Lecture 1

1. KK reduction of SUGRA

Strings: 10D $\longrightarrow$ 4D , compact internal space

$$M^{1,9} = \mathbb{R}^{1,3} \times K^6$$

6dim compact space

$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu + A_{\mu a}\, dx^\mu dy^a + \phi_{ab}\, dy^a dy^b$$

4D: metric, vectors, scalars

if one compact direction: $\Phi(x^\mu, y) = \sum_n \phi_n(x^\mu)\, e^{\frac{iny}{R}}$

$$0 = \nabla_M \nabla^M \Phi \Rightarrow \left(\nabla_\mu \nabla^\mu - \frac{n^2}{R^2}\right)\phi_n = 0 \qquad m_n^2 = \frac{n^2}{R^2}$$

- keep only zero modes (massless) for small R large masses
- mass is provided by other means
  - moduli stabilization
  - brane intersections
  - Higgs mechanism
  - ...

Consider SUGRA in (1+9) dimensions

- supersymmetry : Majorana-Weyl spinors $\bar{\Psi} = \Psi^+ \cdot \Gamma_0 = \Psi^{Tr} C = \tilde{\Psi}$

  can be chosen • real • chiral

  16 - components : $\epsilon_+ \in 16_s$ - positive, $\epsilon_- \in 16_c$ - negative

To keep spin after comp. ≤ 2 (no higher spins in 4D theory)

II A : $\mathcal{N} = (1,1)$ (ϵ_+, ϵ_-)

II B : $\mathcal{N} = (2,0)$ $(\epsilon_+^1, \epsilon_+^2)$

in 4D only Majorana : $\epsilon \in (2,1) \oplus (1,2)$

field content : G_{MN}, B_{MN}, ϕ (NSNS) | C_M, C_{MNK} ; C, C_{MN}, C_{MNKL} (RR) | $\Psi_M^+, \Psi_M^-, \lambda^+, \lambda^-$; $\Psi_M^1, \Psi_M^2, \lambda^1, \lambda^2$ (fermions)

$$S = \int d^{10}x\, e^{-2\phi} \sqrt{-G}\left(\frac{1}{2}R[G] + \frac{1}{4} G^{MN} \partial_M \phi\, \partial_N \phi - \frac{1}{12} H_{MNK} H^{MNK} + \ldots\right.$$

$H_{MNK} = 3\,\partial_{[M} B_{NK]}$ - 3 form flux

2. Reduction on a torus $\mathbb{T}^6$:

$$x^M \to (x^M, y^a) \qquad SO(1,9) \leftarrow SO(1,3)\times SO(6)$$
$$10 \to (4,1) + (1,6)$$

$4, \bar{4}$ - chiral spinors of $SO(6)$

Spinors :
$$16_s \to (2,1,4) + (1,2,\bar{4})$$
$$16_c \to (2,1,\bar{4}) + (1,2,4) \qquad \eta^+, \eta^-$$

$$A = 1,\overline{16} \quad \epsilon^A \Leftrightarrow (\epsilon^{\alpha a}, \epsilon^{\dot\alpha \dot a}) \qquad \epsilon^+ = \varepsilon^+\otimes\eta^+ + \varepsilon^-\otimes\eta^-$$
$$\dot A = 1,\overline{16} \quad \epsilon^{\dot A} \Leftrightarrow (\epsilon^{\alpha \dot a}, \epsilon^{\dot\alpha a}) \qquad \epsilon^- = \varepsilon^-\otimes\eta^+ + \varepsilon^+\otimes\eta^-$$

SUSY in 4D ; $\begin{bmatrix}\epsilon^{\alpha a}\\ \epsilon^{\dot\alpha a}\end{bmatrix}$ 4 spinors ; $\begin{bmatrix}\epsilon^{\alpha \bar a}\\ \epsilon^{\dot\alpha \bar a}\end{bmatrix}$ 4 spinors

SUSY transf in IIA:

$$0 = \delta\psi_M^{\pm} = \nabla_M \epsilon^{\pm} \qquad \epsilon = \begin{bmatrix}\epsilon^+\\ \epsilon^-\end{bmatrix}$$
$$0 = \delta\lambda = \left(\not\partial\Phi + \tfrac{1}{2} H_{MNK}\Gamma^{MNK}\Gamma_{11}\right)\epsilon$$

BPS equ's

SUSY transf do not change the bg.

The problem:
for torus all BPS equations are satisfied for all spinors

$\mathbb{T}^6$ gives $\mathcal{N}=8$ supergravity in 4D.

- non-chiral
- too restricted
- no interesting phenomenology
- flat potential for scalars $\quad V(\Phi_{ab}) = 0$

Ways to more realistic models:

- Manifolds which preserve less SUSY $\Rightarrow$ CY (still non-chiral)
- additional constraints on string spectrum $\Rightarrow$ orientifolds
- D-brane models (break SUSY, add masses)
- fluxes to stabilize scalar moduli ($V(\Phi_{ab}) \neq 0$, has minima)

3. Two-dim. σ-model

1. RNS string and its spectrum.

bosonic action: $S_B = -\frac{1}{4\pi\alpha'}\int d^2\sigma \sqrt{-h}\, h^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X_\mu$

(assume: the string is in a flat space)

introduce SUSY in 2 ways

- target space SUSY: Green-Schwarz superstring
- ✓ world-volume SUSY: Ramond-Neveu-Schwarz superstring

$\hookrightarrow$ add superpartners of the worldvolume scalar fields $X^\mu(\sigma,\tau)$

Consider $N=(1,1)$ in $d=2$; in dimensions $s-t=0$ $\exists$ MW

σ-model with critical dimension $D=10$

$d=(9,1)$; $d=(1,1)$

IIA/B GSO projection
I

Spinors in $d=2$: $\{\rho_\alpha,\rho_\beta\} = 2\eta_{\alpha\beta} = 2\,\mathrm{diag}[-1,+1]$

$$\rho_0 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix};\quad \rho_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix};\qquad \left.\begin{aligned} A^{-1}\rho_\alpha A &= \pm\rho_\alpha^{+} \\ C^{-1}\rho_\alpha C &= \pm\rho_\alpha^{Tr} \end{aligned}\right\}\ A = C = \rho_1$$

Majorana condition: $\bar{\psi} = \tilde{\psi}$

$\psi^{+}A = \psi^{Tr}C \Rightarrow \psi^* = \psi$; $\psi \in \mathbb{R}^2$

$\psi = \begin{bmatrix} \psi_+ \\ \psi_- \end{bmatrix}$; $\omega = \rho_0\rho_1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ - chirality

SUSY: $\{Q_\pm, Q_\pm\} = (H \pm P)$; $N=(p,q)$; $p\,Q_+$, $q\,Q_-$

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in the superconformal gauge (remove gravitino, vielbein, etc.)

$$S = -\frac{1}{8\pi}\int d^2\sigma\left(\frac{2}{\alpha'}\partial_\alpha X^\mu \partial^\alpha X_\mu + 2i\,\bar{\psi}^\mu \rho^\alpha \partial_\alpha \psi_\mu\right)$$

remaining SUSY transformations:

$$\left[\begin{aligned} \sqrt{\tfrac{2}{\alpha'}}\,\delta_\epsilon X^\mu &= i\bar{\epsilon}\psi^\mu \\ \delta_\epsilon \psi^\mu &= \frac{1}{\sqrt{2\alpha'}}\rho^\alpha \partial_\alpha X^\mu \cdot \epsilon \end{aligned}\right.$$

EoM's:
$$\begin{cases} \partial_\alpha \partial^\alpha X^\mu = 0 \\ \rho^\alpha \partial_\alpha \psi^\mu = 0 \end{cases}$$

4. Boundary conditions:

$$\delta S = \text{EoM's} - \frac{1}{8\pi}\int d^2\sigma\, \partial_\alpha\left(\frac{4}{\alpha'}\delta X^\mu \partial^\alpha X_\mu + 2i\,\bar\Psi^\mu \rho^\alpha \delta\Psi_\mu\right)$$

$$\tilde S_B = -\frac{1}{2\pi\alpha'}\int d\tau \left(\delta X^\mu \partial_\sigma X_\mu\big|_{\sigma=\ell} - \delta X^\mu \partial_\sigma X_\mu\big|_{\sigma=0}\right)$$

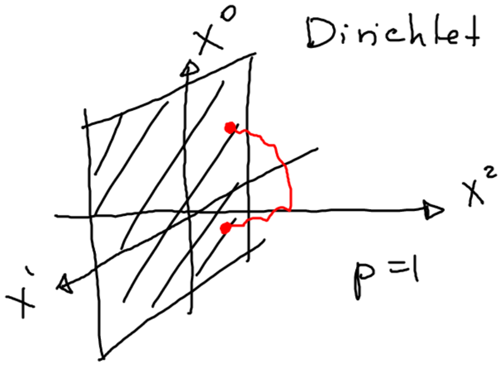
$$\tilde S_F = -\frac{i}{4\pi}\int d\tau \left[\left(\Psi_+^\mu \delta\Psi_{+\mu} - \Psi_-^\mu\delta\Psi_{-\mu}\right)\big|_{\sigma=\ell} - \left(\Psi_+^\mu\delta\Psi_{+\mu} - \Psi_-^\mu\delta\Psi_{\mu-}\right)\big|_{\sigma=0}\right]$$

1) Bosonic

- closed string: $X^\mu(\tau,\sigma+\ell) = X^\mu(\tau,\sigma)$
- open: Neumann: $\partial_\sigma X^a = 0 \quad (a = 0,\dots,p)$

  Dirichlet: $\delta X^i(\tau,\sigma=0/\ell) = 0 \quad (i = p+1,\dots,9)$

  $(\partial_\tau X(\tau,\sigma=0/\ell) = 0)$



$p=1$

2) Fermionic

- closed string: $\Psi_+^\mu(\tau,\sigma+\ell) = \pm\Psi_+^\mu(\tau,\sigma)$ Ramond

  $\Psi_-^\mu(\tau,\sigma+\ell) = \pm\Psi_-^\mu(\tau,\sigma)$ Neveu-Schwarz

+	R	R	NS	NS
−	R	NS	R	NS

(can choose independently)

- open string: $\Psi_+(0) = \eta_1\Psi_-(0)\ ;\ \Psi_+(\ell) = \eta_2\Psi_-(0)$

η_1	+	+	−	−
η_2	+	−	+	−

R and NS boundary conditions

Ex SUSY implies: $\epsilon_+ = \pm\epsilon_-$ – kills half of the total SUSY

BPS objects

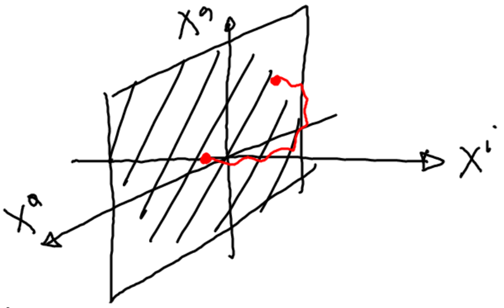
5. Oscillator expansion

1) Open string

boundary conditions:

$$NN: \quad \partial_\sigma X^a(\tau, \sigma = 0) = 0; \quad \partial_\sigma X^a(\tau, \sigma = \ell) = 0$$

$$DD: \quad \delta X^i(\tau, \sigma = 0) = 0; \quad \delta X^i(\tau, \sigma = \ell) = 0$$



$a = \overline{0, p}$

$$NN: X^a(\sigma, \tau) = x^a + \frac{2\pi\alpha'}{\ell} p^a \tau + i\sqrt{2\alpha'} \sum_{n \neq 0} \frac{1}{n} \alpha_n^a e^{-i\frac{\pi n \tau}{\ell}} \cos\left(\frac{\pi n \sigma}{\ell}\right)$$

