QCD phase diagram and its dualities

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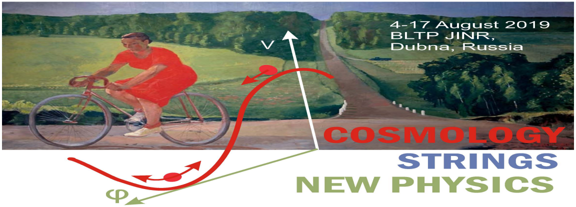

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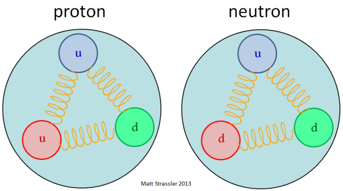
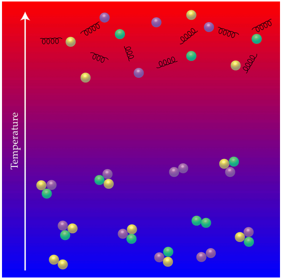
K.G. Klimenko, IHEP T.G. Khunjua, MSU

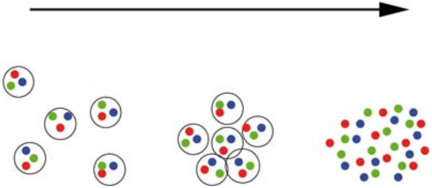
broad group: strong connections with
V. Ch. Zhukovsky, Moscow state University
D. Ebert, Humboldt University of Berlin

details can be found in
JHEP 1906 (2019) 006 arXiv:1901.02855 [hep-ph]
Eur.Phys.J. C79 (2019) no.2, 151, arXiv:1812.00772 [hep-ph],
Phys.Rev. D98 (2018) no.5, 054030 arXiv:1804.01014 [hep-ph],
Phys.Rev. D97 (2018) no.5, 054036 arXiv:1710.09706 [hep-ph]
Phys.Rev. D95 (2017) no.10, 105010 arXiv:1704.01477 [hep-ph]

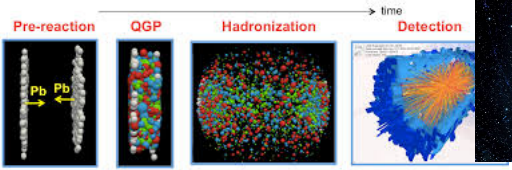
Hadronic, quark matter

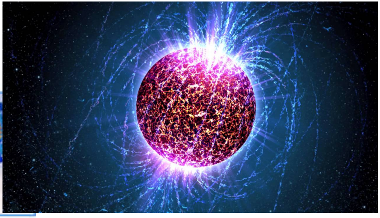


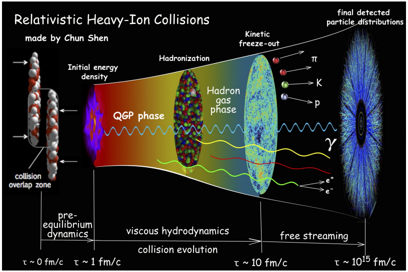


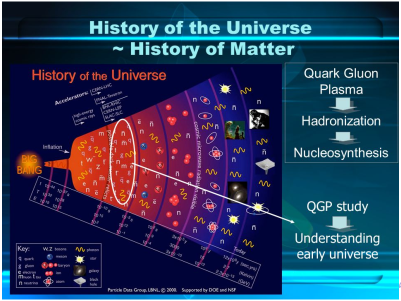


QCD at extreme conditions









QCD Lagrangian

The QCD Lagrangian obtained from the gauge principle reads

$$\mathcal{L}_{\mathrm{QCD}} = \sum_{f=u,d,s} \bar{q}_f (i\not{D} - m_f) q_f - \frac{1}{4} \mathcal{G}_{\mu\nu,a} \mathcal{G}_a^{\mu\nu}. \quad (1)$$

f- quark flavor, the quark field q_f consists of a color triplet (subscripts r, g, and b standing for "red," "green," and "blue"),

$$q_f = \begin{pmatrix} q_{f,r} \\ q_{f,g} \\ q_{f,b} \end{pmatrix}, \quad (2)$$

The covariant derivative is

$$D_\mu \equiv (\partial_\mu - ie\mathcal{A}_\mu), \quad \mathcal{A}_\mu = \mathcal{A}_\mu^a \lambda^a$$

field strength tensor

$$\mathcal{G}_{\mu\nu,a} = \partial_\mu \mathcal{A}_{\nu,a} - \partial_\nu \mathcal{A}_{\mu,a} + g f_{abc} \mathcal{A}_{\mu,b} \mathcal{A}_{\nu,c}, \quad (3)$$

