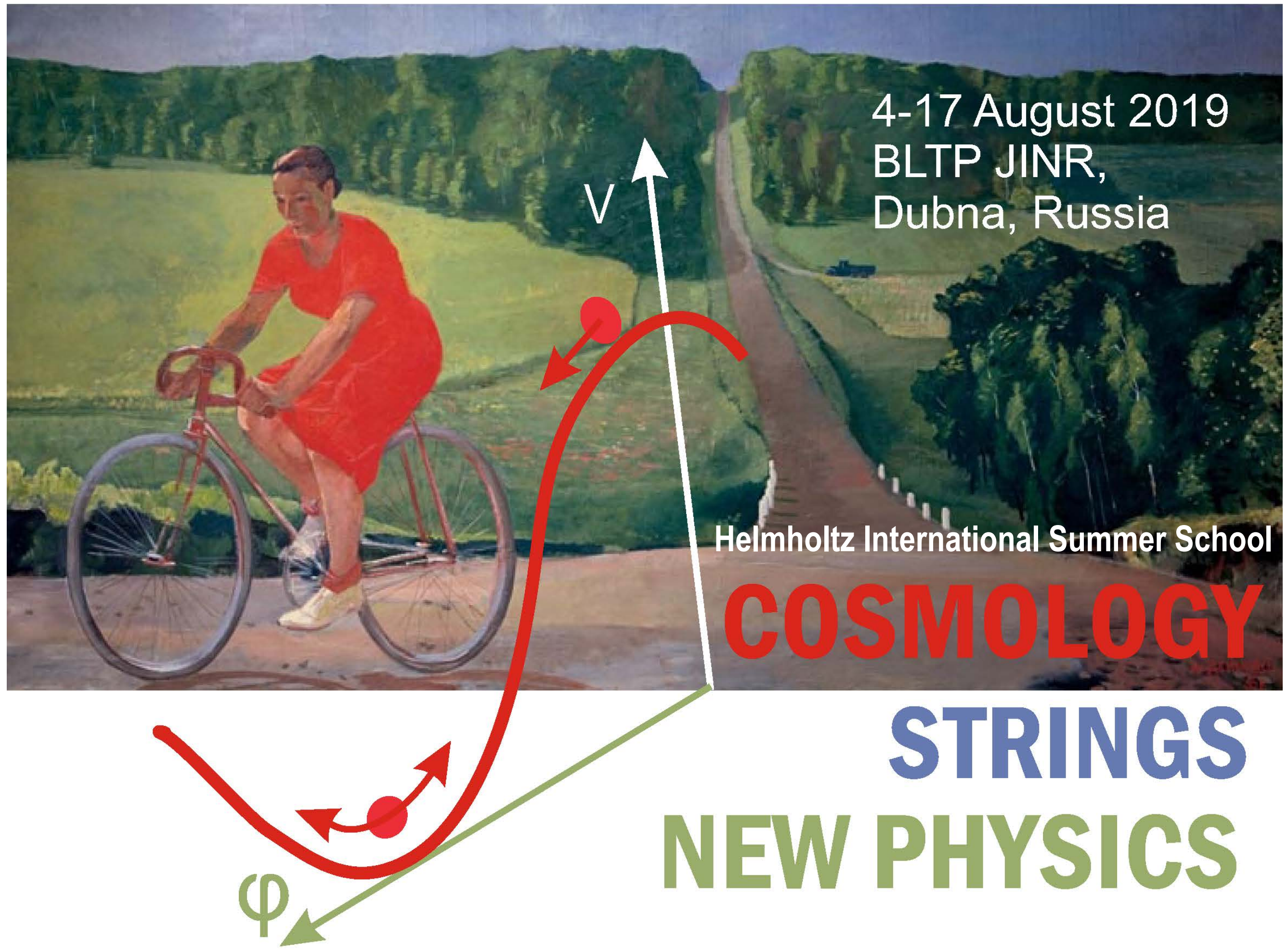
Dubna International Advanced School of Theoretical Physics



Dark Side of the Universe III

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CEICO

EUROPEAN UNION
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MINISTRY OF EDUCATION, YOUTH AND SPORTS

Decoupling vacuum energy from spacetime curvature

$$G_{\mu\nu} - T_{\mu\nu} - \frac{1}{4} g_{\mu\nu} (G - T) = 0$$

invariant under vacuum shifts of energy-momentum

$$T_{\mu\nu} \rightarrow T_{\mu\nu} + \Lambda g_{\mu\nu}$$

Bianchi identity + energy-momentum conservation

$$\nabla_\mu G^{\mu\nu} = 0 \quad + \quad \nabla_\mu T^{\mu\nu} = 0 \quad \Rightarrow \quad \partial_\mu (G - T) = 0$$

