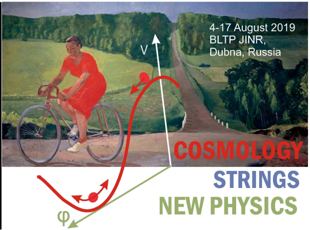
# Growth of Structure in Perturbed Tachyon Dark Energy Model

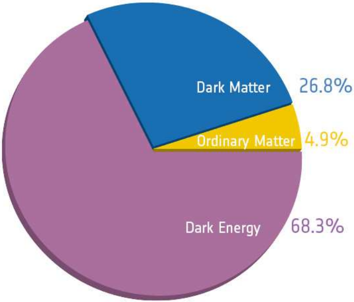
Avinash Singh
IISER, Mohali, India

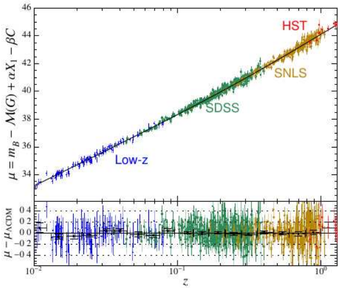
With H. K. Jassal, Archana Sangwan and Manbendra Sharma

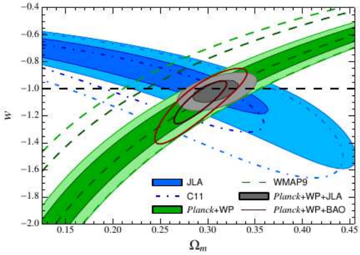



August 7, 2019

# Accelerated expansion




- Appr. 95% cover by **dark** sector.
- Simplest model is $\Lambda CDM$ model.
- Observations do not rule out $w \neq -1$.

Betoule et al., A&A **568** A22 (2014).

**This talk is based on following articles**

- Avinash Singh, Archana Sangwan, H.K. Jassal, *Low redshift observational constraints on tachyon models of dark energy*, JCAP **04** (2019) 047.
- Avinash Singh, H. K. Jassal, Manabendra Sharma, *Perturbations in tachyon dark energy and their effect on matter clustering*. arXiv:1907.13309.

**Lagrangian**

$$\boxed{L = -V(\phi)\sqrt{1 - \partial^\mu\phi\partial_\mu\phi}}$$

**Pressure**

$$\boxed{P_\phi = -V(\phi)\sqrt{1 - \dot{\phi}^2}}$$

**Equation of State**

$$\boxed{w_\phi = \frac{P_\phi}{\rho_\phi} = \dot{\phi}^2 - 1}$$

**Energy density**

$$\boxed{\rho_\phi = \frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}}}$$

**No phantom like EoS**

$$\boxed{-1 \leq w_\phi \leq 0}$$

# Dynamics of Background Universe

**Dynamics of tachyon Scalar Field**

$$\boxed{\ddot{\phi} = -(1-\dot{\phi}^2)\left[3H\dot{\phi} + \frac{1}{V(\phi)}\frac{dV}{d\phi}\right]}$$

**Friedmann Equations for**
$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2)$

$$\boxed{\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho}\,, \quad \boxed{\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P)}$$

**where** $\rho = \rho_m + \rho_r + \rho_\phi$

# Tachyon Potentials

- **Inverse Square Potential**

$$\boxed{V(\phi) = \frac{n}{4\pi G}\left(1 - \frac{2}{3n}\right)^{1/2}\phi^{-2}}$$

- **Exponential Potential**

$$\boxed{V(\phi) = V_a \exp\left(-\phi/\phi_a\right)}$$

**Parameters**

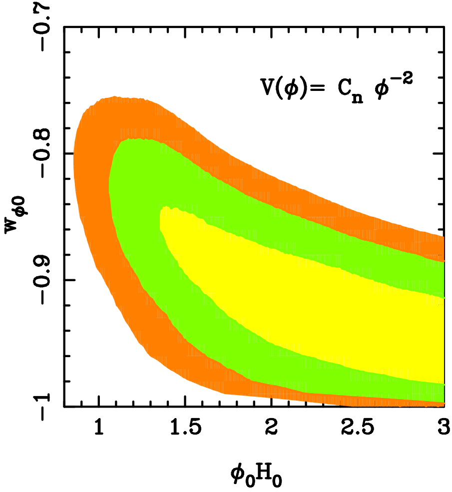
1. $\Omega_{m0}$
2. $\phi_0 H_0$
3. $\dot{\phi}_0$ or $w_{\phi 0}$
4. $\phi_0/\phi_a$