Chiral symmetry

For chiral symmetry it is important that **quark masses are zero**

$$m_f = 0 \quad ---- \quad \text{chiral limit}$$

But if $m_f \neq 0$ **chiral symmetry is broken**

$$SU(2)_L \times SU(2)_R \to SU(2)_{L+R}$$

or

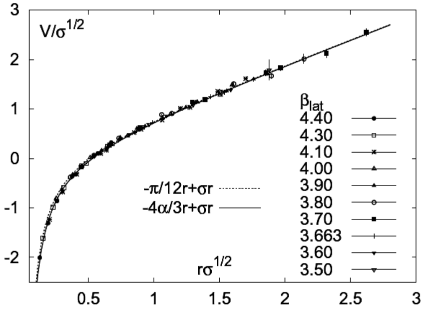
$$SU(2)_V \times SU(2)_A \to SU(2)_V$$

$$SU(2)_V : U = \exp\left(-i\theta_a \frac{\tau_a}{2}\right), \quad SU(2)_A : U = \exp\left(-i\theta_a \gamma^5 \frac{\tau_a}{2}\right)$$

main features of QCD: quark confinement

There is no coloured particles only colourless one, no free quarks

Cornell potential



main features of QCD: chiral symmetry breaking

Unlike the QED , **the QCD vacuum has non-trivial structure** due to non-perturbative interactions among quarks and gluons

lattice simulations $\Rightarrow$ **condensation of quark and anti-quark pairs**

$$\langle \bar{q}q \rangle \neq 0, \quad \langle \bar{u}u \rangle = \langle \bar{d}d \rangle \approx (-250MeV)^3$$

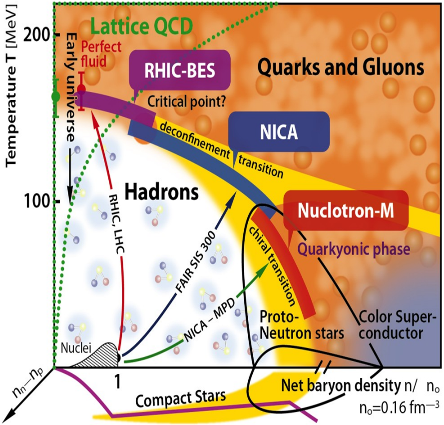
$\langle \bar{q}q \rangle \neq 0$ suggests the existence of the "dynamical mass"

$$\langle \bar{q}q \rangle = -i \lim_{x \to y+0} trS_F(x,y), \quad S_F(p) = \frac{A(p)}{\gamma p - B(p)}$$

If $B(p) = 0 \Rightarrow \langle \bar{q}q \rangle = 0$ due to $tr\gamma^\mu = 0$ (in chiral limit in PT)

NJL and gives $B(p) = M \Rightarrow$ **CSB**

QCD Phase Diagram



Two main phase transitions

- confinement-deconfinement
- chiral symmetry breaking phase—chriral symmetric phase

Methods of dealing with QCD

Methods of dealing with QCD

- perturbative QCD, pQCD, high energy
- First principle calcaltion – lattice Monte Carlo simulations, LQCD
- Effective models

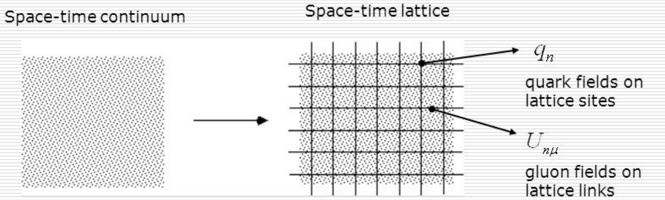
Chiral pertubation theory χPT
Nambu–Jona-Lasinio model NJL
Polyakov-loop extended Nambu–Jona-Lasinio model PNJL
Quark meson model

