Generalized unimodular gravity: constrained dynamics and cosmology

based on PRD **100**, 023542 (2019) (arXiv:1903.09897)

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August 13, 2019

Equation of state

$$p = w\varepsilon$$

Phantom line crossing

$$w < -1$$

leads to negative kinetic energy.

For example,

$$w = \frac{\frac{1}{2}\dot{\varphi}^2 - V(\varphi)}{\frac{1}{2}\dot{\varphi}^2 + V(\varphi)} < -1 \quad \Rightarrow \quad \dot{\varphi}^2 < 0$$

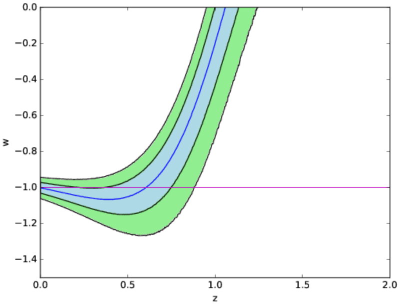

Figure: Deng Wang et al, 2019

Approaches to dark energy problem I

Unimodular gravity[1]

$$S_{\rm UM}[g_{\mu\nu},\lambda] = \int d^4x \Big[\sqrt{g}R - \lambda(\underbrace{\sqrt{g}-1}_{\text{constraint}})\Big]$$

Equations of motion

$$R_{\mu\nu} - \frac{1}{2}\,g_{\mu\nu}R = -\frac{1}{2}\lambda\,g_{\mu\nu}, \qquad \lambda = \frac{1}{2}R$$

Equation of state: $p = -\varepsilon$

Constancy of λ

$$\nabla_\mu \lambda = 0 \;\Rightarrow\; \lambda = \text{const}$$

Symmetries

$$\delta^\xi S_{UM} = 0 \quad \Leftrightarrow \quad \partial_\mu \xi^\mu = 0$$

[1] Henneaux&Teitelboim 1989, Unruh 1989

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Approaches to dark energy problem II

Generalized unimodular gravity[2]

$$S_{\mathrm{GUMG}} = \int d^4x \Big[\sqrt{g}R - \lambda\underbrace{\big((-g^{00})^{-1/2} - N(\gamma)\big)}_{\text{constraint}}\Big], \quad \gamma = \det \gamma_{ij}.$$

Equation of motion

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{1}{2}T_{\mu\nu},$$

$$T_{\mu\nu} = \frac{2}{\sqrt{g}}\frac{\delta}{\delta g^{\mu\nu}}\int d^4x\,\lambda\left((-g^{00})^{-1/2} - N(\gamma)\right) = (\varepsilon + p)\,u_\mu u_\nu + p\,g_{\mu\nu},$$

Perfect fluid parameters

$$\varepsilon = \frac{\lambda}{\sqrt{\gamma}}, \quad p = w\varepsilon, \quad w = 2\,\frac{d\ln N(\gamma)}{d\ln\gamma}, \quad u_\mu = -\delta^0_\mu N(\gamma)$$

[2] Barvinsky&Kamenshchik 2017

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Compatibility with GR

GUMG e.o.m. in terms of projector

$$P_{\mu\nu}{}^{\rho\sigma} G_{\rho\sigma} = 0, \quad P_{\mu\nu}{}^{\rho\sigma} \equiv \delta^{\rho}_{(\mu}\delta^{\sigma}_{\nu)} - \left[\, u_\mu u_\nu + w(u_\mu u_\nu + g_{\mu\nu}) \,\right] u^\rho u^\sigma$$

Comparison with GR e.o.m. solutions

$$G_{\mu\nu} = 0 \quad \Rightarrow \quad P_{\mu\nu}{}^{\rho\sigma} G_{\rho\sigma} = 0.$$

(3+1)-decomposition and symmetries

Equation on gauge parameters [3]

$$\delta^{\xi}\left[(-g^{00})^{-1/2} - N(\gamma)\right]\Big|_{N(\gamma)} = N\left[\partial_t \xi^0 - (1+w)N^k\partial_k\xi^0 - w\,\partial_l\xi^l\right] = 0.$$

Its solutions

$$\xi^{\mu}_{1,2} = \begin{bmatrix} 0 \\ \xi^i_{\perp} \end{bmatrix}, \quad \partial_k \xi^k_{\perp} = 0, \quad \xi^{\mu}_3 = \begin{bmatrix} \xi^0 \\ \partial^i \frac{1}{w\Delta}(\partial_t - (1+w)N^k\partial_k)\xi^0 \end{bmatrix}$$

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[3] (3+1)-decompositions of metric $g_{\mu\nu} \leftrightarrow (N, N^i, \gamma_{ij})$

$$g_{\mu\nu} = \begin{pmatrix} -N^2 + N^i N_i & N_i \\ N_j & \gamma_{ij} \end{pmatrix} \Leftrightarrow \begin{array}{ll} N = (-g^{00})^{-1/2}, & \gamma_{ij} = g_{ij}, \\ N_i = g_{0i}, & N^i = \gamma^{ij}N_j \end{array}$$

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Degrees of freedom count I

GUMG configurational space

$$\delta S/\delta\lambda = 0 \quad \Rightarrow \quad N = N(\gamma), \quad (N^i, \gamma_{ij}) \text{ — 9 conf. variables}$$

$$\pi^{ij} = \delta L/\delta\dot{\gamma}_{ij}, \quad P_i = \delta L/\delta\dot{N}^i \equiv 0 \text{ — primary constraint}$$