With flatness condition $\Omega_{total} = \Omega_{m0} + \Omega_{r0} + \Omega_{\phi 0} = 1.$

Data Used

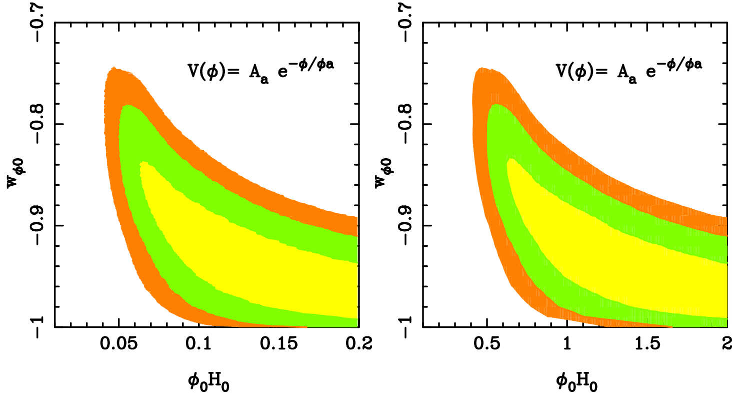
- Supernova-Ia Union 2.1 data.
- Direct measurement of H(z)
- BAO data (from SDSS DR12, 6dFGS, SDSS DR7, WingleZ surveys)

# Background Constraints On $w_{\phi 0} - \phi_0 H_0$ Plane




- $w_{\phi 0}$ and $\phi_0 H_0$ are correlated.
- $\phi_0 H_0 \geq 0.775$ at $3\sigma$ confidence.
- No upper bound on $\phi_0 H_0$

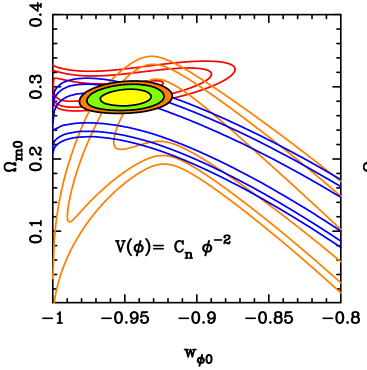
# Background Constraints On $w_{\phi 0} - \phi_0 H_0$ Plane

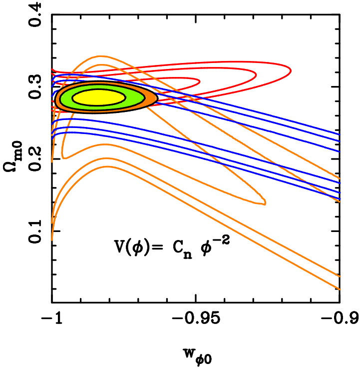



- $w_{\phi 0}$ and $\phi_0 H_0$ are correlated.
- No upper bound on $\phi_0 H_0$

- For $\phi_0/\phi_a = 0.1$ $\phi_0 H_0 \geq 0.045$
- For $\phi_0/\phi_a = 1.0$ $\phi_0 H_0 \geq 0.45$

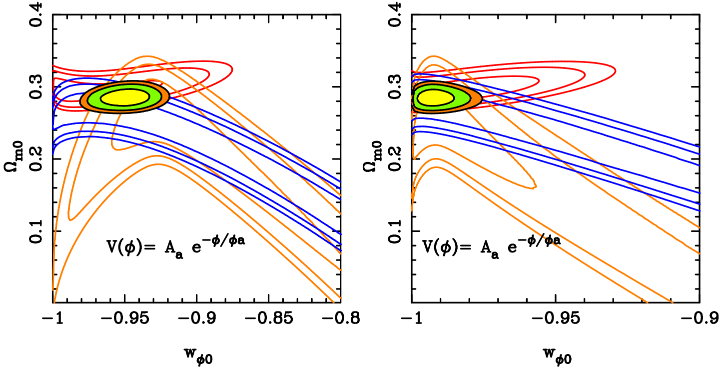
# Background Constraints on $\Omega_{m0}$-$w_{\phi 0}$ Plane



