13th APCTP-BLTP JINR Joint Workshop
“Modern Problems in Nuclear and Elementary Particle Physics”
15-19 July, 2019, Dubna, Russia

YONGSEOK OH (KYUNGPOOK NATIONAL UNIV. / APCTP)

Study of Nuclear Structure with a Generalized Skyrme Functional

KNU KYUNGPOOK NATIONAL UNIVERSITY

APCTP Asia Pacific Center for Theoretical Physics

IN COLLABORATION WITH

- P. Papakonstantinou (IBS, Korea)
- C.-H. Hyun (Daegu Univ.)
- H. Gil (Kyungpook National Univ.)
- Y.-M. Kim (UNIST)

CONTENTS

Ref.
E. Shin, Y. Lim, C.H. Hyun, Y. Oh, PRC 94, 024320 (2016)
Y. Lim, Y. Oh, PRC 95, 034311 (2017)

PHYSICAL REVIEW C **94**, 024320 (2016)

Nuclear isospin asymmetry in α decay of heavy nuclei

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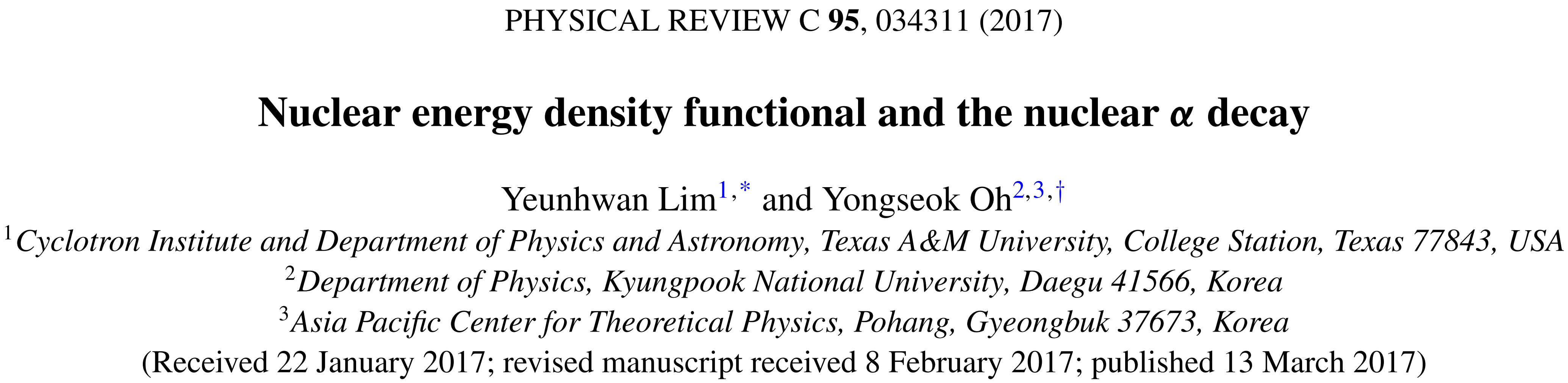

PHYSICAL REVIEW C **95**, 034311 (2017)

Nuclear energy density functional and the nuclear α decay

Yeunhwan Lim[1,*] and Yongseok Oh[2,3,†]
[1] *Cyclotron Institute and Department of Physics and Astronomy, Texas A&M University, College Station, Texas 77843, USA*
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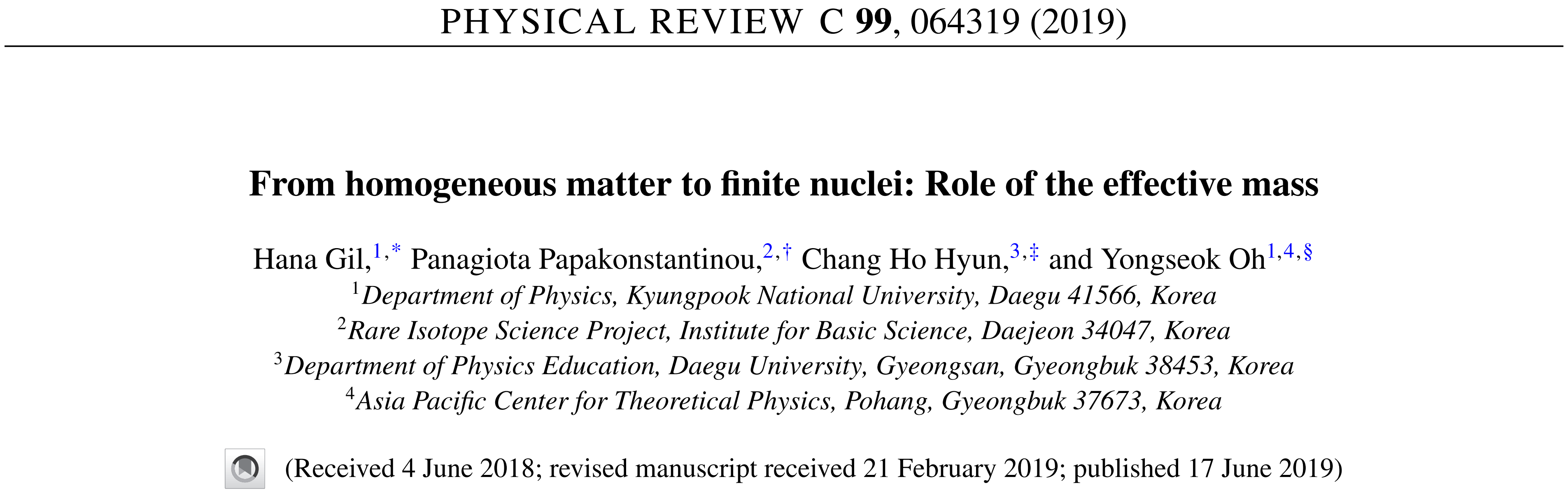
Ref.

