Kerr black holes with synchronised scalar hair and boson stars

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Outline

- Black hole hair
- Kerr black holes with scalar hair
- Further developments
- Problem setup
- Odd-parity scalar hair
- Friedberg-Lee-Sirlin hair
- Nonlinear sigma-model hair
- Skyrme hair
- Dirac stars
- Summary

Black hole uniqueness theorems

Israel: static electrovacuum black hole is spherically symmetric ⇒ Reissner-Nordström black hole is a unique static electrovacuum solution

Carter, Robinson, Mazur & Bunting: Kerr-Newman black hole is a unique stationary electrovacuum solution ⇒ Any electrovacuum black hole is described only by M, Q, J ← all other characteristics are "hair"

⇓

"Black holes have no hair"
J.A. Wheeler

(at least electrovacuum)

No-hair conjecture

"The only allowed characteristics of black hole are those associated with Gauss law"

Bekenstein: no free massive scalar, vector or spin-2 hair

Chase: no massless scalar hair

Teitelboim: no weakly and strongly interacting hair

Sudarsky: no Einstein-Higgs hair with arbitrary number of scalar fields and potential

Finster, Smoller & Yau: no Dirac hair

Heusler: no sigma-model hair

et cetera

"Black holes have hair"

M. Volkov & D. Galtsov,
K. Lee, V. Nair & E. Weinberg,
S. Droz, M. Heusler & N. Straumann,
G. Lavrelashvili & D. Maison,
T. Sotiriou & S.-Y. Zhou,
E. Babichev & C. Charmousis,
C. Herdeiro & E. Radu,
N. Bocharova, K.Bronnikov & V. Melnikov,
B. Kleihaus & J. Kunz
etc.

Yang-Mills hair

Skyrme hair

Yang-Mills-dilaton hair

Horndeski hair

Conformal scalar hair

Yang-Mills-Higgs hair

Galileon hair

Complex scalar and Proca hair

Nonabelian hair on spinning black holes

PRL 112, 221101 (2014) PHYSICAL REVIEW LETTERS week ending 6 JUNE 2014

Kerr Black Holes with Scalar Hair

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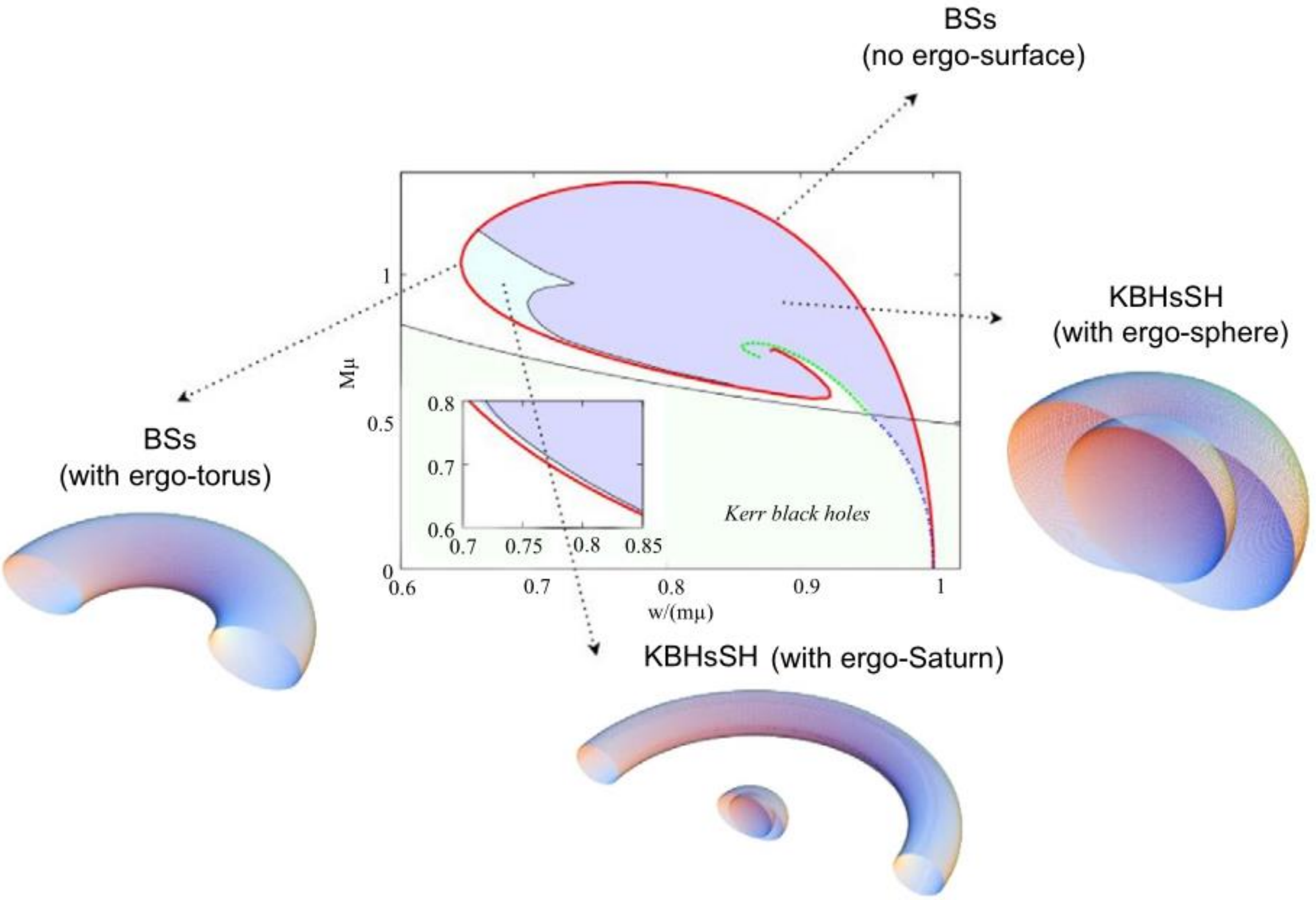
(Received 13 March 2014; revised manuscript received 23 April 2014; published 2 June 2014)

$$R_{\mu\nu} - \frac{1}{2} R \, g_{\mu\nu} = 8\pi T_{\mu\nu}$$
$$\partial_\mu \partial^\nu \Psi = \mu^2 \Psi$$



Further developments

Q-clouds around Kerr black hole

Herdeiro, Radu, Runarsson, 2014

Myers-Perry black hole with scalar hair

Brihaye, Herdeiro, Radu, 2014

Kerr black hole with self-interacting scalar hair

Herdeiro, Radu, Runarsson, 2015

Kerr black holes with Proca hair

Herdeiro, Radu, Runarsson, 2016

Kerr-Newman black hole with scalar hair

Delgado, Herdeiro, Radu, Runarsson, 2016

Dirac stars

Herdeiro, Pombo, Radu, 2017

BTZ black hole with scalar hair

Ferreira, Herdeiro, 2017

Spinning black holes in dynamical Chern-Simons gravity

Delsate, Herdeiro, Radu, 2018

Problem setup

$$\begin{cases} R_{\mu\nu} - \frac{1}{2} R \, g_{\mu\nu} = 8\pi G T_{\mu\nu} \\ \dfrac{\delta\left(\sqrt{-g}\mathcal{L}_{matter}\right)}{\delta\phi} = 0 \end{cases}$$

$$ds^2 = -F_0(r,\theta)dt^2 + F_1(r,\theta)(dr^2 + r^2 d\theta^2) + F_2(r,\theta) r^2 \sin^2\theta \left(d\varphi - \frac{W(r,\theta)}{r} dt\right)^2$$

$F_0 = \frac{\left(1-\frac{r_h}{r}\right)^2}{\left(1+\frac{r_h}{r}\right)^2} e^{f_0}, F_1 = \left(1+\frac{r_h}{r}\right)^4 e^{f_1}, F_2 = \left(1+\frac{r_h}{r}\right)^4 e^{f_2}$ ⇒ solitons and black holes can be treated simultaneously

$f_0|_{r=\infty} = f_1|_{r=\infty} = f_2|_{r=\infty} = W|_{r=\infty} = 0$ — asymptotic flatness
$\partial_r f_0|_{r=r_h} = \partial_r f_1|_{r=r_h} = \partial_r f_2|_{r=r_h} = 0$ — regularity on horizon
$W|_{r=r_h} = r_h\omega$— synchronization condition
$\partial_\theta f_0|_{\theta=0} = \partial_\theta f_1|_{\theta=0} = \partial_\theta W|_{\theta=0} = 0$ — symmetry axis regularity
$f_1|_{\theta=0} = f_2|_{\theta=0} = 0$ — no conical singularity condition
$\partial_\theta f_0|_{\theta=\frac{\pi}{2}} = \partial_\theta f_1|_{\theta=\frac{\pi}{2}} = \partial_\theta f_2|_{\theta=\frac{\pi}{2}} = \partial_\theta W|_{\theta=\frac{\pi}{2}} = 0$ — reflection symmetry

