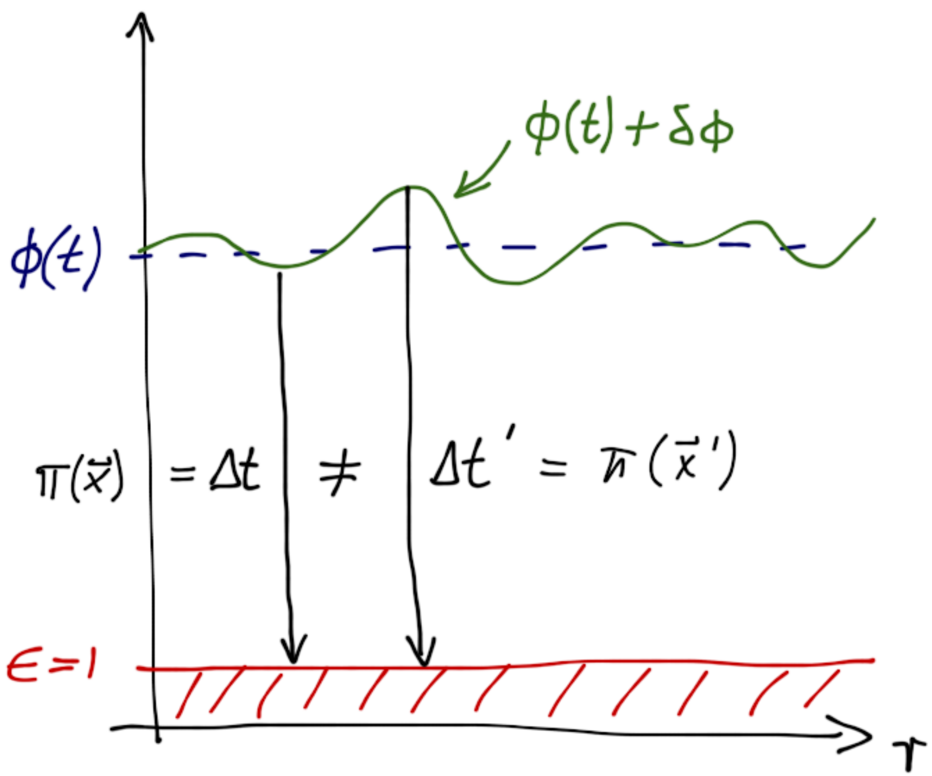
we can display this graphically:



$$ds^2 = dt^2 - a^2 \cdot (1 + 2\zeta + \dots) \cdot \delta_{ij}\, dx^i dx^j$$

things simplify in conformal time:

$$dt = a \cdot d\tau$$

$$\Rightarrow S = M_p^2 \cdot \int d\tau\, d^3x \cdot \underbrace{2a^2 \epsilon_H}_{\equiv z^2} \cdot \left[(\zeta')^2 - (\partial_i \zeta)^2 \right]$$

canonically normalize ζ and go to Fourier modes of field:

$$\zeta \rightarrow v = M_p \cdot z \cdot \zeta$$

$$v = \int \frac{d^3k}{(2\pi)^3} \cdot v_k\, e^{i\vec{k}\vec{r}}$$

$$S = \int \frac{d^3k}{(2\pi)^3} S_k$$

$$S_k = \int d\tau \cdot \left[(v_k')^2 - \left(k^2 - \frac{z''}{z}\right) v_k^2 \right]$$

e.o.m. from $\delta S / \delta v_k$:

$$v_k'' - \left(k^2 - \frac{z''}{z}\right) v_k = 0$$

'Mukhanov - Sasaki eq.'

for quasi-dS :

$$\frac{z''}{z} = \frac{2}{\tau^2}\left(1 + \mathcal{O}(\epsilon, \eta)\right)$$

mode solutions:

$$v_k = \alpha \cdot \frac{e^{-ik\tau}}{\sqrt{2k}}\left(1 - \frac{i}{k\tau}\right) + \beta \cdot \frac{e^{ik\tau}}{\sqrt{2k}}\left(1 + \frac{i}{k\tau}\right)$$

for $\tau = -\frac{1}{aH} \to \infty$ (early times = small distances) curvature negligible & modes should be like Minkowski vacuum modes:

$$\lim_{\tau \to -\infty} v_k = \frac{e^{-ik\tau}}{\sqrt{2k}}$$ 'Bunch-Davies vacuum'

