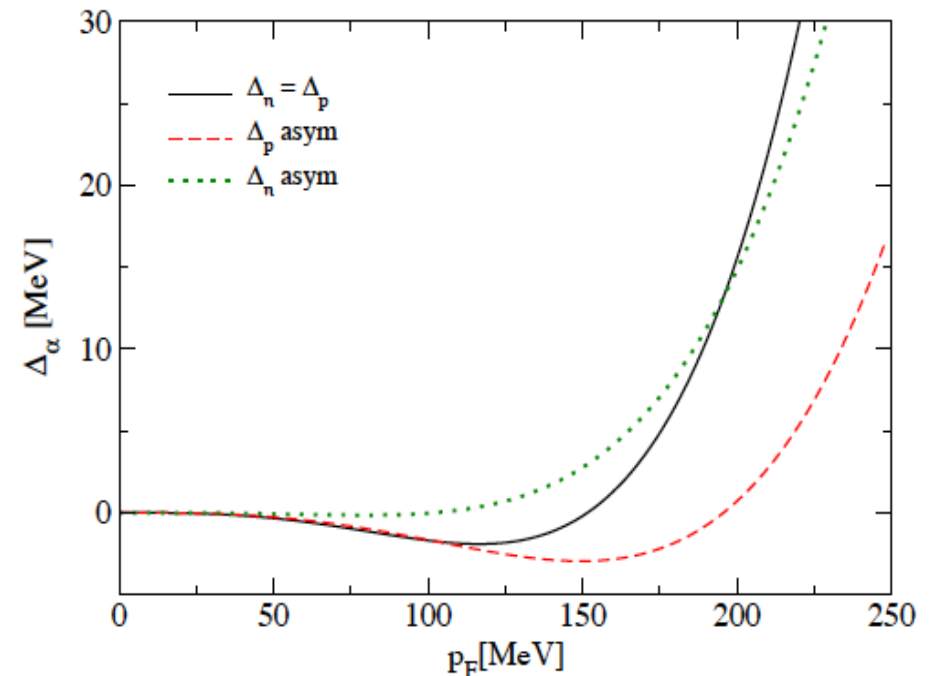
$$m = 350 \text{ MeV} \quad b = 0.6 \text{ fm}$$

$$V_0 = \frac{9\sqrt{3}\pi^{3/2}}{2} \text{ and } \lambda_\alpha = \frac{b}{\sqrt{3}\alpha}$$



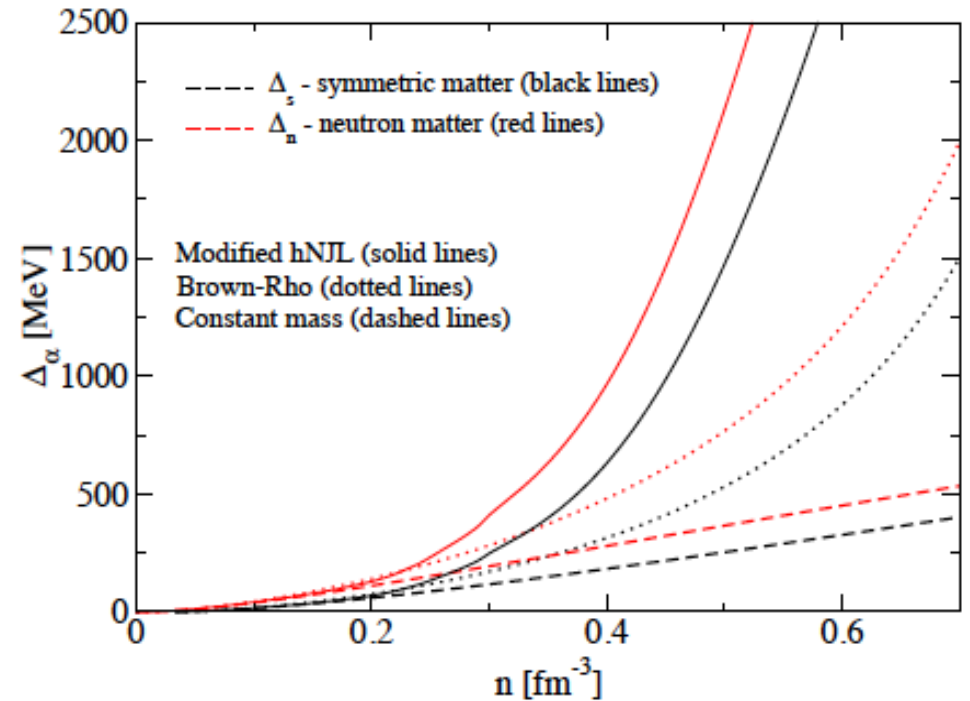
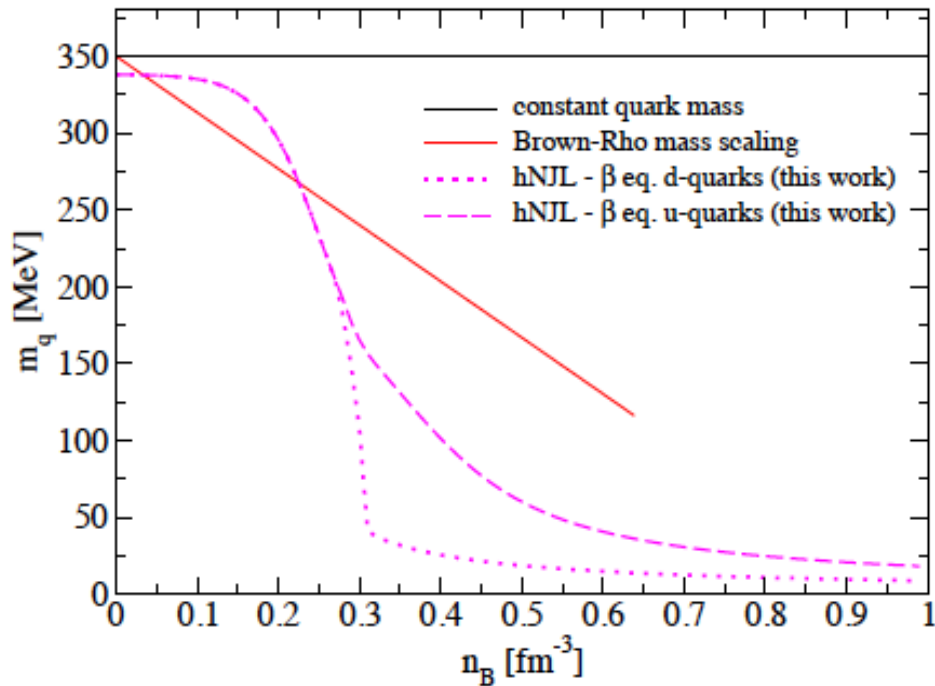
$$W_\alpha(p_{F\nu'}, p) = \frac{V_0 b}{32\pi^2 \lambda_\alpha^4 m} \left\{ 12\lambda_\alpha \sqrt{\pi} [\text{erf}(\lambda_\alpha(p_{F\nu'} - p)) + \text{erf}(\lambda_\alpha(p_{F\nu'} + p))] \right. \\ \left. + \frac{1}{p} [(11 - 2\lambda_\alpha^2 p_{F\nu'}(p_{F\nu'} + p)) e^{-\lambda_\alpha^2(p_{F\nu'} + p)^2} \right. \\ \left. + (11 - 2\lambda_\alpha^2 p_{F\nu'}(p_{F\nu'} - p)) e^{-\lambda_\alpha^2(p_{F\nu'} - p)^2} \right\}$$

$$\Delta_{\nu A, P}^{\text{Pauli}} = \frac{1}{24\sqrt{3}\pi} \frac{b}{m} \sum_{\nu'} [15a_{\nu\nu'} P_F(\nu')^3 + \frac{17}{12} b_{\nu\nu'} b^2 (P^2 + P_F(\nu')^2) P_F(\nu')^3]$$



## 4. Example C: Pauli blocking in NM – details

New aspect: chiral restoration --> dropping quark mass



Increased baryon swelling at supersaturation densities:  
--> dramatic enhancement of the Pauli repulsion !!

## 4. Example C: Pauli blocking in NM – results

**New EoS: Joining RMF (Linear Walecka) for pointlike baryons with chiral Pauli blocking**

$$p = \frac{1}{4\pi^2} \sum_{\nu} \left[ -E_{\nu}^* m_{\nu}^{*2} p_{F\nu} + \frac{2}{3} E_{\nu}^* p_{F\nu}^3 + m_{\nu}^{*4} \log \left( \frac{E_{\nu}^* + p_{F\nu}}{m_{\nu}^*} \right) \right]$$

$$+ \frac{1}{2} \left( \frac{g_{\omega}}{m_{\omega}} \right)^2 n^2 - \frac{1}{2} \left( \frac{g_{\sigma}}{m_{\sigma}} \right)^2 n_s^2 + p_{ex};$$

$$\epsilon = \frac{1}{4\pi^2} \sum_{\nu} \left[ 2 E_{\nu}^{*3} p_{F\nu} - E_{\nu}^* m_{\nu}^{*2} p_{F\nu} - m_{\nu}^{*4} \log \left( \frac{E_{\nu}^* + p_{F\nu}}{m_{\nu}^*} \right) \right]$$

$$+ \frac{1}{2} \left( \frac{g_{\omega}}{m_{\omega}} \right)^2 n^2 + \frac{1}{2} \left( \frac{g_{\sigma}}{m_{\sigma}} \right)^2 n_s^2 + \epsilon_{ex},$$

$$\mu_{ex,\nu} = \Delta_{\nu}(n, x) = \Sigma_{\nu}(p_{F,\nu}; p_{Fn}, p_{Fp}),$$

$$\epsilon_{ex} = \sum_{\nu} \int_0^n dn' \{ x \Delta_p(n', x) + (1-x) \Delta_n(n', x) \},$$

$$p_{ex} = \sum_{\nu} \mu_{ex,\nu} n_{\nu} - \epsilon_{ex},$$

$$n_{s,\nu} = \frac{m_{\nu}^*}{\pi^2} \left[ E_{\nu}^* p_{F\nu} - m_{\nu}^{*2} \log \left( \frac{E_{\nu}^* + p_{F\nu}}{m_{\nu}^*} \right) \right],$$

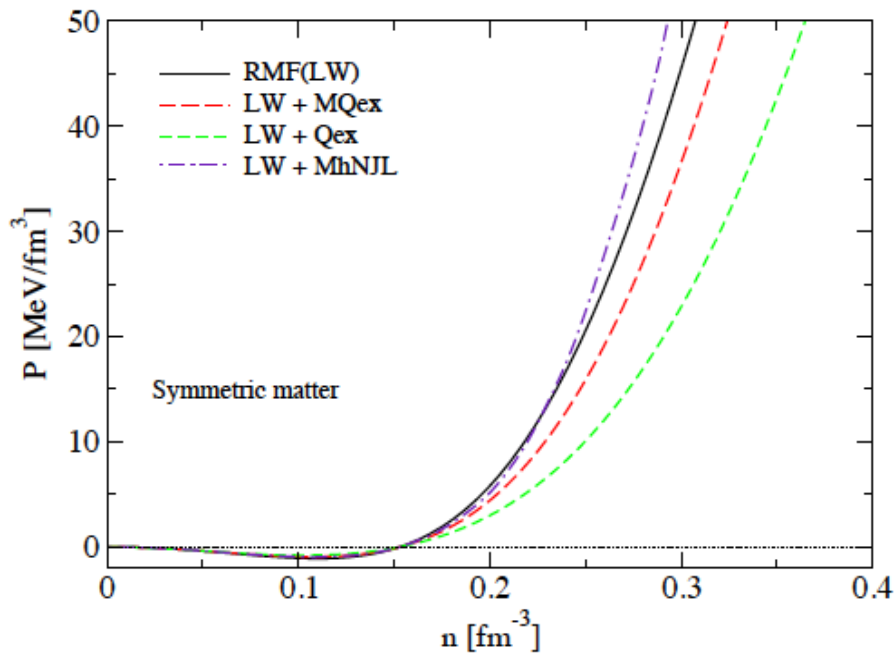
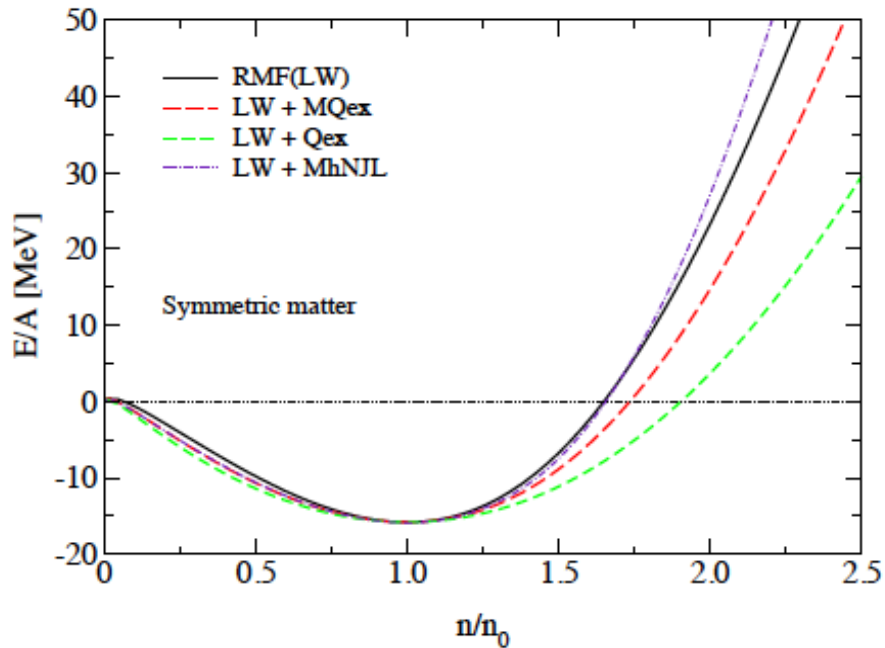
$$E_{\nu}^* = \sqrt{m_{\nu}^{*2} + p_{F\nu}^2}$$

$$n_{\nu} = \frac{p_{F\nu}^3}{3\pi^2},$$

$$m_{\nu}^* = m_{\nu} - \left( \frac{g_{\sigma}}{m_{\sigma}} \right)^2 n_{s,\nu},$$

$$\mu_{\nu} = E_{\nu}^* + \left( \frac{g_{\omega}}{m_{\omega}} \right)^2 n_{\nu} + \mu_{ex,\nu}.$$

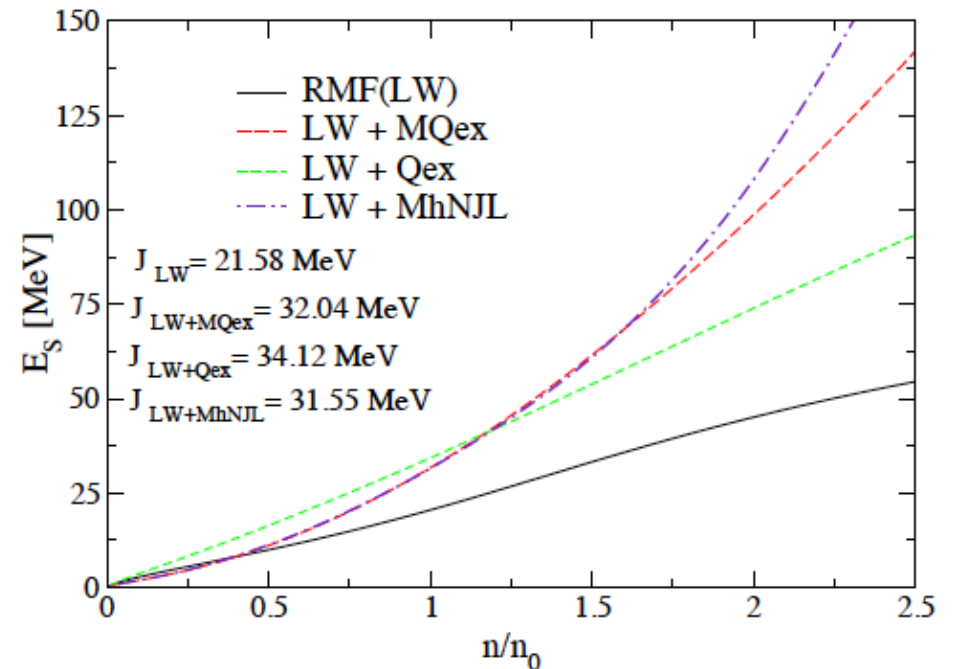
# 4. Example C: Pauli blocking in NM – results



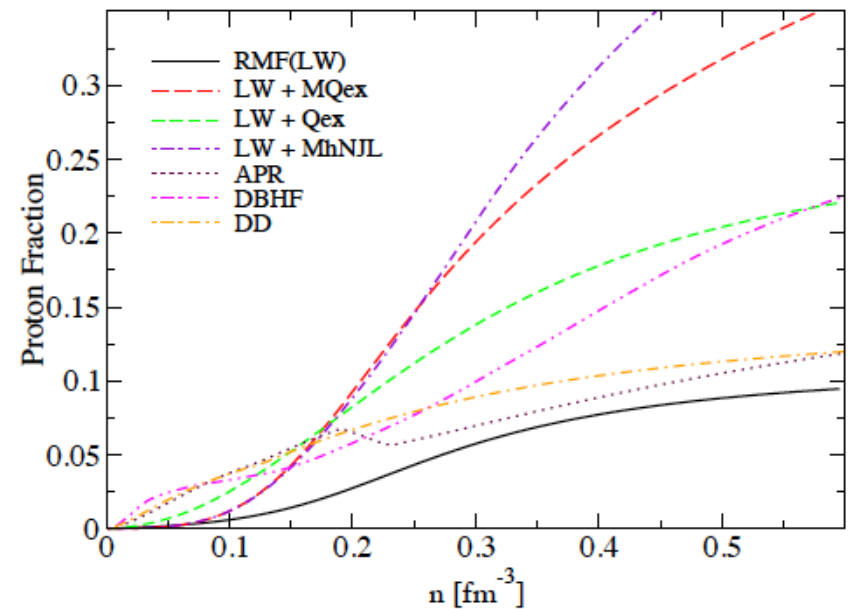
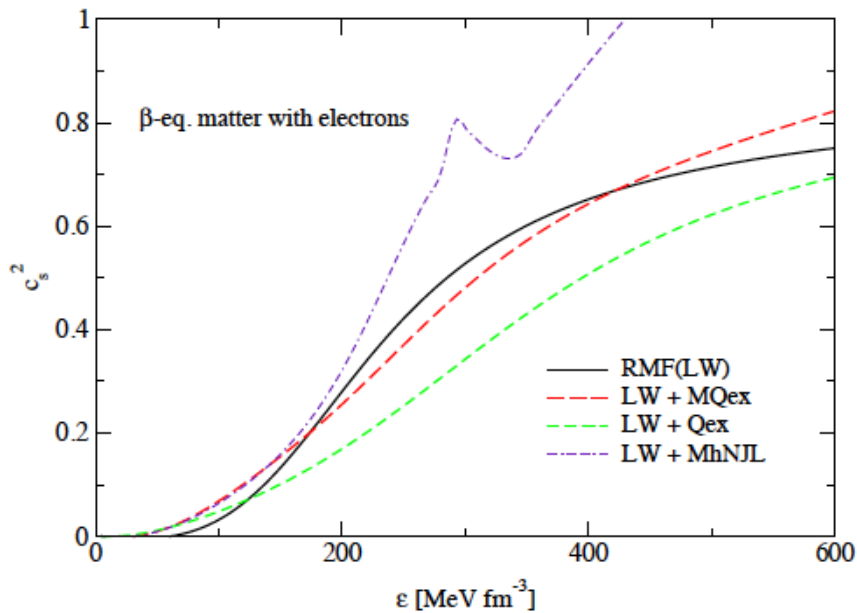
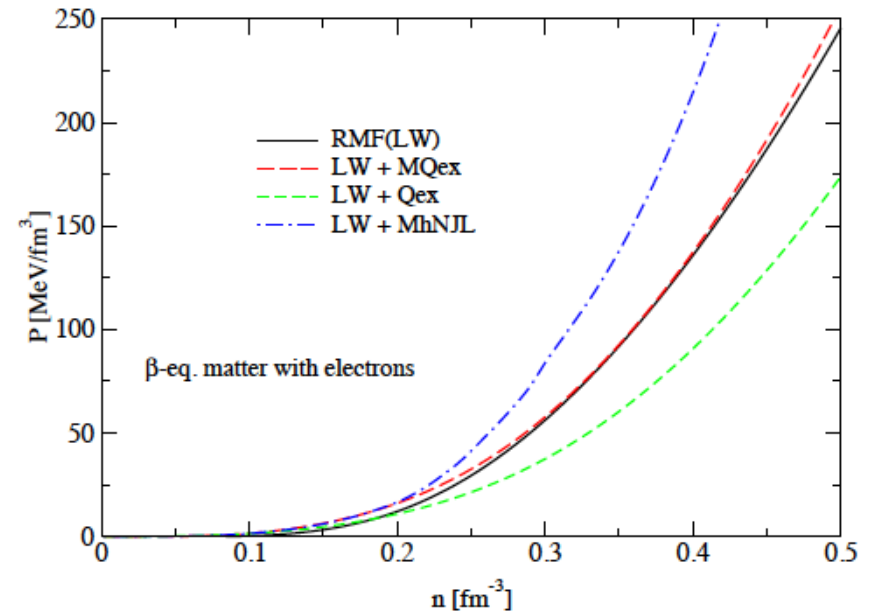
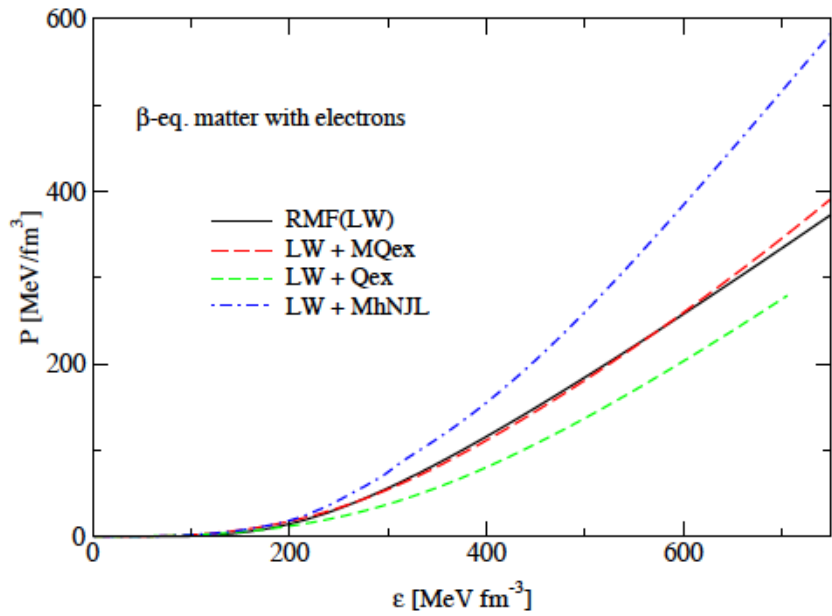
Parametrization: from saturation properties

	$(g_\omega/m_\omega)^2$ [fm <sup>2</sup> ]	$(g_\sigma/m_\sigma)^2$ [fm <sup>2</sup> ]
RMF (LW)	11.6582	15.2883
LW+Qex	6.11035	9.91197
LW+MQex	6.59170	13.29118
LW+MhNJL	9.25683	13.9474

Prediction: symmetry energy

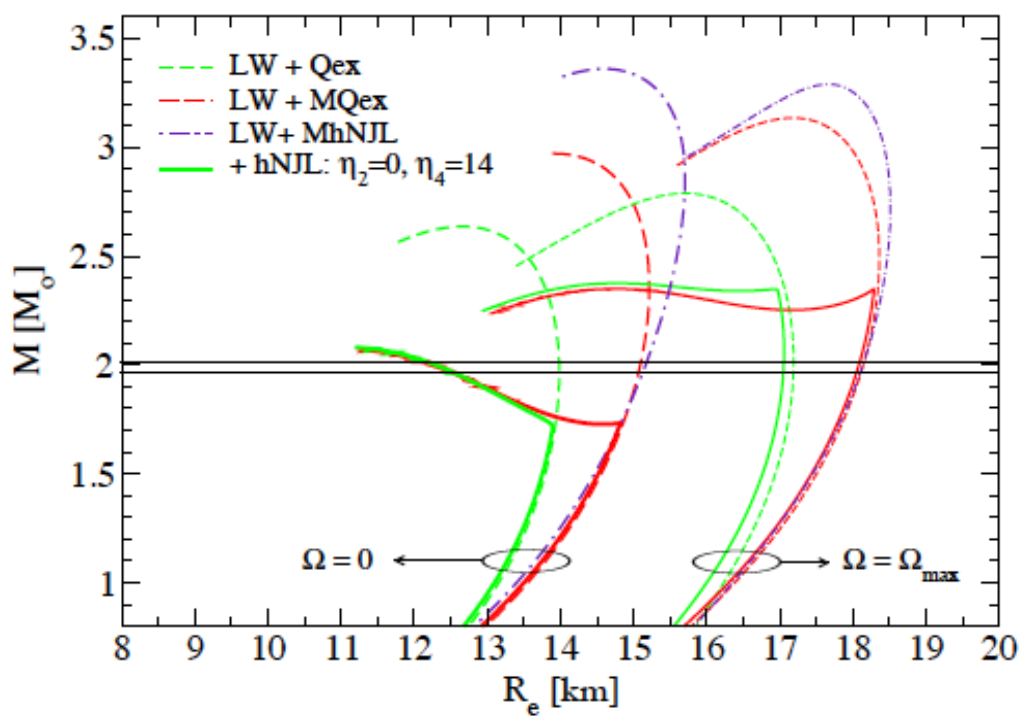
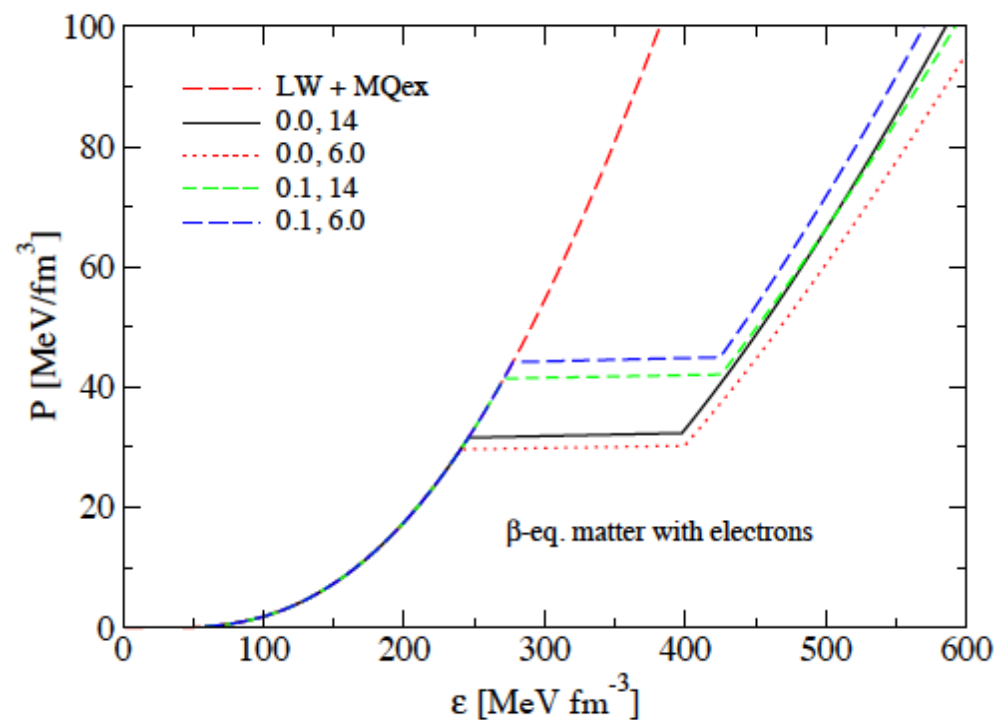
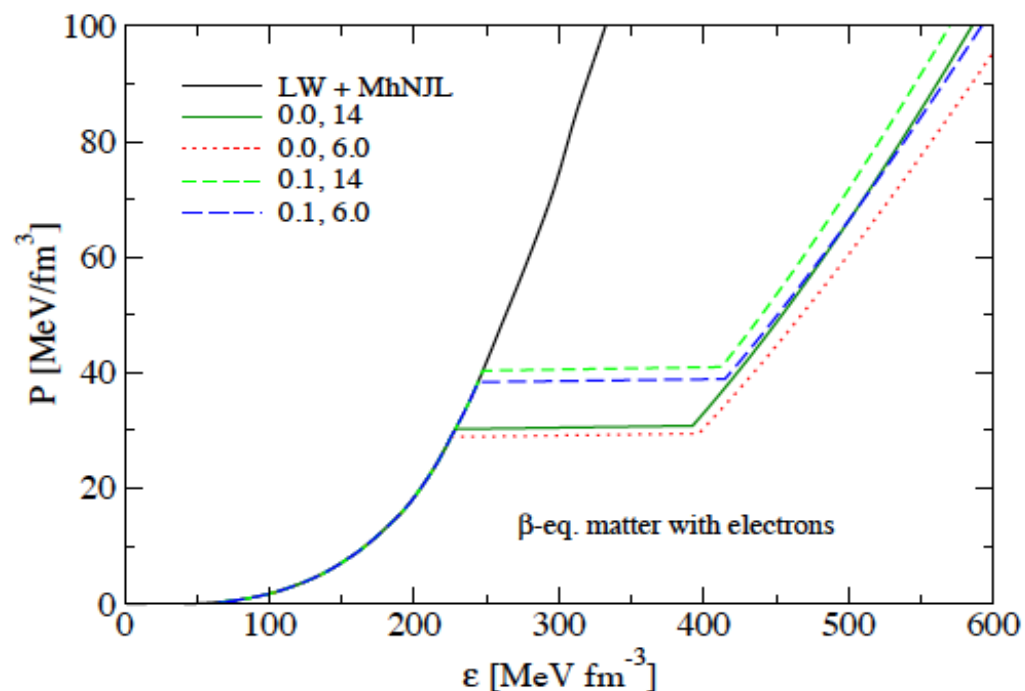
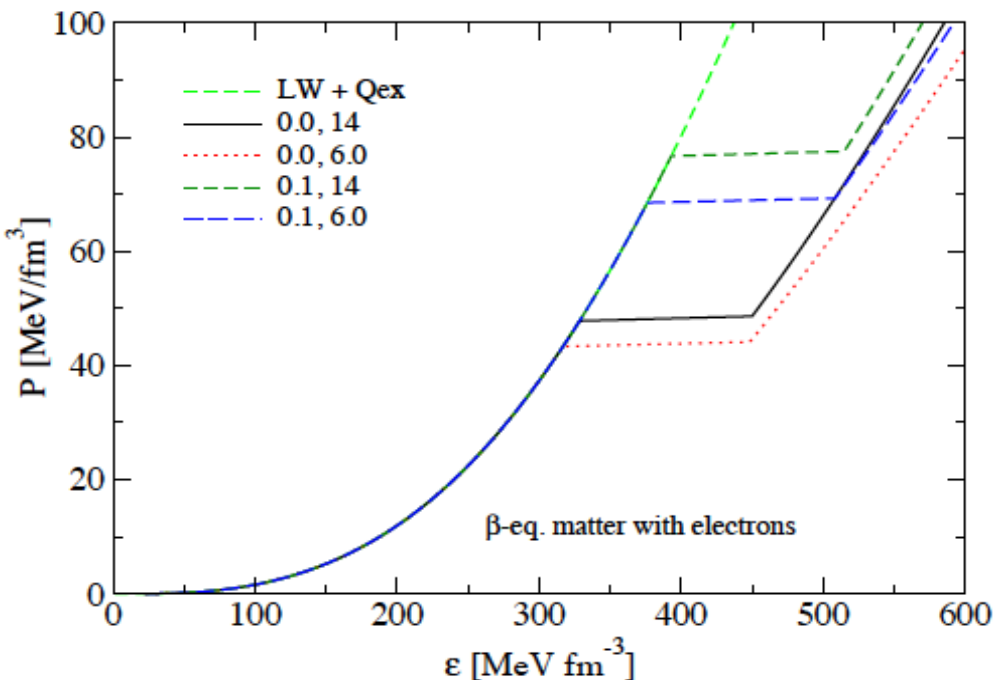


# 4. Example C: Pauli blocking in NM – results





# 4. Example C: Pauli blocking in NM – results



# 4. Example C: Pauli blocking in NM – Summary

Pauli blocking selfenergy (cluster meanfield) calculable in potential models for baryon structure

Partial replacement of other short-range repulsion mechanisms (vector meson exchange)

Modern aspects:

- onset of chiral symmetry restoration enhances nucleon swelling and Pauli blocking at high  $n$
- quark exchange among baryons  $\rightarrow$  six-quark wavefunction  $\rightarrow$  “bag melting”  $\rightarrow$  deconfinement

Chiral stiffening of nuclear matter  $\rightarrow$  reduces onset density for deconfinement

Hybrid EoS:

Convenient generalization of RMF models,

Take care: eventually aspects of quark exchange already in density dependent vertices!