Kerr black holes with odd-parity scalar hair

$$\mathcal{L}_{matter} = \left|\partial_\mu \phi\right|^2 + \mu^2 |\phi|^2$$

$$j^\mu = i(\phi \partial^\mu \phi^* - \phi^* \partial^\mu \phi)$$

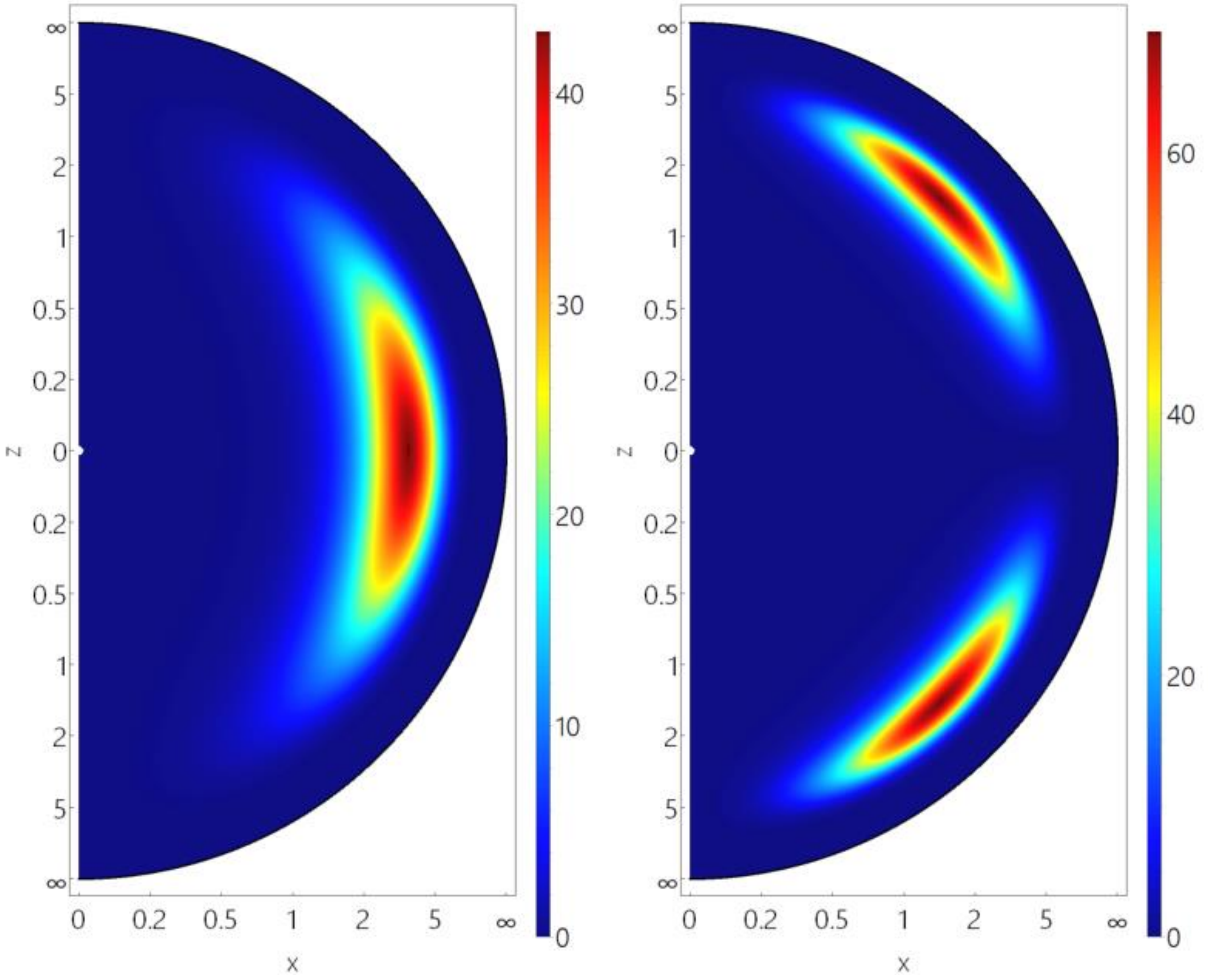
$$\phi = f(r,\theta) e^{i(\omega t + n\varphi)}$$

$$f\Big|_{r=\infty} = 0$$

$$\partial_r f\Big|_{r=r_h} = 0$$

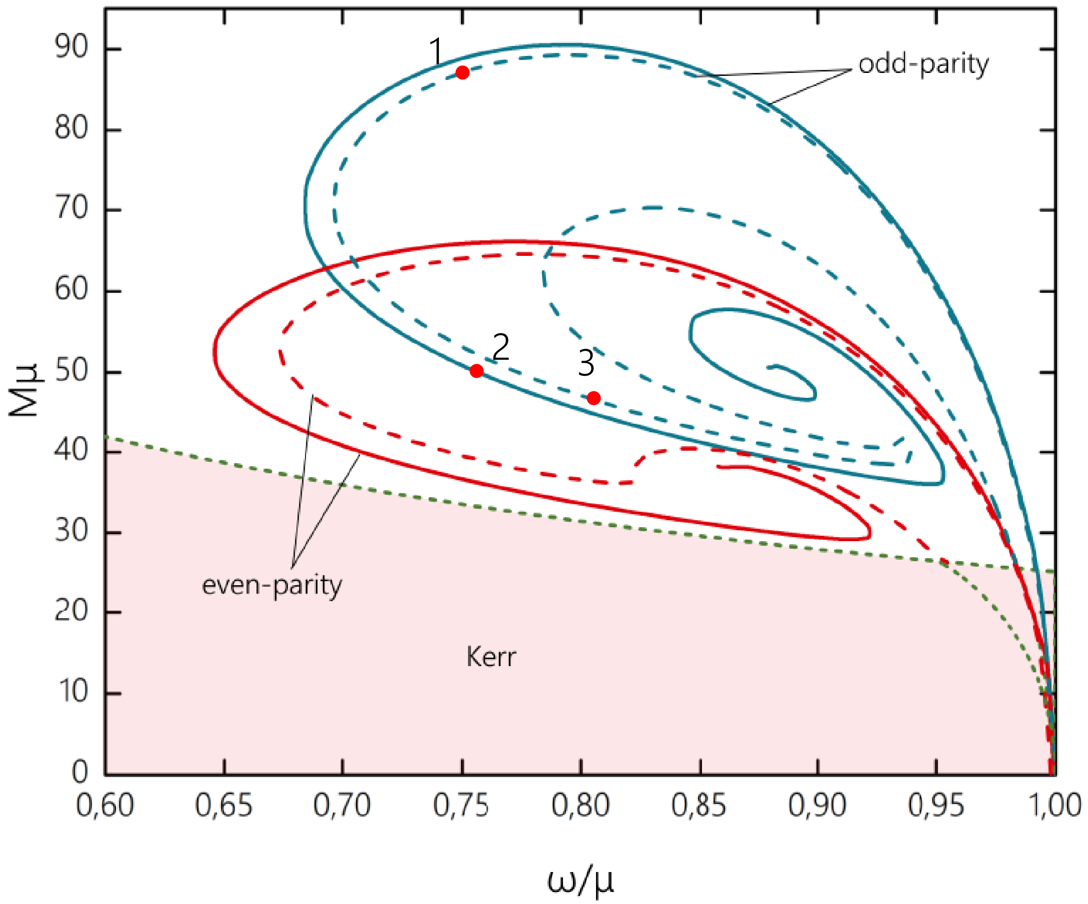
$$f\Big|_{\theta=0,\pi} = 0$$

$$f\Big|_{\theta=\frac{\pi}{2}} = 0$$

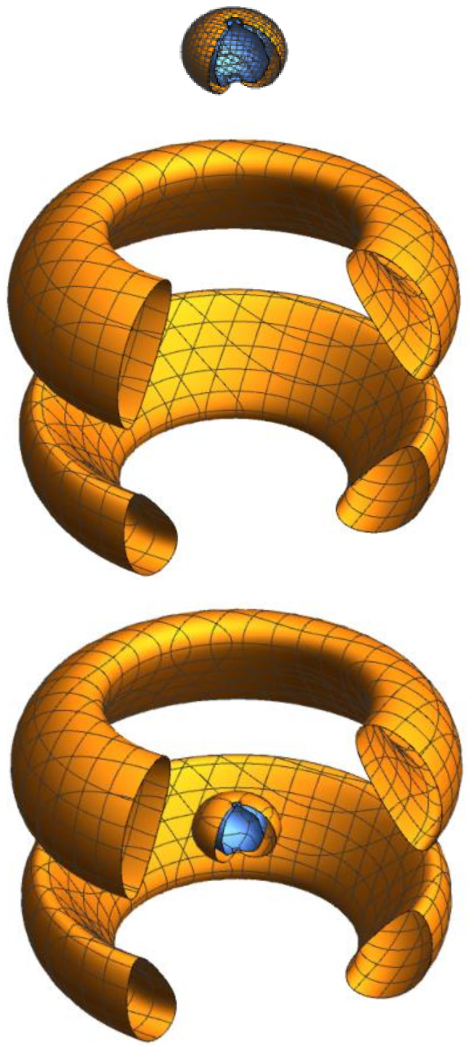


(Kunz, Perapechka, Shnir: arXiv:1904.07630)