H. Gil, P. Papakonstantinou, C.H. Hyun, Y. Oh, PRC 99, 064319 (2019)

H. Gil, Y.-M. Kim, C.H. Hyun, P. Papakonstantinou, Y. Oh, PRC 100 (2019), in press. arXiv:1903.04123



PHYSICAL REVIEW C **99**, 064319 (2019)

From homogeneous matter to finite nuclei: Role of the effective mass

Hana Gil,[1,*] Panagiota Papakonstantinou,[2,†] Chang Ho Hyun,[3,‡] and Yongseok Oh[1,4,§]
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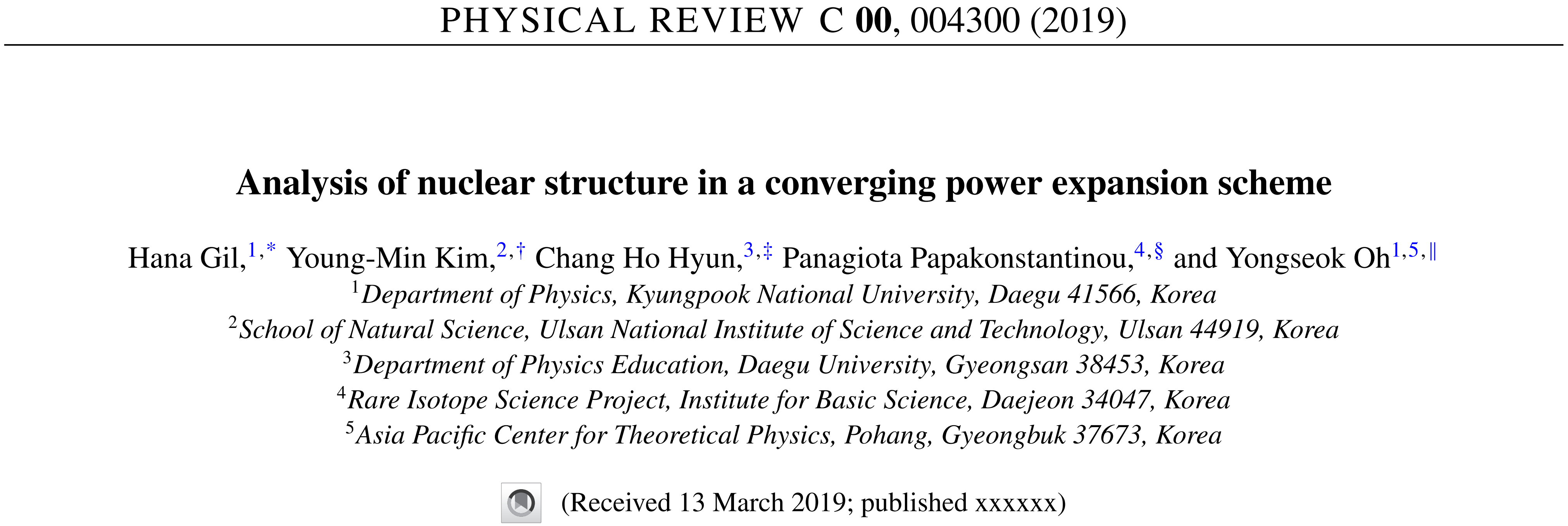

PHYSICAL REVIEW C **00**, 004300 (2019)

Analysis of nuclear structure in a converging power expansion scheme

Hana Gil,[1,*] Young-Min Kim,[2,†] Chang Ho Hyun,[3,‡] Panagiota Papakonstantinou,[4,§] and Yongseok Oh[1,5,‖]
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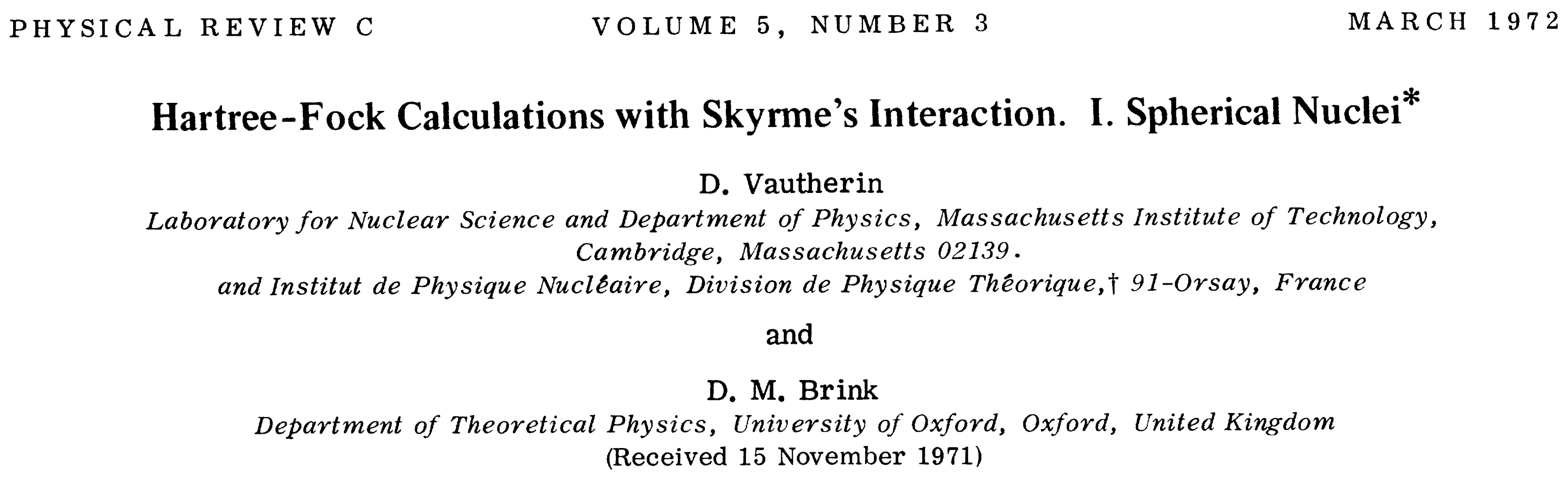
(Received 13 March 2019; published xxxxxx)

