Information loss and recovery in black-hole physics

Lecture I: Black-hole evolution from gravitational collapse

Slava Emelyanov

Institute of Theoretical Physics
Karlsruhe Institute of Technology

viacheslav.emelyanov@kit.edu

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Outline

Particle Physics, the Universe, and the Equivalence Principle

Quantum field theory in the background of collapsing matter

Black-hole evolution after gravitational collapse

Concluding remarks

Particle Physics, the Universe & the Equivalence Principle

▷ **Particle Physics**

Quantum field theory was originally developed to incorporate quantum mechanics with Special Relativity.

The theory of Special Relativity is based on the line element

$$ds^2 = \eta_{ab} dy^a dy^b \, ,$$

which describes global Minkowski spacetime.

Particle Physics, the Universe & the Equivalence Principle

▷ **Particle Physics**

Minkowski spacetime has 10 Killing vectors which give rise to the Poincaré algebra and, thereby, are generators of the Poincaré group.

The Poincaré group is in turn exploited to define elementary-particle states. Namely, physical particle states correspond to its unitary and irreducible representations (characterised by mass and spin).

▷ **The Universe**

The observable Universe is not globally flat, due to dark energy, dark and baryonic matter, as well as radiation.

▷ **The Equivalence Principle**

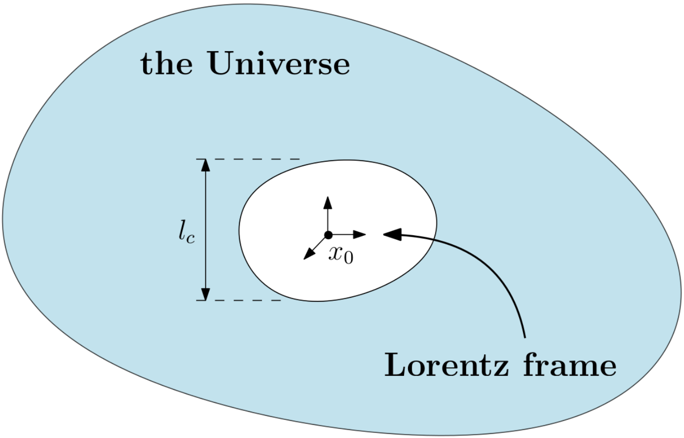
Yet, the Universe locally looks as Minkowski spacetime, in accordance with the Equivalence Principle. It means if l_c is a characteristic curvature length at a given point x_0 in the Universe, then the Universe in the vicinity of x_0 can be approximated by Minkowski spacetime for points x satisfying $|x - x_0| \ll l_c$. Therefore, the actual line element is

$$ds^2 = \left(\eta_{ab} - \frac{1}{3}R_{acbd}(x_0)y^c y^d + \ldots\right)dy^a dy^b \,,$$

where y are, thus, Riemann normal coordinates, such those $y = 0$ corresponds to x_0, and, moreover, $l_c \sim 1/|R_{abcd}R^{abcd}|^{\frac{1}{4}}$.

Particle Physics, the Universe & the Equivalence Principle

▷ **The Equivalence Principle**



▷ **Back to Particle Physics: Feynman propagator**

The scalar-field Feynman propagator in the Universe may then read

$$G_M(x,x') = -\frac{1}{8\pi^2}\left[\frac{u(x,x')}{\sigma(x,x')-i0} - v(x,x')\ln(-\sigma(x,x')+i0) - w(x,x')\right],$$

where $\sigma(x,x')$ is a geodesic distance between x and x', which reduces to $\frac{1}{2}\eta_{ab}(y-y')^a(y-y')^b$ in Minkowski spacetime, and ...

