INSTITUTE FOR RESEARCH IN FUNDAMENTAL SCIENCES
Tehran, Iran

NORTH-WEST UNIVERSITY
YUNIBESITI YA BOKONE-BOPHIRIMA
NOORDWES-UNIVERSITEIT

Finding a remedy to justify the behaviour of accretion disk and jests in perturbed black holes?

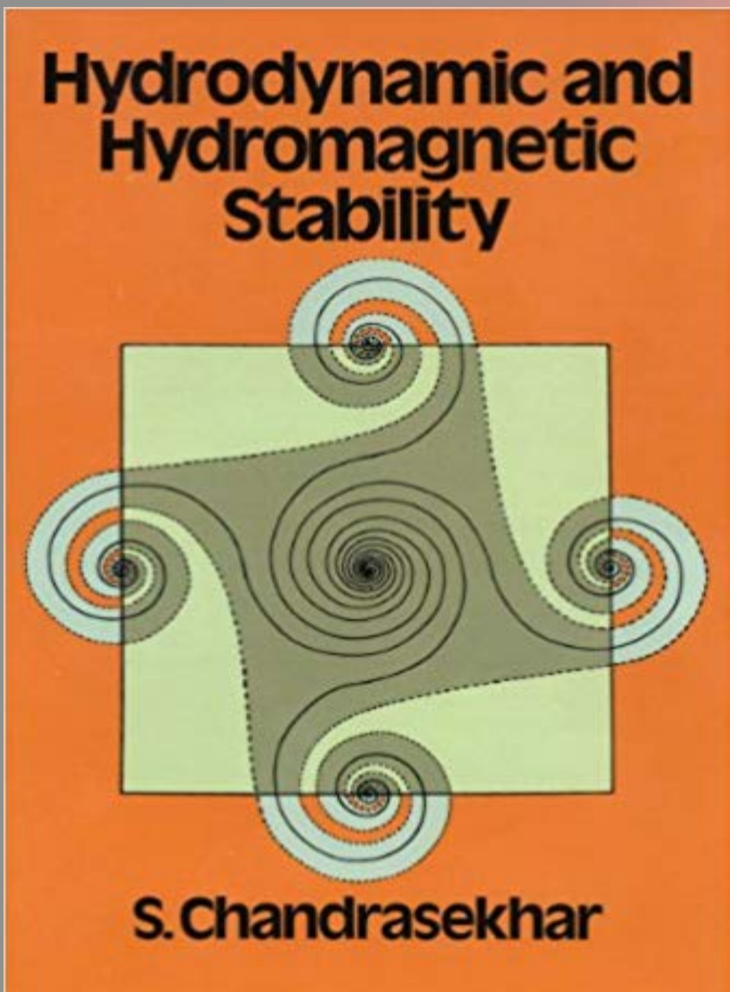
Haidar Sheikhahmadi

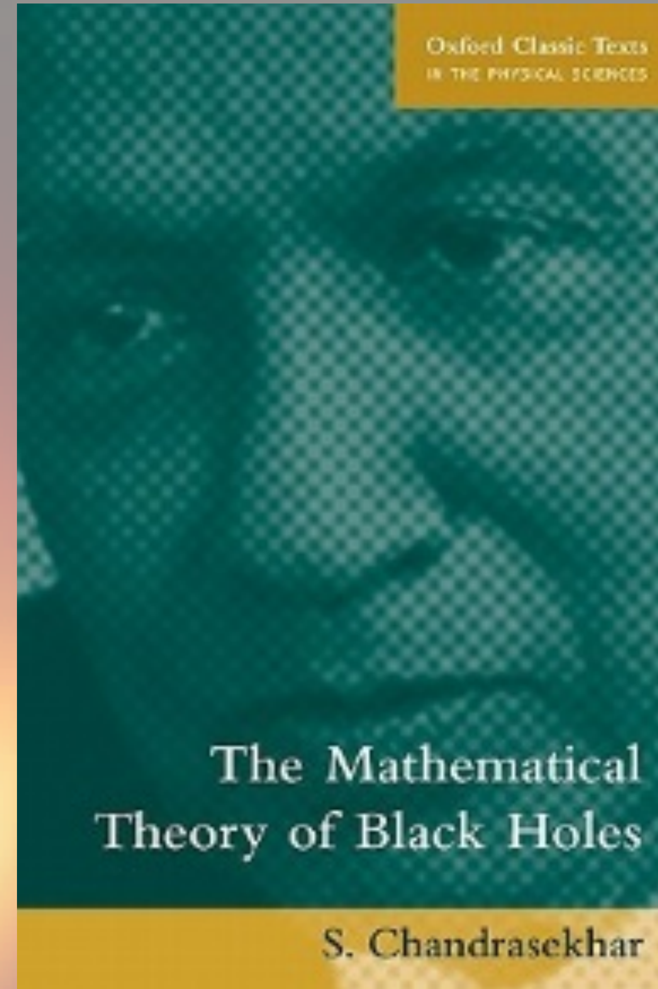
 # Center for Space Research, North-West University, South Africa
&
 # School of Astronomy, Institute for Research in Fundamental Sciences (IPM), Tehran, Iran

09/08/19

4-17/Aug /19

Helmholtz International School
"Cosmology, Strings, New Physics"


Hydrodynamic and Hydromagnetic Stability
S. Chandrasekhar


Oxford Classic Texts
IN THE PHYSICAL SCIENCES
The Mathematical Theory of Black Holes
S. Chandrasekhar