Other baryons:

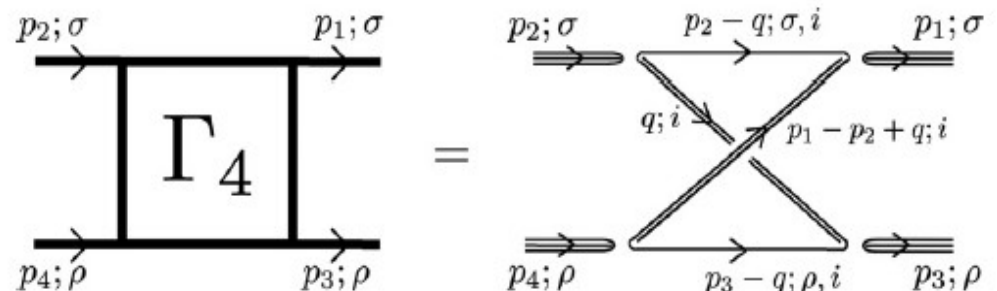
- hyperons
- deltas

Again calculable, partially done in nonrelativistic quark exchange models, chiral effects not yet!

Relativistic generalization:

Box diagrams of quark-diquark model ...

K. Maeda, Ann. Phys. 326 (2011) 1032



# 4a. Pauli blocking effect → Excluded volume

Well known from modeling dissociation of clusters in the supernova EoS:

- excluded volume: Lattimer-Swesty (1991), Shen-Toki-Oyamatsu-Sumiyoshi (1996), ...
- Pauli blocking: Roepke-Grigo-Sumiyoshi-Shen (2003), Typel et al. PRC 81 (2010)
- excl. Vol. vs. Pauli blocking: Hempel, Schaffner-Bielich, Typel, Roepke PRC 84 (2011)

**Here:** nucleons as quark clusters with finite size --> excluded volume effect !

Available volume fraction:  $\Phi = V_{av}/V = 1 - v \sum_{i=n,p} n_i$ ,  $v = \frac{1}{2} \frac{4\pi}{3} (2r_{nuc})^3 = 4V_{nuc}$

Equations of state for T=0 nuclear matter:  $p_{tot}(\mu_n, \mu_p) = \frac{1}{\Phi} \sum_{i=n,p} p_i + p_{mes}$ ,

$$\mathcal{E}_{tot}(\mu_n, \mu_p) = -p_{tot} + \sum_{i=n,p} \mu_i n_i$$

$$p_i = \frac{1}{4} (E_i n_i - m_i^* n_i^{(s)}),$$

$$n_i = \frac{\Phi}{3\pi^3} k_i^3,$$

$$n_i^{(s)} = \frac{\Phi m_i^*}{2\pi^2} \left[ E_i k_i - (m_i^*)^2 \ln \frac{k_i + E_i}{m_i^*} \right],$$

$$E_i = \sqrt{k_i^2 + (m_i^*)^2} = \mu_i - V_i - \frac{v}{\Phi} \sum_{j=p,n} p_j,$$

Effective mass:  $m_i^* = m_i - S_i$ .

Scalar meanfield:  $S_i \sim n_i^{(s)}$

Vector meanfield:  $V_i \sim n_i$

## 4b. Stiff quark matter at high densities

S. Benic, Eur. Phys. J. A 50, 111 (2014)

$$\mathcal{L} = \bar{q}(i\not{\partial} - m)q + \mu_q \bar{q}\gamma^0 q + \mathcal{L}_4 + \mathcal{L}_8, \quad \mathcal{L}_4 = \frac{g_{20}}{\Lambda^2} [(\bar{q}q)^2 + (\bar{q}i\gamma_5\tau q)^2] - \frac{g_{02}}{\Lambda^2} (\bar{q}\gamma_\mu q)^2,$$
$$\mathcal{L}_8 = \frac{g_{40}}{\Lambda^8} [(\bar{q}q)^2 + (\bar{q}i\gamma_5\tau q)^2]^2 - \frac{g_{04}}{\Lambda^8} (\bar{q}\gamma_\mu q)^4 - \frac{g_{22}}{\Lambda^8} (\bar{q}\gamma_\mu q)^2 [(\bar{q}q)^2 + (\bar{q}i\gamma_5\tau q)^2]$$

**Meanfield approximation:**  $\mathcal{L}_{\text{MF}} = \bar{q}(i\not{\partial} - M)q + \tilde{\mu}_q \bar{q}\gamma^0 q - U,$

$$M = m + 2\frac{g_{20}}{\Lambda^2} \langle \bar{q}q \rangle + 4\frac{g_{40}}{\Lambda^8} \langle \bar{q}q \rangle^3 - 2\frac{g_{22}}{\Lambda^8} \langle \bar{q}q \rangle \langle q^\dagger q \rangle^2,$$

$$\tilde{\mu}_q = \mu_q - 2\frac{g_{02}}{\Lambda^2} \langle q^\dagger q \rangle - 4\frac{g_{04}}{\Lambda^8} \langle q^\dagger q \rangle^3 - 2\frac{g_{22}}{\Lambda^8} \langle \bar{q}q \rangle^2 \langle q^\dagger q \rangle,$$

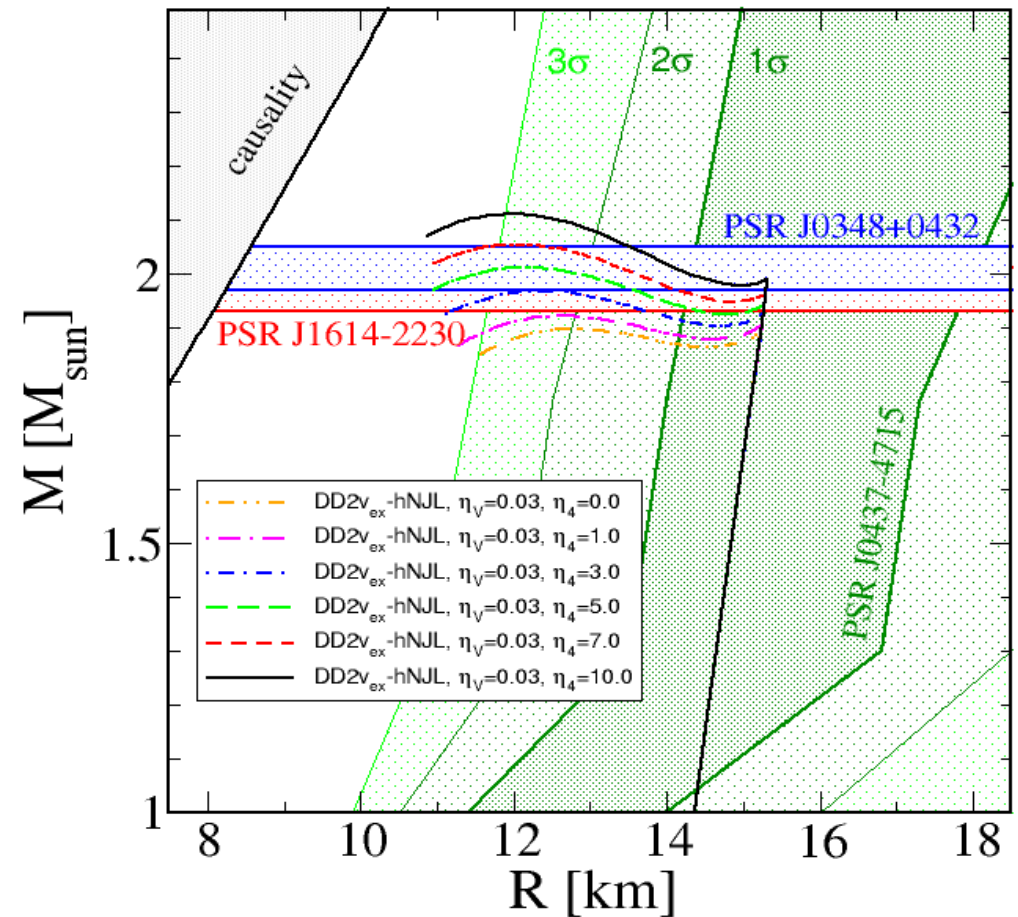
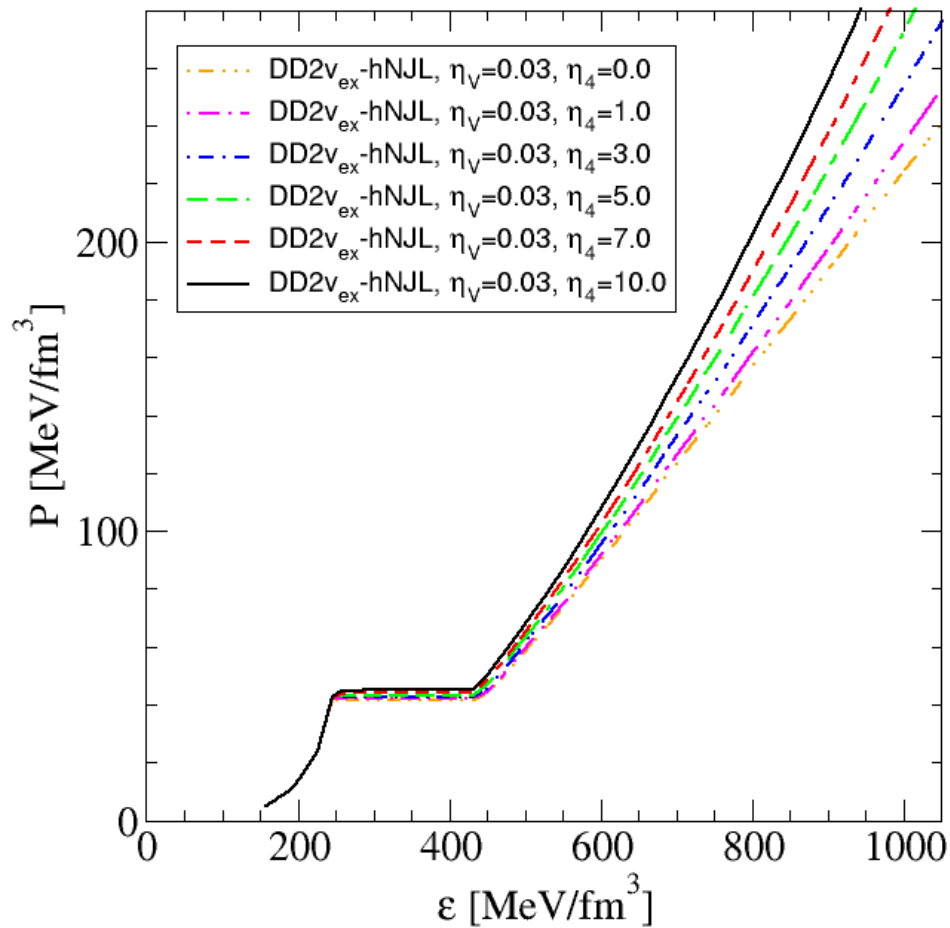
$$U = \frac{g_{20}}{\Lambda^2} \langle \bar{q}q \rangle^2 + 3\frac{g_{40}}{\Lambda^8} \langle \bar{q}q \rangle^4 - 3\frac{g_{22}}{\Lambda^8} \langle \bar{q}q \rangle^2 \langle q^\dagger q \rangle^2 - \frac{g_{02}}{\Lambda^2} \langle q^\dagger q \rangle^2 - 3\frac{g_{04}}{\Lambda^8} \langle q^\dagger q \rangle^4.$$

**Thermodynamic Potential:**

$$\Omega = U - 2N_f N_c \int \frac{d^3 p}{(2\pi)^3} \left\{ E + T \log[1 + e^{-\beta(E - \tilde{\mu}_q)}] + T \log[1 + e^{-\beta(E + \tilde{\mu}_q)}] \right\} + \Omega_0$$

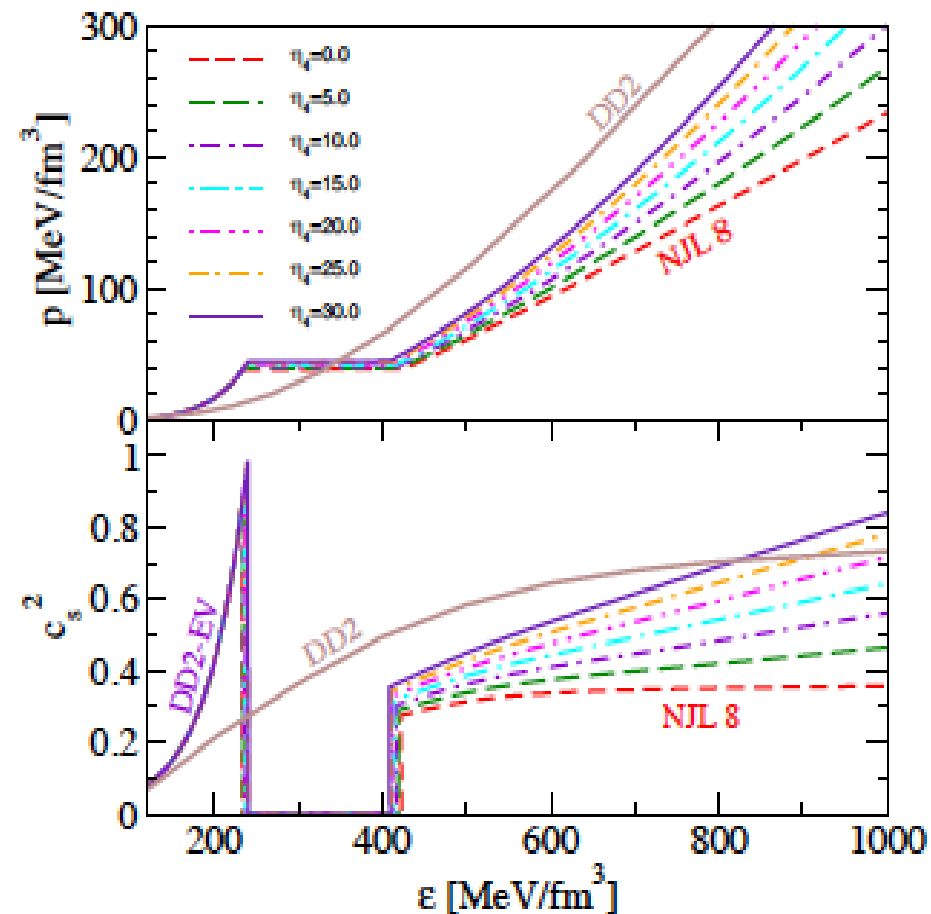
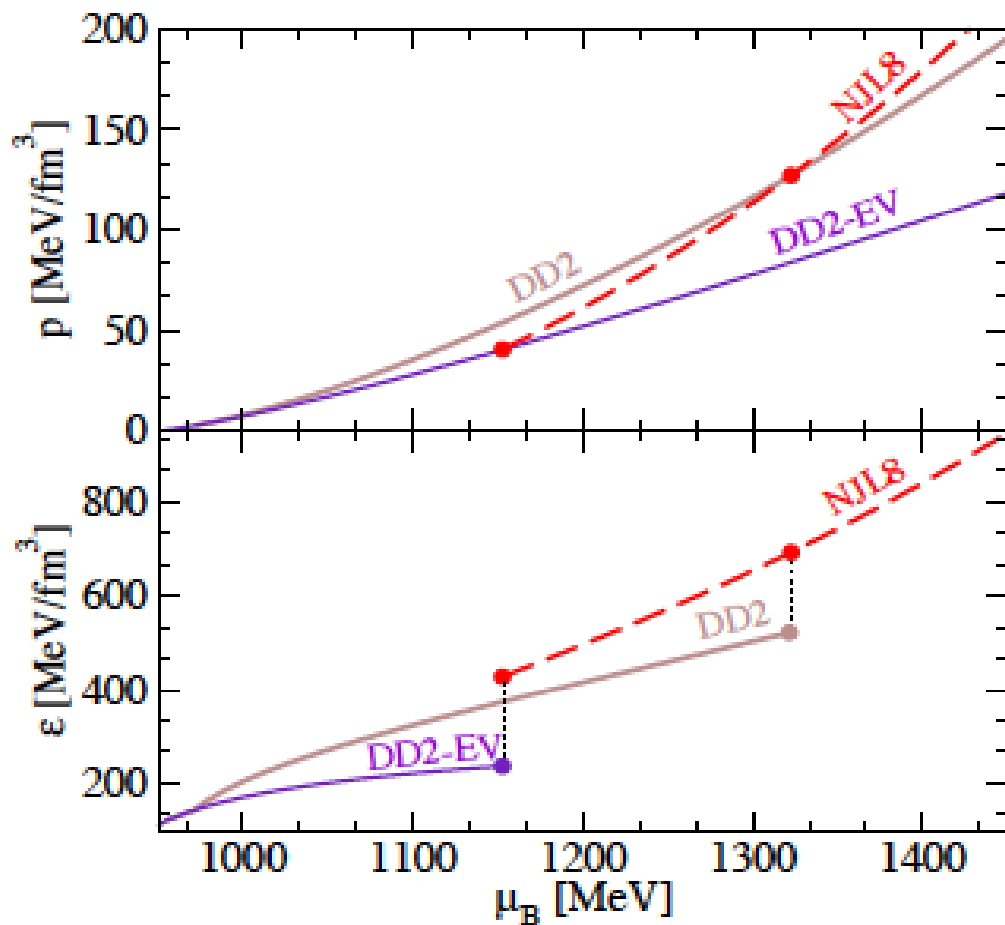
# Result: high-mass twins $\leftrightarrow$ 1st order PT

S. Benic, D. Blaschke, D. Alvarez-Castillo, T. Fischer, S. Typel, arxiv:1411.2856



Hybrid EoS supports  $M$ - $R$  sequences with high-mass twin compact stars

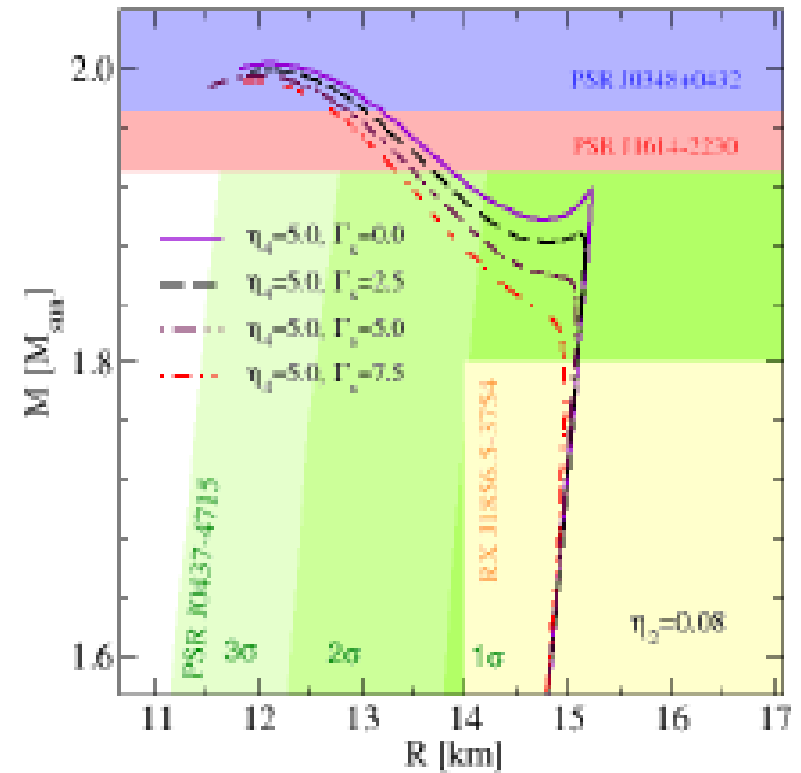
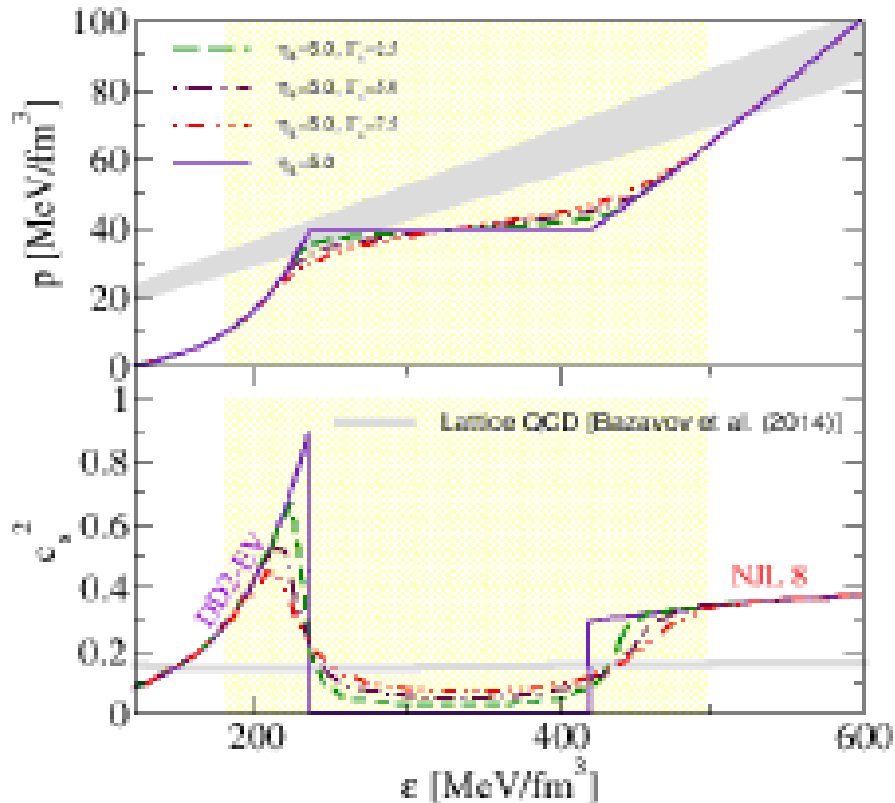
## 4b. Stiff quark matter at high densities



Here: Stiffening of dense hadronic matter by excluded volume in density-dependent RMF

## 4b. Stiff quark matter at high densities

Estimate effects of structures in the phase transition region (“pasta”)



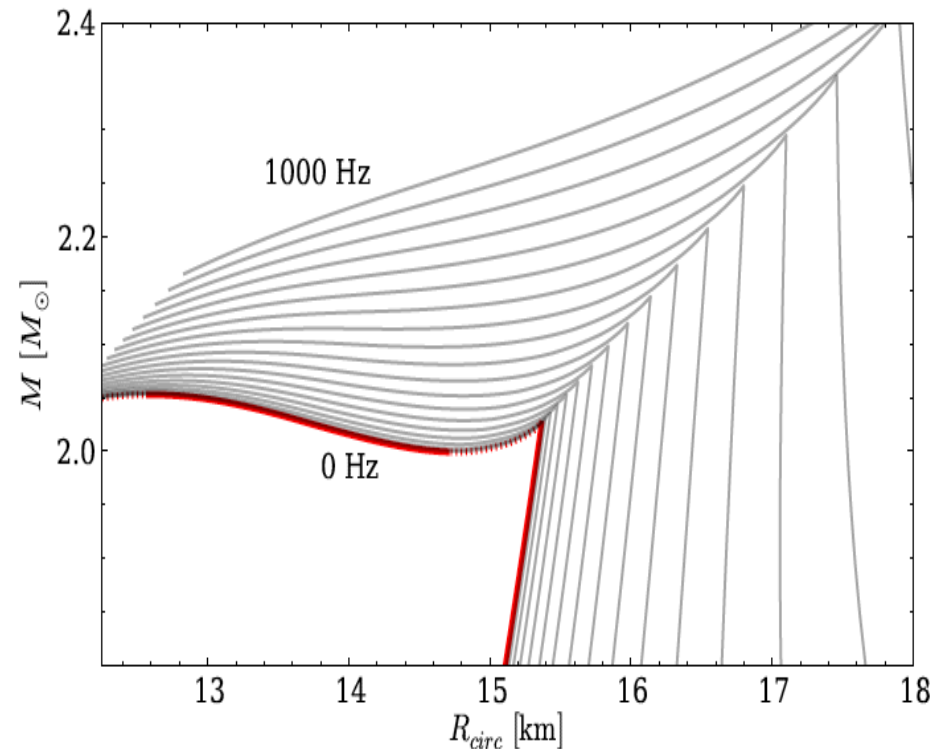
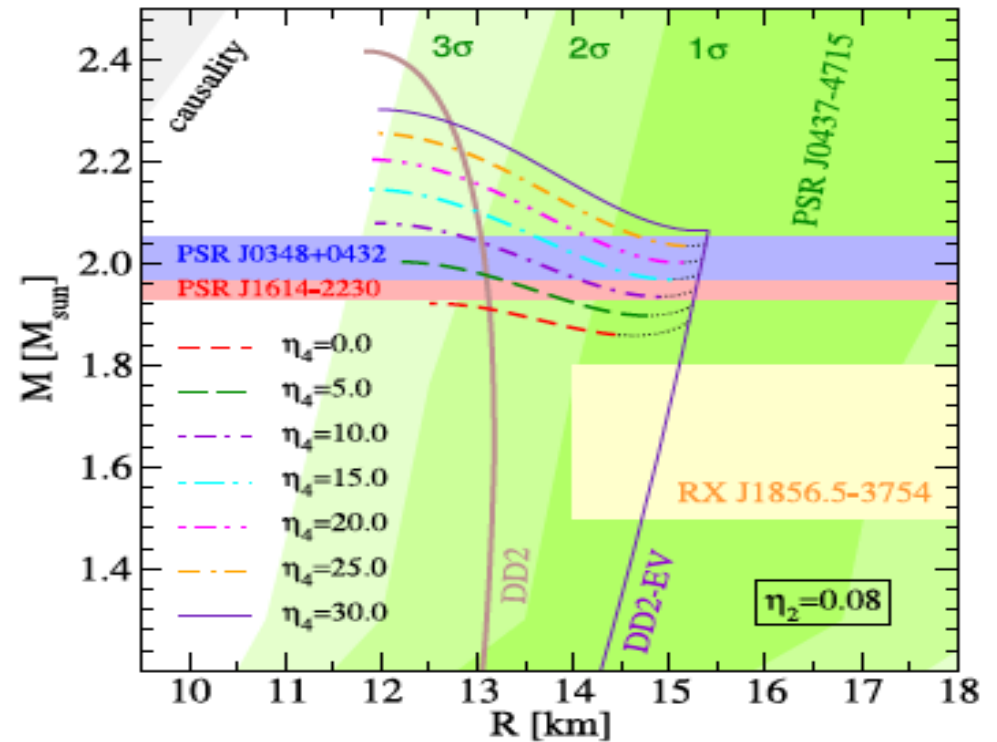
High-mass Twins relatively robust against “smoothing” the Maxwell transition construction

D. Alvarez-Castillo, D.B., arxiv:1412.8463

# 4c. Rotation

- existence of 2  $M_{\text{sun}}$  pulsars and possibility of high-mass twins raises question for their inner structure: (Q)uark or (N)ucleon core ??
  - > degenerate solutions
  - > transition from N to Q branch
- PSR J1614-2230 is millisecond pulsar, period  $P = 3.41$  ms, consider rotation !
- transitions N --> Q must be considered for rotating configurations:
  - > how fast can they be?
- (angular momentum  $J$  and baryon mass should be conserved simultaneously)
- similar scenario as fast radio bursts (Falcke-Rezzolla, 2013) or braking index (Glendenning-Pei-Weber, 1997)

M. Bejger, D.B., work in preparation (2015)





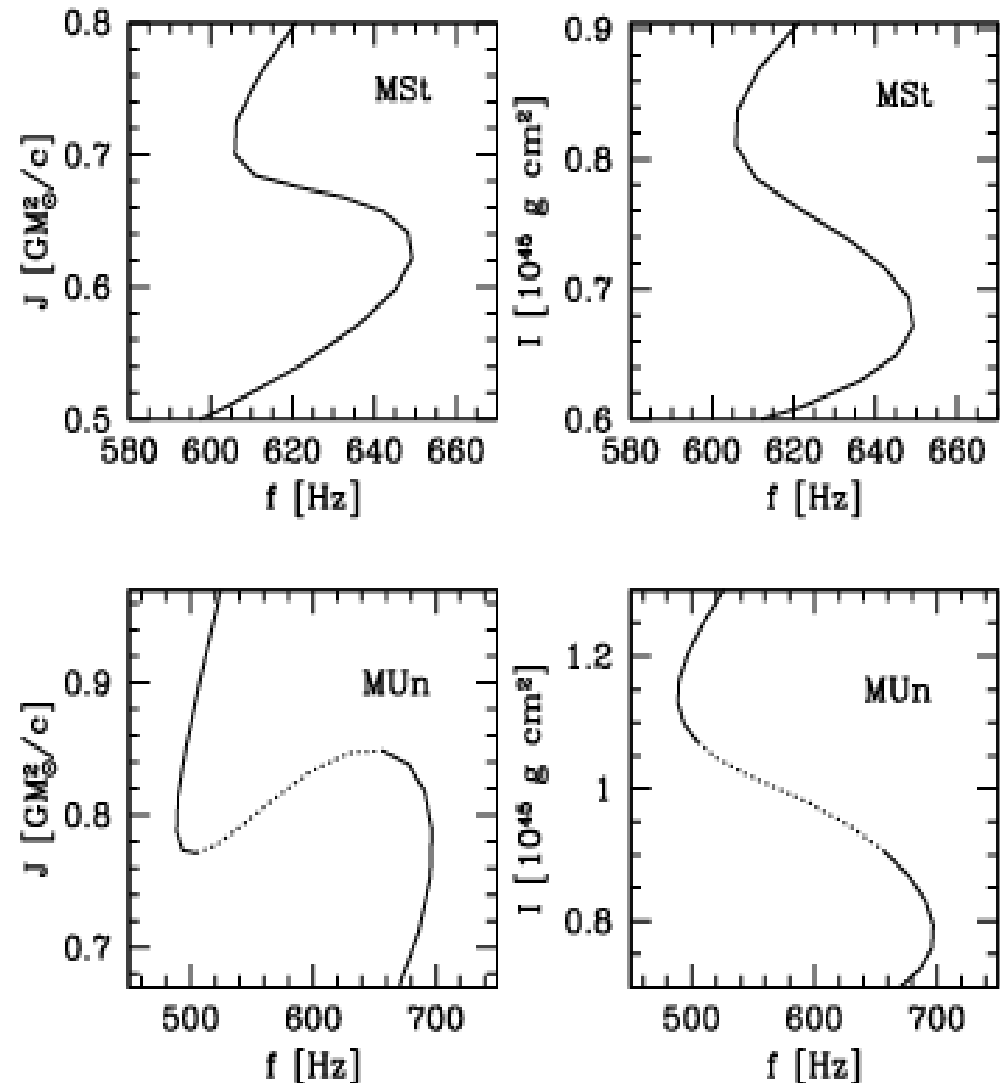
## 4c. Rotation and stability

- ★ Back-bending is connected to the existence of a minimum of  $M_b$  along  $f = \text{const.}$  sequence,
- ★ Change in stability corresponds to extremum of  $M$  or  $M_b$  at fixed  $J$ :

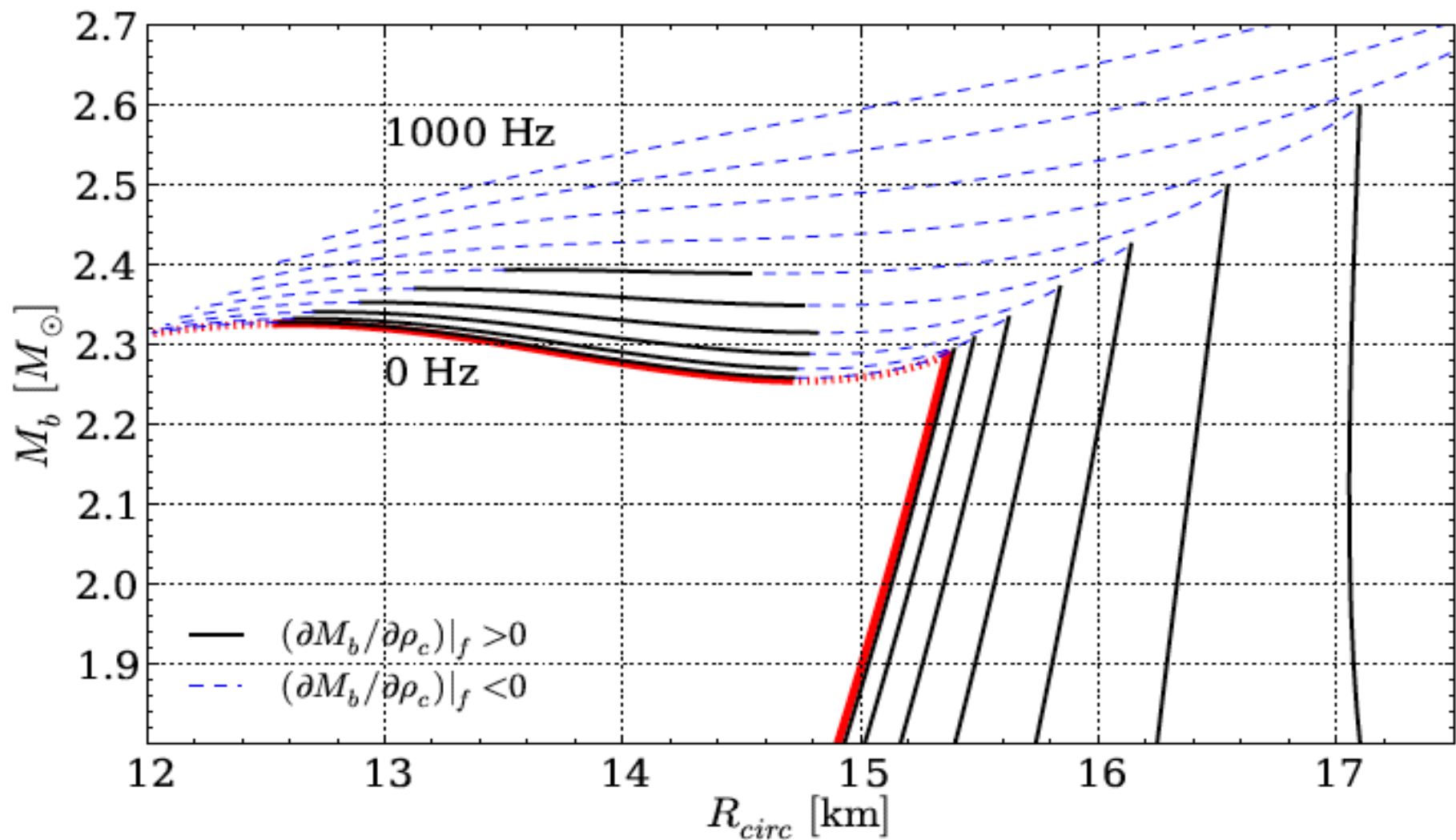
$$\left(\frac{\partial M}{\partial \lambda_c}\right)_J = 0, \quad \left(\frac{\partial M_b}{\partial \lambda_c}\right)_J = 0,$$

or to extremum of  $J$  at fixed either  $M$  or  $M_b$ :

$$\left(\frac{\partial J}{\partial \lambda_c}\right)_M = 0, \quad \left(\frac{\partial J}{\partial \lambda_c}\right)_{M_b} = 0.$$

