Lecture #3
Dark Matter Models

Dmitry Gorbunov

Institute for Nuclear Research of RAS, Moscow

The Helmholtz International School
“Cosmology, Strings, and New Physics”

DIAS-TH Program at BLTP, JINR
Dubna, Russia

Outline

1. Dark Matter properties
2. Thermal Dark Matter
3. Non-thermal candidates
4. Summary

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So far only gravitational evidence for DM

Dark Matter properties from cosmology: $p = 0$

(If) particles:

1. stable on cosmological time-scale

   requires new (almost) conserved quantum number
2. produced in the early Universe

   some time before RD/MD-transition ($T = 0.8\,\text{eV}$)
3. nonrelativistic particles long before RD/MD-transition ($T = 0.8\,\text{eV}$)
   (either Cold or Warm, $v_{RD/MD} \lesssim 10^{-3}$)
   Otherwise no small-size structures, like dwarf galaxies:
   smoothed out by free streaming
   If were in thermal equilibrium: $M_X \gtrsim 1$ keV
4. (almost) collisionless $p = 0$, $v_{\text{sound}} = 0$
5. (almost) electrically neutral CMB distortion
6. all matter inhomogeneities (perturbations) are adiabatic:

$$\delta\left(\frac{n_B}{n_{DM}}\right) = \delta\left(\frac{n_B}{n_\gamma}\right) = \delta\left(\frac{n_\nu}{n_\gamma}\right) = 0$$

Dark Matter properties from astrophysics

1. stable on cosmological time-scale
2. (almost) collisionless — to form ellipsoidal halos
3. (almost) electrically neutral — to be Dark
4. stability of globular stellar clusters — $M_X \lesssim 10^3 M_\odot \approx 10^{61}$ GeV
   otherwise too strong tidal forces
5. confinement in a galaxy: — quantum physics!

   de Broglie wavelength: $\lambda = 2\pi/(M_X v_X) < l_{\text{galaxy}}$, — for bosons

   in a galaxy $v_X \sim 0.5 \cdot 10^{-3}$ $\longrightarrow$ $M_X \gtrsim 3 \cdot 10^{-22}$ eV

   for fermions

   Pauli blocking: $M_X \gtrsim 750$ eV

$$f(\mathbf{p}, \mathbf{x}) = \frac{\rho_X(\mathbf{x})}{M_X} \cdot \frac{1}{\left(\sqrt{2\pi} M_X v_X\right)^3} \cdot e^{-\frac{\mathbf{p}^2}{2M_X^2 v_X^2}} \Big|_{\mathbf{p}=0} \leq \frac{g_X}{(2\pi)^3}$$

Free massive scalar field

$$\mathscr{L} = \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m_\phi^2 \phi^2$$

Homogeneous scalar field in the expanding Universe

$$\ddot{\phi} + 3H\dot{\phi} + m_\phi^2 \phi = 0$$

Two-stage evolution:

$$m_\phi < H(t) \quad \Longrightarrow \quad \phi = \phi_i = \text{const}$$

$$m_\phi > H(t) \quad \Longrightarrow \quad p = \langle E_k \rangle - \langle E_p \rangle = 0\,, \quad \rho \sim m_\phi^2 \phi^2 \propto 1/a^3$$

- dust-like substance in the late Universe, $\Omega \propto m_\phi^{1/2} \phi_i^2$

  depends on initial conditions
- presureless at spatial scales $l > 1/m_\phi$ — fuzzy DM
- isocurvature mode: $\delta\rho_\phi \propto \delta H\,,\ \delta f_i$

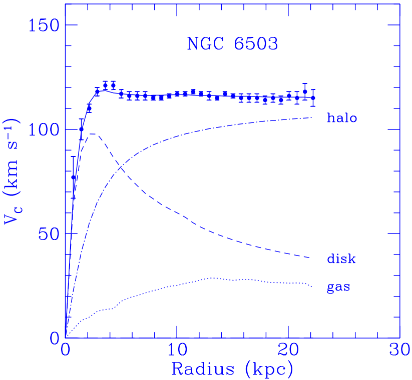
Further strengthening of the astrophysical bounds

- Fermions (collisionless):
  – conservation of the phase space density of DM partciles:
  gives $\gtrsim 5\,$keV when tested with dwarf spheroidal galaxies
  – free streaming gets read of small scale inhomogeneities
  Ly-α forest observations constrain velocity dispersion
- Very light bosons
  – produce preasure, hence non-standard gravitational potentials
  at distances $\propto 1/m$:
  pulsar timing constraints
  – selfinteraction prevents the cusp formation:
  – Supermassive black holes in galactic centers
  – Caustic formation

Galactic dark halos: flat rotation curves

$$v(R) = \sqrt{G\frac{M(R)}{R}}$$

$$M(R) = 4\pi \int_0^R \rho(r) r^2 dr$$



observations: $v(R) \simeq \text{const}$

visible matter:

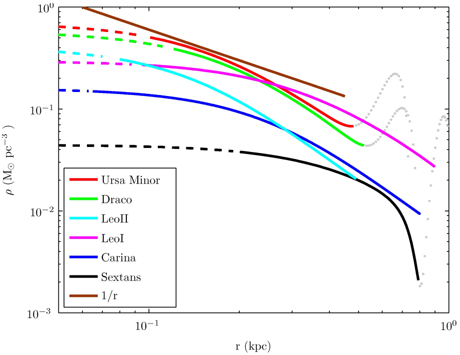
internal regions $v(R) \propto \sqrt{R}$

external ("empty") regions $v(R) \propto 1/\sqrt{R}$

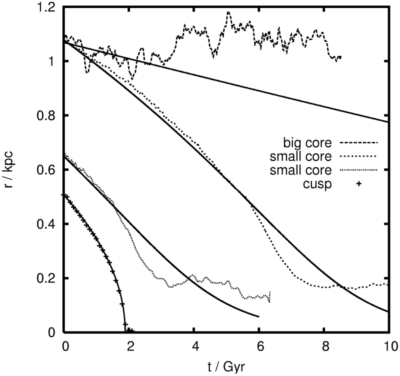
CDM Problems at small-scales ...?

- NFW profile fits nicely DM in galaxy clusters $\rho \propto r^{-1}(r + r_c)^{-2}$
- Dwarf galaxy density profiles: $\rho_M(r) \propto r^{-(0.5-1.5)}$ cusp
  most DM-dominated objects

Cores observed (?)

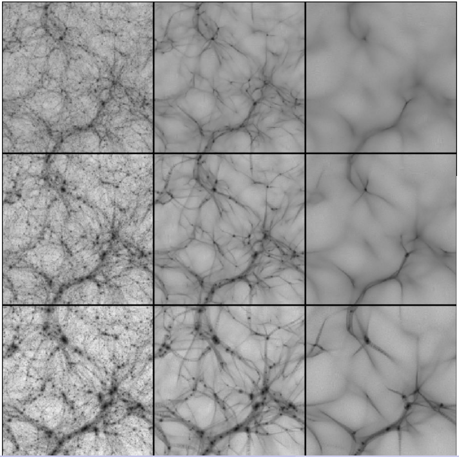


5 Clusters in the Fornax dSph

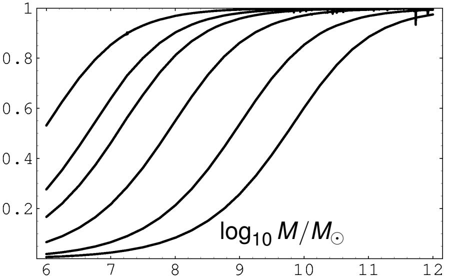


CDM Problems ...?

- Missing satellites: $\frac{dN_{obj}}{d\ln M} \propto \frac{1}{M}$ no-scale 100 instead of 1000
- "Too big to fail" problem
- Solved (?) by Warm Dark Matter (sterile neutrino, gravitino) free-streaming



$\left(\frac{dN_{obj}}{d\ln M}\right)_{WDM} / \left(\frac{dN_{obj}}{d\ln M}\right)_{CDM}$



Dark Matter: many well-motivated candidates

- WIMPs — related to EW scale, SUSY
- sterile neutrinos — active neutrino oscillations
- light scalar field — string theory
- axion — strong CP-problem
- gravitino — local SUSY
- Heavy relics — GUTs
- (Topological) defects — GUTs
- Massive Astrophysical Compact Heavy Objects
- Primordial black hole (remnants) — Phase transitions, exotic inflation, reheating

Multicomponent Dark Matter ?

γ, ν, H, He

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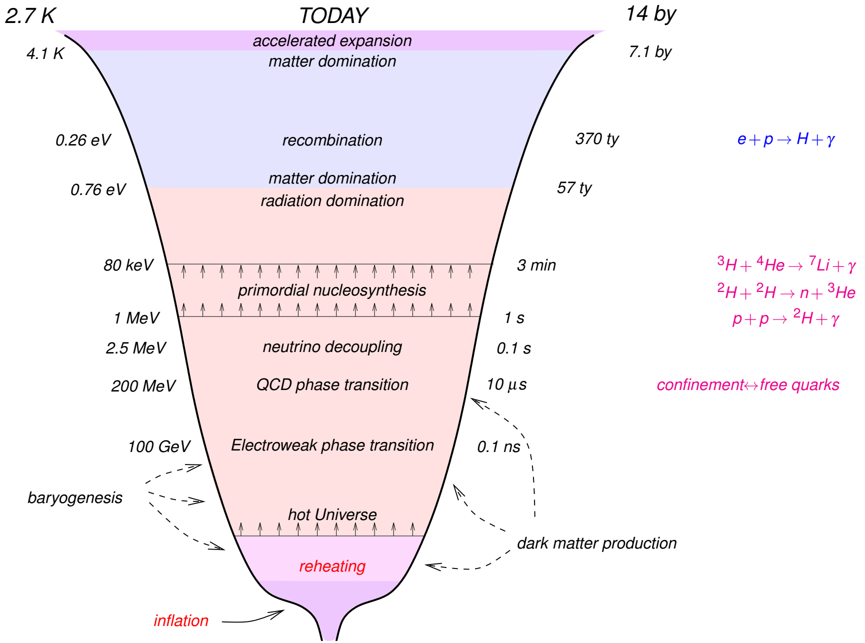