Proceedings of the

NATIONAL ACADEMY OF SCIENCES

Volume 42 · Number 1 · January 15, 1956

ON FORCE-FREE MAGNETIC FIELDS

BY S. CHANDRASEKHAR

UNIVERSITY OF CHICAGO

Communicated November 10, 1955

1. *Introduction.*—Lüst and Schlüter[1] have recently pointed out that cosmic magnetic fields might often satisfy the condition

R. lüst and A. Schlütter, Z. Astrophys., 34, 263, (1954).
S. Chandrasekhar and Donna Elbert, Proc. Cambridge Phil. Soc., 49, 446, (1953).
P. Goldreich and W. H. Julian, The Astrophysical Journal 157, 869 (1969). (Quasarz)
R. D. Blandford and R. L. Znajek, Monthly Notices of the Royal Astronomical Society 179 433 (1977).(AGNs)

Force-Field-Electrodynamic properties (in flat space-time)

The Navier-Stokes equation for a plasma $-\nabla p + \mathbf{j}\times\mathbf{B} = 0$ with $p \ll B^2/2\mu$

$$\mathbf{j}\times\mathbf{B} = 0$$

Which implies

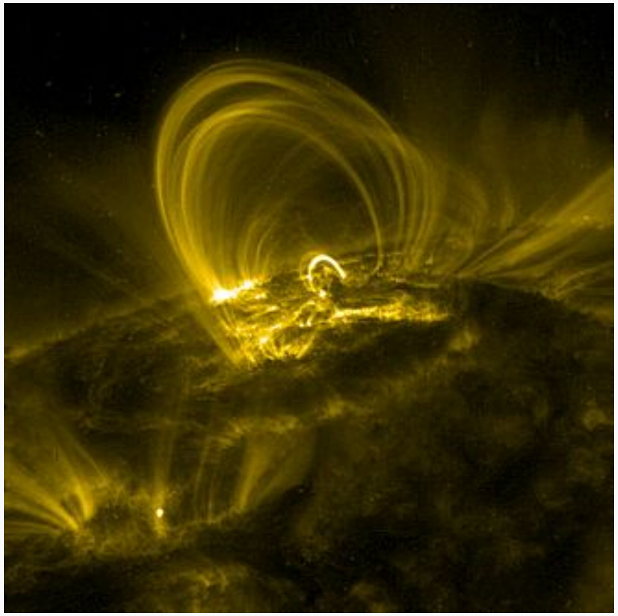
$$\mathbf{j} = \alpha\mathbf{B}$$

Combining this equation with Maxwell equations

$$\nabla\times\mathbf{B} = \mu\mathbf{j}$$
$$\nabla\cdot\mathbf{B} = 0$$

and

$$\nabla\cdot(\nabla\times\mathbf{B}) = 0$$



One immediately has

$$\mathbf{B}\cdot\nabla\alpha = 0,$$
$$\nabla\times\mathbf{B} = \alpha\mathbf{B}$$

$$-\nabla^2\mathbf{B} = \alpha^2\mathbf{B}$$

Constant ← α → Non-constant

$$\nabla^2\mathbf{B} + \alpha^2\mathbf{B} = \mathbf{B}\times\nabla\alpha$$
$$\mathbf{B}\cdot\nabla\alpha = 0$$

null tetrads for Minkowski spherical polar coordinate

$$ds^2 = dt^2 - dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\varphi^2$$

$$l_a = \frac{1}{\sqrt{2}}(1,1,0,0); n_a = \frac{1}{\sqrt{2}}(1,-1,0,0)$$

$$m_a = \frac{1}{\sqrt{2}}(0,0,r,\ ir\sin\theta); \bar{m}_a = \frac{1}{\sqrt{2}}(0,0,r,\ -ir\sin\theta)$$

$$\omega^{\hat{a}} = dx^a$$

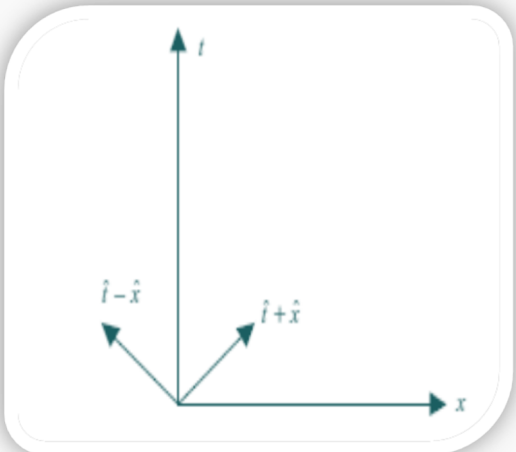
$$l = \frac{\omega^{\hat{t}} + \omega^{\hat{r}}}{\sqrt{2}} = \frac{dt + dr}{\sqrt{2}}, n = \frac{\omega^{\hat{t}} - \omega^{\hat{r}}}{\sqrt{2}}$$

$$m = \frac{\omega^{\hat{\theta}} + i\omega^{\hat{\varphi}}}{\sqrt{2}} = \frac{rd\theta + ir\sin\theta d\varphi}{\sqrt{2}}, \bar{m} = \frac{\omega^{\hat{\theta}} - i\omega^{\hat{\varphi}}}{\sqrt{2}}$$

$$g_{ab} = l_a n_b + l_b n_a - m_a\bar{m}_b - \bar{m}_b m_a;\ l^a = g^{ab} l_b$$

$$g_{\varphi\varphi} = -2\frac{1}{\sqrt{2}}(ir\sin\theta)\frac{1}{\sqrt{2}}(-ir\sin\theta) = r^2\sin^2\theta$$

$$l^a l_a = n^a n_a = m^a m_a = \bar{m}^a \bar{m}_a = l^a m_a$$
$$= n^a m_a = l^a \bar{m}_a = n^a \bar{m}_a = 0,$$
$$l^a n_a = -m^a \bar{m}_a = 1.$$