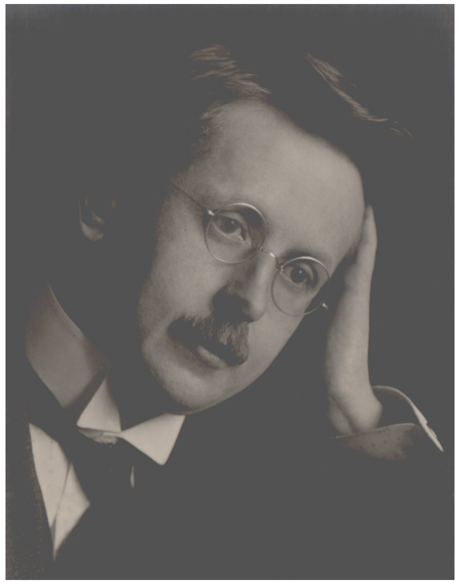
$$\Downarrow$$

Integration constant

$$G_{\mu\nu} - T_{\mu\nu} - \Lambda g_{\mu\nu} = 0$$

What is the action for the *traceless* Einstein field equations ?

$$G_{\mu\nu} - T_{\mu\nu} - \frac{1}{4} g_{\mu\nu} (G - T) = 0$$



No Trace

Weyl invariance?

$$g_{\mu\nu} \rightarrow \Omega^2(x)\, g_{\mu\nu}$$

$$g_{\mu\nu} = \Omega^2 \cdot h_{\mu\nu}$$

$$\frac{\delta S}{\delta \Omega^2} = \frac{g^{\mu\nu}}{\Omega^2} \cdot \frac{\delta S}{\delta g^{\mu\nu}} = 0$$

Mimetic* **vector**-tensor theory

Jiroušek, Vikman (2018)

Ansatz : $$g_{\mu\nu} = h_{\mu\nu} \cdot \left(\nabla^{h)}_{\alpha} V^{\alpha} \right)^{1/2}$$

$$\nabla^{h)}_{\alpha} h_{\mu\nu} = 0$$

Weyl invariance for the *vector field of* ***conformal weight 4***

$$h_{\mu\nu} = \Omega^{2}(x)\, h'_{\mu\nu}$$

$$V^{\mu} = \Omega^{-4}(x)\, V'^{\mu}$$

*Ideologically similar to **scalar**-tensor Mimetic Dark Matter of Chamseddine and Mukhanov (2013)

Action

$$S_g\left[h,V\right] = -\frac{1}{2}\int d^4x\sqrt{-h}\left[\left(\nabla^{h)}_{\alpha}V^{\alpha}\right)^{1/2}R\left(h\right)+\frac{3}{8}\cdot\frac{\left(\nabla^{h)}_{\mu}\nabla^{h)}_{\alpha}V^{\alpha}\right)^{2}}{\left(\nabla^{h)}_{\sigma}V^{\sigma}\right)^{3/2}}\right].$$

Equations of motion

$$\frac{1}{\sqrt{-h}}\cdot\frac{\delta S}{\delta V^{\mu}} = \frac{1}{4}\,\partial_{\mu}\left(T-G\right) = 0$$

$$\frac{1}{\sqrt{-h}}\cdot\frac{\delta S}{\delta h^{\alpha\beta}} = \frac{\sqrt{\nabla^{h)}_{\alpha}V^{\alpha}}}{2}\left[T_{\alpha\beta}-G_{\alpha\beta}-\frac{1}{4}g_{\alpha\beta}\left(T-G-\frac{1}{\nabla^{h)}_{\alpha}V^{\alpha}}V^{\lambda}\partial_{\lambda}\left(T-G\right)\right)\right] = 0$$

$$\Downarrow$$

$$G_{\mu\nu}-T_{\mu\nu}-\frac{1}{4}g_{\mu\nu}\left(G-T\right)=0$$

More Familiar Action

$$S\left[h,\varphi,V,\lambda\right]=\int d^4x\sqrt{-h}\left[-\frac{1}{2}\left(\partial\varphi\right)^2-\frac{1}{12}\varphi^2R\left(h\right)-\frac{\lambda}{72}\varphi^4+\frac{\lambda}{2}\cdot\nabla_\alpha^{h)}V^\alpha\right]$$

wrong sign

Lagrange multiplier

$$\nabla_\alpha^{h)}V^\alpha=\left(\frac{\varphi^2}{6}\right)^2$$

V^μ Stückelberg Freiherr von Breidenbach zu Breidenstein und Melsbach field

Weyl transformations

$$h_{\mu\nu}=\Omega^2\left(x\right)h'_{\mu\nu}$$

$$\varphi=\Omega^{-1}\left(x\right)\varphi'$$

$$V^\mu=\Omega^{-4}\left(x\right)V'^\mu$$

$$\lambda=\lambda'$$

Weyl-invariant variables

$$g_{\mu\nu}=\frac{\varphi^2}{6}h_{\mu\nu}$$

$$\varphi=\varphi$$

$$W^\mu=\left(\frac{\varphi^2}{6}\right)^{-2}V^\mu$$

$$\Lambda=\frac{\lambda}{2}$$

$$S\left[g, W, \Phi_m\right] = \int d^4x \sqrt{-g}\left[-\frac{1}{2}R\left(g\right) + \Lambda\left(\nabla^{g)}_{\mu} W^{\mu} - 1\right)\right] + S_m\left[g, \Phi_m\right]$$

Henneaux–Teitelboim unimodular gravity

Global degree of freedom

$$\mathcal{T}\left(t\right) = \int d^3\mathbf{x}\sqrt{-g}\, W^t\left(t, \mathbf{x}\right)$$

$$\mathcal{T}\left(t_2\right) - \mathcal{T}\left(t_1\right) = \int_{t_1}^{t_2} dt \int d^3\mathbf{x}\sqrt{-g}$$

$$\mathcal{T}\left(t\right) = \int d^3\mathbf{x}\sqrt{-h}\, V^t\left(t, \mathbf{x}\right)$$

