Information loss and recovery in black-hole physics

Lecture II: Across-horizon scattering and charge extraction

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Outline

Retarded versus Feynman propagator

▷ **Maxwell's theory**

Classical electrodynamics is based on the Maxwell equations:

$$\begin{aligned} \nabla\cdot\mathbf{E} &= 4\pi\rho\,, & \nabla\times\mathbf{B} &= +\frac{\partial\mathbf{E}}{\partial t} + 4\pi\mathbf{j}\,, \\ \nabla\cdot\mathbf{B} &= 0\,, & \nabla\times\mathbf{E} &= -\frac{\partial\mathbf{B}}{\partial t}\,, \end{aligned}$$

where **E** and **B** are, respectively, the electric and magnetic fields, and $j^\mu = (\rho, \mathbf{j})$ is the current four-vector, describing electric charges distributed over spacetime.

Retarded versus Feynman propagator

▷ **Maxwell's theory**

These equations give rise to the wave equation:

$$\Box A^{\mu} - \partial^{\mu}\partial_{\nu}A^{\nu} \quad = \quad 4\pi j^{\mu} \,,$$

where $A^{\mu} = (\phi, \mathbf{A})$ is the four-potential of the electromagnetic field:

$$\mathbf{E} \quad = \quad -\frac{\partial \mathbf{A}}{\partial t} - \nabla\phi \,, \qquad \mathbf{B} \quad = \quad \nabla\times\mathbf{A} \,.$$

Retarded versus Feynman propagator

▷ **Retarded propagator**

Fixing the Lorentz-Feynman gauge, $\mathcal{L}_{\mathrm{EM}} \to \mathcal{L}_{\mathrm{EM}} - \frac{1}{2}(\partial A)^2$, one has

$$A^\mu(x) \;=\; \int d^4\bar{x}\, G_R^{\mu\nu}(x,\bar{x})\, j_\nu(\bar{x})\,,$$

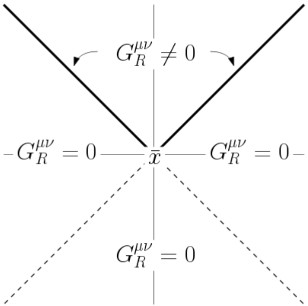
where $G_R^{\mu\nu}(x,\bar{x})$ is the retarded (R) Green's function:

$$\begin{aligned} G_R^{\mu\nu}(x,\bar{x}) &= -\int \frac{d^4q}{(2\pi)^4}\,\frac{\eta^{\mu\nu}}{q^2+iq_0\varepsilon}\,e^{-iq\cdot(x-\bar{x})} \\ &= +\frac{\eta^{\mu\nu}}{2\pi}\,\theta\big(x^0-\bar{x}^0\big)\delta\big((x^0-\bar{x}^0)^2-(\mathbf{x}-\bar{\mathbf{x}}^0)^2\big)\,, \end{aligned}$$

which explicitly shows that $G_R^{\mu\nu}(x,\bar{x})$ has its support on the forward lightcone with origin at $\bar{x}$.

Retarded versus Feynman propagator

- **Retarded propagator**



Support of the retarded propagator $G_R^{\mu\nu}(x, \bar{x})$

Retarded versus Feynman propagator

▷ **Quantum Maxwell theory**

In local quantum field theory, the classical field $A_\mu(x)$ is promoted to a distribution-valued operator which satisfies the following equal-time commutation relations:

$$\begin{aligned} \left[A_\mu(t,\mathbf{x}), A_\nu(t,\bar{\mathbf{x}})\right] &= \left[\dot{A}_\mu(t,\mathbf{x}), \dot{A}_\nu(t,\bar{\mathbf{x}})\right] = 0\,, \\ \left[\dot{A}_\mu(t,\mathbf{x}), A_\nu(t,\bar{\mathbf{x}})\right] &= i\eta_{\mu\nu}\delta\big(\mathbf{x}-\bar{\mathbf{x}}\big)\,, \end{aligned}$$

where the Lorentz-Feynman gauge has been assumed.