Covariant and non-stationary Rissener-Nordeström metric

$$dS^2 = e^{2\nu}(dt)^2 - e^{2\psi}(d\varphi - \omega dt - q_2 dx_2 - q_3 dx_3)^2 - e^{2\mu_2}(dx_2)^2 - e^{2\mu_3}(dx_3)^2$$

$$x_o = t,\ x_1 = \varphi,\ x_2 = r,\ \ x_3 = \theta$$

Useful definitions for null tetrad and spin coefficients and so

$$\lambda_i^a = (l^a, n^a, m^a, \bar{m}^a),$$
$$\nabla_i \equiv \lambda_i^b \nabla_b = (D, \Delta, \delta, \bar{\delta}).$$

$$\eta_{ij} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{bmatrix}.$$

$$\gamma^i{}_{jk} = \lambda_j^b \lambda_k^a \nabla_a \lambda_b^i, \; \gamma_{ijk} = -\gamma_{jik} = \eta_{il}\gamma^l{}_{jk}$$
$$\nabla_{[a}\nabla_{b]}\lambda_c^i = \frac{1}{2} R_{abc}{}^d \lambda_d^i.$$

12 Spin Coef.

$$\pi = -\gamma_{241}, \quad \epsilon = \frac{1}{2}(\gamma_{121} - \gamma_{341}), \quad \kappa = \gamma_{131}$$
$$\lambda = -\gamma_{244}, \quad \alpha = \frac{1}{2}(\gamma_{124} - \gamma_{344}), \quad \rho = \gamma_{134}$$
$$\mu = -\gamma_{243}, \quad \beta = \frac{1}{2}(\gamma_{123} - \gamma_{343}), \quad \sigma = \gamma_{133}$$
$$\nu = -\gamma_{242}, \quad \gamma = \frac{1}{2}(\gamma_{122} - \gamma_{342}), \quad \tau = \gamma_{132}$$

Weyl scalars

$$C_{ab}{}^{cd} = R_{ab}{}^{cd} - 2R_{[a}{}^{[c} g_{b]}{}^{d]} + \frac{1}{3} R g_{[a}{}^{c} g_{b]}{}^{d}.$$
$$\Psi_0 = -C_{abcd} l^a m^b l^c m^d = -C_{1313},$$
$$\Psi_1 = -C_{abcd} l^a n^b l^c m^d = -C_{1213},$$
$$\Psi_2 = -C_{abcd} l^a m^b \bar{m}^c n^d = -C_{1342},$$
$$\Psi_3 = -C_{abcd} l^a n^b \bar{m}^c n^d = -C_{1242},$$
$$\Psi_4 = -C_{abcd} \bar{m}^a n^b \bar{m}^c n^d = -C_{4242}.$$

E. Newman and R. Penrose, Journal of Mathematical Physics 3, 566 (1962).

Oh my goodness, what are these Martian mathematics?!!

Metric coefficients

$$\Delta l^a - Dn^a = (\gamma+\bar\gamma)l^a + (\epsilon+\bar\epsilon)n^a - (\tau+\bar\pi)\bar m^a - (\bar\tau+\pi)m^a,$$
$$\delta l^a - Dm^a = (\bar\alpha+\beta-\bar\pi)l^a + \kappa n^a - \sigma\bar m^a - (\bar\rho+\epsilon-\bar\epsilon)m^a,$$
$$\delta n^a - \Delta m^a = -\bar\nu l^a + (\tau-\bar\alpha-\beta)n^a + \bar\lambda\bar m^a + (\mu-\gamma+\bar\gamma)m^a,$$
$$\bar\delta m^a - \delta\bar m^a = (\bar\mu-\mu)l^a + (\bar\rho-\rho)n^a - (\bar\alpha-\beta)\bar m^a + (\alpha-\bar\beta)m^a.$$

Bianchi identities

$$\bar\delta\Psi_0 - D\Psi_1 = (4\alpha-\pi)\Psi_0 - 2(2\rho+\epsilon)\Psi_1 + 3\kappa\Psi_2,$$
$$\bar\delta\Psi_1 - D\Psi_2 = \lambda\Psi_0 + 2(\alpha-\pi)\Psi_1 - 3\rho\Psi_2 + 2\kappa\Psi_3,$$
$$\bar\delta\Psi_2 - D\Psi_3 = 2\lambda\Psi_1 - 3\pi\Psi_2 + 2(\epsilon-\rho)\Psi_3 + \kappa\Psi_4,$$
$$\bar\delta\Psi_3 - D\Psi_4 = 3\lambda\Psi_2 - 2(\alpha+2\pi)\Psi_3 + (4\epsilon-\rho)\Psi_4,$$
$$\Delta\Psi_0 - \delta\Psi_1 = (4\gamma-\mu)\Psi_0 - 2(2\tau+\beta)\Psi_1 + 3\sigma\Psi_2,$$
$$\Delta\Psi_1 - \delta\Psi_2 = \nu\Psi_0 + 2(\gamma-\mu)\Psi_1 - 3\tau\Psi_2 + 2\sigma\Psi_3,$$
$$\Delta\Psi_2 - \delta\Psi_3 = 2\nu\Psi_1 - 3\mu\Psi_2 + 2(\beta-\tau)\Psi_3 + \sigma\Psi_4,$$
$$\Delta\Psi_3 - \delta\Psi_4 = 3\nu\Psi_2 - 2(\gamma+2\mu)\Psi_3 + (4\beta-\tau)\Psi_4.$$