Global degree of freedom

$$\mathcal{T}(t) = \int d^3\mathbf{x}\sqrt{-h}\, V^t(t, \mathbf{x})$$

For the action

$$S_g[h, V] = -\frac{1}{2}\int d^4x\sqrt{-h}\left[\left(\nabla^{(h)}_\alpha V^\alpha\right)^{1/2} R(h) + \frac{3}{8}\cdot\frac{\left(\nabla^{(h)}_\mu \nabla^{(h)}_\alpha V^\alpha\right)^2}{\left(\nabla^{(h)}_\sigma V^\sigma\right)^{3/2}}\right].$$

The conjugated canonical momentum of

$$\Lambda$$

Ostrogradsky Instability/Ghosts

- *Ostrogradsky's theorem on Hamiltonian instability*
  `Richard P. Woodard`
  `arXiv:1506.02210`

For systems with higher order non-degenerate equations of motion
the Hamiltonian is always linear in canonical momenta!

If there are N order derivatives
in the Lagrangian,
the Hamiltonian is linear in
N-1 canonical momenta!

Hence the Hamiltonian is necessarily unbounded from below!

Conformal weight four is unusual
for a vector field

Can one find a more usual construction?

Axionic Cosmological Constant

Jiroušek, Vikman(2019) in preparation

$$g_{\mu\nu} = h_{\mu\nu} \cdot \sqrt{F_{\alpha\beta}\widetilde{F}^{\alpha\beta}}$$

$$\widetilde{F}^{\alpha\beta} = \frac{1}{2} \cdot \frac{\epsilon^{\alpha\beta\mu\nu}}{\sqrt{-h}} \cdot F_{\mu\nu}$$

Weyl-Invariance for $h_{\mu\nu} = \Omega^2(x)\, h'_{\mu\nu}$

$$g_{\mu\nu} = \frac{h_{\mu\nu}}{(-h)^{1/4}} \cdot \sqrt{\mathscr{P}}$$

Pontryagin Density

$$g_{\mu\nu} = h_{\mu\nu} \cdot \sqrt{F_{\alpha\beta}F^{\alpha\beta}}$$

Mukohyama et all (2018)

$$F_{\alpha\beta}\widetilde{F}^{\alpha\beta} = \nabla_{\alpha}^{h)} C^{\alpha}$$

Chern-Simons Current

$$C^{\alpha} = \mathrm{tr}\frac{\varepsilon^{\alpha\beta\gamma\delta}}{\sqrt{-h}}\left(\boldsymbol{F}_{\beta\gamma}\boldsymbol{A}_{\delta} - \frac{2}{3} ig\boldsymbol{A}_{\beta}\boldsymbol{A}_{\gamma}\boldsymbol{A}_{\delta}\right)$$

composite vector variable
of conformal weight four!

$$S_g\left[h,A\right] = -\frac{1}{2}\int d^4x\sqrt{-h}\left[\left(F_{\alpha\beta}\widetilde{F}^{\alpha\beta}\right)^{1/2} R\left(h\right) + \frac{3}{8}\cdot\frac{\left(\nabla_\mu^{h)}\left(F_{\alpha\beta}\widetilde{F}^{\alpha\beta}\right)\right)^2}{\left(F_{\sigma\rho}\widetilde{F}^{\sigma\rho}\right)^{3/2}}\right]$$

matter couples to $g_{\mu\nu} = \frac{\varphi^2}{6}\cdot h_{\mu\nu}$ **where** $F_{\alpha\beta}\widetilde{F}^{\alpha\beta} = \left(\frac{\varphi^2}{6}\right)^2$

$$\frac{1}{\sqrt{-h}}\cdot\frac{\delta S}{\delta A_\nu} = \widetilde{F}^{\mu\nu}\partial_\mu\left(T-G\right) = 0$$

$$\frac{1}{\sqrt{-h}}\cdot\frac{\delta S}{\delta h^{\alpha\beta}} = \frac{\varphi^2}{12}\left[T_{\alpha\beta} - G_{\alpha\beta} - \frac{1}{4}\left(T-G\right)g_{\alpha\beta}\right] = 0$$

Benign Higher Derivatives

$$S_g\left[h,A\right] = -\frac{1}{2}\int d^4x\sqrt{-h}\left[\left(F_{\alpha\beta}\widetilde{F}^{\alpha\beta}\right)^{1/2} R\left(h\right) + \frac{3}{8}\cdot\frac{\left(\nabla_\mu^{h)}\left(F_{\alpha\beta}\widetilde{F}^{\alpha\beta}\right)\right)^2}{\left(F_{\sigma\rho}\widetilde{F}^{\sigma\rho}\right)^{3/2}}\right]$$

$$F_{\alpha\beta}\widetilde{F}^{\alpha\beta} = \left(\frac{\varphi^2}{6}\right)^2$$

$$S\left[h,\varphi,A,\lambda\right] = \int d^4x\sqrt{-h}\left[-\frac{1}{2}\left(\partial\varphi\right)^2 - \frac{1}{12}\varphi^2 R\left(h\right) - \frac{\lambda}{72}\varphi^4 + \frac{\lambda}{2}\cdot F_{\alpha\beta}\widetilde{F}^{\alpha\beta}\right]$$

wrong sign

Lagrange multiplier

Axionic Cosmological Constant?