NUCLEAR STRUCTURE WITH SKYRME EDF



PHYSICAL REVIEW C VOLUME 5, NUMBER 3 MARCH 1972

Hartree-Fock Calculations with Skyrme's Interaction. I. Spherical Nuclei*

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and Institut de Physique Nucléaire, Division de Physique Théorique,† 91-Orsay, France

and

D. M. Brink
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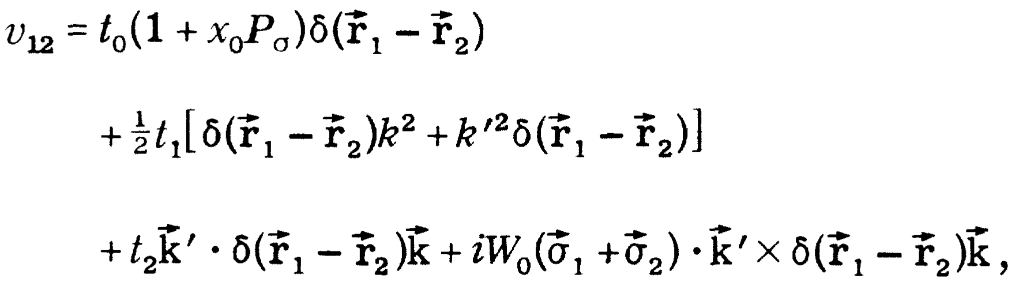
(Received 15 November 1971)



$$v_{12} = t_0(1 + x_0 P_\sigma)\delta(\vec{r}_1 - \vec{r}_2) + \tfrac{1}{2}t_1[\delta(\vec{r}_1 - \vec{r}_2)k^2 + k'^2\delta(\vec{r}_1 - \vec{r}_2)] + t_2\vec{k}' \cdot \delta(\vec{r}_1 - \vec{r}_2)\vec{k} + iW_0(\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \vec{k}' \times \delta(\vec{r}_1 - \vec{r}_2)\vec{k},$$

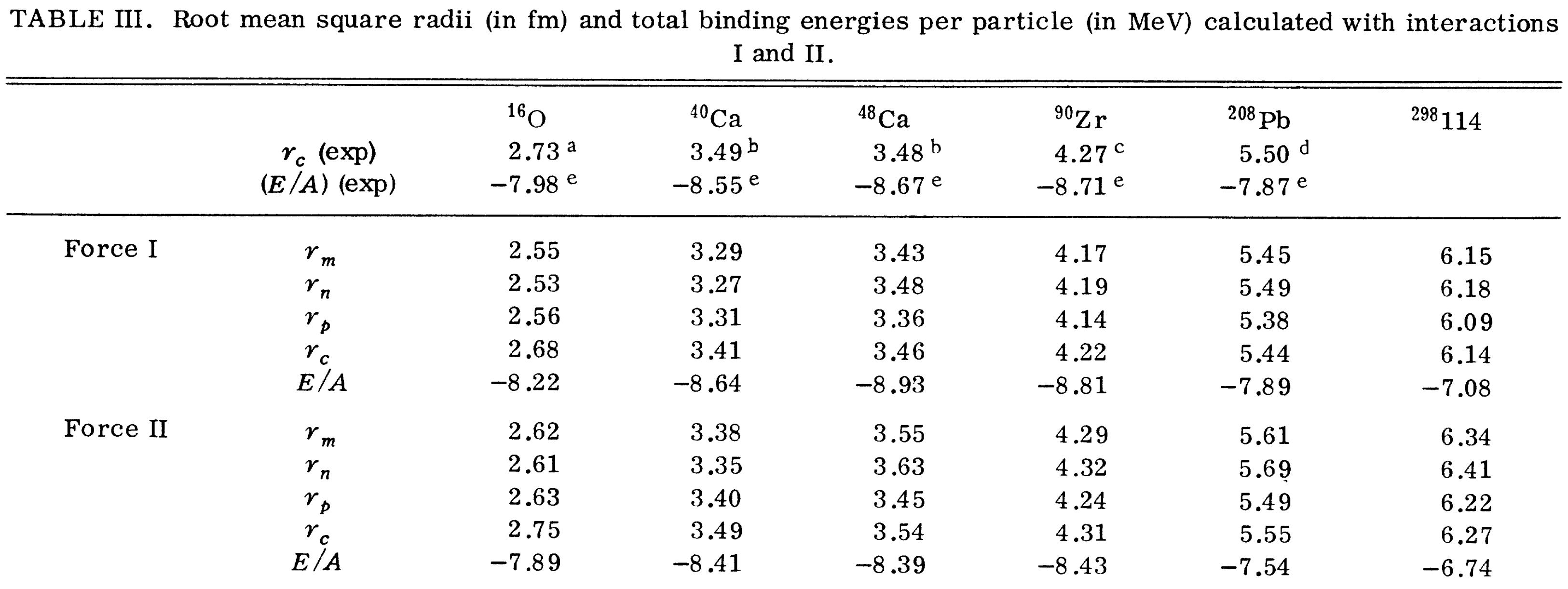
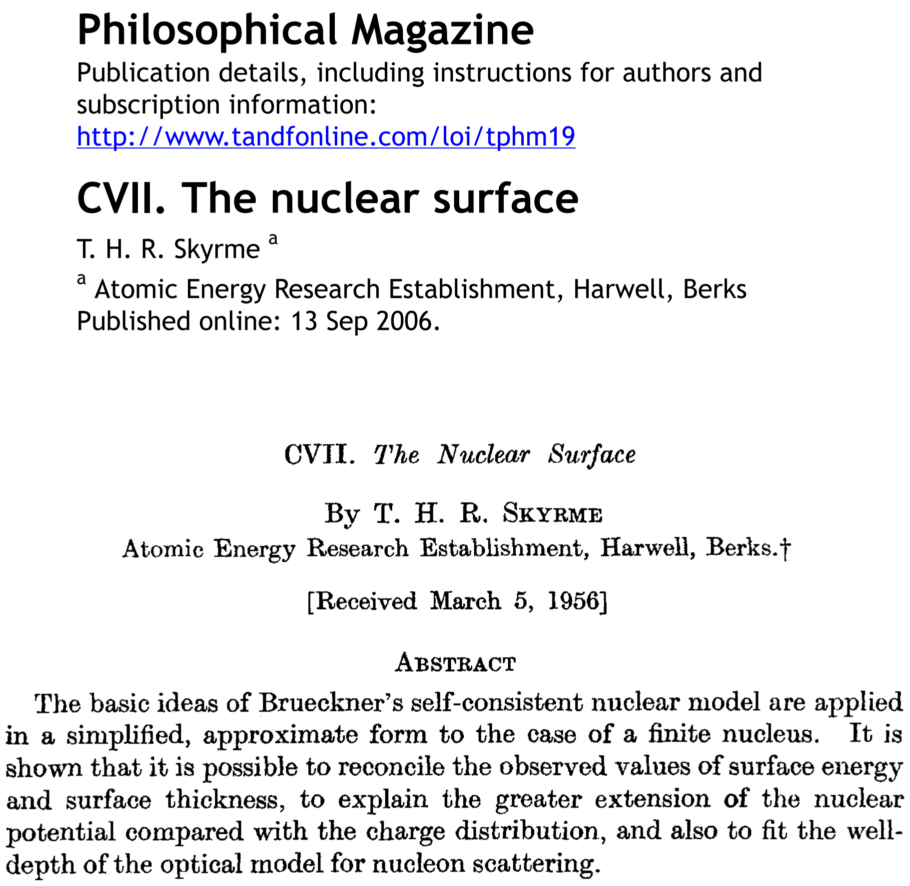