Retarded versus Feynman propagator

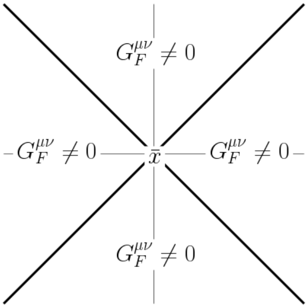
▷ **Feynman propagator**

The Feynman (F) propagator is a basic ingredient of Feynman rules:

$$
\begin{aligned}
G_F^{\mu\nu}(x,\bar{x}) &\equiv \theta(x^0-\bar{x}^0)\langle A^\mu(x)A^\nu(\bar{x})\rangle + \theta(\bar{x}^0-x^0)\langle A^\nu(\bar{x})A^\mu(x)\rangle \\
&= \int \frac{d^4q}{(2\pi)^4}\,\frac{-i\eta^{\mu\nu}}{q^2+i\varepsilon}\,e^{-iq\cdot(x-\bar{x})} \\
&= +\frac{\eta^{\mu\nu}}{(2\pi)^2}\,\frac{1}{(x-\bar{x})^2-i\varepsilon}\,.
\end{aligned}
$$

Retarded versus Feynman propagator

- **Feynman propagator**



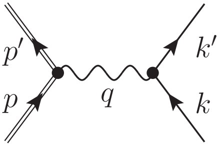
Support of the Feynman propagator $G_F^{\mu\nu}(x, \bar{x})$.

Retarded versus Feynman propagator

▷ **Electron-muon scattering: Causality**

The leading-order probability amplitude for the Coulomb scattering of two electrically charged elementary particles, electron and muon, is represented by the following Feynman diagram:

$$_{\text{out}}\langle \mu,\, p' \,;\, e,\, k' | \mu,\, p \,;\, e,\, k \rangle_{\text{in}} \quad \propto$$



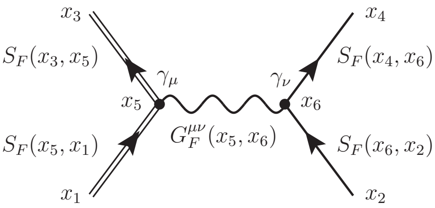
,

with definition $q \equiv p' - p$ and arrows showing the flow of negative electric charge (and also the flow of positive lepton number).

Retarded versus Feynman propagator

- **Electron-muon scattering: Causality**

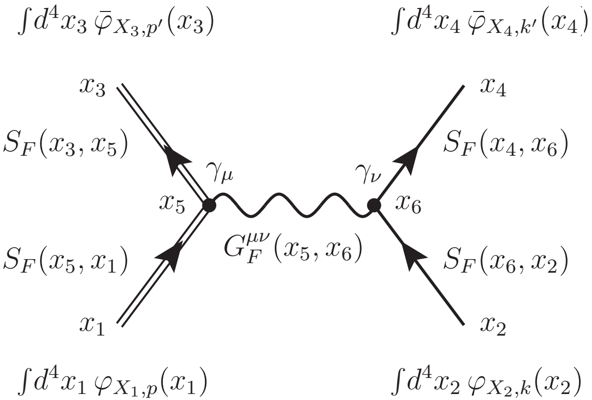
In position space, the electron-muon scattering is described by



This amplitude is non-normalisable - one should take into account that particles are represented by wave packets which are well-localised in both coordinate and momentum spaces. Namely, one has...

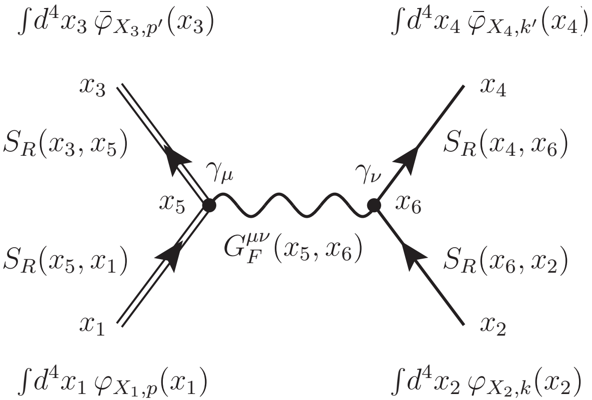
Retarded versus Feynman propagator

▷ **Electron-muon scattering: Causality**



Retarded versus Feynman propagator

▷ **Electron-muon scattering: Causality**



Retarded versus Feynman propagator

▷ **Electron-muon scattering: Causality**

The Feynman propagator does not imply a non-causal propagation of on-shell particles:

- $x_5 \in J^+(x_1)$ and $x_3 \in J^+(x_5)$, thus the out-muon is in the causal future of the in-muon.
- $x_6 \in J^+(x_2)$ and $x_4 \in J^+(x_6)$, thus the out-electron is in the causal future of the in-electron.
- $x_3, x_4 \in J^+(x_1) \cup J^+(x_2)$ generically holds, there is, however, no guarantee $x_4 \in J^+(x_1)$ and/or $x_3 \in J^+(x_2)$. This circumstance does not lead to causality violation, as the virtual photon is non-observable.