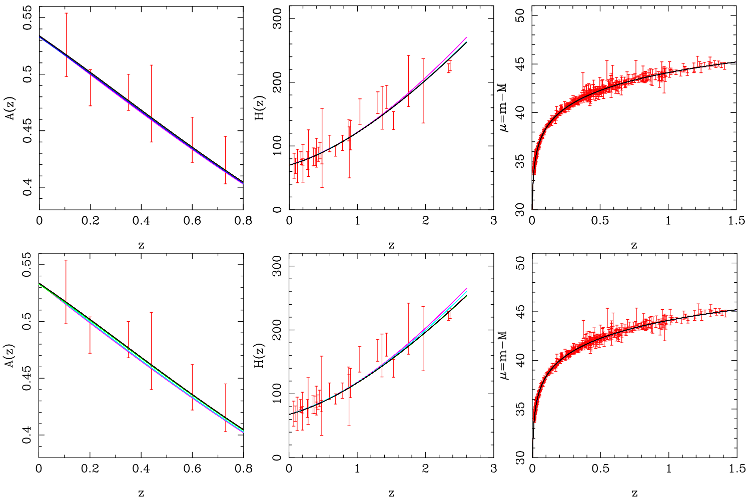

- $w_{\phi 0} \to -1$ as $\phi_0 H_0$ increase
  - $\phi_0 H_0 = 2.0$ (left panel) $w_{\phi 0} = -0.950^{+0.033}_{-0.031}$
  - $\phi_0 H_0 = 4.0$ (right panel) $w_{\phi 0} = -0.984^{+0.021}_{-0.015}$
- $\Omega_{m0} = 0.285^{+0.023}_{-0.022}$ at $3\sigma$ confidence.

# Background Constraints on $\Omega_{m0}$-$w_{\phi 0}$ Plane



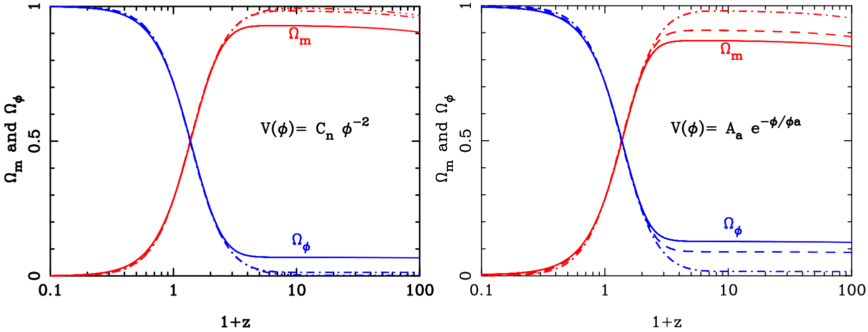

- $w_{\phi 0} \to -1$ as $\phi_0 H_0$ increase
  - $\phi_0 H_0 = 0.1$ (left panel) $w_{\phi 0} = -0.949^{+0.030}_{-0.032}$
  - $\phi_0 H_0 = 0.3$ right panel $w_{\phi 0} = -0.993^{+0.017}_{-0.007}$
- $\Omega_{m0} = 0.285^{+0.023}_{-0.022}$ at $3\sigma$ confidence
- $\phi_0/\phi_a = 0.1$

# Comparison between data and theory



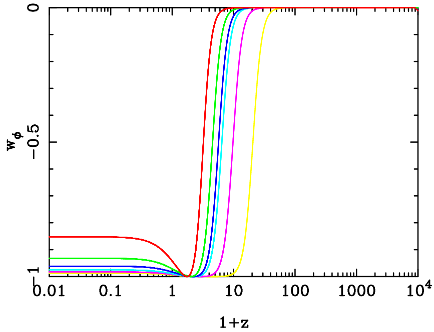

Row-1 for $V(\phi) \propto \phi^{-2}$ and row-2 for $V(\phi) \propto \exp(-\phi/\phi_a)$.

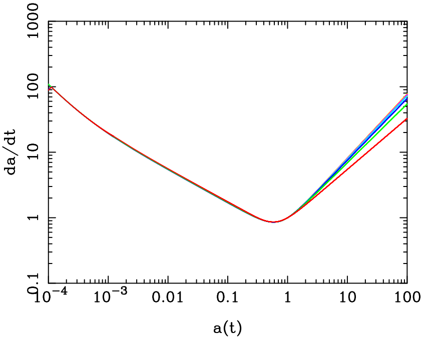
# Background Cosmology- Evolution




Evolution of density parameters.

# Background Cosmology- Evolution for $V(\phi) \propto \phi^{-2}$




Evolution of cosmological phase and equation of state $w_\phi$.