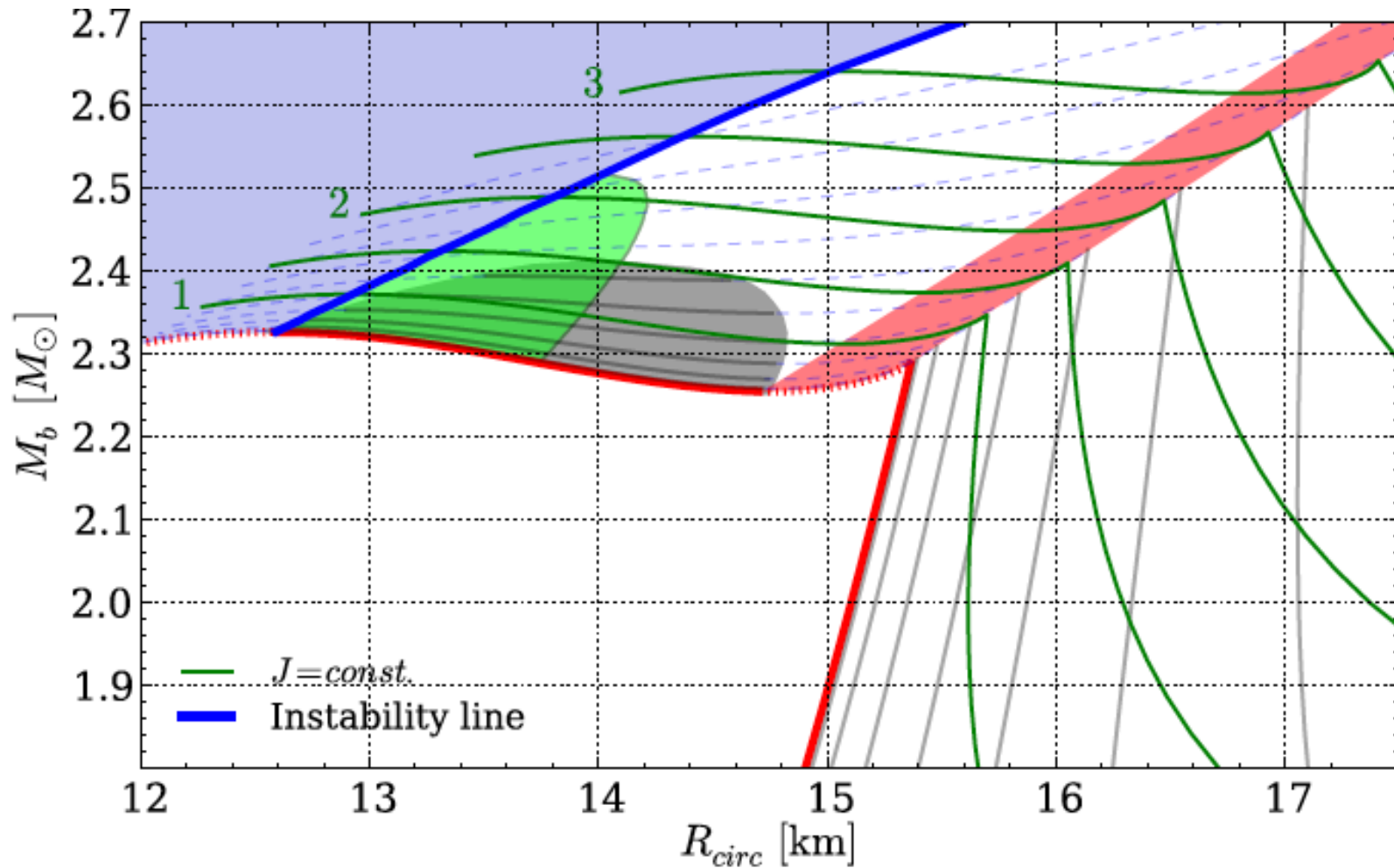


## 4c. Rotation and stability



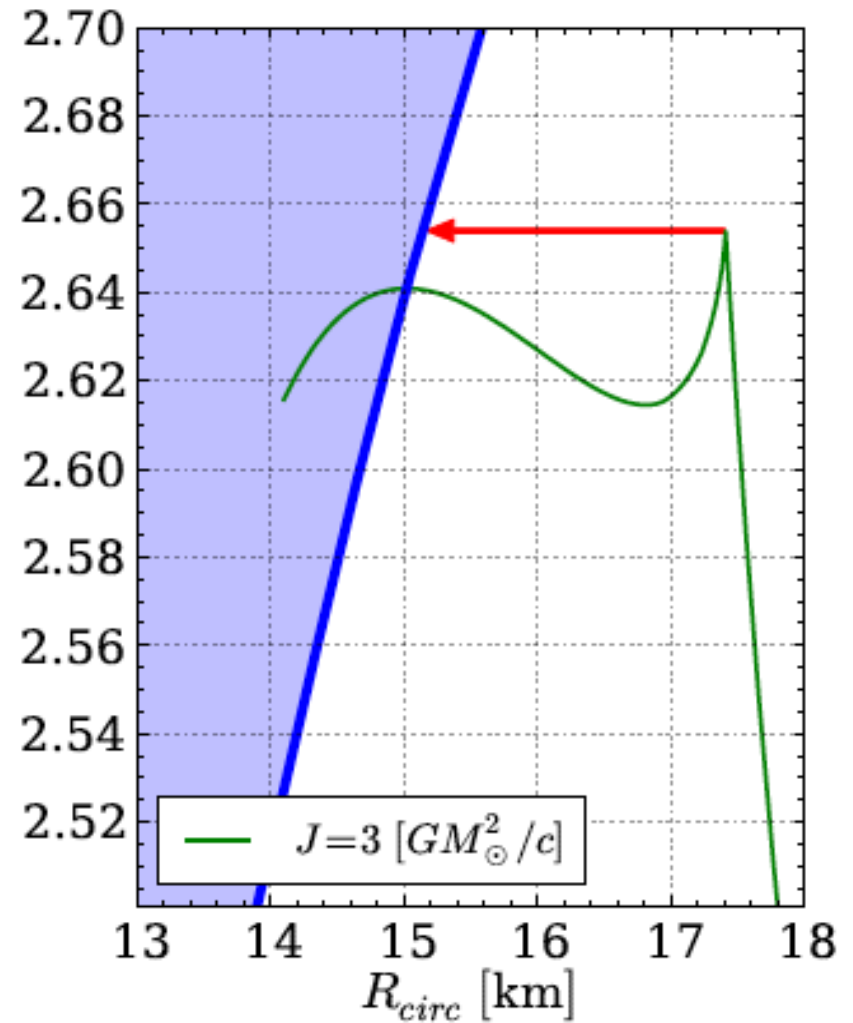
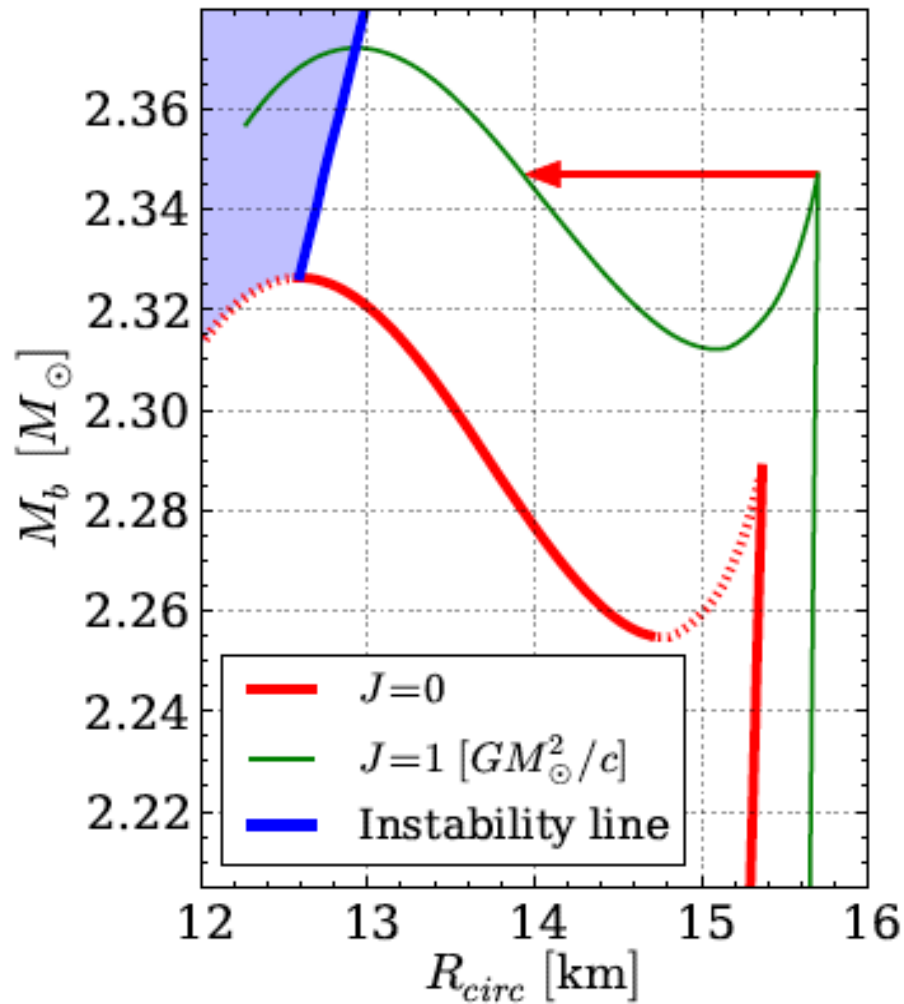
Large regions of backbending phenomenon  
(NS spins up while losing angular momentum due to the dense matter EoS)

## 4c. Rotation and stability



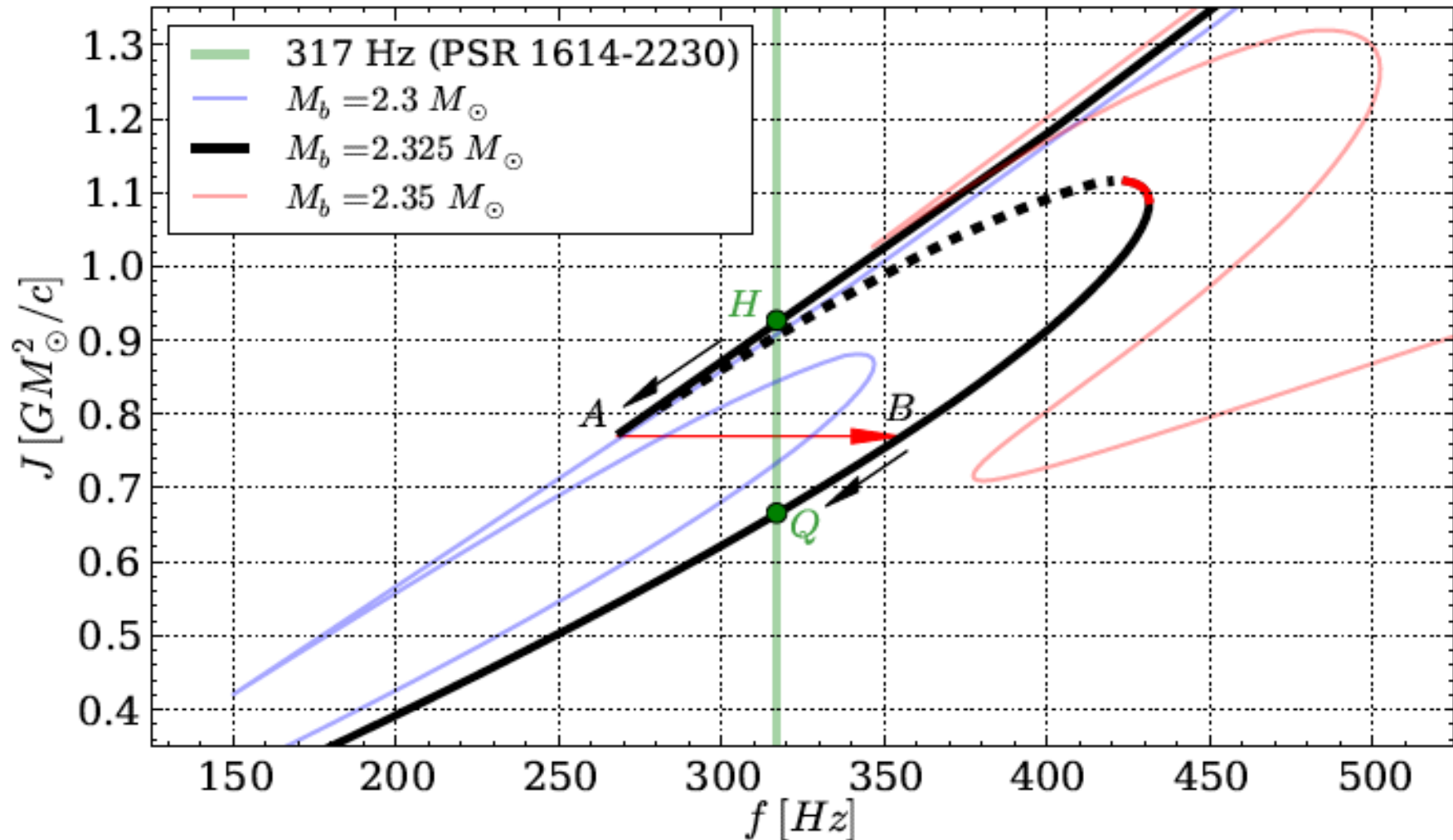
- Red region** - strong phase-transition instability,
- Blue region** - unstable w.r.t axisymmetric oscillations,
- Grey region - no back-bending,
- Green region** - stable twin branch reached after the mini-collapse from the tip of  $J = const.$  curve, along  $M_b = const.$

## 4c. Rotation and stability



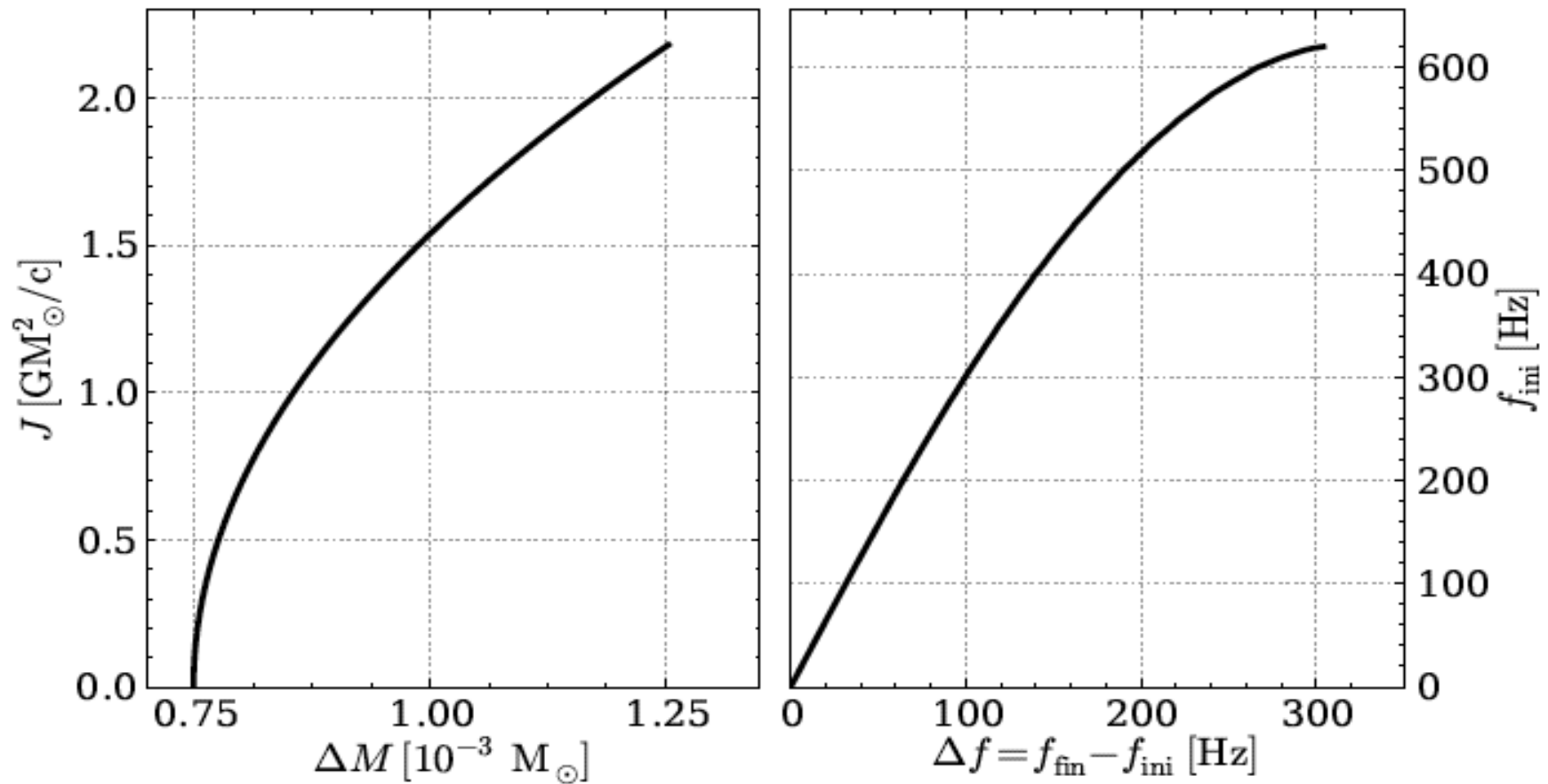
Stars with too much angular momentum *e.g.*, *spun-up by accretion* end up in the instability.

## 4c. Constraints from mass and frequency



For NSs with measured gravitational mass  $M$  and frequency - possibility to put limits on  $M_b$ ,  $J$ , moment of inertia  $I$ , core EOS composition etc.

## 4c. Energy release and spin-up (glitch)



**Left panel:** energy release (difference in the gravitational mass) vs  $J$  of the configuration entering the strong phase-transition instability.

**Right panel:** spin-up  $\Delta f$  (difference between the final and initial spin frequency) against the spin frequency of the initial configuration.

## 4c. Rotation - summary

This type of instability EOS provides a "natural" explanation for:

- ★ Lack of back-bending in radiopulsar timing,
- ★ Spin frequency cut-off at some moderate (but  $>716$  Hz) frequency,
- ★ Falcke & Rezzolla Fast Radio Burst (FRB) engine
  - ★ catastrophic mini-collapse to the second branch (or to a black hole),
  - ★ massive rearrangement of the magnetic field  $\rightarrow$  energy emission.

Astrophysical predictions:

- ★ Way to constraint on  $M_b$ ,  $J$ ,  $I$ , core EOS etc.,
- ★ Specific shape of NS-BH mass function (no mass gap?)
- $\rightarrow$  population of massive, low B-field NSs (radio-dead?),
- $\rightarrow$  population of massive, high B-field NSs (collapse enhances the field?),
- ★ Characteristic burst-like signature in GW emission during the mini-collapse.

## 4d. New Bayesian Analysis scheme



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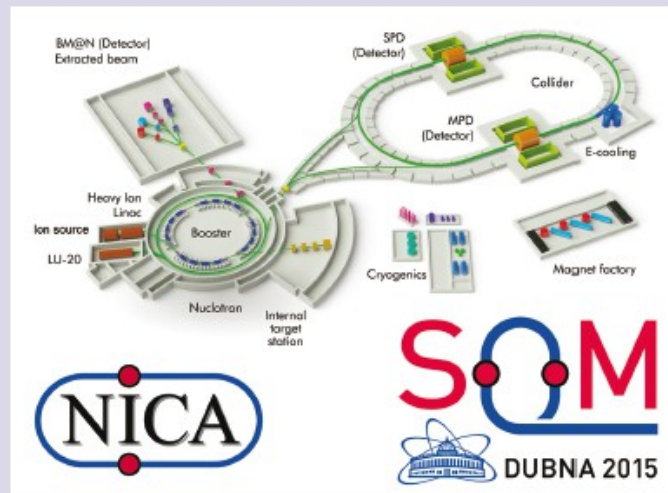
## 15th International Conference on Strangeness in Quark Matter (SQM2015)

Dubna, Russia  
6–11 July 2015

**Editors:** David E. Alvarez-Castillo, David Blaschke, Vladimir Kekelidze,  
Victor Matveev and Alexander Sorin

Volume 668 2016

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**IOP Publishing**

# Strategy towards event simulations testing PT signal

## Two alternative approaches:

### I) Direct approach based on transport codes:

Particle trajectories are followed;

Properties of the medium are encoded in propagators and cross sections

→ UrQMD (Aichelin et al.),

→ PHSD (Bratkovskaya, Cassing, et al.),

→ PHSD + SACA (Bratkovskaya, Aichelin, LeFevre, et al.)

### II) Hybrid approach:

Joins hydrodynamic evolution of a (multi-)fluid system described by an **EoS** with

Particle transport via a procedure called “**particlization**” (Karpenko)

Particularly suitable for studying effects of a strong phase transition in model EoS

a) Sandwich: UrQMD + hydro + hadronic cascade (H. Petersen et al.)

→ PT in hydro stage only

b) **3-fluid hydro**dynamics (Ivanov) + particlization (Karpenko)

→ PT in baryon stopping regime already!

(main difference to sandwich; appropriate for energy range of NICA / CBM)

Both approaches provide the inputs for the simulation of the **detector response**  
(GEANT-MPD: Rogachevsky, Merts, Batyuk, Wielanek, et al.)

# Hydrodynamic modelling for NICA / FAIR

**More complicated for lower energies:**

- baryon stopping effects,
- finite baryon chemical potential,
- EoS unknown from first principles

We want to simulate the effects of, and ultimately discriminate different EoS/PT types  
The model has to be coupled to a detector response code to simulate detector events



Initial state



3-fluid hydro,  
(Yu. Ivanov)

hydrodynamic evolution



adapt the procedure  
from existing hybrid model  
(Iu. Karpenko)

particlization

hadronic  
corona

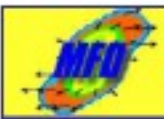


(optionally) cascade:  
PHSD, UrQMD, etc  
(E. Bratkovskaya,  
H. Petersen)

detector  
response



GEANT  
MPD, BM @N  
(O. Rogachevsky,  
P. Batyuk,  
S. Merts, et al.)



# 3-Fluid Dynamics

Baryon Stopping

JINR, 24.08.10

Model

Rapidity Density

Fit

Reduced curvature

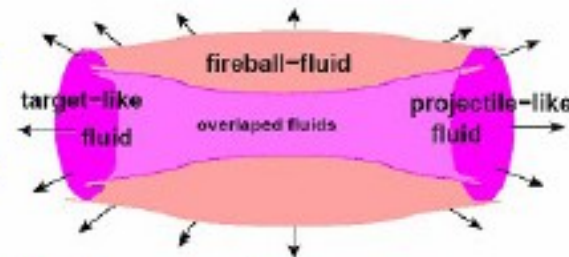
Trajectories

Crossover

Summary

Produced particles populate mid-rapidity  
 $\Rightarrow$  **fireball** fluid

distribution function



momentum along beam

**Target-like fluid:**

$$\partial_\mu J_t^\mu = 0$$

Leading particles carry bar. charge

$$\partial_\mu T_t^{\mu\nu} = -F_{tp}^\nu + F_{ft}^\nu$$

exchange/emission

**Projectile-like fluid:**

$$\partial_\mu J_p^\mu = 0,$$

$$\partial_\mu T_p^{\mu\nu} = -F_{pt}^\nu + F_{fp}^\nu$$

**Fireball fluid:**

$$J_f^\mu = 0,$$

Baryon-free fluid

$$\partial_\mu T_f^{\mu\nu} = F_{pt}^\nu + F_{tp}^\nu - F_{fp}^\nu - F_{ft}^\nu$$

Source term      Exchange

The **source term** is delayed due to a formation time  $\tau \sim 1 \text{ fm}/c$

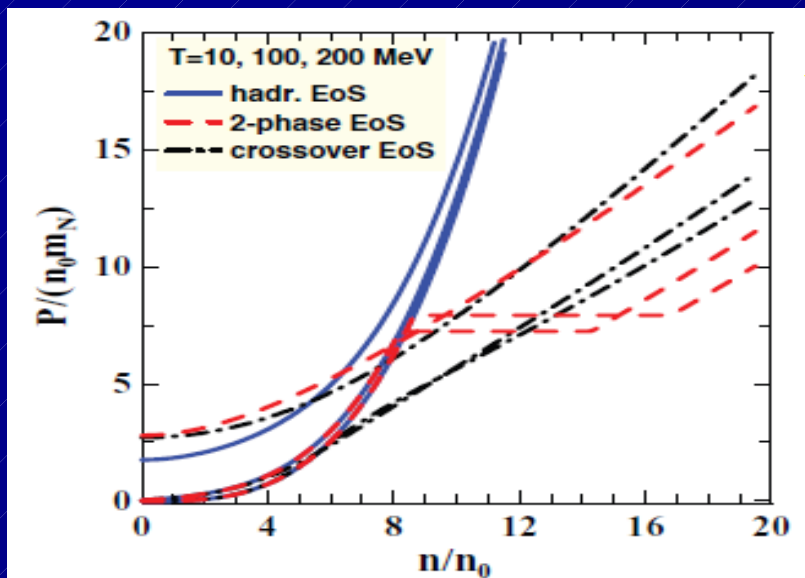
**Total energy-momentum conservation:**

$$\partial_\mu (T_p^{\mu\nu} + T_t^{\mu\nu} + T_f^{\mu\nu}) = 0$$

<http://theory.gsi.de/~ivanov/mfd/>

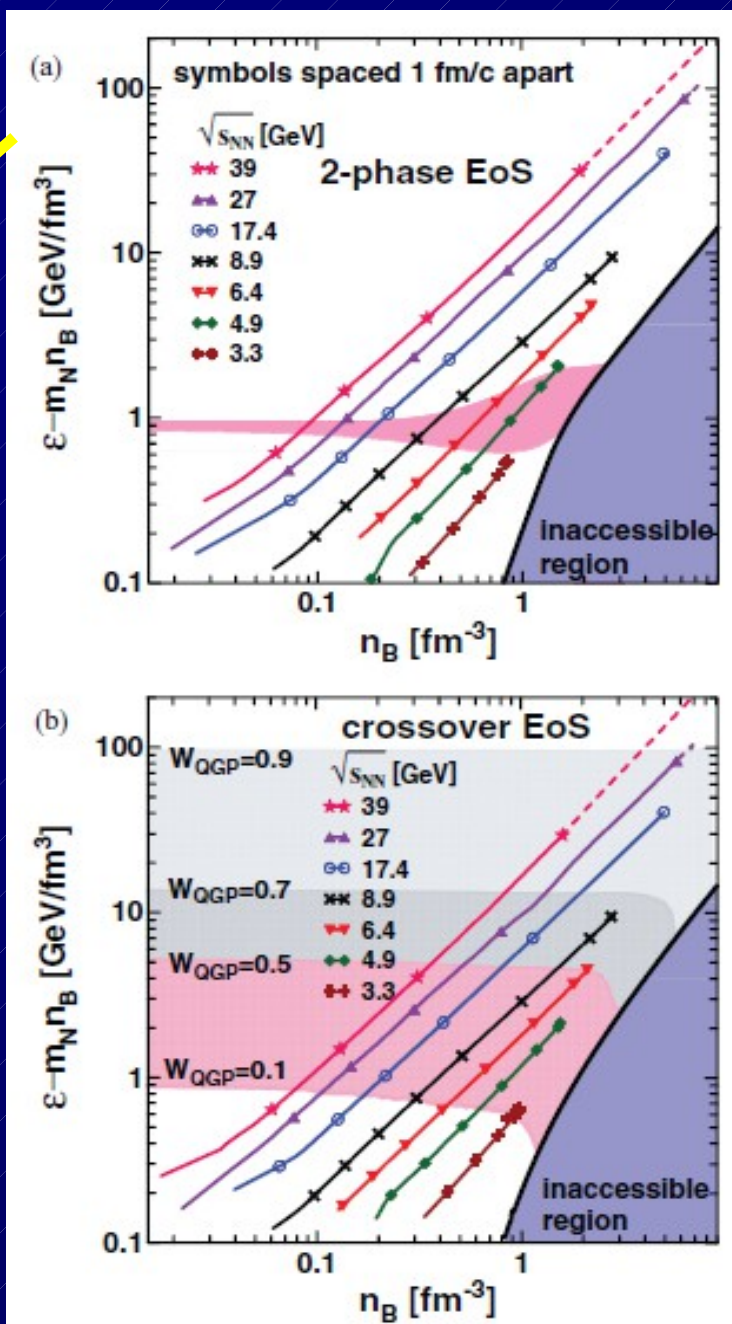
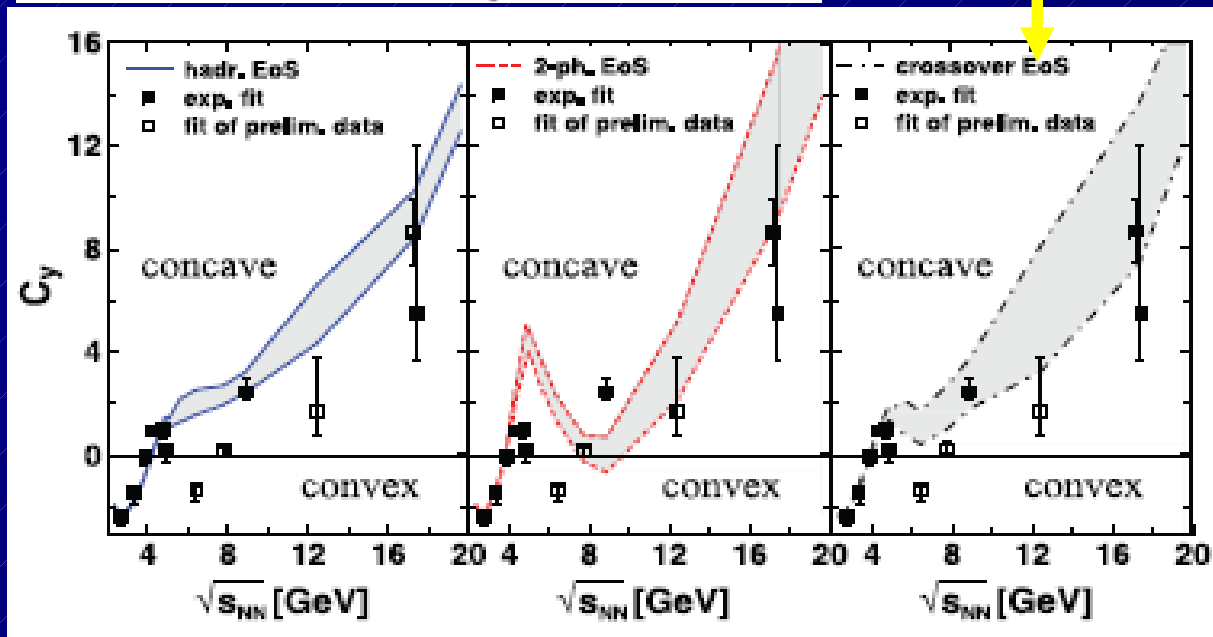
# Net proton rapidity distribution – test case for a 1<sup>st</sup> order PT signal

**Theory:** Yu.B. Ivanov, Phys. Rev. C 87, 064904 (2013)



**EoS**

3-fluid hydro  
Evolution ...