Kerr black holes with odd-parity scalar hair



ergosurfaces: $g_{tt} = 0$

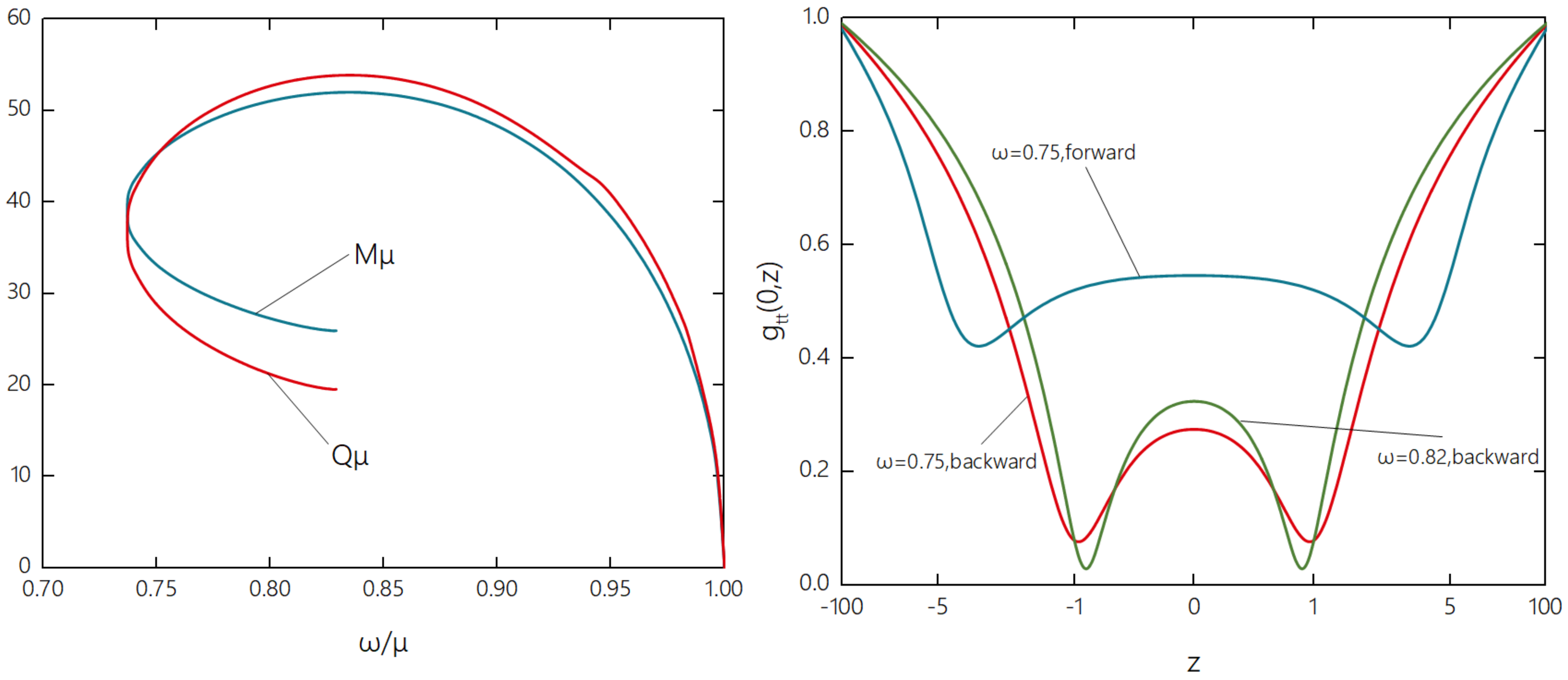


ergosphere $= S^2$

ergo-double-tori
$= (S^1 \times S^1) \oplus (S^1 \times S^1)$

ergo-double-Saturn
$= S^2 \oplus (S^1 \times S^1) \oplus (S^1 \times S^1)$

Double boson stars



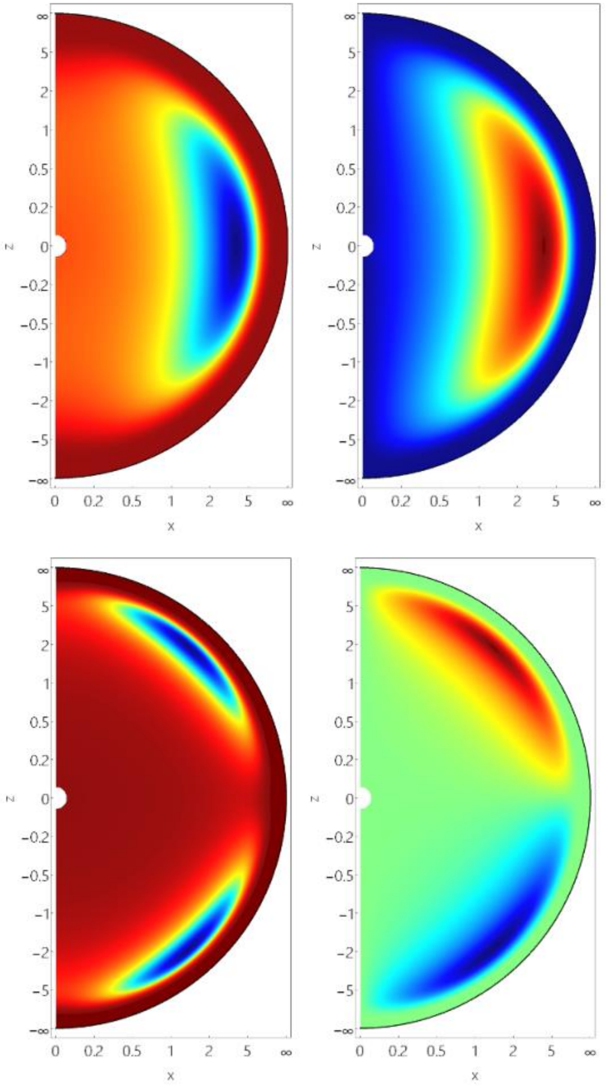
Kerr black holes with Friedberg-Lee-Sirlin hair

$$\mathcal{L}_{matter} = \frac{1}{2}\left(\partial_\mu \psi\right)^2 + \left|\partial_\mu \phi\right|^2 + m^2\psi^2|\phi|^2 - \mu^2(\psi^2 - v^2)^2$$

$$j^\mu = i(\phi\partial^\mu\phi^* - \phi^*\partial^\mu\phi)$$

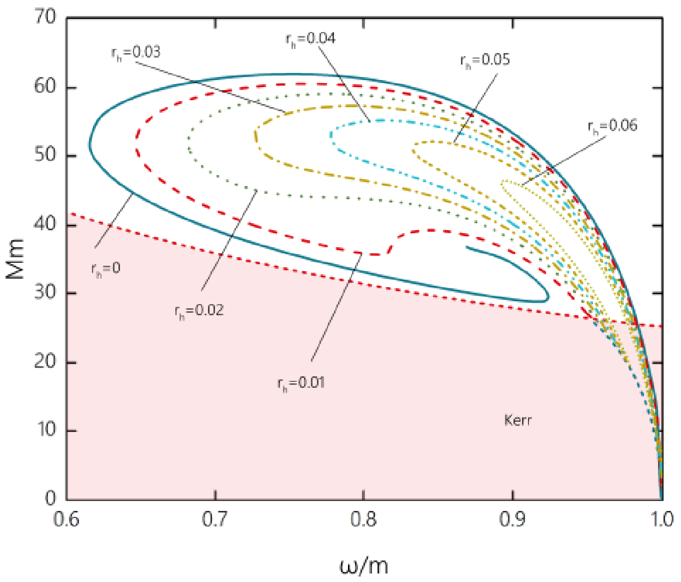
$$\psi = X(r,\theta)$$
$$\phi = Y(r,\theta)e^{i(\omega t + n\varphi)}$$