Commutation for *spin coefficients*

$$\Delta\lambda - \bar\delta\nu = -(\mu+\bar\mu+3\gamma-\bar\gamma)\lambda + (3\alpha+\bar\beta+\pi-\bar\tau)\nu - \Psi_4$$
$$\delta\rho - \bar\delta\sigma = \rho(\bar\alpha+\beta) - \sigma(3\alpha-\bar\beta) + (\rho-\bar\rho)\tau + (\mu-\bar\mu)\kappa - \Psi_1$$
$$\delta\alpha - \bar\delta\beta = \mu\rho - \lambda\sigma + \alpha\bar\alpha + \beta\bar\beta - 2\alpha\beta + \gamma(\rho-\bar\rho) + \epsilon(\mu-\bar\mu) - \Psi_2$$
$$\delta\lambda - \bar\delta\mu = (\rho-\bar\rho)\nu + (\mu-\bar\mu)\pi + \mu(\alpha+\bar\beta) + \lambda(\bar\alpha-3\beta) - \Psi_3$$
$$\delta\nu - \Delta\mu = \mu^2 + \lambda\bar\lambda + \mu(\gamma+\bar\gamma) - \bar\nu\pi + \nu(\tau-3\beta-\bar\alpha)$$
$$\delta\gamma - \Delta\beta = \gamma(\tau-\bar\alpha-\beta) + \mu\tau - \sigma\nu - \epsilon\bar\nu - \beta(\gamma-\bar\gamma-\mu) + \alpha\bar\lambda$$
$$\delta\tau - \Delta\sigma = \mu\sigma + \rho\bar\lambda + \tau(\tau+\beta-\bar\alpha) - \sigma(3\gamma-\bar\gamma) - \kappa\bar\nu$$
$$\Delta\rho - \bar\delta\tau = -(\rho\bar\mu+\sigma\lambda) + \tau(\bar\beta-\alpha-\bar\tau) + (\gamma+\bar\gamma)\rho + \kappa\nu - \Psi_2$$
$$\Delta\alpha - \bar\delta\gamma = \nu(\rho+\epsilon) - \lambda(\tau+\beta) + \alpha(\bar\gamma-\bar\mu) + \gamma(\bar\beta-\bar\tau) - \Psi_3$$
$$D\rho - \bar\delta\kappa = \rho^2 + \sigma\bar\sigma + (\epsilon+\bar\epsilon)\rho - \bar\kappa\tau - \kappa(3\alpha+\bar\beta-\pi)$$
$$D\sigma - \delta\kappa = (\rho+\bar\rho)\sigma + (3\epsilon-\bar\epsilon)\sigma - (\tau-\bar\pi+\bar\alpha+3\beta)\kappa + \Psi_0$$
$$D\tau - \Delta\kappa = (\tau+\bar\pi)\rho + (\bar\tau+\pi)\sigma + (\epsilon-\bar\epsilon)\tau - (3\gamma+\bar\gamma)\kappa + \Psi_1$$
$$D\alpha - \bar\delta\epsilon = (\rho+\bar\epsilon-2\epsilon)\alpha + \beta\bar\sigma - \bar\beta\epsilon - \kappa\lambda - \bar\kappa\gamma + (\epsilon+\rho)\pi$$
$$D\beta - \delta\epsilon = (\alpha+\pi)\sigma + (\bar\rho-\bar\epsilon)\beta - (\mu+\gamma)\kappa - (\bar\alpha-\bar\pi)\epsilon + \Psi_1$$
$$D\gamma - \Delta\epsilon = (\tau+\bar\pi)\alpha + (\bar\tau+\pi)\beta - (\epsilon+\bar\epsilon)\gamma - (\gamma+\bar\gamma)\epsilon + \tau\pi - \nu\kappa + \Psi_2$$
$$D\lambda - \bar\delta\pi = \rho\lambda + \bar\sigma\mu + \pi^2 + (\alpha-\bar\beta)\pi - \nu\bar\kappa - (3\epsilon-\bar\epsilon)\lambda$$
$$D\mu - \delta\pi = \bar\rho\mu + \sigma\lambda + \pi\bar\pi - (\epsilon+\bar\epsilon)\mu - \pi(\bar\alpha-\beta) - \nu\kappa + \Psi_2$$
$$D\nu - \Delta\pi = (\bar\tau+\pi)\mu + (\tau+\bar\pi)\lambda + (\gamma-\bar\gamma)\pi - (3\epsilon+\bar\epsilon)\nu + \Psi_3$$