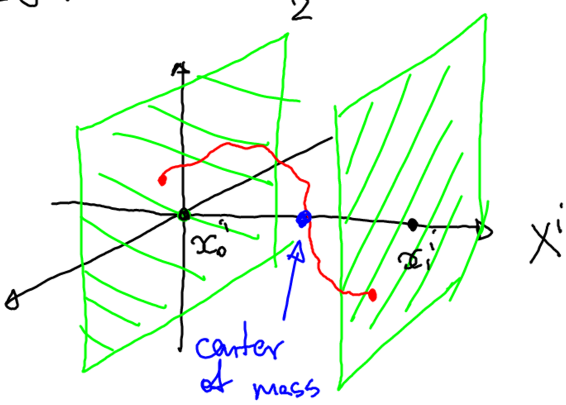
$$DD: X^i(\sigma, \tau) = x_0^i + \frac{1}{\ell}(x_1^i - x_0^i)\sigma + \sqrt{2\alpha'} \sum_{n \neq 0} \frac{1}{n} \alpha_n^i e^{-i\frac{\pi n \tau}{\ell}} \sin\left(\frac{\pi n \sigma}{\ell}\right)$$

center of mass momentum:

$$p^i = \int_0^\ell d\sigma \, \partial_\tau X^i = 0$$

center of mass position

$$q^i = \frac{1}{\ell} \int_0^\ell d\sigma \, X^i = \frac{x_0^i + x_1^i}{2}$$



For a single Dp-brane states are stuck to the brane

$$R: \quad \psi_+^a(0) = +\psi_-^a(0)$$

$$NS: \quad \psi_+^a(0) = -\psi_-^a(0)$$

$$\psi_\pm^\mu = \sum_{r \geq 0} b_r^\mu e^{\frac{i n \sigma_\pm \pi}{\ell}} \qquad \begin{array}{l} r \in \mathbb{Z} \;; R \\ r \in \mathbb{Z} + \frac{1}{2} : NS \end{array}$$

2) Closed string

$\sigma_\pm = \sigma^0 \pm \sigma^1$

$\partial_\pm = \frac{1}{2}(\partial_0 \pm \partial_1)$

- bosons

$$\partial_\alpha \partial^\alpha X^\mu = 0 \quad \Rightarrow \quad \partial_{+-} X^\mu = 0$$

$$X^\mu(\sigma, \tau) = X^\mu_R(\sigma_-) + X^\mu_L(\sigma_+)$$

$$X^\mu_R(\sigma_-) = \frac{1}{2} x^\mu + \frac{\pi\alpha'}{\ell} p^\mu \sigma_- + i\sqrt{\frac{\alpha'}{2}} \sum_{n\neq 0} \frac{1}{n} \alpha^\mu_n e^{-\frac{2\pi}{\ell} i \sigma_- n}$$

$$X^\mu_L(\sigma_+) = \frac{1}{2} x^\mu + \frac{\pi\alpha'}{\ell} p^\mu \sigma_+ + i\sqrt{\frac{\alpha'}{2}} \sum_{n\neq 0} \frac{1}{n} \bar{\alpha}^\mu_n e^{-\frac{2\pi}{\ell} i \sigma_+ n}$$

$\alpha^{\mu *}_n = \alpha^\mu_{-n}$; $\bar{\alpha}^{\mu *}_n = \bar{\alpha}^\mu_{-n}$ — to have real functions

Canonical momentum: $\Pi_\mu = \dfrac{\partial \mathcal{L}}{\partial \dot{X}^\mu}$

EX check:

$$P_\mu = \int_0^\ell d\sigma\, \Pi_\mu = p_\mu \; ; \quad q^\mu(\tau) = \frac{1}{\ell}\int_0^\ell d\sigma\, X^\mu = x^\mu + \frac{2\pi\alpha'}{\ell} p^\mu \tau$$

movement of the center of mass

$$J^{\mu\nu} = \int_0^\ell d\sigma \left(X^\mu \Pi^\nu - X^\nu \Pi^\mu \right) = \ell^{\mu\nu} + E^{\mu\nu} + \bar{E}^{\mu\nu}$$

$\ell^{\mu\nu} = x^\mu p^\nu - x^\nu p^\mu$ — momentum of the center of mass

$$E^{\mu\nu} = -i \sum_{n=1}^{\infty} \frac{1}{n} \left(\alpha^\mu_{-n} \alpha^\nu_n - \alpha^\nu_{-n} \alpha^\mu_n \right)$$

- fermionic

EoM's: $\partial_+ \psi_-^\mu = 0$; $\partial_- \psi_+^\mu = 0$

R:
$$\begin{bmatrix} \psi_+ \\ \psi_- \end{bmatrix} = \sqrt{\frac{2\pi}{\ell}} \sum_{n\in\mathbb{Z}} \begin{bmatrix} \bar{b}^\mu_n \\ b^\mu_n \end{bmatrix} e^{-\frac{2\pi i n}{\ell}\sigma_\pm}$$

NS:
$$\begin{bmatrix} \psi_+ \\ \psi_- \end{bmatrix} = \sqrt{\frac{2\pi}{\ell}} \sum_{r\in\mathbb{Z}+1/2} \begin{bmatrix} \bar{b}^\mu_r \\ b^\mu_r \end{bmatrix} e^{-\frac{2\pi i r}{\ell}\sigma_\pm}$$

adding of ℓ to $\sigma_\pm$ multiplies by $e^{i\pi} = -1$

Summary:

- Toroidal models have to many susy's and cannot stabilize moduli
- One needs more compl. manifolds and/or add. structures
- strings can end on D-branes
- various boundary conditions are possible

Lecture 2 Quantization

1. Light cone quantization

- covariant canonical approach → amplitudes
- path integral → geometric aspects, non-pert. corrections etc.
- light cone framework → easy and fast for mass spectrum

The problem with covariant quantization:

vector field: $$[A_\mu(t,\vec{x}), \dot{A}_\nu(t,\vec{y})] = i\,\eta_{\mu\nu}\,\delta(\vec{x}-\vec{y})$$

$$\downarrow$$

$$[a_\mu(\vec{p}), a_\nu^\dagger(\vec{q})] = 2\omega\,\eta_{\mu\nu}\,\delta(\vec{p}-\vec{q})$$

$|\vec{p}\rangle_0 = a_0^\dagger|0\rangle \quad {}_0\langle\vec{p}|\vec{p}\rangle_0 = \langle 0|a_0 a_0^\dagger|0\rangle = -2\omega < 0$

negative norm state

One has to project out unphys states or the corresponding operators

Choose light-cone gauge

$$X^\mu = (X^0, X^1, \dots, X^{D-1}) = (X^+, X^-, X^i) \qquad i = 2,\dots, D-1$$

$$X^\pm = \frac{1}{\sqrt{2}}(X^0 \pm X^1) \quad ; \quad \psi^\pm = \frac{1}{\sqrt{2}}(\psi^0 \pm \psi^1)$$

fix: $$X^+ = \frac{2\pi\alpha'}{\ell} p^+ \tau \quad : \quad \alpha_0^+ = \tilde{\alpha}_0^+ = \sqrt{\frac{\alpha'}{2}}\, p^+$$

$$\psi^+ = 0 \qquad ; \qquad b_n^+ = 0$$

ex Commutation relations for oscillation modes:

$$[q^-, p^+] = -i \quad , \quad [q^i, p^j] = i\,\delta^{ij} \quad \leftarrow \alpha_0^i$$

$$n,m \neq 0 \quad [\alpha_n^i, \alpha_m^j] = n\,\delta^{ij}\,\delta_{n+m,0}\ ;$$

$$\{b_r^i, b_s^j\} = \delta^{ij}\,\delta_{r+s,0}$$

Ground state of NS string: $\alpha_m^i|0\rangle_{NS} = 0 \qquad m = 1,2,\dots$

$b_r^i|0\rangle_{NS} = 0 \qquad r = \frac{1}{2}, \frac{3}{2}, \dots$

R string: $\alpha_m^i|0\rangle_R = 0 \qquad m = 1,2,\dots$

$b_r^i|0\rangle_R = 0 \qquad r = 1,2,\dots$

$\{b_0^i, b_0^j\} = \delta^{ij}$ – Clifford algebra of $\mathbb{R}^8$ $\qquad$ r=0 – special care needed

$|0\rangle_R$ fall into irreps of Spin(8)

2. Mass spectrum

- open string between 2 Dp-branes

$$\boxed{\alpha' m^2 = \sum_{n=1}^{\infty} \alpha_{-n}^{\mu}\alpha_n^{\mu} + \sum_{r>0} r\, b_{-r}^{\mu} b_r^{\mu} + \frac{1}{4\pi^2\alpha'}(\Delta X)^2 + a}$$

$a = -\frac{1}{2}$ NS

$a = 0$ R

- closed string

$$\boxed{\begin{aligned} \frac{1}{2}\alpha' m_L^2 &= \sum_{n>0} \alpha_{-n}^{i}\alpha_n^{i} + \sum_{r>0} r\, b_{-r}^{i} b_r^{i} + a \\ \frac{1}{2}\alpha' m_R^2 &= \sum_{n>0} \bar{\alpha}_{-n}^{i}\bar{\alpha}_n^{i} + \sum_{r>0} r\, \bar{b}_{-r}^{i} \bar{b}_r^{i} + a \end{aligned}}$$

$a =$ const

$m_L^2 = m_R^2$ – level matching constraint

$D = 10$ from $SO(1, D-1)$ Poincaré invariance

$\Rightarrow$ NS: $a = -\frac{1}{2}$

R: $a = 0$

$[b_0^i, m^2] = 0 \Rightarrow$ mass states carry a repr. w.r.t. the Clifford algebra, generated by b_0^i.

3. Open string spectrum, D-brane

NS sector: $\alpha_n^\mu |0;p^a\rangle_{NS} = 0$; $b^\mu_{n-\frac{1}{2}}|0;p^a\rangle_{NS} = 0$ $\quad n = 1,2,\dots$

R sector: $\alpha_n^\mu |0;p^a\rangle_R = 0$; $b^\mu_n |0;p^a\rangle_R = 0$ $\quad n = 1,2,\dots$

$\{b_0^i, b_0^j\} = \delta^{ij} \Rightarrow$ ground state is an SO(8) spinor

$b_0^i = \frac{1}{\sqrt{2}}\Gamma^i$ – Clifford algebra of SO(8) – has MW spinors

real

$$\Gamma^i = \begin{bmatrix} 0 & \gamma^{i a}{}_{\dot a} \\ \tilde\gamma^{i \dot b}{}_b & 0 \end{bmatrix} \quad ; \quad \text{spinors} \quad \psi = \begin{bmatrix} \psi^a \\ \psi^{\dot a} \end{bmatrix} \begin{matrix} \text{chirality} + & 8_c \\ \text{chirality} - & 8_s \end{matrix}$$

Ramond ground state can be labelled : $\left.\begin{matrix} |a\rangle_R \\ |\dot a\rangle_R \end{matrix}\right\} |A\rangle_R$

$$b_0^i |a\rangle_R = \frac{1}{\sqrt{2}}\gamma^{ia}{}_{\dot a}|\dot a\rangle_R \qquad \alpha_n^i |A\rangle_R = 0$$
$$b_0^i |\dot a\rangle_R = \frac{1}{\sqrt{2}}\gamma^{i\dot a}{}_{a}|a\rangle_R \qquad b_n^i |A\rangle_R = 0 \qquad n>0$$

$\alpha' m^2$	state	irrep of SO(8)		$(-1)^F$	
	NS sector				
$\frac{1}{4\pi\alpha'}(\Delta X)^2 - \frac{1}{2}$	$\|0;p^a\rangle_{NS}$	1	χ	$-$	tachyon
0	$b^i_{-1/2}\|0;p^a\rangle$	8_v	A^μ	$+$	GSO: keep +'s
	R sector				
0	$\|a;p^a\rangle_R$	8_s	ψ	$+$	
0	$\|\dot a;p^a\rangle_R$	8_c	$\dot\psi$	$-$	

GSO projection: 1) N Dp branes $\quad (-1)^F = +1$: $\quad p=9$ $\quad A^\mu{}_{IJ}$; ψ_{IJ} $\quad U(N)$ SYM $\quad \mathcal{N}=(1,0)$ SUSY in $d=1+9$

2) Dp-$\overline{\text{D}p}$ brane $\quad (-1)^F = -1 \quad \chi, \dot\psi \quad$ no SUSY

instability

attraction and annihilation of the brane-antibrane system.