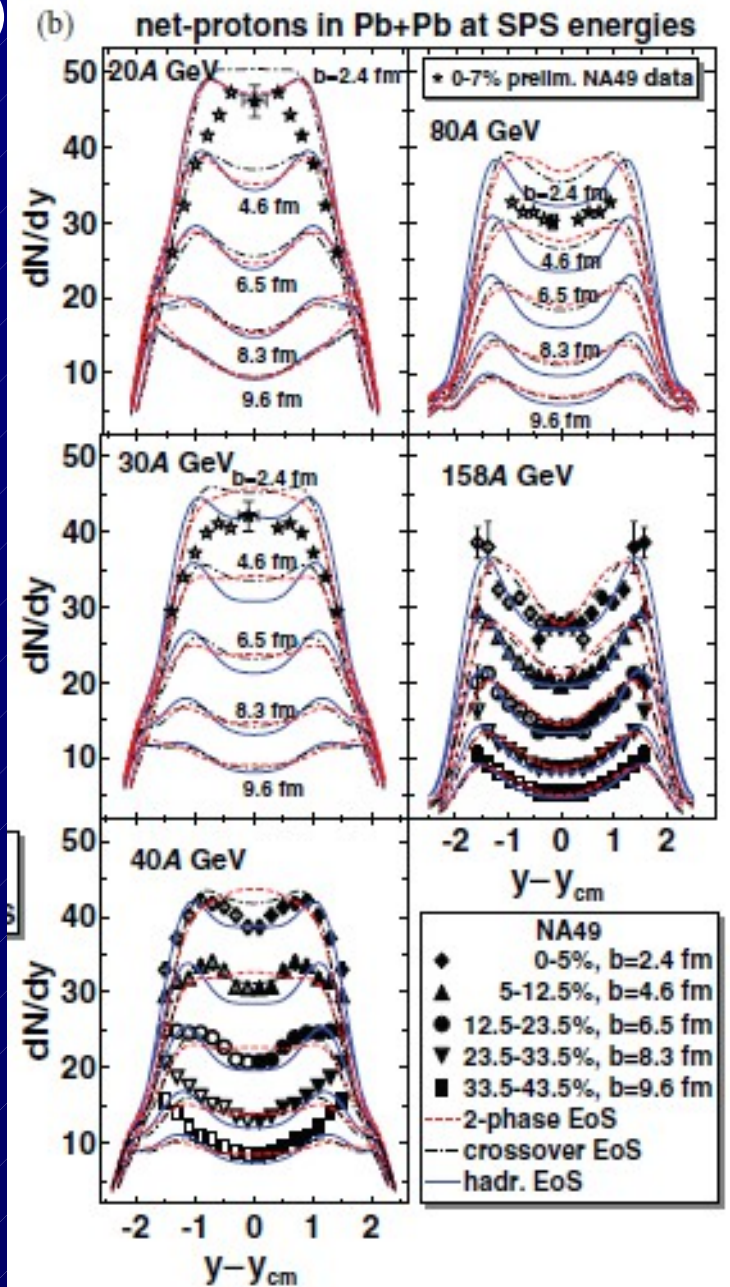
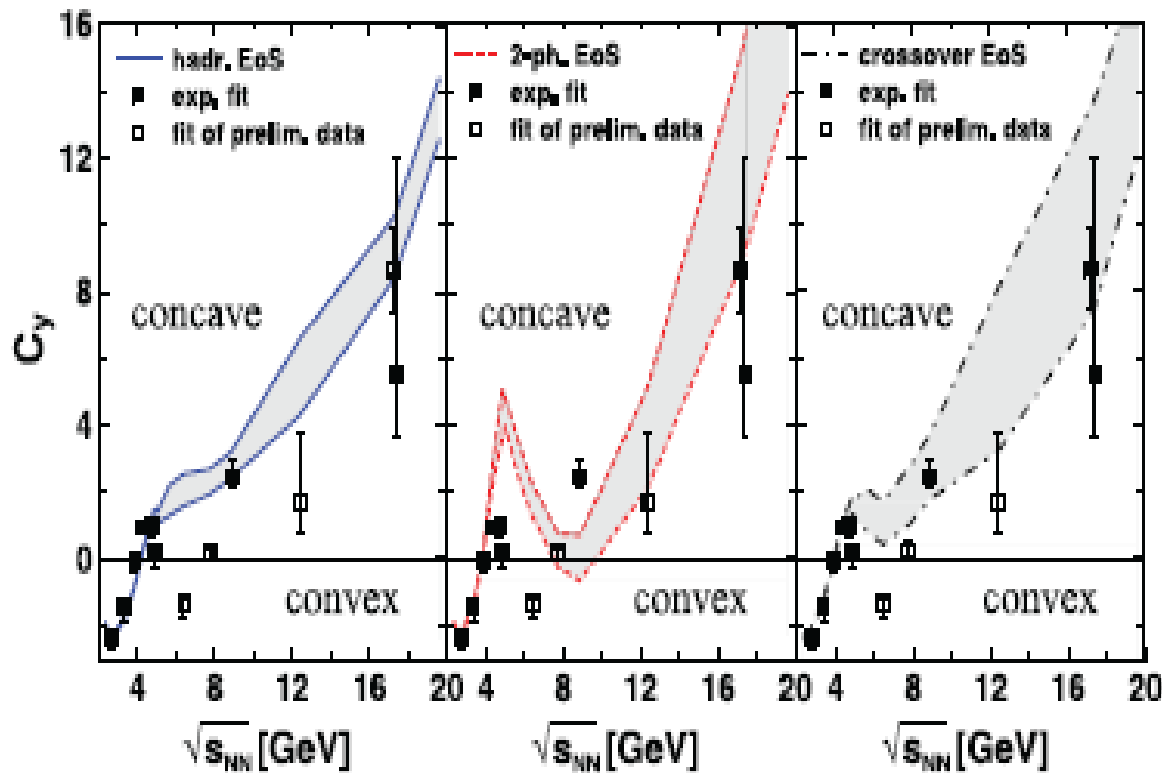


# Net proton rapidity distribution – test case for a 1<sup>st</sup> order PT signal

**Theory:** Yu.B. Ivanov, Phys. Rev. C 87, 064904 (2013)

$$C_y = \left( y_{c.m.}^3 \frac{d^3 N}{dy^3} \right)_{y=y_{c.m.}} / \left( y_{c.m.} \frac{dN}{dy} \right)_{y=y_{c.m.}}$$

$$= (y_{c.m.}/w_s)^2 (\sinh^2 y_s - w_s \cosh y_s).$$



# Net proton rapidity distribution – test case for a 1<sup>st</sup> order PT signal

## Event set:

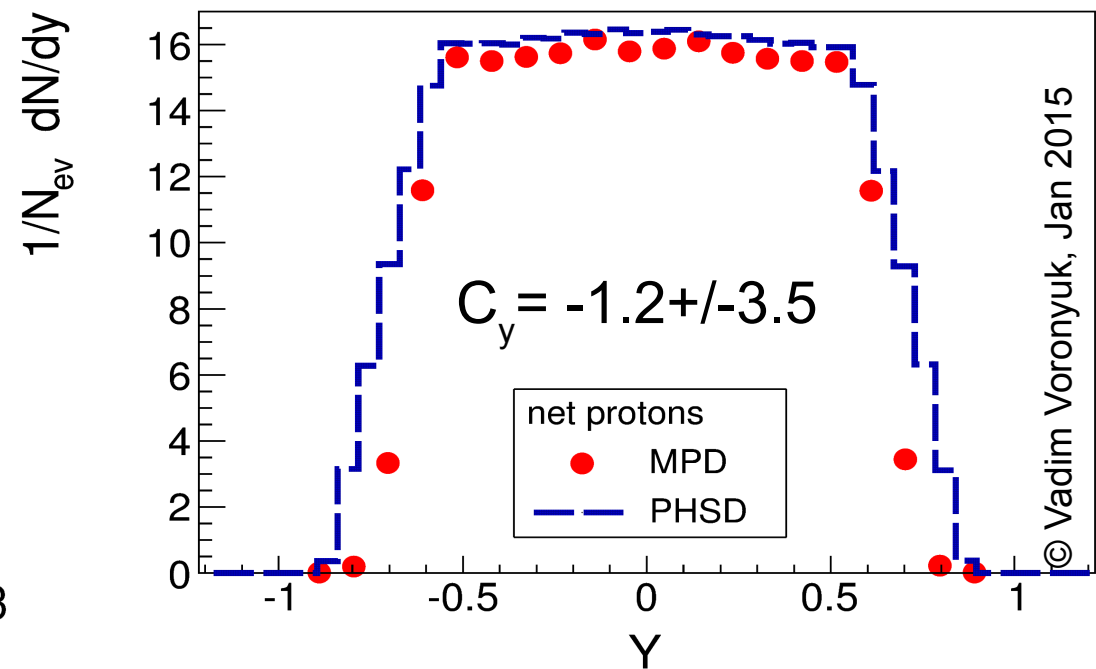
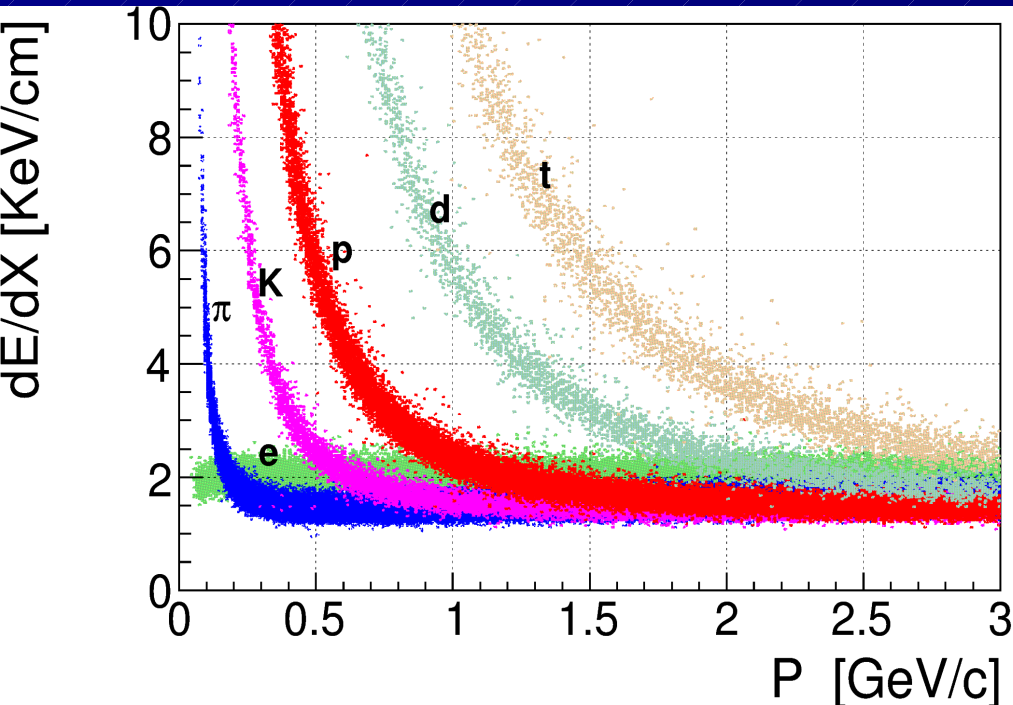
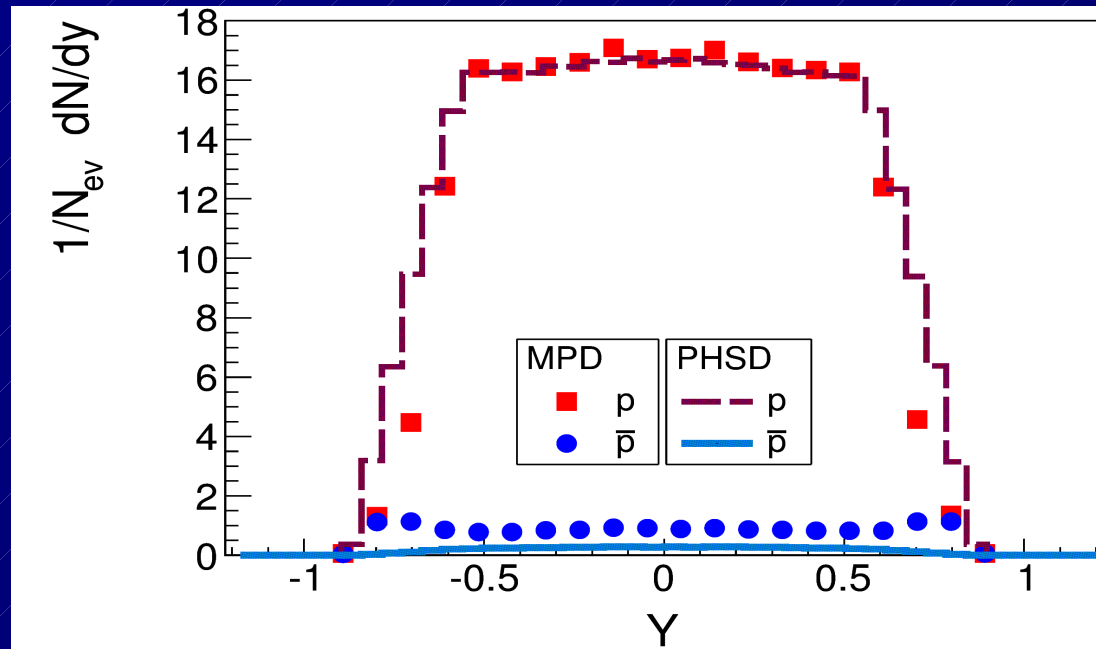
40k AuAu @  $\sqrt{s}_{NN} = 9$  GeV [0-5%]  
 The most reliable region  
 $|\eta| < 1.2$  ;  $0.4 < p_t [\text{GeV}/c] < 0.8$

## Result:

PHSD input  $\rightarrow$  GEANT+MPD  
 detector reproduces the rapidity distribution !  
 (previous concerns not confirmed !!)

## Signal:

$$C_y = \left( y_{cm}^3 \frac{d^3 N}{dy^3} \right)_{y=0} / \left( y_{cm} \frac{dN}{dy} \right)_{y=0}$$



# Net proton rapidity distribution – test case for a 1<sup>st</sup> order PT signal

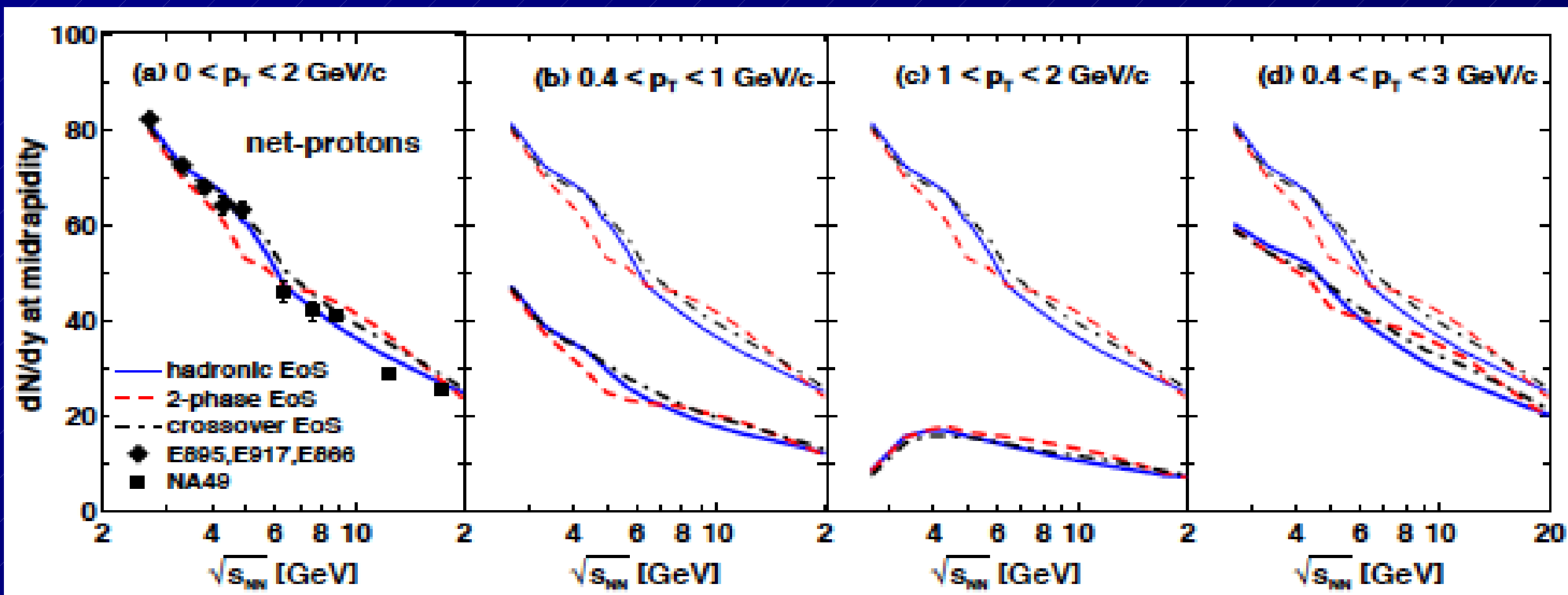
## Investigation of $p_T$ cuts:

Yu. Ivanov & D. Blaschke, arxiv:1504.03992

$$C_y = \left( y_{\text{beam}}^3 \frac{d^3 N}{dy^3} \right)_{y=0} / \left( y_{\text{beam}} \frac{dN}{dy} \right)_{y=0}$$

$$= (y_{\text{beam}}/w_s)^2 (\sinh^2 y_s - w_s \cosh y_s).$$

- i.  $0 < p_T < 2 \text{ GeV}/c$  and a very unrestrictive constraint to the rapidity range  $|y| < 0.7 y_{\text{beam}}$ , where  $y_{\text{beam}}$  is the beam rapidity in the collider mode, which is practically equivalent to the full acceptance;
- ii.  $0.4 < p_T < 1 \text{ GeV}/c$  and  $|y| < 0.5$ , the expected MPD acceptance [17];
- iii.  $1 < p_T < 2 \text{ GeV}/c$  and  $|y| < 0.5$ , an acceptance range where low-momentum particles witnessing collective behaviour are largely eliminated;
- iv.  $0.4 < p_T < 3 \text{ GeV}/c$  and  $|y| < 0.5$ , the range of the STAR acceptance [18].





# Net proton rapidity distribution – test case for a 1<sup>st</sup> order PT signal

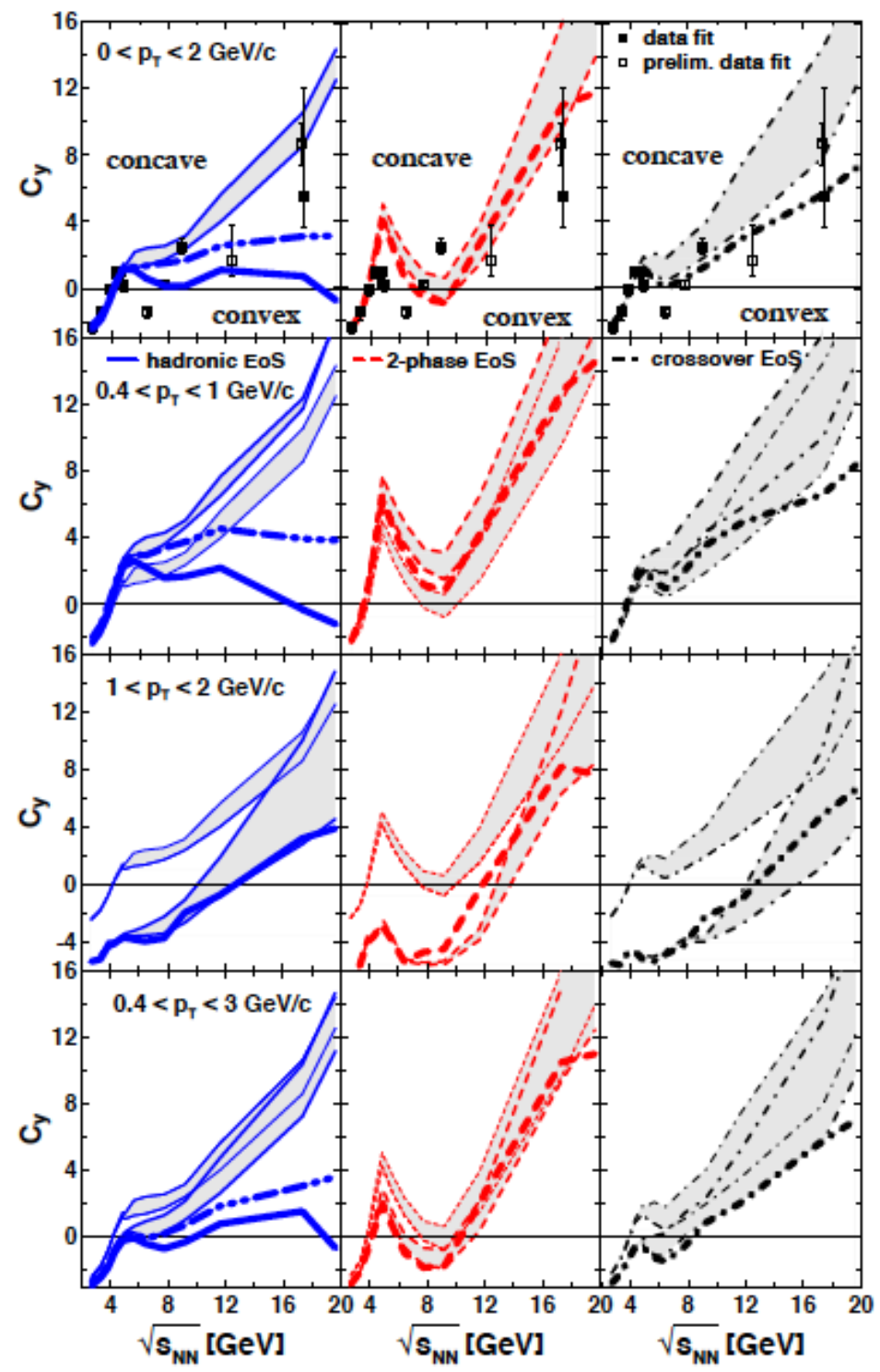
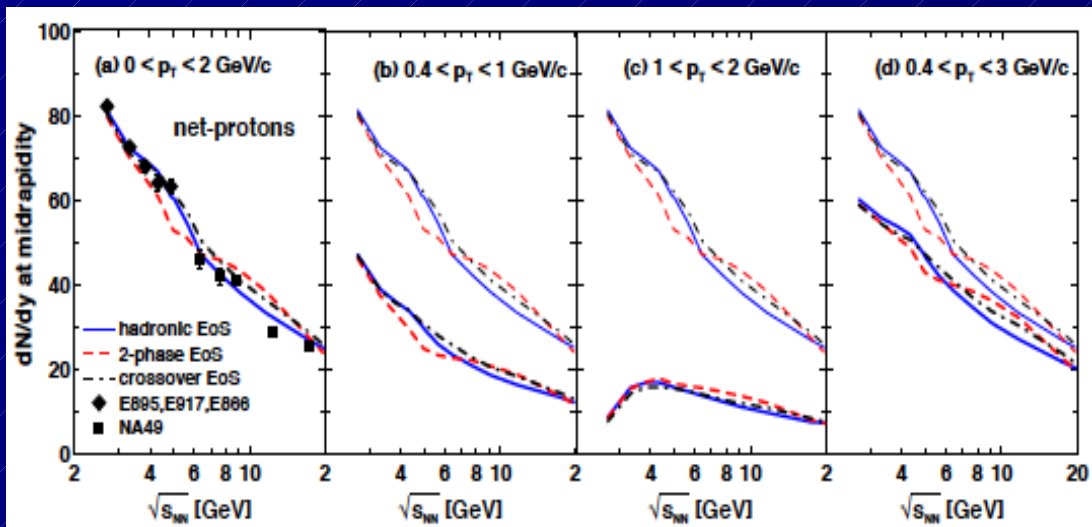
## Investigation of $p_T$ cuts:

Yu. Ivanov & D. B., PRC 92, 024916 (2015)

$$C_y = \left( y_{\text{beam}}^3 \frac{d^3 N}{dy^3} \right)_{y=0} / \left( y_{\text{beam}} \frac{dN}{dy} \right)_{y=0}$$

$$= (y_{\text{beam}}/w_s)^2 (\sinh^2 y_s - w_s \cosh y_s)$$

- “wiggle” formed in the nonequilibrium compression stage of the collision, where  $p_T$  **only in 3FH**
- robust against serious  $p_T$  cuts
- at high  $p_T$  (1 - 2 GeV/c) in convex region
- at low  $p_T$  (0.2 - 1 GeV/c) in concave region
- required accuracy in  $C_y$  determination:  $\Delta C_y < 2$



# Net proton rapidity distribution – test case for a 1<sup>st</sup> order PT signal

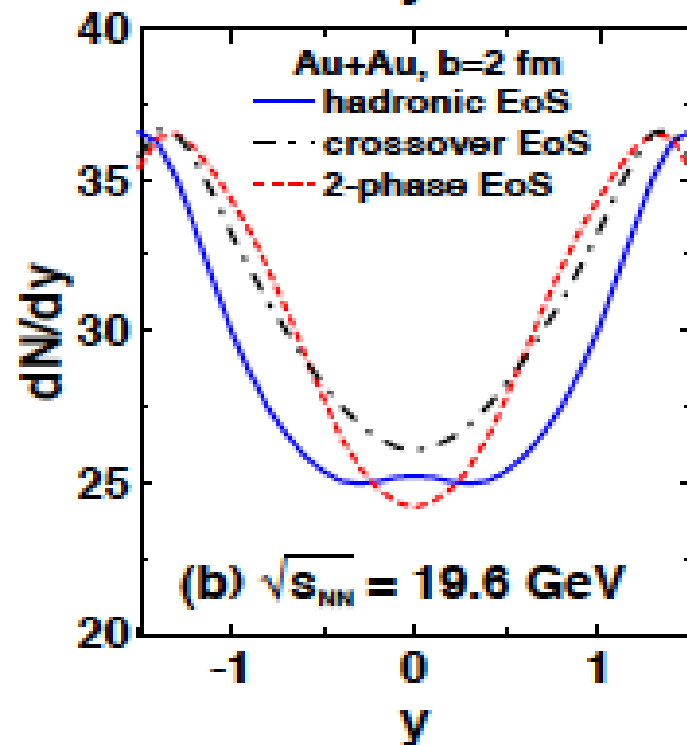
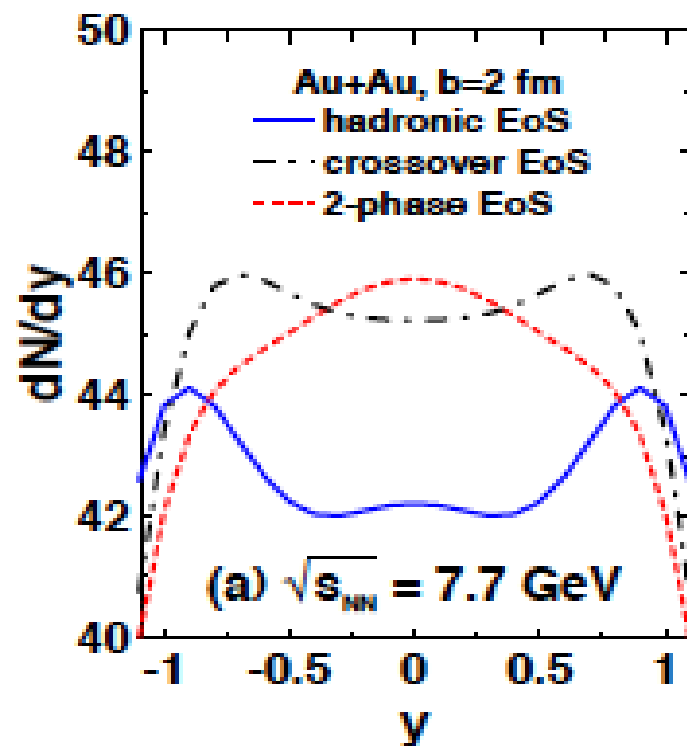
## Investigation of $p_T$ cuts:

Yu. Ivanov & D. Blaschke, arxiv:1504.03992

$$\frac{dN}{dy} = a \left( \exp \left\{ -\left(1/w_s\right) \cosh(y - y_s) \right\} + \exp \left\{ -\left(1/w_s\right) \cosh(y + y_s) \right\} \right),$$

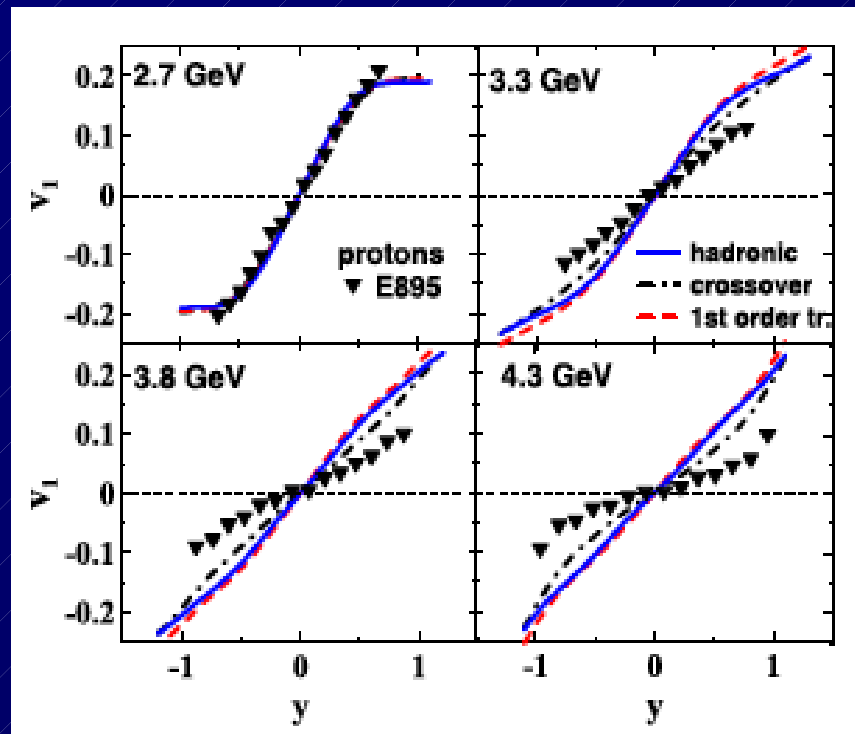
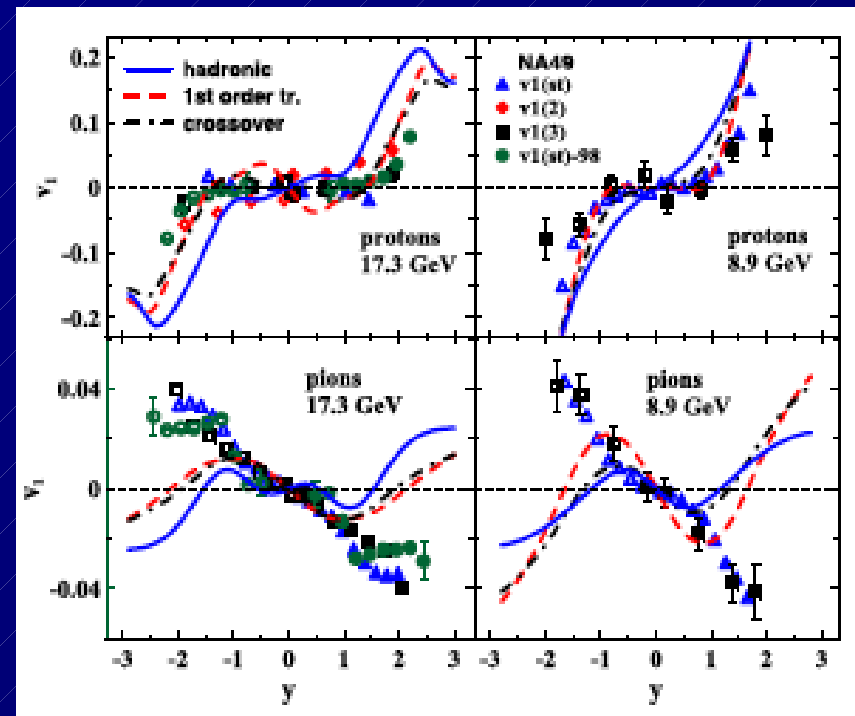
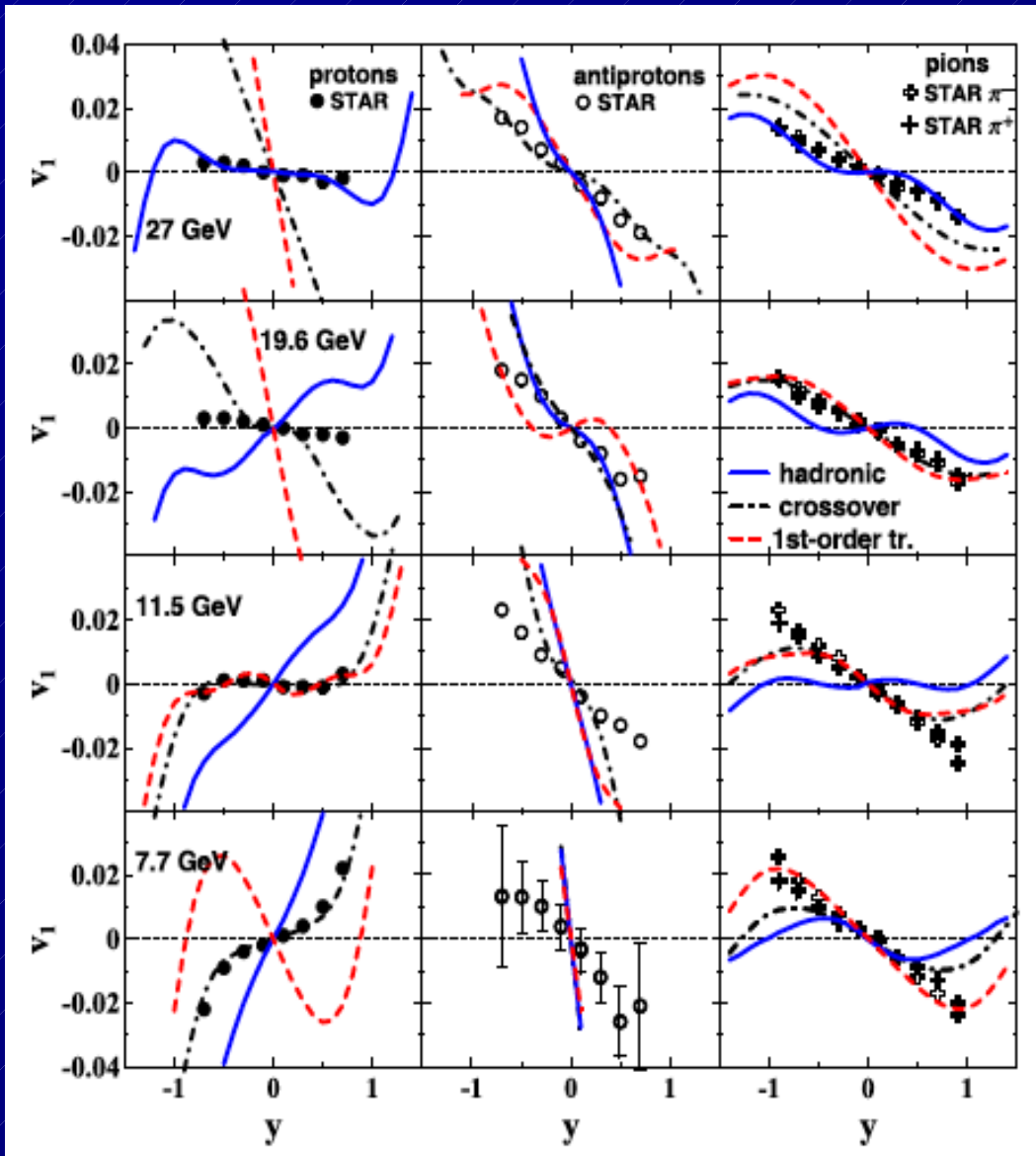
$$C_y = \left( y_{\text{beam}}^3 \frac{d^3 N}{dy^3} \right)_{y=0} / \left( y_{\text{beam}} \frac{dN}{dy} \right)_{y=0} = (y_{\text{beam}}/w_s)^2 (\sinh^2 y_s - w_s \cosh y_s).$$