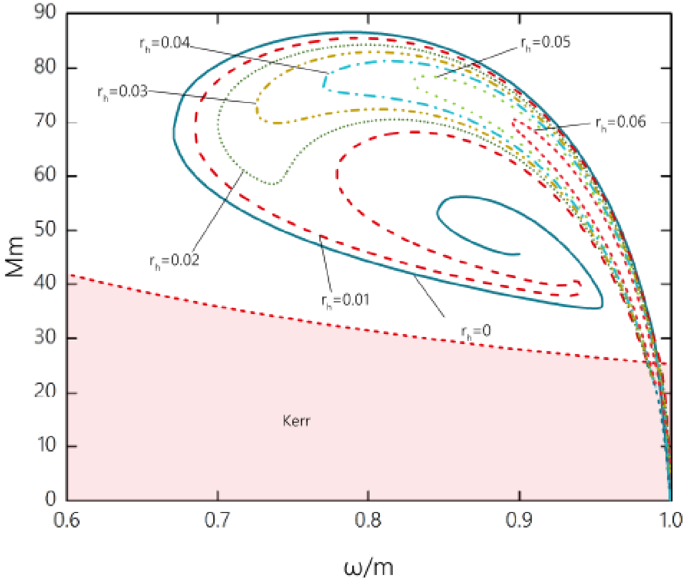
$X\Big\vert_{r=\infty} = v$	$Y\Big\vert_{r=\infty} = 0$
$\partial_r X\Big\vert_{r=r_h} = 0$	$\partial_r Y\Big\vert_{r=r_h} = 0$
$\partial_\theta X\Big\vert_{\theta=0,\pi} = 0$	$Y\Big\vert_{\theta=0,\pi} = 0$
$\partial_\theta Y\Big\vert_{\theta=\frac{\pi}{2}} = 0$	$\partial_\theta Y\Big\vert_{\theta=\frac{\pi}{2}} = 0$ or $Y\Big\vert_{\theta=\frac{\pi}{2}} = 0$

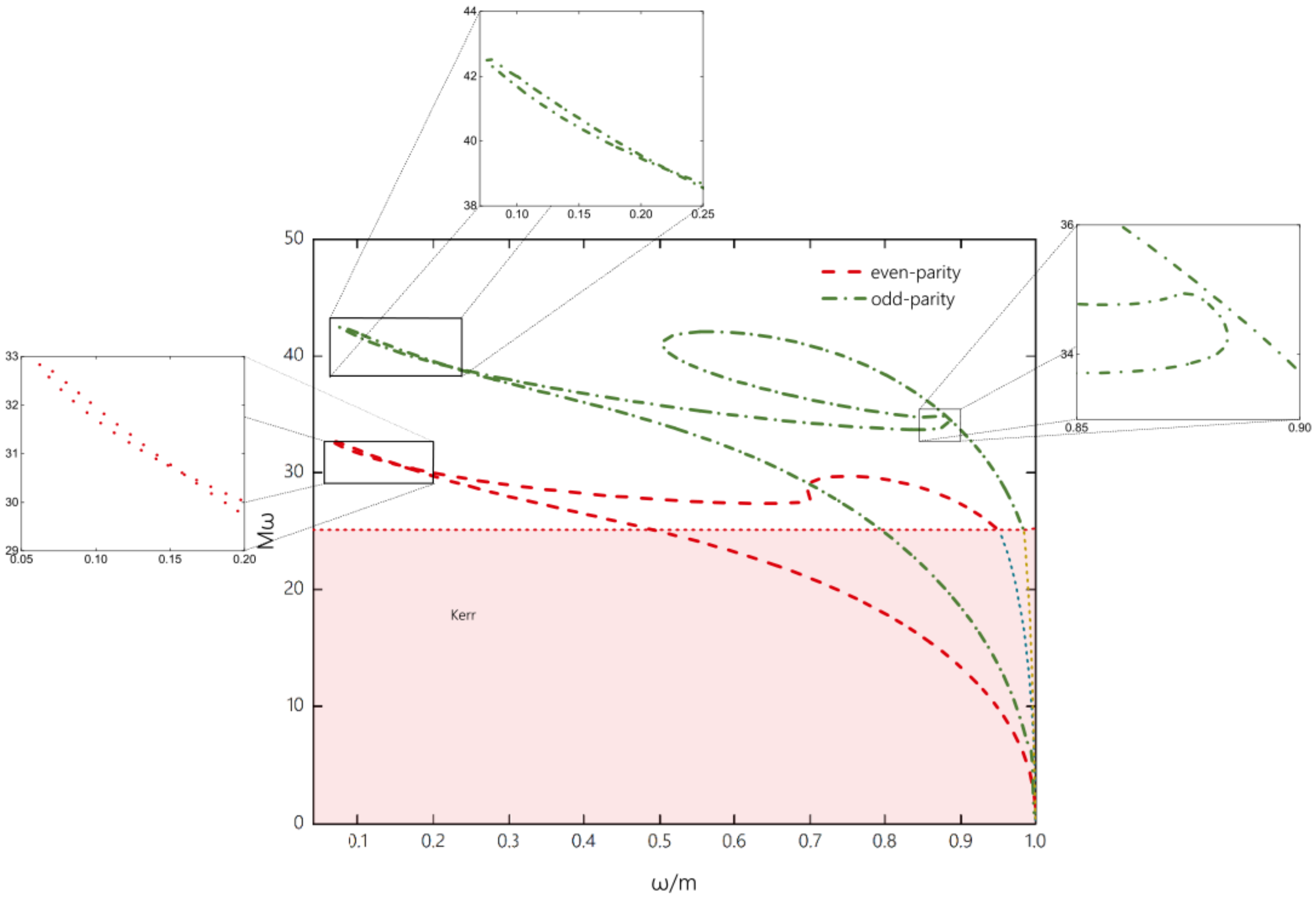


(Kunz, Perapechka, Shnir: JHEP 07 (2019) 109)

Kerr black holes with Friedberg-Lee-Sirlin hair







Kerr black holes with $O(3)$ sigma-model hair

$$\mathcal{L}_{matter} = \frac{1}{2}\left(\partial_\mu \phi^a\right)^2 + m^2(1-\phi^3)$$

$$\phi_a \phi^a = 1$$

$$j^\mu = \phi^2 \partial_\mu \phi^1 - \phi^1 \partial^\mu \phi^2$$

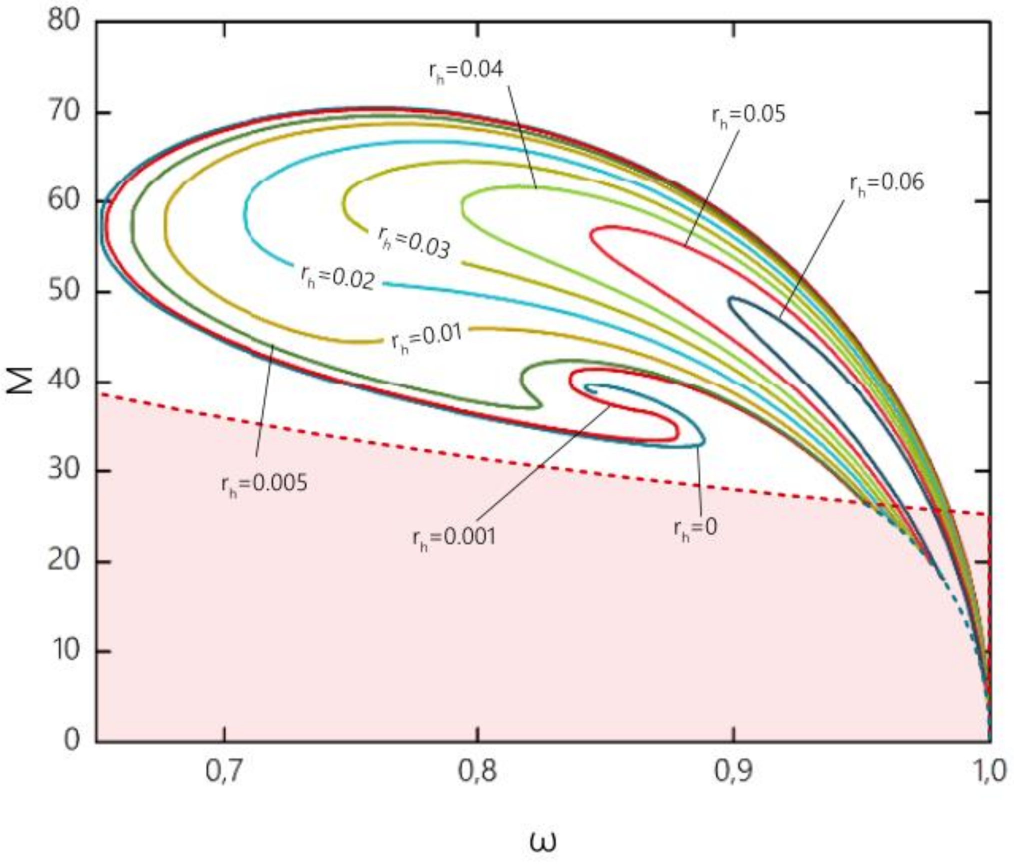
$$\phi = (\sin f(r,\theta)\cos(m\varphi - \omega t), \sin f(r,\theta)\sin(m\varphi - \omega t), \cos f(r,\theta))$$