Weyl-invariant variables

$$g_{\mu\nu} = \frac{\varphi^2}{6} h_{\mu\nu} \qquad \Lambda = \frac{\lambda}{2} \qquad A_\mu$$

$$S\left[g, A, \Lambda\right] = \int d^4x \sqrt{-g} \left[-\frac{1}{2} R\left(g\right) + \Lambda \left(F_{\alpha\beta} \widetilde{F}^{\alpha\beta} - 1 \right) \right]$$

PHYSICS REPORTS (Review Section of Physics Letters) 104, Nos. 2–4 (1984) 143–157. North-Holland, Amsterdam

Foundations and Working Pictures in Microphysical Cosmology

Frank WILCZEK

I would like to briefly mention one idea in this regard, that I am now exploring. It is to do something for the Λ-parameter very similar to what the axion does for the θ-parameter in QCD, another otherwise mysteriously tiny quantity. The basic idea is to promote these parameters to dynamical variables, and then see if perhaps small values will be chosen dynamically. In the case of the

Another way of thoughts

Cleaning up the cosmological constant

2012

Ian Kimpton and Antonio Padilla

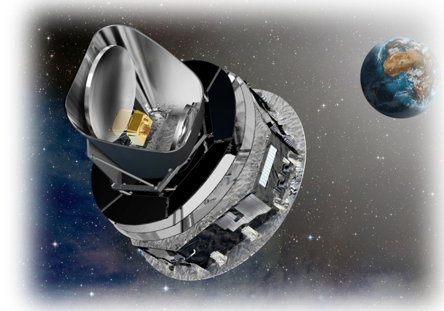
School of Physics and Astronomy, University of Nottingham, Nottingham NG7 2RD, UK

We now observe that the vacuum energy coming from particle physics enters the action via a term of the form $-2\Lambda \int d^4x\sqrt{-\tilde{g}}$. This has no effect on the dynamics provided

$$\frac{\delta}{\delta\phi_i}\int d^4x\sqrt{-\tilde{g}} = 0. \tag{2.1}$$

This is only possible when $\tilde{g}_{ab}$ is a composite field, for which $\sqrt{-\tilde{g}}$ is the integrand of a topological invariant, and/or a total derivative. Note that our method is distinct from unimodular gravity in which the metric determinant is *constrained* to be unity [13].