2.7 K
TODAY
14 by
accelerated expansion
4.1 K
matter domination
7.1 by
0.26 eV
recombination
370 ty
$e + p \to H + \gamma$
0.76 eV
matter domination
57 ty
radiation domination
80 keV
3 min
$^3H + ^4He \to ^7Li + \gamma$
primordial nucleosynthesis
$^2H + ^2H \to n + ^3He$
1 MeV
1 s
$p + p \to ^2H + \gamma$
2.5 MeV
neutrino decoupling
0.1 s
200 MeV
QCD phase transition
10 μs
confinement↔free quarks
100 GeV
Electroweak phase transition
0.1 ns
baryogenesis
hot Universe
dark matter production
reheating
inflation

Decoupling of relativistic Dark Matter

Assumptions

1. DM particles are in equlibrium in plasma
2. DM decouple from plasma at temperature $T_d \gtrsim M_X$,
   so they are relativistic (e.g. neutrino)

$$n_X(T_d) = g_X \cdot \left(\begin{array}{c} 1 \\ \frac{3}{4} \end{array}\right) \cdot \frac{\zeta(3)}{\pi^2}\, T_d^3$$

Later on useful

$$n_X a^3 = \text{const}\,, \qquad s a^3 = \text{const} \qquad \Longrightarrow \qquad \frac{n_X}{s} = \text{const} = \# \frac{g_X}{g_*(T_d)}$$

DM particle mass M_X fixes Ω_X:

$$\Omega_X = \frac{M_X \cdot n_{X,0}}{\rho_c} = \frac{M_X \cdot s_0}{\rho_c} \frac{n}{s} \approx 0.2 \times \frac{M_X}{100\text{ eV}} \left(\frac{g_X}{2}\right) \cdot \left(\frac{100}{g_*(T_d)}\right)$$

– NO heavy stable feebly coupled to SM particles !

– NO realistic DM models:

Pauli blocking prevents fermionic DM

$$\frac{p_X}{M_X} \propto \frac{a_d}{a} \sim \frac{3T}{M_X} \left(\frac{g_*(T)}{g_*(T_d)}\right)^{1/3}$$

too energetic for the proper structure formation

Decoupling of relativistic Dark Matter

Can we save the relativistic Dark Matter ??

one can try, say, nonstandard cosmological evolution

with entropy production

1. hot stage (radiation domination) $\rho \propto 1/a^4$
2. add new nonrelativistic particles decoupled from plasma $\rho \propto 1/a^3$
3. later they start to dominate
   intermediate stage of matter domination terminates before BBN !!
4. both relativistic DM density and entropy density decrease
5. new nonrelativistic particles decay reheating the Universe $T > 3\,\text{MeV}$
   entropy production

Decoupling of nonrelativistic Dark Matter

Assumptions:

1. no $X-\bar{X}$ asymmetry — either $X=\bar{X}$ or $n_X = n_{\bar{X}}$
2. @ $T \lesssim M_X$ in thermal equilibrium with plasma — (e.g. neutrons)

$$n_X = n_{\bar{X}} = g_X \left(\frac{M_X T}{2\pi}\right)^{3/2} e^{-M_X/T}$$

$$X\bar{X} \longrightarrow \text{light particles}$$

freeze-out temperature T_f — $H \equiv T^2/M_{Pl}^*$

$$n_X \langle \sigma_{ann} v \rangle = H(T_f) \longrightarrow T_f = \frac{M_X}{\ln\left(\frac{g_X M_X M_{Pl}^* \sigma_0}{(2\pi)^{3/2}}\right)} .$$

Bethe formula: — s-wave: $\sigma_{ann} = \frac{\sigma_0}{v}$

Weakly Interacting Massive Particles

density after freeze-out: $$n_X(T_f) = \frac{T_f^2}{M_{Pl}^* \sigma_0}$$

present density: $$n_X(T_0) = \left(\frac{a(T_f)}{a(T_0)}\right)^3 n_X(T_f) = \left(\frac{s_0}{s(T_f)}\right) n_X(T_f) \propto \frac{1}{T_f}$$

$X + \bar{X}$ contribution to critical density:

$$\Omega_X = 2\frac{M_X n_X(T_0)}{\rho_c} = 7.6 \frac{s_0 \ln\left(\frac{g_X M_{Pl}^* M_X \sigma_0}{(2\pi)^{3/2}}\right)}{\rho_c \sigma_0 M_{Pl} \sqrt{g_*(T_f)}}$$

$$= 0.1 \cdot \left(\frac{(10\ \text{TeV})^{-2}}{\sigma_0}\right) \frac{10}{\sqrt{g_*(T_f)}} \ln\left(\frac{g_X M_{Pl}^* M_X \sigma_0}{(2\pi)^{3/2}}\right) \cdot \frac{1}{2h^2}$$