$$f\Big|_{r=\infty} = 0$$

$$\partial_r f\Big|_{r=r_h} = 0$$

$$f\Big|_{\theta=0,\pi} = 0$$

$$\partial_\theta f\Big|_{\theta=\frac{\pi}{2}} = 0$$

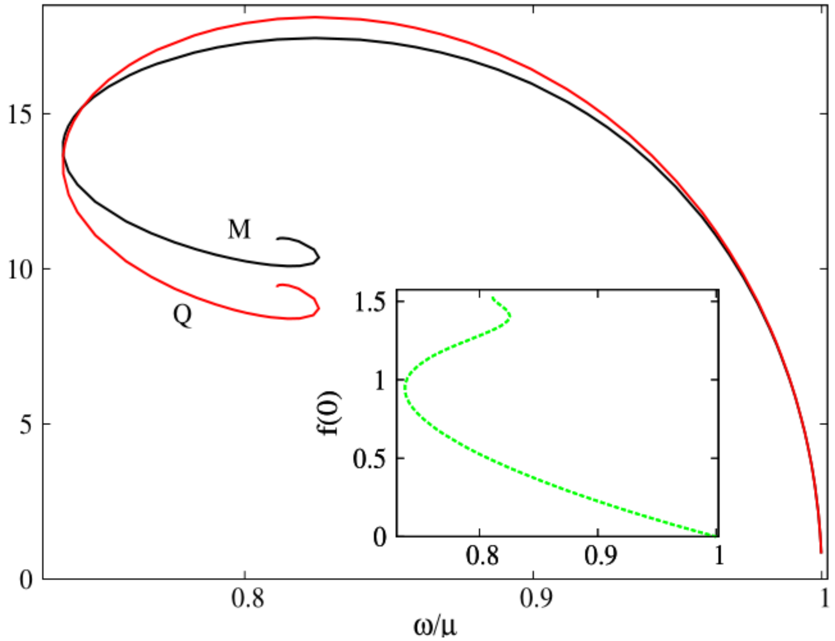


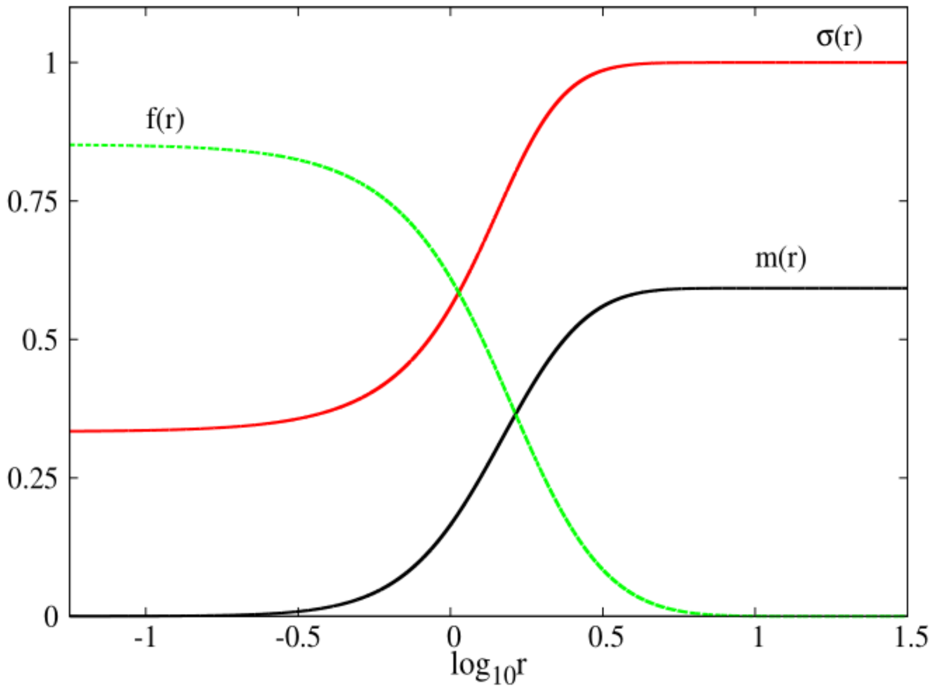
(Herdeiro, Perapechka, Radu, Shnir: JHEP 02 (2019) 111)

$O(3)$ sigma-model stars

$$ds^2 = -N(r)\sigma^2(r)dt^2 + \frac{dr^2}{N(r)} + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

$$\phi = (\sin f(r)\cos(\omega t), -\sin f(r)\sin(\omega t), \cos f(r))$$





Skyrme model

(T.H.R. Skyrme, 1961)

Skyrme field: $U(\boldsymbol{x}) \in \mathrm{SU}(2) \xrightarrow[\boldsymbol{x}\to\infty]{} \mathbb{1} \Rightarrow U: S^3 \to S^3$

Left $\mathfrak{su}(2)$-current: $L_\mu = U^\dagger \partial_\mu U$

Topological charge: $B = \pi_3(S^3) = \frac{1}{24\pi^2}\int \varepsilon^{ijk}\mathrm{Tr}\left[\left(U^\dagger\partial_i U\right)\left(U^\dagger\partial_j U\right)\left(U^\dagger\partial_k U\right)\right]d^3x = \frac{1}{24\pi^2}\int \varepsilon^{ijk}\mathrm{Tr}\left[L_i L_j L_k\right]d^3x$

corresponds to mass number of nuclei

Topological current: $\mathcal{B}^\mu = \frac{1}{24\pi^2\sqrt{-g}}\varepsilon^{\mu\nu\rho\sigma}\mathrm{Tr}\left(L_\nu L_\rho L_\sigma\right) \Rightarrow B = \int \mathcal{B}^0\sqrt{-g}\,d^3x$

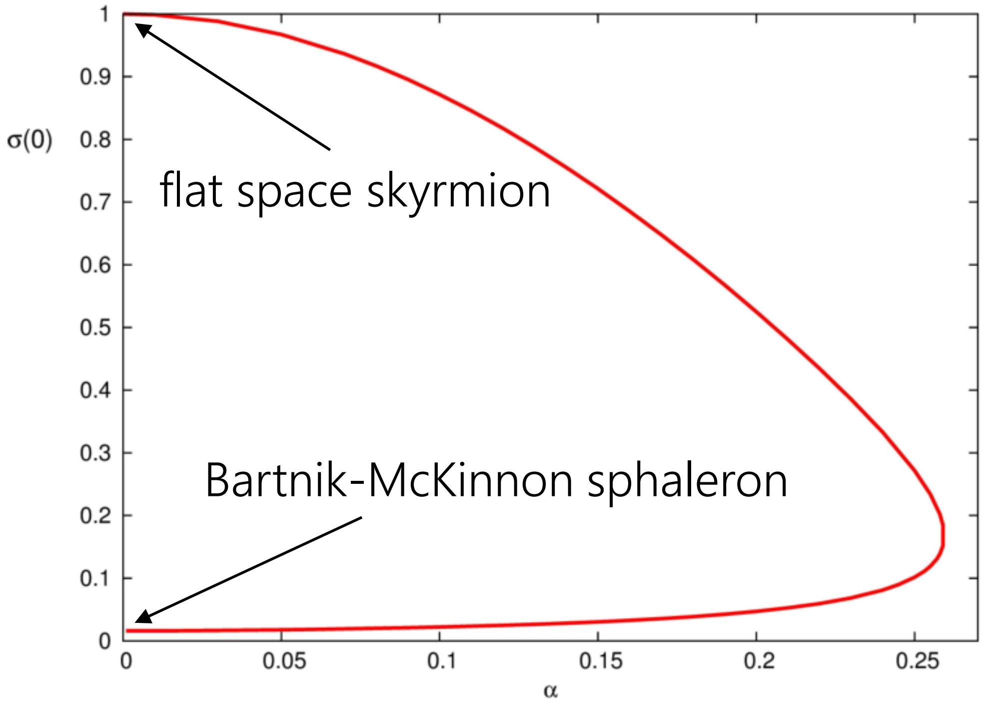
$$\mathcal{L}_{024} = \underbrace{\frac{1}{2}\mathrm{Tr}\left(L_\mu L^\mu\right)}_{\text{nonlinear sigma-model term}} + \underbrace{\frac{1}{16}\mathrm{Tr}\left(\left[L_\mu, L_\nu\right]\left[L^\mu, L^\nu\right]\right)}_{\text{Skyrme term}} + \underbrace{m^2\mathrm{Tr}(U - \mathbb{1})}_{\text{potential term}}$$