# Background Cosmology- Evolution for $V(\phi) \propto exp(-\phi/\phi_a)$

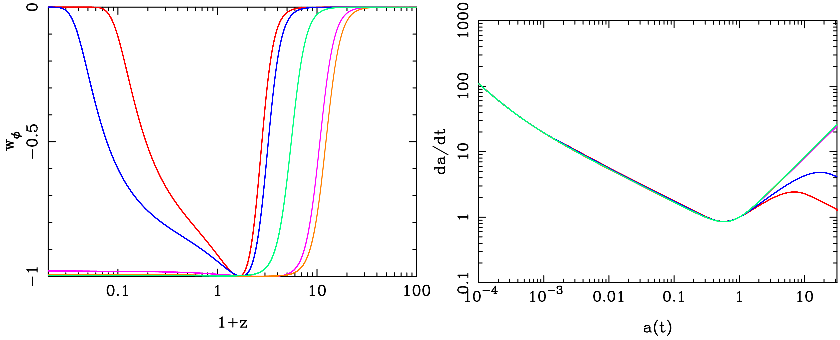



Figure: Evolution of cosmological phase and equation of state $w_\phi$.

# Perturbations in matter and scalar field

In matter field

$$\boxed{\begin{aligned}\rho(t,\vec{x}) &= \rho_b(t) + \delta\rho(t,\vec{x}),\\ p(t,\vec{x}) &= p_b(t) + \delta p(t,\vec{x}),\\ u^\mu &= u^\mu_b + \delta u^\mu,\end{aligned}}$$

In scalar field

$$\boxed{\phi(t,\vec{x}) = \phi_b(t) + \delta\phi(t,\vec{x}).}$$

Subscript 'b' stands for average background values.

# Perturbed Einstein equations

Growth of Structure can be studied by solving

$$\boxed{\delta G^{\mu}_{\nu} = 8\pi G \delta T^{\mu}_{\nu}}$$

- Geometrical part

$$\delta G^{\mu}_{\nu}$$

$$\Downarrow$$

$$\boxed{ds^2 = -(1+2\Phi)dt^2 + a^2(t)(1-2\Phi)[dx^2+dy^2+dz^2]}$$

- Source part

$$\delta T^{\mu}_{\nu} = \delta T^{\mu}_{\nu_{(matter)}} + \delta T^{\mu}_{\nu_{(\phi)}}$$

$$\Downarrow$$

$$\boxed{T^{\mu}_{\nu_{(matter)}} = (\rho + p)u^{\mu}u_{\nu} - p g^{\mu}_{\nu}}$$

and

$$\boxed{T^{\mu\nu}_{(\phi)} = \frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}L_{\phi})}{\delta g_{\mu\nu}}}$$

# Solution of perturbed equations

Background dynamics

$\boxed{a(t), \;\; \phi_b(t)}$

$+$

Perturbed dynamics

$\boxed{\Phi(t,\vec{x}), \;\; \delta\phi(t,\vec{x})}$

Consider all perturbed quantities $\Phi$, $\delta\phi$, $\delta\rho$, $\delta p$ and $\delta u^\mu$ up to first order.
Solved system of equations in Fourier space; using relation $k = \frac{2\pi}{\lambda_p}$, for a fixed $k$ or $\lambda_p$.

Initial conditions at $z_{in} = 1000$

- $w_{\phi_{in}} = -1$ or $\dot{\phi}_{in} = 0$.
- $\delta\phi_{in} = 0$ and $\delta\dot{\phi}_{in} = 0$

Parameters

- $\Omega_{m0} = 0.285$
- $\phi_{in} H_0 \approx \phi_0 H_0$

# Solution of perturbed equations

- The dynamical equation for gravitational potential-

$$\ddot{\Phi} + 4\frac{\dot{a}}{a}\dot{\Phi} + \left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right)\Phi = -4\pi G V(\bar{\phi})\sqrt{1-\dot{\bar{\phi}}^2}\left(\frac{\Phi\dot{\bar{\phi}}^2 - \dot{\delta\phi}\dot{\bar{\phi}}}{1-\dot{\bar{\phi}}^2}\right)$$
$$-4\pi G\left(\frac{\partial V}{\partial\phi}\right)_{\bar{\phi}}\delta\phi\sqrt{1-\dot{\bar{\phi}}^2}$$

- Dynamical equation for scalar field perturbation-

$$\frac{\ddot{\delta\phi}}{(1-\dot{\bar{\phi}}^2)} + \left[3H + \frac{2\dot{\bar{\phi}}\ddot{\bar{\phi}}}{(1-\dot{\bar{\phi}}^2)^2}\right]\dot{\delta\phi}$$
$$+\left[3H\dot{\bar{\phi}}\frac{V'}{V} + \frac{k^2}{a^2} + \frac{\ddot{\bar{\phi}}}{(1-\dot{\bar{\phi}}^2)}\left(\frac{V'}{V}\right) + \frac{V''}{V}\right]\delta\phi$$
$$-\left[12H\dot{\bar{\phi}} + \frac{2(2-\dot{\bar{\phi}}^2)\ddot{\bar{\phi}}}{(1-\dot{\bar{\phi}}^2)} + \frac{2V'}{V} + \frac{2\dot{\bar{\phi}}^4\ddot{\bar{\phi}}}{(1-\dot{\bar{\phi}}^2)^2}\right]\Phi + \frac{5\dot{\bar{\phi}}^3 - 4\dot{\bar{\phi}}}{(1-\dot{\bar{\phi}}^2)}\dot{\Phi} = 0$$