4. Closed string spectrum, supergravity

NS sector: α^i_{-n} ; $b^i_{-n+\frac{1}{2}}$; $n = 1, 2, \dots$ ground state $|0\rangle_{NS}$ scalar

R sector: α^i_{-n} ; b^i_{-n} ; $n = 1, 2, \dots$ ground stat $|0\rangle_R$ - spinor

NS $\quad \frac{1}{2}\alpha' m^2 = \sum_{n=1}^{\infty} \alpha^i_{-n}\alpha^i_n + \sum_{n=1}^{\infty}(n-\frac{1}{2}) b^i_{-n+\frac{1}{2}} b^i_{n-\frac{1}{2}} - \frac{1}{2}$

- $\frac{1}{2}\alpha' m^2 |0\rangle = -\frac{1}{2}|0\rangle \Rightarrow \boxed{m^2(|0\rangle) = -\frac{1}{\alpha'}}$ tachyon

- $\frac{1}{2}\alpha' m^2 b^i_{-1/2}|0\rangle = \frac{1}{2} b^j_{-\frac{1}{2}} \underbrace{b^j_{\frac{1}{2}} b^i_{-1/2}}_{\{b^j_{\frac{1}{2}}, b^i_{-1/2}\}} |0\rangle - \frac{1}{2} b^i_{-1/2}|0\rangle \boxed{= 0}$

  massless

  $\{b^i_{-n}, b^j_n\} = \delta^{ij}$

- $\frac{1}{2}\alpha' m^2 \alpha^i_{-1}|0\rangle = \alpha^j_{-1}\underbrace{\alpha^j_1 \alpha^i_{-1}}_{[\alpha^j_1, \alpha^i_{-1}]}|0\rangle - \frac{1}{2}\alpha^i_{-1}|0\rangle = \frac{1}{2}\alpha^i_{-1}|0\rangle$

  $[\alpha^i_n, \alpha^j_{-n}] = n\,\delta^{ij}$

  $[\alpha^i_1, \alpha^j_{-1}] = \delta^{ij}$

  $\boxed{m^2(\alpha^i_{-1}|0\rangle) = \frac{1}{\alpha'}}$

Massless state in the NS sector: $\alpha^i_{-1/2}|0\rangle$ (only R mode)!

R $\quad \frac{1}{2}\alpha' m^2 = \sum_{n=1}^{\infty} \alpha^i_{-n}\alpha^i_n + \sum_{n=1}^{\infty} n\, b^i_{-n} b^i_n$;

- $\frac{1}{2}\alpha' m^2 |A\rangle_R = 0$ - massless
- $b^i_{-1}|A\rangle_R$ & $\alpha^i_{-1}|A\rangle_R$ - mass $\frac{2}{\alpha'}$

Massless states of the closed string (both R & L modes)

$\alpha' m^2$	State	irrep of $SO(8)$	$(-1)^F$	$(-1)^{\tilde F}$
	(NS,NS) - sector			
-2	$\lvert 0\rangle_L \times \lvert 0\rangle_R$	$\mathbf{1}$	$-$	$-$
0	$\tilde b^i_{-1/2}\lvert 0\rangle_L \times b^i_{-1/2}\lvert 0\rangle_R$ $8_v \quad\quad 8_v$	$8_v \times 8_v = 1 + 28 + 35_v$ $\Phi \quad B_{ij} \quad g_{ij}$	$+$	$+$
	(R,R) - sector			
0	$\lvert a\rangle_L \times \lvert b\rangle_R$ $8_s \quad\quad 8_s$	$8_s \times 8_s = 1 + 28 + 35_s$ $C_0 \quad C_2 \quad C_4^+$	$+$	$+$
	$\lvert a\rangle_L \times \lvert \dot b\rangle_R$ $8_s \quad\quad 8_c$	$8_s \times 8_c = 8_v + 56_v$ $C_1 \quad C_3$	$+$	$-$
	$\lvert \dot a\rangle_L \times \lvert b\rangle_R$ $8_c \quad\quad 8_s$	$8_c \times 8_s = 8_v + 56_v$ $C_1 \quad C_3$	$-$	$+$
	$\lvert \dot a\rangle_L \times \lvert \dot b\rangle_R$ $8_c \quad\quad 8_c$	$8_c \times 8_c = 1 + 28 + 35_c$ $C_0 \quad C_2 \quad C_4^-$	$-$	$-$
	(NS,R)			
0	$\tilde b^i_{-1/2}\lvert 0\rangle_L \times \lvert a\rangle_R$ $8_v \times 8_s$	$8_v \times 8_s = 8_c + 56_c$ $\dot\lambda \quad \psi^i$	$+$	$+$
	$\tilde b^i_{-1/2}\lvert 0\rangle_L \times \lvert \dot a\rangle_R$ $8_v \times 8_c$	$8_v \times 8_c = 8_s + 56_s$ $\lambda \quad \dot\psi^i$	$+$	$-$
	(R,NS)			
0	$\lvert a\rangle_L \times b^i_{-1/2}\lvert 0\rangle_R$ $8_s \quad\quad 8_v$	$8_s \times 8_v = 8_c + 56_c$ $\dot\lambda \quad \psi^i$	$+$	$+$
	$\lvert \dot a\rangle_L \times b^i_{-1/2}\lvert 0\rangle_R$ $8_c \quad\quad 8_v$	$8_c \times 8_v = 8_s + 56_s$ $\lambda \quad \dot\psi^i$	$-$	$+$

$$\left.\begin{aligned} (-1)^F \lvert 0\rangle_{NS} &= -\lvert 0\rangle_{NS} \\ (-1)^F \lvert a\rangle_R &= +\lvert a\rangle_R \\ (-1)^F \lvert \dot a\rangle_R &= -\lvert a\rangle_R \end{aligned}\right\}$$

(assign)

F - world sheet fermion number

GSO projection

$(-1)^F = 1, \quad (-1)^{\tilde F} = 1$ NS sector

$(-1)^F = 1, \quad (-1)^{\tilde F} = \pm 1$ R sector

$$|a\rangle_L \times |b\rangle_R \Longleftrightarrow \quad \Psi_{ab} = \sum C_{i_1\cdots i_n} (\gamma^{i_1\cdots i_n})_a{}^c \, \mathcal{C}_{cb}$$

$$\Gamma^i = \begin{bmatrix} 0_a{}^b & \gamma_{a\dot a} \\ \tilde\gamma^{\dot b b} & 0_{\dot a}{}^{\dot b} \end{bmatrix} \quad ; \quad \gamma^{i_1}{}_{a\dot a}\, \gamma^{i_2 \dot b b} \cdot \Psi_b{}^a \sim C^{i_1 i_2}$$

5. Gauge potentials and branes

II A :	$\phi,\ B_{ij},\ g_{ij}$	$C_1,\ C_3$	$\lambda,\ \psi^i\ ;\ \lambda',\ \psi'^i$	non-chiral SUSY
II B :	$\phi,\ B_{ij},\ g_{ij}$	$C_0,\ C_2,\ C_4^+$	$\lambda_1, \lambda_2\ ;\ \psi_1^i, \psi_2^i$	chiral SUSY

interact with D-branes

Interaction with $(p+1)$-forms:

- $p=0$, particle $\quad S = -mc\int ds + \int A_\mu j^\mu d\tau$

$$= e\int A_\mu \dot x^\mu d\tau = e\int A_\mu dx^\mu = e\int_\gamma A_{(1)}$$

- $p=1$, string $\quad S = \int d^2\sigma \left(\sqrt{-h}\, h^{ab} G_{\mu\nu} + \epsilon^{ab} B_{\mu\nu}\right)\partial_a X^\mu \partial_b X^\nu$

$$= \int d^2\sigma \sqrt{-h}\, h^{ab} G_{\mu\nu}\, \partial_a X^\mu \partial_b X^\nu + \int B_{(2)}$$

- $\forall p \quad$ p-brane $\quad S = \int d^{p+1}\xi \sqrt{-\det h_{ab}} + \int C_{(p+1)}$

gauge invariance: $\int C_{(p+1)}$ is not gauge inv. if the brane is not closed

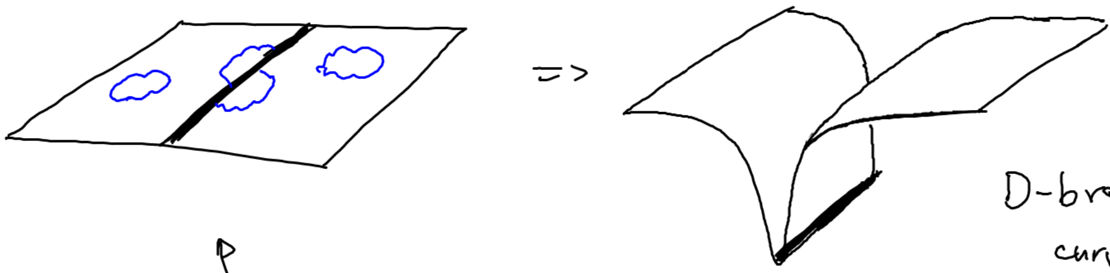
II A :		D0		D2		D4		D6	
II B :	D(-1) instanton		D1		D3		D5		D7

<u>Summary</u>:

- open strings can end on multiple D-branes producing U(N) SYM on D-brane worldvolume
- open strings are bound to the w-v of D-branes
- GSO projection removes tachyon
- tachyon is a sign of D-$\bar D$ instability
- closed strings interact with D-branes via RR gauge fields

Lecture 3: Geometry and dynamics of D-branes

1. Geometry of D-branes



p — described by open & closed string partition function

described by supergravity background solutions

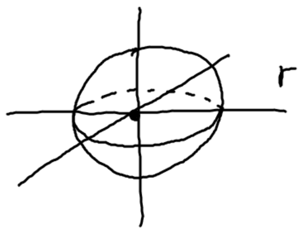
D-brane generates curved geometry around its world-volume

Backreaction on geometry $\to$ supergravity description.

- $p=0$ example: charged black hole in 4d (Reissner-Nordström solution)

$$\begin{cases} ds^2 = f(r)\,dt^2 - f(r)^{-1}dr^2 - r^2 d\Omega^2 \quad ; \quad f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} \\ A_0 = \frac{Q}{r} \quad - \text{ electromagnetic potential} \end{cases}$$

charge: $q_e = \int_{\Sigma_{(2)}} *F_{(2)}$ (electric), $q_m = \int_{\Sigma_{(2)}} F_{(2)}$ (magnetic) ;



$r = \text{const}$

$$q_e = *\int F_{(2)} = \int \epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma} dx^\mu \wedge dx^\nu = \int \underbrace{\sqrt{g}}_{r^2\sin\theta}\ \underbrace{\epsilon_{0r\theta\varphi}}_{1}\ \underbrace{F^{0r}}_{\frac{Q}{r^2}}\, d\theta\, d\varphi = 4\pi Q$$

charge of the source

$q_m = 0$ (no magn. charge)

Solves EoM's with sources:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = T_{\mu\nu} \qquad T^{00} = (2M\,\delta^{(3)}(\vec{x}), 0, 0, 0)$$
$$\nabla_\mu F^{\mu\nu} = j^\nu \qquad j^\mu = (q_e\,\delta^{(3)}(\vec{x}), 0, 0, 0)$$

equations of motion with sources

Effective action: $S_{p=0} = -M \int_\gamma ds + q_e \int_\gamma A_{(1)}$

$$S_{\text{Full}} = \int d^4x \sqrt{g}\left(R[g] + F_{\mu\nu}F^{\mu\nu}\right) + S_{p=0}$$

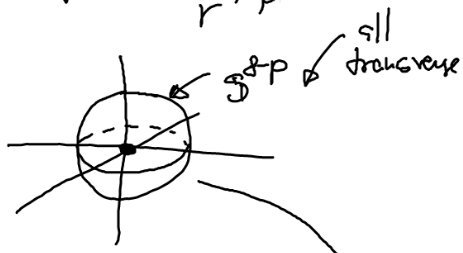
BPS: $Q = M$ $\quad f(r) = \left(1 - \frac{M}{r}\right)^2 = \left(1 + \frac{M}{\rho}\right)^{-2}$; $r = M + \rho$

$$ds^2 = H^{-2} dt^2 - H^2\left(d\rho^2 + \rho^2 d\Omega^2\right) \ ; \quad \big(\underbrace{\partial_1^2 + \partial_2^2 + \partial_3^2}_{\rho,\,\theta,\,\varphi}\big) H(\rho) = \delta^{(3)}(\vec{x})$$

Harmonic function

• 100: In supergravity: critical black p-brane solution (preserve $\frac{1}{2}$ of supersymmetry)

- have $9-p$ transverse directions
- are defined by harmonic function $H_p = 1 + \frac{\mu}{r^{7-p}}$
- interact with $(p+1)$-form potential



charge $- \; q_e = \int_{S^{8-p}} \omega_{(8-p)} = \int_{S^{8-p}} *_{10} F_{(p+2)}$

