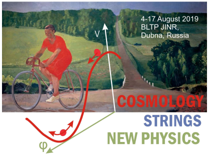
Road to Emergent Spacetime

All the Way From Quantum Mechanics to Quantum Gravity



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BLTP JINR Summer School Presentation
August 13, 2019

What does "emergent" mean?

Ordinary approach to quantum gravity:

$$\text{Classical gravity} \xrightarrow{\text{quantization}} \text{Quantum gravity}$$

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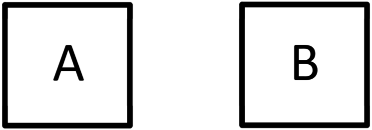
Emergent approach to quantum gravity:

$$\text{Quantum theory in Hilbert space} \xrightarrow{\text{geometrization}} \text{Classical gravity}$$

using the main idea that "entanglement = geometry"

History: entanglement

What is entanglement?



Classical thermodynamics: $S_A + S_B \geq S_{AB} \geq min(S_A, S_B)$

QM is nonlocal, so we have $S_A + S_B \geq S_{AB} \geq 0$
Meaning that subsystems of a completely determined system may have entropy
In QFT, subsystem decomposition is enforced by event horizons

History: black hole entropy

Black Hole entropy was discovered by Hawking and Bekenstein

$$S = \frac{A}{4G}$$

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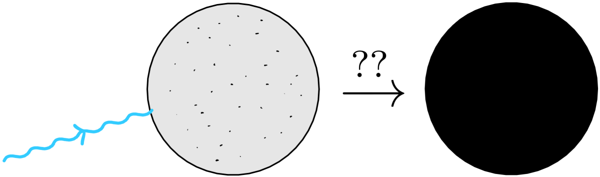
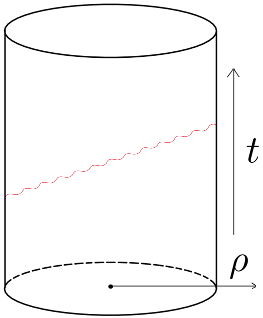
Not extensive! $\Rightarrow$ Holographic principle:

$$S \geq \frac{A}{4G} \xrightarrow{??} S = \frac{A}{4G}$$



History: AdS/CFT
arXiv: 9711200 and 9802150

$\Rightarrow$ Anti de Sitter/CFT
(AdS/CFT)
Correspondence



Important result:

$$\langle \mathcal{O}(t_1, l)\mathcal{O}(t_2, 0)\rangle \sim e^{-\frac{\Delta L}{L_{AdS}}},$$

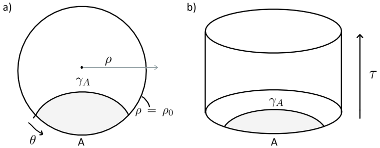
"*geodesic approximation*"

History: Ryu-Takayanagi

arXiv: 0603001

More general relation between (entanglement) entropy and areas by Ryu and Takayanagi

$$S_A = \frac{\text{Area}[\gamma_A]}{4G}$$

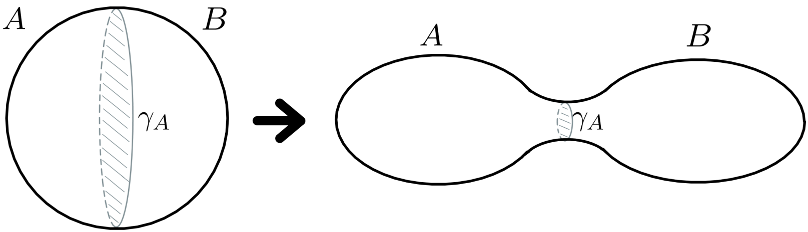


Conjecture proven by Lewkowycz and Maldacena in 2013 and again with different formalism by Lewkowycz and Parrikar in 2018.

History: Disentangling experiment

arXiv: 1609.00026

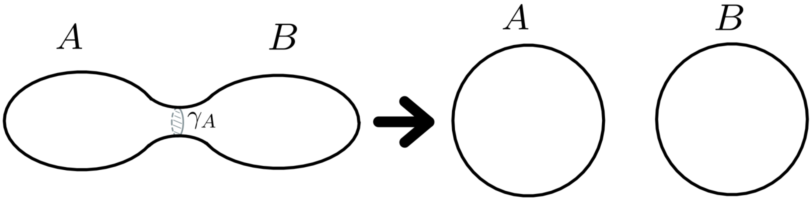
Disentangling experiment:



- Pinching implied by Ryu-Takayanagi formula
- Separation implied by geodesic approximation plus information theoretical result: $2S_A \geq \langle \mathcal{O}_A(t_1, l)\mathcal{O}_B(t_2, 0)\rangle \sim e^{-\frac{\Delta L}{L_{AdS}}}$

History: entanglement builds geometry
arXiv: 1609.00026

Disentangling experiment:



$\Rightarrow$ With no entanglement, spacetime disconnects!