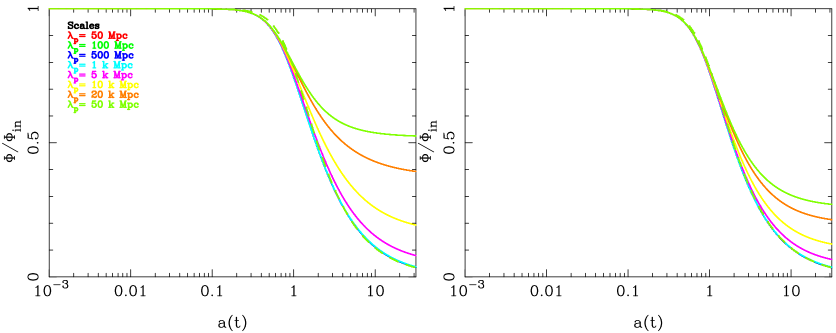
# Solution of perturbed equations

Equations for dark energy density contrast and matter density constrast.

$$\delta_\phi = \frac{V'(\bar{\phi})}{V(\bar{\phi})}\delta\phi - \left(\Phi\dot{\bar{\phi}}^2 - \dot{\bar{\phi}}\dot{\delta\phi}\right),$$

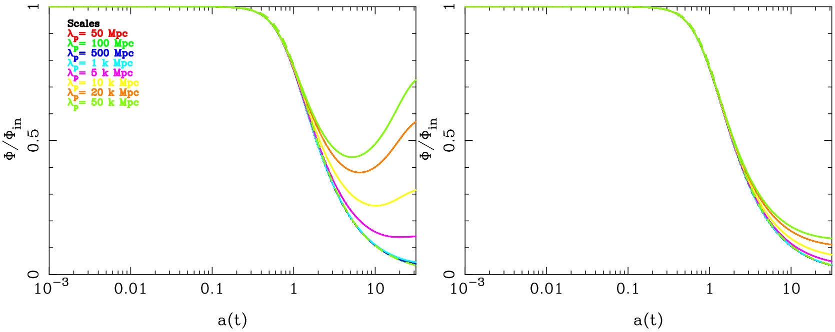
$$\delta_m = -\frac{1}{4\pi G\rho_m a^{-3}}\left[3\frac{\dot{a}^2}{a^2}\Phi + 3\frac{\dot{a}}{a}\dot{\Phi} + \frac{k^2\Phi}{a^2}\right] - \frac{\delta_\phi}{\rho_m a^{-3}}\frac{V(\bar{\phi})}{\sqrt{1-\dot{\bar{\phi}}^2}}.$$

# Evolution of Gravitational Potential $\Phi$ for $V(\phi) \propto \phi^{-2}$



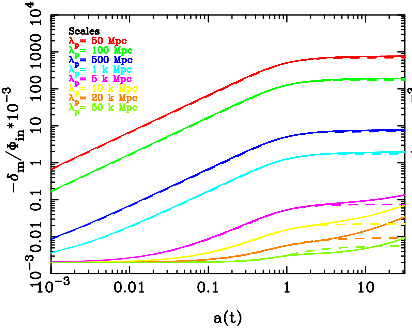

For left panel $\phi_{in} H_0 = 1.0$ and for right panel $\phi_{in} H_0 = 2.0$.

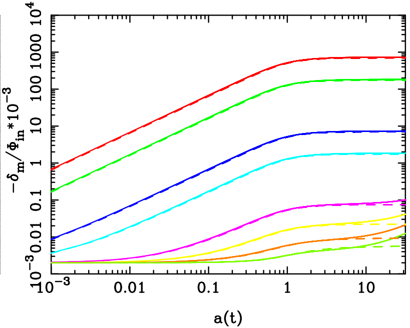
# Evolution of Gravitational Potential $\Phi$ for $V(\phi) \propto exp(-\phi/\phi_a)$




For left plot $\phi_{in} H_0 = 1.0$ and for right plot $\phi_{in} H_0 = 2.0$.