Bifurcation in the canonical constraints

$$\underbrace{P_i = 0}_{\text{primary}} \Rightarrow \underbrace{H_i = 0}_{\text{secondary}} \Rightarrow \underbrace{\partial_i T = 0 \Leftrightarrow \begin{cases} T = Q(t) \\ T = 0 \not\Rightarrow \end{cases}}_{\text{tertiary}} \Rightarrow \underbrace{\partial_i S = 0}_{\text{quaternary}}$$

Two branches:

$\underline{T = 0 \text{ branch}}$

- (P_i, H_i, T) — 7 constraints
- All of them — *first class*

$\underline{T \neq 0 \text{ branch}}$

- $(P_i, H_i, \partial_i T, \partial_i S)$ — 8 constraints
- 4 are of the first class
- 4 are of the second class

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Degrees of freedom count II

General formula for d.o.f.

$$2\times\begin{pmatrix}\text{\# of phys.}\\ \text{d.o.f.}\end{pmatrix} = \begin{pmatrix}\text{\# of canon.}\\ \text{variables}\end{pmatrix} - 2\times\begin{pmatrix}\text{\# of 1-st cl.}\\ \text{constraints}\end{pmatrix} - \begin{pmatrix}\text{\# of 2-nd cl.}\\ \text{constraints}\end{pmatrix}$$

Our case

- $T = 0$ branch: $(2 \times 9 - 2 \times 7)/2 = 2$ d.o.f. — graviton
- $T \neq 0$ branch: $(2 \times 9 - 2 \times 4 - 4)/2 = 3$ d.o.f. — graviton + scalar

Role of 3-rd d.o.f. ?

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Role of 3-rd d.o.f. ?

Background equations

Background metric

$$ds^2 = -N^2(a)\, d\tau^2 + a^2(\tau)\, \sigma_{ij}(x)\, dx^i dx^j$$

Background e.o.m.

$$\dot{H} - \frac{k}{a^2} = -\frac{3}{2}(1+w)\Big(H^2 + \frac{k}{a^2}\Big), \quad w(a) = \frac{1}{3}\frac{d\ln N(a)}{d\ln a}$$

"Friedmann" equation

$$H^2 + \frac{k}{a^2} = \frac{1}{3}\frac{C}{Na^3},$$
$$\dot{H} - \frac{k}{a^2} = -\frac{1}{2}(1+w)\frac{C}{Na^3}.$$

C is an integral of motion

Energy density

$$\varepsilon = \frac{C}{Na^3}, \qquad \frac{d\varepsilon}{da} = -3\,(1+w)\,\frac{\varepsilon}{a}$$

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Metric perturbations

$$\delta\gamma_{ij} = a^2\left(t_{ij} + 2\nabla_{(i}F_{j)} - 2\psi\,\sigma_{ij} + 2\nabla_i\nabla_j E\right), \quad t^i{}_i = \nabla^i t_{ij} = \nabla_i F^i = 0,$$
$$\delta N^i = \nabla^i B + V^i, \quad \nabla_i V^i = 0.$$

Physical Lagrangian $S = S_s + S_t$, $(k = 0)$

$$S_t[t_{ij}] = \frac{1}{4}\int d\eta\, d^3x\, a^2 (t'^2_{ij} - (\partial_k t_{ij})^2),$$
$$S_s[\psi] = \frac{1}{2}\int d\eta\, d^3x\, z^2\left(\psi'^2 + c_s^2\,\psi\Delta\psi\right).$$

where

$$z^2 = \frac{3\Omega}{2w}a^2, \quad c_s^2 = \frac{w}{\Omega}(1+w), \quad \Omega = 1 + w + \frac{1}{3}\frac{d\ln w}{d\ln a}$$

Mukhanov-Sasaki form of equation $(\vartheta = z\psi)$

$$S_s = \frac{1}{2}\int d\eta\, d^3x\left(\vartheta'^2 + c_s^2\,\vartheta\Delta\vartheta + \frac{z''}{z}\vartheta^2\right),$$

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Inflation model

Exponential growth condition

$$w(a) \to -1 \quad \Rightarrow \quad N(a) \to \frac{1}{a^3}, \quad a \to 0$$

Ansatz satisfying the condition above

$$N(a) = \frac{1}{a^3}\left[1 + \left(\frac{a}{a_0}\right)^3 + B\left(\frac{a}{a_0}\right)^{6\beta} + \ldots\right], \quad \beta > \frac{1}{2},$$

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Crucial observable parameters

$$n_s \simeq 1 - \frac{3}{4}\Delta\beta, \qquad \beta = \frac{3}{2} - \Delta\beta,$$

$$\delta_s^2 \simeq \frac{\sqrt{6B}H_0^2}{27\pi^2 M_P^2}\left(\frac{a}{a_0}\right)^{-3\Delta\beta}, \qquad H_0^2 = \frac{C}{3}$$

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Conclusions

GUMG is ...

- Complicated gauge model, which d.o.f. number depends on initial (boundary) conditions.
- Failure attempt to obtain phantom line crossing without inestabilities

$$z^2 \propto \frac{\Omega}{w} > 0, \quad c_s^2 = \frac{w(1+w)}{\Omega} > 0$$

- Gravitational model, whose additional d.o.f. — scalar graviton can be responsible for inflationary perturbations with the parameters close to observable

$$\begin{aligned} n_s - 1 &\simeq -0.04, \\ \delta_s^2 &\simeq 10^{-10}, \\ r &\ll 1, \end{aligned} \quad \Rightarrow \quad \begin{aligned} B &= O(1), \\ \Delta\beta &\simeq 0.05. \end{aligned}$$

Thank you for your attention!