TABLE III. Root mean square radii (in fm) and total binding energies per particle (in MeV) calculated with interactions I and II.

		^{16}O	^{40}Ca	^{48}Ca	^{90}Zr	^{208}Pb	^{298}114
	r_c (exp)	2.73 [a]	3.49 [b]	3.48 [b]	4.27 [c]	5.50 [d]	
	(E/A) (exp)	−7.98 [e]	−8.55 [e]	−8.67 [e]	−8.71 [e]	−7.87 [e]	
Force I	r_m	2.55	3.29	3.43	4.17	5.45	6.15
	r_n	2.53	3.27	3.48	4.19	5.49	6.18
	r_p	2.56	3.31	3.36	4.14	5.38	6.09
	r_c	2.68	3.41	3.46	4.22	5.44	6.14
	E/A	−8.22	−8.64	−8.93	−8.81	−7.89	−7.08
Force II	r_m	2.62	3.38	3.55	4.29	5.61	6.34
	r_n	2.61	3.35	3.63	4.32	5.69	6.41
	r_p	2.63	3.40	3.45	4.24	5.49	6.22
	r_c	2.75	3.49	3.54	4.31	5.55	6.27
	E/A	−7.89	−8.41	−8.39	−8.43	−7.54	−6.74

SKYRME MODEL



Philosophical Magazine

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CVII. The nuclear surface

T. H. R. Skyrme [a]

[a] Atomic Energy Research Establishment, Harwell, Berks
Published online: 13 Sep 2006.

