Towards de Sitter and Inflation in String Theory

(A. Westphal, DESY)

literature:

- Inflation and String Theory
  (textbook by Baumann & McAllister)
  also on arXiv: 1404.2601

- reviews:

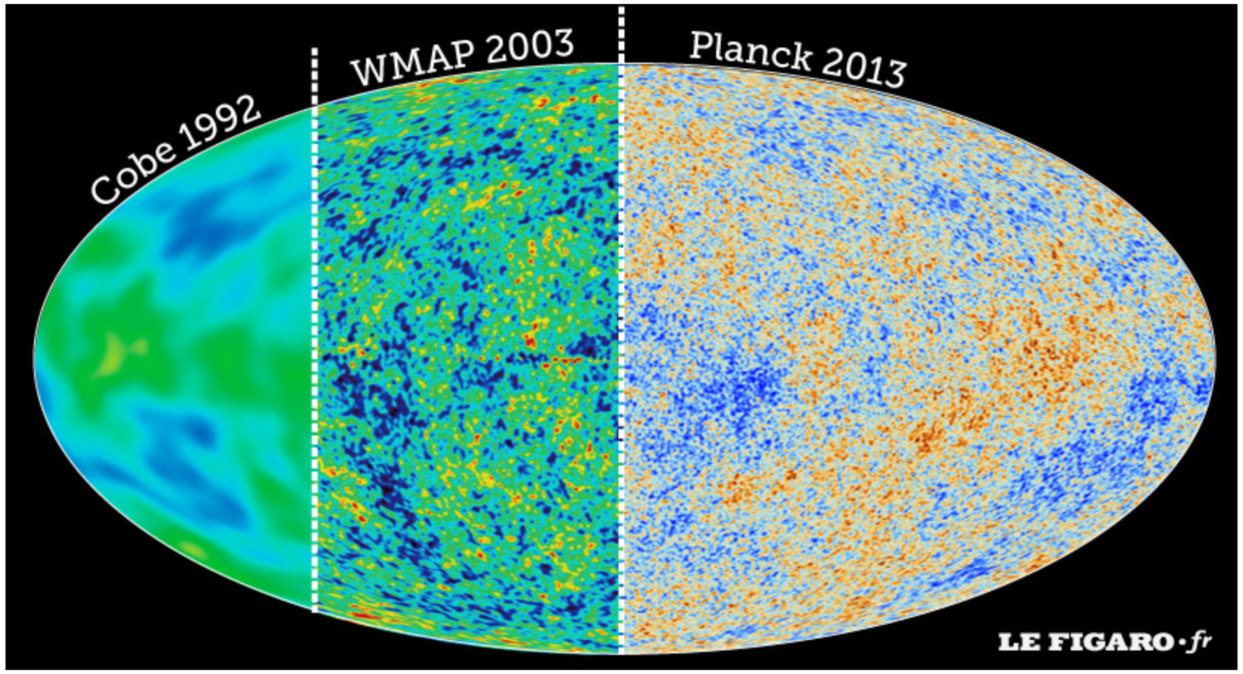
  arXiv: 1409.5350
  (Westphal)

  arXiv: 1306.3512
  (Burgess, Cicoli & Quevedo)

  arXiv: 0803.1194
  (Denef)

- String Theory and Particle Physics
  – An Introduction to String Pheno
  (textbook by Ibanez & Uranga)

INFLATION SEEDS STRUCTURE FORMATION



Power spectrum of density fluctuations at the moment of horizon re-entry

$$C_\ell = \int \mathrm{d}\ln k \; P_\zeta(k) T_\ell^2(k)$$

Angular power spectrum of CMB temperature fluctuations

Transfer function

$\Downarrow$ CMB: spatially isotropic & spatial homogeneity

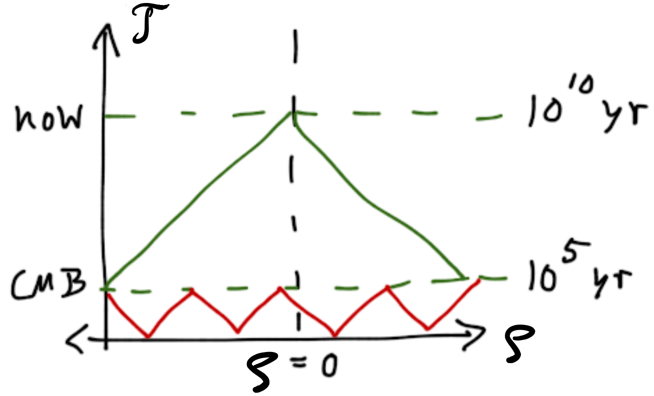
FRW space-time:

$$ds^2 = dt^2 - a^2(t)\cdot\left[d\rho^2 + f^2(\rho)\cdot d\Omega_2^2\right]$$

$$\left.\frac{S^3,\ H^3,\ \mathbb{R}^3}{k = +1,\ -1,\ 0}\right\}$$ maximally symm. 3-space