- $1/N$ expansion (large number of colors) G.t'Hooft.
  the predictions of $\frac{1}{N_c}$ expansions for QCD are mostly of a qualitative nature
- Holographic methods, Gauge/gravity or gauge/string duality AdS/CFT conjecture

Lattice QCD

QCD on a space-time lattice

K. G. Wilson 1974



- Feynman path integral
  - Action $S_{QCD} = \frac{1}{g_s^2}\sum_P tr(UUUU) + \sum_f \bar{q}_f(\gamma \cdot U + m_f)q_f$
  - Physical quantities as integral averages

$$\langle O(U,\bar{q},q)\rangle = \frac{1}{Z}\int \prod_{n\mu} dU_{n\mu} \prod_n d\bar{q}_n dq_n \, O(U,(U,\bar{q},q))e^{-S_{QCD}}$$

→ *Monte Carlo Evaluation of the path integral*

lattice QCD at non-zero baryon chemical potential μ_B

It is well known that **at non-zero baryon chemical potential** μ_B **lattice simulation** is quite challenging due to the **sign problem** complex determinant

$$(Det(D(\mu)))^\dagger = Det(D(-\mu^\dagger))$$

Methods of dealing with QCD

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- **Effective models**

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Nambu–Jona-Lasinio model, NJL

Polyakov-loop extended Nambu–Jona-Lasinio model PNJL

Quark meson model

NJL model

NJL model can be considered as **effective field theory for QCD**.

the model is **nonrenormalizable**
Valid up to $E < \Lambda \approx 1$ GeV

Parameters G, Λ, m_0

chiral limit $m_0 = 0$

in many cases chiral limit is a very good approximation

dof– **quarks**
no gluons only **four-fermion interaction**
attractive feature — dynamical CSB
the main drawback – lack of confinement (PNJL)

Nambu–Jona-Lasinio model

Nambu–Jona-Lasinio model

$$\mathcal{L} = \bar{q}\gamma^{\nu}\mathrm{i}\partial_{\nu}q + \frac{G}{N_c}\Big[(\bar{q}q)^2 + (\bar{q}\mathrm{i}\gamma^5 q)^2\Big]$$

$$q \to e^{i\gamma_5\alpha}q$$

continuous symmetry

$$\widetilde{\mathcal{L}} = \bar{q}\Big[\gamma^{\rho}\mathrm{i}\partial_{\rho} - \sigma - \mathrm{i}\gamma^5\pi\Big]q - \frac{N_c}{4G}\Big[\sigma^2 + \pi^2\Big].$$

Chiral symmetry breaking

$1/N_c$ expansion, leading order

$$\langle\bar{q}q\rangle \neq 0$$

$$\langle\sigma\rangle \neq 0 \quad \longrightarrow \quad \widetilde{\mathcal{L}} = \bar{q}\Big[\gamma^{\rho}\mathrm{i}\partial_{\rho} - \langle\sigma\rangle\Big]q$$

Methods of dealing with QCD

QCD at T and μ
(QCD at extreme conditions)

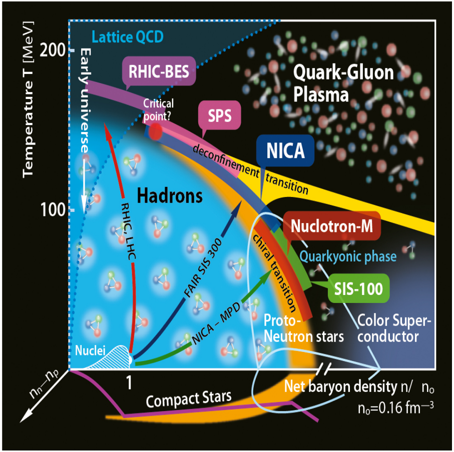
- neutron stars
- heavy ion collisions
- Early Universe

Methods of dealing with QCD

- First principle calcultion – lattice QCD

- Effective models

  Nambu–Jona-Lasinio model
  NJL



Different types of chemical potentials: dense matter with isotopic imbalance

Baryon chemical potential μ_B

Allow to consider systems with non-zero baryon densities.