- point-like in transverse

Supergravity: $g_{MN}, \; b_{MN}, \; \Phi \; ; \; C_M, \quad C_{MNK}$ IIA

$\qquad C \qquad C_{MN} \qquad C_{MNKL}$ IIB

need to specify all these fields

<u>Solutions</u>:
$$ds^2 = H_p^{-\frac{1}{2}} \underbrace{\left(dt^2 - dx^{1\,2} - \ldots - dx^{p\,2}\right)}_{\text{world volume}} - H_p^{\frac{1}{2}} \underbrace{\left(dr^2 + r^2 d\Omega^2_{(8-p)}\right)}_{\text{transverse}}$$

the only non-vanishing gauge field $\} \; C_{01..p} = H^{-1} - 1$

$$e^{-2\Phi} = H^{\frac{p-3}{2}}$$

Charge: $q = \int_{S^{8-p}} * F_{p+2} = \int_{S^{8-p}} \epsilon_{\mu_1 \ldots \mu_{8-p} \nu_1 \ldots \nu_{p+2}} F^{\nu_1 \ldots \nu_{p+2}} dx^{\mu_1} \wedge \ldots \wedge dx^{\mu_{8-p}}$

$$F_{012\ldots pr} = -H^{-2} \partial_r H \; ; \quad F^{012\ldots pr} = H^{\frac{p}{2}} F_{012\ldots pr}$$

$$q \sim \int \underbrace{r^{8-p} d\Omega_{(8-p)}}_{\text{volume}} \cdot \underbrace{H^{-\frac{p}{2}+2}}_{\sqrt{-g}} \cdot \underbrace{\left(-H^{-2}\partial_r H\right) \cdot H^{\frac{p}{2}}}_{F} = (7-p) \cdot \mu \cdot \Omega_{(8-p)}$$

$p = 7$ - special case, log divergence

Such solutions preserve $\frac{1}{2}$ of the total amount of SUSY:

IIA: $\quad g_{\mu\nu} = e_\mu^a e_\nu^b \eta_{ab} \; ; \quad B_{\mu\nu} \; ; \quad \Phi ; \quad C_{(1)}, C_{(3)}$

$$\psi_\mu = \begin{bmatrix} \psi_\mu^+ \\ \psi_\mu^- \end{bmatrix} ; \quad \lambda = \begin{bmatrix} \lambda^+ \\ \lambda^- \end{bmatrix} \; - 32\text{-comp non-chiral spinors}$$

SUSY:

$$\delta_\epsilon e_\mu^a = -i \bar{\epsilon} \Gamma^a \psi_\mu$$
$$\delta_\epsilon B_{\mu\nu} = -2i \bar{\epsilon} \Gamma_\mu \underline{\Gamma_{11} \psi_\nu}$$
$$\vdots$$

$$\delta_\epsilon \psi_\mu = \nabla_\mu \epsilon - \frac{1}{8} \Gamma_{11} H_{\mu\nu\rho} \Gamma^{\nu\rho} \epsilon + \ldots$$
$$\delta \lambda = \left(\not{\partial} \Phi + \frac{1}{12} \Gamma_{11} \not{H}\right) \epsilon + \ldots$$

IIA: $(1 \pm \Gamma_0 \dots \Gamma_p \mathcal{O}_p)\epsilon = 0$ $\qquad \mathcal{O}_p = i(-\Gamma_{11})^{\frac{p+2}{2}}$

IIB: $(1 \pm \Gamma_0 \dots \Gamma_p \mathcal{O}_p)\epsilon^{1,2} = 0$ $\qquad \mathcal{O}_p = P_{\frac{p+3}{2}}$; $P_n = \begin{cases} \sigma^1 & \text{even} \\ i\sigma^2 & \text{odd} \end{cases}$

EG

	0	1	2	3	4	5	6	7	8	9
D3	x	x	x	x	•	•	•	•	•	•

breaks $SO(1,9) \leftarrow SO(1,3) \times SO(6)$
w-v th. $\qquad$ R-sym

$16_s \to (2,1,4) + (1,2,\bar{4})$; $\epsilon^{+}_{1,2} = (\varepsilon^+ \otimes \eta^+)_{1,2} + (\varepsilon^- \otimes \eta^-)_{1,2}$

Gammas:

$4: \hat{\Gamma}_m = \gamma_m \otimes \mathbb{1}$; $\gamma_5 = -i\gamma^0\gamma^1\gamma^2\gamma^3$ $\qquad \gamma_5 \varepsilon^\pm = \pm \varepsilon^\pm$

$6: \hat{\Gamma}_a = \gamma_5 \otimes \Gamma_a$ $\qquad \hat{\Gamma}_0\hat{\Gamma}_1\hat{\Gamma}_2\hat{\Gamma}_3 = i\gamma_5 \otimes \mathbb{1}$; $\gamma_5^2 = 1$

$$\mathcal{O}_3 = P_3 = i\sigma^2$$

$$\underbrace{(1 \pm (i\gamma_5 \otimes \mathbb{1}) \cdot P_3)}_{2\mathbb{P}_\pm} \begin{bmatrix} \epsilon^1 \\ \epsilon^2 \end{bmatrix} = 0$$

$\left.\begin{array}{l} \mathbb{P}_\pm \cdot \mathbb{P}_\pm = \mathbb{P}_\pm \\ \mathbb{P}_+ \cdot \mathbb{P}_- = 0 \end{array}\right\}$ a projector $\Rightarrow$ kills half of the space

the condition implies:

$$\epsilon^1 = i\gamma_5 \otimes \mathbb{1}\, \epsilon^2$$

only one spinor survives

$\epsilon^{1+} = \varepsilon_1^+ \otimes \eta^+ + \varepsilon_1^- \otimes \eta^-$

$\epsilon^A = \begin{bmatrix} \varepsilon_1^{+A} \\ \varepsilon^-_{1A} \end{bmatrix}$ $\qquad \mathcal{N}=4$ SUSY with $SU(4)$ R-symmetry

Theory on D3: $\mathcal{N}=4$ $D=1+3$ SYM

- can we see this dynamically?
- why p can be > 3 if the string spectrum produces only C_0, C_1, C_2, C_3, C_4?

2. Dynamics of D-branes

$$S_{F1} = \int d^2\sigma \left(\sqrt{-h}\, h^{ab} G_{\mu\nu} \partial_a X^\mu \partial_b X^\nu + \epsilon^{ab} B_{\mu\nu} \partial_a X^\mu \partial_b X^\nu \right)$$

gauge transformation: $\delta_\Lambda B_{\mu\nu} = 2\partial_{[\mu}\Lambda_{\nu]}$ – sugra field

$$\delta_\Lambda S_{F1} = 2\int d^2\sigma\, \epsilon^{ab} \partial_\mu \Lambda_\nu \partial_a X^\mu \partial_b X^\nu = 2\int d^2\sigma\, \epsilon^{ab} \partial_a \Lambda_\nu \partial_b X^\nu =$$

$$= 2\int d^2\sigma\, \epsilon^{ab} \partial_a (\Lambda_\nu \partial_b X^\nu) = 2\int d\tau\, \Lambda_\nu \partial_\tau X^\nu \Big|_{\sigma=0}^{\sigma=\pi} =$$

$$= 2\int d\tau\, \Lambda_\alpha \partial_\tau X^\alpha$$; Λ_α – gauge transformation localized on Dp

Compensating field: $\Delta S_{F1} = \int d\tau\, A_\alpha \partial_\tau X^\alpha$

$\delta_\Lambda A_\alpha = -\Lambda_\alpha$; $\delta_\lambda A_\alpha = \partial_\alpha \lambda$ – w-v gauge transf.

$F_{\alpha\beta} = 2\underbrace{\partial_\alpha A_\beta}$; – not gauge inv.

$\mathcal{F}_{\alpha\beta} = 2\underbrace{\partial_\alpha A_\beta} + B_{\alpha\beta}$ – gauge inv. $\quad B_{\alpha\beta} = B_{\mu\nu}\dfrac{\partial X^\mu}{\partial y^\alpha}\dfrac{\partial X^\nu}{\partial y^\beta}$ – projection on the w–v.

$\mathcal{F}_2 = dA_1 + B_2$; A_1 interacts with open string ends

Open string action:

Flux does not give simply charge of the endpoint

$$S = \frac{1}{4\pi\alpha'}\int d^2 z\, \partial X^\mu \bar\partial X_\mu - i\int_0^{2\pi} d\theta\, \dot x^\mu A_\mu\big|_{r=1}$$

Calculate effective action for disk string diagrams

Background field method:

$$X^\mu(z,\bar z) = X_B^\mu(z,\bar z) + X_b^\mu(\theta) =$$
$$= X_{\hat B}^\mu(z,\bar z) + \xi_0^\mu + y(\theta) \,;$$

On the boundary $A_\mu(\xi + y) = -\frac{1}{2}F_{\mu\nu}(\xi)\, y^\nu + \dots$

In the partition function: $\mathcal{D}X = \prod_{z,\bar z\in D\setminus\partial D} dX^\mu \cdot d\xi \cdot \prod_{z,\bar z\in\partial D} dy$

Bulk d.o.f. — center of mass — boundary stringy d.o.f.

Integrate out string oscillations: dX^μ, dy^μ.

Effective action (exact in α', one-disk)

$$S_{eff} = \frac{1}{(4\pi^2\alpha')^5 g_s}\int d^{10}\xi\, \sqrt{\det(\delta_{\mu\nu} + 2\pi\alpha' F_{\mu\nu})}\,;$$

static D9 with w.-v. A_μ field

for general Dp on general bg:

$$S_{Dp} = -T_p\int d^{p+1}\xi\,\sqrt{\det(g_{ab} + 2\pi\alpha'\mathcal{F}_{ab})}\,; \qquad T_p = \frac{1}{\sqrt{\alpha'}}\frac{1}{(2\pi\sqrt{\alpha'})^p}$$

w.-v. $\quad \mathcal{F}_{ab} = 2\underbrace{\partial_a A_b} + \frac{1}{2\pi\alpha'}B_{ab}$;

Pull back: $\quad \left.\begin{aligned} g_{ab} &= \partial_a X^\mu \partial_b X^\nu g_{\mu\nu} \\ B_{ab} &= \partial_a X^\mu \partial_b X^\nu B_{\mu\nu}\end{aligned}\right\}$ interaction with closed strings

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3. Worldvolume gauge theory

choose gauge for w-v diffeos: $X^\mu(\xi) = (\xi^0,\dots,\xi^p, \underbrace{\Phi^1(\xi)\dots\Phi^{9-p}(\xi)}_{\Phi^I})$

background: $G_{\mu\nu} = \eta_{\mu\nu}$, $B_{\mu\nu} = 0$ (bending brane in flat space)

$$g_{ab} = \partial_a\Phi^\alpha\partial_b\Phi^\beta\,\delta_{\alpha\beta} + \eta_{ab}$$

consider D3

$$S = -\frac{1}{(4\pi^2\alpha')^2}\frac{1}{g_s}\int d^4\xi \sqrt{-\det\left(\eta_{ab} + \partial_a\phi^\alpha\partial_b\phi^\beta\delta_{\alpha\beta} + 2\pi\alpha' F_{ab}\right)}$$

$= T_p$; $T_p = \frac{1}{(4\pi^2\alpha')^2 g_s}$ – tension of the brane

Using $\log\det M = \mathrm{Tr}\log M$ and expanding in field values:

$$S = -T_p\pi^2\alpha'^2\int d^4\xi\left(F^2 + \frac{1}{2\pi^2\alpha'^2}\partial\phi^2\right) - T_p\cdot V_{p+1}$$

$\Rightarrow$ $\boxed{\left(\frac{g_{YM}}{2\pi\alpha'}\right)^2 = T_p}$

On D3-brane w.-v. :
- small field fluctuations give SYM
- BG is flat (closed string space)

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4. Coupling to RR potentials

IIA action for: $S_{RR} = \int_{M_{10}}\left[-G_2\wedge * G_2 - G_4\wedge * G_4 + dC_3\wedge dC_3\wedge B_2\right]$

RR fields

$G_2 = dC_1$; $G_4 = dC_3 + H_3\wedge C_1$

gauge transf: $\delta B_2 = d\Lambda_1$

$\delta C_1 = d\lambda_0 \Rightarrow \delta G_2 = 0$

$\delta C_3 = d\lambda_2 + H_3\wedge\lambda_0 \Rightarrow \delta G_4 = 0$

the action is invariant

- $\delta S^{D2}_{WZ} = \int \delta C_3 = \int H_3\wedge\lambda_0 \neq 0$ not gauge inv.
- What are gauge potentials for D4, D6?

new potential

EOM for C_3: $d * G_4 - dC_3\wedge H_3 = 0$ $\quad * G_4 - C_3\wedge H_3 = -dC_5$ D4