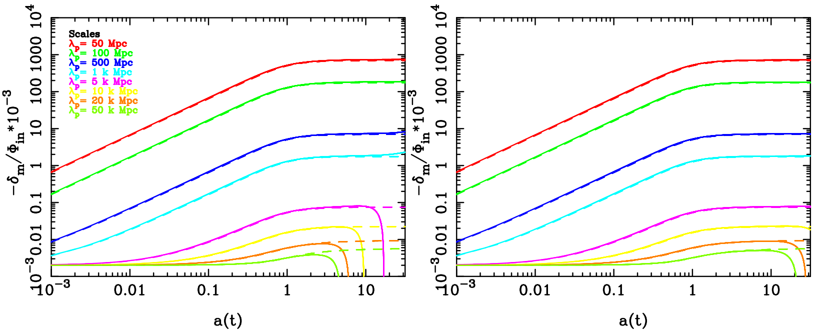
# Evolution of $\delta_m = \frac{\delta\rho_m}{\rho_m}$ for $V(\phi) \propto \phi^{-2}$



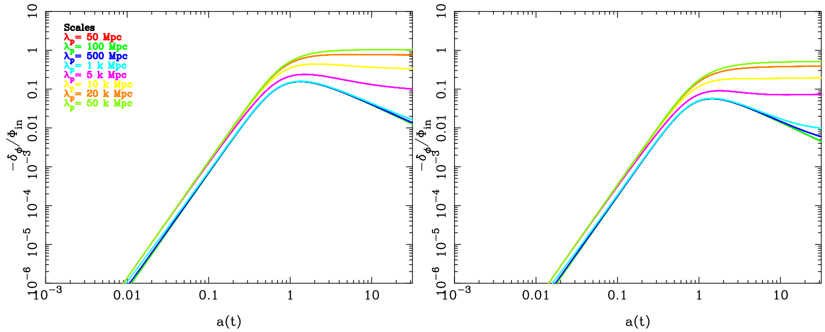

For left plot $\phi_{in} H_0 = 1.0$ and for right plot $\phi_{in} H_0 = 2.0$.

# Evolution of $\delta_m = \frac{\delta\rho_m}{\rho_m}$ for $V(\phi) \propto exp(-\phi/\phi_a)$



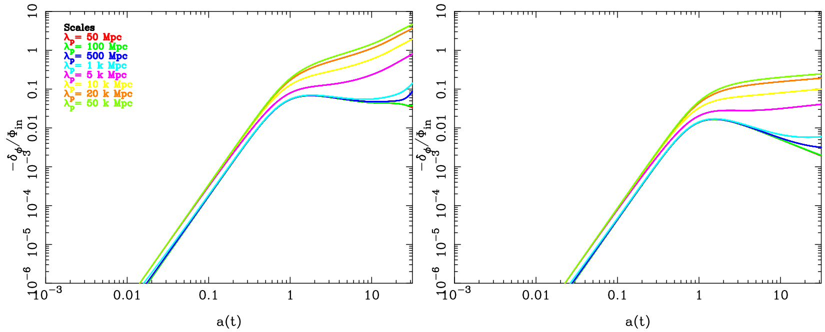

For left plot $\phi_{in}H_0 = 1.0$ and for right plot $\phi_{in}H_0 = 2.0$.

# Evolution of $\delta_\phi = \frac{\delta\rho_\phi}{\rho_\phi}$ for $V(\phi) \propto \phi^{-2}$



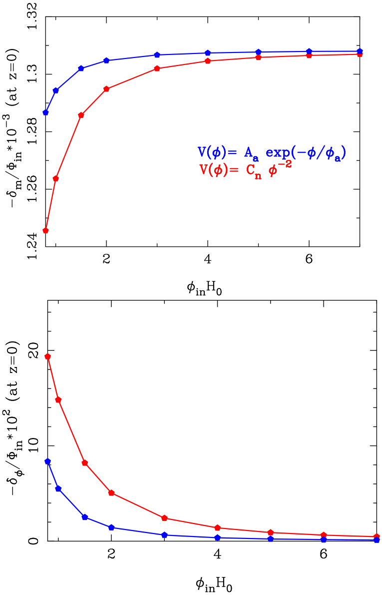

For left plot $\phi_{in} H_0 = 1.0$ and for right plot $\phi_{in} H_0 = 2.0$.

# Evolution of $\delta_\phi = \frac{\delta\rho_\phi}{\rho_\phi}$ for $V(\phi) \propto exp(-\phi/\phi_a)$



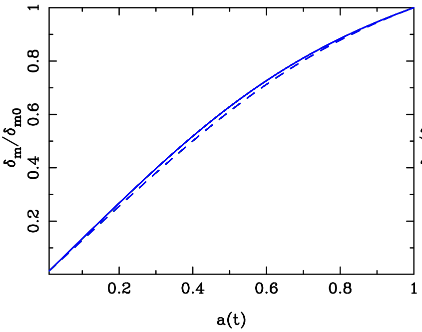

For left plot $\phi_{in}H_0 = 1.0$ and for right plot $\phi_{in}H_0 = 2.0$.