▷ **Back to Particle Physics: Feynman propagator**

... with

$$\begin{aligned} u(x,x') &= 1 + C\sigma^{\frac{1}{2}}(x,x') + C^2\sigma(x,x') + \dots, \\ v(x,x') &= \tfrac{1}{2}\big(m^2 + (\xi - \tfrac{1}{6})C\big) + C\sigma^{\frac{1}{2}}(x,x') + \dots, \\ w(x,x') &= \tfrac{3}{16}\big(m^4 + m^2 C + \Box C + C^2\big)\sigma(x,x') + \dots, \end{aligned}$$

in symbolic notations, where C stands for the curvature tensor, m is the scalar-field mass and ξ is the scalar-field non-minimal coupling to gravity:

$$\Big(\Box + m^2 - \xi R\Big) G_M(x,x') = -i\frac{\delta(x-x')}{(-g(x))^{\frac{1}{2}}},$$

where R is the Ricci scalar.

▷ **Back to Particle Physics: Feynman propagator**

In global Minkowski spacetime, $l_c \to \infty$, we thus have

$$G_M\big(x(y), x(y')\big) = \frac{m^2}{4\pi^2} \frac{K_1\big(m\sqrt{-2(\sigma_M(y,y')-i0)}\big)}{m\sqrt{-2(\sigma_M(y,y')-i0)}} = \int \frac{d^4k}{(2\pi)^4} \frac{i\, e^{-ik_\mu (y-y')^\mu}}{k^2 - m^2 + i0},$$

explaining the meaning of the index M.

▷ **Back to Particle Physics: Feynman propagator**

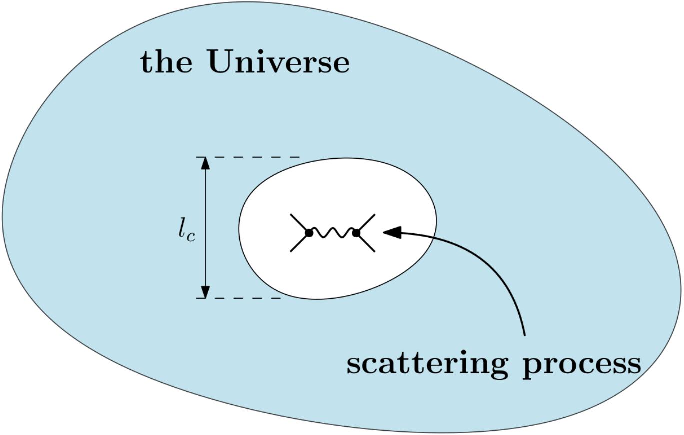
But, strictly speaking, $l_c < \infty$, i.e. we actually have

$$G_M\big(x(y), x(y')\big) \;=\; \int \frac{d^4k}{(2\pi)^4}\, \frac{i\, e^{-ik_\mu (y-y')^\mu}}{k^2 - m^2 + i0} + \mathrm{O}(l_c^{-2})\,,$$

where l_c is the characteristic curvature length at the point x_0.

Particle Physics, the Universe & the Equivalence Principle

▷ **Back to Particle Physics: Feynman propagator**



Particle Physics, the Universe & the Equivalence Principle

▷ **Back to Particle Physics: Elementary particles are localised states**

Why does the Minkowski-spacetime approximation work in practice?

In short: Elementary particles correspond to localized states, meaning that they are described by wave packets which are essentially non-vanishing only in a small space-time region in the observable Universe. If l_p is a characteristic size of a particle wave packet, then this elementary particle can be related to a unitary and irreducible representation of the local Poincaré group if $l_p \ll l_c$. But, if $l_p \gtrsim l_c$, then the standard-model definition of particles looses its meaning.

▷ **Back to Particle Physics: Locally Minkowski vacua**

The Minkowski vacuum of elementary particle physics is a no-particle state defined in local Lorentz frames.

Particle Physics, the Universe & the Equivalence Principle

▷ **Back to Particle Physics: Locally Minkowski vacua**

However, one can think about quantum states described by

$$G(x,x') \;=\; G_M(x,x') + G_V(x,x')\,,$$

where $G_V(x,x')$ solves the scalar-field equation:

$$\Big(\Box + m^2 - \xi R\Big)\,G_V(x,x') \;=\; 0\,,$$

such those

$$G\big(x(y),x(y')\big) \;=\; G_M\big(x(y),x(y')\big) + \mathrm{O}\big(l_c^{-2}\big)\,.$$

Particle Physics, the Universe & the Equivalence Principle

▷ **Back to Particle Physics: Locally Minkowski vacua**

Thereby, we cannot distinguish

$$G_M(x, x') \quad \text{from} \quad G_M(x, x') + G_V(x, x')$$

in particle colliders on Earth.