$* G_4 = -G_6$; $G_6 = dC_5 + H_3\wedge C_3$

EoM for C_1: $d * G_2 - H_3\wedge * G_4 = 0 = d(\underbrace{* G_2 - H_3\wedge C_5}_{= dC_7}) = 0$ new potential D6

$* G_2 = G_8$

$G_8 = dC_7 + H_3\wedge C_5$

BI for G_{8} (G_2) $\Leftrightarrow$ EoM for G_2 (G_8)

EM duality: MW EoM : $d * F_2 = 0 \leftrightarrow \partial_\mu F^{\mu\nu} = 0$

BI : $dF_2 = 0 \leftrightarrow F_2 = dA_1$

define $* F_2 = \tilde{F}_2 = d\tilde{A}_1$

$dF_2 = 0 \Leftrightarrow d * \tilde{F}_2 = 0$

$d\tilde{F}_2 = 0 \Leftrightarrow d * F_2 = 0$

$q_e = \int * F_2$; $q_m = \int * \tilde{F}_2 = \int F_2$

electric $\qquad$ magnetic

Democratic formulation (pseudo-action)

$$S = \int_{M_{10}} \left[- G_2 \wedge * G_2 - G_4 \wedge * G_4 - G_6 \wedge * G_6 - G_8 \wedge * G_8 \right]$$

1) δC_1 : $\quad -d * G_2 + H_3 \wedge * G_4 = 0$

2) δC_3 : $\quad -d * G_4 + H_3 \wedge * G_6 = 0$

3) δC_5 : $\quad -d * G_6 + H_3 \wedge * G_8 = 0$

4) δC_7 : $\quad -d * G_8 = 0 \longrightarrow d G_2 = 0 \Rightarrow \boxed{G_2 = d C_1}$

additionally impose: $\quad * G_8 = - G_2$, $\quad * G_6 = G_4$, $\quad * G_4 = - G_6$, $\quad * G_2 = G_8$

③ : $0 = - d G_4 - H_3 \wedge G_2 = - d G_4 - H_3 \wedge d C_1 \Rightarrow \boxed{G_4 = d C_3 + H_3 \wedge C_1}$

② : $0 = d G_6 + H_3 \wedge G_4 = d G_6 + H_3 \wedge d C_3 \Rightarrow \boxed{G_6 = d C_5 + H_3 \wedge C_3}$

① : $0 = - d G_8 - H_3 \wedge G_6 = - d G_8 - H_3 \wedge d C_5 \Rightarrow \boxed{G_8 = d C_7 + H_3 \wedge C_5}$

The same equations of motion

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naive expression: $\quad \tilde{S}_{WZ} = \int d^{p+1}\xi \; C_{p+1}$ not gauge invariant under

$$\delta_\lambda C_{p+1} = d\lambda_p + H_3 \wedge \lambda_{p-2}$$

$$\delta_\lambda \tilde{S}_{WZ} = \int d^{p+1}\xi \; H_3 \wedge \lambda_{p-2}$$

$$\hat{S}_{WZ} = \int d^{p+1}\xi \left(C_{p+1} + B_2 \wedge C_{p-1} + \tfrac{1}{2} B_2 \wedge B_2 \wedge C_{p-3} + \dots \right)$$

$$\delta_\lambda \hat{S}_{WZ} = \int d^{p+1}\xi \left(d\lambda_p + \cancel{H_3 \wedge \lambda_{p-2}} + \underbrace{B_2 \wedge d\lambda_{p-2}}_{= d(B_2 \wedge \lambda_{p-2}) - \cancel{H_3 \wedge \lambda_{p-2}}} + B_2 \wedge H_3 \wedge \lambda_{p-4} \right.$$

$\hat{S}_{WZ}$ not gauge inv. under $\quad \delta_\Lambda B_2 = d\Lambda$

fix: $B_2 \longrightarrow \mathcal{F}_2 \longleftarrow$ completely gauge invariant

$$\boxed{S_{WZ} = \int d^{p+1}\xi \left[e^{\mathcal{F}_2} \wedge C \right]_{p+1}}$$ — gauge inv. WZ term.

Lecture 4 Open string on D-branes intersecting at angles

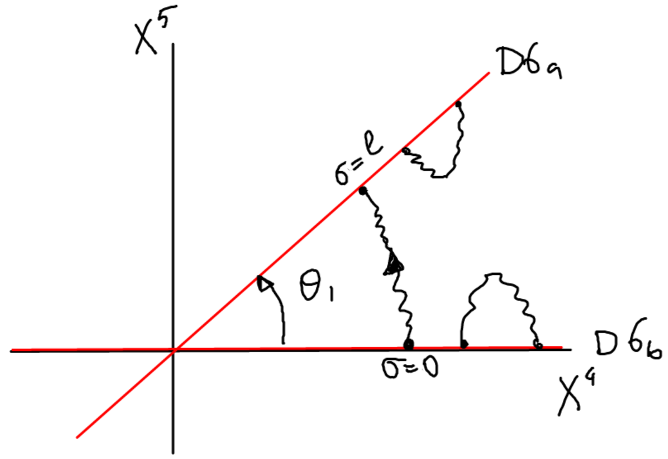
1. Boundary conditions

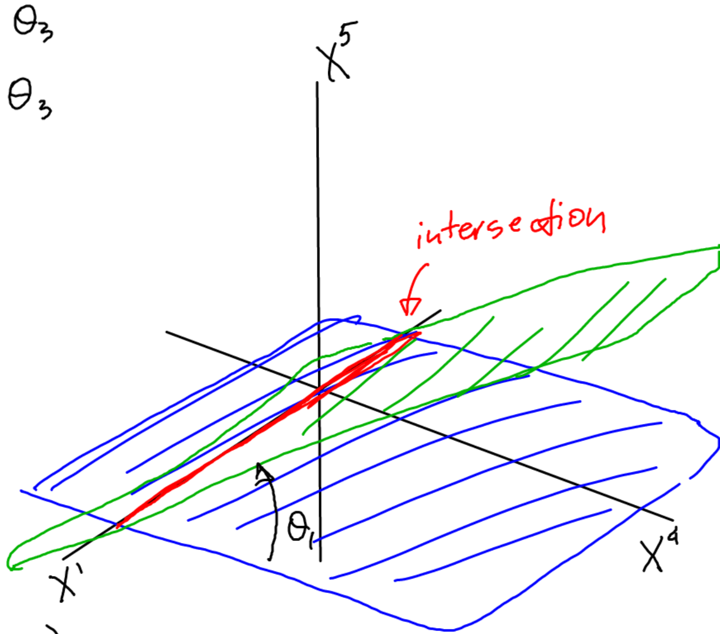
D_p branes carries $D=p+1$ SYM (maximal)

Possible 4dim truncations:
- take D3 → non-chiral theory with too much SUSY
- multiple branes which intersect at $\frac{\pi}{2}$ (possibly unstable)
- intersection at general angle.

2 D6 branes on $\mathbb{R}^{1,9}\times T^2\times T^2\times T^2$

	0	1	2	3	4 5	6 7	8 9
D6a	x	x	x	x	θ_1	θ_2	θ_3
D6b	x	x	x	x	θ_2	θ_3	θ_3





Boundary conditions (string b/w two branes)

$$\left.\begin{array}{ll}\partial_\sigma X^4=0 & N\\ \partial_\tau X^5=0 & D\end{array}\right\}\ \sigma=0$$

← rotated boundary conditions

⇓

twisted string spectrum

$$\left.\begin{array}{ll}\partial_\sigma\left(X^4\cdot\cos\theta_1+X^5\sin\theta_1\right)=0 & N\\ \partial_\tau\left(-X^4\cdot\sin\theta_1+X^5\cos\theta_1\right)=0 & D\end{array}\right\}\ \sigma=\ell$$

at $\sigma=0$: satisfied

$$X^4=i\sqrt{2\alpha'}\sum_n\frac{1}{n}\alpha_n\cos\left(\frac{n\sigma\pi}{\ell}\right)e^{\frac{in\tau}{\ell}}$$

$$X^5=\sqrt{2\alpha'}\sum_n\frac{1}{n}\beta_n\sin\left(\frac{n\sigma\pi}{\ell}\right)e^{\frac{in\tau}{\ell}}$$

$n\in\mathbb{Z}$ does NOT solve

at $\sigma=\ell$:

$$-\alpha_n\sin(\pi n)\cdot\cos\theta_1+\beta_n\cos(\pi n)\sin\theta_1=0$$

$$-\alpha_n\cos(\pi n)\sin\theta_1+\beta_n\sin(\pi n)\cos\theta_1=0$$

looks like sum of angles: ⟶

1) $\alpha_n=+\beta_n \Rightarrow \alpha_n\sin(\pi n-\theta)=0 \Rightarrow n_+=m+\frac{\theta}{\pi},\ m\in\mathbb{Z}$

2) $\alpha_n=-\beta_n \Rightarrow \alpha_n\sin(\pi n+\theta)=0 \Rightarrow n_-=m-\frac{\theta}{\pi}\ \ m\in\mathbb{Z}$

2. Oscillator expansion:

$$X^4 = i\sqrt{2\alpha'} \sum_{n_\pm \neq 0} \left(\frac{\alpha_{n+}}{n_+} \cos\frac{n_+ \sigma \pi}{\ell}\, e^{\frac{i n_+ \tau \pi}{\ell}} + \frac{\alpha_{n-}}{n_-} \cos\frac{n_- \sigma\pi}{\ell}\, e^{-\frac{i n_- \tau}{\ell}\pi} \right)$$

$$X^5 = \sqrt{2\alpha'} \sum_{n_\pm \neq 0} \left(\frac{\alpha_{n+}}{n_+} \sin\frac{n_+ \sigma\pi}{\ell}\, e^{\frac{i n_+ \tau}{\ell}} - \frac{\alpha_{n-}}{n_-} \sin\frac{n_- \sigma\pi}{\ell}\, e^{-\frac{i n_- \tau \pi}{\ell}} \right)$$

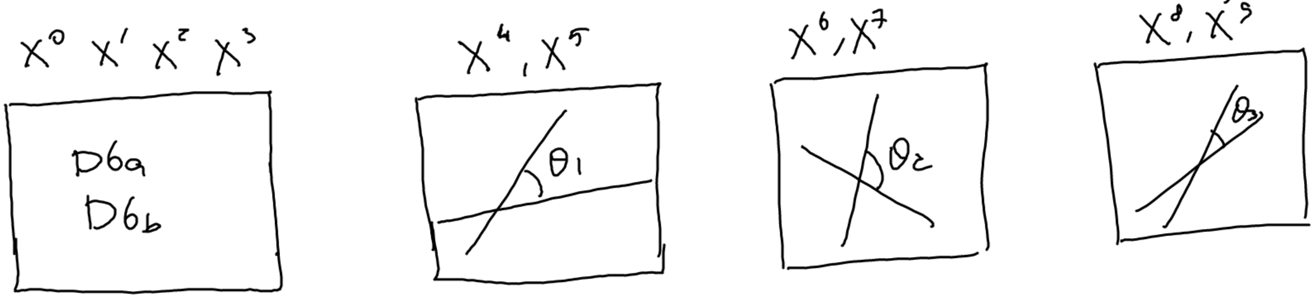
Check: $\theta = 0$: $\alpha_{n+} \pm \alpha_{n-}$ will be osc. in N and D directions

In total one has

1) X^0, X^1, X^2, X^3; $\alpha^a_{-n}, \alpha^a_n$ - Neum. directions, $n \in \mathbb{Z}$
ψ^a_{-r}, ψ^a_r, $r \in \mathbb{Z} + {}^{1/2}_{0}\ {}^{NS}_{R} \to$ ground state ferm.