WIMPs: discussion

$$\Omega_X = 0.1 \cdot \left(\frac{(10\ \text{TeV})^{-2}}{\sigma_0} \right) \frac{10}{\sqrt{g_*(T_f)}} \ln\left(\frac{g_X M_{Pl}^* M_X \sigma_0}{(2\pi)^{3/2}} \right) \cdot \frac{1}{2h^2}$$

- natural DM: subweak-scale cross section $\sigma_0 \sim 0.01 \times \sigma_W$
  say, $M_X \sim 1$ TeV or X is not a weak gauge eigenstate
- naturaly "light" unitarity $\sigma_0 \lesssim \frac{4\pi}{M_X^2} \longrightarrow M_X \lesssim 100$ TeV
- all stable particles with smaller σ_0 are forbidden !!
- WIMPs remain in kinetic equilibrium with plasma till $T \sim 10$ MeV

  this is Cold Dark Matter, $v_{RD/MD} \ll 10^{-3}$

  WIMPs may form dark halos (clumps) much lighter than dwarf galaxies

Weakly IMPs are mostly welcome (e.g. LSP in SUSY)

We can fully explore the model !! lectures by S.Demidov

- Direct searches for Galactic Dark Matter ($v \sim 10^{-3}$)

$$X + \text{nuclei} \to X + \text{nuclei} + \Delta E$$

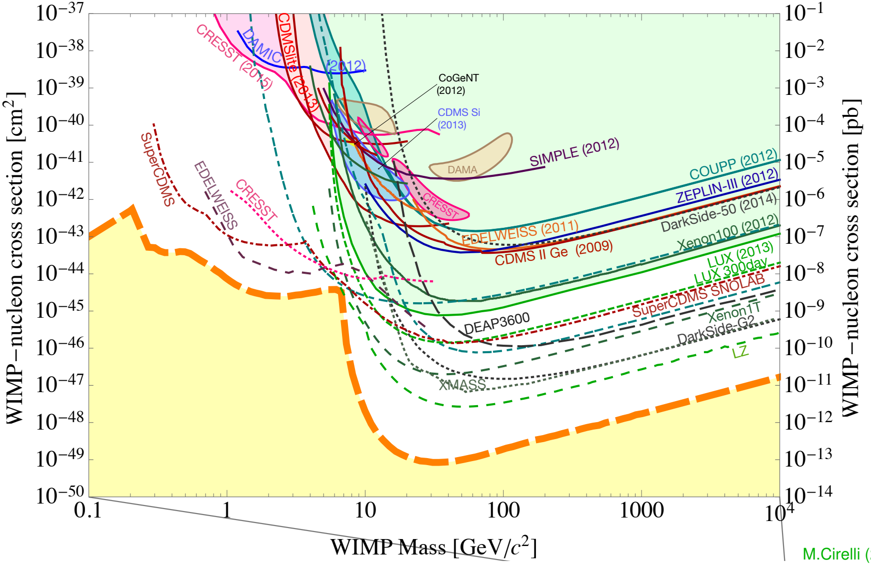
- Can search for WIMPs in cosmic rays: products of WIMPs annihilation (in Galactic center, dwarf galaxies, Sun)

$$X + \bar{X} \to p\bar{p}\,,\;\; e^+e^-\,,\;\; \nu, \gamma, \dots$$

- Can search for WIMPs in collision experiments (LHC):

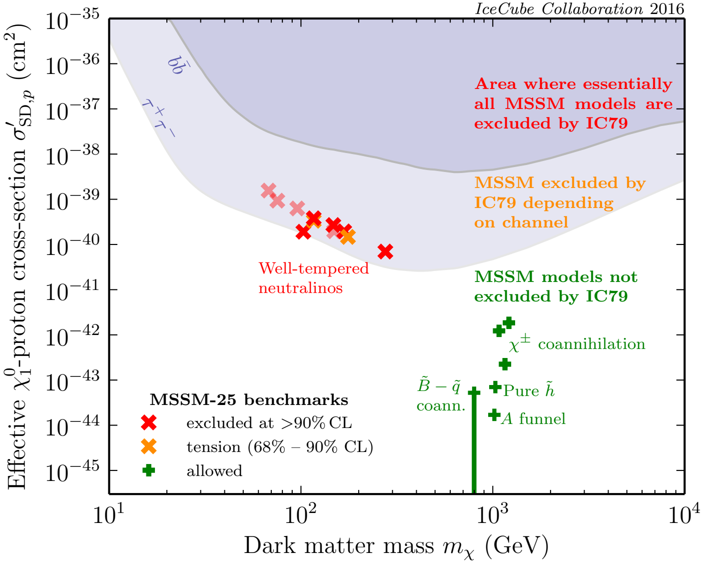
$$X + \bar{X} \leftrightarrow \text{SM} + \text{SM'} + \dots$$