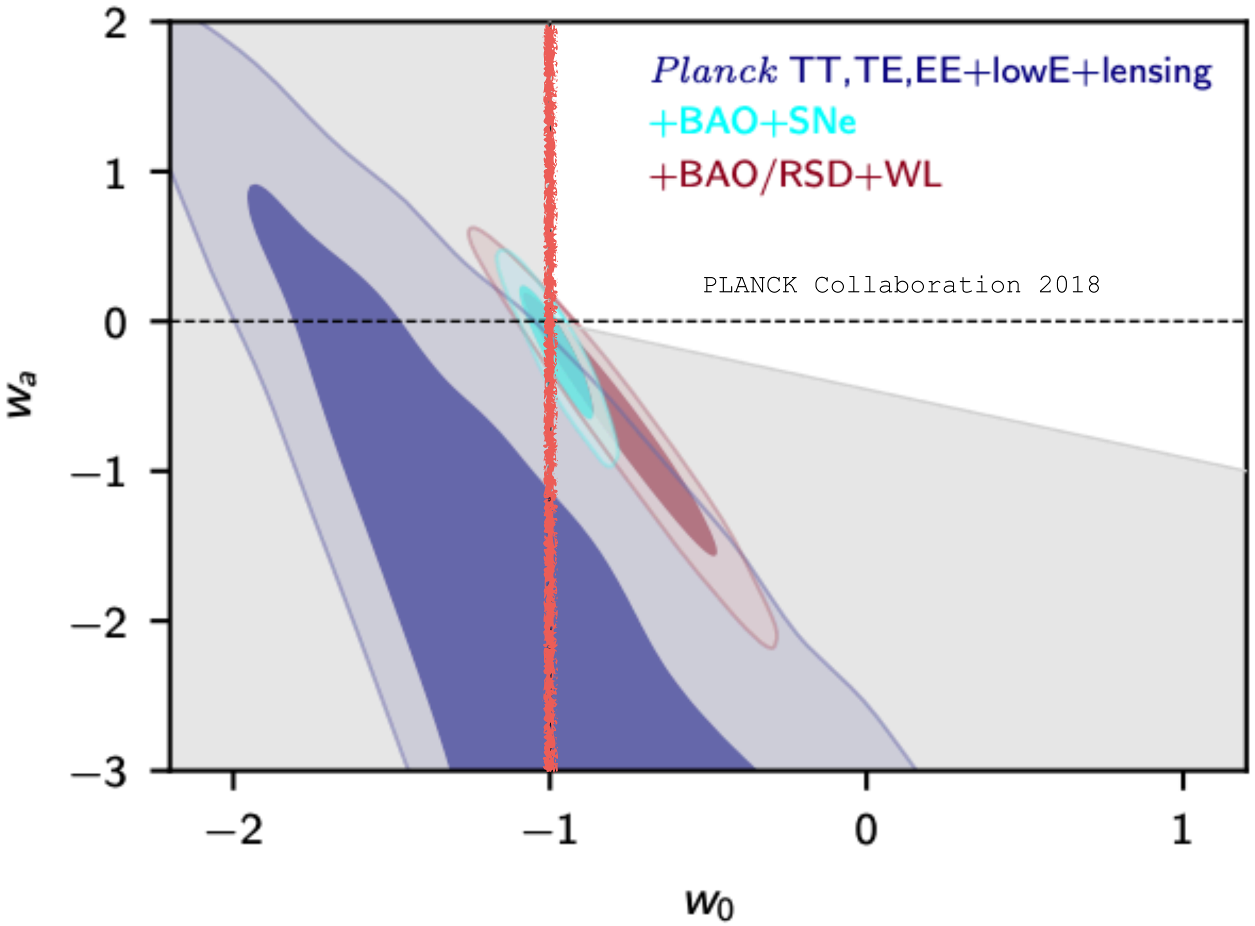
Dark Energy equation of state

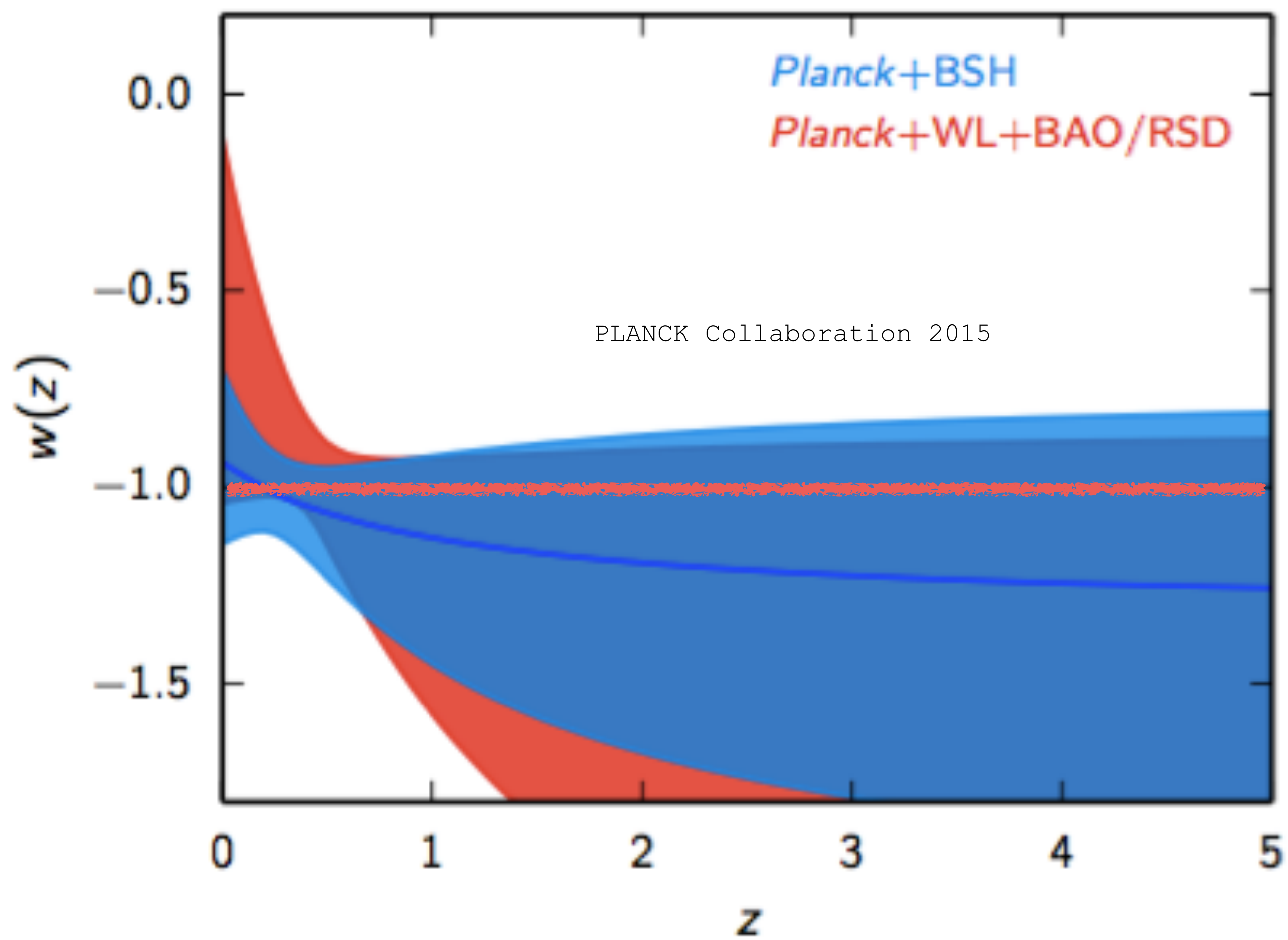


$$w = p/\varepsilon$$

$$w\left(a\right) = w_0 + \left(1 - a\right) w_a$$

PLANCK Collaboration 2013





Maybe DE is locally dynamical similarly to Inflation?