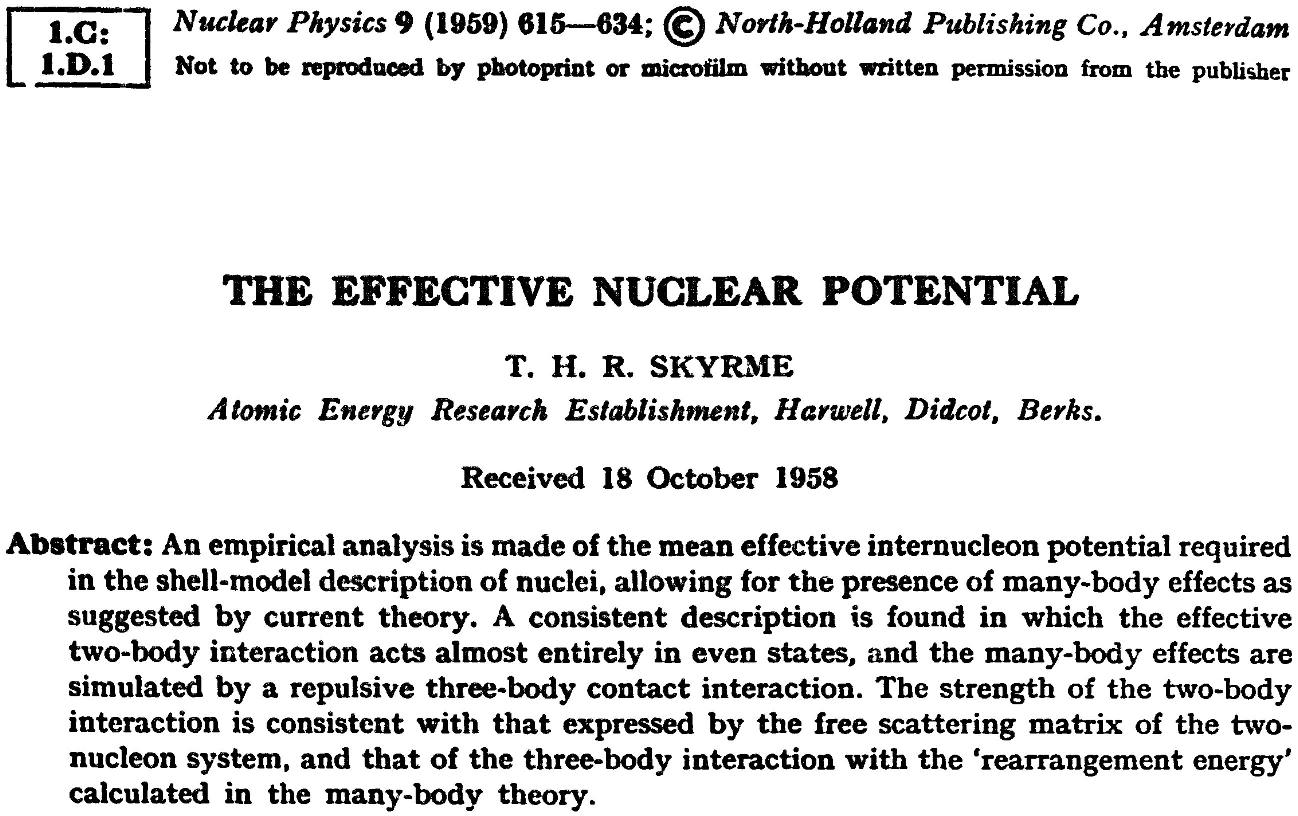
CVII. *The Nuclear Surface*

By T. H. R. Skyrme
Atomic Energy Research Establishment, Harwell, Berks.†

[Received March 5, 1956]

Abstract

The basic ideas of Brueckner's self-consistent nuclear model are applied in a simplified, approximate form to the case of a finite nucleus. It is shown that it is possible to reconcile the observed values of surface energy and surface thickness, to explain the greater extension of the nuclear potential compared with the charge distribution, and also to fit the well-depth of the optical model for nucleon scattering.



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Nuclear Physics **9** (1959) 615—634; © *North-Holland Publishing Co., Amsterdam*
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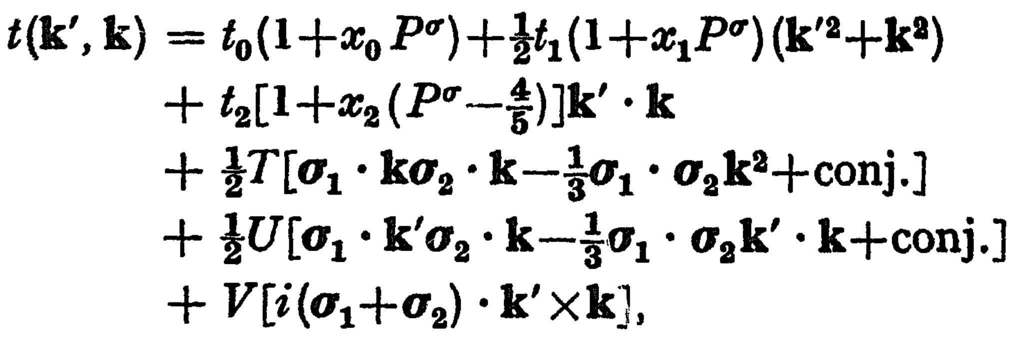
THE EFFECTIVE NUCLEAR POTENTIAL

T. H. R. SKYRME
Atomic Energy Research Establishment, Harwell, Didcot, Berks.

Received 18 October 1958

Abstract: An empirical analysis is made of the mean effective internucleon potential required in the shell-model description of nuclei, allowing for the presence of many-body effects as suggested by current theory. A consistent description is found in which the effective two-body interaction acts almost entirely in even states, and the many-body effects are simulated by a repulsive three-body contact interaction. The strength of the two-body interaction is consistent with that expressed by the free scattering matrix of the two-nucleon system, and that of the three-body interaction with the 'rearrangement energy' calculated in the many-body theory.



$$\begin{aligned} t(\mathbf{k}', \mathbf{k}) &= t_0(1+x_0 P^\sigma)+\tfrac{1}{2}t_1(1+x_1 P^\sigma)(\mathbf{k}'^2+\mathbf{k}^2) \\ &+ t_2[1+x_2(P^\sigma-\tfrac{4}{5})]\mathbf{k}'\cdot\mathbf{k} \\ &+ \tfrac{1}{2}T[\boldsymbol{\sigma}_1\cdot\mathbf{k}\,\boldsymbol{\sigma}_2\cdot\mathbf{k}-\tfrac{1}{3}\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\mathbf{k}^2+\text{conj.}] \\ &+ \tfrac{1}{2}U[\boldsymbol{\sigma}_1\cdot\mathbf{k}'\boldsymbol{\sigma}_2\cdot\mathbf{k}-\tfrac{1}{3}\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\mathbf{k}'\cdot\mathbf{k}+\text{conj.}] \\ &+ V[i(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\mathbf{k}'\times\mathbf{k}], \end{aligned}$$

Tony Skyrme
Former Fellow, Trinity College, Cambridge. Reprinted with permission
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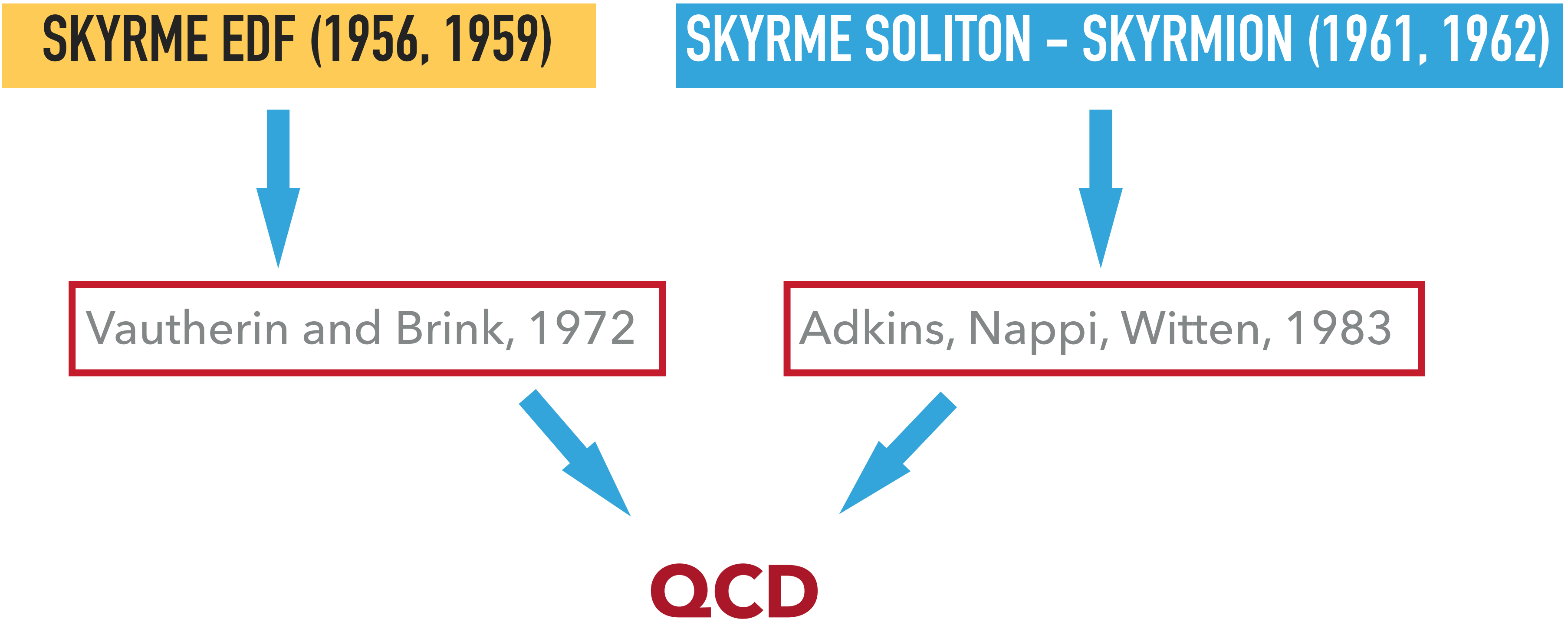


