

On Bandwidth On Demand Problem

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This work is supported by Russian Ministry of Science and Higher Education, grant #05.613.21.0088, unique ID RFMEFI61318X0088.

Background



NEC 2019: «MC2E: Meta-Cloud Computing Environment for HPC» [Ruslan SMELIANSKY]



Problems:

- To recognize tasks that can be solved in HPC-C

- To effectively share resources in HPC-C NEC 2019: «Improving Resource Usage in HPC Clouds» [Andrey CHUPAKHIN]

- To provide required flow throughput with the service «Bandwidth On Demand»







Fault tolerance, overhead expenses, reservation opportunity

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Resource aggregation: protocols





Resource aggregation: FDMP





Flow distribution: one flow



Problems:

What routes should be used?

Menger's theorem: Let *G* be a finite undirected graph and *x* and *y* two distinct vertices. Then the size of the minimum edge cut for *x* and *y* (the minimum number of edges whose removal disconnects *x* and *y*) is equal to the maximum number of pairwise edge-independent paths from *x* to *y*.



Stepanov E., Smeliansky R. On analysis of traffic flow demultiplexing effectiveness // MoNeTec-2018

Flow distribution: one flow

Problems:

- Is it possible to distribute a flow?
- How should it be done if it is so?

Mathematical model

Input:

flow

G = (V, E) - directed graph without loops $C = \{c_{ij}\} - \text{ set of the available bandwidth for each } e_{ij} \in E$ R - required bandwidth for the flow $P = \{P_k\} - \text{ set of non-intersecting paths without loop for the}$ $\Delta_j = \min_{e_{km} \in P_j} c_{ij}, \ \Delta = \sum_j \Delta_j$

Output:

 $\Delta \ge R$ – there is the solution $\Delta < R$ – there is no solution

 $\Delta = R - \text{single load distribution}$ $\Delta > R - \text{set of distributions (progressive filling algorithm)}$

Flow distribution: multi-flows

Mathematical model

Input:

G = (V, E) – directed graph without loops

 $C = \{c_{ij}\} - \text{ set of the available bandwidth for each } e_{ij} \in E$ $F = \{f_i\} - \text{ set of flows}$

 $R = \{R_i\}$ - set of bandwidth requirements for each $f_i \in F$ $P_i = \{P_{ik}\}$ - set of non-intersecting paths without loop for flow f_i (previous slide)

This problem is similar to the fair bandwidth allocation problem in a network

Max-Min Fairness: allocation is max-min fair if no rates can be increased without decreasing an already smaller rate

If max-min fair allocation exists, it can be done with the *progressive filling algorithm*

$$\Delta_{ij} = \min_{e_{km} \in P_{ij}} c_{ij}, \ \Delta_i = \sum_j \Delta_{ij}$$

$$r_i = \{r_{ik}\} - \text{throughput of } f_i \text{ on each path } P_{ik}$$

$$r_i \le R_i, r_{ik} \ge 0$$

Output:

$$\exists i: \Delta_i < R_i - \text{there is no solution}$$

$$r_{A}=0.5$$

 $r_{B}=1.5$
 $r_{c}=0.5$
 $r_{c}=0.5$



Conclusion



The bandwidth on demand service breaks up into two problems: resources aggregation and flows distribution.

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Resources aggregation can be solved with the various techniques or its combination.

Flows distribution problem was formulated and was solved for the case of one flow.

Flows distribution for the case of many flows is our next step