$d\rho = \frac{dr}{a}$ 'comoving distance'

use conformal time: $dt \to d\tau = \frac{dt}{a}$



horizon problem

Einstein's equations

$$\Rightarrow\ H^2 = \frac{1}{3M_p^2}\,\rho - \frac{k}{a^2}$$ Friedmann equation

energy density: matter, radiation, ...

spatial curvature

ρ drives expansion ...

flatness problem: $\Omega_i \equiv \frac{\rho_i}{\rho_{flat}}$

$$\left.\frac{\rho_k}{\rho_{rad.}}\right|_{now} = \left.\frac{\Omega_k}{\Omega_{rad.}}\right|_{now} \sim e^{120}\left.\frac{\rho_k}{\rho_{rad.}}\right|_{t_{GUT}} \lesssim 0.01$$

$$\Rightarrow \text{need } \left.\frac{\rho_k}{\rho_{rad}}\right|_{t_{GUT}} \simeq \Omega_k\Big|_{t_{GUT}} < e^{-120}\,!$$

solution? $\longrightarrow$ $\ddot{a} > 0$ acceleration early on comoving particle horizon:

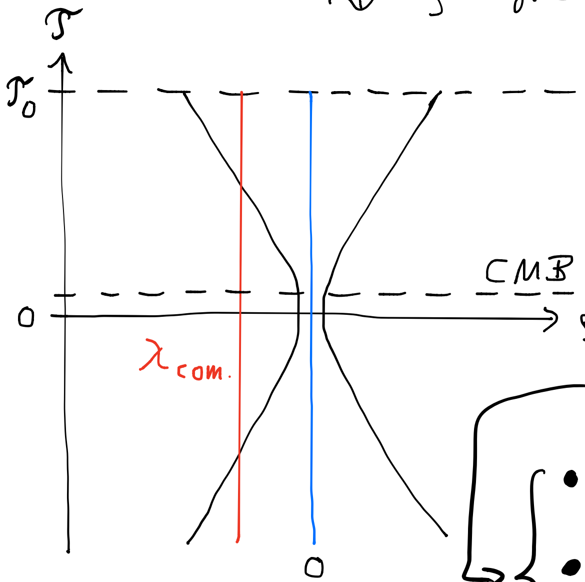
$$\rho = \int_0^{\tau_0} d\tau = \int_0^{t_0} \frac{dt}{a} = \int_0^{a_0} \frac{d\ln a}{aH}$$

comoving spatial distance

and $\frac{1}{aH} \sim a^{\frac{1}{2}(1+3w)}$

$\Rightarrow$ for $w > -\frac{1}{3}$: $\frac{1}{aH}$ dominated at large a

$\rightsquigarrow$ ρ growing with a



$\frac{d}{dt}\left(\frac{1}{aH}\right) > 0$

$\frac{d}{dt}\left(\frac{1}{aH}\right) < 0$

- $\dot{a}, H < 0$: bounce

  or

- $\ddot{a}, H > 0$: inflation

  $\epsilon_H = -\frac{\dot{H}}{H^2} < 1$

if ρ mostly:	c.c.	matter	radiation
$a(t)$	e^{Ht}	$t^{2/3}$	$\sqrt{t}$
$H(t)$	const.	$\sim \frac{1}{t}$	$\sim \frac{1}{t}$
p	$-\rho$	0	$\frac{1}{3}\rho$

inflation!

$\frac{\rho_k}{\rho_{c.c.}} \sim a^{-2} \Rightarrow$ need $e^{-N_e} \equiv e^{-Ht} \sim e^{-60}$

I slow-roll inflation (Linde | Albrecht & Steinhardt '82)

1 scalar field ϕ, action:

$$S = \int d^4x \sqrt{-g}\left[\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - V(\phi)\right]$$

scalar potential

$\delta S / \delta g^{\mu\nu}$

$T_{\mu\nu}$: $\rho = \frac{1}{2}(\partial\phi)^2 + V$, $p = \frac{1}{2}(\partial\phi)^2 - V$

consider: $\vec{\nabla}\phi = 0$, only $\dot{\phi}$

$\leftarrow$ redshifts fast, if $a \sim e^{H\cdot t}$

then if: $\boxed{\dot{\phi}^2 \ll V}$ $\Rightarrow p = -\rho$

motion dominated by potential energy

$$\begin{cases} a \sim e^{H\cdot t} \\ H \simeq \text{const.} \end{cases}$$

can ensure this, if slow-roll:

e.o.m. for ϕ:

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{\partial V}{\partial \phi} \equiv -V'$$

slow-roll: $|\ddot{\phi}| \ll |3H\dot{\phi}|, |V'|$

$$\Rightarrow 3H\dot{\phi} = -V' \quad \text{slow-roll e.o.m.} \quad (*)$$

then i): $p \simeq -\rho$

$$\Rightarrow 1 \gg \frac{\dot{\phi}^2}{V} = \frac{V'^2}{9VH^2}, \quad H^2 \simeq \frac{V}{3}$$

$$= \frac{1}{3}\left(\frac{V'}{V}\right)^2 = \frac{2}{3}\epsilon, \quad \boxed{\epsilon \equiv \frac{1}{2}\left(\frac{V'}{V}\right)^2}$$

$$\Rightarrow \boxed{\epsilon \ll 1 \text{ ensures } p \simeq -\rho.}$$

<u>1st slow-roll condition</u>

ii) ensure slow-roll for long time:

$\hookrightarrow$ maintain: $\ddot{\varphi} \ll 3H\dot{\varphi}$

from (*) $\Rightarrow$ $\dot{\varphi}^2 = \frac{V'^2}{3V}$

and using (*) compute:

$$\frac{\ddot{\varphi}}{3H\dot{\varphi}} = 2\epsilon - \frac{1}{3}\eta \,, \qquad \boxed{\eta \equiv \frac{V''}{V}}$$

$\Rightarrow \dfrac{\ddot{\varphi}}{3H\dot{\varphi}} \ll 1$ implies $\boxed{\eta \ll 1}$ if $\epsilon \ll 1$

2nd slow-roll condition

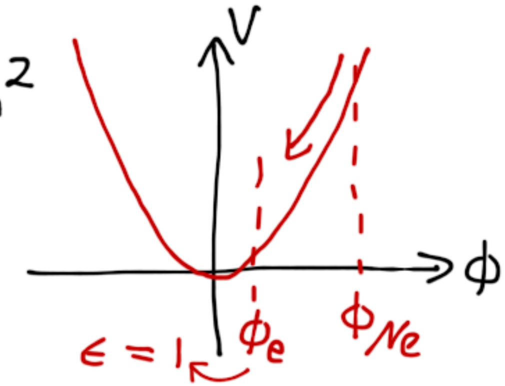
2 classes of $V(\phi)$

i) large-field models

examples: $V(\phi) \sim \phi^p$, $p \geq 2$

e.g. $V(\phi) = \frac{m^2}{2}\phi^2$

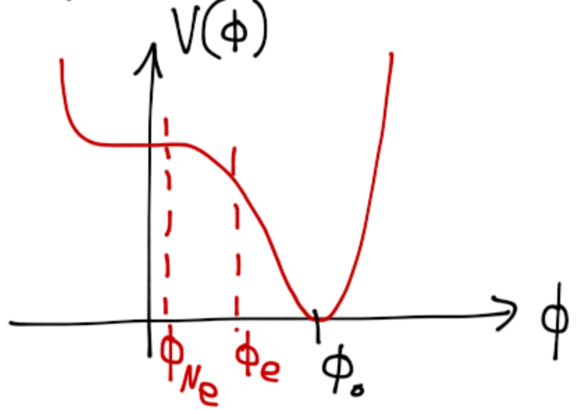
$\phi_e : \epsilon(\phi_e) = 1$



$$\epsilon = \frac{p^2}{2\phi^2} \Rightarrow \boxed{\phi_e \sim M_p}$$

$$N_e = \int_{t_e}^{t_{N_e}} H\,dt = \int_{\phi_e}^{\phi_{N_e}} \frac{H}{\dot{\phi}}\,d\phi = \int_{\phi_e}^{\phi_{N_e}} \frac{d\phi}{\sqrt{2\epsilon}} \simeq \frac{\phi_{N_e}^2}{2p} \Rightarrow \boxed{\phi_{N_e} = \sqrt{2p\,N_e} \gg M_p}$$

ii) small-field models



$$V(\phi) = V_0\left(1 - \sqrt{2\epsilon_0}\cdot\phi - \frac{4}{\phi_0^3}\phi^3 + \frac{3}{\phi_0^4}\phi^4\right)$$

$V'(\phi_0) = 0$, $V(\phi_0) = 0$

$$\Rightarrow \sqrt{2\epsilon} = \left|\frac{V'}{V}\right| = \frac{\sqrt{2\epsilon_0} - \frac{12}{\phi_0^3}\phi^2}{1-\sqrt{2\epsilon_0}\phi}$$

for $\phi \ll 1$, $\epsilon = 1$:

$\Rightarrow \phi_e \ll 1$ if $\phi_0 \ll 1$.

$$\frac{\phi_e}{\phi_0} = \frac{2^{1/4}}{2\sqrt{3}}\sqrt{\frac{\phi_0}{M_P}}$$

Why $\phi \sim M_p$ as discriminator?