- “wiggle” formed in the nonequilibrium compression stage of the collision, where  $p_T$  **only in 3FH**
- robust against serious  $p_T$  cuts
- at high  $p_T$  (1 - 2 GeV/c) in convex region
- at low  $p_T$  (0.2 - 1 GeV/c) in concave region
- required accuracy in  $C_y$  determination:  $\Delta C_y < 2$



# Directed flow indicates a cross-over Deconfinement transition ...

Ivanov & Soldatov, Phys. Rev. C 91 (2015)



# 3+1D viscous hydro-cascade model (Yu. Karpenko, FIAS)

3+1D viscous hydro+cascade model was applied for A+A collisions at RHIC Beam Energy Scan energies ( $\sqrt{s} = 7.7 - 39$  GeV), and for SPS energy points

Cascade-hydro-cascade approach:

Initial state: UrQMD cascade

S.A. Bass et al., Prog. Part. Nucl. Phys. 41 255-369, 1998

Hydrodynamic phase: numerical 3+1D hydro solution via original relativistic viscous hydro code

Iu. Karpenko, P. Huovinen, M. Bleicher, arXiv:1312.4160

Hydro starts at  $\tau = \sqrt{t^2 - z^2} = \tau_0$  (red curve):

$$\tau_0 = \frac{2R}{\gamma v_z}$$

$\{T^{0\mu}, N_b^0, N_q^0\}$  of fluid = averaged  $\{T^{0\mu}, N_b^0, N_q^0\}$  of particles

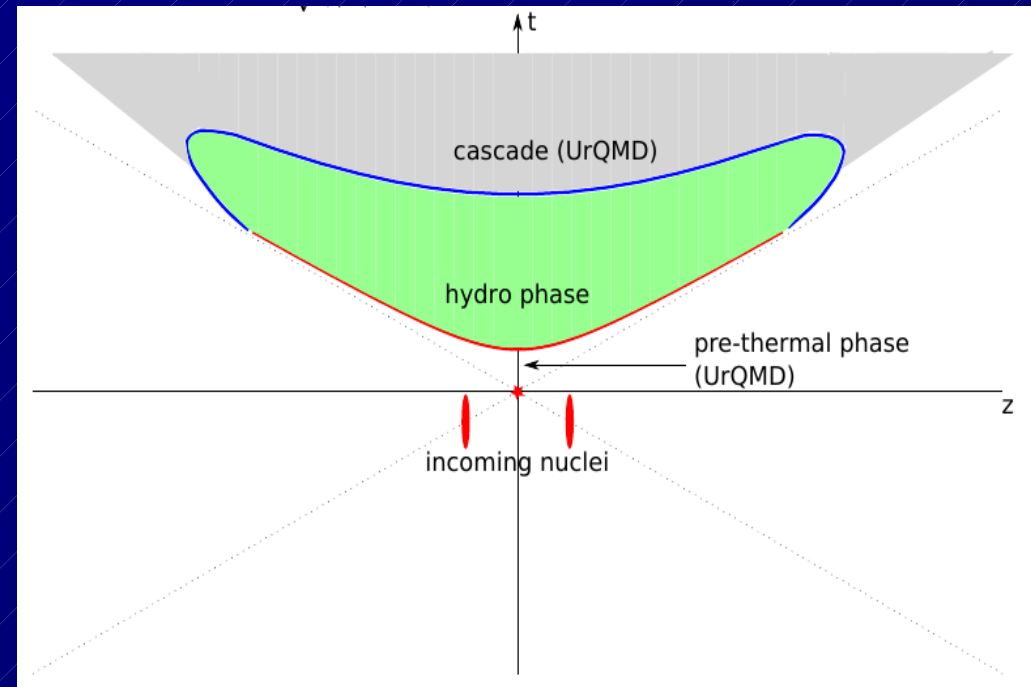
Fluid  $\rightarrow$  particle transition

$\varepsilon = \varepsilon_{SW} = 0.5 \text{ GeV/fm}^3$  (blue curve):

$\{T^{0\mu}, N_b^0, N_q^0\}$  of hadron-resonance gas =  $\{T^{0\mu}, N_b^0, N_q^0\}$  of fluid

Hadronic cascade: UrQMD

P. Huovinen, H. Petersen: "Particlization in hybrid models", Eur. Phys. J. A48 (2012) 171; arxiv:1206.3371



Equations of state for hydrodynamic phase

- Chiral model

- ▶ coupled to Polyakov loop to include the deconfinement phase transition
- ▶ good agreement with lattice QCD data at  $\mu_B = 0$ , also applicable at finite baryon densities
- ▶ (current version) has **crossover type PT** between hadron and quark-gluon phase at all  $\mu_B$

- Hadron resonance gas + Bag Model (a.k.a. EoS Q)

- ▶ hadron resonance gas made of  $u, d$  quarks including repulsive meanfield
- ▶ the phases matched via Maxwell construction, resulting in **1<sup>st</sup> order PT**

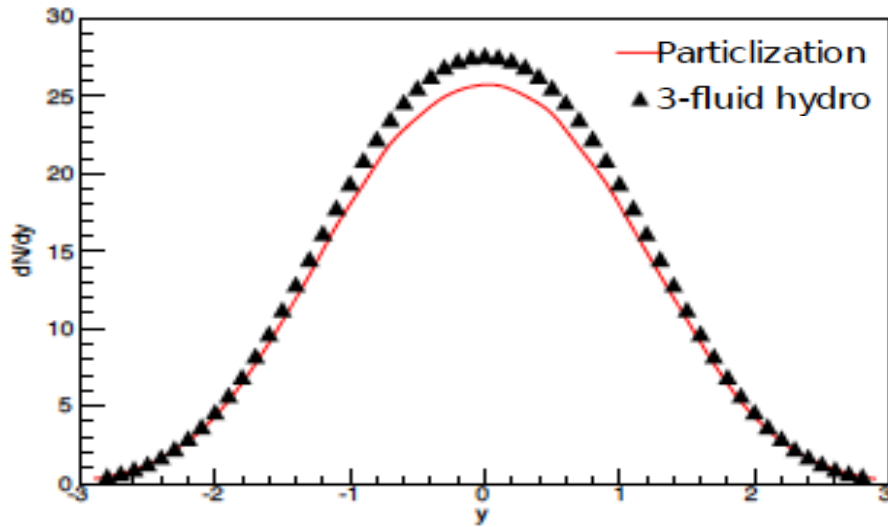
J. Steinheimer, S. Schramm and H. Stocker, J. Phys. G 38, 035001 (2011);  
P.F. Kolb, J. Sollfrank, and U. Heinz, Phys.Rev. C 62, 054909 (2000).

# Preview: Particlization of 3-fluid Hydrodynamics model

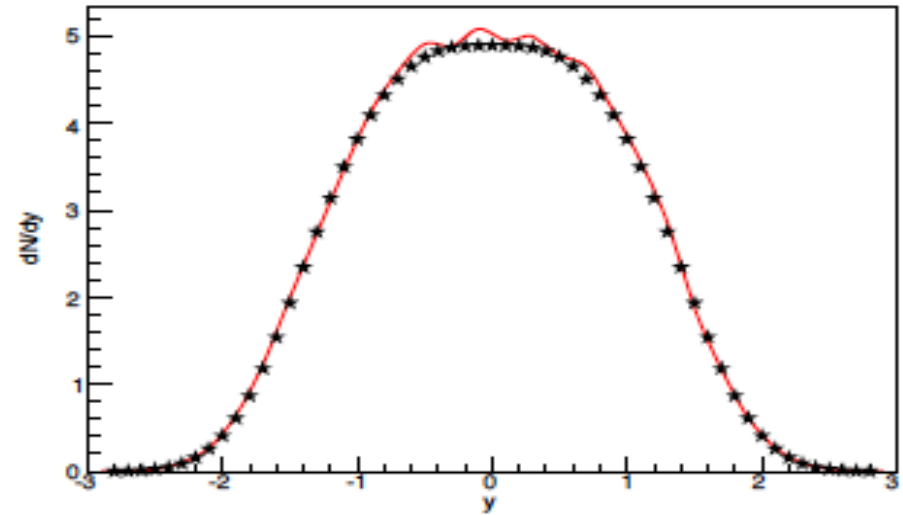
Yu. Karpenko & Yu. Ivanov: May 2014 - May 2015

Rapidity distribution

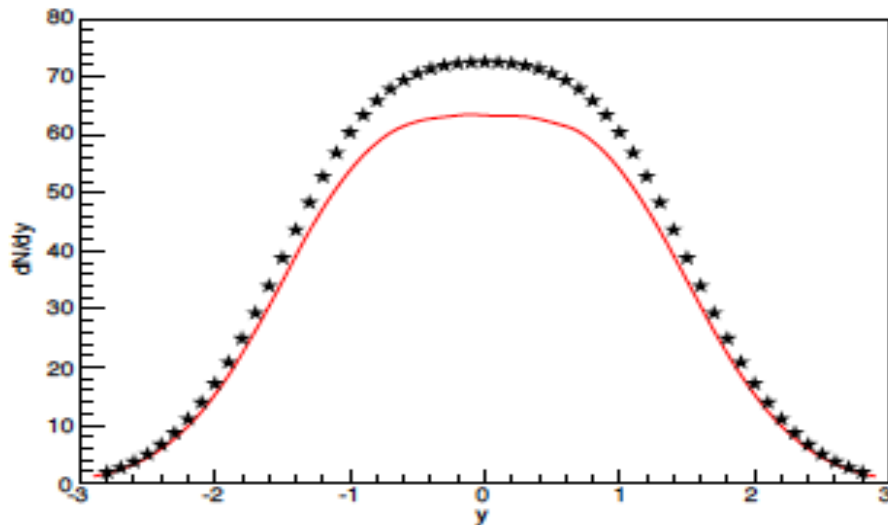
pions, fireball



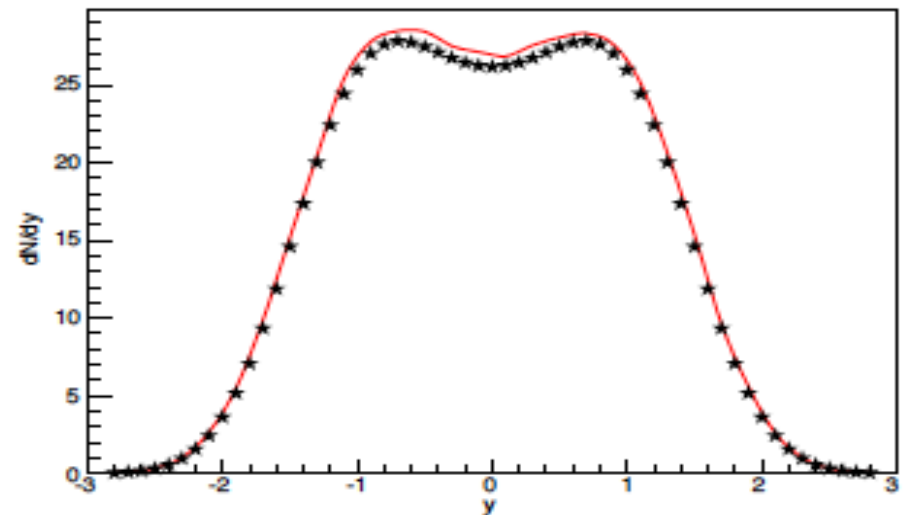
kaons, fireball



pions, baryon-rich



kaons, baryon-rich

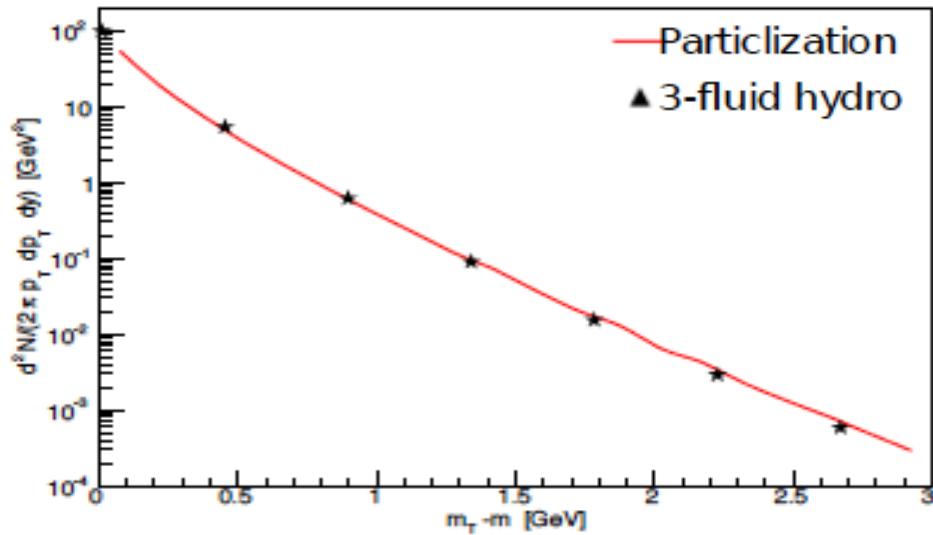


# Preview: Particlization of 3-fluid Hydrodynamics model

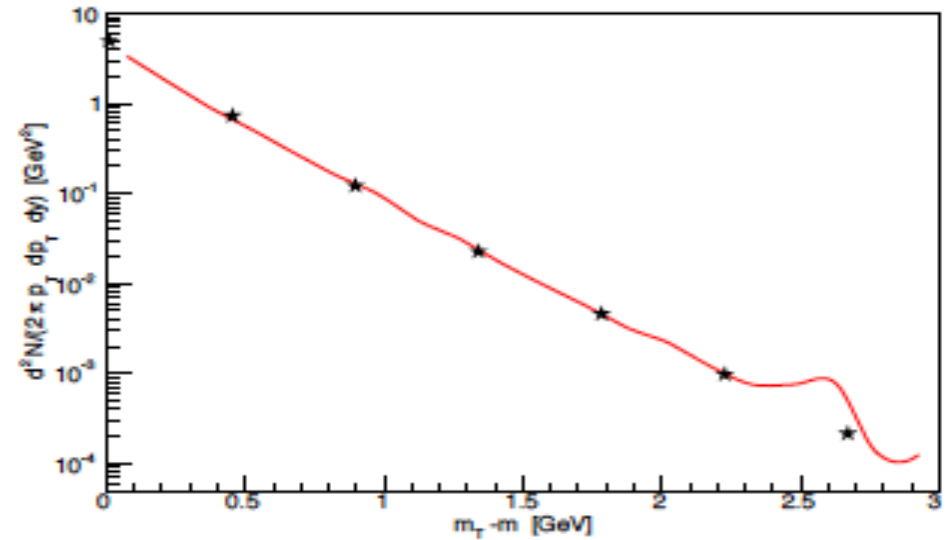
Yu. Karpenko & Yu. Ivanov: May 2014 - May 2015

$P_T / m_T$  distribution

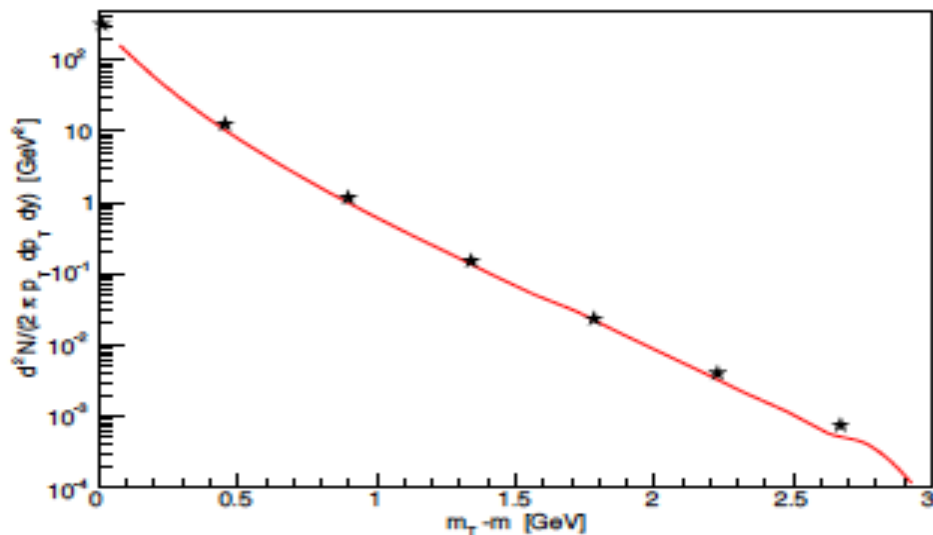
pions, fireball



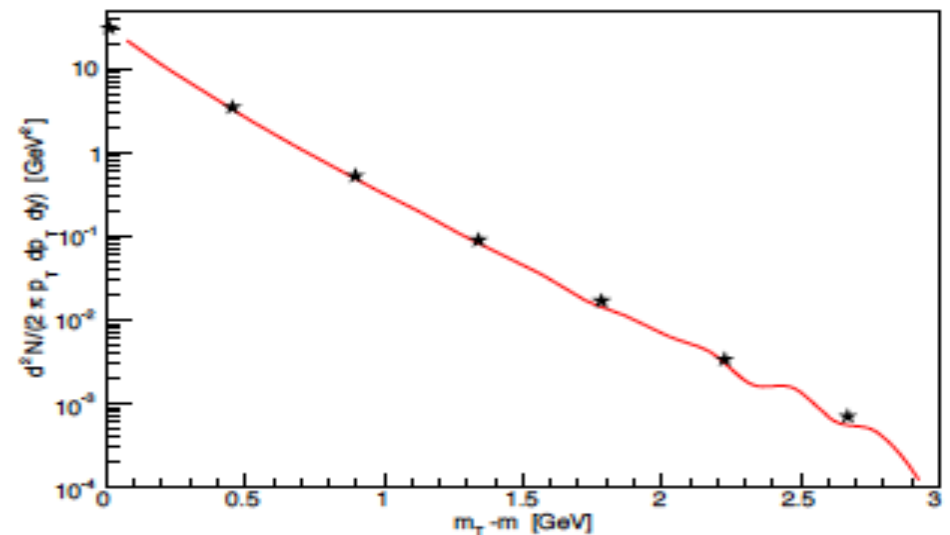
kaons, fireball



pions, baryon-rich



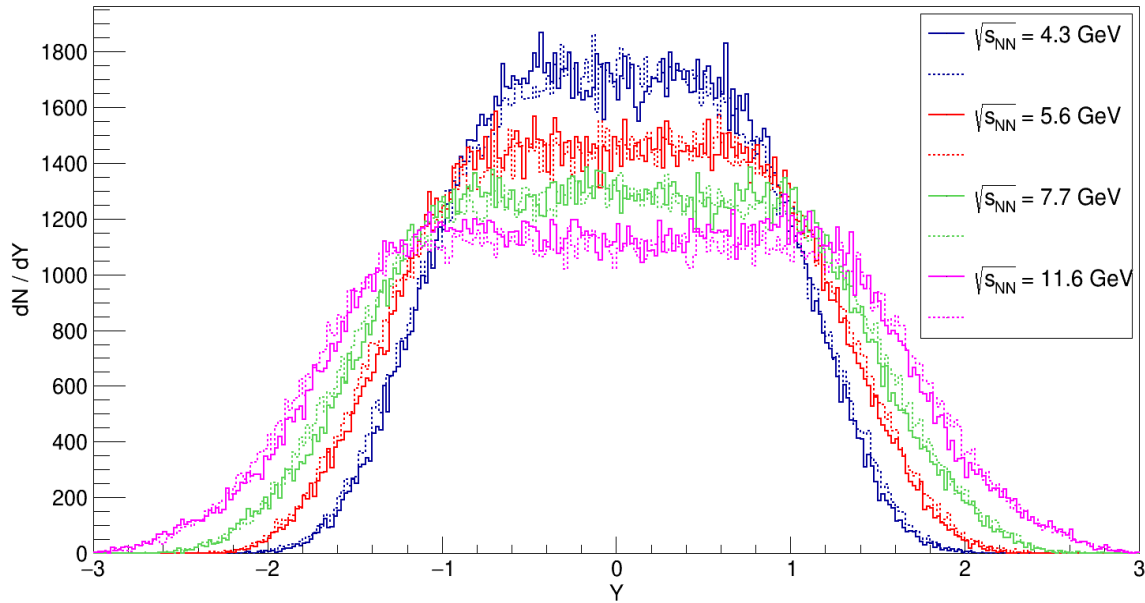
kaons, baryon-rich



**Baryon stopping signal for first order phase transition ?**

# Baryon stopping signal for first order phase transition ?

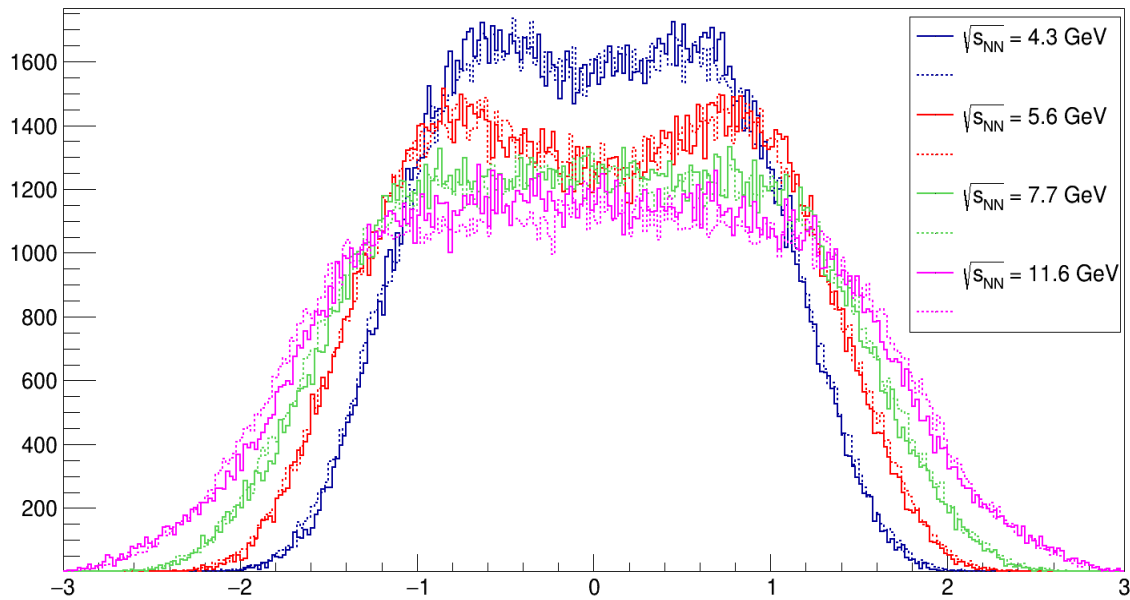
i1, EoS - crossover, solid line - 3FD, dashed line - 3FD + UrQMD



EoS: Crossover

Impact parameter:  $b=2$  fm

i1, EoS - 1PT, solid line - 3FD, dashed line = 3FD + UrQMD



EoS: First order PT

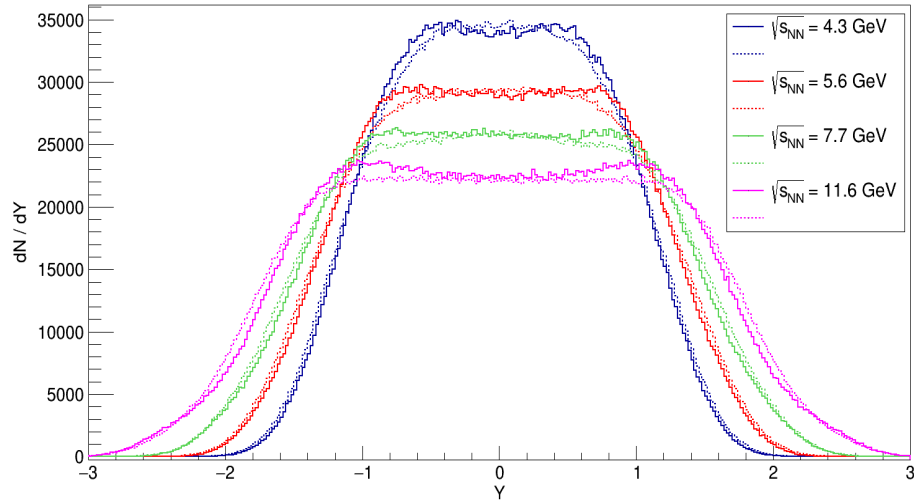
Impact parameter:  $b=2$  fm



# Baryon stopping signal for first order phase transition ?

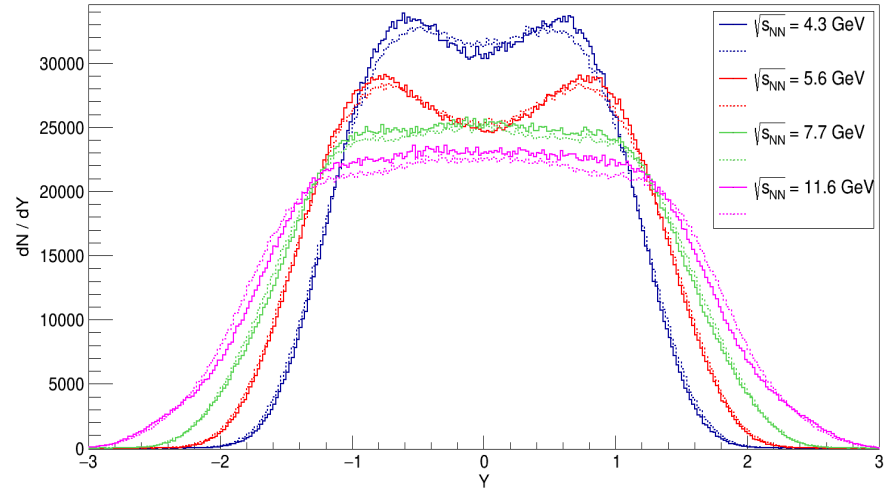
EoS: Crossover

b = 2 fm, EoS - crossover, 3FD (solid line), 3FD + UrQMD (dashed line)



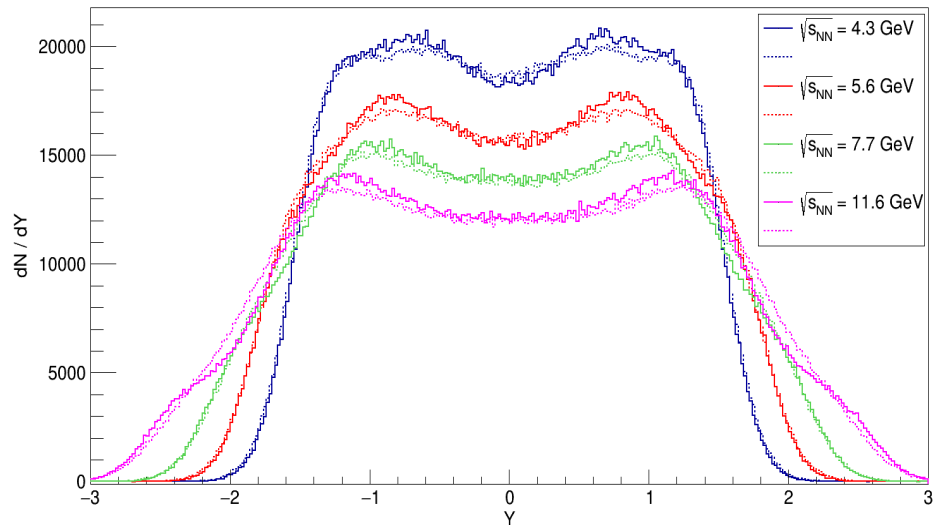
EoS: First order PT

b = 2 fm, EoS - 1PT, 3FD (solid line), 3FD+ UrQMD (dashed line)

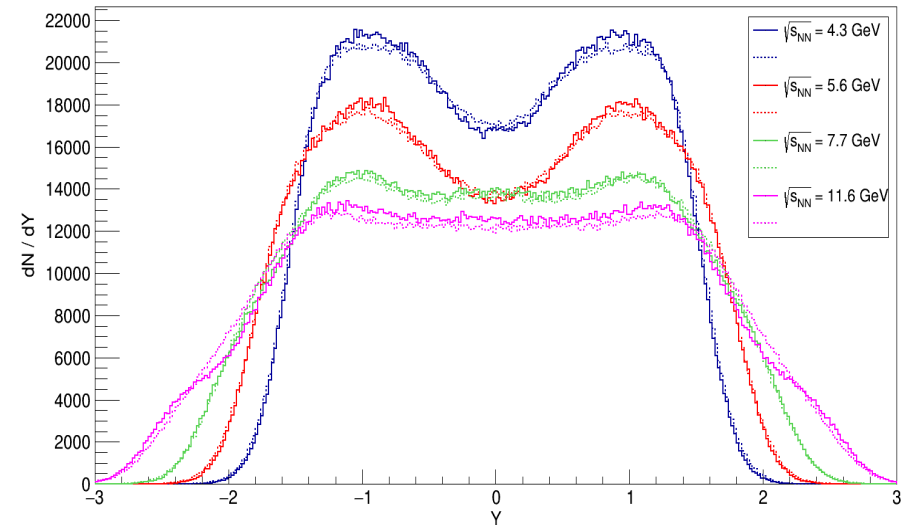


i1:  
b=2 fm

b = 6 fm, EoS - crossover, 3FD (solid line), 3FD + UrQMD (dashed line)

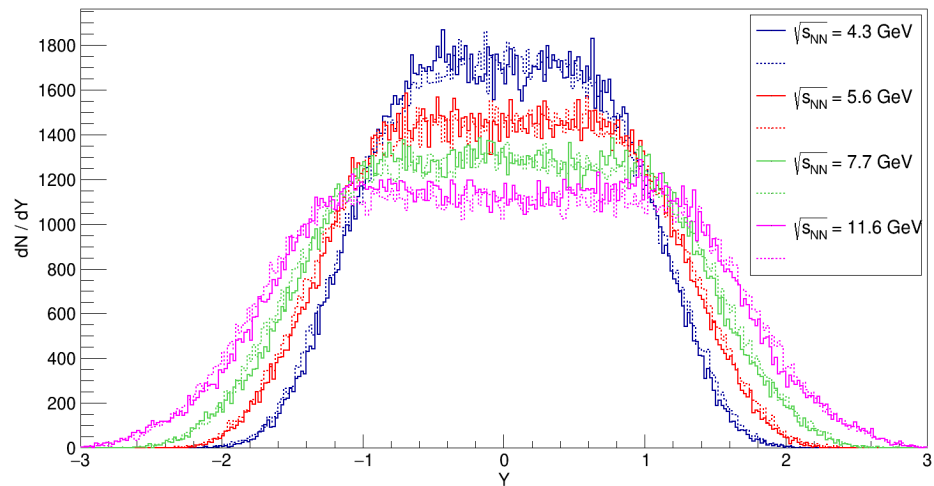


b = 6 fm, EoS - 1PT, 3FD (solid line), 3FD + UrQMD (dashed line)