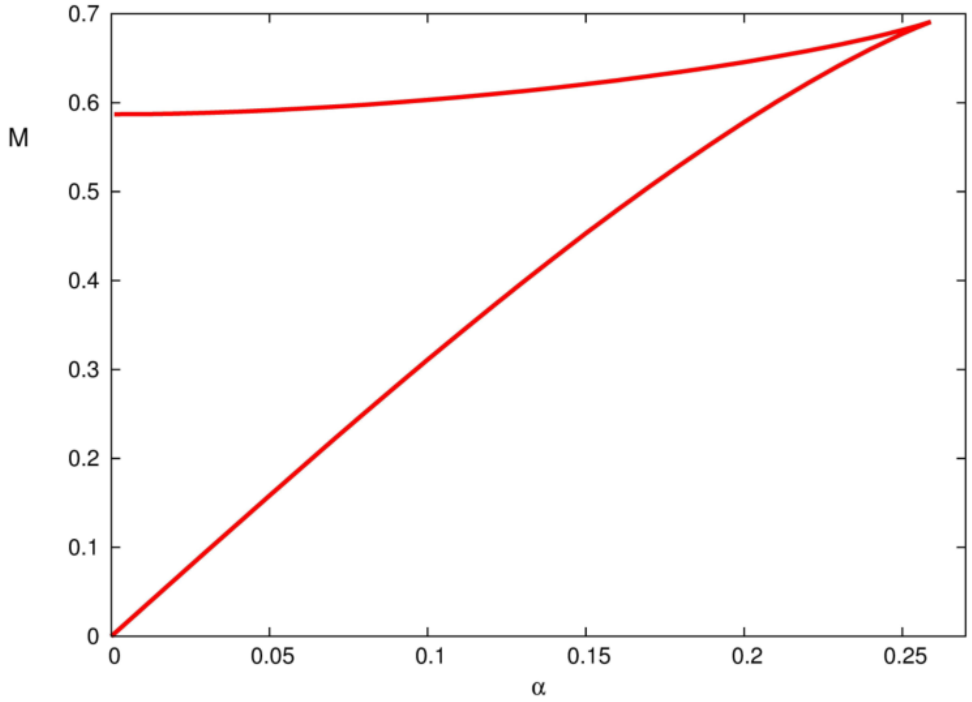
Gravitating skyrmions

(P. Bizon & T. Chmaj, 1992)

$$S = \int \left(\frac{R}{\alpha^2} + \mathcal{L}_{024} \right) \sqrt{-g}\, d^4x$$

$$ds^2 = -\sigma(r)^2 N(r) dt^2 + \frac{dr^2}{N(r)} + r^2 d\Omega^2$$ — spherically-symmetric ansatz



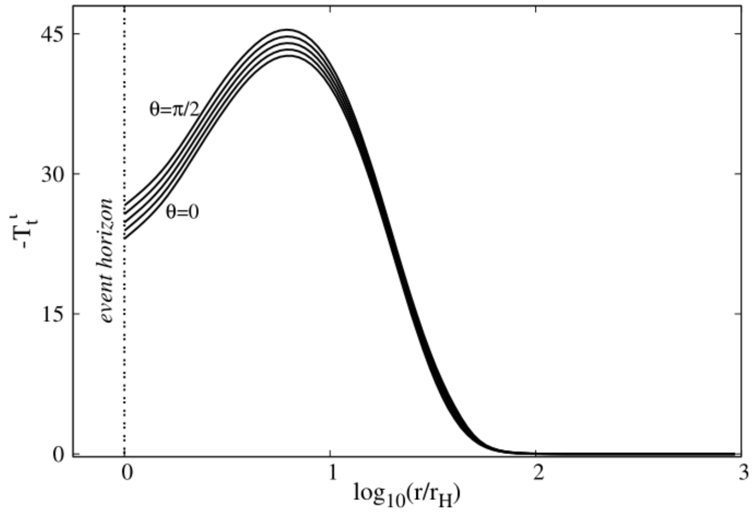


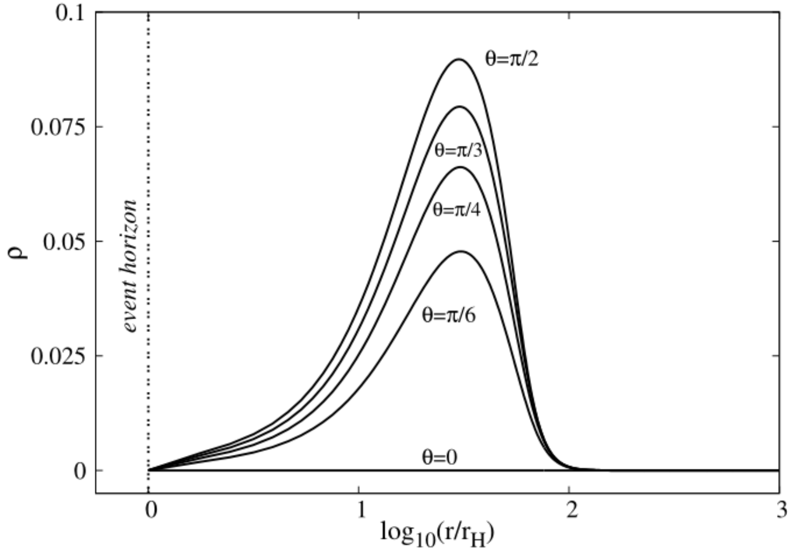
Kerr black holes with Skyrme hair

$$\mathcal{L}_{matter} = \frac{1}{2}\left(\partial_\mu \phi^a\right)^2 - \frac{1}{4}\left[\left(\partial_\mu \phi^a \partial_\nu \phi^a\right)^2 - \left(\partial_\mu \phi^a\right)^4\right] + m^2(1-\phi^3)$$
$$\phi_a \phi^a = 1$$

$$\phi = \left(\phi_1(r,\theta)\cos(m\varphi - \omega t), \phi_1(r,\theta)\sin(m\varphi - \omega t), \phi_2(r,\theta), \phi_3(r,\theta)\right)$$

$$\phi_1\Big|_{r=\infty} = \phi_2\Big|_{r=\infty} = \phi_3\Big|_{r=\infty} = 0$$
$$\partial_r\phi_1\Big|_{r=r_h} = \partial_r\phi_2\Big|_{r=r_h} = \partial_r\phi_3\Big|_{r=r_h} = 0$$
$$\phi_1\Big|_{\theta=0,\pi} = \partial_\theta\phi_2\Big|_{\theta=0,\pi} = \partial_\theta\phi_3\Big|_{\theta=0,\pi} = 0$$
$$\partial_\theta\,\phi_1\Big|_{\theta=\frac{\pi}{2}} = \phi_2\Big|_{\theta=\frac{\pi}{2}} = \partial_\theta\,\phi_3\Big|_{\theta=\frac{\pi}{2}} = 0$$