→ consider higher-dim. operators correcting V ...

⇒ generically we get among them:

$$\Delta V_d \sim V \cdot \left(\frac{\phi}{M_p}\right)^{d-4}$$

$$\Rightarrow \Delta\eta = \frac{\Delta V_d''}{V} \sim (\phi/M_p)^{d-6}$$

$$\Rightarrow \Delta\epsilon \sim \left(\frac{\phi}{M_p}\right)^{2(d-5)} \to 1 \text{ at } \phi \sim M_p$$

→ destroys slow-roll: "eta-problem"

2 ways out:

- keep $\phi \ll M_p$ "small-field" and look for UV-theory to enumerate finite # of dim-6 corrections — and tune
- find a symmetry, that forbids all higher terms → then $\phi \gg M_p$ possible...

∼ <u>example:</u> $V(\phi) \sim \phi^p$ + gravity

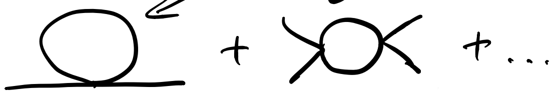
graviton vertex $\sim T_{\mu\nu} \sim V, V''$

$\Rightarrow \Delta V \sim V^2, V''\cdot V$ <u>not</u> $V \cdot \frac{\phi^2}{M_p^2}$

$\rightarrow$ <u>shift symmetry</u> of gravity...

∼ look at scalar corrections as well – 'daisy' diagrams e.g. for $V_0 = \lambda \cdot \phi^4$:

individually catastrophic!



$+ \ldots =$

$$= (-1)^n \lambda \phi^4 \left(\frac{\phi}{M_p}\right)^{2n-4}$$

benign alternating series

$$= V_0(\phi) \cdot \left(1 + c \cdot \ln \frac{\phi}{M_p}\right)$$

$\rightarrow$ $\boxed{\text{Clearly need UV completion!}}$

<u>Quantum description $\rightarrow$ perturbations</u>

generalized Hawking radiation:

$\rightarrow$ QF in grav. system with event horizon of size R_h

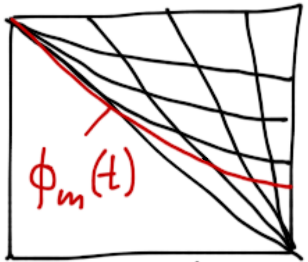
$\Rightarrow$ light fields radiate quanta w/ thermal distro and: $T \sim \frac{1}{R_h}$

- BHs: $R_h = 2GM \Rightarrow T_{BH} \sim \frac{1}{M}$
- dS: $R_h \sim H^{-1}$

$\Rightarrow T_{dS} \sim H$

$\Rightarrow \langle \delta\phi_k^2 \rangle, \langle \delta g_{\mu\nu_k}^2 \rangle \sim H^2$

define time-slicing by $\phi_m(t)$:



- slow-roll inflation: time translation inv. of dS <u>slightly broken</u>

  ⇝ Goldstone boson $\pi(\vec{x},t)$ due to spontaneous time transl. symm. breaking:

$$t \to U(\vec{x},t) = t + \pi(\vec{x},t)$$

- fluctuations of background fields:

$$\delta\phi_m(t) = \phi_m\big(t + \pi(\vec{x},t)\big) - \phi_m(t)$$

Einstein eq.s couple $\delta g_{\mu\nu}$ to π.
Use gauge freedoms of the metric:

i) spat. flat gauge: $g_{ij} = a^2 \cdot \delta_{ij}$

δg_{00}, δg_{0i} linked to π by Einstein eq.s
$\Rightarrow$ only fluctuation mode is π

ii) comoving gauge: $\delta\phi_m = 0$

π <u>eaten</u> by $\delta g_{\mu\nu}$, $g_{ij} = a^2 \cdot e^{2\zeta(\vec{x},t)}\delta_{ij}$

$\Rightarrow$ relation: $\zeta = -H\pi + \dots$

<u>eff. action</u>: 'chiral lagrangian' of dS inv. breaking

$$S = \int d^4x\sqrt{-g}\left(U, (\partial_\mu U)^2, \Box U, \dots\right)$$

$$= \int d^4x\sqrt{-g}\, M_p^2\,\dot{H}\cdot\left[\dot{\pi}^2 - \frac{1}{a^2}(\partial_i\pi)^2 + 3\epsilon_H H^2\pi^2 + \dots\right]$$

$$= \int d^4x \cdot a^3 \cdot 2M_p^2\epsilon_H\left[\dot{\zeta}^2 - \frac{1}{a^2}(\partial_i\zeta)^2 + \dots\right]$$