$$\frac{\mu_B}{3}\bar{q}\gamma^0 q = \mu\bar{q}\gamma^0 q,$$

Different types of chemical potentials: dense matter with isotopic imbalance

Baryon chemical potential μ_B

Allow to consider systems with non-zero baryon densities.

$$\frac{\mu_B}{3}\bar{q}\gamma^0 q = \mu\bar{q}\gamma^0 q,$$

Isotopic chemical potential μ_I

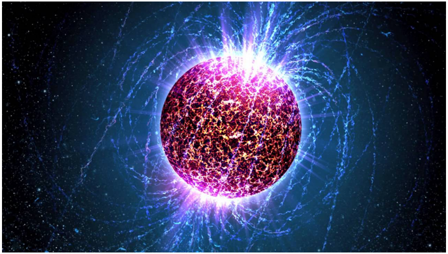
Allow to consider systems with isotopic imbalance.

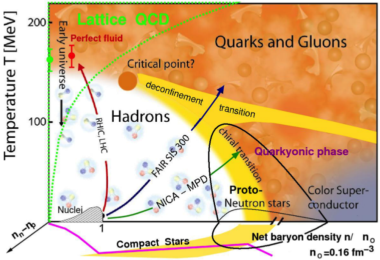
$$n_I = n_u - n_d \quad \longleftrightarrow \quad \mu_I = \mu_u - \mu_d$$

The corresponding term in the Lagrangian is $\frac{\mu_I}{2}\bar{q}\gamma^0\tau_3 q$

QCD phase diagram with isotopic imbalance

neutron stars, heavy ion collisions have isotopic imbalance





Different types of chemical potentials: chiral imbalance

chiral (axial) chemical potential

Allow to consider systems with chiral imbalance (difference between between densities of left-handed and right-handed quarks).

$$n_5 = n_R - n_L \quad \longleftrightarrow \quad \mu_5 = \mu_R - \mu_L$$

The corresponding term in the Lagrangian is

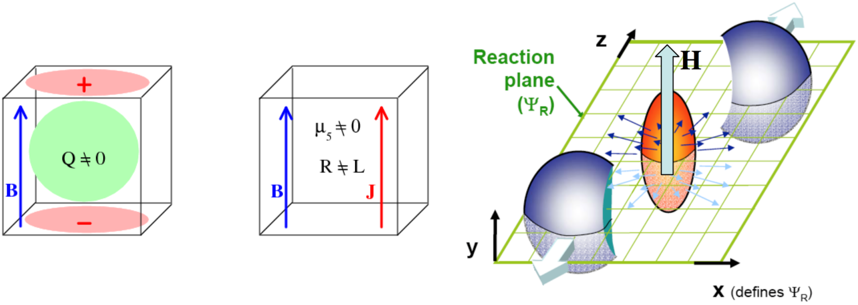
$$\mu_5 \bar{q} \gamma^0 \gamma^5 q$$

chiral isospin imbalance $\mu_{I5} = \mu_{u5} - \mu_{d5}$

$$n_{I5} = n_{u5} - n_{d5}, \qquad n_{I5} \quad \longleftrightarrow \quad \mu_{I5}$$

Term in the Lagrangian — $\frac{\mu_{I5}}{2} \bar{q} \tau_3 \gamma^0 \gamma^5 q$

Chiral magnetic effect



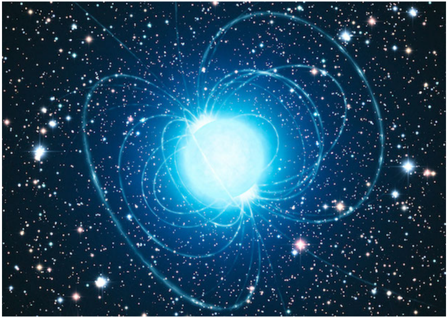
$$\vec{J} = c\mu_5\vec{B}, \qquad c = \frac{e^2}{2\pi^2}$$

A. Vilenkin, PhysRevD.22.3080,

K. Fukushima, D. E. Kharzeev and H. J. Warringa, Phys. Rev. D **78** (2008) 074033 [arXiv:0808.3382 [hep-ph]].

Chiral imbalance in dense matter

Chiral imbalance could appear in compact stars



- Chiral separation effect
- Chiral vortical effect

Order parameters, condensates

Condensates $\langle \sigma(x) \rangle$ and $\langle \pi_a(x) \rangle$
Order parameters

$$\langle \sigma(x) \rangle = M, \quad \langle \pi_\pm(x) \rangle = \Delta, \quad \langle \pi_3(x) \rangle = 0. \tag{4}$$