Null tetrads for RN metric

$$l^p = (l^t, l^r, l^\theta, l^\varphi) = (\frac{r^2}{\Delta}, 1, 0, 0),$$

$$n^p = (n^t, n^r, n^\theta, n^\varphi) = (1, -\frac{r^{-2}}{2\Delta^{-1}}, 0, 0),$$

$$m^p = (m^t, m^r, m^\theta, m^\varphi) = (0, 0, -\frac{r^{-1}}{\sqrt{2}}, \frac{ir^{-1}}{\sqrt{2}}\csc\theta).$$

$$l \cdot n = 0, \ \ m \cdot \bar{m} = -1$$

\+

Force Free Field equations

$$\nabla_a F^{ba} = 4\pi J^a$$

$$\nabla_{[a} F_{bc]} = 0; \ F_{ab} J^b = 0$$

$$\Delta = r^2 - 2Mr + Q^2 = r^2 e^{2\nu} = r^2 e^{-2\mu_2}$$

Maxwell equations

$$(D - 2\rho)\phi_1 - (\delta^* + \pi - 2\alpha)\phi_0 + \kappa\phi_2 = 0,$$
$$(\delta - 2\tau)\phi_1 - (\Delta + \mu - 2\gamma)\phi_0 + \sigma\phi_2 = 0,$$
$$(\delta^* + 2\pi)\phi_1 - (D - \rho + 2\varepsilon)\phi_2 - \lambda\phi_0 = 0,$$
$$(\Delta + 2\mu)\phi_1 - (\delta - \tau + 2\beta)\phi_2 - \nu\phi_0 = 0.$$

Maxwell's scalars

$$2\phi_1 = F_{pq}(l^p n^q + m^p \bar{m}^q)$$

$$\phi_0 = F_{ab} l^a m^b = \frac{e^{-\upsilon}}{\sqrt{2}}[i(F_{01} + F_{21}) + (F_{03} + F_{23})]$$

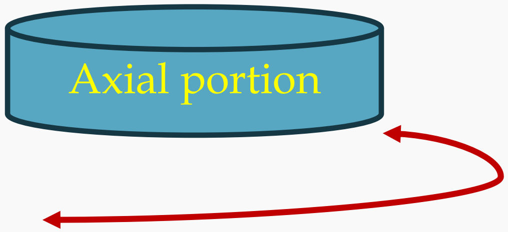
$$\phi_2 = F_{ab} \bar{m}^a m^b = \frac{e^{\upsilon}}{2\sqrt{2}}[i(F_{01} + F_{21}) + (F_{03} + F_{23})]$$

Some of Linearized Maxwell equations

$$(e^{\psi+\mu_2}F_{12})_{,3}+(e^{\psi+\mu_3}F_{31})_{,2}=0;$$

$$(e^{\mu_2+\mu_3}F_{01})_{,0}+(e^{\upsilon+\mu_3}F_{12})_{,2}+(e^{\upsilon+\mu_2}F_{13})_{,3}$$

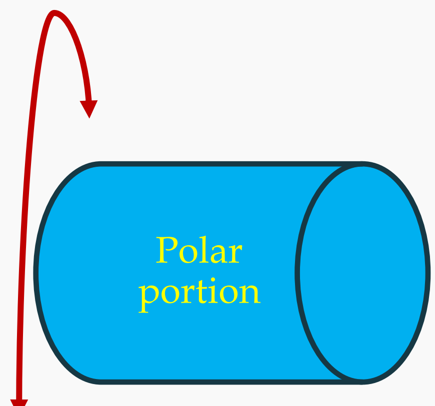
$$=e^{\psi+\mu_3}F_{02}(\omega_{,2}-q_{2,0})+e^{\psi+\mu_2}F_{13}(\omega_{,3}-q_{3,0})-e^{\psi+\nu}F_{23}(q_{2,3}-q_{3,2})$$



$$(e^{\psi+\mu_3}F_{02})_{,2}+(e^{\psi+\mu_2}F_{03})_{,3}=0;$$

$$(e^{\mu_2+\mu_3}F_{01})_{,0}+(e^{\upsilon+\mu_3}F_{12})_{,2}+(e^{\upsilon+\mu_2}F_{13})_{,3}$$

$$=e^{\psi+\upsilon}F_{01}(q_{2,3}-q_{3,2})+e^{\psi+\mu_2}F_{12}(\omega_{,3}-q_{3,0})-e^{\psi+\mu_3}F_{13}(\omega_{,2}-q_{2,0})$$



Axial perturbations:

Gegenbauer function

$$(r^2 e^{2\upsilon}\tilde{Q}_{23}\sin^3\theta)_{,3} + r^4\tilde{Q}_{02,0}\sin^3\theta = 2(r^3 e^{\upsilon}\sin^2\theta)\delta R_{12} = 4Qre^{\upsilon}F_{01}\sin^2\theta$$

$$(r^2 e^{2\upsilon}\tilde{Q}_{23}\sin^3\theta)_{,2} - r^2 e^{-2\upsilon}\tilde{Q}_{03,0}\sin^3\theta = -(2r^2\sin^2\theta)\delta R_{13} = 0$$

$$C_{l+2}^{-3/2}(\theta) = \sin^3\theta \frac{d}{d\theta}\frac{1}{\sin\theta}\frac{dP_l(\theta)}{d\theta}$$

$$C_{l+2}^{-3/2}(\theta) = \sin^2\theta(P_{l,\theta,\theta} - P_{l,\theta}\cot\theta)$$

$$\tilde{Q}(t,r,\theta) = \Delta\tilde{Q}_{23}\sin^3\theta = \Delta(q_{2,3} - q_{3,2})\sin^3\theta = \tilde{Q}(r)C_{l+2}^{-3/2}(\theta)$$