2) X^4, X^5, θ_1 $\alpha^1_{n_\pm}, \psi^1_{r_\pm}$, $n_\pm = m \pm \frac{\theta_1}{\pi}$, $r_\pm = r \pm \theta_1 \ {}^{+\frac12}_{+0}$
X^6, X^7, θ_2 $\alpha^2_{n_\pm}, \psi^2_{r_\pm}$, $n_\pm = m \pm \frac{\theta_2}{\pi}$, $r_\pm = r \pm \theta_2 + \frac12/0$
X^8, X^9, θ_3 $\alpha^3_{n_\pm}, \psi^3_{r_\pm}$, $n_\pm = m \pm \frac{\theta_3}{\pi}$, $r_\pm = r \pm \theta_3 + {}^{1/2}_{0}$

} in twisted directions $\nu^I = \frac{\theta^I}{\pi}$



Commutation relations: $[\alpha_n, \alpha_m] = n\,\delta_{n+m} \Rightarrow [\alpha_n, \alpha_{-n}] = n > 0$
$\{\psi_r, \psi_s\} = \delta_{r+s}$ raising

$$[\alpha^I_{n_\pm}, \alpha^J_{n'_\mp}] = n_\pm \delta_{n+n'} \delta^{IJ} \quad n \in \mathbb{Z},\ I,J = 1,2,3$$

$$\{\psi^I_{r_\pm}, \psi^J_{s_\mp}\} = \delta_{r+s}\delta^{IJ}; \quad r \in \mathbb{Z}\ {}^{+1/2}_{+0}\ {}^{NS}_{R}$$

string types: aa, bb - localized on D6a & D6b, produce $D=1+6$ SYM
ab, ba - localized on the intersection
produce $D=1+3$ theory

3. Spectrum

$$\alpha' M^2 = N_B + N_F + a\sum_I \nu_I - a; \quad a = \tfrac12\ NS,\ a=0\ R$$

$$N_B = \overbrace{\sum_{n>0} \alpha^a_{-n}\cdot\alpha^a_n}^{d=1+3} + \sum_{I=1}^{3} \left[\sum_{n>0} \left(\alpha^I_{-n_+}\alpha^I_{n_+} + \alpha^I_{-n_-}\alpha^I_{n_-}\right) + \alpha^I_{-\nu I}\alpha^I_{\nu I} \right]$$

$$N_F^R = \sum_{r\in\mathbb{Z}+\frac12} r\,\psi^a_{-r}\psi^a_r + \sum_{I=1}^{3}\left[\sum_{r>0} \left(r_+\psi^I_{-r_+}\psi^I_{r_+} + r_-\psi^I_{-r_-}\psi^I_{r_-}\right) + \theta^I \psi^I_{-\nu^I}\psi^I_{\nu^I} \right]$$

$$N_F^{NS} = \sum_{r>0} r\,\psi^a_{-r}\psi^a_r + \sum_{I=1}^{3} \sum \left(r_+\psi^I_{-r_+}\psi^I_{r_+} + r_-\psi^I_{-r_-}\psi^I_{r_-}\right) \qquad r_\pm = n \pm \nu\ {}^{+\frac12}_{+0}\ {}^{NS}_{R}$$

- NS sector: $\left(\alpha_n^a,\ \psi^a_{n+\frac{1}{2}},\ \alpha^I_{n+},\ \alpha^I_{n-},\ \psi^I_{n\pm\nu+\frac{1}{2}}\right)|0\rangle_{NS}=0$ ground state

Mass $\alpha' M^2$	State	Field	$(-1)^F$
$\frac{1}{2}(\sum_I \nu^I - 1)$	$\|0\rangle_{NS}$	massive scalar τ	-1
$\frac{1}{2}\sum_I \nu^I$	$\psi^a_{-1/2}\|0\rangle_{NS}$	massive vector B^a	$+1$
$\frac{1}{2}(-\nu^1+\nu^2+\nu^3)$	$\psi^1_{-\frac{1}{2}+}\|0\rangle_{NS}$	$\Phi^{1,2,3,4}$ massive scalars	$+1$
$\frac{1}{2}(\nu^1-\nu^2+\nu^3)$	$\psi^2_{-\frac{1}{2}+}\|0\rangle_{NS}$		$+1$
$\frac{1}{2}(\nu^1+\nu^2-\nu^3)$	$\psi^3_{-\frac{1}{2}+}\|0\rangle_{NS}$		$+1$
$1-\frac{1}{2}\sum_I \nu^I$	$\psi^1_{-\frac{1}{2}+\nu^1}\psi^2_{-\frac{1}{2}+\nu^2}\psi^3_{-\frac{1}{2}+\nu^3}\|0\rangle_{NS}$		$+1$

$$\alpha' M^2 \psi^I_{-\underbrace{(1/2\mp\nu^I)}_{r_\pm}}|0\rangle_{NS} = \left(\frac{1}{2}\mp\nu^I\right)+\frac{1}{2}(\nu^1+\nu^2+\nu^3)-\frac{1}{2} =$$

$$=\frac{1}{2}\sum_I \nu^I \mp \nu^I \quad \text{with } - \text{ has lower mass}$$

- R sector

$\{\psi^a_0,\psi^b_0\}=\delta^{ab}$ – 1+3 dim γ-matrices

ground state: $|\alpha\rangle_R$, $|\dot\alpha\rangle_R$ – 1+3 dim chiralities

Mass	State	Field	$(-1)^F$
0	$\|\alpha\rangle_R$	4d spinor left ψ	$+1$
0	$\|\dot\alpha_R\rangle$	4d spinor right $\dot\psi$	-1
$\sum_I \nu^I$	$\psi^1_{-1/2}\psi^2_{-1/2}\psi^3_{-1/2}\|\dot\alpha\rangle_R$	4d massive spinor $\dot\xi$	$+1$
$\sum_I \nu^I$	$\psi^1_{-1/2}\psi^2_{-1/2}\psi^3_{-1/2}\|\alpha\rangle_R$	4d massive spinor ξ	-1

- Open string between $D6_a - D6_b$ provides chiral massless fermion!

- Condition for supersymmetry: D6$_b$: $\epsilon_L = \Gamma^0\Gamma^1\Gamma^2\Gamma^3\Gamma^4\Gamma^6\Gamma^8\epsilon_R$

$\Gamma^{4'}$ is Γ^4 rotated by θ^1 D6$_a$: $\epsilon_L = \Gamma^0\Gamma^1\Gamma^2\Gamma^3\Gamma^{4'}\Gamma^{6'}\Gamma^{8'}\epsilon_R$

$\exists$ SO(6) rotation Θ: D6$_a \longrightarrow$ D6$_b$ $\Rightarrow$ $\Gamma^{4'}\Gamma^{6'}\Gamma^{8'} = \Theta^{-1}\Gamma^{468}\Theta$

SUSY: $\Theta\epsilon = \epsilon$

Subgroup of SO(6) which preserves 1 spinor is SU(3) $\Rightarrow$ $(\theta_1, \theta_2, \theta_3)$ must give an SU(3) element inside SO(6)

$(\theta_1 \pm \theta_2 \pm \theta_3) \bmod 2\pi = 0$ $\boxed{(\nu_1 \pm \nu_2 \pm \nu_3) \bmod 2 = 0}$

1. $\nu_1 + \nu_2 + \nu_3 = 0 \Rightarrow$ massless: (B^a, Ψ, ξ) $\mathcal{N}=1$ vector

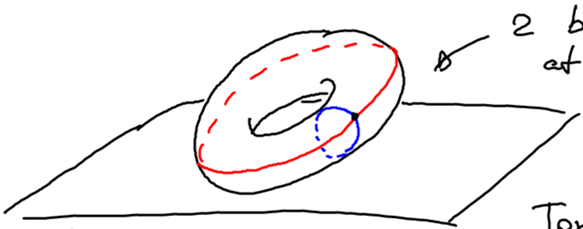
2. other signs & $\nu_1 + \nu_2 + \nu_3 = 2$ massless $\boxed{(\Phi^{1,2,3,4}, \Psi) \quad \mathcal{N}=1 \text{ chiral}}$
   what is needed for SM building

Summary 1:
- brane intersections provide massive particles
- D6 model gives $d=4$ theory with realistic spectrum
- D6 can model $\mathcal{N}=1$ SUSY $\Rightarrow$ MSSM-like models

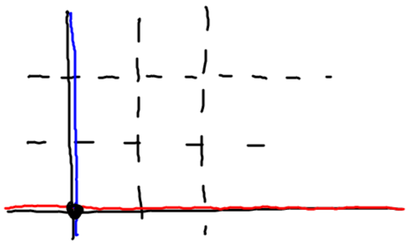
4. Brane wrappings and generations

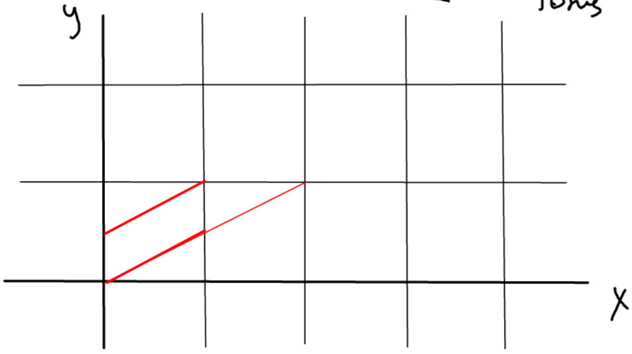
The space: $\mathbb{R}^{1,4} \times T^2 \times T^2 \times T^2$

branes on tori can intersect multiple times, depending on winding numbers



2 branes intersecting at $\frac{\pi}{2}$ angle

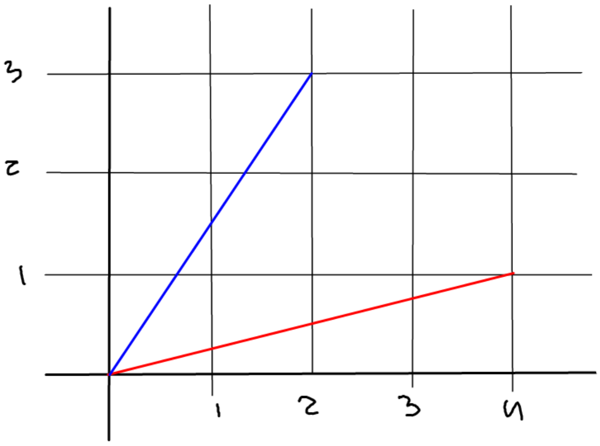




Torus $x \sim x+1$
$y \sim y+1$

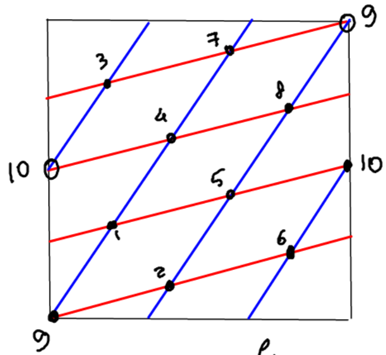
wraps x 2 times
wraps y 1 time

Multiple branes:



denote branes by integer primes $(m,n) \in \mathbb{Z}^2$

$\ell_1 = (4,1)$, $\ell_2(2,3)$ how many intersections?



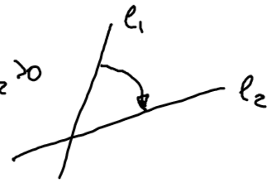
10 intersections

Coincident branes have 0 intersections

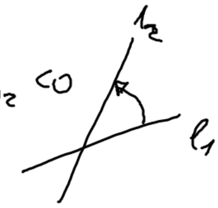
$$I_{12} = \det\left[\ell_2 \;\; \ell_1\right] = \det\begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix}$$

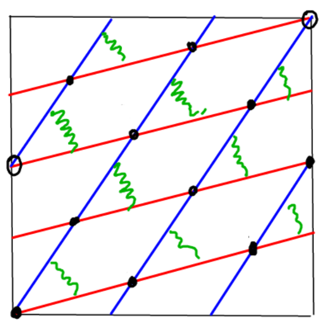
a topological number of the configuration

$I_{12} > 0$



$I_{12} < 0$





- 10 different types of strings b/w 2 branes
- each has the same spectrum
- I_{12} generations
- all localized in 1+3 dimension since tori are small