SKYRME EDF (1956, 1959)

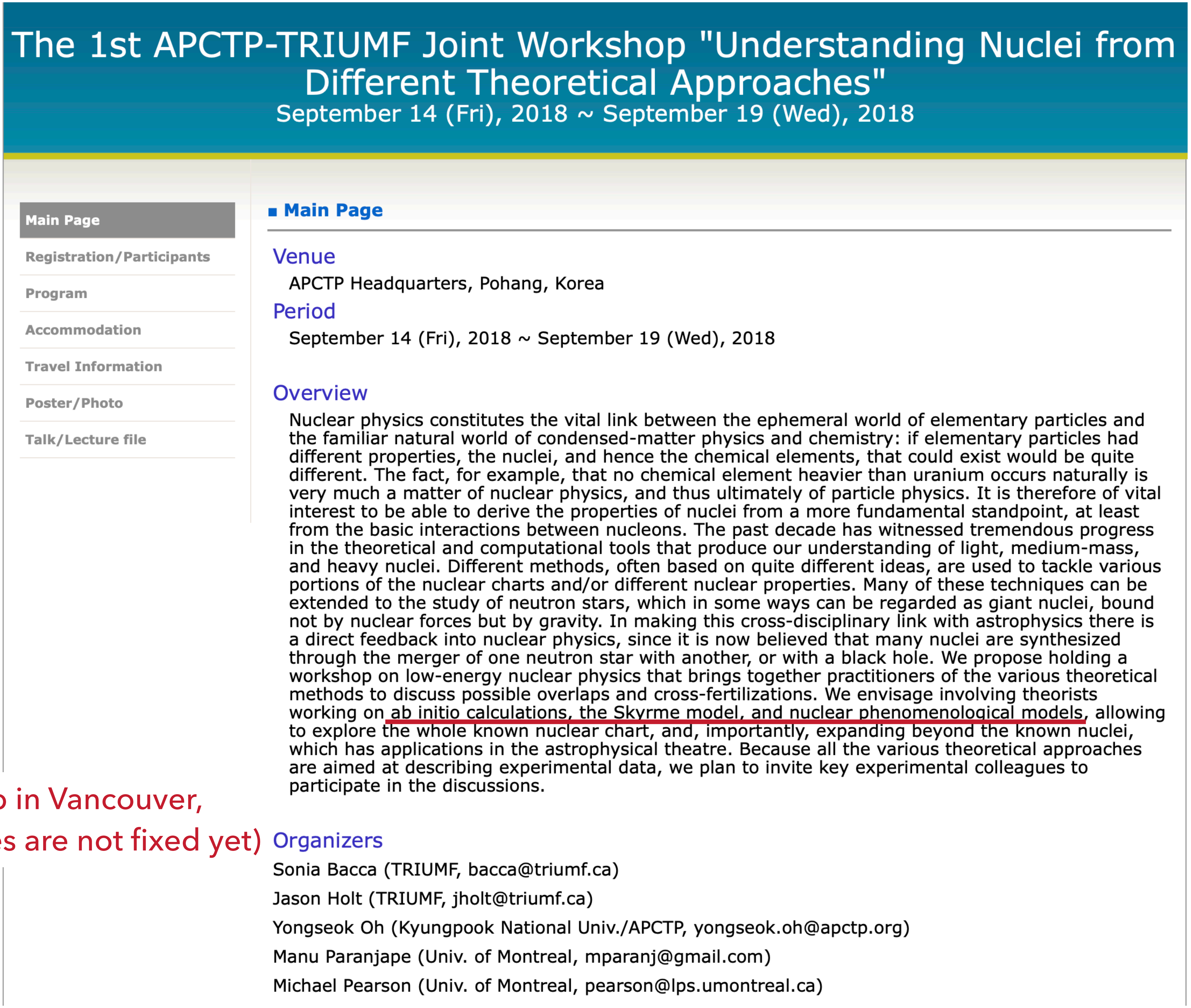
SKYRME SOLITON – SKYRMION (1961, 1962)

Vautherin and Brink, 1972

Adkins, Nappi, Witten, 1983

QCD

Skyrme's ideas on nucleon and nuclear structure



The 1st APCTP-TRIUMF Joint Workshop "Understanding Nuclei from Different Theoretical Approaches"
September 14 (Fri), 2018 ~ September 19 (Wed), 2018

