Simple playground

$$L = F(Y) \cdot \mathrm{e}^{4\pi} + K(Y) \cdot \Box\pi \cdot \mathrm{e}^{2\pi}$$

$$\Box\pi \equiv \partial_\mu \partial^\mu \pi \,, \qquad Y = \mathrm{e}^{-2\pi} \cdot (\partial_\mu \pi)^2$$

- Second order equations of motion
- Scale invariance: $\pi(x) \to \pi'(x) = \pi(\lambda x) + \ln\lambda$.

  (technically convenient)

Homogeneous solution in Minkowski space (attractor)

$$e^{\pi_c} = \frac{1}{\sqrt{Y_*}\,|t|}\,, \qquad t < 0$$

- $Y \equiv e^{-2\pi_c} \cdot (\partial_\mu \pi_c)^2 = Y_* =$ const, a solution to

$$Z(Y_*) \equiv -F + 2Y_* F_Y - 2Y_* K + 2Y_*^2 K_Y = 0$$

$$F_Y = dF/dY\ .$$

Energy density

$$\rho = e^{4\pi_c} Z = 0$$

Effective pressure T_{11}:

$$p = e^{4\pi_c} \left(F - 2Y_* K\right)$$

Can be made negative by suitable choice of $F(Y)$ and $K(Y)$
$\Longrightarrow$ $\rho + p < 0$, violation of the Null Energy Condition.

Turning on gravity

$$p = \mathrm{e}^{4\pi_c}\,(F - 2Y_* K) = -\frac{M^4}{Y_*^2\,|t|^4}\,, \qquad \rho = 0$$

M: mass scale characteristic of π

- Use $\dot{H} = -4\pi G(p+\rho) \Longrightarrow$

$$H = \frac{4\pi}{3}\frac{M^4}{M_{Pl}^2 Y_*^2\,|t|^3}$$

NB:

$$\rho \sim M_{Pl}^2 H^2 \propto \frac{1}{M_{Pl}^2\,|t|^6}$$

Genesis.

NB: Early times $\Longrightarrow$ weak gravity, $\rho \ll p$. Expansion, $H \neq 0$, is negligible for dynamics of π.

Perturbations about homogeneous Minkowski solution

$$\pi(x^\mu) = \pi_c(t) + \delta\pi(x^\mu)$$

- Quadratic Lagrangian for perturbations:

$$L^{(2)} = \mathrm{e}^{2\pi_c} Z_Y (\partial_t \delta\pi)^2 - B(\vec{\nabla}\delta\pi)^2 + W(\delta\pi)^2$$

$B = B[Y; F, K, F_Y, K_Y, K_{YY}]$. Absence of ghosts:

$$Z_Y \equiv dZ/dY > 0 \quad \text{at } Y = Y_*$$

Absence of gradient instabilities and of superluminal propagation

$$B > 0\,; \qquad B < \mathrm{e}^{2\pi_c} Z_Y$$

Can be arranged.

- Bounce:

  (1) early contraction dominated by another matter; Galileon takes over and reverses sign of H

  (2) Judicial choice of Lagrangian functions F and K.

- Both regimes can be made healthy: neither ghosts nor gradient instabilities

So far, so good

What about more complete cosmologies

with conventional expansion in the end (inflationary or not)?

Early examples: either Big Rip singularity in future,
$\pi = \infty$, $H = \infty$ at $t < \infty$

Creminelli, Nicolis, Trincherini ’2010

or gradient instability

Cai, Easson, Brandenberger ’2012;
Koehn, Lehners, Ovrut ’2014;
Pirtskhalava, Santoni, Trincherini, Uttayarat ’2014;
Qiu, Wang ’2015;
Kobayashi, Yamaguchi, Yokoyama ’2015;
Sosnovikov ’2015

Is instability generic
or just a drawback of models constructed so far?

Can one construct healthy bounce and/or Genesis
within the original theory?

No-go for Horndeski

To make long story short

Consider cubic theory

$$L = \frac{1}{2\kappa}R + F(\pi, X) - K(\pi, X)\Box\pi$$

Assume that there exists bounce or Genesis solution (spatially flat).