5. Standard model particle content:

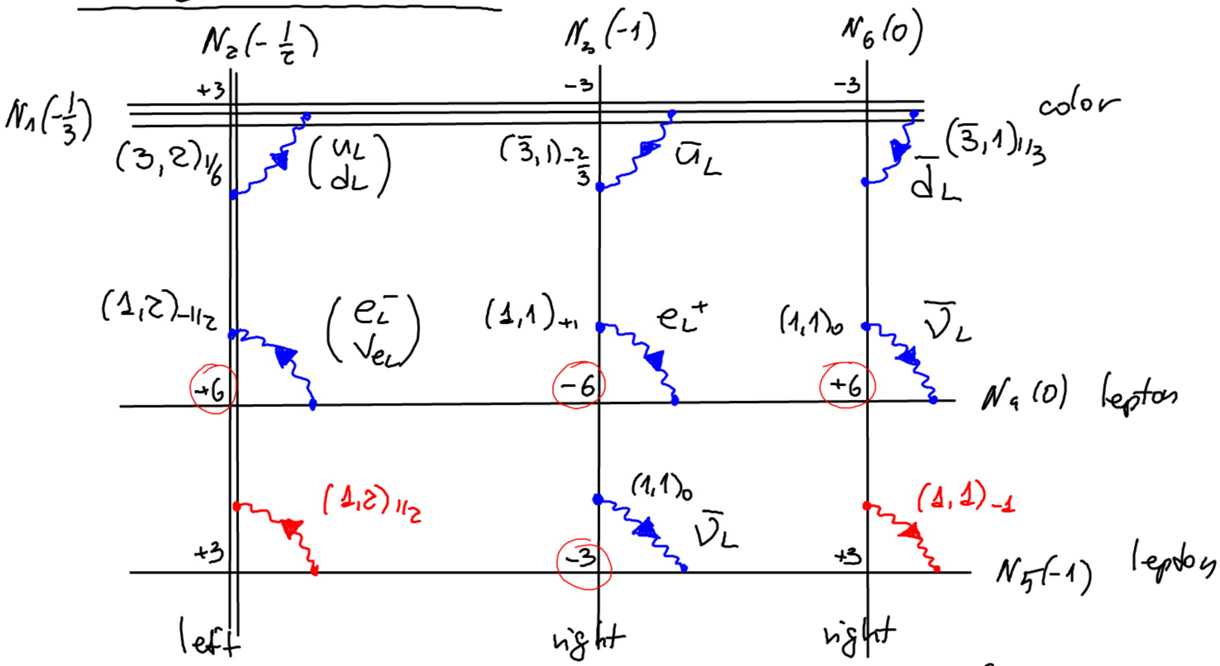
the group is $SU(3)_c \times SU(2) \times U(1)$

$(q_c, q_w)_Y$

multiplet	fields and generations		
$(3,2)_{1/6}$	$\begin{bmatrix} u_L \\ d_L \end{bmatrix}$	$\begin{bmatrix} c_L \\ s_L \end{bmatrix}$	$\begin{bmatrix} t_L \\ d_L \end{bmatrix}$
$(\bar{3},1)_{-2/3}$	$\bar{u}_L$	$\bar{c}_L$	$\bar{t}_L$
$(\bar{3},1)_{1/3}$	$\bar{d}_L$	$\bar{s}_L$	$\bar{d}_L$
$(1,2)_{-1/2}$	$\begin{bmatrix} \nu_{eL} \\ e_L^- \end{bmatrix}$	$\begin{bmatrix} \nu_{\mu L} \\ \mu_L^- \end{bmatrix}$	$\begin{bmatrix} \nu_{\tau L} \\ \tau_L^- \end{bmatrix}$
$(1,1)_1$	e_L^+	μ_L^+	τ_L^+
$(1,1)_0$	$\bar{\nu}_{eL}$	$\bar{\nu}_{\mu L}$	$\bar{\nu}_{\tau L}$
$(1,2)_{-1/2}$	h		

- 3 generations
- 3 gauge groups
- left and right have different properties
- Y is different

6. D6 brane model



$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$: • ends on $N_1 = 3$ color branes, starts on $N_2 = 2$ left branes $\Big\}$ $(3,2)$

• $Y = -\frac{1}{3} - \left(-\frac{1}{2}\right) = -\frac{1}{3} + \frac{1}{2} = \frac{1}{6}$

• intersection numbers:

branes		T_1^2	T_2^2	T_3^2
color	$N_1 = 3$	$(1,2)$	$(1,-1)$	$(1,-2)$
left	$N_2 = 2$	$(1,1)$	$(1,-2)$	$(-1,5)$
right	$N_3 = 1$	$(1,1)$	$(1,0)$	$(-1,5)$
leptonic	$N_4 = 1$	$(1,2)$	$(-1,1)$	$(1,1)$
leptonic	$N_5 = 1$	$(1,2)$	$(-1,1)$	$(2,-7)$
right	$N_6 = 1$	$(1,1)$	$(3,-4)$	$(1,-5)$

$$I_{12} = \det\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \times \det\begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix} \times \det\begin{bmatrix} 1 & -1 \\ -2 & 5 \end{bmatrix} =$$

$$= (1-2) \times (-2+1) \times (5-2) = 3$$ generations of left quarks

- Spectrum is reproduced
- RR cancellation condition is satisfied
- Adding orientifolds gives image branes ⇒ reduces extra states
- need electroweak symmetry breaking (brane rotation and recombination)
- still huge choice in brane wrapping numbers, non-dynamical
- sizes of tori and moduli stabilization
- Hierarchy problem

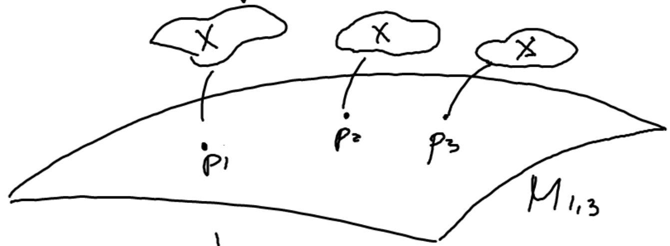
Lecture 5. Cosmological models with branes

String + SUGRA compactification problem : $10 = 4 + 6$

- issues with compactification
- cosmological issues

1) Issues with compactification

Choose space as $M_{1,3} \times X_6$

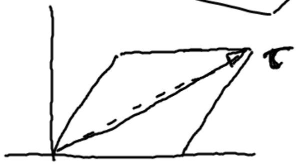


Moduli $\to$ fields on $M_{1,3}$

EG: $\mathrm{Vol}(X)\,(x^0, x^1, x^2, x^3)$

$\uparrow$ volume modulus

EG: $\mathbb{T}^2$

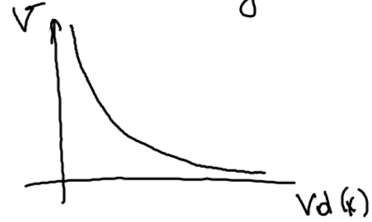


$\leftarrow$ complex structure modulus on $\mathbb{T}^2$

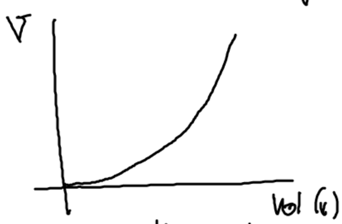
the way of defining complex coord. on $\mathbb{T}^2$

moduli for CY's :
- complex structure
- Kähler $\leftarrow$ way of defining $\omega_{(2)}$ on the manifold

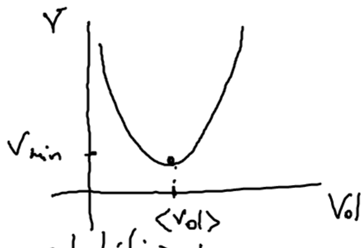
$\searrow$ in 4d theory = scalar fields with a potential



decompactify



breaks the model



stabilized

One (or many) of the moduli acts as the inflaton

2) Cosmological constraints

- $N \sim 60$ e-foldings $\frac{a}{a_0} \sim e^{60}$
- slow-roll single field inflation

$$\mathcal{L} = \sqrt{-g}\left(-\frac{1}{2}(\partial\varphi)^2 - V(\varphi)\right)$$

slow-roll: $\begin{cases} \frac{1}{2}\dot{\varphi} \ll V & \text{- kinetic term does not contr.} \\ \ddot{\varphi} \ll H|\dot{\varphi}| & \text{- large friction} \end{cases}$

$$\left.\begin{aligned} \varepsilon &= \frac{1}{2}\left(\frac{V' M_{Pl}}{V}\right)^2 \ll 1 \\ \eta &= \frac{V'' M_{Pl}^2}{V} \; ; \; |\eta| \ll 1 \end{aligned}\right\} \text{ conditions for slow-roll inflation}$$

Inflaton candidates: fields from

- distance between branes
- orientation of branes

} open string (brane inflation)

- size and deformations of the internal manifold
- axion field

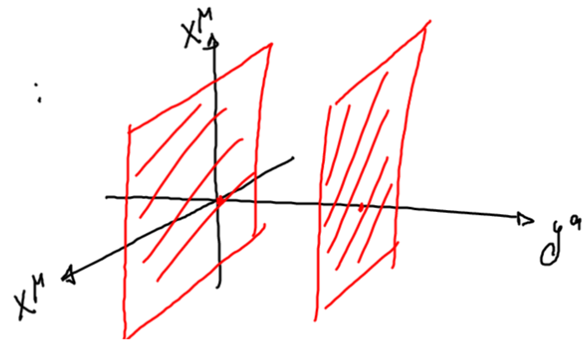
} closed string (moduli inflation)

2. D3-$\overline{D3}$ brane inflation

open string between D3 branes

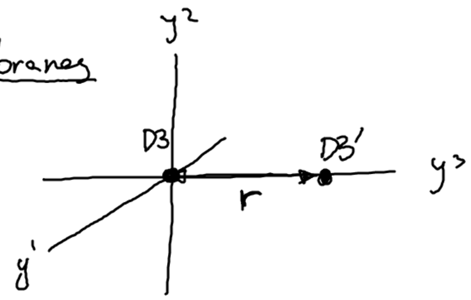
Mass	field	GSO
$\frac{1}{4\pi\alpha'}(\Delta X)^2 - \frac{1}{2}$	X	$-$
0	A^μ	$+$
0	ψ	$+$
0	$\dot{\psi}$	$-$

set-up:



GSO $=-1$: D3-$\overline{D3}$ unstable config.

Estimate potential between D3 ($\overline{D3}$) branes



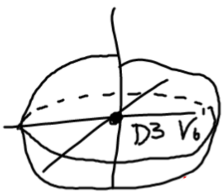
- gravitational:

$$g_{00} = H^{-1/2} \sim 1 - \frac{M}{2r^4}$$

$$V_g = -\frac{M}{4r^4}$$

- from RR charge

$$*d*F_5 = j_4 \Rightarrow \int_{V_6} dF_5 = \int_{V_6} *j_4 = \sum_i Q_i \text{ inside } V_6$$



1. If V_6 is compact $\Rightarrow \partial V_6 = 0 \Rightarrow \sum_i Q_i = 0$
charges should be compensated inside the compact space (transverse)

2. If the transverse space is non-compact
The potential $V_R = \pm \frac{M}{4r^4}$;

$$V_{gR} \sim -\frac{M}{r^4}(1 \pm 1)$$ D3-$\overline{D3}$ / D3-D3 do not interact

$\overline{D3}$ moves towards the D3 brane
distance r can be inflaton

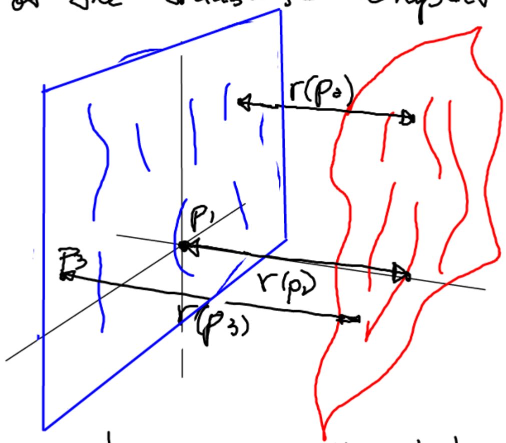
total: D3-$\overline{D3}$ $\boxed{V_{tot} = T_3\left(1 - \frac{1}{8\pi^2 r^4 T_3}\right)}$

$M_s^2 = \alpha'^{-1}$ (mass); $l_s = 2\pi\sqrt{\alpha'}$ (length); $T^{-1} = 2\pi\alpha'$ (tension of string) $T_3 = \frac{1}{(2\pi)^3 \alpha'^2 g_s}$ — tension of the D3 brane

The potential is valid when:

$M_s^{-1} \ll r \ll d$, d - size of the transverse compact space

M_s^{-1}: instability; d: simple geometry



brane is allowed to have small fluctuations

Kinetic term for the inflaton

inflaton field $r(x^\mu)$

DBI action:

$$S_{DBI} = -T_3 \int d^4\xi \sqrt{|\det h_{\alpha\beta}|}$$

$$h_{\alpha\beta} = \partial_\alpha X^M \partial_\beta X^N G_{MN}$$

G_{MN}: Minkowski external space

gauge fix $\xi^\alpha = x^\mu$

$X^M = (x^\mu, \bar{y}^a)$

$\bar{y}^a$: coord. of the antibrane

$$\det\left(\eta_{\alpha\beta} + \partial_\alpha \bar{y}^a \partial_\beta \bar{y}^b \delta_{ab}\right)$$

$$= 1 + \eta^{\alpha\beta}\partial_\alpha \bar{y}^a \partial_\beta \bar{y}^b \delta_{ab} + \dots$$