Einstein's Equivalence Principle and black holes

▷ **The Equivalence Principle**

There is no way, by experiments confined to infinitesimally small regions of spacetime, to distinguish one local Lorentz frame in one region of spacetime from any other local Lorentz frame in the same or any other region.

C.W. Misner *et.al.*, *Gravitation* (1973)

Einstein's Equivalence Principle and black holes

▷ **Characteristic time scales**

In the near-horizon region, curvature length & time scales are

$$l_c \quad \sim \quad 3\times10^3 \text{ m} \left(\frac{M}{M_\odot}\right) \quad \text{and} \quad \tau_c \quad \sim \quad 10^{-5} \text{ s} \left(\frac{M}{M_\odot}\right).$$

In quantum field theory, interaction length & time scales are

$$l_i \quad \sim \quad 3\times10^{-15} \text{ m} \left(\frac{\text{GeV}}{\mathcal{E}}\right) \quad \text{and} \quad \tau_i \quad \sim \quad 10^{-23} \text{ s} \left(\frac{\text{GeV}}{\mathcal{E}}\right).$$

Einstein's Equivalence Principle and black holes

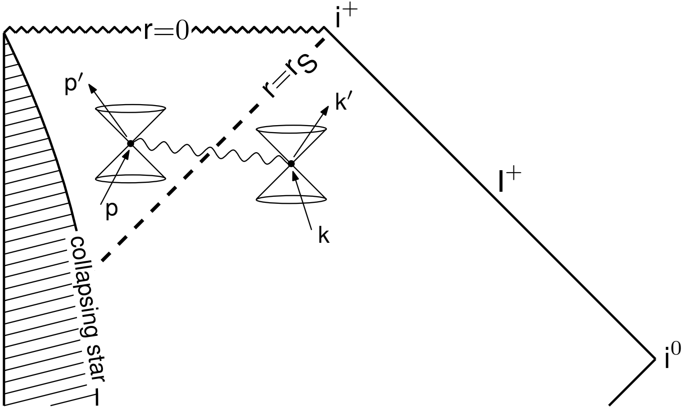
Therefore, if

$$\tau_c \quad \gg \quad \tau_i \quad \text{or} \quad \mathcal{E} \quad \gg \quad 10^{-18}\ \text{GeV}\left(\frac{M_\odot}{M}\right),$$

then standard-model QFTs properly describe particle reactions, even near a black-hole horizon.

Scattering across the black-hole horizon

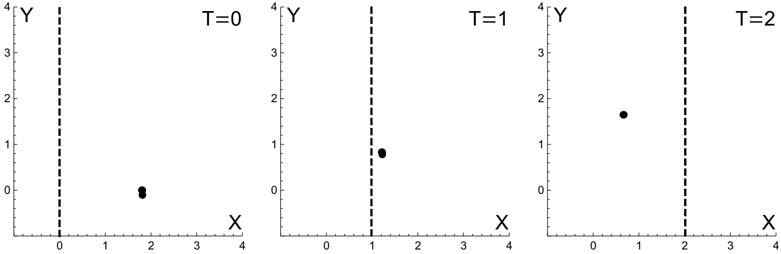
▷ **Muon-electron scattering near the horizon**



k for the electron and p for the muon, r_S is the Schwarzschild radius

Scattering across the black-hole horizon

▷ **Case with zero probability for electron to recoil**

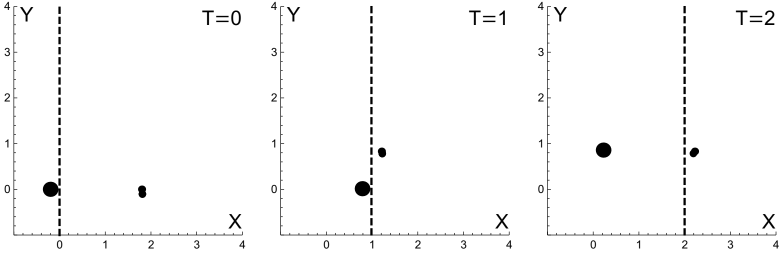


The dashed line stands for the projected black-hole horizon in LICS.