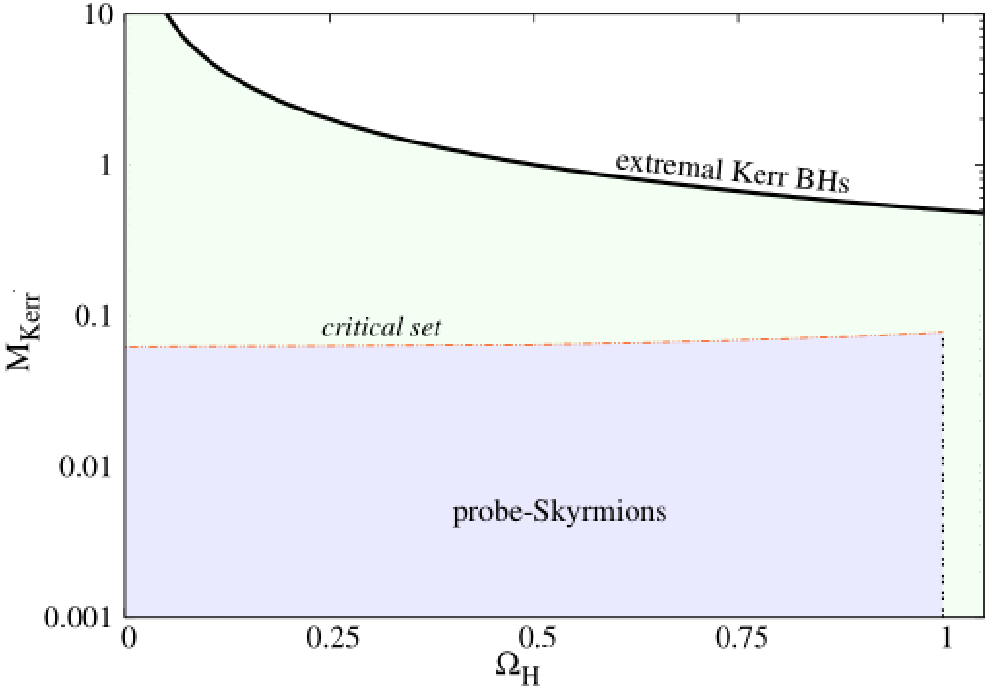


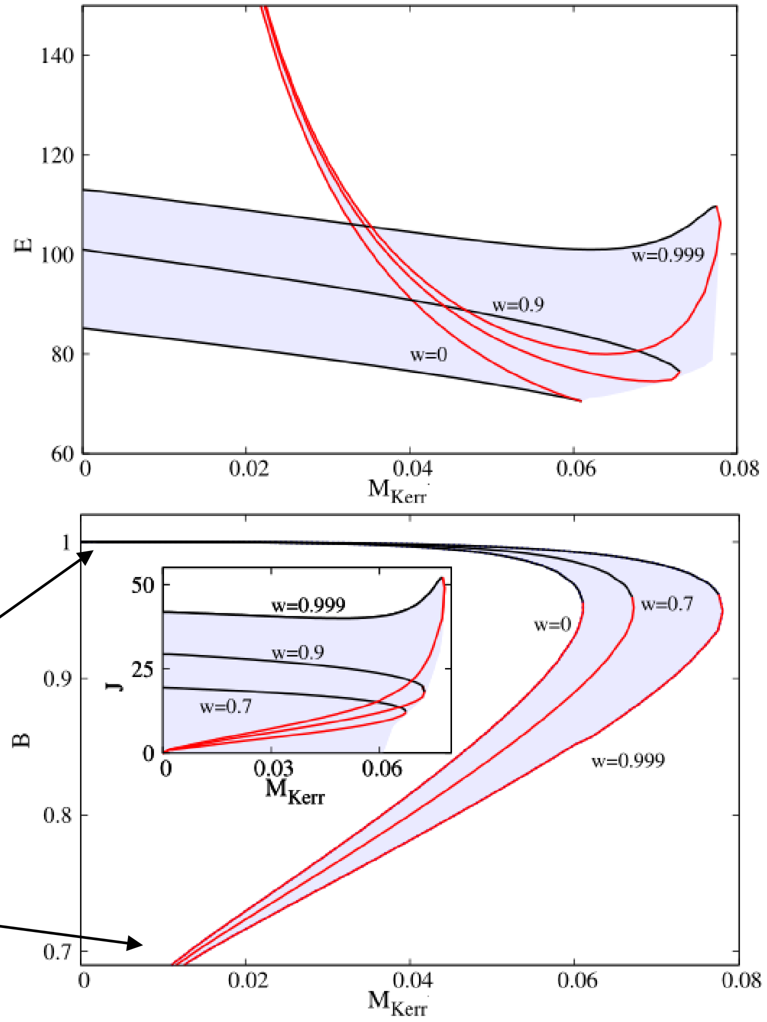


(Perapechka, Shnir: Phys. Rev. D 96 (2017) 125006;
Herdeiro, Perapechka, Radu, Shnir: JHEP 10 (2018) 119)

Skerrmions

i.e. skyrmions on Kerr background

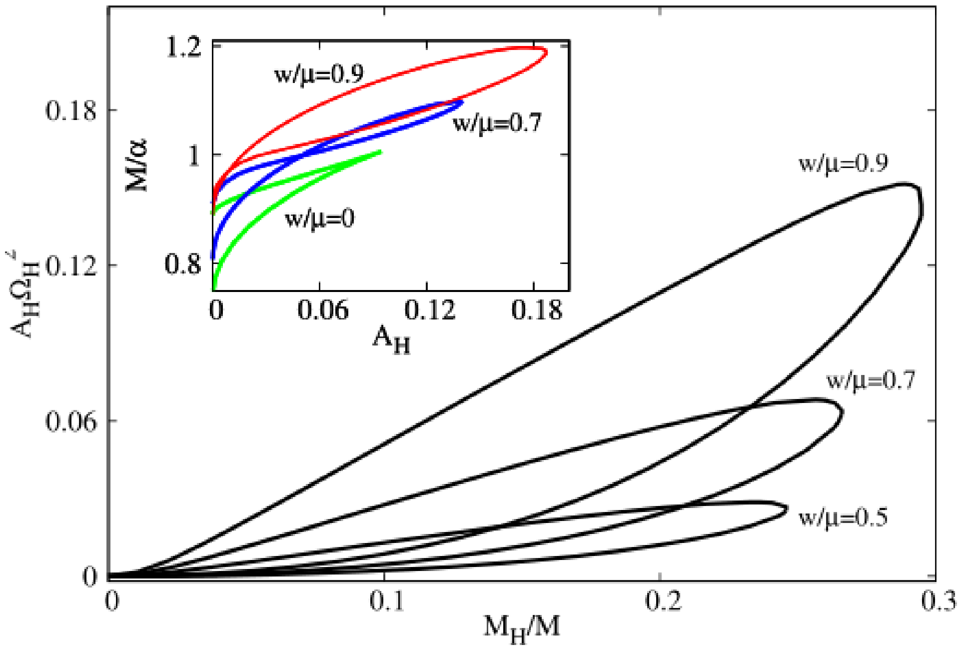


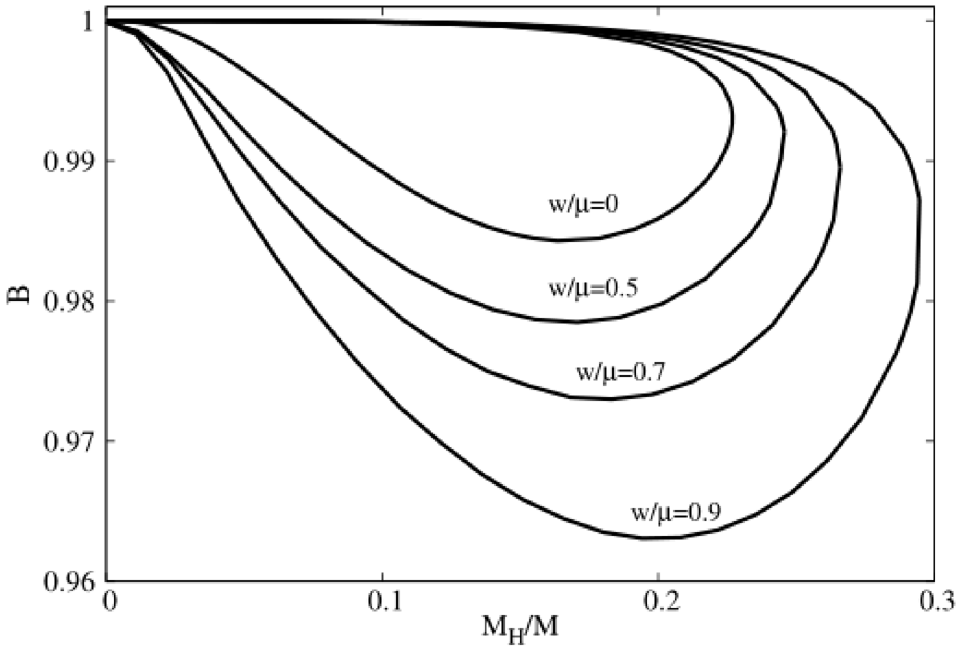


flat space skyrmion

Bartnik-McKinnon sphaleron

Kerr black holes with Skyrme hair





Four branches exist:

- Skyrmion ($G \to 0 \Rightarrow$ flat space skyrmion, $\omega \to 0 \Rightarrow$ static black hole with Skyrme hair, first branch)
- BM ($G \to 0 \Rightarrow$ Bartnik-McKinnon sphaleron, $\omega \to 0 \Rightarrow$ static black hole with Skyrme hair, second branch)
- Cloudy Skyrmion ($G \to 0 \Rightarrow$ diverges, requires high enough ω)
- Cloudy BM ($G \to 0 \Rightarrow$ diverges, requires high enough ω)