Calculate quadratic Lagrangian for salar perturbations (metric included)

$$L^{(2)} = A\dot{\chi}^2 - \frac{1}{a^2}B(\partial_i\chi)^2 + \dots$$

No ghosts, gradient instabilities:

$$A > 0\,, \quad B > 0$$

$$\frac{B\dot{\pi}^2}{a} = \dot{\mathscr{R}} - \kappa a \mathscr{R}^2 \,, \qquad \mathscr{R} = a^{-1}\left(K_X \dot{\pi}^3 - \frac{1}{\kappa} H\right)$$

$B > 0 \implies \dot{\mathscr{R}} - \kappa a \mathscr{R}^2 > 0$. Integrate $\dot{\mathscr{R}}/\mathscr{R}^2 - \kappa a > 0$:

$$\frac{1}{\mathscr{R}(t_i)} - \frac{1}{\mathscr{R}(t_f)} > \kappa \int_{t_i}^{t_f} dt \; a(t) \,.$$

Bouncing scenario, Genesis: $\int_{-\infty}^{t_f} dt \; a(t) = \infty \,, \quad \int_{t_i}^{\infty} dt \; a(t) = \infty \,.$

- Suppose $\mathscr{R}(t_i) > 0$. Then at $t > t_i$ one has $\mathscr{R}(t) > 0$ (since $\dot{\mathscr{R}} > 0$).

$$\frac{1}{\mathscr{R}(t_f)} < \frac{1}{\mathscr{R}(t_i)} - \kappa \int_{t_i}^{t_f} dt \; a(t) \,.$$

  Right hand side changes sign at some $t_f \implies \mathscr{R}(t_f) = \infty$, singularity in future.

- Case $\mathscr{R}(t) < 0$: singularity in past. QED

- Similar argument forbids wormholes (in that case problem is with $A \Longleftrightarrow$ ghosts)

- Argument intact in presence of extra matter (obeying NEC) which interacts with Galileon only gravitationally:

$$\frac{B\dot{\pi}^2}{a} = \dot{\mathscr{R}} - \kappa a \mathscr{R}^2 - \frac{\rho_M + p_M}{2a} \ ,$$

even worse.

- Extends to general Horndeski theories with all four allowed terms present in Lagrangian (below)

Kobayashi '2016

- Extends to model with extra conventional scalar ϕ and

$$L = -\frac{1}{2\kappa} R + F(\pi, X, \phi, X_{\pi\phi}, X_\phi) + K(\pi, X, \phi) \Box \pi$$

where $X_{\pi\phi} = \nabla_\mu \pi \cdot \nabla^\mu \phi, \quad X_\phi = (\nabla\phi)^2$.

Kolevatov, Mironov '2016

General Horndeski theory

$$
\begin{aligned}
L =& F(\pi, X) - K(\pi, X)\Box\pi \\
&+ G_4(\pi, X)R + G_{4,X}\left[(\Box\pi)^2 - (\nabla_\mu\nabla_\nu\pi)^2\right] \\
&+ G_5 \cdot G^{\mu\nu}\nabla_\mu\nabla_\nu\pi - \frac{1}{6}G_{5,X}\left[(\Box\pi)^3 - 3\Box\pi\cdot(\nabla_\mu\nabla_\nu\pi)^2 + 2(\nabla_\mu\nabla_\nu\pi)^3\right]
\end{aligned}
$$

- Modified gravity (scalar-tensor). Second order field eqs (!)
- Again instability of Genesis and bounce.

Kobayashi '2016; Ijjas, Steinhardt '2016

Choose unitary gauge $\delta\pi = 0$.

$$
ds^2 = N^2 dt^2 - a^2 e^{2\zeta}(\delta_{ij} + h_{ij} + \frac{1}{2}h_{ik}k_{kj})(N^i dt + dx^i)(N^j dt + dx^j)
$$

Dynamical variables in scalar sector: transverse traceless h_{ij} and ζ.

$$L_\zeta = A_\zeta \dot{\zeta}^2 - a^{-2} B_\zeta (\partial_i \zeta)^2\,, \qquad L_h = A_h \dot{h_{ij}}^2 - a^{-2} B_h (\partial_k h_{ij})^2$$