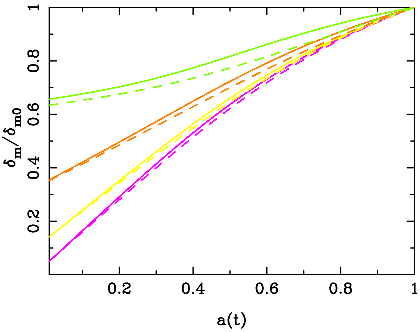
# Dependency of $\delta_m$ and $\delta_\phi$ at $z = 0$ on $\phi_{in} H_0$




- Results for scale $\lambda_p = 1k\ Mpc$
- For fixed $\phi_{in} H_0$, $(\delta_m)_{expo} > (\delta_m)_{inverse}$.
- Whereas $(\delta_\phi)_{expo} < (\delta_\phi)_{inverse}$
- As $\phi_{in} H_0$ increases and $w_\phi \to -1$, $\delta_\phi \to 0$ and both models coincide.

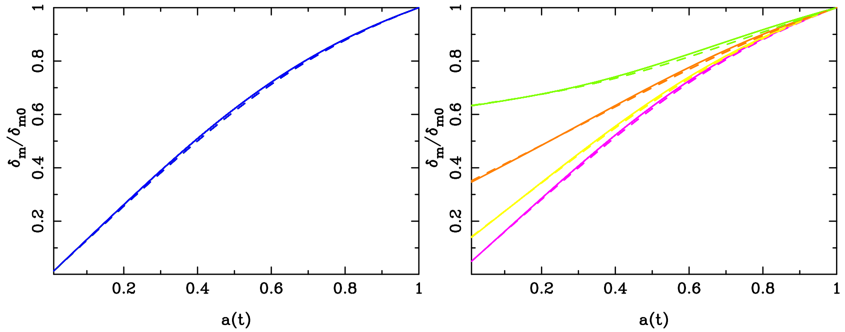
# Evolution of Linear Growth function $D_m^+ = \frac{\delta_m}{\delta_{m0}}$ for $V(\phi) \propto \phi^{-2}$



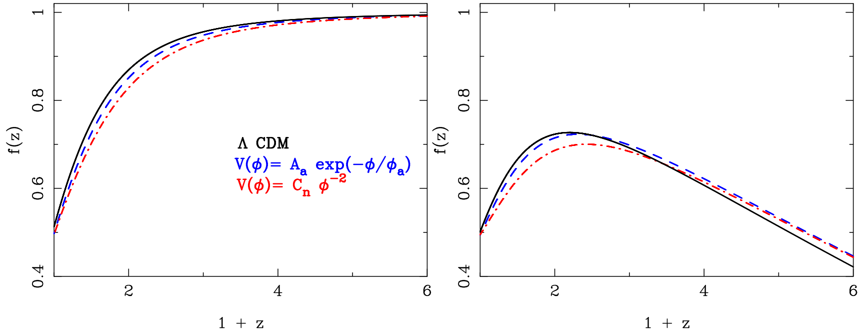

Evolution of $D_m^+$ at sub-Hubble scales (left panel) and super-Hubble scales (right panel) for $\phi_{in} H_0 = 1.0$ and $\Omega_{m0} = 0.285$.

# Evolution of Linear Growth function $D_m^+ = \frac{\delta_m}{\delta_{m0}}$ for $V(\phi) \propto exp(-\phi/\phi_a)$



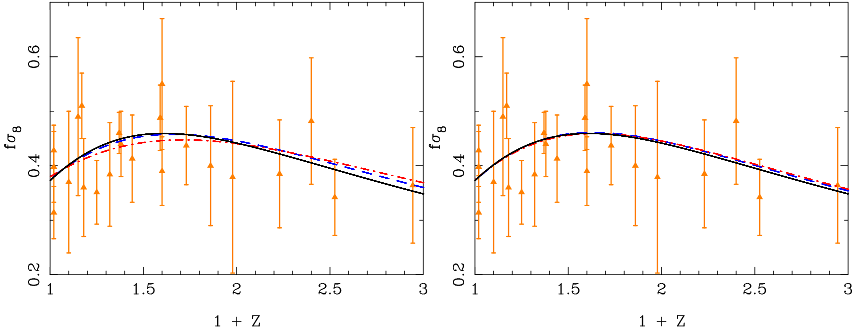

Evolution of $D_m^+$ at sub-Hubble scales (left panel) and super-Hubble scales (right panel) for $\phi_{in}H_0 = 1.0$ and $\Omega_{m0} = 0.285$.