We did not care about CC during the early universe acceleration (inflation), maybe this is problem can be ignored for the current stage of acceleration as well?

We need a theory to compare with CC!

“Best” models of Inflation so far

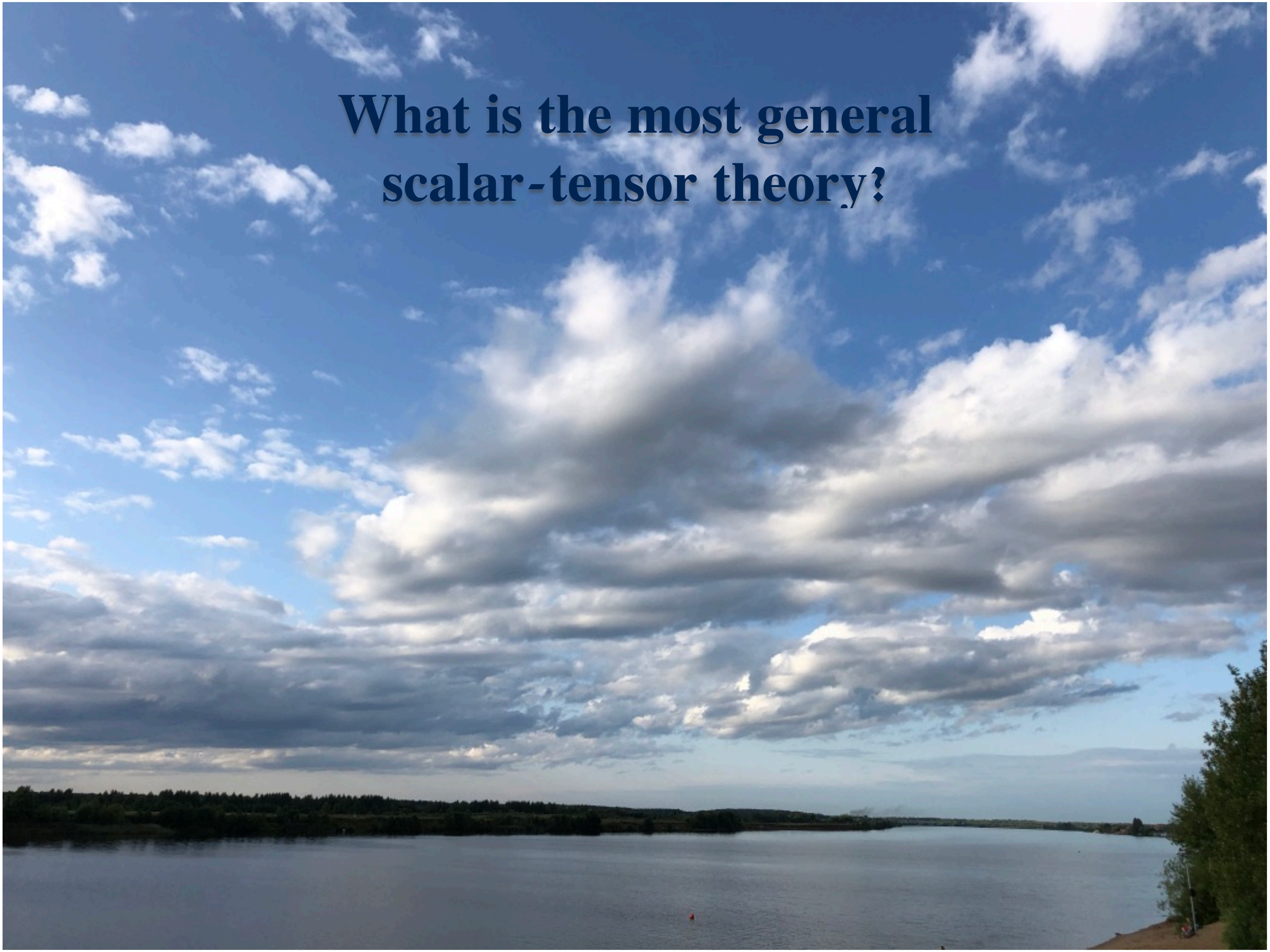
$$S = \int d^4x\sqrt{-g}\left(-\left(1+\xi\left|\mathcal{H}\right|^2\right)R + \left|\partial\mathcal{H}\right|^2 - V\left(\mathcal{H}\right)\right)$$

Bezrukov, Shaposhnikov, 2008

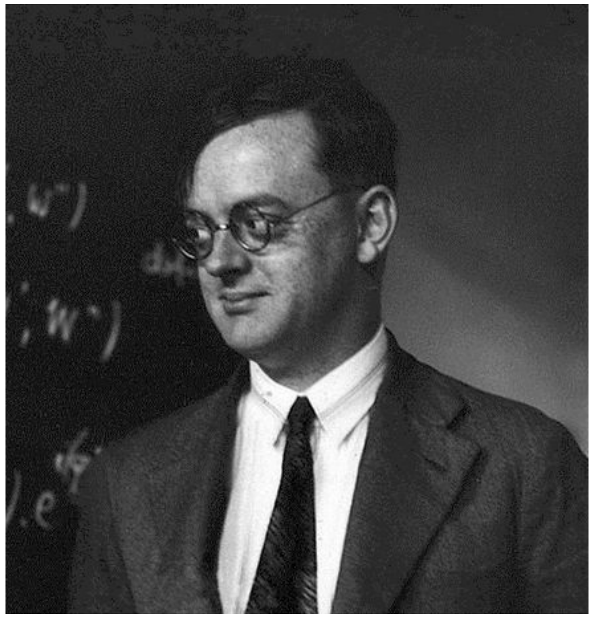
$$S = \frac{1}{2}\int d^4x\sqrt{-g}\left(-R + \frac{R^2}{6M^2}\right)$$

Starobinsky, 1980

do modify gravity!