Dualities of the phase diagram

The TDP (phase daigram) is invariant under

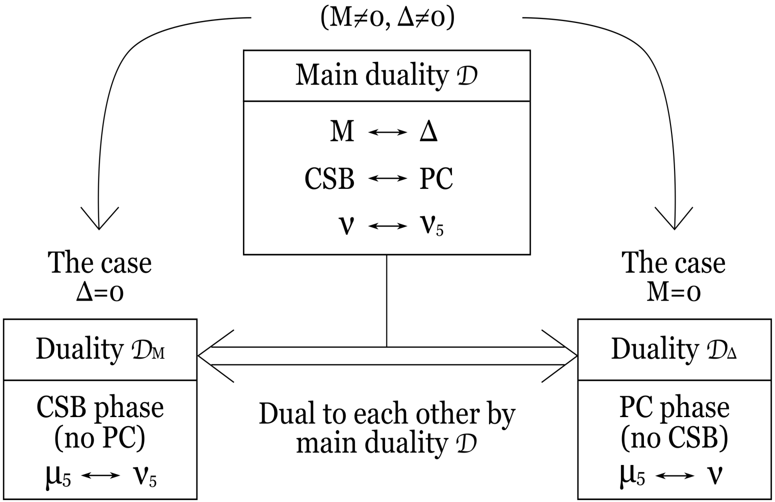
- Interchange of condensates

- matter content

$$\Omega(C_1, C_2, \mu_1, \mu_2)$$

$$\Omega(C_1, C_2, \mu_1, \mu_2) = \Omega(C_2, C_1, \mu_2, \mu_1)$$

Dualities of the phase diagram

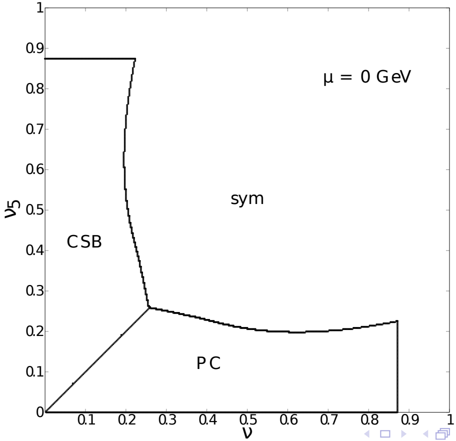


(ν, ν_5) phase portrait of NJL

Duality between chiral symmetry breaking and pion condensation

$$\mathcal{D}: \;\; M \longleftrightarrow \Delta, \quad \nu \longleftrightarrow \nu_5$$

$$\text{PC} \longleftrightarrow \text{CSB} \quad \nu \longleftrightarrow \nu_5$$



Dualities on the lattice

Dualities on the lattice

$\mu_B \neq 0$ impossible on lattice but if $\mu_B = 0$

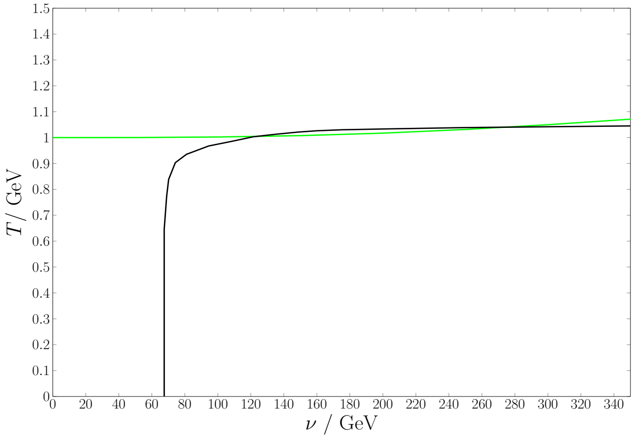
- **QCD at** μ_5 has **no sign problem** and can be considered **on lattice**