Regge-Wheeler like equation

$$\Lambda^2 Z_k^- = V^- Z_k^-; V_j^{(-)} = \frac{\Delta}{r^5}((\mu_C^2 + 2)r - q_j(1 + \frac{q_k}{\mu_C^2 r})), (j,k = 1,2, j \neq k)$$

$$q_1 = 3M + \sqrt{(3M)^2 + (2Q\mu_C)^2},$$

$$q_2 = 3M - \sqrt{(3M)^2 + (2Q\mu_C)^2},$$

$$\mu_C = 2n = (l-1)(l+2)$$

T. Reege and J. A. Wheeler, Phys. Rev. 108, 1063-9 (1957).

Some necessary definitions have appeared in formulas

$$Z_1^{(-)} = \sqrt{(q_1 - q_2)q_1}\,(H_1^{(-)}\cos\overline{\psi} + H_2^{(-)}\sin\overline{\psi}),$$
$$Z_2^{(-)} = \sqrt{(q_1 - q_2)q_1}\,(H_2^{(-)}\cos\overline{\psi} - H_1^{(-)}\sin\overline{\psi}),$$

$$\rho = -1/r, \beta = -\alpha = \frac{1}{2\sqrt{2}}\frac{\cot\theta}{r},$$
$$\mu = \frac{-\Delta}{2r^3}, \gamma = \mu + \frac{r-M}{2r^2} = \frac{Mr - Q^2}{2r^3},$$

$$B_{23} = \frac{-Q\mu_C}{r^2}H_1^{(+)} - \frac{2Q^2e^{\nu}}{r^3}\Phi$$
$$B_{03} = \frac{-Q\mu_C}{r^2}H_{1,r}^{(+)} - \frac{2Q^2e^{\nu}}{r^4\varpi}(nrH_2^{(+)} + Q\mu_C H_1^{(+)}) + \frac{2Q^2e^{-\nu}}{r^6}(2Q^2 + r^2 + 3Mr)$$

$$\sin 2\overline{\psi} = \frac{2i\sqrt{q_1q_2}}{q_1 - q_2} = \frac{2Q\mu_C}{\sqrt{(3M)^2 + (2Q\mu_C)^2}},$$

$$r_* = \int\frac{r^2}{\Delta}dr = r + \frac{r_+^2}{r_+ - r_-}\lg|r_- \times r_+| - \frac{r_-^2}{r_+ - r_-}\lg|r_- \times r_-|$$

Polar perturbations:

$$(\delta\psi+\delta\upsilon)_{,r,\theta}+(\delta\psi-\delta\mu_3)_{,r}\cot\theta+(-\frac{1}{r}+\upsilon_{,r})\delta\upsilon_{,\theta}-(\frac{1}{r}+\upsilon_{,r})\delta\mu_{2,\theta}$$
$$=-e^{\mu_2+\mu_3}\delta R_{23}=-2\frac{Qe^{-\nu}}{r}F_{03}$$

$$\delta\nu=N(r)P_l(\theta),\quad \delta\mu_2=L(r)P_l(\theta),$$
$$\delta\mu_3=(T(r)P_l(\theta)+V(r)P_{l,\theta,\theta}(\theta))$$
$$\delta\psi=(T(r)P_l(\theta)+V(r)P_{l,\theta}(\theta)\cot\theta)$$

$$e^{2\nu}[\delta\psi_{,r,r}+2(\frac{1}{r}+\upsilon_{,r})\delta\psi_{,r}+\frac{1}{r}(\delta\psi+\delta\upsilon-\delta\mu_2-\delta\mu_3)_{,r}-2(\frac{1}{r}+2\upsilon_{,r})\frac{\delta\mu_2}{r}]$$
$$\frac{1}{r^2}[\delta\psi_{,\theta,\theta}+\delta\psi_{,\theta}\cot\theta+(\delta\psi+\delta\upsilon-\delta\mu_2-\delta\mu_3)_{,\theta}\cot\theta+2\delta\mu_3]$$
$$-e^{-2\nu}\delta\psi_{,0,0}=-\delta R_{11}=2\frac{Q}{r^2}\delta F_{02}$$

$$\delta F_{02}=\frac{r^2e^{2\nu}}{2Q}B_{02}(r)P_l(\theta),$$
$$F_{03}=-\frac{re^{\nu}}{2Q}B_{03}(r)P_{l,\theta}(\theta),$$
$$F_{23}=-i\sigma\frac{re^{-\nu}}{2Q}B_{23}(r)P_{l,\theta}(\theta),$$

Zerrili like equation

$$\Lambda^2Z_k^+=V^+Z_k^+;\ V_1^{(+)}=\frac{\Delta}{r^5}(U+\frac{(q_1-q_2)}{2}W);\ V_2^{(+)}=\frac{\Delta}{r^5}(U-\frac{(q_1-q_2)}{2}W),$$

$$\Delta=r^2-2Mr+Q^2=r^2e^{2\nu}=r^2e^{-2\mu_2}$$

$$Z_1^{(+)}=q_1H_1^{(+)}+i\sqrt{q_1q_2}H_2^{(+)},$$
$$Z_2^{(+)}=-i\sqrt{q_1q_2}H_1^{(+)}+q_1H_2^{(+)}.$$