Conjecture: spacetime connectivity is built by entanglement

Recovering spacetime dynamics
Base idea

Two things are fundamental:

- The Ryu-Takayanagi relation $S = A_\gamma/4G$ is true
- "Fundamental" theory is a quantum theory, understood as a theory of operator algebras in some Hilbert space, with no a priori geometrical meaning.

Dynamics of entanglement in fundamental theory give dynamics of dual (emergent) geometry.

Roadmap

Three main topics of modern developments:

- The translation of the dynamics of entanglement in CFT to the AdS geometry via the Ryu-Takayanagi relation (conservative)

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Three main topics of modern developments:

- The translation of the dynamics of entanglement in CFT to the AdS geometry via the Ryu-Takayanagi relation (conservative)
- Ingredients beyond entanglement (moderate)
- Non-field theory boundary (extreme)

Geometry from entanglement
arXiv: 1308.3716

Strategy: characterize *dynamics* of entanglement in CFT

1st law of entanglement dynamics

$$\delta S_A = \delta \langle H_A \rangle$$

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Find constraints on geometry M such that:

- Variation of Ryu-Takayanagi areas $\delta A_\gamma / 4G$ correctly compute δS_A.
- "asymptotic energy" of spacetime E_{hyp} satisfies $\delta E_{hyp} = \delta \langle H_A \rangle$

Result: possible for all boundary balls in all Lorentz frames *if and only if* M satisfies linearized Einstein field equations

Geometry from entanglement

Second order (basics) arXiv: 1705.03026

CFT side: dynamics given by "Fisher information" $F(\delta\rho, \delta\rho)$. To n:th order, CFT dynamics described by n:th order perturbation of *relative entropy*. Coupling to stress energy given by relation to "modular Hamiltonian".

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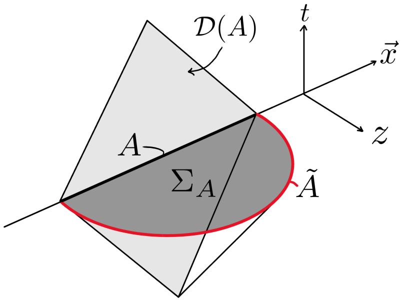
Gravity side: second order dynamics given by variation of "Wald functional" δS_{Wald}.



Geometry from entanglement

Second order (results) arXiv: 1705.03026

Results at second order:

- Equality

$$\delta S_{Wald} = F(\delta\rho, \delta\rho)$$

  for all choices of A, in all Lorentz frames, *if and only if* geometry M satisfies EFE everywhere.

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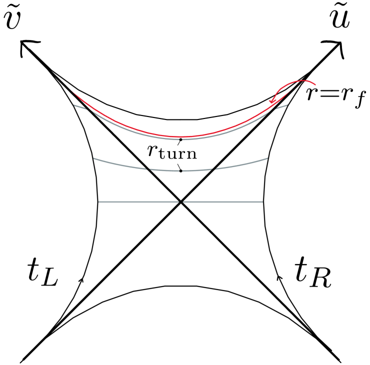
  for all choices of A, in all Lorentz frames, *if and only if* geometry M satisfies EFE everywhere.
- Reeh-Schlieder theorem of QFT ⇒ universal coupling of fields to geometry in dual geometry
- Positivity of relative entropy ⇒ dual geometry satisfies null energy condition

"Physicality axioms" in gravity are fundamental theorems of the CFT!

Entanglement is not enough
arXiv: 1411.0690

Three problems:

- Ryu-Takayanagi surfaces can not cross black hole horizons.
- Entangled particle pairs have constant entanglement independent of separation
- Wormholes grow while having constant area.



Dual of maximally extended Schwarzschild
arXiv: 0106112

CFT dual of simple wormhole: "thermofield double"

$$|\Psi^{\beta}_{TFD}\rangle = \frac{1}{Z_\beta}\sum_i e^{\frac{-\beta E_i}{2}}\,|E_i\rangle_L \otimes |E_i\rangle_R$$

Hamiltonian time evolution $\rightarrow$ extra (relative) phases.
Longer wormhole $\rightarrow$ more complex state?