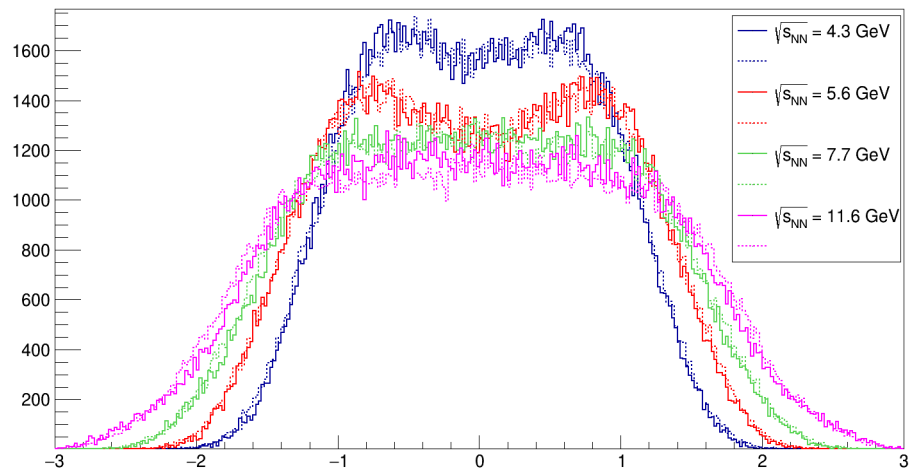


i3:  
b=6 fm

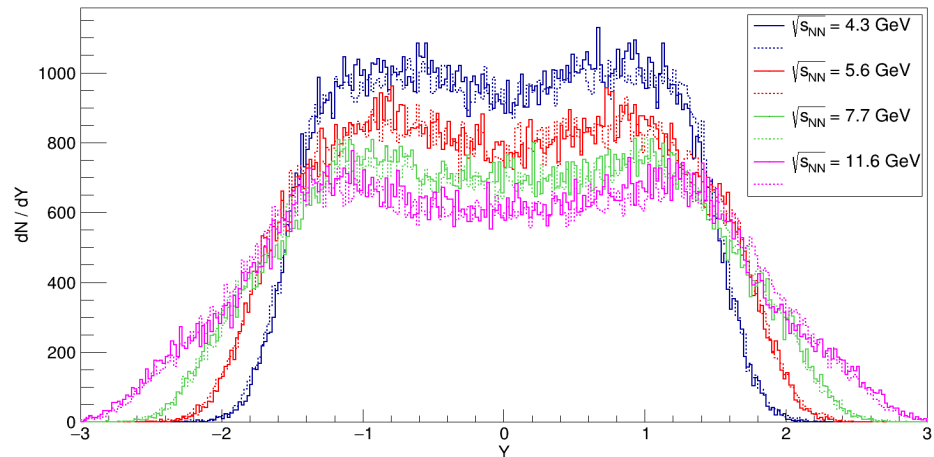
i1, EoS - crossover, solid line - 3FD, dashed line - 3FD + UrQMD



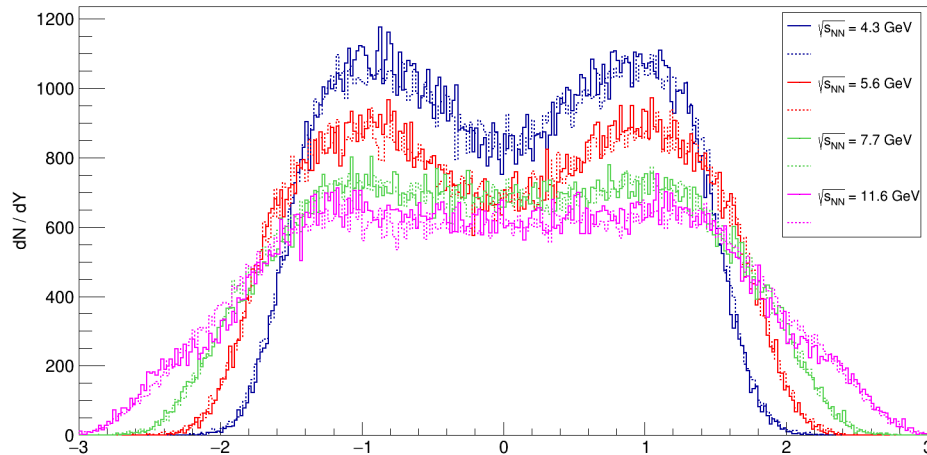
i1, EoS - 1PT, solid line - 3FD, dashed line = 3FD + UrQMD



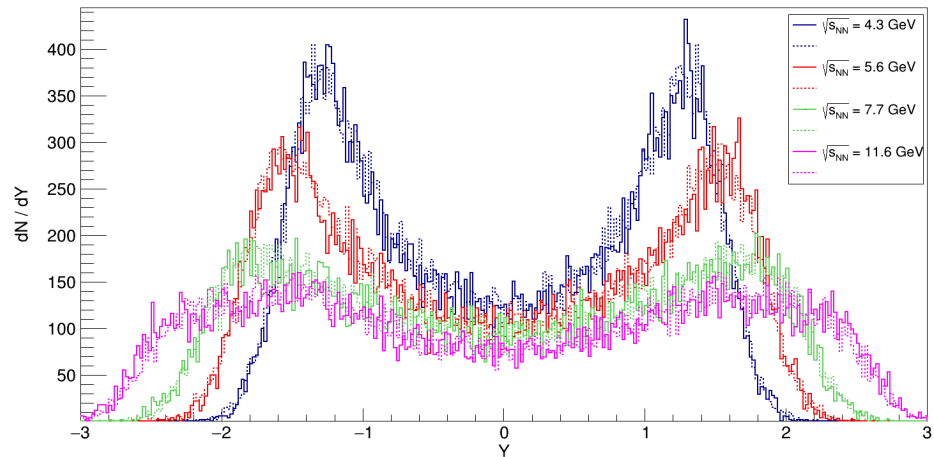
i3, EoS - crossover, solid line - 3FD, dashed line - 3FD + UrQMD



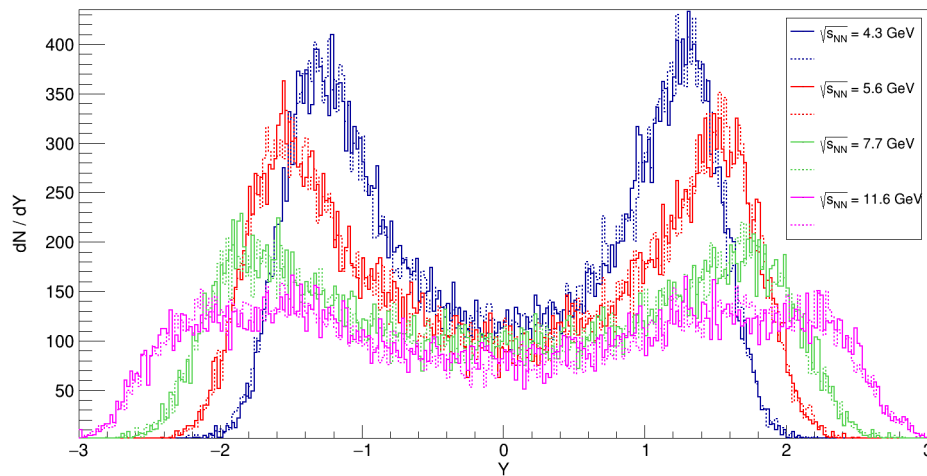
i3, EoS- 1PT, solid line - 3FD, dashed line - 3FD + UrQMD



i5, EoS - crossover, solid line - 3FD, dashed line - 3FD + UrQMD

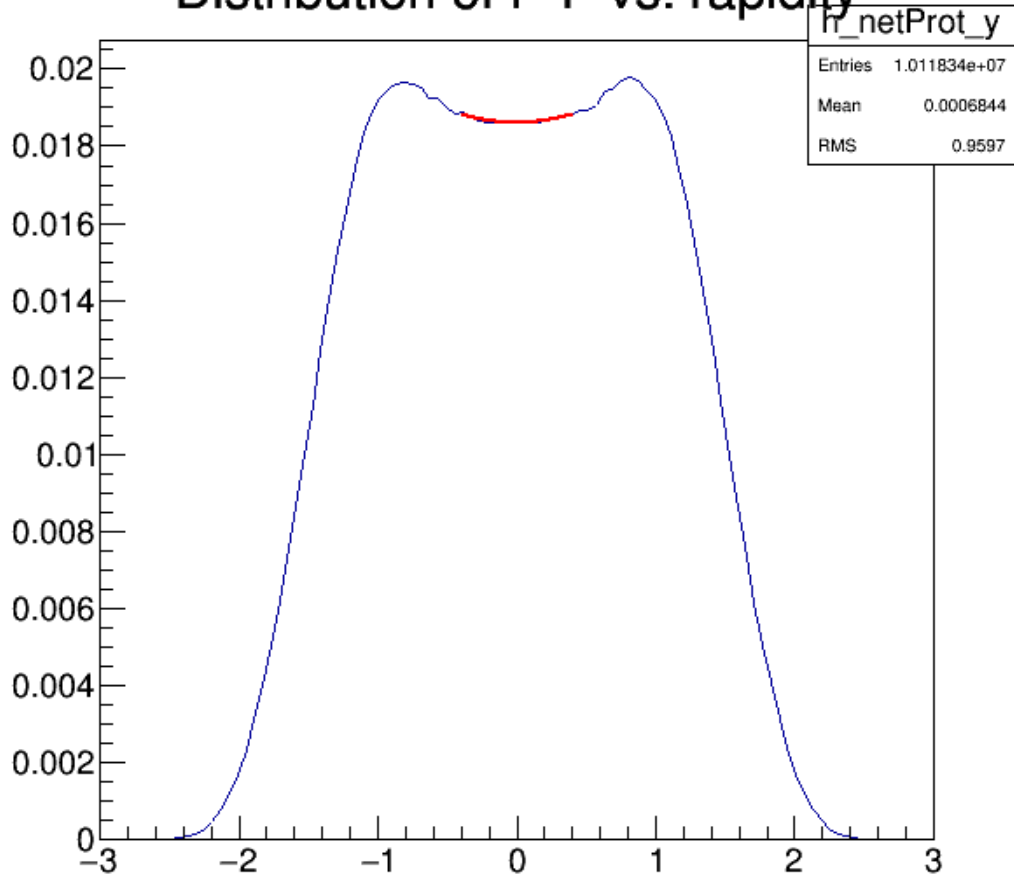


i5, EoS - 1PT, solid line - 3FD, dashed line - 3FD + UrQMD

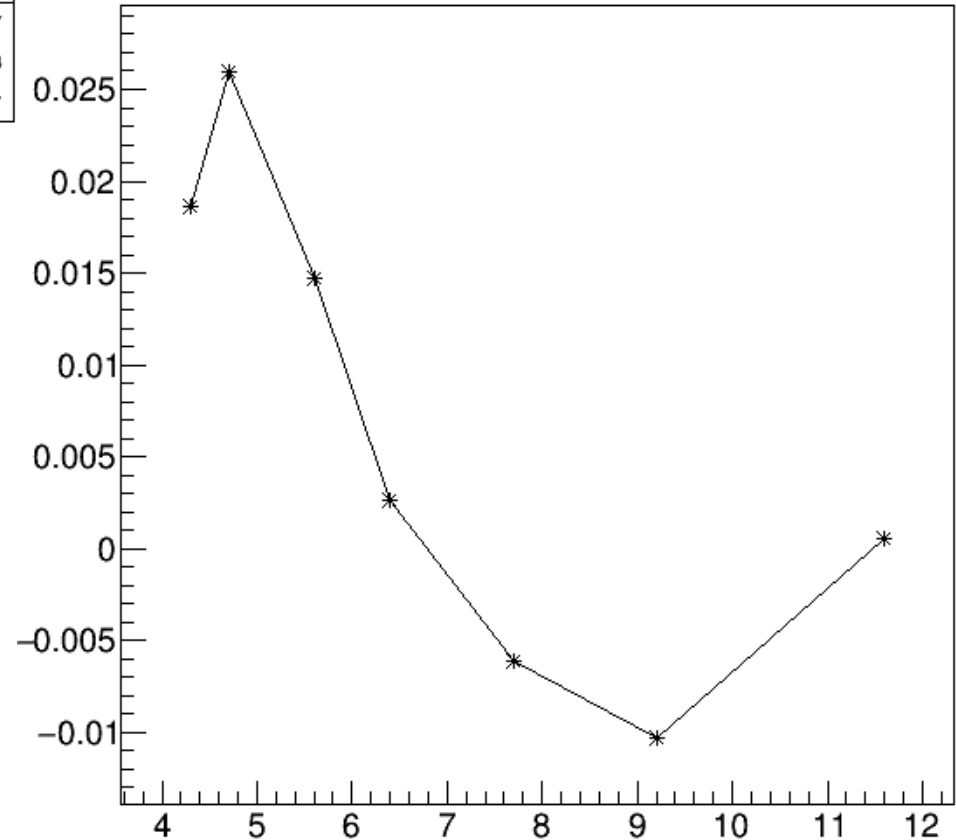


# Baryon stopping signal: extract the curvature at midrapidity !

Distribution of  $P-\bar{P}$  vs. rapidity



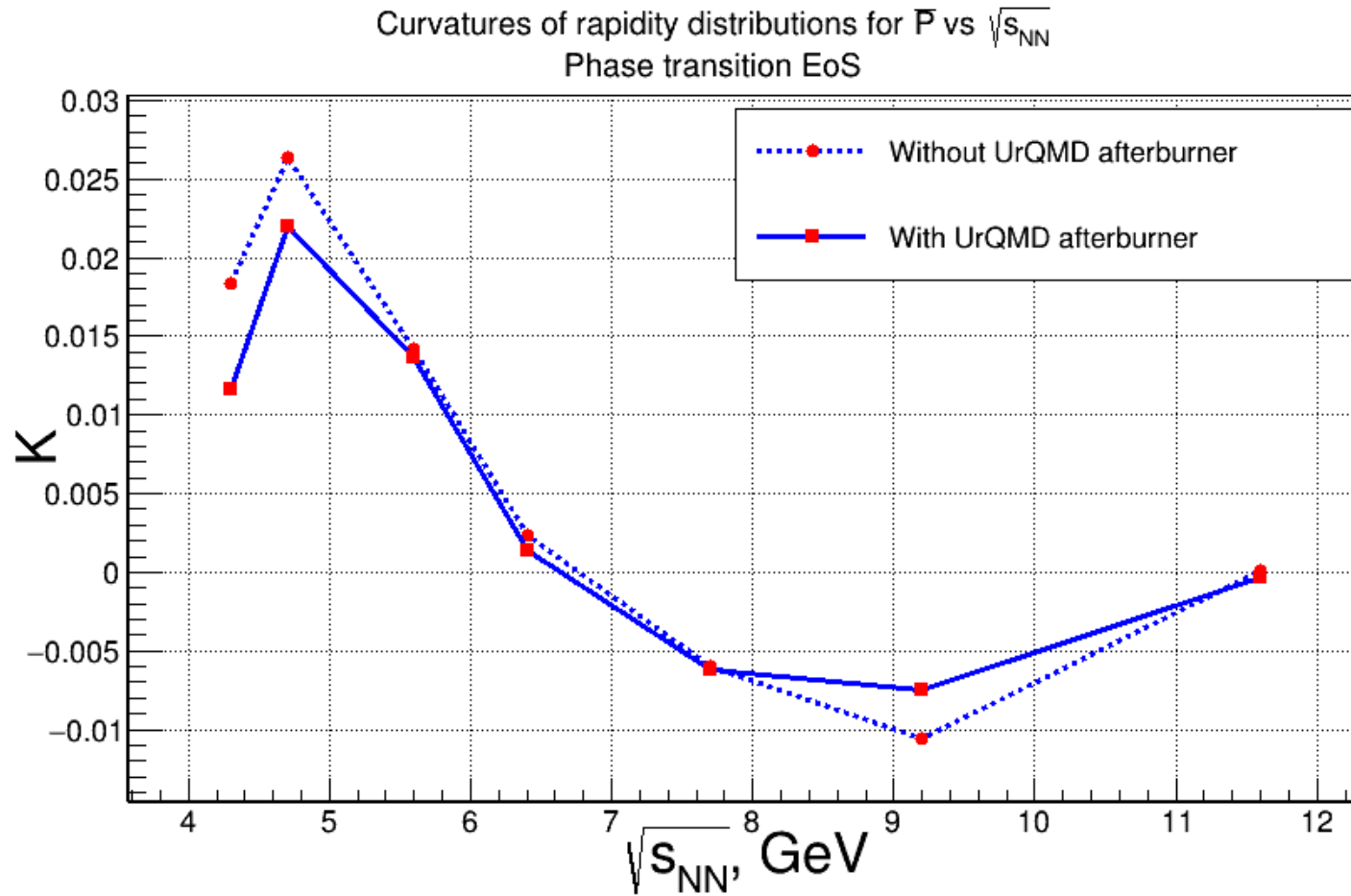
Curvatures of rapidity distributions for  $\bar{P}$  vs  $\sqrt{s_{NN}}$



“Wiggle” in the excitation function (energy scan) confirmed !

Particlization of 3FH works satisfactorily → join with UrQMD “afterburner”

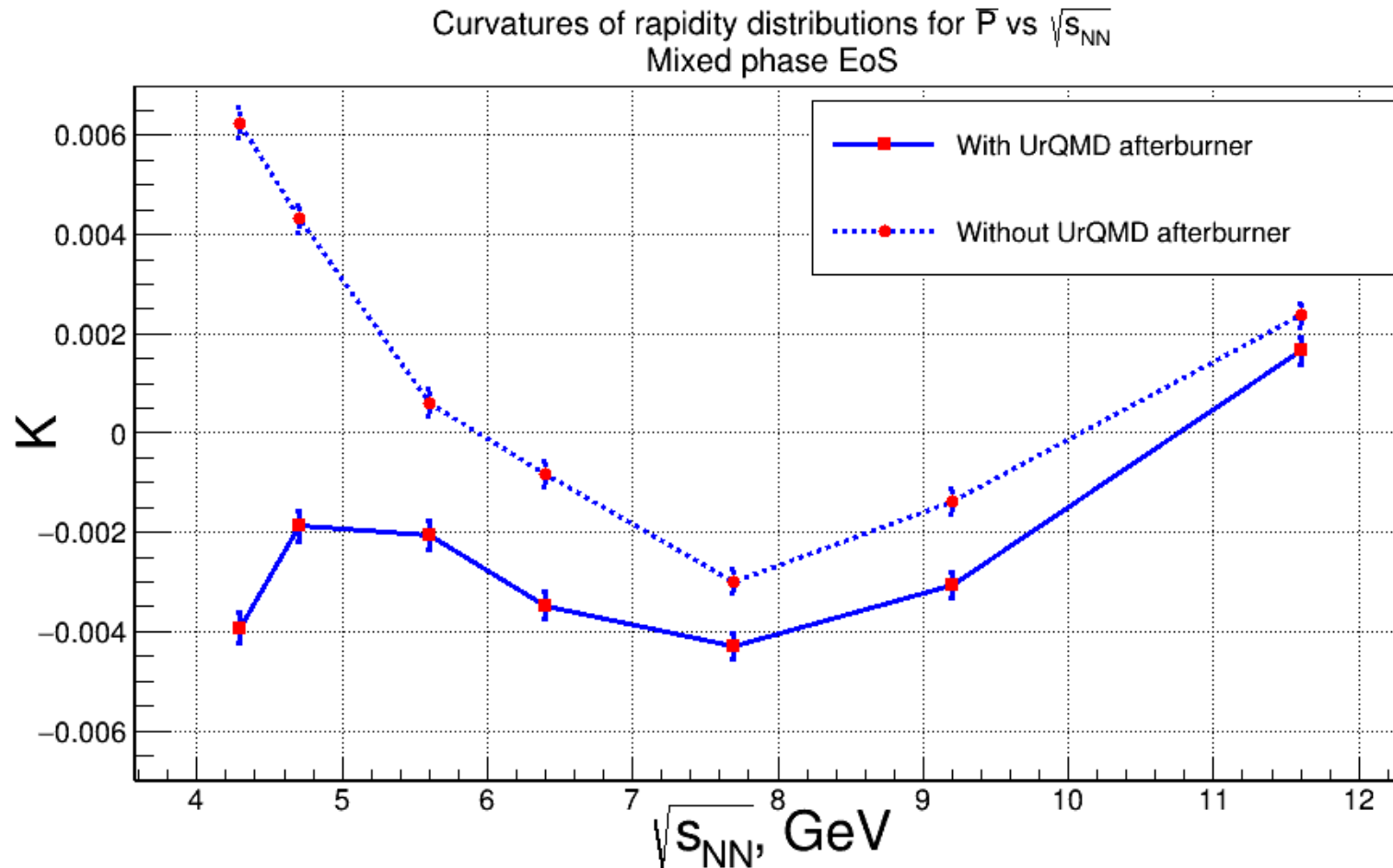
# Baryon stopping signal: effect of UrQMD afterburner !



“Wiggle” in the excitation function (energy scan) confirmed !

UrQMD afterburner almost no effect on the “wiggle” for the 2PT EoS

# Baryon stopping signal: effect of EoS & UrQMD afterburner !

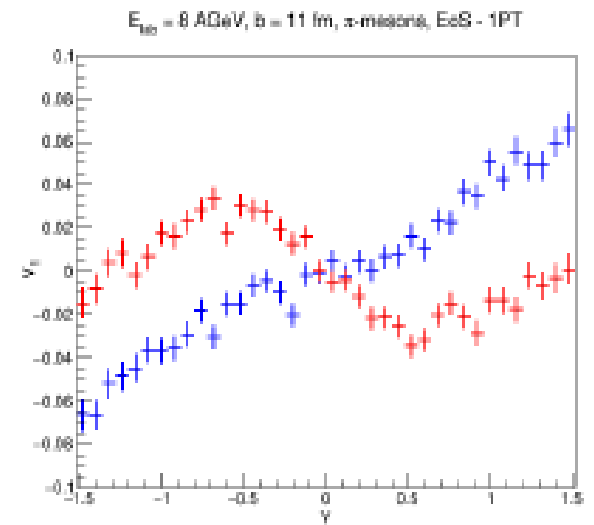
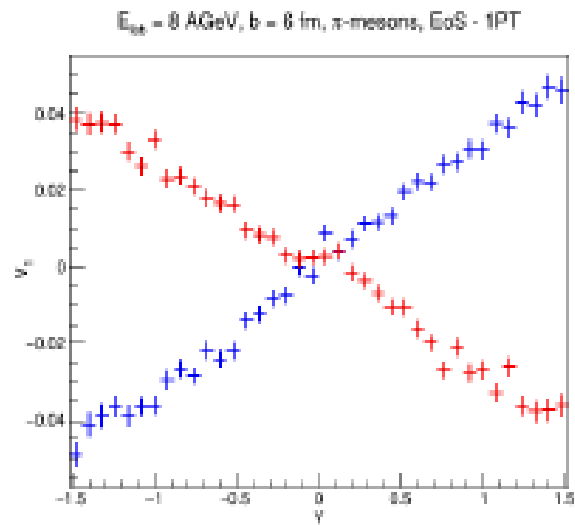
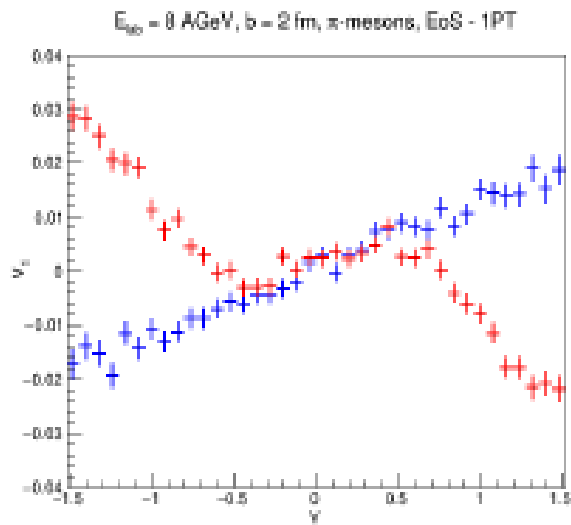
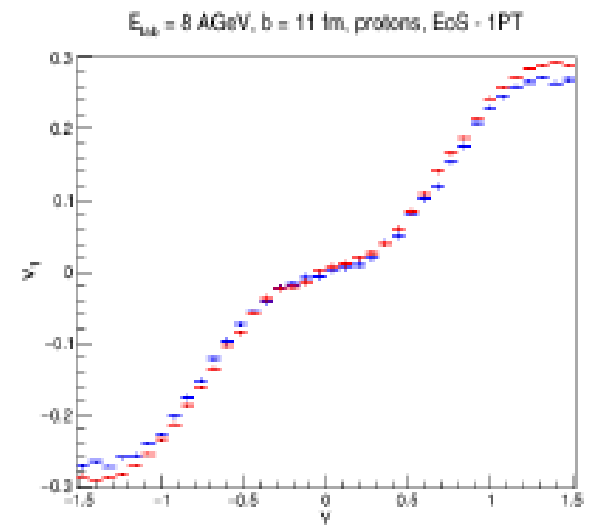
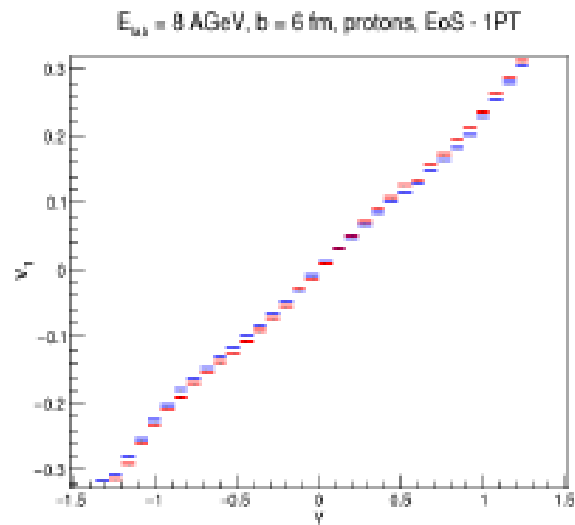
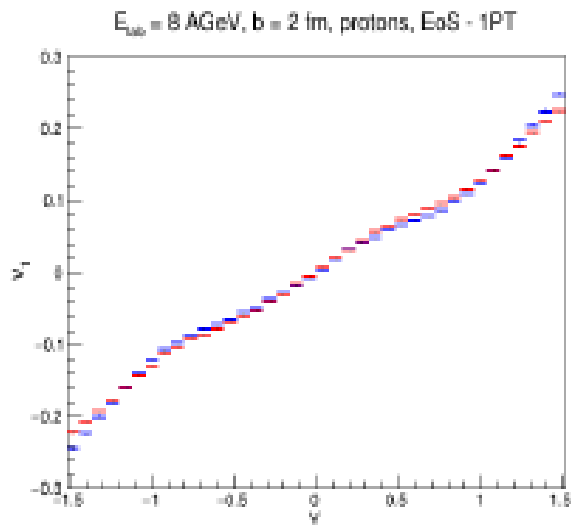


“Wiggle” in the excitation function (energy scan) confirmed !

UrQMD afterburner smoothens the “wiggle” for the crossover EoS !

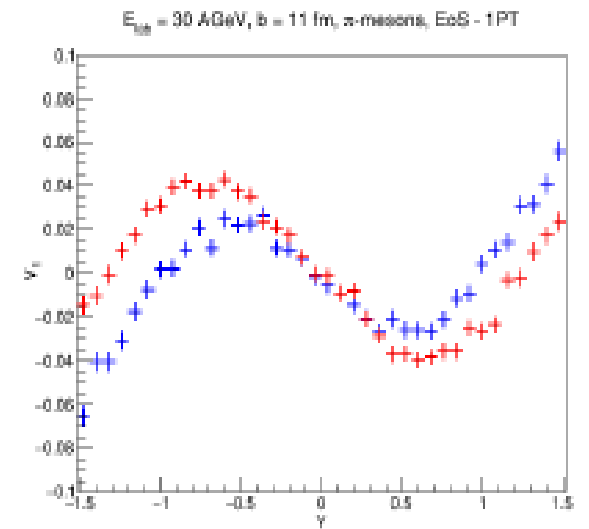
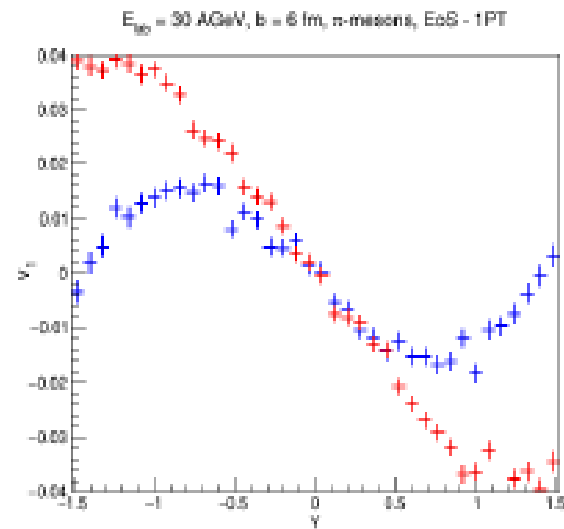
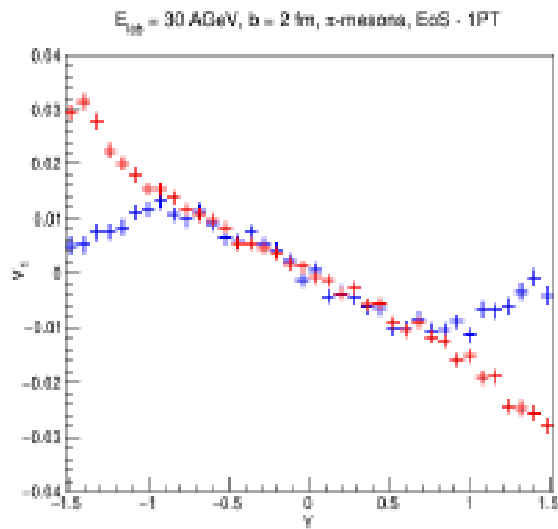
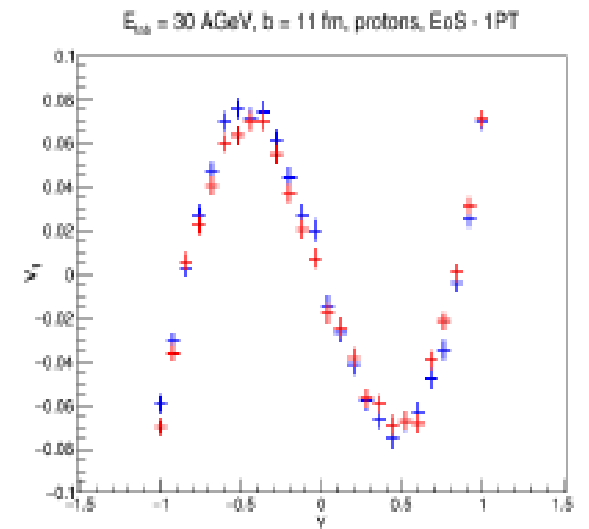
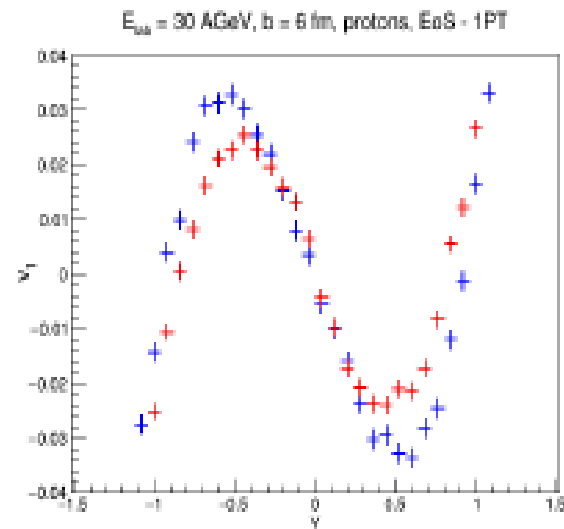
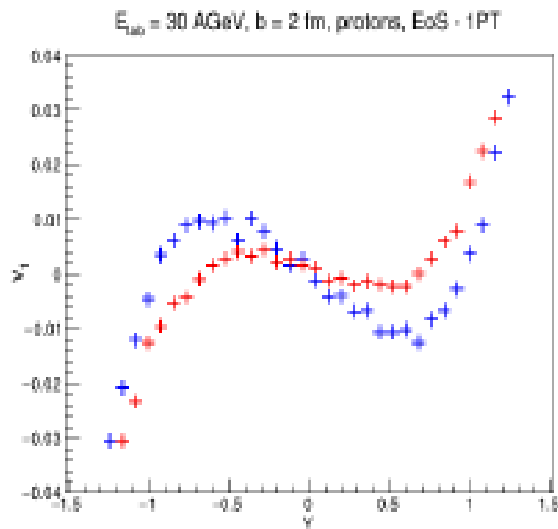
# What about flow? First results (v1) ....

$E_{lab} = 8 \text{ AGeV}$



Blocking of pions by hadronic rescattering (red plusses), upper w/o UrQMD !

# What about flow? First results(v1) .... $E_{lab} = 30$ AGeV



Antiflow of protons  $\rightarrow$  pions follow !