$\Rightarrow \alpha = 1 \,,\; \beta = 0$

quantization now straight forward:

$$\downarrow \quad v_k \to \hat{v}_k = \hat{a}_{\vec{k}} v_k + \hat{a}^{+}_{-\vec{k}} v^*_{-k}$$

compute 2-point function:

$$\langle \hat{v}_k \hat{v}_{k'} \rangle = \langle 0 | \hat{v}_k \hat{v}_{k'} | 0 \rangle$$

$$= \langle 0 | (a_{\vec{k}} v_k + a^{+}_{-\vec{k}} v^*_k)(a_{\vec{k}'} v_{k'} + a^{+}_{-\vec{k}'} v^*_{k'}) | 0 \rangle$$

$$= v_k v^*_{k'} \langle 0 | [a_{\vec{k}}, a^{+}_{-\vec{k}'}] | 0 \rangle$$

$$= |v_k|^2 \cdot \delta^{(3)}(\vec{k} + \vec{k}')$$

$$\Rightarrow |v_k|^2 = \frac{1}{2k^3}\cdot\frac{1}{\tau^2} = \frac{a^2H^2}{2k^3}$$

$$v = z\zeta\,, \quad \Delta_v^2 \equiv \frac{k^3}{2\pi^2}|v_k|^2 \Big| (M_p = 1)$$

↖ dim.-less power spectru

$$\Rightarrow \Delta_\zeta^2 = \frac{1}{z^2}\Delta_v^2 = \frac{1}{8\pi^2}\cdot\frac{H^2}{\epsilon} \quad (\epsilon_H \approx \epsilon)$$

$$\leftarrow \dot\phi = -\frac{V'}{3H} = -H\sqrt{2\epsilon}$$

$$= \frac{1}{4\pi^2}\cdot\frac{H^4}{\dot\phi^2}$$

similarly, tensor fluctuations → primordial gravitational waves:

$$ds^2 = dt^2 - a^2\cdot\left(e^{2\zeta}\cdot\delta_{ij} + h_{ij}\right)\cdot dx^i dx^j$$

$$\Rightarrow S = \int d\tau\, d^3x \cdot \frac{M_p^2}{4} a^2 \cdot \left[(h_{ij}')^2 - (\vec\nabla h_{ij})^2\right]$$

$$u_{ij} = M_p \frac{a}{2} h_{ij}$$

$$= \int \frac{d^3k}{(2\pi)^3}\int d\tau \cdot \sum_\gamma \left[(u_{k,\gamma}')^2 - \left(k^2 - \frac{a''}{a}\right)u_{k,\gamma}^2\right]$$

↖ 2 graviton polarizations

$$\Rightarrow \langle \hat u_{k,\gamma}\hat u_{k',\gamma}\rangle = |u_{k,\gamma}|^2 \cdot \delta^{(3)}(\vec k + \vec k')$$

$$\Rightarrow \Delta_T^2 = \underset{\uparrow \sum_\gamma}{2}\cdot\frac{4}{M_p^2 a^2}\Delta_u^2 = \frac{4k^3}{\pi^2}\cdot\frac{1}{M_p^2 a^2}\cdot|u_{k,\gamma}|^2$$

$$= \frac{2}{\pi^2}\cdot\frac{H^2}{M_p^2}$$

$$\Rightarrow \boxed{r \equiv \frac{\Delta_T^2}{\Delta_\zeta^2} = 16\epsilon}$$

'tensor-to-scalar ratio'

in slow-roll: $\dot{\phi} = -\frac{V'}{3H}$

$$\Rightarrow \Delta_\zeta^2 = \frac{1}{12\pi^2} \cdot \frac{V^3}{V'^2} = \frac{1}{24\pi^2} \cdot \frac{V}{\epsilon}$$

spectral tilt n_s of Δ_ζ^2:

$$n_s = 1 + \frac{d \ln \Delta_\zeta^2}{d \ln k}$$

use: • $k = a\, k_{ph}\big|_{hor.\ cross.} = e^{-N_e} H$

N_e e-folds before end of infl.

$\Rightarrow d \ln k = - d N_e$

• $\frac{d\phi}{dN_e} = \frac{\dot{\phi}}{H} = -\frac{V'}{V}$

$$\Rightarrow n_s = 1 - 6\epsilon + 2\eta$$

2nd significance of r:

compute $N_e = \int H dt = \int \frac{d\varphi}{\sqrt{2\epsilon}}$

$$\Rightarrow N_e \simeq \frac{\Delta\phi}{M_p} \cdot \frac{1}{\sqrt{2\epsilon}}$$

$$\Leftrightarrow r = 16\epsilon \simeq \frac{8}{N_e^2} \cdot \left(\frac{\Delta\phi}{M_p}\right)^2$$

$$\Rightarrow \boxed{r \simeq 0.003 \cdot \left(\frac{50}{N_e}\right)^2 \cdot \left(\frac{\Delta\phi}{M_p}\right)^2}$$

'Lyth bound'

$\rightsquigarrow$ $r \sim 0.01$ corresponds to boundary between large-field and small-field inflation.

slow-roll inflation ...