Prospects in WIMP searches



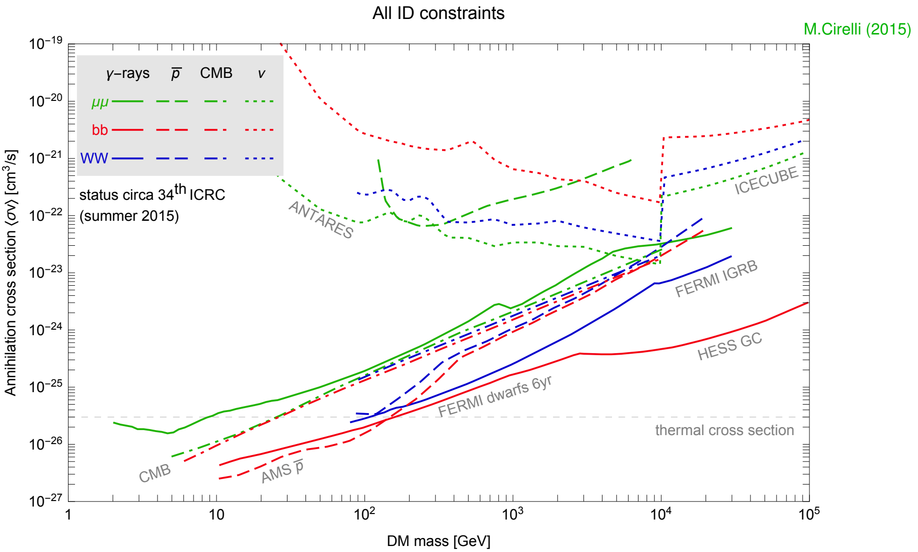
M.Cirelli (2015)

Constraining the DM model parameter space



M.G. Aartsen at al (2016)

Present indirect limits on DM annihilation (clumps..)



LHC limits for annihilation

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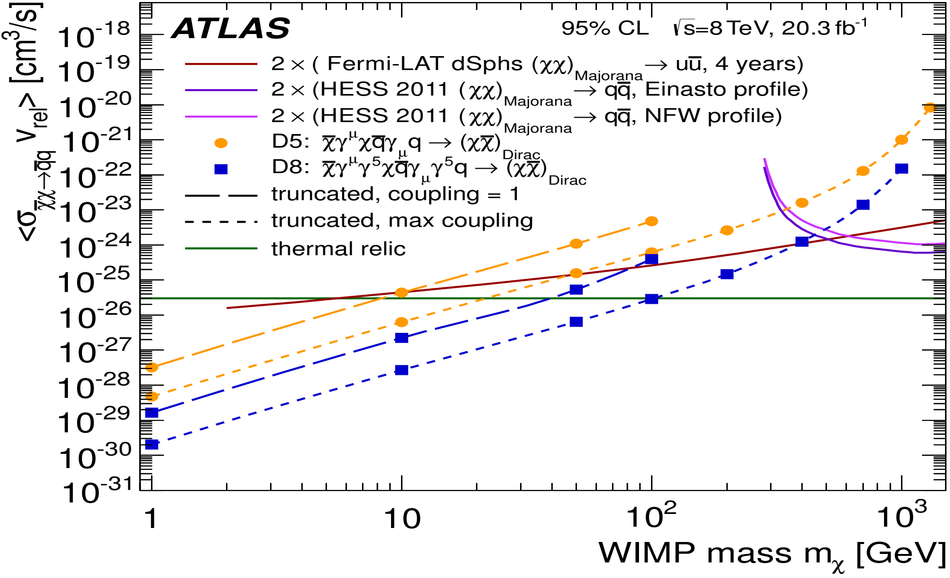

If thermal CDM but not Weakly IMPs?

We still can study the model if DM annihilates (partly) into SM particles

- But DM particle X can be light and feebly coupled (t-channel)

$$\sigma_0 \sim \frac{\xi^4}{M_X^2}$$

ξ is not a gauge coupling within GUT !

- With small σ_0 one needs entropy production
- σ_0 may be increased by s-channel resonance, $M_Y \approx 2M_X$
- annihilation can be amplified by co-annihilation channels, $X + A \to SM$
- With light messangers between Dark and Visible sectors many estimates change, say $\sigma_0 = \sigma_0(v)$
- DM interaction at freeze-out and now are not the same
  say, Sommerfield enhancement of the annihilation of slow particles $v \sim 10^{-3}$

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Dark Matter: non-thermal production

1. in the primordial plasma of SM particles (via scatterings, oscillations):
   gravitino
   sterile neutrino of 1-50 keV
2. at phase transitions:
   axion of $10^{-4} - 10^{-7}$ eV
   Q-balls
   strangelets (?)
3. during reheating (after inflation?): black holes
   any guy coupled (only) to inflaton
   - perturbatively: inflaton decays
     production by external (inflaton) field
   - non-perturbatively: Bose-enhancement of
     coherent production by external field
4. while the Universe expands:
   gravity produces any particles at $H \sim M_X$