Key relation

$$\frac{d}{dt}\left(\frac{a(t) A_h^2(t)}{\Theta(t)}\right) = -a(t)(B_\zeta + B_h)$$

where $\Theta(t) = -2HG_4 + \dot{\pi} X K_X + \dots$, a complicated expression involving backrgound $\pi(t)$ and $H(t)$. Same story:

$$\frac{a(t_f) A_h^2(t_f)}{\Theta(t_f)} - \frac{a(t_i) A_h^2(t_i)}{\Theta(t_i)} = -\int_{t_i}^{t_f} dt\ a(t)(B_\zeta + B_h)$$

Impossible for $B_\zeta > 0$, $B_h > 0$, finite A_h, Θ and

$$\int_{-\infty}^{t_f} dt\ a(t)(B_\zeta + B_h) = \infty\,, \quad \int_{t_i}^{+\infty} dt\ a(t)(B_\zeta + B_h) = \infty\,.$$

$$\frac{\Theta(t)}{a(t) A_h^2(t)} = \infty \quad \text{at some time} \quad t$$

Beyond Horndeski theories

Zumalacárregui, Gacia-Bellido' 2014

Gleyzes, Langlois, Piazza, Vernizzi' 2014

- Give up requirement of second order field equations
- Require that there remains one scalar degree of freedom + tensor

  Allowed terms

$$G_4(\pi, X)R + F_4(\pi, X)\left[(\Box\pi)^2 - (\nabla_\mu\nabla_\nu\pi)^2\right]$$

  F_4 and G_4 no longer related.

- Way to understand: disformal transformation

$$g_{\mu\nu} \to \Omega(\pi, X)g_{\mu\nu} + \Lambda(\pi, X)\partial_\mu\pi\partial_\nu\pi$$

  Horndeski $\to$ beyond Horndeski

  NB: This is formal trick. Ω, Λ may be singular

Now

$$a(t)(B_\zeta + B_h) = -\frac{d}{dt}\left[\frac{aA_h(A_h - \Delta)}{\Theta}\right]$$

$(A_h - \Delta)$ can cross zero without singularity.

No-go theorem no longer holds

Effective field theory: Cai et.al.' 2016, Creminelli et.al.'2016

Covariant formalism: Kolevatov et.al.' 2017, Cai, Piao' 2017

NB: $\Theta = 0$ not a problem, gauge artifact

Ijjas'2017;

Mironov, V.R., Volkova' 2018

Bounce: proof of principle

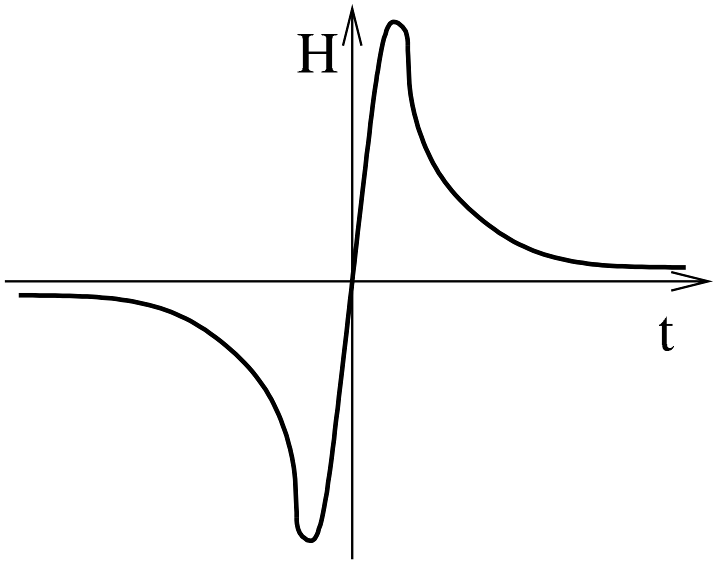
"Inverse method"

Term by Ijjas, Steinhardt '2016

- Choose background $\pi(t) = t$, no loss of generality

  Then $X = (\partial\pi)^2 = 1$. Field equations and stability conditions involve $f_0(t) = F(\pi(t))$, $f_1(t) = F_X(\pi(t))$, etc., all at $X = 1$.