[Guth, Linde, Starobinsky, Albrecht, Steinhardt '80s]

$$\cancel{\ddot{\phi}} + 3H\dot{\phi} = -\partial_\phi V \equiv -V' \quad , \quad H^2 = \frac{1}{3}\left(\frac{1}{2}\cancel{\dot{\phi}^2} + V\right)$$

$$a \sim e^{Ht}$$

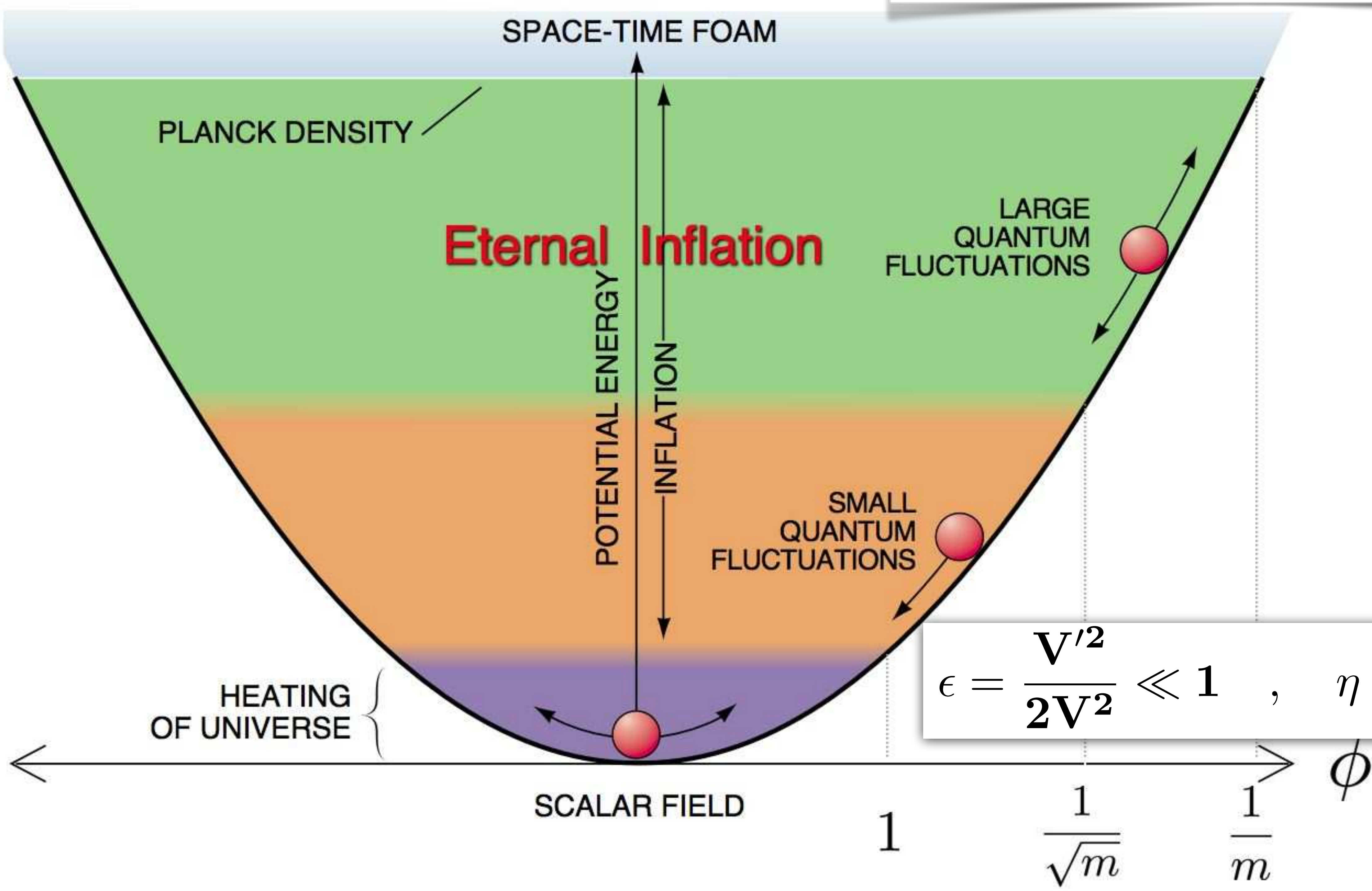
$$\epsilon = \frac{\mathbf{V'^2}}{\mathbf{2V^2}} \ll \mathbf{1} \quad , \quad \eta = \frac{\mathbf{V''}}{\mathbf{V}} \ll \mathbf{1}$$

slow-roll inflation ...

[Guth, Linde, Starobinsky, Albrecht, Steinhardt '80s]

$$V(\phi) = \frac{m^2}{2}\phi^2$$

$$3H\dot{\phi} \simeq -V' \; , \; H^2 \simeq \frac{1}{3}V$$



$$\epsilon = \frac{\mathbf{V'^2}}{\mathbf{2V^2}} \ll \mathbf{1} \quad , \quad \eta = \frac{\mathbf{V''}}{\mathbf{V}} \ll \mathbf{1}$$