■ Main Page

Venue

APCTP Headquarters, Pohang, Korea

Period

September 14 (Fri), 2018 ~ September 19 (Wed), 2018

Overview

Nuclear physics constitutes the vital link between the ephemeral world of elementary particles and the familiar natural world of condensed-matter physics and chemistry: if elementary particles had different properties, the nuclei, and hence the chemical elements, that could exist would be quite different. The fact, for example, that no chemical element heavier than uranium occurs naturally is very much a matter of nuclear physics, and thus ultimately of particle physics. It is therefore of vital interest to be able to derive the properties of nuclei from a more fundamental standpoint, at least from the basic interactions between nucleons. The past decade has witnessed tremendous progress in the theoretical and computational tools that produce our understanding of light, medium-mass, and heavy nuclei. Different methods, often based on quite different ideas, are used to tackle various portions of the nuclear charts and/or different nuclear properties. Many of these techniques can be extended to the study of neutron stars, which in some ways can be regarded as giant nuclei, bound not by nuclear forces but by gravity. In making this cross-disciplinary link with astrophysics there is a direct feedback into nuclear physics, since it is now believed that many nuclei are synthesized through the merger of one neutron star with another, or with a black hole. We propose holding a workshop on low-energy nuclear physics that brings together practitioners of the various theoretical methods to discuss possible overlaps and cross-fertilizations. We envisage involving theorists working on ab initio calculations, the Skyrme model, and nuclear phenomenological models, allowing to explore the whole known nuclear chart, and, importantly, expanding beyond the known nuclei, which has applications in the astrophysical theatre. Because all the various theoretical approaches are aimed at describing experimental data, we plan to invite key experimental colleagues to participate in the discussions.

Organizers

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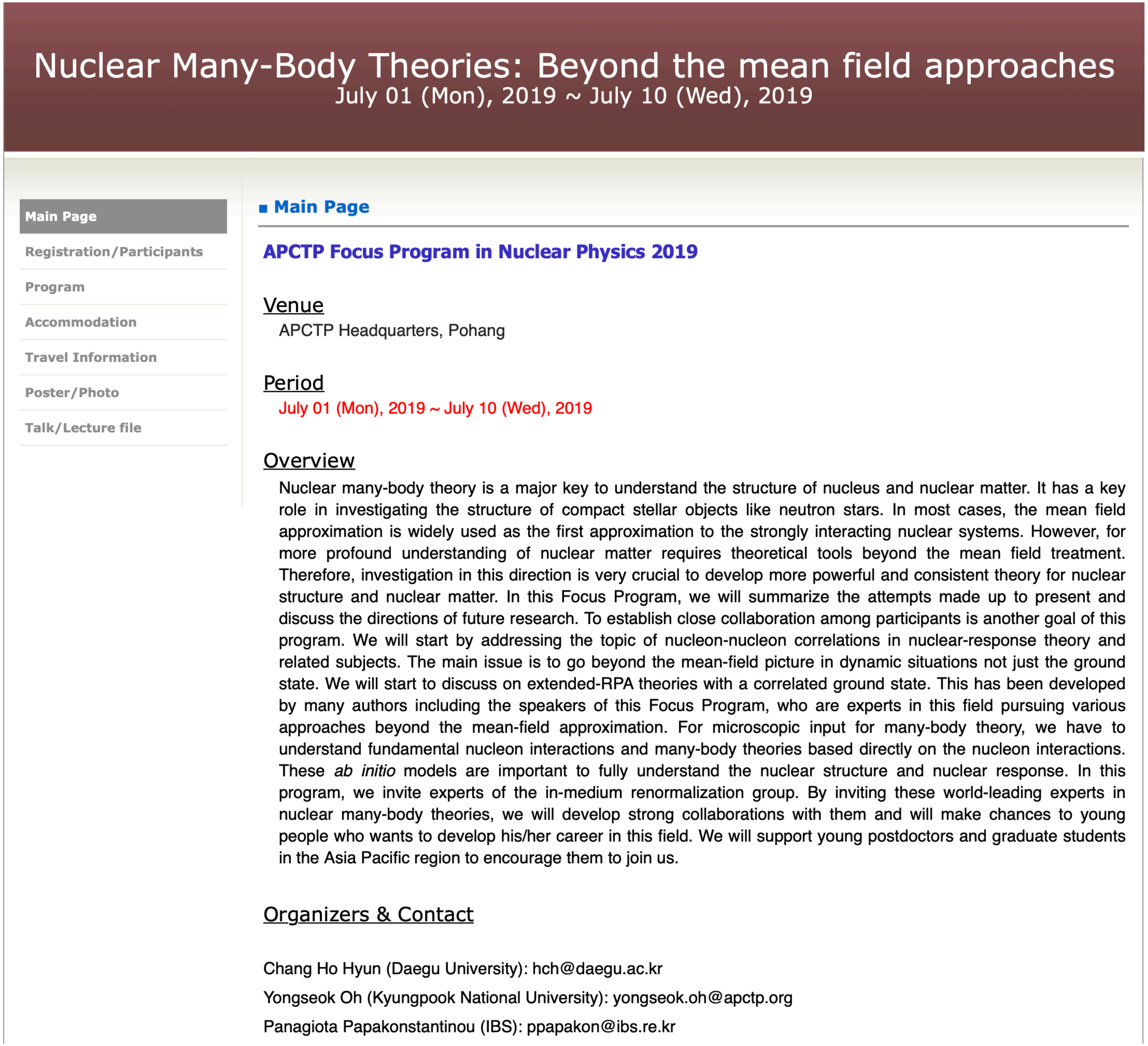
2nd Joint Workshop in Vancouver,
Summer 2020 (dates are not fixed yet)