- Choose your favorite $H(t)$ such that $H(t) \to \frac{1}{3t}$ as $|t| \to \infty$
GR + Galileon = conventional massless scalar.

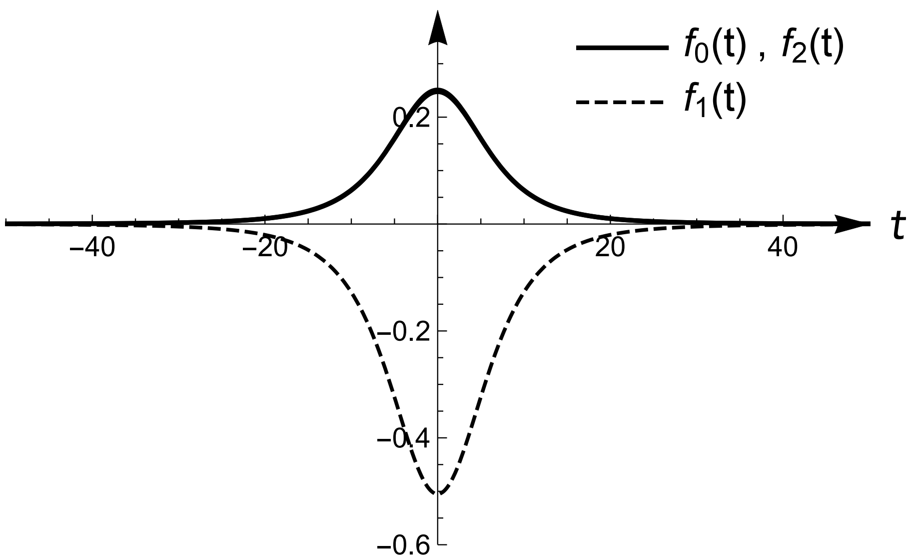


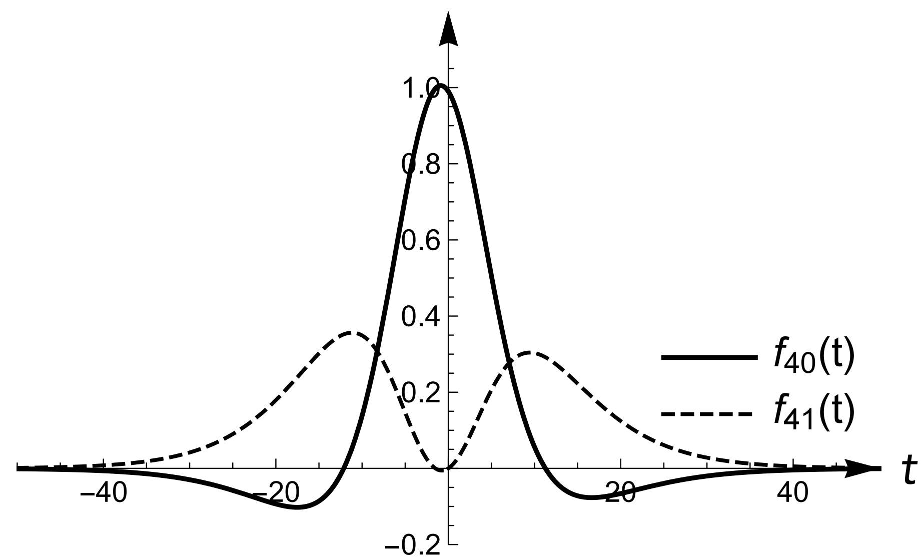
- Asymptotics of Lagrangian functions as $|t| \to \infty$:

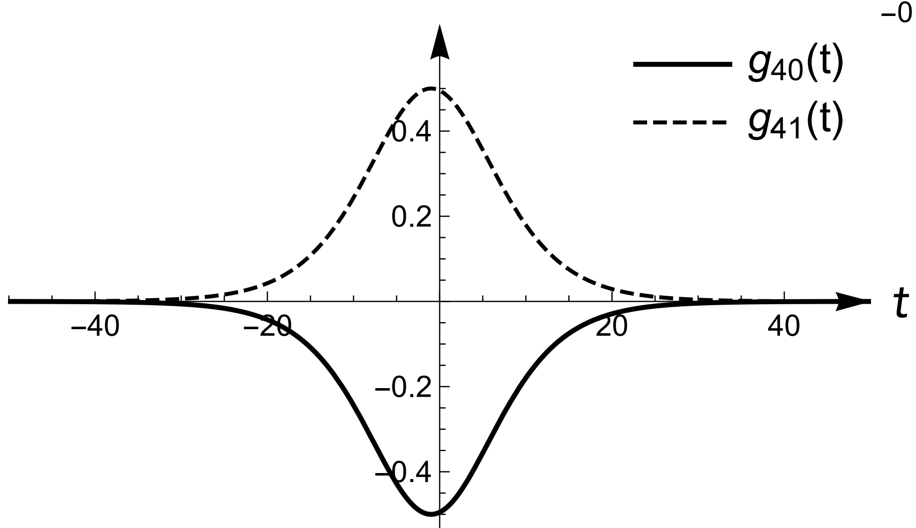
$$F(t) = \frac{1}{t^2}, \quad F_X(t) = \frac{1}{t^2} \Longrightarrow F = \frac{(\partial\pi)^2}{\pi^2} = (\partial \log \pi)^2$$

$$G_4 = \frac{M_{Pl}^2}{16\pi}, \qquad K = F_4 = 0$$

- Cook up Lagrangian functions in such a way that
  - Field equation are satisfied
  - Stability conditions are satisfied at all times

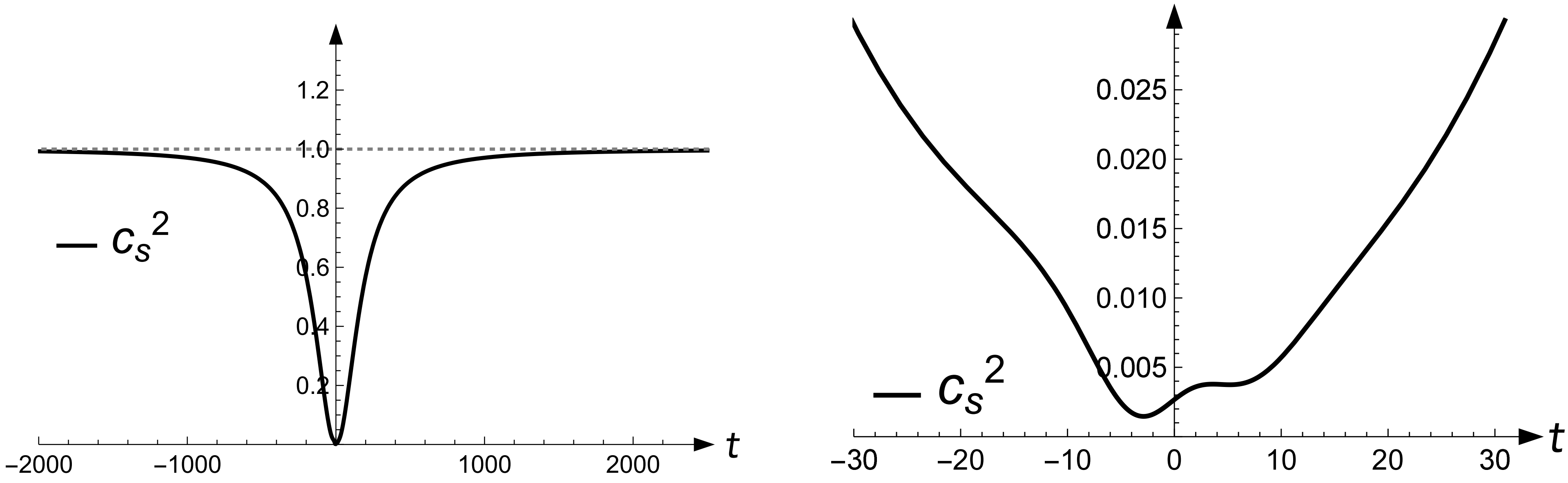






No kidding: speed of gravity waves is always 1.