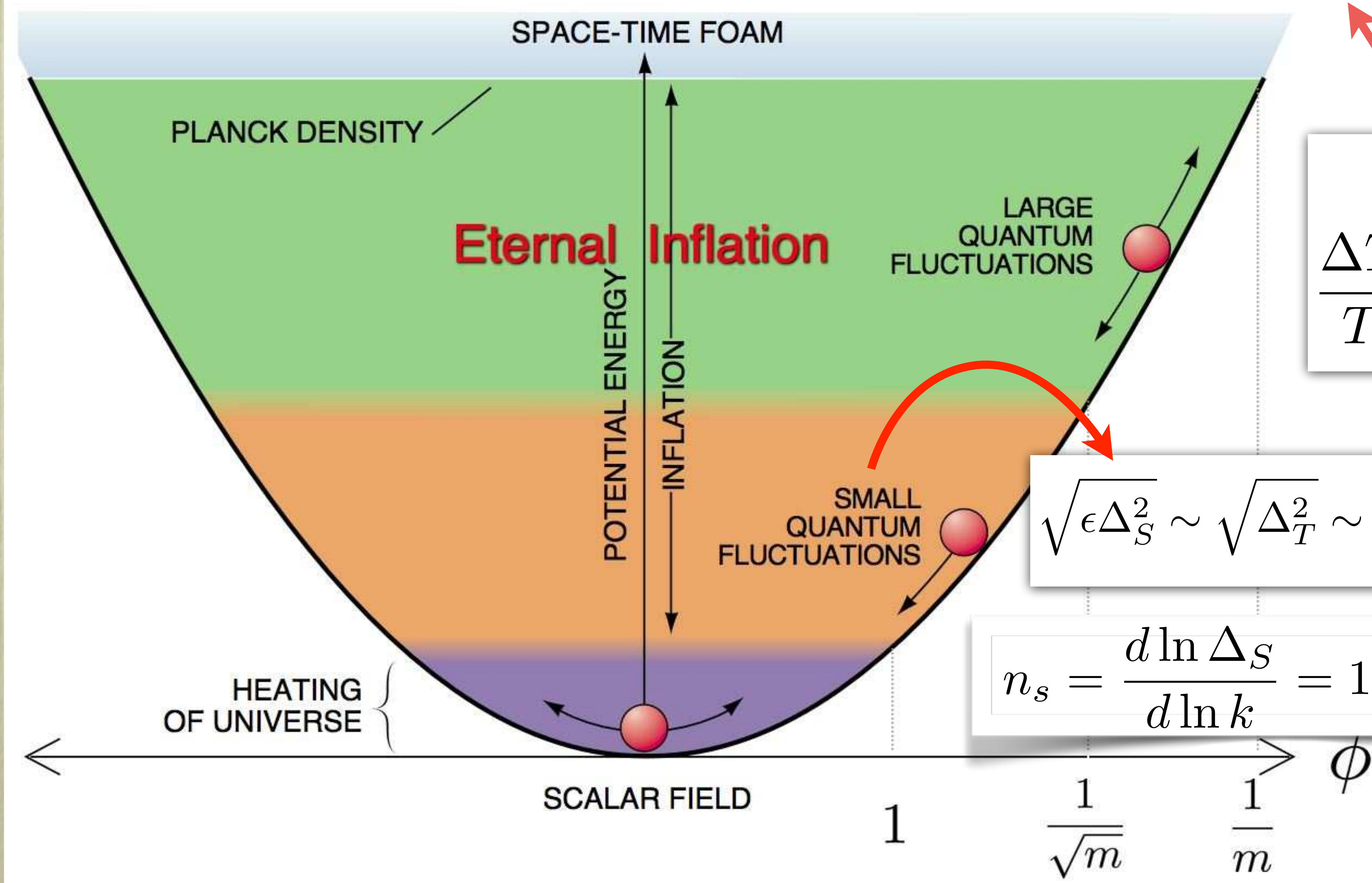
[picture from lecture notes: Linde '07]

slow-roll inflation ...

[Guth, Linde, Starobinsky, Albrecht, Steinhardt '80s]

$$V(\phi) = \frac{m^2}{2}\phi^2$$

$$m \sim 10^{13}\,\mathrm{GeV}$$



CMB:

$$\frac{\Delta T}{T} \sim 10^{-5}$$

$$\sqrt{\epsilon \Delta_S^2} \sim \sqrt{\Delta_T^2} \sim \frac{H}{M_P} \sim 10^{-5}$$

$$n_s = \frac{d \ln \Delta_S}{d \ln k} = 1 - 6\epsilon + 2\eta$$

[picture from lecture notes: Linde '07]