From the point of view of elementary particles physics, all these states are no-particle states or quantum vacua.

QFT in the background of collapsing matter

▷ **Quantum-field model**

For the sake of simplicity, we shall study a linear scalar-field model with conformal coupling to gravity:

$$\left(\Box - \frac{1}{6}\,R\right)\phi(x) \quad = \quad 0\,,$$

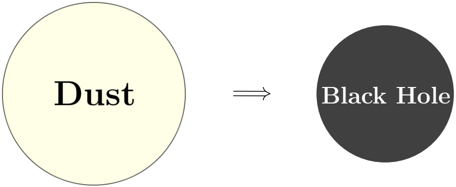
where R is the Ricci scalar.

QFT in the background of collapsing matter

▷ **Oppenheimer-Snyder model**

Gravitational collapse is a complicated process.

Yet, there is a comparably simple model by Oppenheimer and Snyder, which captures main features of this process:



QFT in the background of collapsing matter

- **Outside stars before the collapse**

The space-time geometry of non-rotating stars is described by

$$ds_O^2 = f(r)dt^2 - \frac{dr^2}{f(r)} - r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right),$$

with the lapse function

$$f(r) = 1 - \frac{r_H}{r},$$

where $r_H \equiv 2M$ is the Schwarzschild radius with the stellar mass M.

QFT in the background of collapsing matter

▷ **Outside stars before the collapse: Quantum vacua**

From symmetries of the Schwarzschild geometry, the most general form of the stress-energy tensor of the scalar field reads

$$\langle \hat{\Theta}_\nu^\mu(x) \rangle_O = \frac{p(r)}{r^4}\,\mathrm{diag}\big[-1,+1,0,0\big] + \frac{s(r)}{r^4}\,\mathrm{diag}\big[-3,+1,+1,+1\big]\,,$$

where we have from the stress-tensor conservation, $\nabla_\mu \langle \hat{\Theta}_\nu^\mu(x) \rangle = 0$:

$$\begin{aligned} p(r) &= +2k(r) - \frac{rf(r)k'(r)}{2f(r) - rf'(r)}\,, \\ s(r) &= -k(r) + \frac{rf(r)k'(r)}{2f(r) - rf'(r)}\,, \end{aligned}$$

with the arbitrary function $k(r)$.

QFT in the background of collapsing matter

▷ **Outside stars before the collapse: Quantum vacua**

The Boulware (B) vacuum is a state looking empty at large radii. This unique global asymptotically-empty state is described by

$$k_B(r) \approx +\frac{1}{1440\pi^2}\frac{1+6f(r)-63f^2(r)}{16f^2(r)}\left(\frac{r_H}{r}\right)^2.$$

The vacuum energy density in this state is negative outside the star!

QFT in the background of collapsing matter

▷ **Outside stars before the collapse: Quantum vacua**

But any quantum state described by

$$k_V(r) \;=\; \mathrm{O}\left(\left(\frac{r_H}{r}\right)^2\right) \quad \text{at} \quad r \to \infty$$

also looks empty at large radii!

These states correspond to the local vacua with $G = G_M + G_V$.

QFT in the background of collapsing matter

▷ **Inside stars just before the collapse**

The space-time geometry of cold stars is described by

$$ds_I^2 = a^2(\eta)\Big(d\eta^2 - d\chi^2 - \sin^2\chi\big(d\theta^2 + \sin^2\theta d\phi^2\big)\Big),$$

where the scale factor

$$a(\eta) = a_0 \begin{cases} 1\,, & \eta \leq 0\,, \\ \frac{1}{2}(1+\cos\eta)\,, & \eta \in (0,\,\pi)\,, \end{cases}$$

where

$$\begin{aligned} r_S &= a(\eta)\sin\chi_0 & - \quad & \text{the stellar radius,} \\ r_H &= a_0\sin^3\chi_0 & - \quad & \text{the gravitational radius,} \end{aligned}$$

with χ_0 being the stellar-surface location in the FRW frame.