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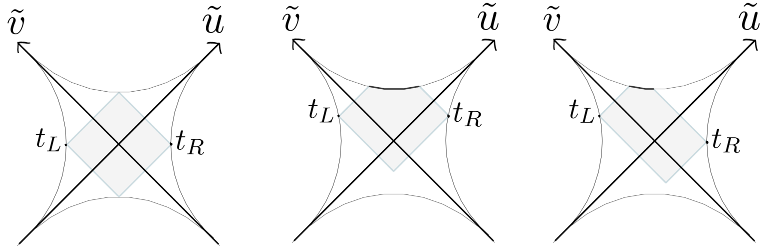
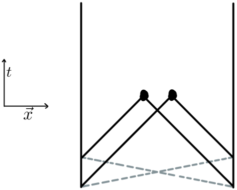
More covariantly, we may consider *WdW action*:
$S_{WdW} = \int_{WdW} \mathcal{L}_{EH}$

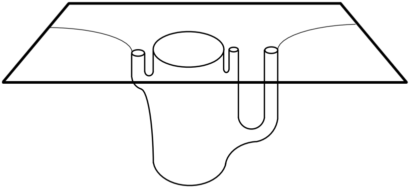


Entangled black holes are connected by wormholes
arXiv: 1306.0533

Wormholes related to nonlocal entanglement on boundary ∼ *local entanglement in bulk*.



Entangled black holes produced by evaporation



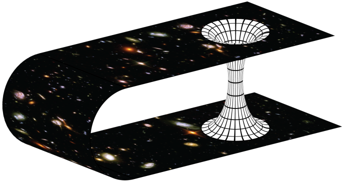
In limit of small emissions, particles

ER = EPR

Conclusion:

$$\text{ER} = \text{EPR}$$

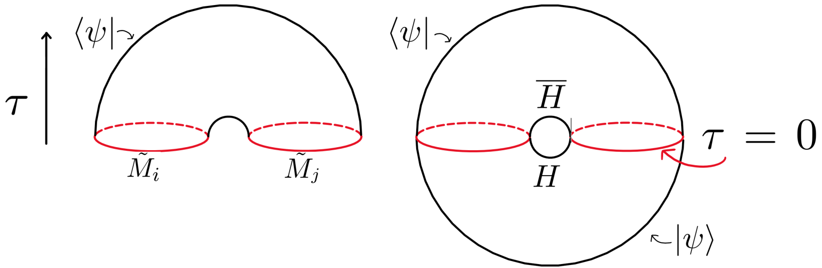
or more understandably, wormholes equal entanglement:



$$= \frac{1}{\sqrt{2}} \left(|1\rangle \otimes |0\rangle + |0\rangle \otimes |1\rangle \right)$$

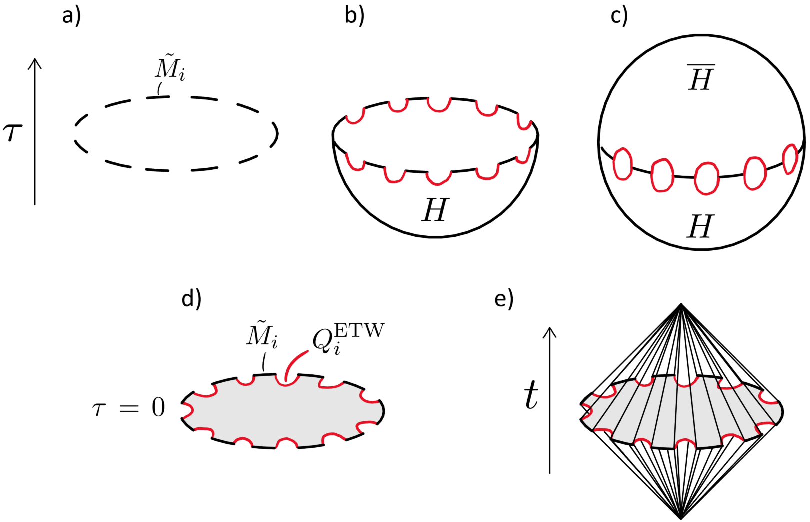
Beyond Holography
Intro

We are able to split the boundary into disjoint patches and prepare them in entangled states via path integral in Euclidean time



Beyond Holography

arXiv: 1809.01197



What did I do?

- Write a review with a lot of technical details.
- I do some original work, trying to extend the "conservative" approach further. I find explicit expressions for the third order variation of the relative entropy of unbounded operators, and conditions on the metric implementing a second order "Hollands-Wald" gauge that is necessary in the gravitational analysis.

Conclusion

- There is considerable evidence that gravity may emerge from entanglement information (plus complexity).
- AdS/CFT holography is not strictly necessary - in the sense that the boundary need not be geometric. Thus tensor network models may be considered somewhat sane.
- Lots of work to do
  - Third order conservative approach
  - Bulk quantum corrections to conservative approach
  - Microstate holography (and complexity)
  - Tensor network type emergent gravities
  - Axiomatic treatment?