## **Further developments:**

- MPD Detector simulation (Oleg Rogachevsky et al.)**
- New 2-phase EoS (Wroclaw group, see talk Bastian)**



**A new class of 2-phase EoS:  
Motivation from Astrophysics**

# Modern 2-phase EoS in Astrophysics of Compact Stars

## 1. Pauli blocking effect → Excluded volume

Well known from modeling dissociation of clusters in the supernova EoS:

- excluded volume: Lattimer-Swesty (1991), Shen-Toki-Oyematsu-Sumiyoshi (1996), ...
- Pauli blocking: Roepke-Grigo-Sumiyoshi-Shen (2003), Typel et al. PRC 81 (2010)
- excl. Vol. vs. Pauli blocking: Hempel, Schaffner-Bielich, Typel, Roepke PRC 84 (2011)

**Here:** nucleons as quark clusters with finite size --> excluded volume effect !

Available volume fraction:  $\Phi = V_{av}/V = 1 - v \sum_{i=n,p} n_i$ ,  $v = \frac{1}{2} \frac{4\pi}{3} (2r_{nuc})^3 = 4V_{nuc}$

Equations of state for T=0 nuclear matter:  $p_{tot}(\mu_n, \mu_p) = \frac{1}{\Phi} \sum_{i=n,p} p_i + p_{mes}$ ,

$$p_i = \frac{1}{4} (E_i n_i - m_i^* n_i^{(s)}),$$

$$\varepsilon_{tot}(\mu_n, \mu_p) = -p_{tot} + \sum_{i=n,p} \mu_i n_i,$$

$$n_i = \frac{\Phi}{3\pi^3} k_i^3,$$

$$n_i^{(s)} = \frac{\Phi m_i^*}{2\pi^2} \left[ E_i k_i - (m_i^*)^2 \ln \frac{k_i + E_i}{m_i^*} \right],$$

Effective mass:  $m_i^* = m_i - S_i$ .

Scalar meanfield:  $S_i \sim n_i^{(s)}$

$$E_i = \sqrt{k_i^2 + (m_i^*)^2} = \mu_i - V_i - \frac{v}{\Phi} \sum_{j=p,n} p_j,$$

Vector meanfield:  $V_i \sim n_i$

# Modern 2-phase EoS in Astrophysics of Compact Stars

## 2. Stiff quark matter at high densities

S. Benic, Eur. Phys. J. A 50, 111 (2014)

$$\mathcal{L} = \bar{q}(i\cancel{\partial} - m)q + \mu_q \bar{q}\gamma^0 q + \mathcal{L}_4 + \mathcal{L}_8, \quad \mathcal{L}_4 = \frac{g_{20}}{\Lambda^2} [(\bar{q}q)^2 + (\bar{q}i\gamma_5\tau q)^2] - \frac{g_{02}}{\Lambda^2} (\bar{q}\gamma_\mu q)^2,$$
$$\mathcal{L}_8 = \frac{g_{40}}{\Lambda^8} [(\bar{q}q)^2 + (\bar{q}i\gamma_5\tau q)^2]^2 - \frac{g_{04}}{\Lambda^8} (\bar{q}\gamma_\mu q)^4 - \frac{g_{22}}{\Lambda^8} (\bar{q}\gamma_\mu q)^2 [(\bar{q}q)^2 + (\bar{q}i\gamma_5\tau q)^2]$$

**Meanfield approximation:**  $\mathcal{L}_{\text{MF}} = \bar{q}(i\cancel{\partial} - M)q + \tilde{\mu}_q \bar{q}\gamma^0 q - U,$

$$M = m + 2\frac{g_{20}}{\Lambda^2} \langle \bar{q}q \rangle + 4\frac{g_{40}}{\Lambda^8} \langle \bar{q}q \rangle^3 - 2\frac{g_{22}}{\Lambda^8} \langle \bar{q}q \rangle \langle q^\dagger q \rangle^2,$$

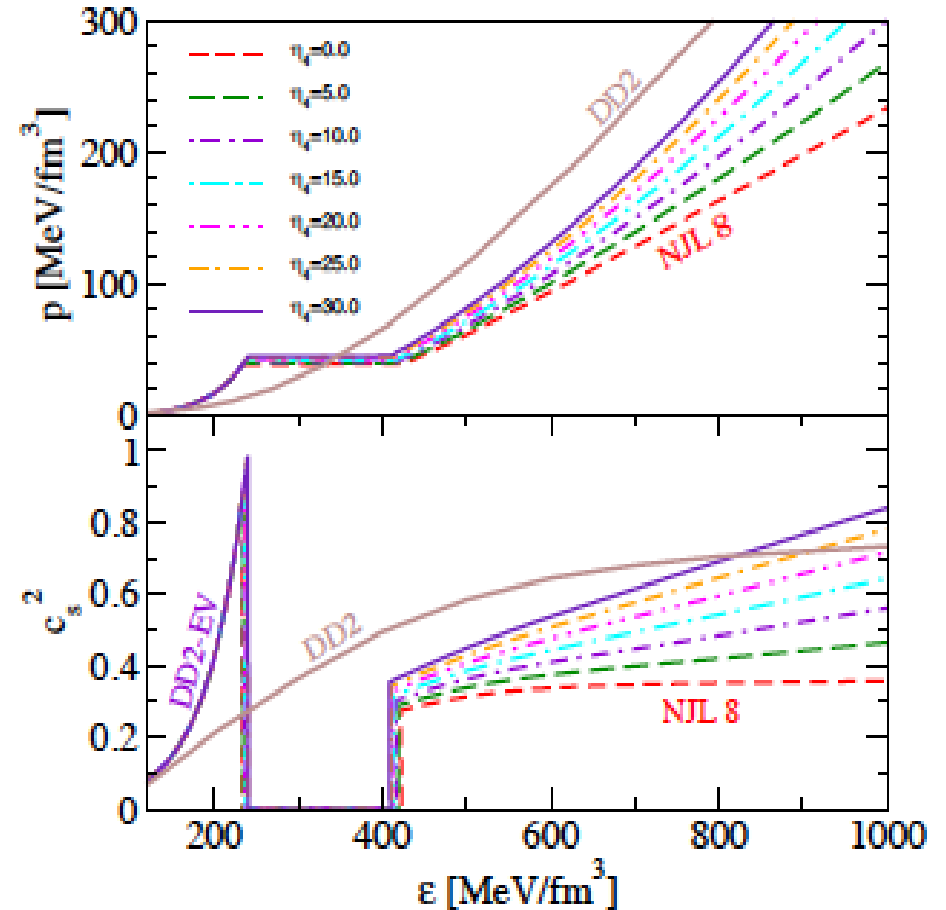
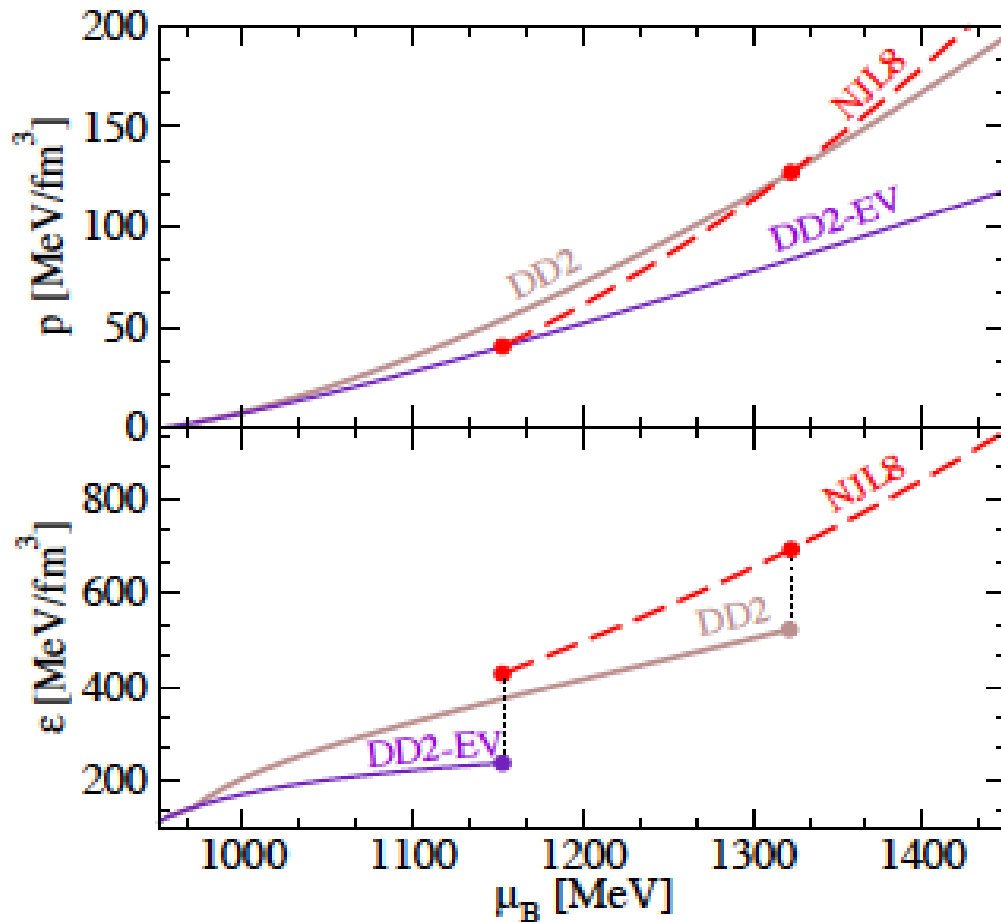
$$\tilde{\mu}_q = \mu_q - 2\frac{g_{02}}{\Lambda^2} \langle q^\dagger q \rangle - 4\frac{g_{04}}{\Lambda^8} \langle q^\dagger q \rangle^3 - 2\frac{g_{22}}{\Lambda^8} \langle \bar{q}q \rangle^2 \langle q^\dagger q \rangle,$$

$$U = \frac{g_{20}}{\Lambda^2} \langle \bar{q}q \rangle^2 + 3\frac{g_{40}}{\Lambda^8} \langle \bar{q}q \rangle^4 - 3\frac{g_{22}}{\Lambda^8} \langle \bar{q}q \rangle^2 \langle q^\dagger q \rangle^2 - \frac{g_{02}}{\Lambda^2} \langle q^\dagger q \rangle^2 - 3\frac{g_{04}}{\Lambda^8} \langle q^\dagger q \rangle^4.$$

**Thermodynamic Potential:**

$$\Omega = U - 2N_f N_c \int \frac{d^3 p}{(2\pi)^3} \left\{ E + T \log[1 + e^{-\beta(E - \tilde{\mu}_q)}] + T \log[1 + e^{-\beta(E + \tilde{\mu}_q)}] \right\} + \Omega_0$$

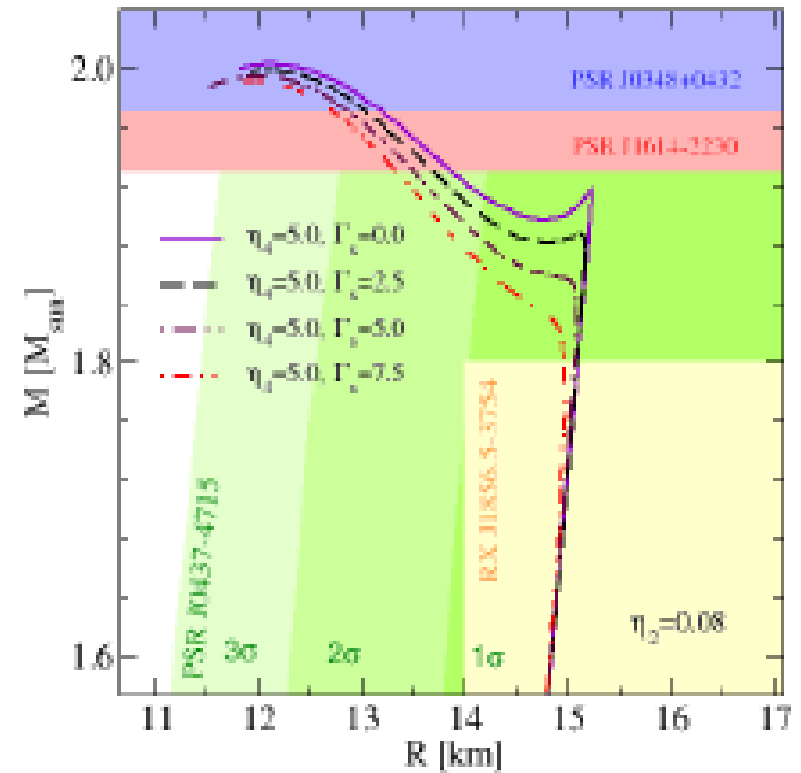
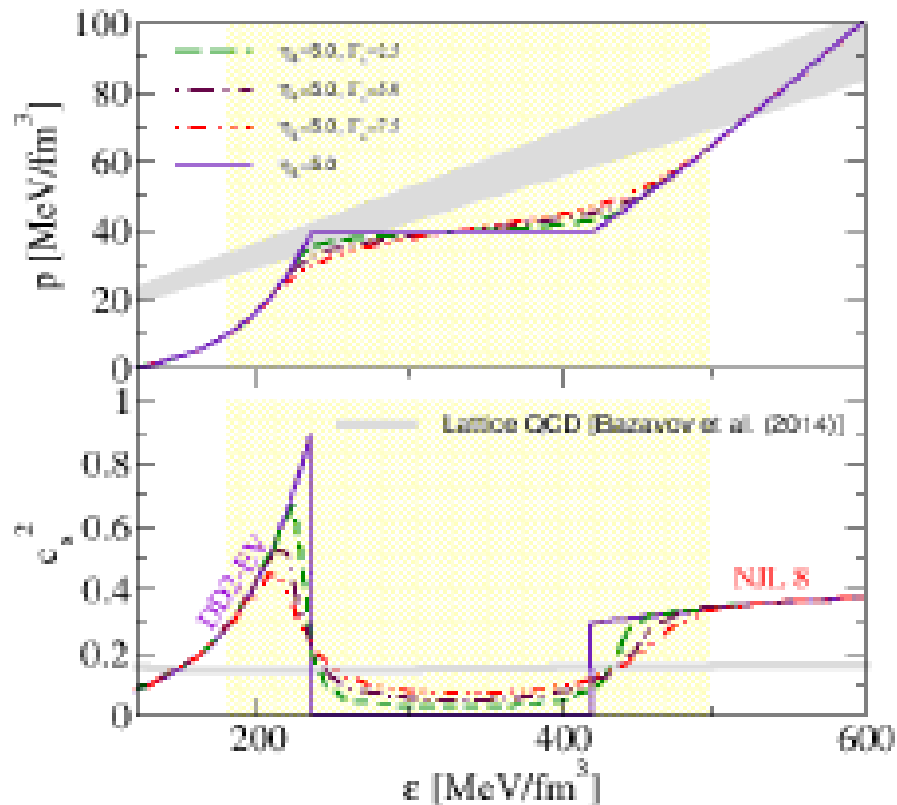
# Modern 2-phase EoS in Astrophysics of Compact Stars



Here: Stiffening of dense hadronic matter by excluded volume in density-dependent RMF  
 Stiffening of dense quark matter by higher order quark vector current interactions ( $\eta_4$ )

# Modern 2-phase EoS in Astrophysics of Compact Stars

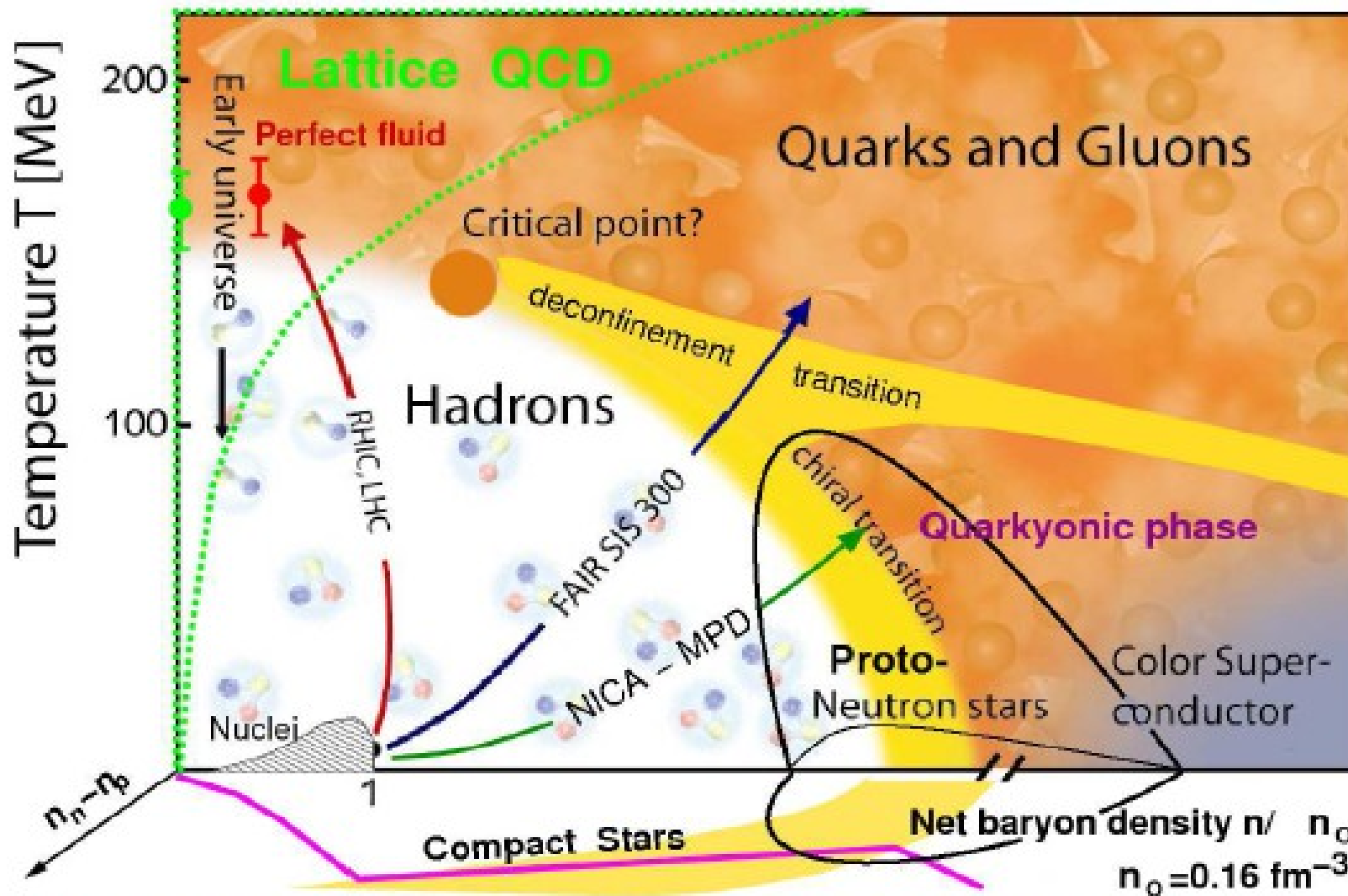
Estimate effects of structures in the phase transition region (“pasta”)



High-mass Twins relatively robust against “smoothing” the Maxwell transition construction

D. Alvarez-Castillo, D.B., [arxiv:1412.8463](https://arxiv.org/abs/1412.8463)

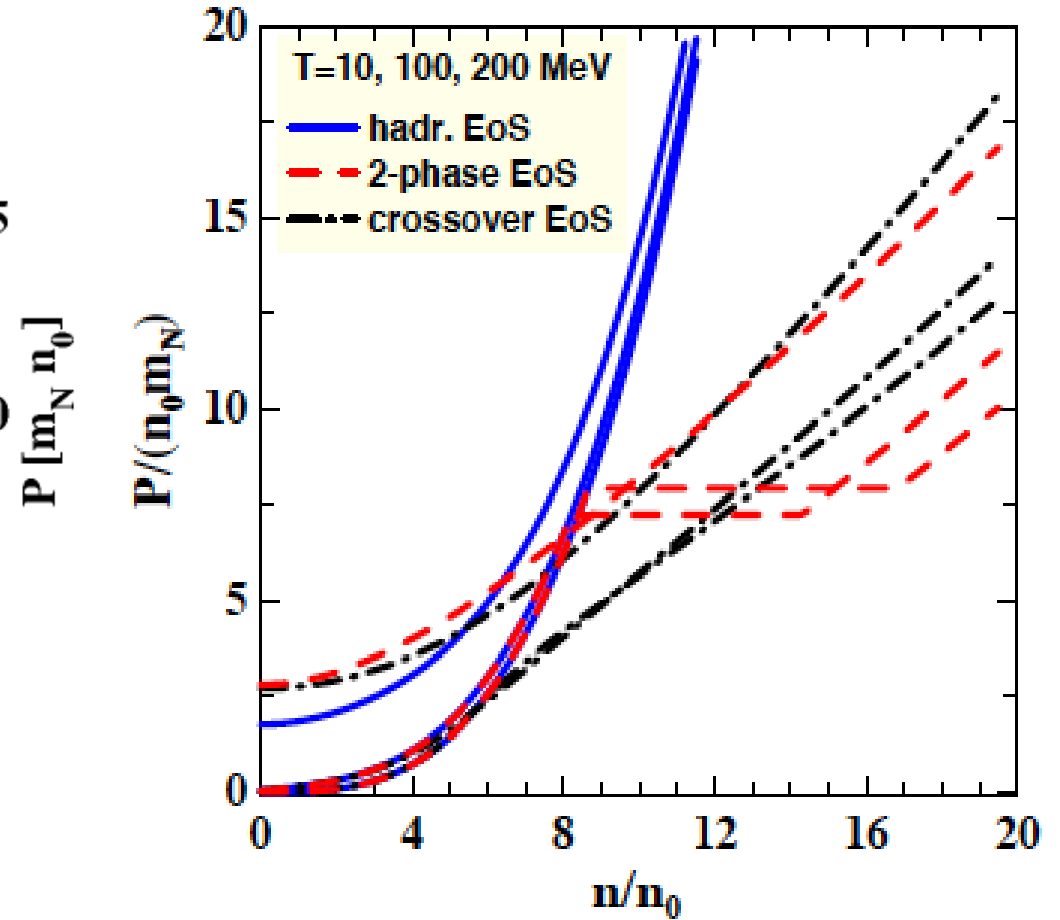
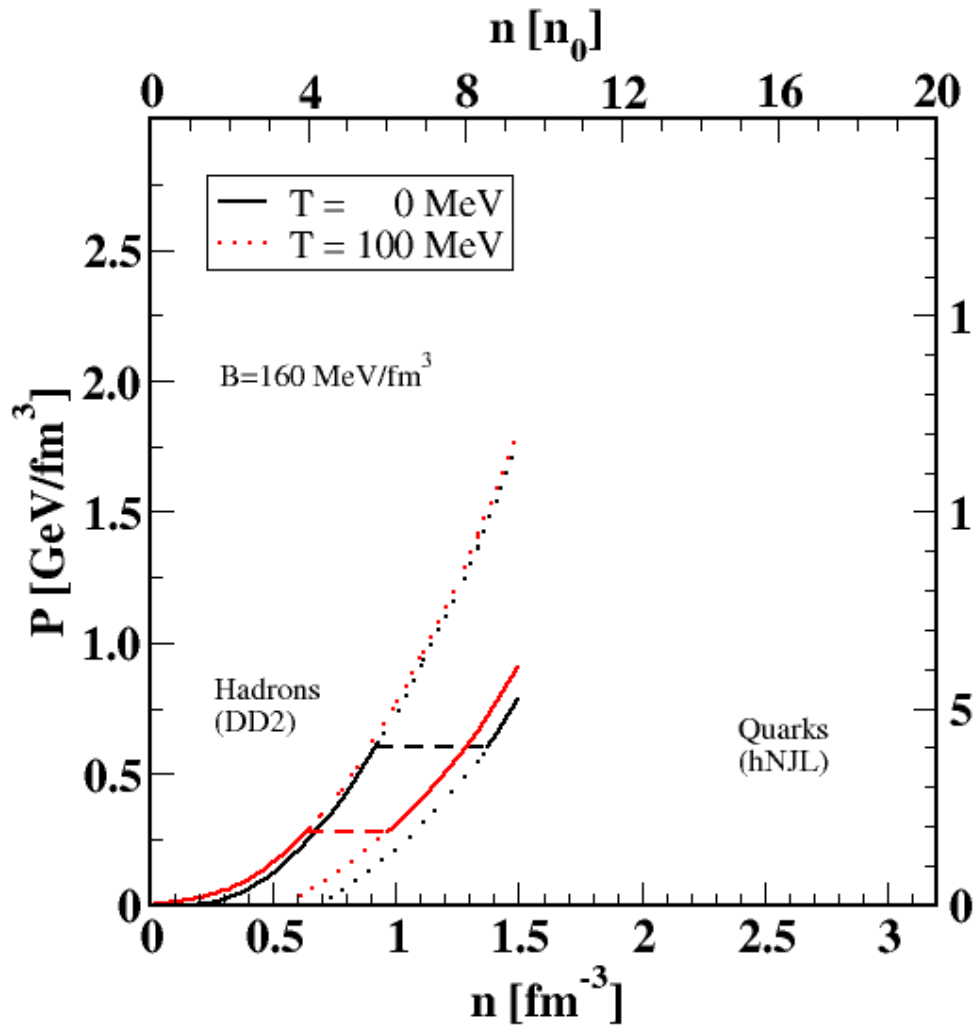
# Support a CEP in QCD phase diagram with Astrophysics?



NICA White Paper, <http://theor.jinr.ru/twiki-cgi/view/NICA/WebHome>

Crossover at finite  $T$  (Lattice QCD) + First order at zero  $T$  (Astrophysics) = Critical endpoint exists!

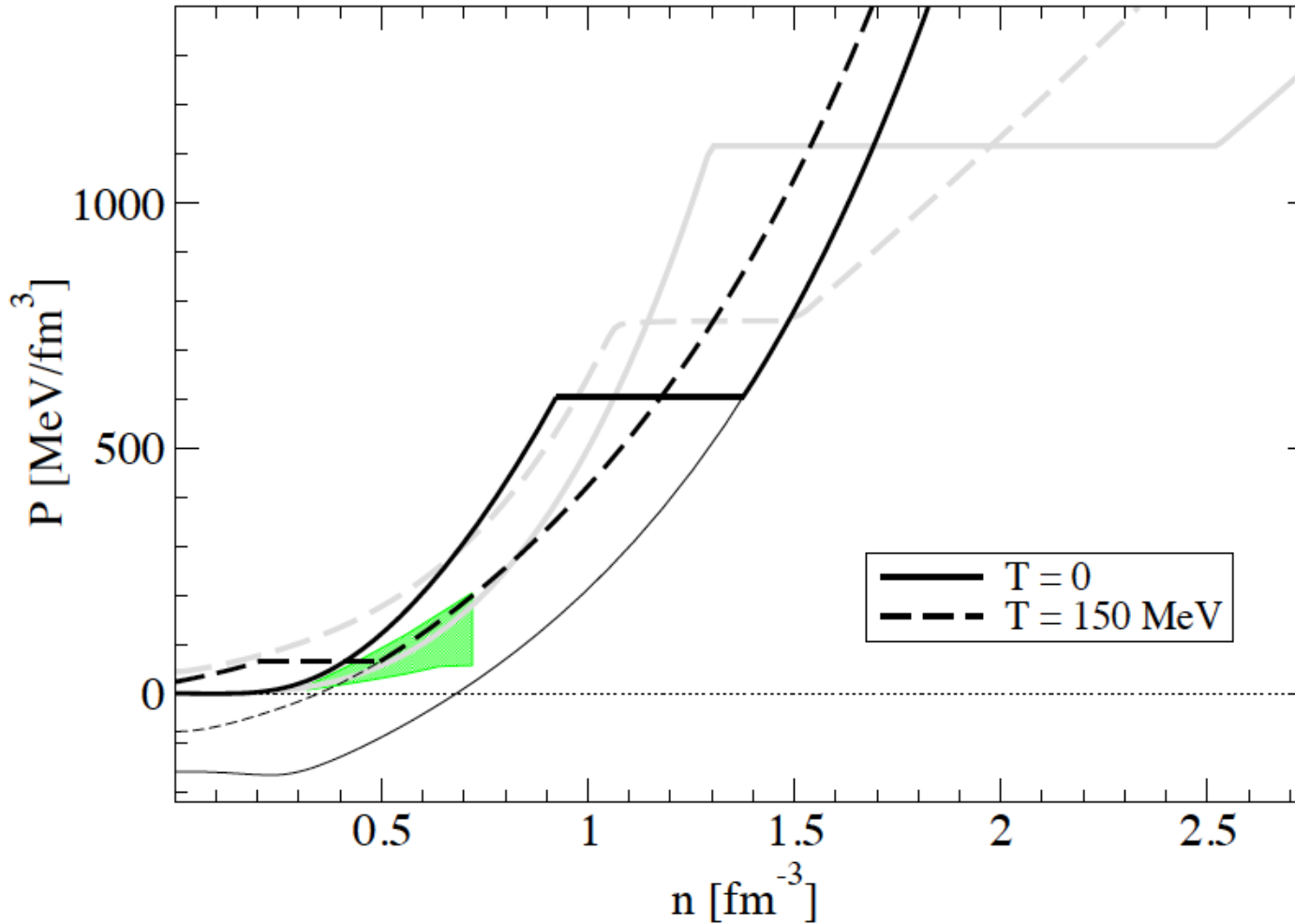
# Comparison 2-phase EoS



N.-U. Bastian, D. Blaschke (S. Benic, S. Typel),  
In progress (2015)

A. Khvorostukhin et al. EPJC 48 (2006) 531  
Yu. Ivanov, D. Blaschke, arxiv:1504.03992

# Comparison 2-phase EoS



Grey: Ivanov (2010)

Black: DD2 vs. HNJLb

$B = 160$  MeV/fm<sup>3</sup>



## Summary / Outlook:

- Baryon stopping signal (“wiggle”) remains a robust signal for 1<sup>st</sup> order PT also under severe cuts in transverse momentum !
- Discrimination between hadronic phase and crossover transition ambiguous
- Position of the “wiggle” in the beam energy scan is EoS dependent – new EoS ?!
- Particlization of 3-Fluid Hydrodynamics model works !
- UrQMD “afterburner” works too !
  
- Detector simulation in progress
- Systematic study of modern 2-phase EoS (Bayesian analysis) in progress



# Critical Point and Onset of Deconfinement 2016

and

Working Group Meeting of COST Action MP1304

Wrocław, Poland

May 30th - June 4th, 2016

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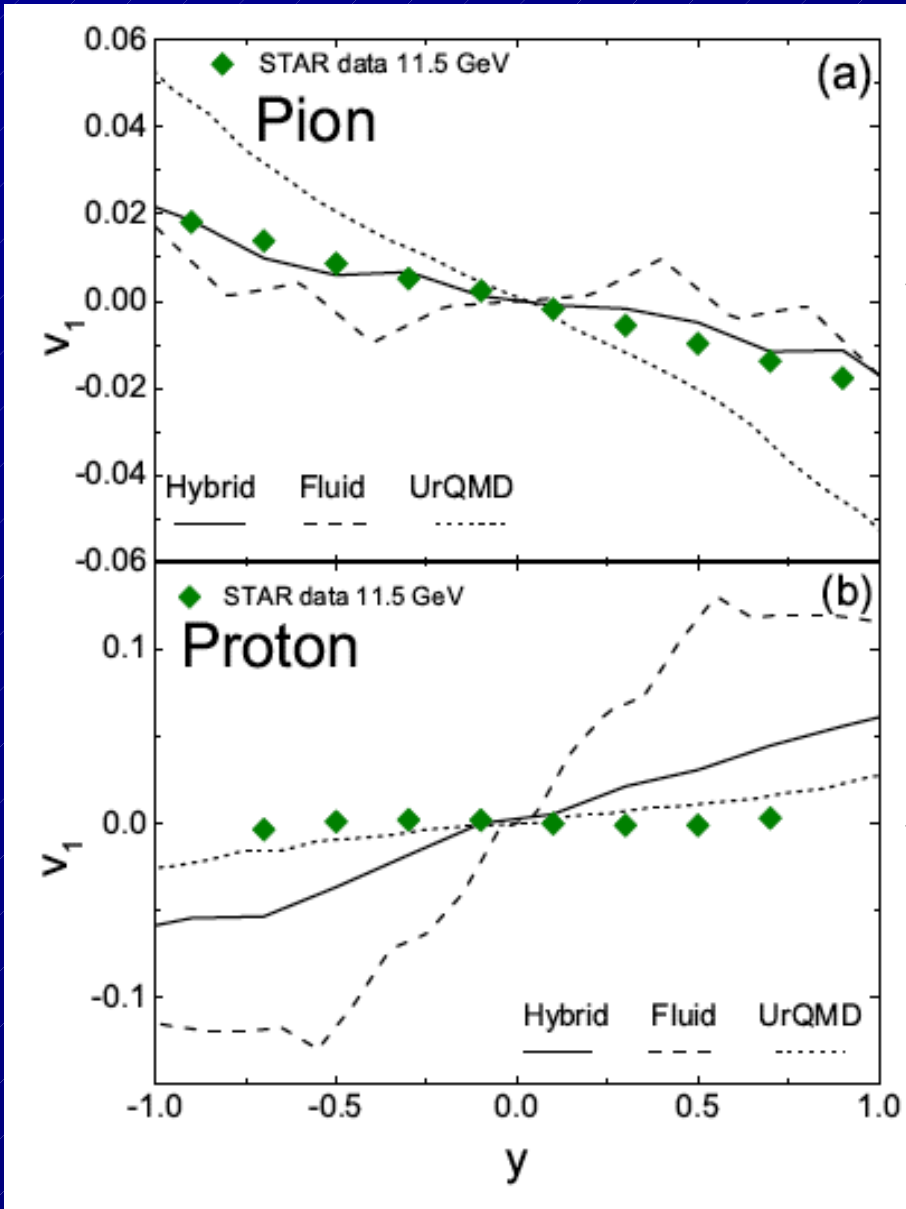
Springer



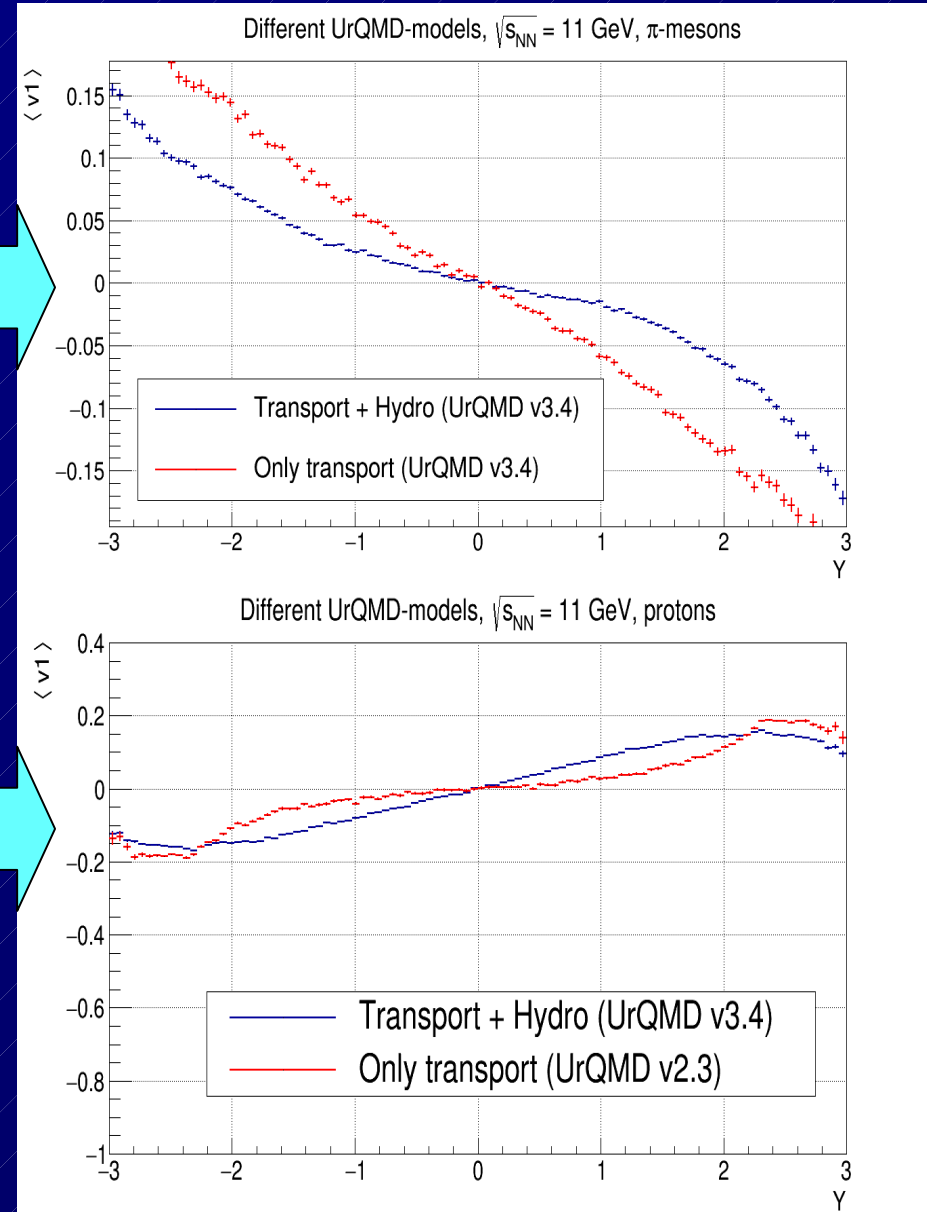
# **Additional Slides**

# Further results of test simulations – flow observables

J. Steinheimer et al., Phys. Rev. C89 (2014) 054913



Reproduced!