  V. Braguta, A. Kotov et al, ITEP

- **QCD at** μ_I

  G. Endrodi group, B. Brandt et al,
  earlier lattice simulation.

Dualities on the lattice



The Strength of Duality

Dualities in different approaches: in large N_c orbifold equivalences

- **Large N_c orbifold equivalences** connect gauge theories with different gauge groups and **matter content** in the large N_c limit.
  M. Hanada and N. Yamamoto,
  JHEP 1202 (2012) 138, arXiv:1103.5480 [hep-ph],
  PoS LATTICE **2011** (2011), arXiv:1111.3391 [hep-lat]

two gauge theories with gauge groups G_1 and G_2 with μ_1 and μ_2

Duality

$$G_1 \longleftrightarrow G_2, \; \mu_1 \longleftrightarrow \mu_2$$

Dualities can be used in circumvent the sign problem

two gauge theories with gauge groups G_1 and G_2 with μ_1 and μ_2

Duality

$$G_1 \longleftrightarrow G_2,\ \mu_1 \longleftrightarrow \mu_2$$

- G_2 is sign problem free
- G_1 has sign problem, can not be studied on lattice

Investigations of $G_2 \rightarrow$ properties of phase structure of G_1

Circumvent the sign problem

Duality

QCD at $\mu_1 \longleftrightarrow$ QCD at μ_2

QCD with μ_2 —- sign problem free,

QCD with μ_1 —- sign problem (no lattice)

A number of papers predicted **anticatalysis** (T_c decrease with μ_5) of dynamical chiral symmetry breaking

A number of papers predicted **catalysis** (T_c increase with μ_5) of dynamical chiral symmetry breaking

(Could even depend on the scheme of regularization)

V. Braguta, ITEP, lattice results show the **catalysis**

But unphysically large pion mass

Phase diagram at μ_I is now well studied

simulations of Endrodi group, earlier lattice simulation,
ChPT has similar predictions
D.T. Son, M.A. Stephanov Phys.Rev.Lett. 86 (2001) 592-595
arXiv:hep-ph/0005225, Phys.Atom.Nucl.64:834-842,2001;
Yad.Fiz.64:899-907,2001 arXiv:hep-ph/0011365

Duality $\Rightarrow$ catalysis of chiral symmetry beaking

In vacuum the quantities $\langle\sigma(x)\rangle$ and $\langle\pi_a(x)\rangle$ do not depend on space coordinate x.

in a dense medium the ground state expectation values of bosonic fields might depend on spatial coordinates

CDW ansatz for CSB
the single-plane-wave LOFF ansatz for PC

$$\langle\sigma(x)\rangle = M\cos(2kx^1), \quad \langle\pi_3(x)\rangle = M\sin(2kx^1),$$
$$\langle\pi_1(x)\rangle = \Delta\cos(2k'x^1), \quad \langle\pi_2(x)\rangle = \Delta\sin(2k'x^1)$$

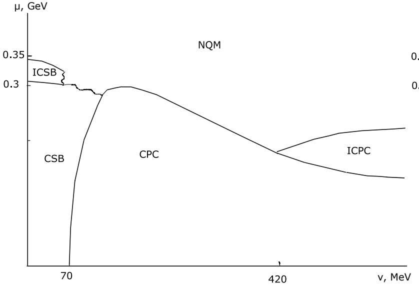
equivalently

$$\langle\pi_\pm(x)\rangle = \Delta e^{\pm 2k'x^1}$$

Duality

Duality in inhomogeneous case is shown

$$\mathcal{D}_I : \qquad M \longleftrightarrow \Delta, \quad \nu \longleftrightarrow \nu_5, \quad k \longleftrightarrow k'. \tag{5}$$



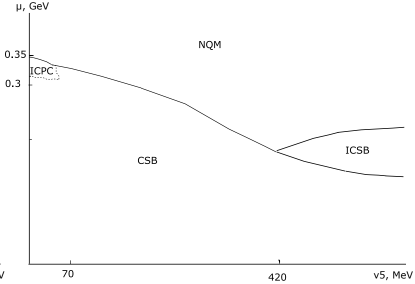


Figure: (ν, μ)-phase diagram

Figure: (ν_5, μ)-phase diagram

They are dualy conjugated to each other

It is also shown that duality is the property of QCD itself and is valid not only in effective models and $1/N_c$ approximation

Thanks for the attention