Gravitino production

$$\mathscr{L} = \frac{1}{F}\partial^\mu \psi \cdot J_\mu^{SUSY} \,, \quad \tilde{G}_\mu \to \tilde{G}_\mu + i\sqrt{4\pi}\frac{M_{Pl}}{F}\partial_\mu \psi$$

$$m_{3/2} = \sqrt{\frac{8\pi}{3}}\frac{F}{M_{Pl}} \quad \longleftrightarrow \quad \Lambda = 0$$

$$1\text{ TeV} \lesssim \sqrt{F} \lesssim M_{Pl} \,, \quad 2\cdot 10^{-4}\text{ eV} \lesssim m_{3/2} \lesssim M_{Pl}$$

LSP in low scale SUSY breaking models

$$2\cdot 10^{-4}\text{ eV} \lesssim m_{3/2} \lesssim 100\text{ GeV} \longrightarrow \sqrt{F} \lesssim 10^{10}\text{ GeV}$$

Thermal equilibrium is forbidden (fermion; would be hot DM):

$$\Omega_{3/2} = \frac{m_{3/2}\cdot n_{3/2}}{\rho_c} = 0.2\left(\frac{m_{3/2}}{200\,\text{eV}}\right)\left(\frac{g_{3/2}}{2}\right)\left(\frac{210}{g_*(T_d)}\right)\frac{1}{2h^2}$$

Gravitino production in scatterings and decays

$$\tilde{X}_i \to \tilde{G} + X_i\,, \quad X_i + X_j \to \tilde{X}_k + \tilde{G}$$

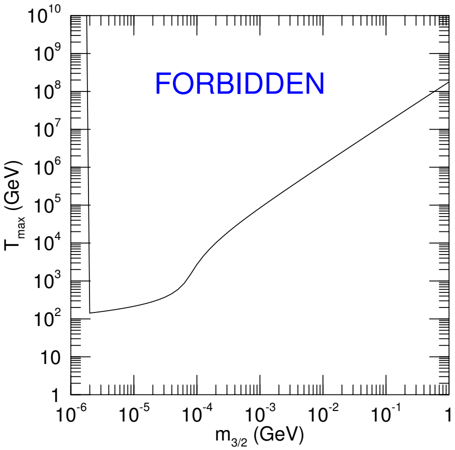
$$\Gamma \propto \frac{1}{F^2} \propto \frac{1}{m_{3/2}^2}\,, \qquad \sigma \propto \frac{1}{F^2} \propto \frac{1}{m_{3/2}^2}$$

$$\frac{dn_{3/2}}{dt} + 3Hn_{3/2} = \sum_i \Gamma_{\tilde{X}_i} \cdot \gamma_i^{-1} \cdot n_{\tilde{X}_i} + \sum_{i,j} \langle \sigma_{ij} \rangle \cdot n_{X_i} n_{X_j}\,,$$

$$\frac{d}{dT}\left(\frac{n_{3/2}}{s}\right) = -\sum_i \Gamma_{\tilde{X}_i} \cdot \frac{n_{\tilde{X}_i}}{\gamma_i \cdot sHT} - \sum_{i,j} \frac{\langle \sigma_{ij} \rangle \cdot n_{X_i} n_{X_j}}{sHT}\,,$$

$$\propto \frac{1}{T^3} \qquad\qquad \propto \text{const}$$

$$\Omega_{3/2} \sim \left(\frac{200\ \text{keV}}{m_{3/2}}\right) \cdot \left(\frac{T_{max}}{10\ \text{TeV}}\right) \times \left(\frac{M_S}{1\ \text{TeV}}\right)^2 \cdot \left(\frac{15}{\sqrt{g_*(T_{max})}}\right) \cdot \frac{1}{2h^2}$$



...issues of QFT description

of a gauge theory at finite temperature

Outcome depends on initial conditions !!!

Axion: Natural but fine-tuned

Theory and Nature:

$$\Delta\mathscr{L} \propto \theta G_{\mu\nu} G_{\lambda\rho} \varepsilon^{\mu\nu\lambda\rho}$$

$$\theta < 10^{-9}$$

nonantropic parameter!

$$\theta \to \theta + a(x)/f_a$$

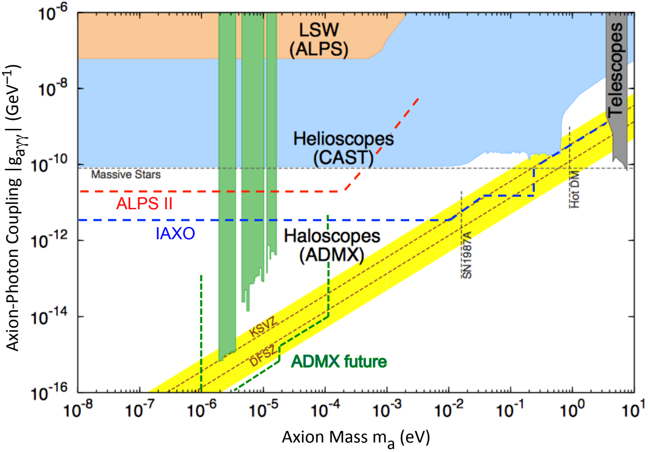
$$m_{\text{axion}} \simeq f_\pi m_\pi / f_a$$