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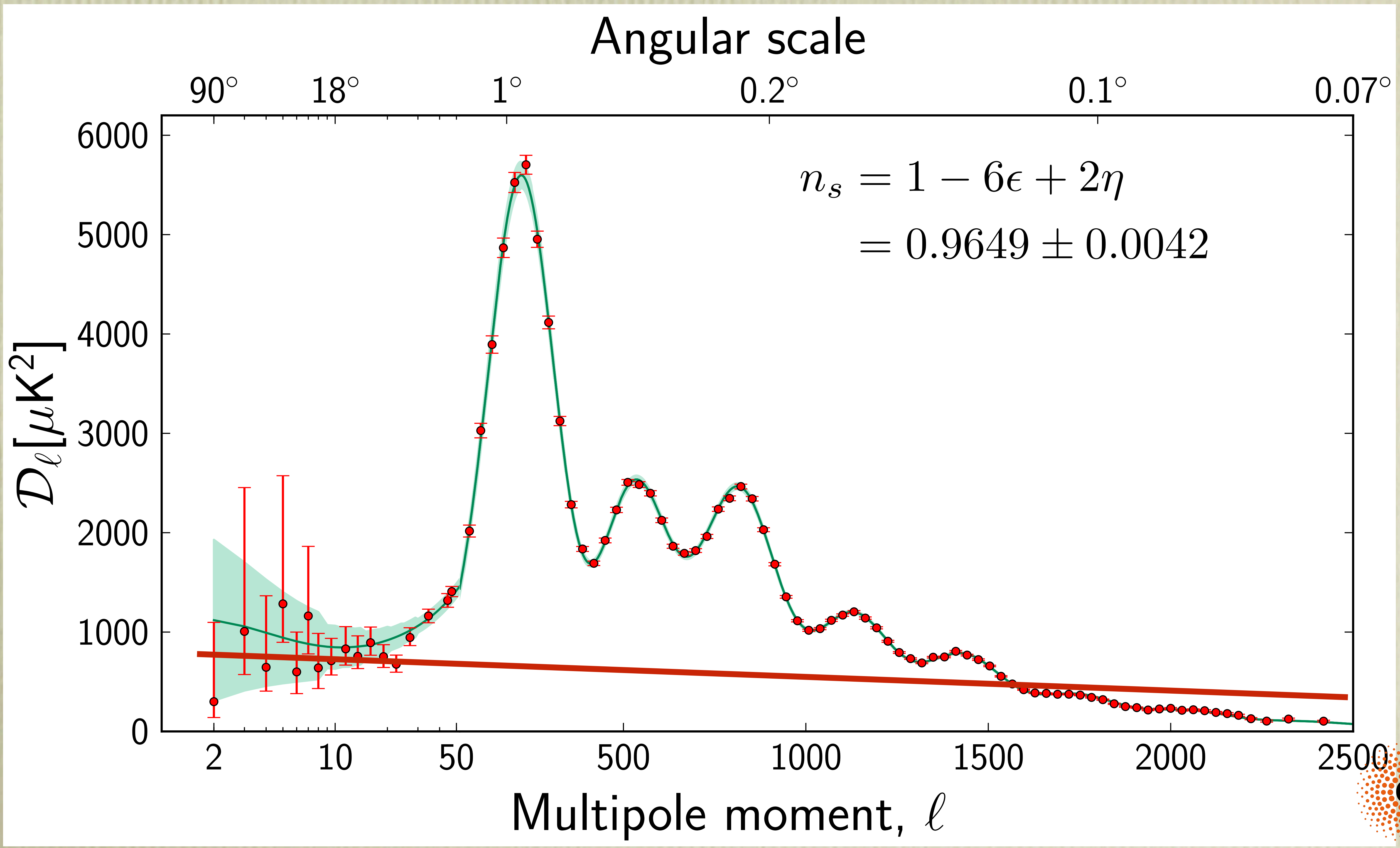
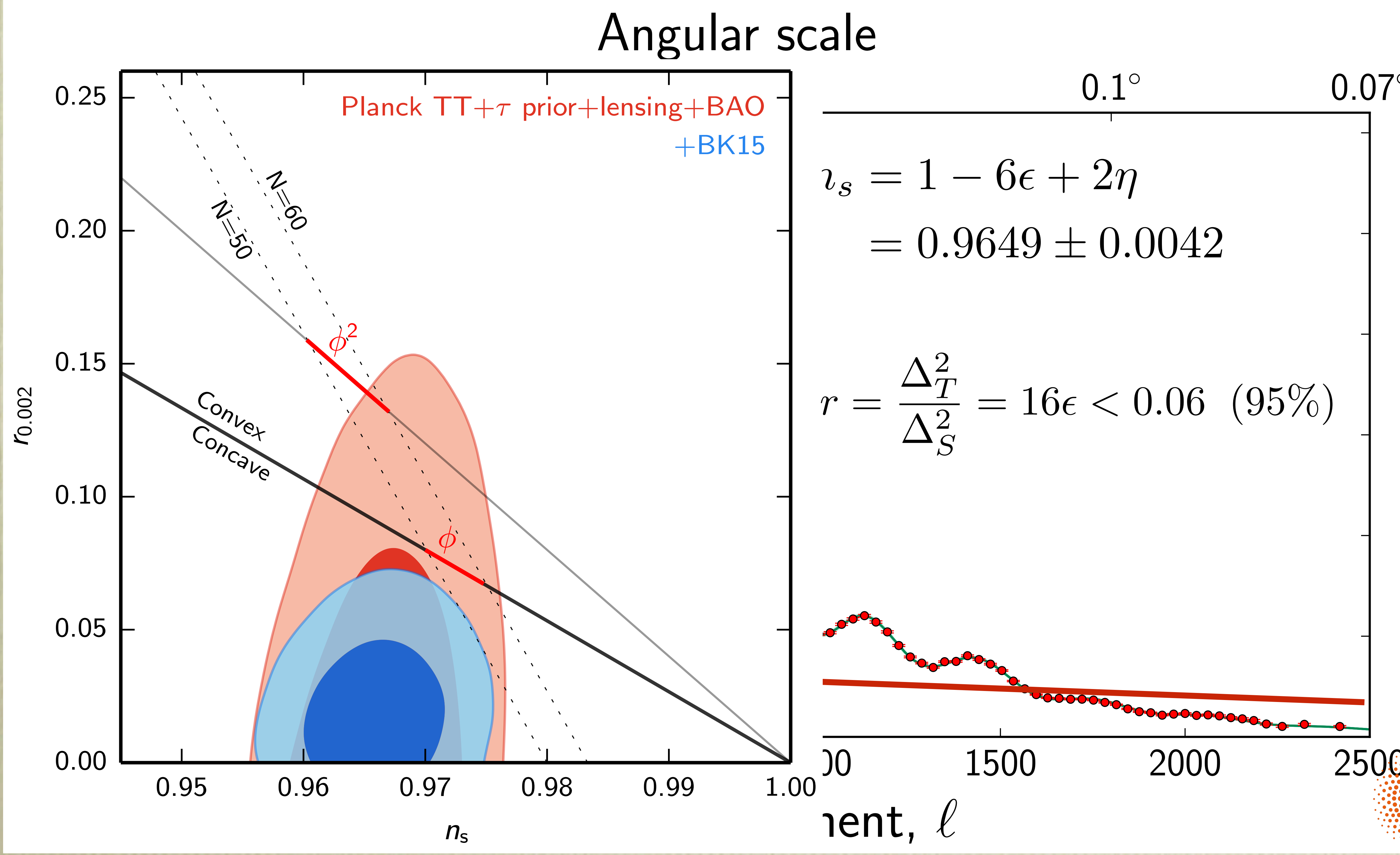
slow-roll inflation ...