What is the most general
scalar-tensor theory?

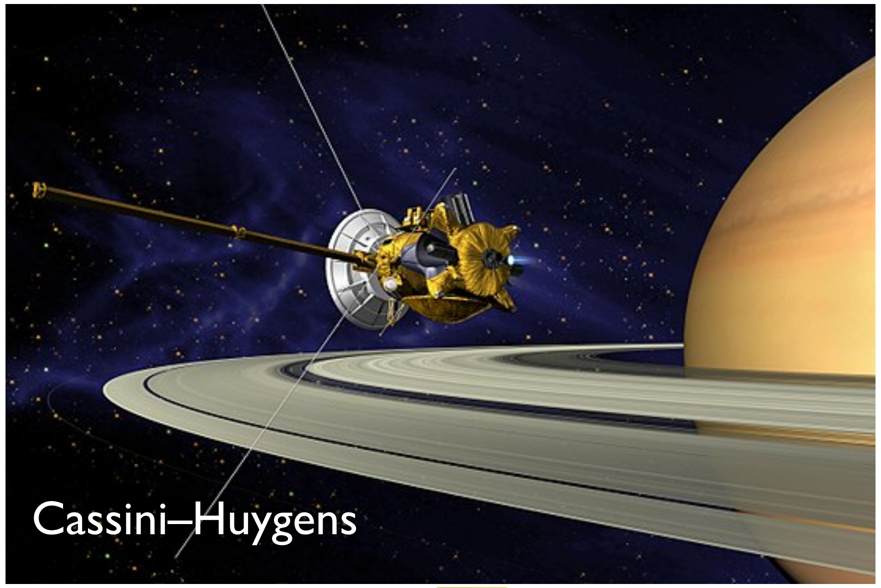
Jordan - Brans - Dicke



1955-1959

1961

$$G_N^{-1} \to \phi$$



$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left(-\phi R + \frac{\omega}{\phi} (\partial\phi)^2 \right)$$

2003

$$\omega > 40\ 000$$

$$S = \int d^4x \sqrt{-g} \left(f(\phi) R + (\partial\phi)^2 - V(\phi) \right)$$

scalar-tensor theory

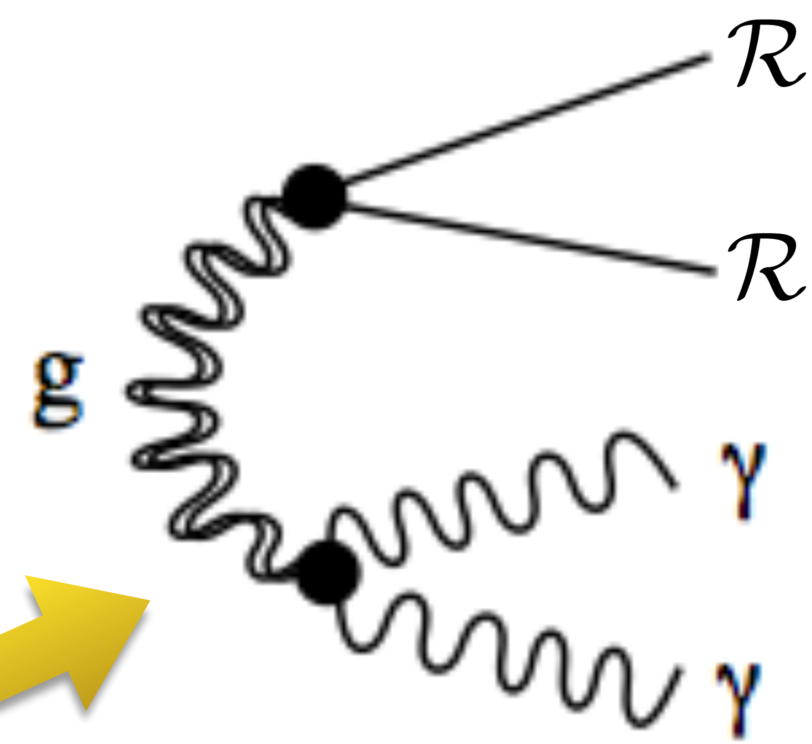
Fighting with **ghosts** and **gradient** instabilities

$$S[\mathcal{R}] = \frac{1}{2}\int d\tau d^3\mathbf{x}\, Z\left(\left(\mathcal{R}'\right)^2 - c_{\rm S}^2\left(\partial_i\mathcal{R}\right)^2\right) \qquad \mathcal{R} = \Phi + H\frac{\delta\varphi}{\dot\varphi}\,,$$

$$H_{\mathbf{k}} = \frac{|P_{\mathbf{k}}|^2}{2Z} + \frac{Zc_S^2k^2\,|\mathcal{R}_{\mathbf{k}}|^2}{2}$$

$$R_{\mathbf{k}} \sim \exp\left(|c_s|\,k\tau\right)$$

Gradient instability $c_s^2 < 0$



ghost $Z(t) < 0$

ghosts - modes (oscillators) with the ***negative mass***

$$\Gamma_{0\to2\gamma2\phi} \sim \frac{\Lambda^8}{M_{\rm Pl}^4}$$

Cline, Jeon, Moore, (2003)

What is the most general scalar-tensor theory with one scalar dof on top the graviton, and second order equations of motion?