Speed of scalar perturbation $0 < c_s^2 \leq 1$

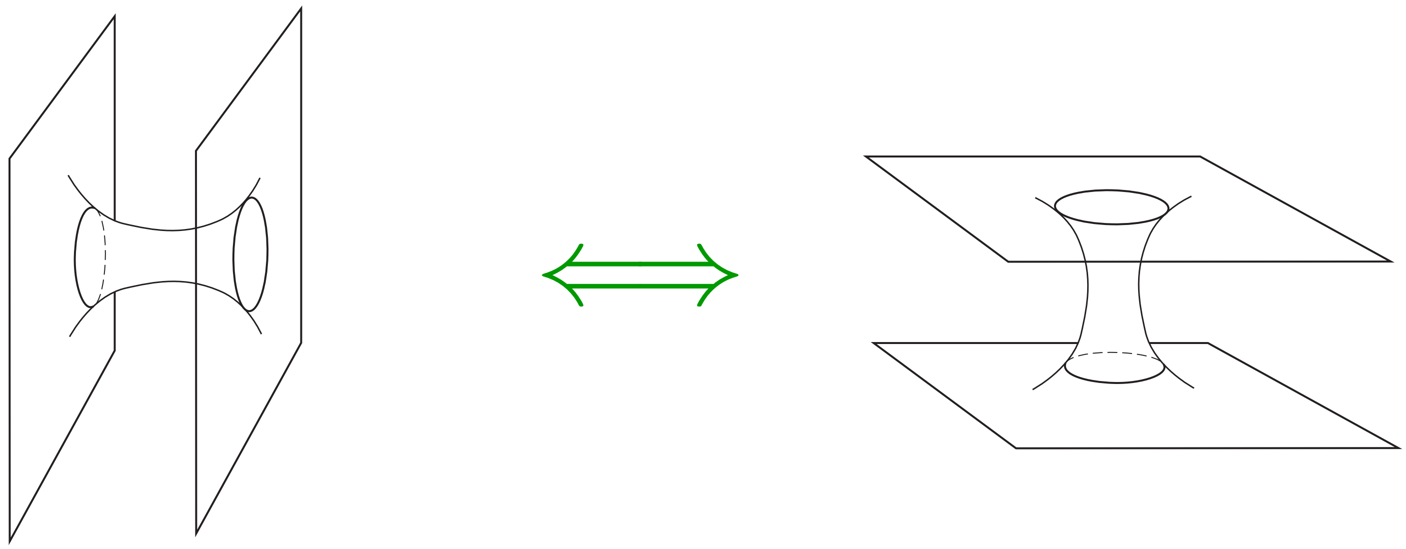


Completely stable bounce

Similar construction for Genesis.

What about wormholes?

Static wormhole $\Longleftrightarrow$ Bouncing Universe



No-go for Horndeski: no stable, static, spherically symmetric wormholes: always ghosts.

V.R. ’2016

Evseev, Melichev’ 2018

Theorem does not hold beyond Horndeski

Mironov, V.R., Volkova ’2018

Franciolini, Hui, Penco, Santoni, Trincherini’ 2018

Work in progress

Instead of conclusion

- Constructing bouncing or Genesis cosmology is a non-trivial task. Even harder than originally thought.
- Exotic fields are needed. It is “beyond Horndeski” that does the job.
    - UV completion not known (and may not exist)
- Fully consistent bouncing and Genesis cosmologies possible at classical field theory level
- Wormholes, creation of a universe in lab: open issues.
    - NB: wormhole $\Longleftrightarrow$ time machine

Morris, Thorne, Yurtsever’ 1988

- Ahead: more to understand

Reading

- V. A. Rubakov, “The Null Energy Condition and its violation,” Phys. Usp. **57**, 128 (2014) [Usp. Fiz. Nauk **184**, 137 (2014)] [arXiv:1401.4024 [hep-th]]

- D. Langlois, “Dark energy and modified gravity in degenerate higher-order scalar-tensor (DHOST) theories: A review,” Int. J. Mod. Phys. D **28**, 1942006 (2019) [arXiv:1811.06271 [gr-qc]]

- T. Kobayashi, “Horndeski theory and beyond: a review,” Rept. Prog. Phys. **82**, 086901 (2019) [arXiv:1901.07183 [gr-qc]]