[Guth, Linde, Starobinsky, Albrecht, Steinhardt ‘80s]



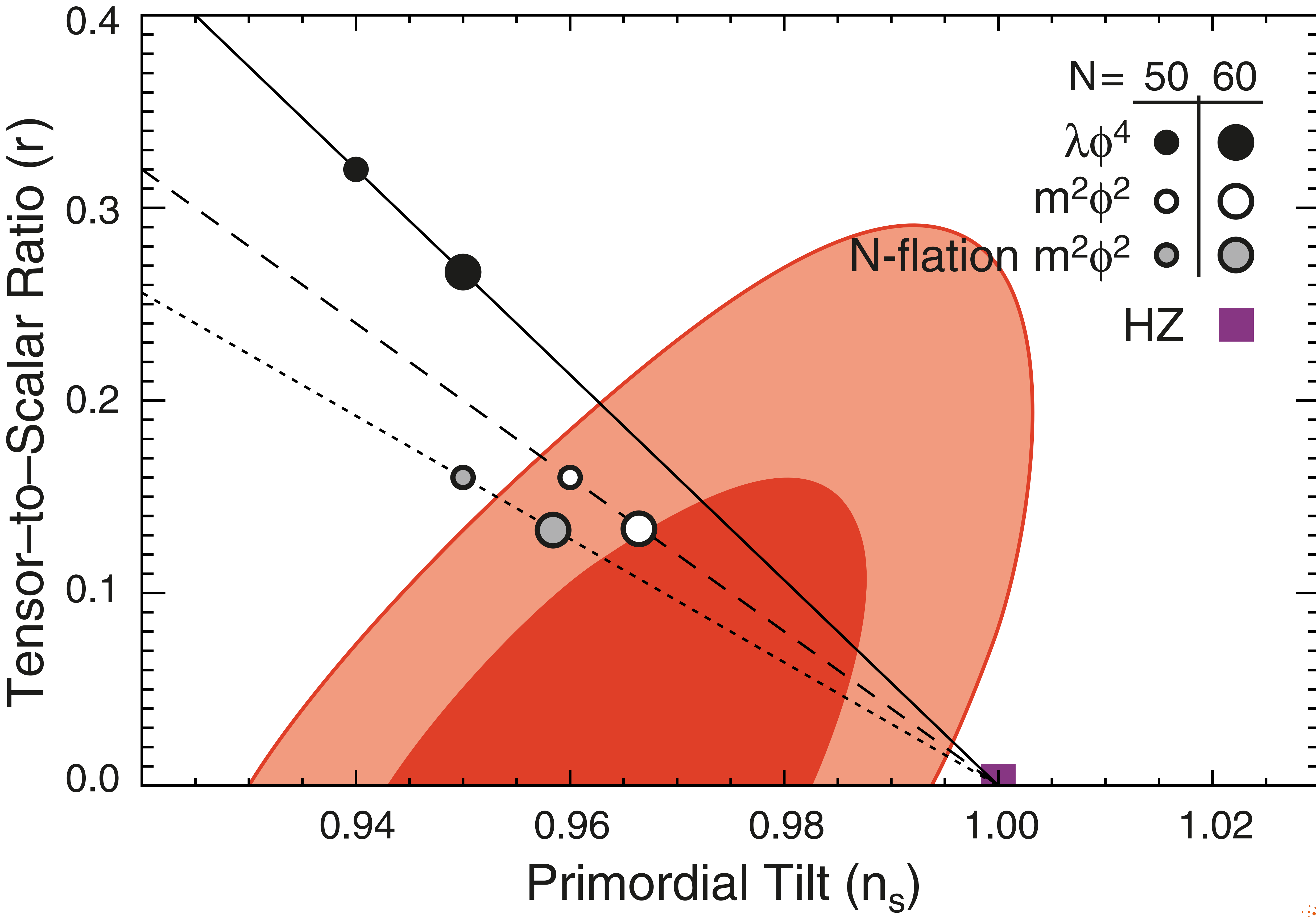
slow-roll inflation ...