Gregory Horndeski

Horndeski scalar-tensor theory (1974)

$$X = \frac{1}{2} g^{\mu\nu} \phi_{;\mu} \phi_{;\nu}$$

k-essence, perfect fluid c_s

Armendariz-Picon, Damour, Mukhanov, Garriga (1999)

$$S = \int d^4x \sqrt{-g} \left[K(X,\phi) + \right.$$

$$+ G(X,\phi)\, \Box\phi +$$

$w = p/\varepsilon < -1$

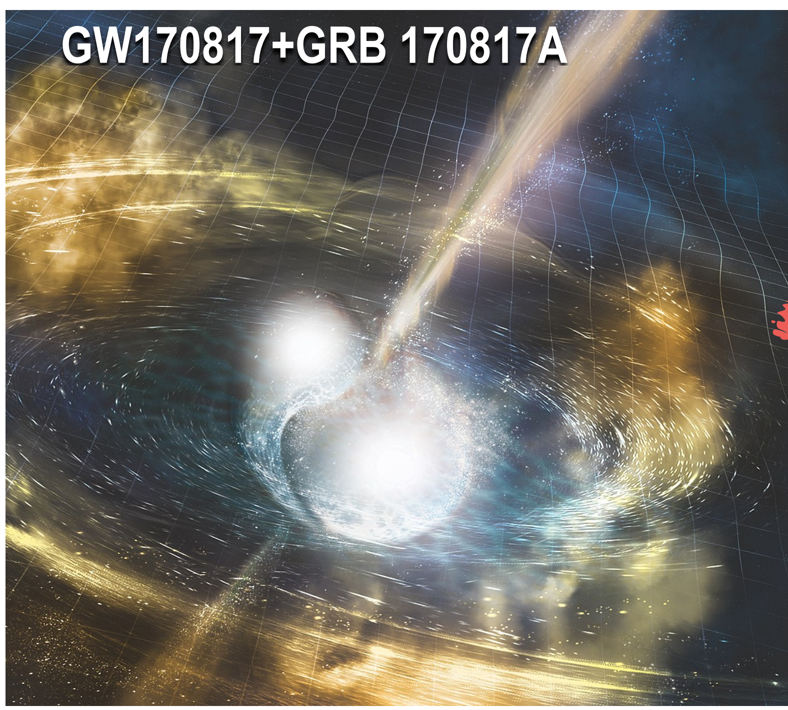
Kinetic Gravity Braiding

Imperfect fluid

$c_s(\rho_{ext}, p_{ext})$

Deffayet, Pujolas, Sawicki, Vikman (2010)

$|c_T - 1| \lesssim 10^{-15}$



$G_N(X,\phi)$ $\quad$ c_T

$$G_4(X,\phi) R + G_{4,X}(X,\phi) \left[\left(\phi^{;\mu}_{;\mu}\right)^2 - \left(\phi^{;\mu}_{;\nu}\right)^2 \right] +$$

$$+ G_5(X,\phi) G^{\mu\nu} \phi_{;\mu;\nu} + \frac{1}{6} G_{5,X}(X,\phi) \left[\left(\phi^{;\mu}_{;\mu}\right)^3 - 3\phi^{;\mu}_{;\mu} \left(\phi^{;\alpha}_{;\beta}\right)^2 + 2\left(\phi^{;\mu}_{;\nu}\right)^3 \right] \Big]$$

$c_T(\rho_{ext}, p_{ext})$

Kinetic Gravity Braiding is the only survivor of GW170817+GRB 170817A!

$$S = \int d^4x\sqrt{-g}\left(-R + K\left(\varphi, X\right) + G\left(\varphi, X\right)\Box\varphi + \mathscr{L}_m\left(g\right)\right) \qquad X = \left(\partial\varphi\right)^2$$

In ***Kinetic Gravity Braiding*** the GW's propagate with the speed of light on all backgrounds!

$$S = \int d^4x\sqrt{-g}\left(-f\left(\varphi\right)R + K\left(\varphi, X\right) + G\left(\varphi, X\right)\Box\varphi + \mathscr{L}_m\left(g\right)\right)$$

$$g_{\mu\nu} \to f\left(\varphi\right)^{-1} g_{\mu\nu}$$

One of the Possible Couplings to Matter

$$S = \int d^4x\sqrt{-g}\left(-R + K'\left(\varphi, X\right) + G'\left(\varphi, X\right)\Box\varphi + f^{-2}\left(\varphi\right)^2\mathscr{L}_m\left(f^{-1}\left(\varphi\right)g\right)\right)$$

Speed of GW does not change under this field redefinition/ "frame change"!

Good news for you:

There are still
a lot of hard unsolved important problems
in cosmology and gravitational physics!