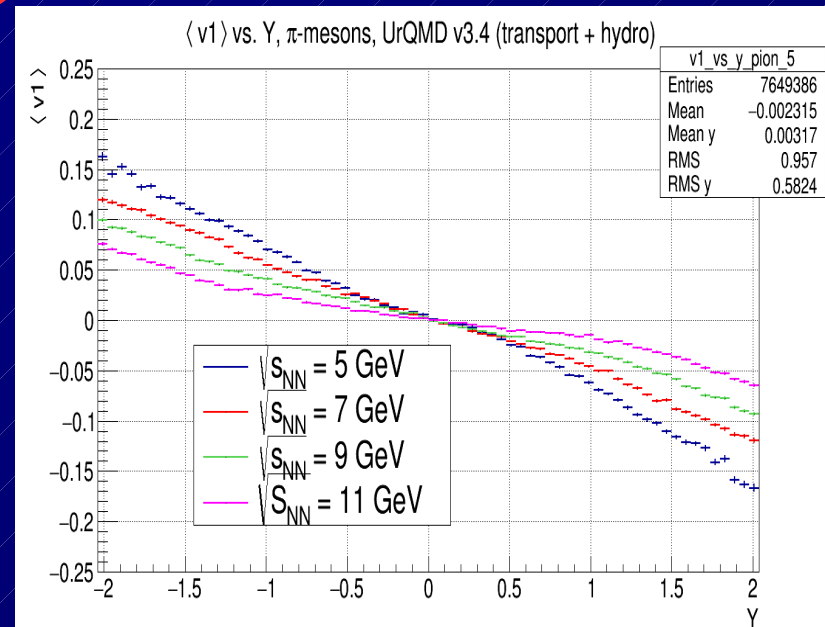
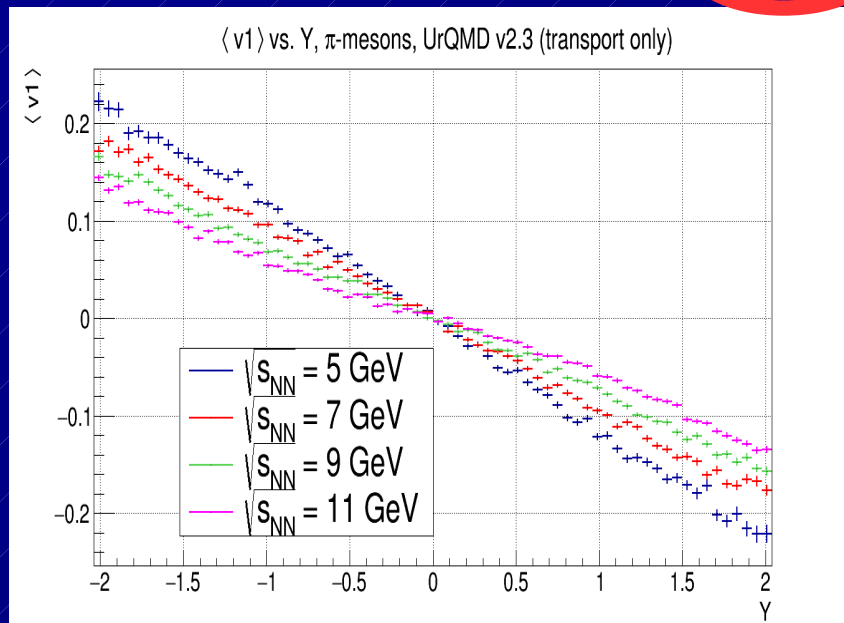
# Further results of test simulations – flow observables

NICA energy scan: **UrQMD**

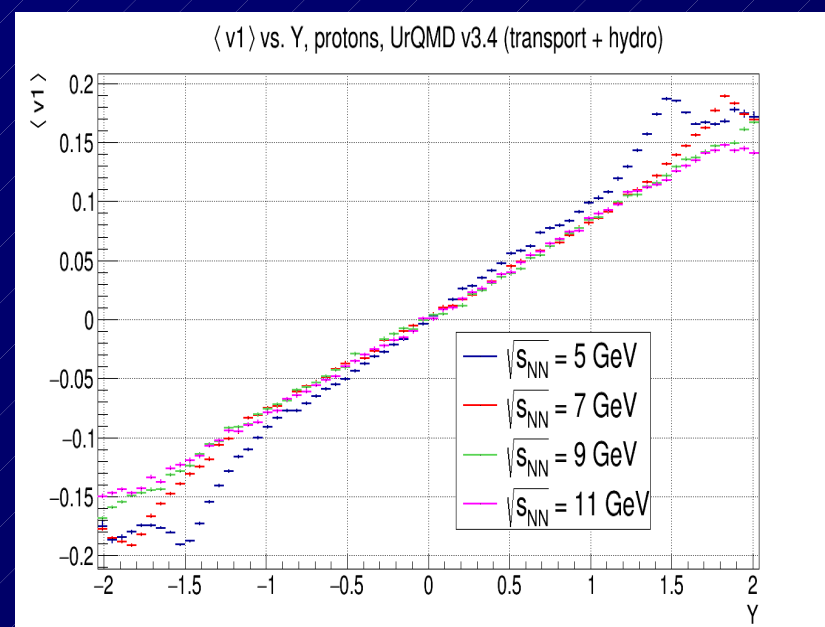
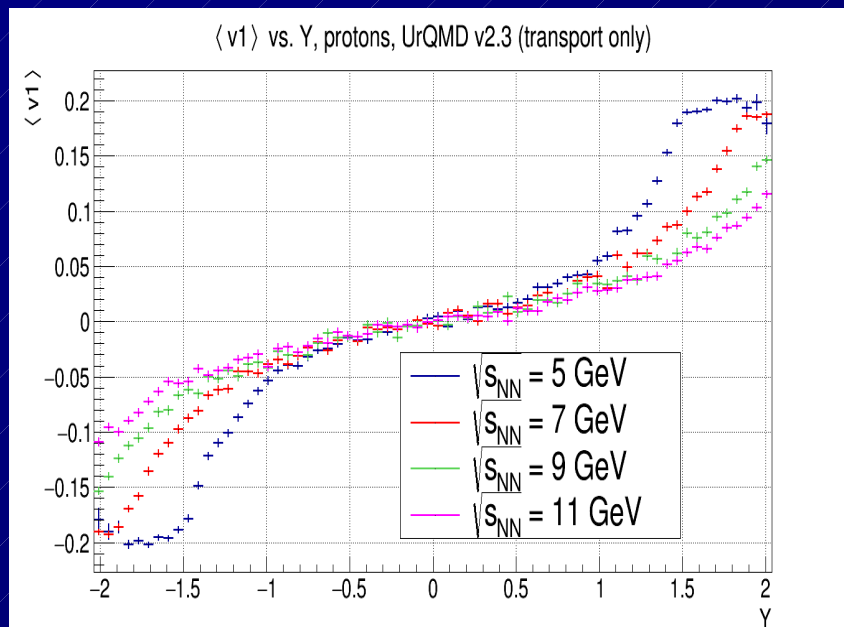
$\langle v_1 \rangle$

UrQMD + hydro (1<sup>st</sup> order PT)

Pions



Protons



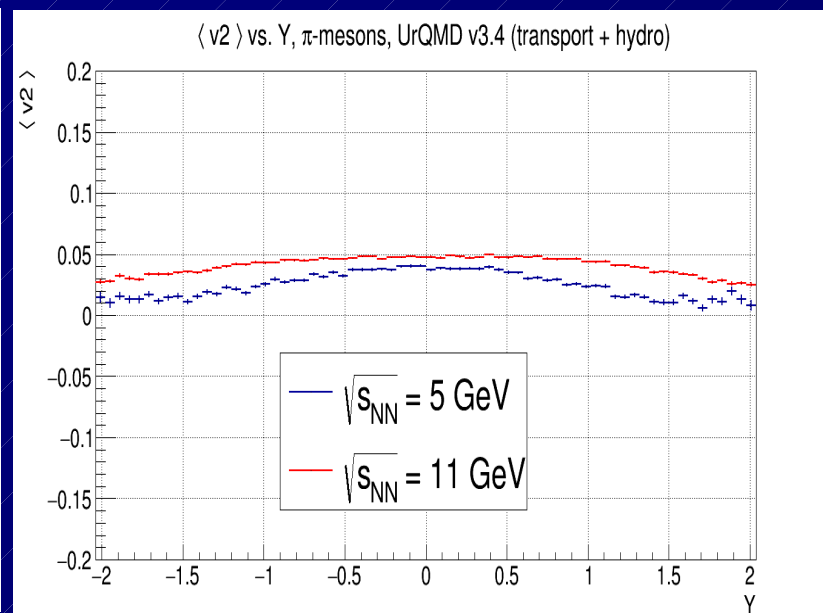
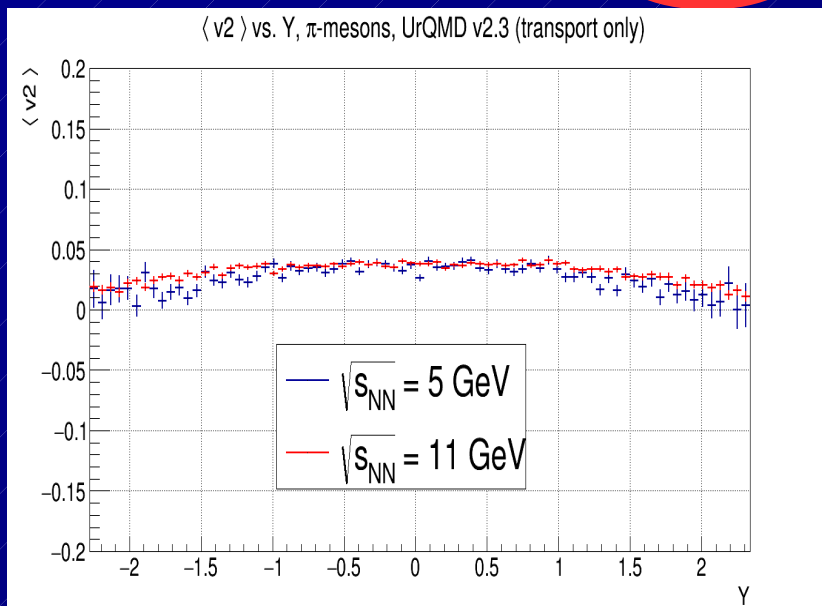
# Further results of test simulations – flow observables

NICA energy scan: UrQMD

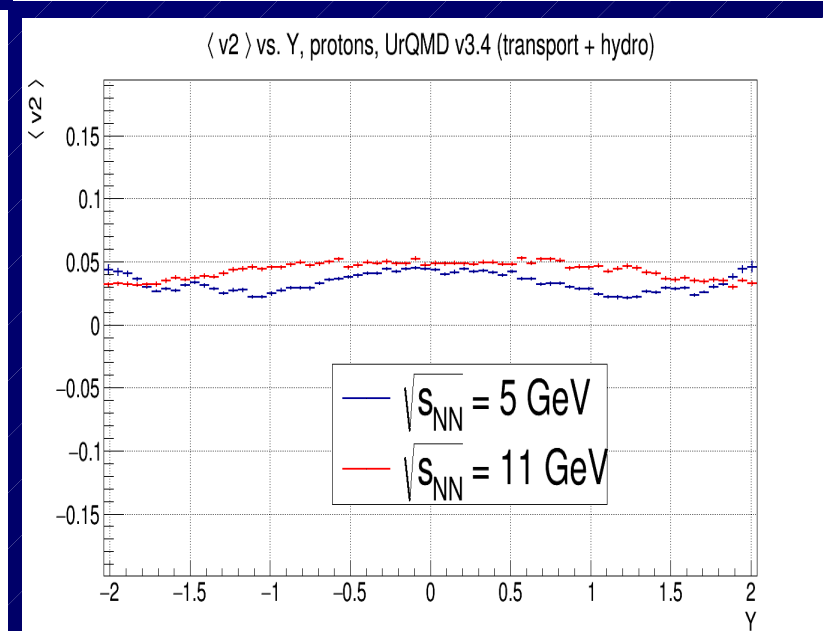
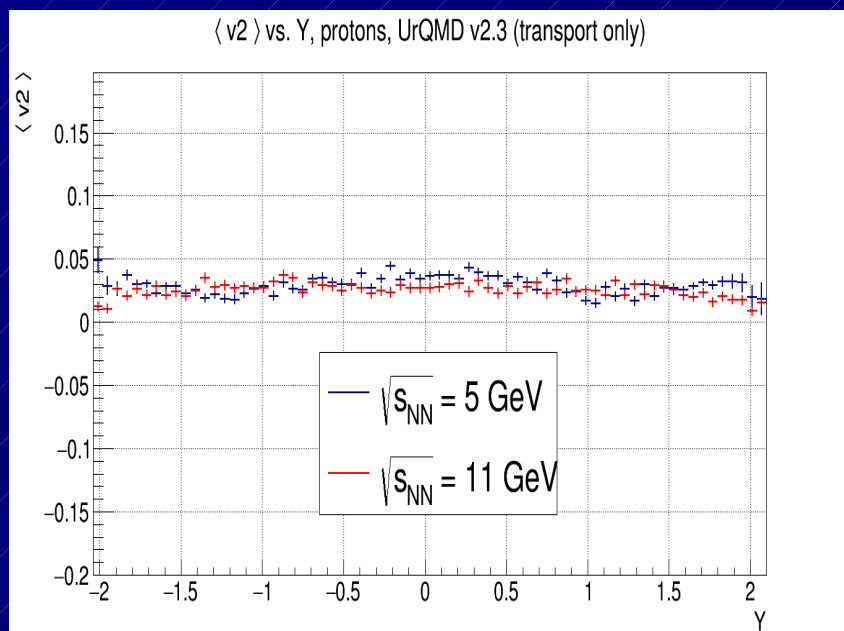
$\langle v_2 \rangle$

UrQMD + hydro (1<sup>st</sup> order PT)

Pions



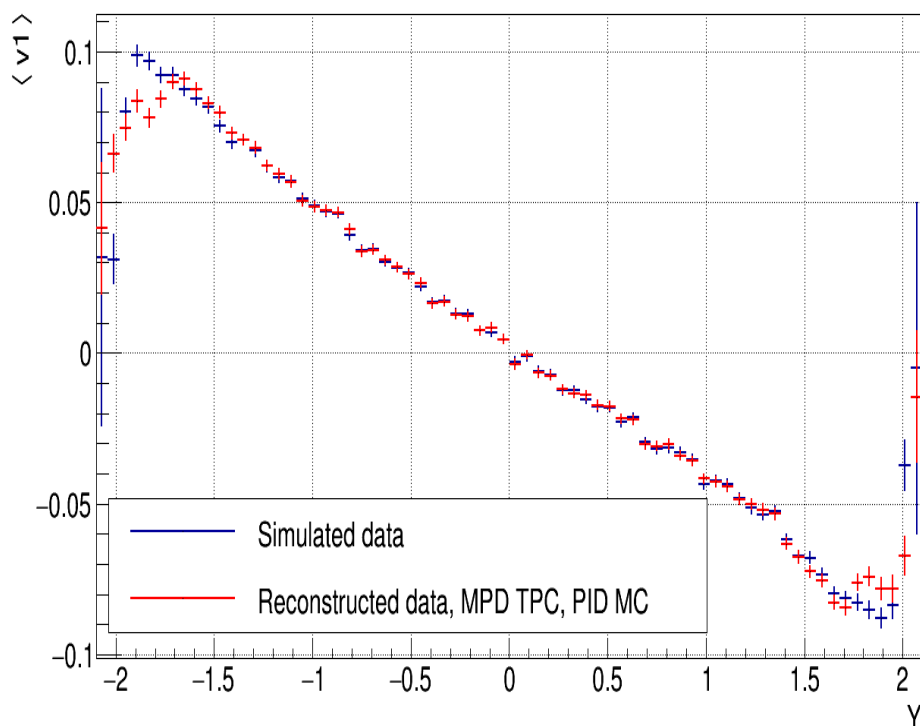
Protons



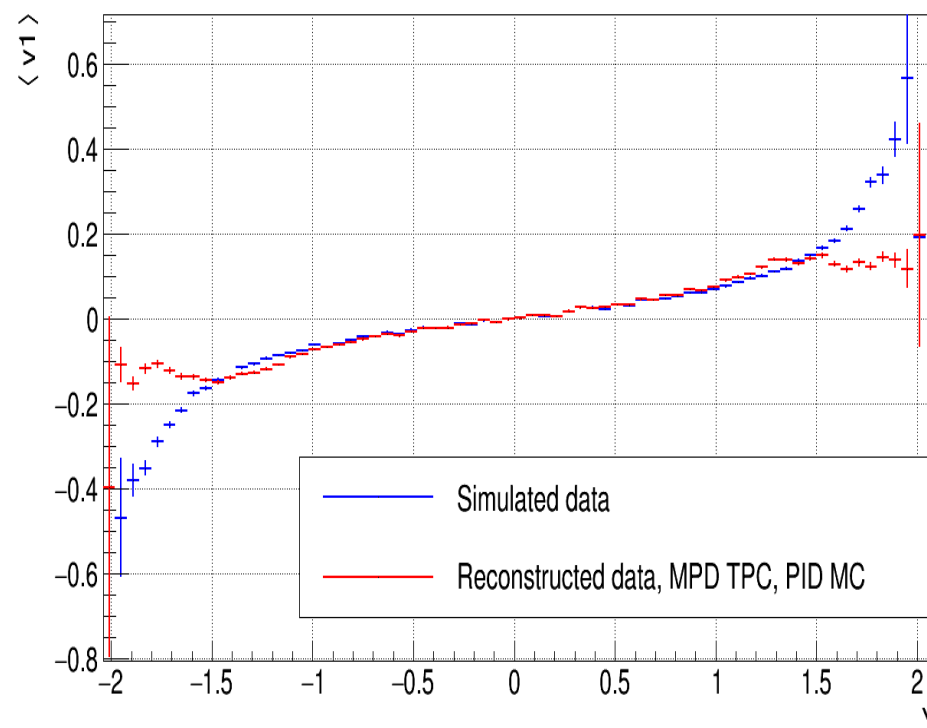
# Further results of test simulations – flow observables

Detector Simulation with GEANT : Excellent reproduction of simulation results !

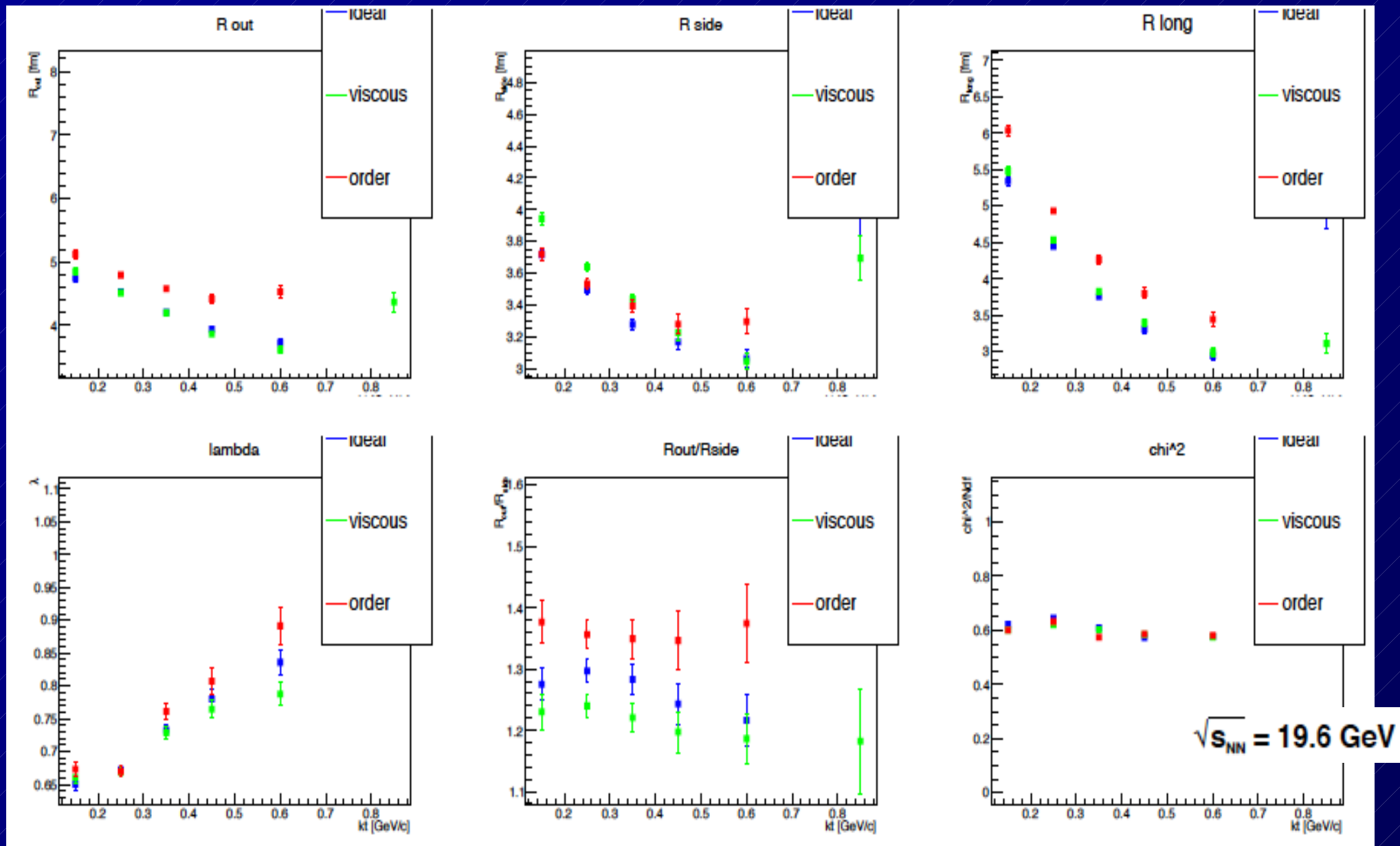
$\langle v_1 \rangle$  vs.  $Y$ ,  $\pi$ -mesons,  $\sqrt{s_{NN}} = 9$  GeV



$\langle v_1 \rangle$  vs.  $Y$ , protons,  $\sqrt{s_{NN}} = 9$  GeV

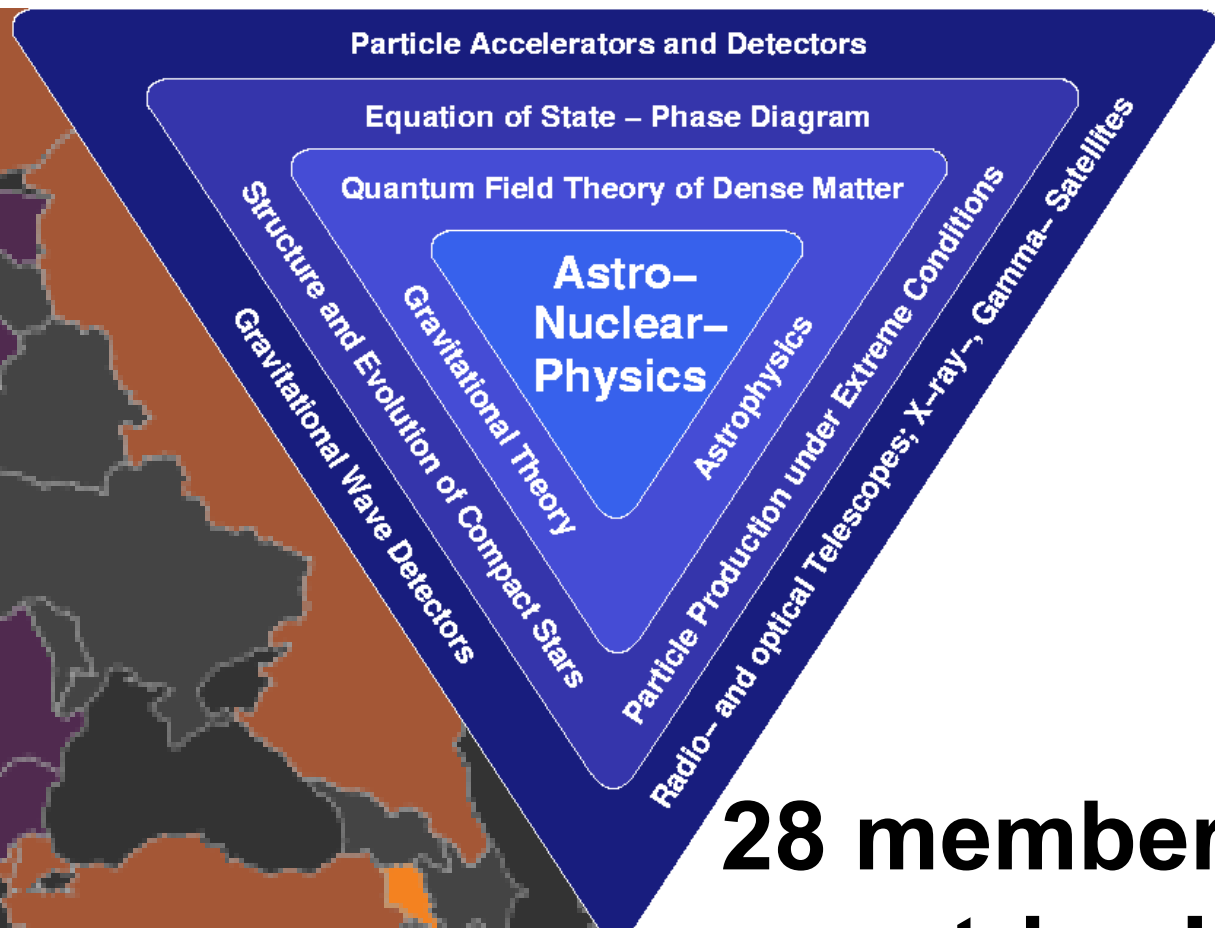
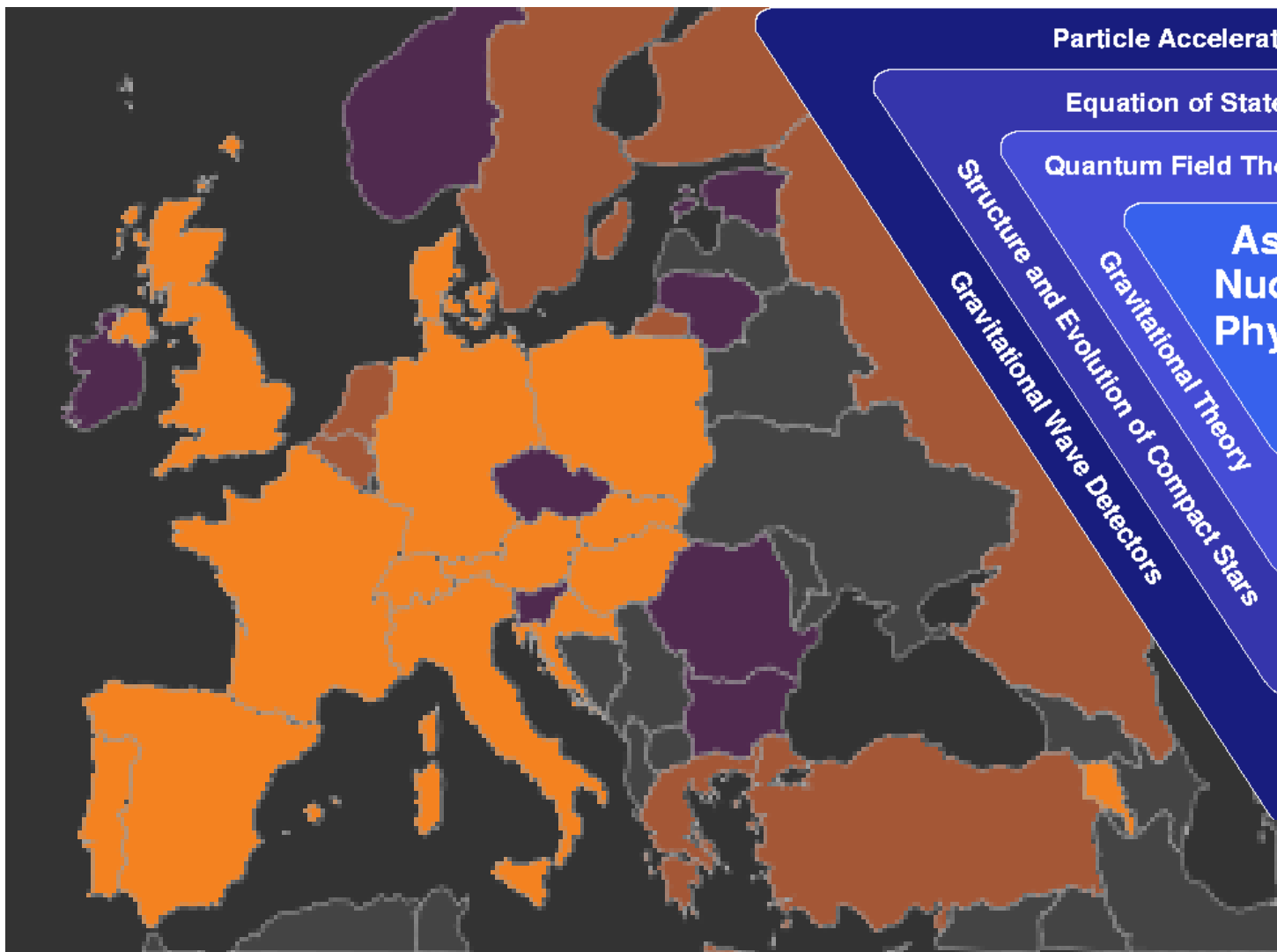


# Further results of test simulations – HBT radii



$\sqrt{s_{NN}} = 19.6$  GeV





**28 member  
countries !!  
(MP1304)**

**New**



**Kick-off: Brussels, November 25, 2013**



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