[Guth, Linde, Starobinsky, Albrecht, Steinhardt '80s]

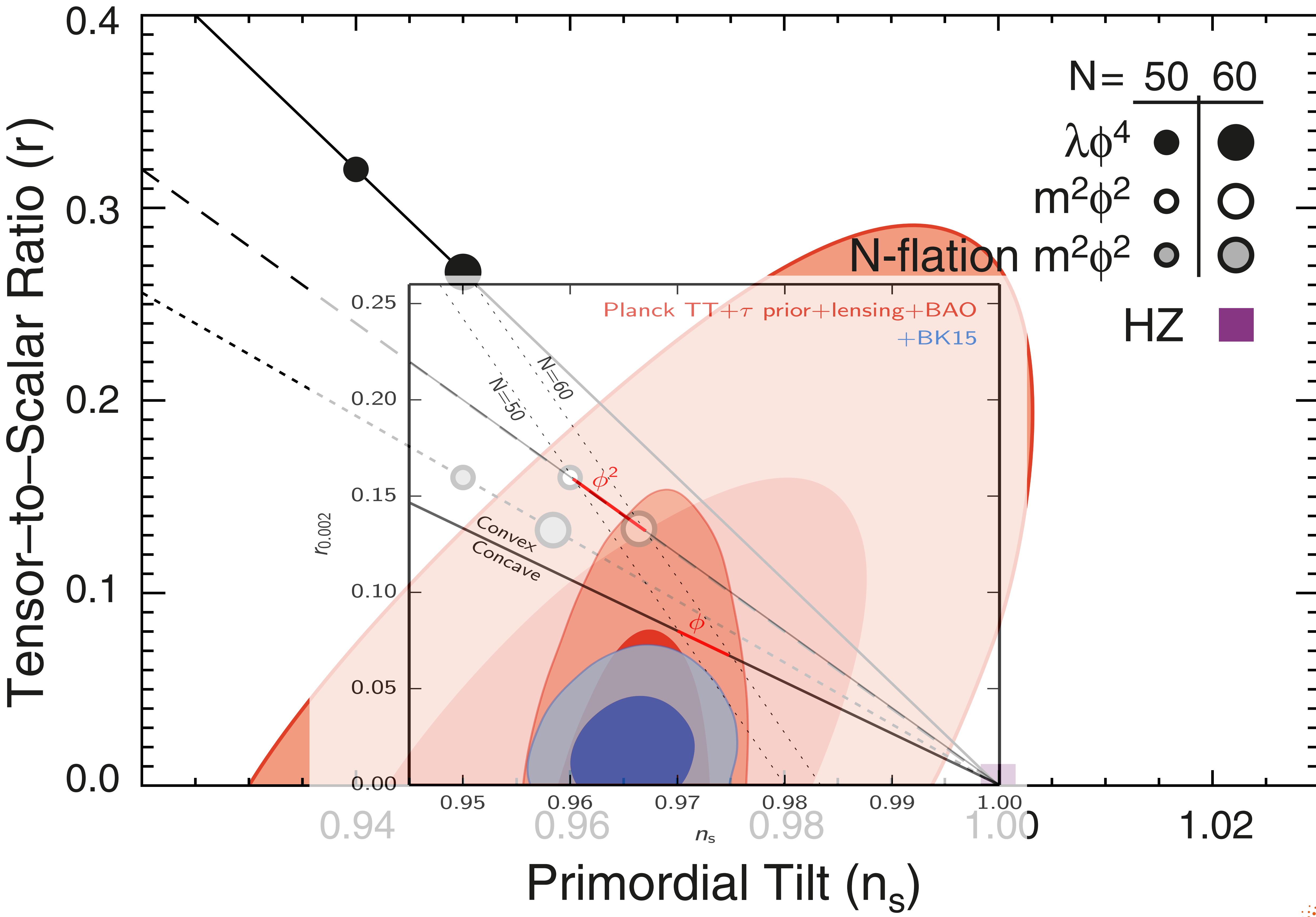


$$n_s = 1 - 6\epsilon + 2\eta$$
$$= 0.9649 \pm 0.0042$$

$$r = \frac{\Delta_T^2}{\Delta_S^2} = 16\epsilon < 0.06 \ \ (95\%)$$



Tensor–to–Scalar Ratio (r)
0.4
0.3
0.2
0.1
0.0
0.94
0.96
0.98
1.00
1.02
Primordial Tilt (n_s)
N= 50 60
$\lambda\phi^4$
$m^2\phi^2$
N-flation $m^2\phi^2$
HZ