F. J. Zerrili, ibid., 2, 2141-60 (1970).
F. J. Zerrili, Phys. Rev. letters., 24, 737-8 (1970).

$$U = (2nr + 3M)W + (\varpi - nr - M) - \frac{2n\Delta}{\varpi}$$

$$W = \frac{\Delta}{r\varpi^2}(2nr + 3M) + \frac{1}{\varpi}(nr + M)$$

$$X = \frac{ne^{\nu}}{r}\Phi + \frac{n}{r}H_2^{(+)},$$

$$L = \frac{e^{\nu}}{r^3}(3Mr - 4Q^2)\Phi - \frac{(nrH_2^{(+)} + Q\mu_C H_1^{(+)})}{r^2}$$

$$N = \frac{e^{\nu}}{r^2}(M - \frac{r}{\Delta}(M^2 - Q^2 + (r^2\sigma)^2))\Phi + 2\frac{ne^{2\nu}}{\varpi}H_2^{(+)}$$
$$+ \frac{(nrH_2^{(+)} + Q\mu_C H_1^{(+)})}{r\varpi^2}\{e^{2\nu}[\varpi - 2nr - 3M] - (n+1)\varpi\}$$
$$- \frac{e^{2\nu}}{\varpi}(nrH_2^{(+)} + Q\mu_C H_1^{(+)})_{,r}$$

$$H_2^{(+)} = \frac{r}{n}X - \frac{r^2}{\varpi}(L + X - B_{23}),$$

$$H_1^{(+)} = \frac{-1}{Q\mu}(r^2 B_{23} + 2\frac{Q^2}{r}(\frac{r}{n}X - H_2^{(+)})),$$

$$\Phi = \int (nrH_2^{(+)} + Q\mu_C H_1^{(+)})\frac{e^{-\nu}}{\varpi r}dr,$$

$$\varpi = nr + 3M - \frac{2Q^2}{r}, \mu_C^2 = 2n = (l-1)(l+2)$$

$$\frac{1}{r} - \nu_{,r} = \frac{1}{r\Delta}(r^2 - 3Mr + 2Q^2),$$

$$\mu_C X_j = \mp q_j Z_i^{(\pm)} + \mu_C^2 \frac{r^4}{\Delta}(1 + \frac{q_j}{\mu_C^2 r})\Lambda_+ Z_i^{(+)}$$

Definition of Energy-Momentum tensor in curved space-time

$$T_{\mu\nu} = F_{\mu\alpha}F_{\nu}^{\alpha} - \frac{1}{4} g_{\mu\nu} F_{\theta\rho} F^{\theta\rho}$$

The contravariant orthonormal basis are as

$$\begin{aligned} e_0^a &= (e^{-\nu}, \omega e^{-\nu}, 0, 0)\,, \\ e_1^a &= (0, e^{-\psi}, 0, \ 0)\,, \\ e_2^a &= (0, q_2 e^{-\mu_2}, e^{-\mu_2}, 0)\,, \\ e_3^a &= (0, q_3 e^{-\mu_3}, 0, e^{-\mu_3}). \end{aligned}$$

Non-zero components of Energy-momentum tensors in both tetrads and curved space-time

$$\overset{t}{T}_{11} = -2\phi_0\phi_0^* ; \overset{t}{T}_{13} = -2\phi_0\phi_1^* ; \overset{t}{T}_{12} + \overset{t}{T}_{30} = -4\phi_1\phi_1^*$$

$$\overset{t}{T}_{23} = -2\phi_1\phi_2^* ; \overset{t}{T}_{22} = -2\phi_2\phi_2^* ; \overset{t}{T}_{33} = -2\phi_0\phi_2^* \ .$$

t for tetrads
B for Bend S-T

$$\overset{B}{T}_{11} = e^{2\psi} \ \overset{t}{T}_{11} \ ,$$

$$\overset{B}{T}_{13} = (e^{\psi+\mu_3} \ \overset{t}{T}_{13} - q_3 e^{2\psi} \ \overset{t}{T}_{11}) \ ,$$

$$\overset{B}{T}_{12} = (e^{\psi+\mu_2} \ \overset{t}{T}_{12} - q_2 e^{2\psi} \ \overset{t}{T}_{11}) \ ,$$

$$q_3 \ \overset{B}{T}_{10} + \overset{B}{T}_{30} = e^{\mu_3-\nu} \ \overset{t}{T}_{30} \ .$$

Weyl and
Maxwell scalars

$$-2i\sigma(q_1[q_1-q_2])^2\,\mathrm{Im}\,\Psi_0 = \frac{r^3}{\Delta^2}(Y_{+2}\cos\psi + Y_{+1}\sin\psi)\frac{C_{l+2}^{-3/2}}{\sin^2\theta}$$

$$-2i\sigma\,\mathrm{Im}\,r^4\Psi_4 = \frac{r^3\,\dfrac{C_{l+2}^{-3/2}}{\sin^2\theta}}{4(q_1[q_1-q_2])^2\Delta^2}(Y_{-2}\cos\psi + Y_{-1}\sin\psi)$$

$$\frac{2}{3}i\sigma(2q_1[q_1-q_2])^{1/2}\,\mathrm{Im}\,\phi_0 = \frac{r}{\Delta}\Lambda_+(Z_1^{(-)}\cos\psi - Z_2^{(-)}\sin\psi)P_l^\theta$$

$$2\sqrt{2}\mathrm{Re}\,\phi_0 = \mu\frac{r}{\Delta}\Lambda_+(H_1^{(+)}\cos\psi + \frac{2Q}{\mu r}\Phi)P_l^\theta$$

$$\sqrt{(2q_1[q_1-q_2])}\mathrm{Im}\phi_2 = \frac{3r}{2i\sigma\Delta}\Lambda_-(Z_1^{(-)}\cos\psi - Z_2^{(-)}\sin\psi)P_l^\theta\ ,$$

This is the first exact solution for ingoing (outgoing) solutions??