$$\mathscr{L} \propto g_{a\gamma\gamma} \times a(x) F_{\mu\nu} F^{\mu\nu}$$

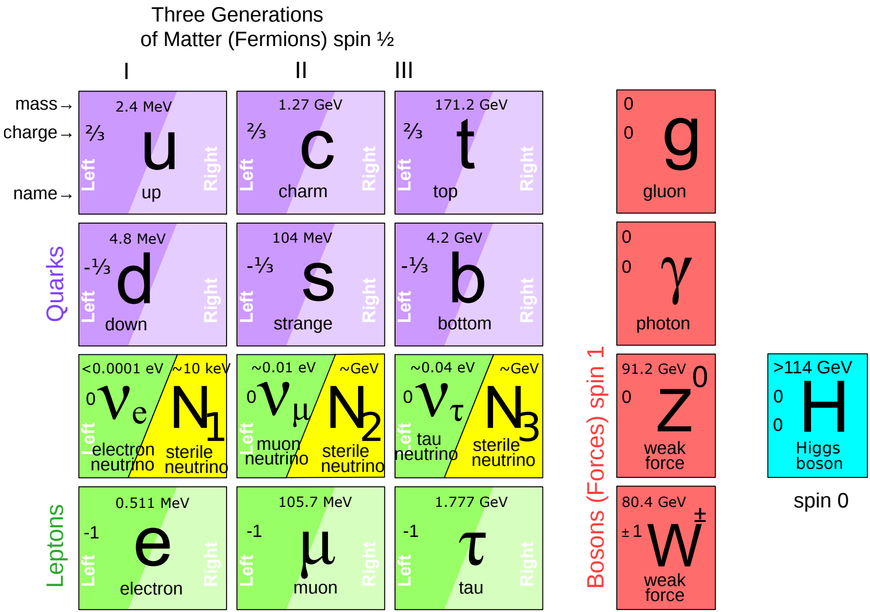
Dark Matter region

$$\frac{\Omega_{\text{axion}}}{\Omega_{DM}} = \bar{\theta}_i^2 \cdot \left(\frac{4 \cdot 10^{-6}\,\text{eV}}{m_{\text{axion}}}\right)^{1.2}$$

$\hat{O}_{n+4}/M_{Pl}^n$ make axion superheavy



P.W. Graham et al (2016)


Three Generations
of Matter (Fermions) spin ½
I
II
III
mass →
charge →
name →
2.4 MeV
⅔
u
up
1.27 GeV
⅔
c
charm
171.2 GeV
⅔
t
top
Left
Right
Quarks
4.8 MeV
-⅓
d
down
104 MeV
-⅓
s
strange
4.2 GeV
-⅓
b
bottom
<0.0001 eV
0
νe
electron neutrino
~10 keV
N1
sterile neutrino
~0.01 eV
0
νμ
muon neutrino
~GeV
N2
sterile neutrino
~0.04 eV
0
ντ
tau neutrino
~GeV
N3
sterile neutrino
Leptons
0.511 MeV
-1
e
electron
105.7 MeV
-1
μ
muon
1.777 GeV
-1
τ
tau
0
0
g
gluon
0
0
γ
photon
91.2 GeV
0
Z0
weak force
80.4 GeV
±1
W±
weak force
Bosons (Forces) spin 1
>114 GeV
0
0
H
Higgs boson
spin 0

Seesaw mechanism: $M_N \gg 1\,\mathrm{eV}$

With $m_{active} \lesssim 1\,\mathrm{eV}$ we work in the seesaw (type I) regime:

$$\mathcal{L}_N = \overline{N} i \partial\!\!\!/ N - f \overline{L}_e^c \tilde{H} N - \frac{M_N}{2} \overline{N}^c N + \text{h.c.}$$

Higgs gains $\langle H \rangle = v/\sqrt{2}$ and then

$$\mathscr{V}_N = \frac{1}{2} \left(\overline{\nu}_e , \overline{N}^c \right) \begin{pmatrix} 0 & v \frac{f}{\sqrt{2}} \\ v \frac{f}{\sqrt{2}} & M_N \end{pmatrix} \begin{pmatrix} \nu_e \\ N \end{pmatrix} + \text{h.c.}$$

For a hierarchy $M_N \gg M^D = v \frac{f}{\sqrt{2}}$ we have

flavor state $\nu_e = U \nu_1 + \theta\, N$ with $U \approx 1$ and

active-sterile mixing: $$\theta = \frac{M^D}{M_N} = \frac{v f}{2 M_N} \ll 1$$

and mass eigenvalues

$$\approx M_N \quad \text{and} \quad -m_{active} = \theta^2 M_N \lll M_N$$

Sterile neutrino: well-motivated keV-mass Dark Matter

- massive fermions giving mass to active neutrino through mixing (seesaw)

$$m_a \sim \frac{f^2 v^2}{M_N^2} M_N \sim \theta^2 M_N$$

- unstable, $N \to \nu\nu\nu$ is always open
  but exceeding the age of the Universe if