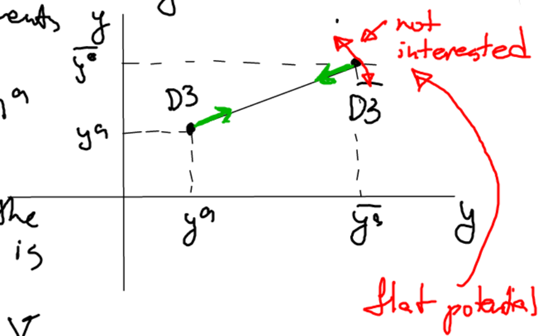
$$= 1 + \eta^{\alpha\beta}\partial_\alpha r\, \partial_\beta r$$

$r(x)^2 = (\bar{y}^a - y^a)^2$;

interested only in transverse movements

$\bar{y}^a - y^a = r \cdot n^a$

only one modulus of the configuration is relevant



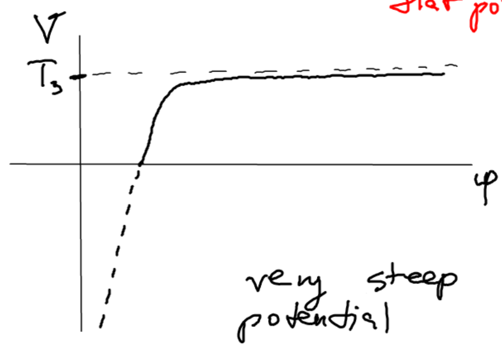
$$\mathcal{L} = -T_3\left(1 + \frac{1}{2}(\partial r)^2\right) + \frac{T_3}{8\pi^4 r^4}$$

$$\phi = \sqrt{T_3} \cdot r$$

$$\boxed{\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - \left(T_3 - \frac{C}{\phi^4}\right)}$$

$C = \dfrac{T_3^2}{8\pi^2}$

chaotic inflation



The tachyon: $\alpha' m_\chi^2 = \dfrac{1}{4\pi\alpha' T_3}\varphi^2 - \dfrac{1}{2}$

brane-antibrane annihilation ends inflation and kills the inflaton

Slow-roll parameter $\eta = \dfrac{V'' M_{Pl}^2}{V} \simeq -\dfrac{20 C M_{Pl}^2}{T_3 \phi^6}$

4 dim Planck mass: $M_{Pl}^2 = (8\pi G_N)^{-1}$; $\dfrac{1}{16\pi G_{10}}\int d^{10}x\sqrt{-g_{10}}R_{10} \to \dfrac{1}{16\pi G_N}\int d^4x \sqrt{-g_4}R_4$

$G_{10}^{-1} = G_N^{-1} \cdot \mathrm{Vol}_6$; $G_{10}^{-1} = T_3^2 2\pi$ ← from field theory limit of ST

$M_{Pl}^2 = \pi^2 T_3^2 \mathrm{Vol}_6$

all together: $\eta \approx 2\dfrac{\mathrm{Vol}_6}{r^6} \simeq \left(\dfrac{L}{r}\right)^6$ $\quad r \gg L$ brane movement larger than space size

3. Warped brane models

Way out of the large η problem: consider warped geometries
- closed string moduli (fluxes, geometry)
- additional sources (D-branes, O-planes)

Full 10D geometry: $ds^2 = h(y)^{-1/2} g_{\mu\nu} dx^\mu dx^\nu + \underbrace{h(y)^{1/2} \tilde{g}_{mn}(y)}_{g_{mn}(y)} dy^m dy^n$ internal metric

IIB theory:

$$S_{IIB} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{g} \left[R - \frac{\partial_\mu \tau \partial^\mu \bar{\tau}}{2(\mathrm{Im}\,\tau)^2} - \frac{G_3 \wedge * \bar{G}_3}{12\,\mathrm{Im}\,\tau} - \frac{F_5 \wedge * F_5}{4 \cdot 5!} \right]$$

$$+ \frac{1}{8 i \kappa_{10}^2} \int \frac{1}{\mathrm{Im}\,\tau} C_4 \wedge G_3 \wedge \bar{G}_3 \; ; \qquad G_3 = F_3 - \tau H_3; \quad \tau = C_0 + i e^{-\phi}$$

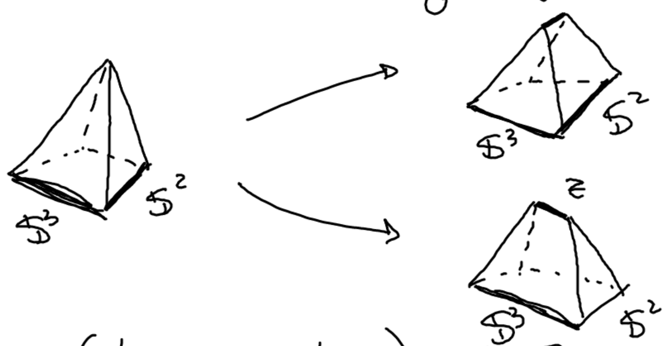
$H_3 = dB_2$; $F_3 = dC_2$; $F_5 = dC_4 + \frac{1}{2} B_2 \wedge F_3 - \frac{1}{2} C_2 \wedge H_3$ – gauge inv. field str.

$dF_5 = H_3 \wedge F_3$ – important for brane back reaction

Very particular class of CY_3: with long throat

Conifold $\sum_{i=1}^{4} w_i^2 = 0$ conical singularity

resolution:



$\Rightarrow \sum_{i=1}^{4} w_i^2 = z \neq 0$ complex structure modulus

Approximately:
$ds^2 = h^{1/2}(r)\left(dr^2 + r^2 ds^2_{X_5}\right)$ $\quad \underbrace{X_5 = S^3 \times S^2}$ $= \frac{SU(2)\times SU(2)}{U(1)} = T^{1,1}$ coset space

To keep EoM's solved:

$$\frac{1}{4\pi^2\alpha'} \int_A F_3 = M \in \mathbb{Z} \; ; \qquad \frac{1}{4\pi^2\alpha'} \int_B H_3 = -K \in \mathbb{Z}$$

A, B – 3-cycles; $\quad A: \underbrace{\sum_{i=1}^{4} x_i^2 = z}_{3\text{ dim}} \quad ; \quad B: \underbrace{x_4^2 - \sum_{i=1}^{3} y_i^2 = z}_{3\text{ dim}}$;

$w_i = x_i + i y_i$

These stabilize $z = e^{-\frac{2\pi K}{M g_s}} =: a^3(r_0)$

The conifold geometry becomes precisely

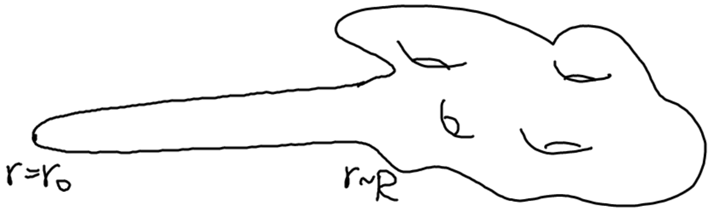
$$ds^2 = h(r)^{-1/2}\, dx^\mu dx_\mu + h(r)^{1/2}\left(dr^2 + r^2 ds^2_{X_5}\right) \quad ; \quad h(r) \simeq \frac{R^4}{r^4}$$

$$R^4 = \frac{27}{4}\pi g_s M K \alpha'^2$$

New information in the model

- M, K – amount of flux
- $a(r_0)$ – value of the complex structure modulus z

} change potential for the inflaton



$r_0 < r < R$ – one is in the throat

As before: • add D3 } moves
• add $\overline{D3}$

1) D3 in the throat does not feel a force

D3 interacts with $F_5 \Rightarrow$ $dF_5 = H_3 \wedge F_3 \neq 0$!

$$S = -\tau_3 \int d^4x\; h(r)^{-1}\sqrt{1 - h(r)(\partial r)^2} \pm \tau_3 \int d^4x\; C_{0123}$$

EoMs imply: $\quad C_{0123} = h(r)^{-1}\epsilon_{0123}$

for static brane: $\quad S \sim -T_3(1 \pm 1)\left(\frac{r_1}{R}\right)^4 \int d^4x$ ← the potential

D3 brane floats

$\overline{D3}$ drains into the throat

2) Interaction energy beetween D3 and $\overline{D3}$ branes:

branes perturb the background: $\quad h(r) \to h(r) + \delta h(r)$

$\qquad C_4(r) \to C_4(r) + \delta C_4(r)$

EoM's: $\quad \Delta_6 \delta h = \frac{R^4}{N}\delta^6(\vec{r} - \vec{r}_1) \quad ; \quad C_{0123} = h(r)^{-1}\epsilon_{0123}$

↳ $\quad \delta h = \frac{R^4}{N r_1^4} \quad ; \quad \delta C_4 = -\frac{\delta h}{h^2} \; ; \quad$ define $\varphi = \sqrt{T_3}\, r_1$

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - 2T_3 a_0^4\left(1 - a_0^4 \frac{R^4 T_3^2}{N\phi^4}\right)$$

$\eta = a_0^4 \cdot \eta_0 \; ; \quad \epsilon = a_0^8 \epsilon_0$ ← warped parameters

a_0 can be chosen to keep η & ϵ within the slow-roll range

4. <u>η - problem</u>

Stabilize moduli of the model

- U^a - complex structure of CY → stabilized by fluxes F_3, H_3
- τ - axio-dilaton → stabilized by F_3, H_3
- T - Kähler modulus; $\mathrm{Im}\, T \sim (\mathrm{Vol}\, M_6)^{2/3}$ unstabilized

↳ massless scalar field.

$\underbrace{\text{KKLT} \to \text{instanton corrections} \to \eta \sim \frac{2}{3}}_{\eta\text{-problem}}$ → need a mechanism to fix this

Addendum. SU(4) and SO(6)

Consider $\mathbb{C}^6 \simeq so(4,\mathbb{C}) := \{ A \in Mat_{4\times 4}\,\mathbb{C}\,;\ A^{Tr} = -A \}$

$$\underset{\underset{\mathbb{C}\quad\mathbb{C}}{\uparrow\quad\uparrow}}{\overset{\shortparallel}{(x^a, y^a)}} \quad \text{complex. coord.} \qquad \text{defined as} \begin{cases} A^{ab} = \epsilon^{abc}(x_c - i y_c) \\ A^{a4} = x^a + i y^a \end{cases}$$

group action of $SL(4,\mathbb{C})$:

$SL(4,\mathbb{C}) : so(4;\mathbb{C}) \longrightarrow so(4;\mathbb{C})$; action on $\overset{\mathbb{C}^6}{so(4,\mathbb{C})}$

$$\rho(g)(A) := g A g^{Tr}$$

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two signs are possible

$$\mathbb{R}^6 \longleftrightarrow \underset{\text{Hodge}}{*A} = \underbrace{\pm A^{+}}_{= A^*} \Rightarrow \left.\begin{array}{l} (*A)^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}A_{\rho\sigma} \\ \shortparallel \\ -A^{\nu\mu *} = A^{\mu\nu *} \end{array}\right\} so^{-}(4,\mathbb{C})$$

$$\Downarrow$$

Restriction induces action of SU(4) on $\mathbb{R}^6$ and covering of $SO(6,\mathbb{R})$

$$x^a + i y^a = x^{a*} + i y^{a*}$$

can choose $x^a, y^a \in \mathbb{R}$

$$(x^1, y^1, x^2, y^2, x^3, y^3) \to \mathbb{R}^6$$

We are interested in rotations: θ_1 in (x_1, y_1), θ_2 in (x_2, y_2), θ_3 in (x_3, y_3)

$A'_{\mu\nu}$ is made of $\quad x'_a + i y'_a = e^{i\theta_a}(x_a + i y_a) \leftarrow$ SO(6) rotation

The corresponding SU(4) matrix is recovered as $A' = g . A . g^{Tr}$, which is

$$g = \mathrm{diag}\left[e^{\frac{1}{2} i(\theta_1 - \theta_2 - \theta_3)},\ e^{-\frac{i}{2}(\theta_1 - \theta_2 + \theta_3)},\ e^{-\frac{i}{2}(\theta_1 + \theta_2 - \theta_3)},\ e^{\frac{i}{2}(\theta_1 + \theta_2 + \theta_3)} \right]$$

setting any of the entries to 1 gives an SU(3) matrix