Maybe So astonishing!!

$$\sqrt{8}\Delta\pi J = (\cot\theta\partial_{,\theta} + \partial_{,\theta,\theta} + \frac{1}{\sin^2\theta}\partial_{,\varphi,\varphi})\Omega(v,\theta,\varphi) \; .$$

$$Im\,\phi_0 = \frac{-ir}{\Delta\sin\theta}\Omega_{,\phi}(v,\theta,\varphi),$$

$$Re\,\phi_0 = \frac{-r}{\Delta}\Omega_{,\theta}(v,\theta,\varphi)\;,$$

$$\Omega_{axial}(v,\theta,\varphi) = \int_{\varphi_1}^{\varphi_2}\frac{3\sin\theta}{\sigma(2q_1[q_1-q_2])^{1/2}}\times\Lambda_+(Z_1^{(-)}\cos\bar{\psi} - Z_2^{(-)}\sin\bar{\psi})P_{l,\theta}d\varphi\;,$$

$$\Omega_{polar}(v,\theta,\varphi) = \frac{-\mu_C}{2\sqrt{2}}\Lambda_+(H_1^{(+)}\cos\psi + \frac{2Q}{\mu_C r}\Phi)P_l(\theta)\;,$$

Concluding Remarks

- The EHT ultimately proved many of important theories in GR
- M87 and M87* now are our existent laboratory to learn more about
- We can not call any singularities Black Hole!!
- Any extra orders or sources have no sense of developed hair conjecture
- no hair conjecture, cosmic censorship, besides dyonic theory can be discussed
- Zerrili and Reege-Wheeler like equations for RN-BH have been expressed again
- To span all ST, and cause of causality concepts, we had to re-scale the coordinates
- Null tetrads and Newman-Penrose formalism have had very important role in development of GR and have resulted in golden age of it
- Einstein-Maxwell theory and force-free mechanism have been evaluated
- The accretion disc and jets can be examined by virtue of Blanford-Znajek theory
- For the first time an exact solution for the Maxwell's scalars have been introduced
- The E-M tensor in both tetrad and bend spaces have been obtained

References and bibliography

[1] The Event Horizon Telescope Collaboration The Astrophysical Journal Letters, 875:L1 (17pp), (2019).
[2] R. lüst and A. Schlütter, Z. Astrophys., 34, 263, (1954).
[3] S. Chandrasekhar and Donna Elbert, Proc. Cambridge Phil. Soc., 49, 446, (1953).
[4] S. Chandrasekhar, Proceedings of the National Academy of Sciences, 42 1, (1956).
[5] S. Chandrasekhar, Hydrodynamic and Hydromagnetic Stability, Oxford at the Clarendon Press, (1961).
[6] P. Goldreich and W. H. Julian, The Astrophysical Journal 157, 869 (1969).
[7] R. D. Blandford and R. L. Znajek, Monthly Notices of the Royal Astronomical Society 179 433 (1977).
[8] D. MacDonald and K. S. Thorne, Mon. Not. R. Astron. Soc. 198, 345 (1982).
[9] K. S. Thorne and D. MacDonald, Mon. Not. R. Astron. Soc. 198, 339 (1982).
[11] K. S. Thorne, R. H. Price, and D. A. MacDonald, Black Holes: The Membrane Paradigm (Yale, New Haven,1986).
[12] B. Punsly, Black Hole Gravitohydromagnetics (Springer, Berlin, 2001).
[13] V. Semenov, S. Dyadechkin, and B. Punsly, Science 305, 978 (2004).
[14] J. C. McKinney and R. Narayan, Mon. Not. R. Astron. Soc. 375, 513 (2007).
[15] T. D. Brennan, S. E. Gralla, T. Jacobson [arXiv:1305.6890[gr-qc]].
[16] S. Chandrasekhar, Proc. Roy. Soc. (London) A, 365, 453 (1979).
[17] S. Chandrasekhar, Proc. Roy. Soc. (London) A, 369, 425 (1980).
[18] E. Newman and R. Penrose, Journal of Mathematical Physics 3, 566 (1962).
[19] S. A. Teukolsky and W. Press W, The Astrophysical Journal 193 , 443 (1974).
[20] F. J. Zerilli, IBID, 9, 860 (1974).
[21] S. Chandrasekhar and B. C. Xanthopoulos, Proc. Roy. Soc. (London) A, 367, 1 (1979).
[22] B. C. Xanthopoulos, Proc. Roy. Soc. (London) A, 378, 73 (1981).
[23] C. W. Misner, K. S. Thorne, and J. A. Wheeler, Gravitation, W. H. Company, 21 th printing, New York (1998).
[24] T. Padmanabhan, Gravitation Foundations and Frontiers, Cambridge University Press, 1st printing, Cambridge (2010).
[25] S. Chandrasekhar and P. C. Kendall, Astrophys. J. 126, 457C (1957).

I would like to appreciate

Prof. Y. Sobouti for introducing this topic to me

Prof. H. Firouzjahi to let me correct some of my wrong postulations

Prof. G. Ellis for our very creative and enlightening discussion aboutNewman-Penrose formalism and perturbations in metric

Prof. A. Weltman and Dr. R. Costa for their helpful debates on

Prof. T. Harko for his enlightening and constructive discussions

and you all researchers and participants