Tensor–to–Scalar Ratio (r)
Primordial Tilt (n_s)
0.4
0.3
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N= 50 60
$\lambda\phi^4$
$m^2\phi^2$
N-flation $m^2\phi^2$
HZ
Planck TT+τ prior+lensing+BAO
+BK15
$r_{0.002}$
n_s
0.25
0.20
0.15
0.10
0.05
0.00
0.95
0.96
0.97
0.98
0.99
1.00
N=60
N=50
ϕ^2
ϕ
Convex
Concave
erc

UV completion – large fields in string theory

strings : 1-dim closed or open vibrating objects with length $\sqrt{\alpha'}$ & tension $\frac{1}{\alpha'}$

action : fluctuations of 2-dim worldvolume

$$S = \frac{1}{\alpha'}\int d^2\sigma \sqrt{-g}$$

space-time d.o.f. : always $g_{\mu\nu}, B_{\mu\nu}, \phi$

⊕ SUSY $\Rightarrow$ $D = 10$

dualities : e.g. T-duality

theory on $\underset{R}{\bigcirc}\ S^1$ $\hat{=}$ theory on $\underset{1/R}{\bigcirc}\ \tilde{S}^1$

$\Rightarrow$ Dp-branes from end-of-open strings Dirichlet boundary conditions

$\Leftrightarrow$ fluctuating 'hyperplanes' with $(p+1)$-dim. worldvolume

$\downarrow$

10D effective spacetime action dictated by 10D $\mathcal{N} = 1,2$ SUSY :

$$S = \frac{1}{2\alpha'^4}\int d^{10}x \sqrt{-g}\, e^{-2\phi}\left(R + 4(\partial_M\phi)^2 + |H_3|^2\right)$$

$$+ S_{matter}$$

$$S_{\text{matter}} = \frac{1}{\alpha'^4} \int d^{10}x \sqrt{-g} \cdot \Big[\sum_p |\tilde{F}_p|^2 + CS$$

localized objects like: Dp-branes & Op-planes (orientifold planes)

$$\left\{ + \sum_p \left(T_p^B \frac{\delta^{9-p}(x_\perp)}{\sqrt{-g_\perp}} - T_p^O \frac{\delta^{9-p}(x_\perp)}{\sqrt{-g_\perp}} \right) \right.$$

$$+ \text{ higher-derivative} \Big]$$

$$\tilde{F}_p = F_p + B_2 \wedge F_{p-2}$$ CS couplings

$$F_p = dC_{p-1}, \quad H_3 = dB_2$$

10D → 4D: compactification

$$\mathcal{M}_{10} \to \mathcal{M}_4 \times X_6$$

$\text{vol}(X_6) \sim L^6$ small, but bigger than unity in units of $\sqrt{\alpha'}$ (string length) for control.

$\sim$ many choices for X_6

$\sim$ X_6 has deformation modes: massless 4D scalars – "moduli"

$\rightsquigarrow$ correspond to volumes of p-dim. sub-manifolds of X_6 : p-cycles Σ_p

$\rightsquigarrow$ e.g. CY_3 : 2- and 3-cycle moduli

CY_3 : $\dim_{\mathbb{C}} = 3$ Kähler space

with $c_1(CY_3) \sim \underbrace{R_{i\bar{j}}}_{=0} dx^i d\bar{x}^{\bar{j}}$

Calabi-Yau's are 'Ricci-flat'

CY_3 have metric deformations δg_{mn} which cannot be removed by local coord. trafos :

$\delta g^a_{i\bar{j}}$: Kähler moduli $a = 1, \dots, h^{1,1}$ } volumes of $h^{1,1}$ 2-cycles Σ_a

δg^α_{ij} : complex structure moduli $\alpha = 1, \dots, h^{2,1}$ | volumes of $h^{2,1}$ 3-cycles Σ_α

expand $g_{mn} \to g_{mn} + t_a \delta g^a_{m\bar{n}} + u_\alpha \delta g^\alpha_{mn}$

and plug into :

$$S = \int d^{10}x \sqrt{-g}\, R$$

$$R \to R + \underbrace{G_{a\bar{b}}\, \partial t^a \partial \bar{t}^{\bar{b}} + G_{\alpha\bar{\beta}}\, \partial u^\alpha \partial \bar{u}^{\bar{\beta}}}$$

t^a, u^α massless 4D scalar fields: 'moduli'

string theory compactified on a CY_3 gives as EFT in 4D an $\mathcal{N}=1$ supergravity:

t^a, u^α part of chiral superfields