QFT in the background of collapsing matter

▹ **Inside stars before the collapse: Quantum vacua**

From symmetries of the FRW geometry and the regularity at the stellar center, the most general form of the stress tensor reads

$$\langle \hat{\Theta}_\nu^\mu(x) \rangle_I = \frac{\pi(\chi)}{a^4(\eta)}\,\mathrm{diag}\big[-1, +1, 0, 0\big] + \frac{\sigma(\chi)}{a^4(\eta)}\,\mathrm{diag}\big[-3, +1, +1, +1\big]\,,$$

where we have from the stress-tensor conservation, $\nabla_\mu \langle \hat{\Theta}_\nu^\mu(x) \rangle = 0$:

$$\begin{aligned} \pi(\chi) &= \kappa(\chi)\csc^2\chi\,, \\ \sigma(\chi) &= \sigma(0) - \int_0^\chi d\bar{\chi}\,\csc^2\bar{\chi}\,\frac{d\kappa(\bar{\chi})}{d\bar{\chi}}\,, \end{aligned}$$

where $\kappa(\chi)$ is any function of χ, such that $\kappa(\chi)/\chi^2 \to 0$ at $\chi \to 0$.

QFT in the background of collapsing matter

▷ **Inside stars just before the collapse: Quantum vacua**

There is a "preferred" quantum state in the scalar-field model considered, that is known as the conformal (C) vacuum, described by

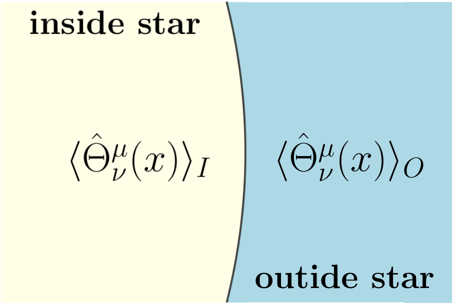
$$\kappa_C(\chi) \;=\; 0 \quad \text{and} \quad \sigma_C(0) \;=\; -\frac{1}{1440\pi^2}\,.$$

The vacuum energy density in this state is <u>positive</u> inside the star!

QFT in the background of collapsing matter

- **Matching condition**

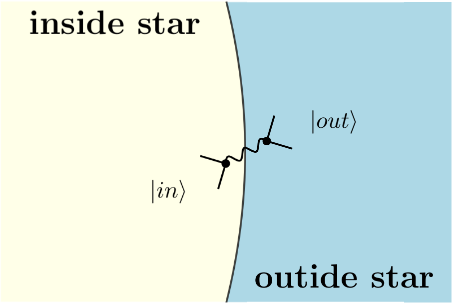
We have two space-time regions – inside and outside of the star, – but how the inside vacuum is related to the outside one?



QFT in the background of collapsing matter

▷ **Matching condition**

We have two space-time regions – inside and outside of the star, – but how the inside vacuum is related to the outside one?



QFT in the background of collapsing matter

▷ **Matching condition**

Basic idea: No **Q**uantum **P**hase **T**ransition across the interface.

The no-QPT condition leads to

$$\langle \hat{\Theta}^{\mu}_{\nu}(x) \rangle_I \Big|_{r=r_S} = \langle \hat{\Theta}^{\mu}_{\nu}(x) \rangle_O \Big|_{r=r_S} .$$

QFT in the background of collapsing matter

▷ **Matching condition**

Thus, in a frame co-moving with the stellar surface, we have

$$\langle \hat{\Theta}^A_B(x) \rangle_O \Big|_{r=r_S} = \frac{p(r_{S,0})}{r_S^4}\,\mathrm{diag}\big[-1,+1,0,0\big] + \frac{s(r_{S,0})}{r_S^4}\,\mathrm{diag}\big[-3,+1,+1,+1\big]\,,$$

where $r_{S,0}$ is the initial stellar radius.