• for the electron and ● for the muon

Scattering across the black-hole horizon

▷ **Case with nonzero probability for electron to recoil**



The dashed line stands for the projected black-hole horizon in LICS.
• for the electron and ⬤ for the muon

Information transfer across the black-hole horizon

▷ **Gedankenexperiment**

Two experimenters Castor and Pollux. Pollux stays outside the event horizon and Castor moves inside the horizon.

Pollux starts to emit, at regular time intervals, appropriate electrons and sends out a finite number N of electrons in total $(1 \ll N < \infty)$.

Castor either emits N appropriate muons for a “yes” message or sends no muons for a “no” message.

Information transfer across the black-hole horizon

- **Gedankenexperiment**

If Castor has emitted N muons, then Pollux's detector has a nonzero chance to register a momentum change of the exterior electron, but if Castor has emitted no muons at all, then Pollux's detector will never register a recoil electron.

After N measurements, Pollux summarizes: a sequence of N no-recoil electrons is written as "NO" and a sequence with at least one recoil electron is written as "YES".

Information transfer across the black-hole horizon

- **Gedankenexperiment**

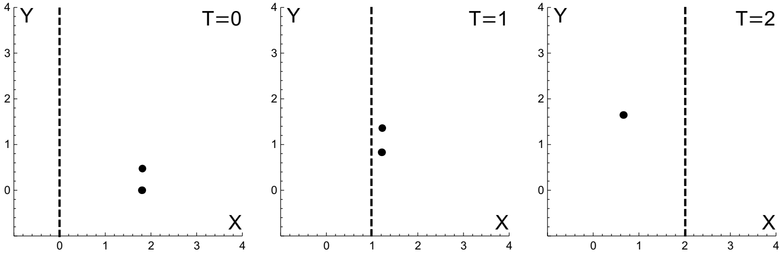
Thus, Castor can send a message "yes" or "no", which is read by Pollux as "YES" or "NO".

Pollux can transmit this message to distant observers by classical means such as pulses of electromagnetic radiation.

Information transfer across the black-hole horizon

▷ **Gedankenexperiment**

▷▷ **“no” message from Castor (no muons emitted)**



The dashed line stands for the projected black-hole horizon in LICS.
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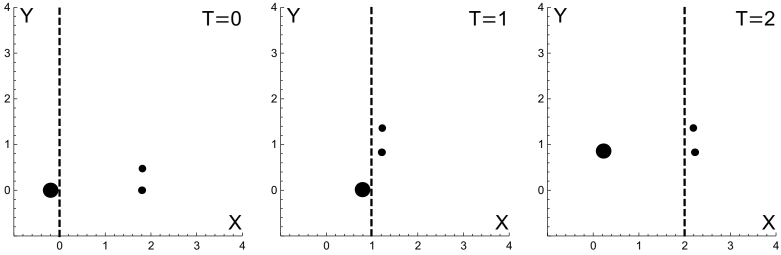
Information transfer across the black-hole horizon

▷ **Gedankenexperiment**

▷▷ **“yes” message from Castor (mouns emitted)**



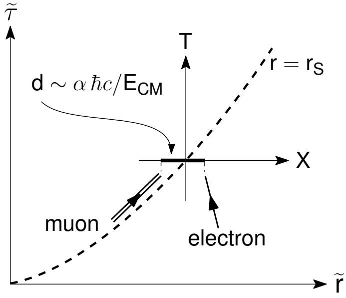
The dashed line stands for the projected black-hole horizon in LICS.
• for the electron and ⬤ for the muon

Information transfer across the black-hole horizon

▷ **Scattering as entanglement harvesting**

Nature is quantum, $\hbar \neq 0$, which gives rise to nonlocal correlations.