} nonlinear superposition of skyrmion or BM solution and pion cloud

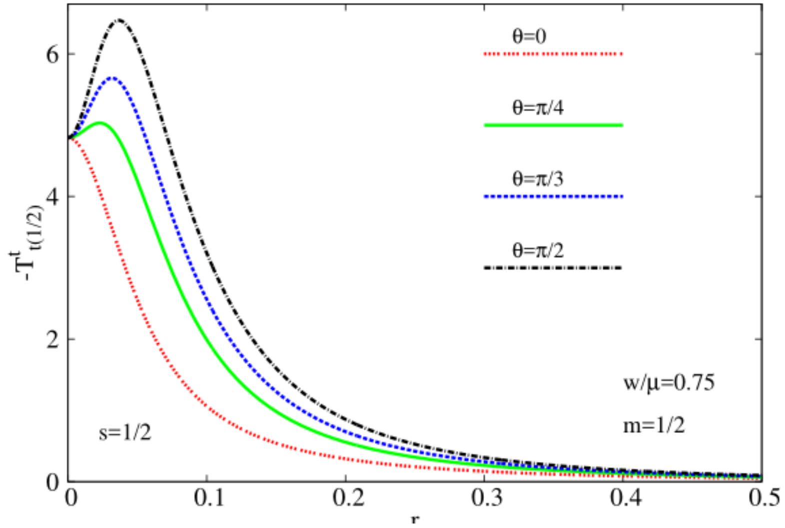
Dirac stars

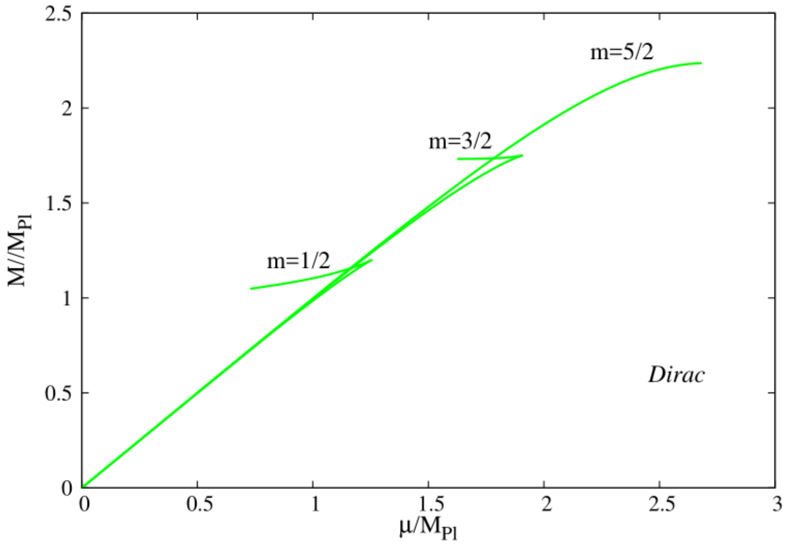
$$\mathcal{L}_{matter} = -i\left[\frac{1}{2}\left(\gamma^{\mu} D_{\mu}\overline{\Psi}\Psi - \overline{\Psi}\gamma^{\mu} D_{\mu}\Psi\right) + \mu\overline{\Psi}\Psi\right]$$

$$j^{\mu} = \overline{\Psi}\gamma^{\mu}\Psi$$

$$\Psi = e^{i(m\varphi-\omega t)}(\psi_1, \psi_2, -i\psi_1^*, -i\psi_2^*)$$
$$\psi_1 = P(r,\theta) + iQ(r,\theta)$$
$$\psi_2 = X(r,\theta) + iY(r,\theta)$$

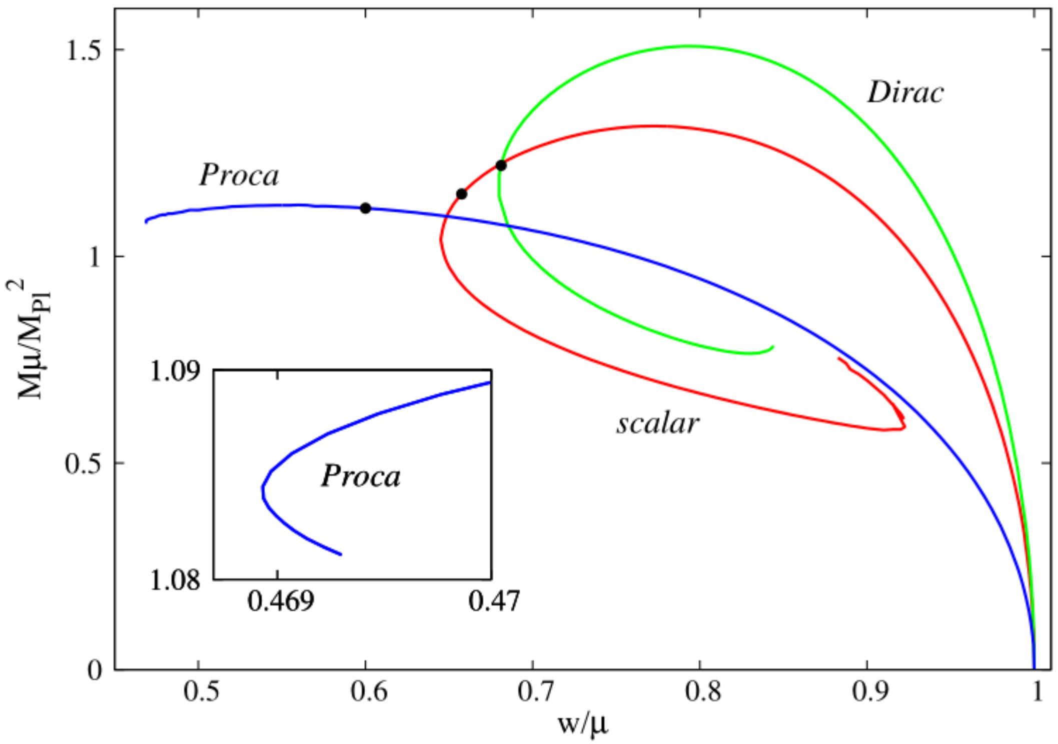
$$P\Big|_{r=\infty} = Q\Big|_{r=\infty} = X\Big|_{r=\infty} = Y\Big|_{r=\infty} = 0$$
$$P\Big|_{r=0} = Q\Big|_{r=0} = X\Big|_{r=0} = Y\Big|_{r=0} = 0$$
$$\partial_\theta P\Big|_{\theta=0} = \partial_\theta Q\Big|_{\theta=0} = X\Big|_{\theta=0} = Y\Big|_{\theta=0} = 0$$
$$P\Big|_{\theta=\pi} = Q\Big|_{\theta=\pi} = \partial_\theta X\Big|_{\theta=\pi} = \partial_\theta Y\Big|_{\theta=\pi} = 0$$

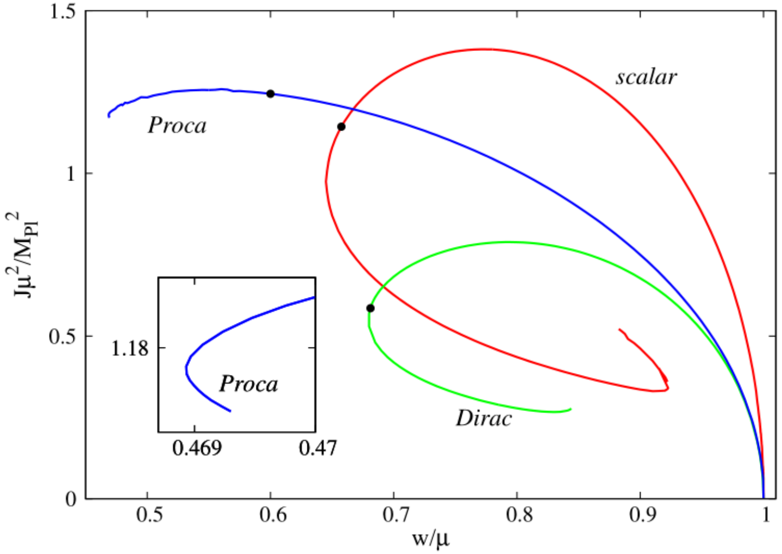


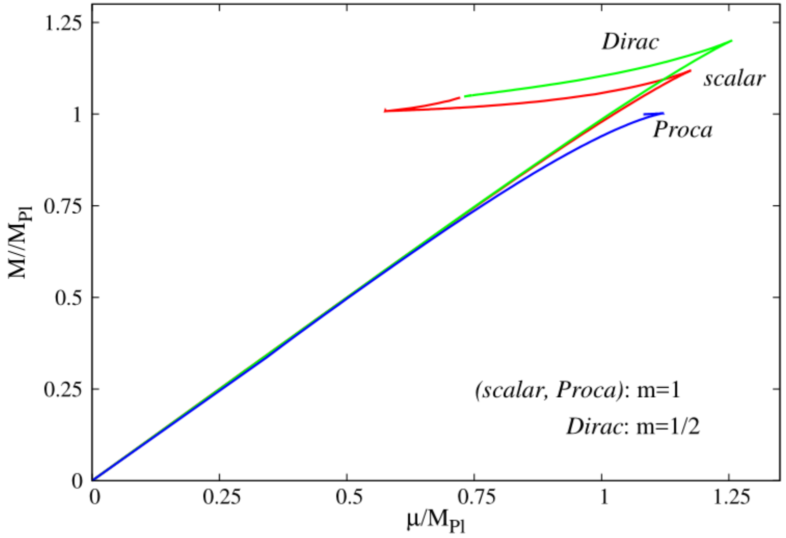


(Herdeiro, Perapechka, Radu, Shnir: arXiv:1906.05386, to appear in PLB)

Comparison of various spin field stars







Summary

- Spinning boson stars and Kerr black holes with scalar hair were constructed. All types of nontopological field configurations considered share a great deal of common properties.
- Odd-parity boson stars and hairy black holes were constructed. They can be considered as double boson stars (or double hair clouds around black hole). It appears that in static case branch of boson stars is connected to double black hole solution.
- Kerr black holes with Skyrme hair were constructed as an example of black holes with topological hair. Their properties greatly differ from those with nontopological hair.
- Dirac stars were constructed. It appears that when considered as classical field theory solutions, boson and fermion stars share the same universal pattern, insensitive to the fermionic/bosonic nature of the fields. It is interesting to further check this result for spin-3/2 and spin-2 stars.

Thank you for attention