QFT in the background of collapsing matter

▷ **Matching condition**

But, in the Schwarzschild frame, we have

$$\langle \hat{\Theta}^{\mu}_{\nu}(x) \rangle_O \Big|_{r=r_S} \quad = \quad e^{\mu}_{A}(x) e^{B}_{\nu}(x) \langle \hat{\Theta}^{A}_{B}(x) \rangle_O \Big|_{r=r_S}$$

on the stellar surface, where

$$\begin{aligned} e^{\mu}_{T}(x)\partial_{\mu} &= +\frac{f(r_{S,0})}{f(r)}\,\partial_t - f(r_{S,0})\sqrt{1 - f(r)/f(r_{S,0})}\,\partial_r\,, \\ e^{\mu}_{R}(x)\partial_{\mu} &= -\frac{\sqrt{1 - f(r)/f(r_{S,0})}}{f(r)}\,\partial_t + \partial_r\,. \end{aligned}$$

QFT in the background of collapsing matter

▷ **Matching condition**

and, in the limit $r_S \to r_H$, we obtain

$$\langle \hat{\Theta}^{\mu}_{\nu}(x) \rangle_O \Big|_{r_S \to r_H} = -\frac{L_{OS}}{4\pi r_H^2}\, n^{\mu} n_{\nu} \Big|_{r_S \to r_H} + \frac{s(r_{S,0})}{r_H^4}\, \mathrm{diag}\big[-1, -1, +1, +1\big]\,,$$

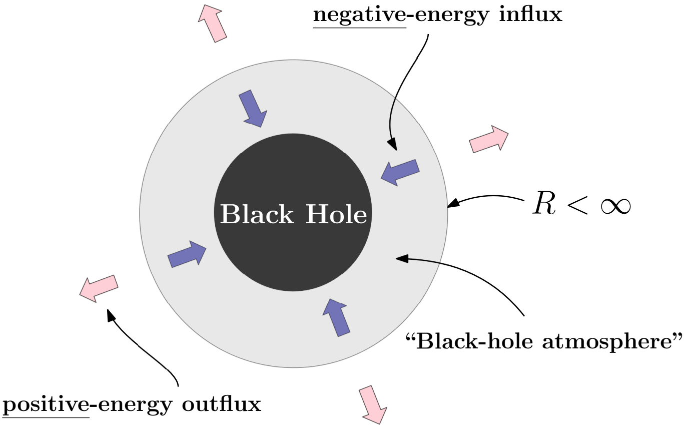
where $n^{\mu} \equiv (1/f(r), -1, 0, 0)$ and

$$L_{OS} \equiv +\frac{8\pi}{r_H^2} f(r_{S,0})\big(p(r_{S,0}) + 2s(r_{S,0})\big)$$

is the black-hole luminosity in the **O**ppenheimer-**S**nyder collapse.

Black-hole evolution after gravitational collapse

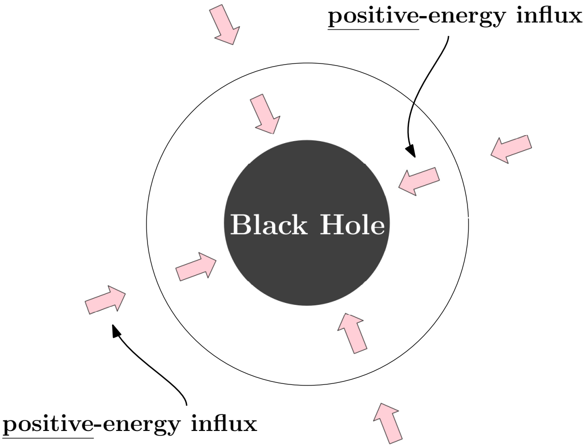
- **Black-hole contraction:** $L_{OS,B} \approx +(21.26\times10^{-5}/r_H^2)\times(5r_H/r_{S,0})^2$