  (applicable for $M_N < M_W$)

$$\theta^2 < 1.5 \times 10^{-7} \left(\frac{50\,\text{keV}}{M_N}\right)^5$$

- with seesaw constraint $m_a \sim \theta^2 M_N$

$$\tau_{N \to 3\nu} \sim 1/\left(G_F^2 M_N^5 \theta_{\alpha N}^2\right) \sim 1/\left(G_F^2 M_N^4 m_\nu\right) \sim 10^{11}\,\text{yr}\,(10\,\text{keV}/M_N)^4$$

Sterile neutrino: indirect searches

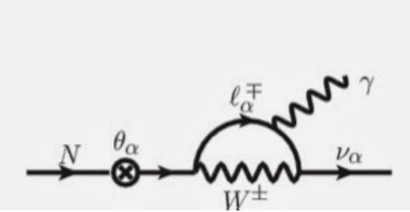
$$m_a \sim \frac{f^2 v^2}{M_N^2} M_N \sim \theta^2 M_N$$

- unstable, but exceeding the age of the Universe if

$$\frac{\theta^2}{3 \times 10^{-3}} < \left(\frac{10\,\text{keV}}{M_N}\right)^5$$

- DM sterile neutrinos can be searched at X-ray telescopes because of two-body radiative decay

give limits in absence of the feature

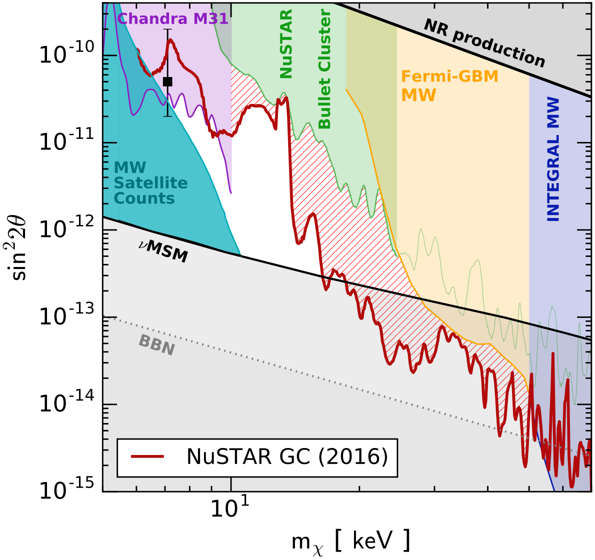


a narrow line $(\delta E_\gamma / E_\gamma \sim v \sim 10^{-3})$
at photon frequency $E_\gamma = M_N/2$

$$\frac{\theta^2}{10^{-11}} \lesssim \left(\frac{10\text{ keV}}{M_N}\right)^4$$

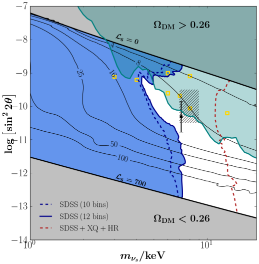
. . . present searches

1609.00667, 1706.03118

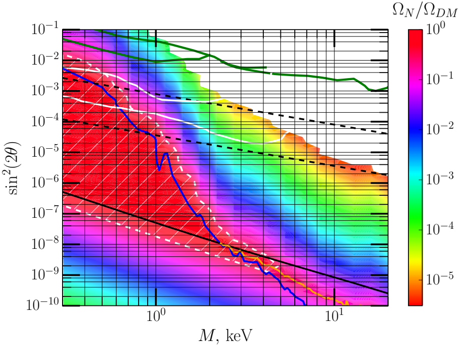


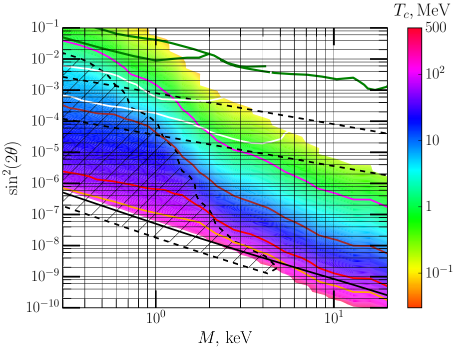
- upper limits on mixing: from X-ray searches
- lower limits on mass: from structure formation with $p_N \sim T$, DM free streaming

  too fast at $T = 1$ eV



Results for details see 1705.02184





Important:

$m_a \sim \theta^2 M_N$

1. seesaw light sterile neutrino (dashed lines)
2. can be directly tested !! (green and white lines)

Outline

1 Dark Matter properties

2 Thermal Dark Matter

3 Non-thermal candidates

4 Summary

Summary (I)

1. We need DM both in past (cosmology)
and at present (astrophysics)
2. For stability a symmetry is needed
3. There are claimed discrepancies between CDM simulations and observations of small scale structures, observations of central regions of dwarf galaxies
4. WDM? selfinteracting DM? no proof
5. Structures: DM cannot be hot (e.g. SM neutrinos can not help)
6. WIMPs (neutralino) are natural candidates for Cold Dark Matter
7. Much more options for WIMP-like candidates. . .
8. Generally, heavy and/or feebly coupled thermal relics
are forbidden !!