# Evolution of Linear Growth Rate



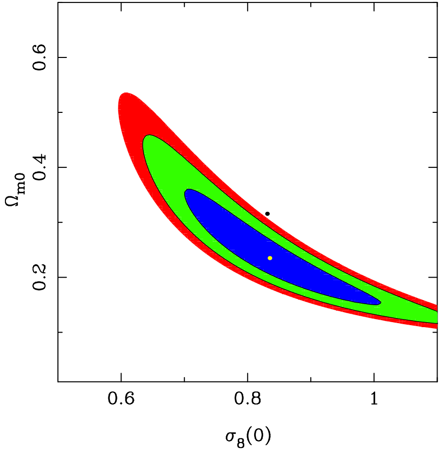

Evolution of linear growth rate $f = \frac{d \ln \delta_m}{d \ln a}$ for $\phi_{in} H_0 = 1.0$ at 50 $Mpc$(left panel) and 5000 $Mpc$ (right panel).

# Comparison between Theory and RSD Data



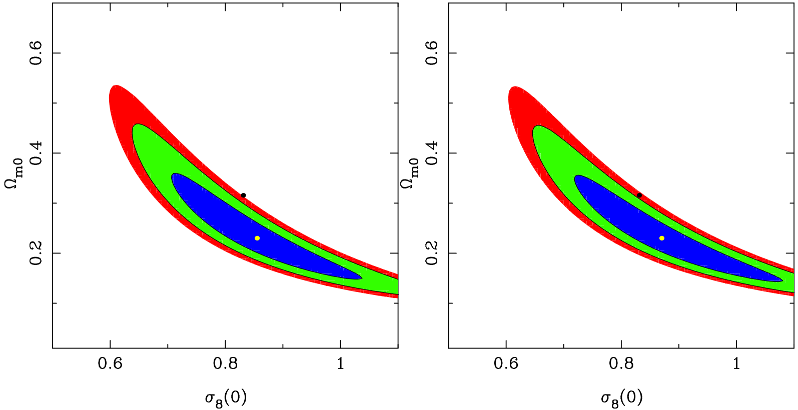

Here, $\sigma_8(z) = \sigma_8(z=0)\frac{\delta_m}{\delta_{m0}}$. Left panel is for $\phi_{in}H_0 = 0.8$ and right panel is for $\phi_{in}H_0 = 2.0$. Other parameters are taken to be the corresponding best fit values.

# Constraints using RSD data: $\Lambda CDM$ **model**



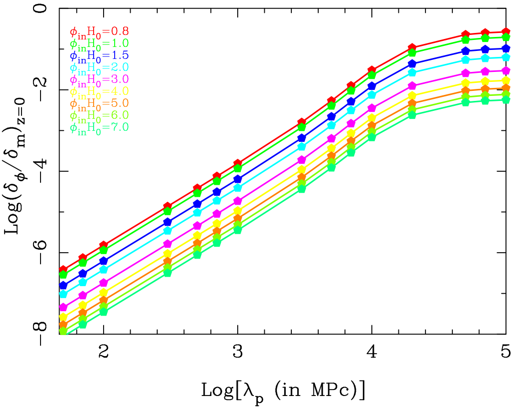

Constraints on $\Omega_{m0} - \sigma_8(0)$ for $\Lambda CDM$ model. Black dot in represents the best fit value for Plank-2018.

# Constraints using RSD data: **tachyon models**




Plot on left and right are for exponential and inverse square potentials respectively. Here, we have fixed $\phi_{in} H_0 = 0.8$. Black dot in each plot represents the best fit value for Plank-2018.

# Summery and Conclusions




- At fixed scale as $\phi_{in}H_0$ increases and $w_{\phi 0} \rightarrow -1$ dark energy become more and more homogeneous.

- $\delta_\phi < 10^{-4}\delta_m$ at scales $\lambda_p < 10^3$ $Mpc$; dark energy can be considered homogeneous.
- At $\lambda_p > 10^3$ $Mpc$ $\delta_\phi$ become significant.
- At $\lambda_p = 10^5$ $Mpc$, for $\phi_{in}H_0 = 0.8$ the ratio $(\delta_\phi/\delta_m)_{z=0} =$ 0.2645 and 0.1060.
- If $w_{\phi 0} \neq -1$ then at Hubble and super-Hubble scales, $\delta_\phi$ become significant.