Black-hole evolution after gravitational collapse

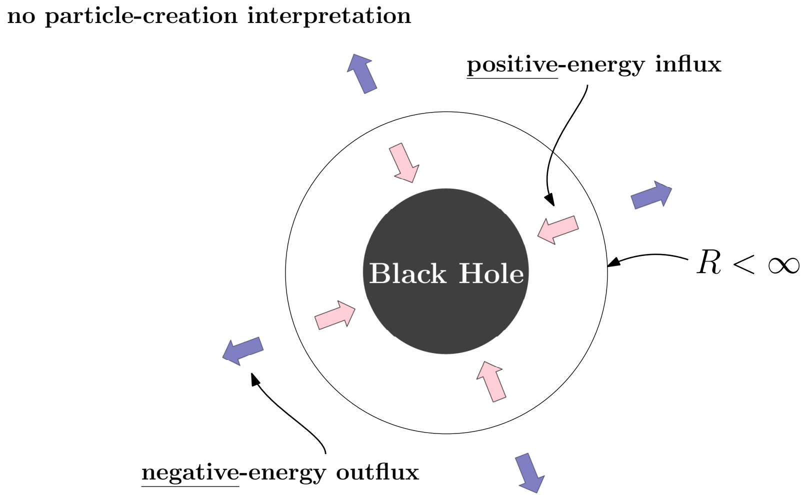
- **Black-hole expansion I:** $L_{OS,C} \approx -(11.32\times10^{-5}/r_H^2)\times(5r_H/r_{S,0})^2$

particle-creation interpretation



Black-hole evolution after gravitational collapse

▷ **Black-hole expansion II:** $L_{OS,C} \approx -(11.32\times10^{-5}/r_H^2)\times(5r_H/r_{S,0})^2$



Concluding remarks

▷ **(1/5) The Hawking effect from Bekenstein-entropy argument**

The main feature of the Hawking effect is a thermal spectral profile of the positive-energy outflux.

Yet, Bekensteins black-hole entropy implies that the Hawking temperature can be attributed to black holes, although black-hole evaporation does not follow from the thermodynamic argument by itself.

The no-QPT condition with Bekenstein's entropy leads to the Hawking effect.

Concluding remarks

▷ **(2/5) Evasion of the quantum no-hair theorem**

The luminosity in the Unruh-vacuum state, $L_U \approx +29.75\times10^{-5}/r_H^2$, turns out to be entirely determined by the black-hole mass only. For this reason, it is tempting to generalize the no-hair theorem to the statement "a black hole has neither classical nor quantum hair".

We have found that the "generalised no-hair theorem" can be evaded, by directly showing that the Unruh state is not a unique quantum vacuum describing evaporating black holes.

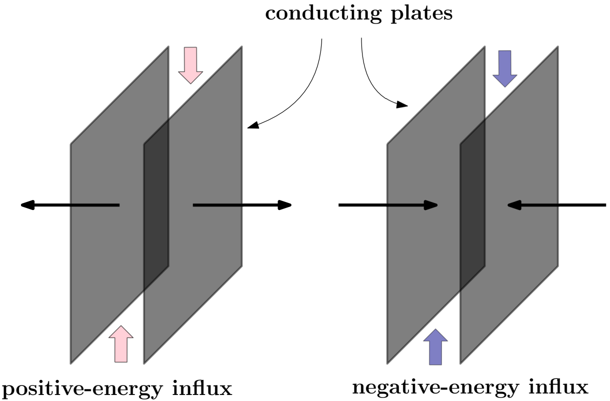
Concluding remarks

▷ **(3/5) The Hawking and anti-Hawking effects**

We have found that some black holes may be expanding in nature due to the absorption of positive vacuum energy, while evaporating black holes absorb negative vacuum energy.

Concluding remarks

▷ **(4/5) A lab probe of the Hawking and anti-Hawking effects: Sketch**



Concluding remarks

▷ **(5/5) Information-loss problem in black-hole physics**

If a black hole completely evaporates via the Hawking process, then the information about, e.g., matter composition of the star collapsed to that black hole is lost as the Hawking outflux is featureless.

This kind of process is not compatible with unitarity.

Many proposals to resolve this problem have been suggested in the literature, including the idea that semi-classical physics may be not enough to preserve the unitarity.

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