Optimal separation of the muon and the electron is $d \sim \alpha\,\hbar c/E_{\mathrm{CM}}$ which guarantees having a significant scattering probability:



$\widetilde{\tau}$ and $\widetilde{r}$ are Novikov's infalling-clock time and radial coordinates

Information transfer across the black-hole horizon

- **Scattering as entanglement harvesting**

Our scattering setup is entirely analogous to

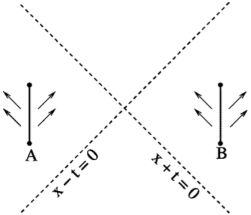
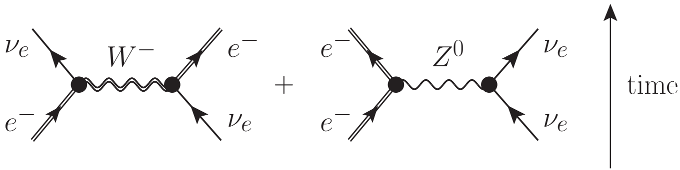


FIG. 1. The world lines of detectors A and B are shown for the duration of the interaction. The horizontal and vertical axes are space and time, respectively. The arrows denote the emitted radiation. Notice that the radiation emitted by detector $A(B)$ does not affect detector $B(A)$, since for $t > T$ the interaction is switched off.

Credit to Reznik *et.al.*, Phys. Rev. A **71** (2005) 042104, who calculated how two causally disconnected detectors get entangled with time.

Charge extraction from a black hole

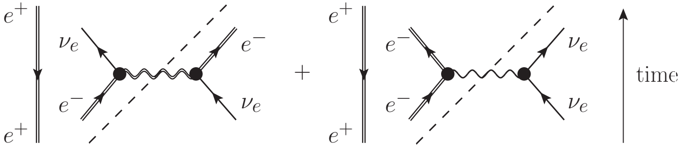
▷ **Electric-charge extraction**



Position-space Feynman diagrams for $e^- \, \nu_e$ scattering at tree level in Minkowski spacetime. The arrows show the flow of lepton number. The double lines indicate that electric charge is transported.

Charge extraction from a black hole

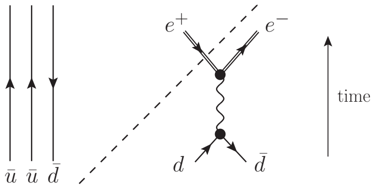
▷ **Electric-charge extraction**



Across-horizon $e^- \, \nu_e$ scattering allows for electric-charge extraction from a static nonrotating black hole. Specifically, the electric-charge-extraction process follows from the charged-current position-space Feynman diagram on the left. The projected black-hole horizon is indicated by the dashed line with the black-hole center to its left. The positron inside the black-hole horizon does not participate in the scattering.

Charge extraction from a black hole

▷ **Electric-charge reduction**



Position-space Feynman diagram for electric-charge reduction of a static nonrotating charged black hole. Shown is a black hole with an initial negative electric charge $Q = -e$, which is changed to $Q = 0$ by an infalling positron, while the corresponding electron escapes to spatial infinity. The double line denotes an electron and the single line a quark. The projected black-hole horizon is shown by the dashed line with the black-hole center to its left.

Concluding remarks

▷ **(1/3) Across-horizon scattering**

Causally disconnected particles can interact with each other through non-local vacuum fluctuations in quantum theory.

The momentum exchange q between inside- and outside-particles is the only measurable observable.

An outside-observer can measure, in principle, the momentum change of the outside-particle, but *not* the charge of the inside-particle or its initial momentum.

Concluding remarks

- **(2/3) Across-horizon information transfer**

As a matter of principle, the across-horizon-scattering effect can be used to encode a message which can be sent by an inside-observer to an outside-observer.

Concluding remarks

▷ **(3/3) Charge extraction from a black hole**

A quantum scattering process not only allows for the extraction of information from a static nonrotating black hole but also for the extraction of electric charge.

No tension with causality: Electric charge is not just a measure of how strongly two electrons scatter with each other, but, rather, a measure of how the corresponding photon propagates. This becomes clear if we examine the structure of radiative corrections contributing to charge renormalization. It turns that out charge renormalization depends only on the photon propagator (including vacuum-polarization effects). One can, therefore, say that the electric charge belongs to the gauge field, rather than to the matter field.

References

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