Nuclear Many-Body Theories: Beyond the mean field approaches

July 01 (Mon), 2019 ~ July 10 (Wed), 2019

■ **Main Page**

APCTP Focus Program in Nuclear Physics 2019

Venue

APCTP Headquarters, Pohang

Period

July 01 (Mon), 2019 ~ July 10 (Wed), 2019

Overview

Nuclear many-body theory is a major key to understand the structure of nucleus and nuclear matter. It has a key role in investigating the structure of compact stellar objects like neutron stars. In most cases, the mean field approximation is widely used as the first approximation to the strongly interacting nuclear systems. However, for more profound understanding of nuclear matter requires theoretical tools beyond the mean field treatment. Therefore, investigation in this direction is very crucial to develop more powerful and consistent theory for nuclear structure and nuclear matter. In this Focus Program, we will summarize the attempts made up to present and discuss the directions of future research. To establish close collaboration among participants is another goal of this program. We will start by addressing the topic of nucleon-nucleon correlations in nuclear-response theory and related subjects. The main issue is to go beyond the mean-field picture in dynamic situations not just the ground state. We will start to discuss on extended-RPA theories with a correlated ground state. This has been developed by many authors including the speakers of this Focus Program, who are experts in this field pursuing various approaches beyond the mean-field approximation. For microscopic input for many-body theory, we have to understand fundamental nucleon interactions and many-body theories based directly on the nucleon interactions. These *ab initio* models are important to fully understand the nuclear structure and nuclear response. In this program, we invite experts of the in-medium renormalization group. By inviting these world-leading experts in nuclear many-body theories, we will develop strong collaborations with them and will make chances to young people who wants to develop his/her career in this field. We will support young postdoctors and graduate students in the Asia Pacific region to encourage them to join us.

Organizers & Contact

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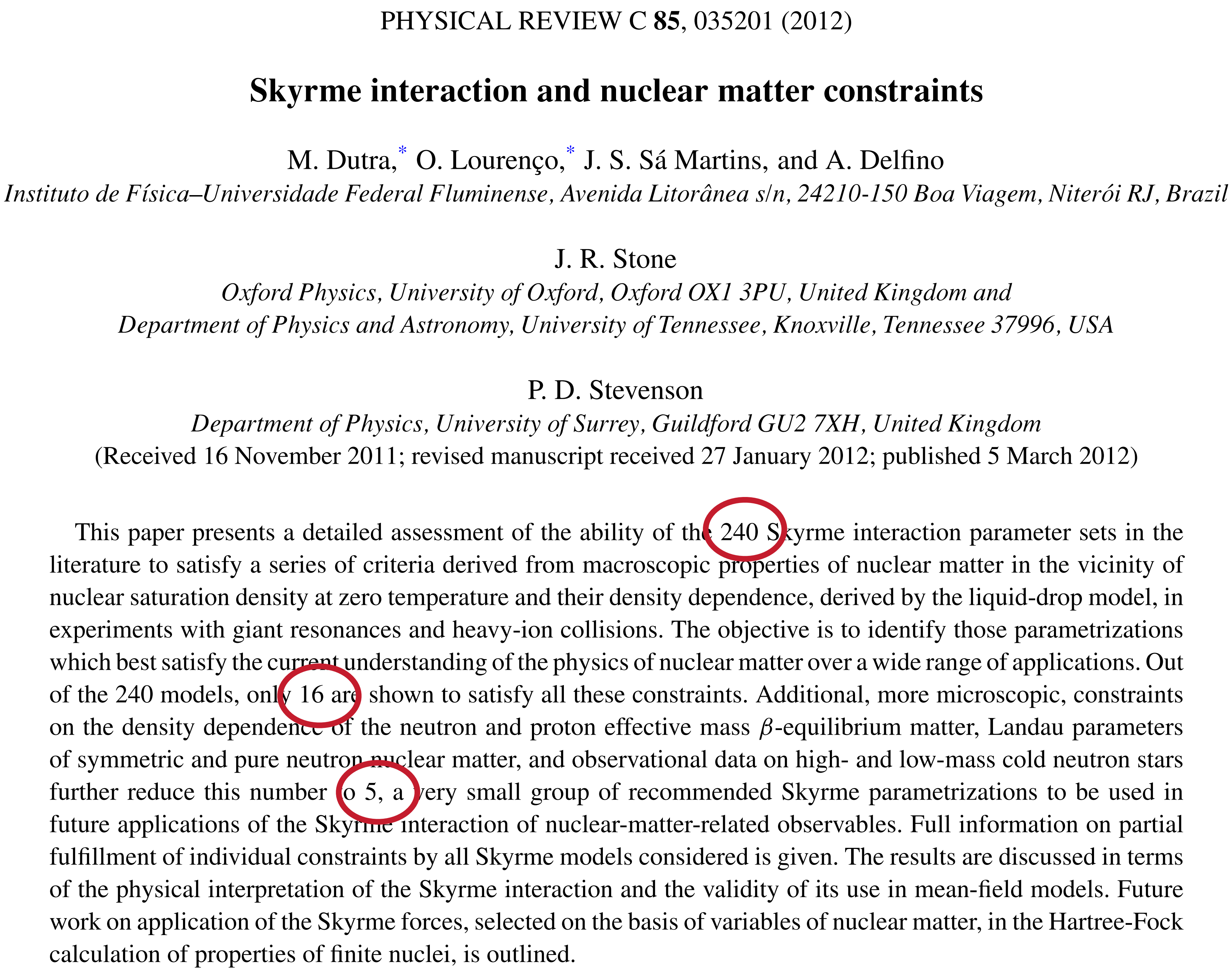
VARIOUS SKYRME MODELS

From 1972 to 2012



PHYSICAL REVIEW C **85**, 035201 (2012)

Skyrme interaction and nuclear matter constraints

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(Received 16 November 2011; revised manuscript received 27 January 2012; published 5 March 2012)

This paper presents a detailed assessment of the ability of the 240 Skyrme interaction parameter sets in the literature to satisfy a series of criteria derived from macroscopic properties of nuclear matter in the vicinity of nuclear saturation density at zero temperature and their density dependence, derived by the liquid-drop model, in experiments with giant resonances and heavy-ion collisions. The objective is to identify those parametrizations which best satisfy the current understanding of the physics of nuclear matter over a wide range of applications. Out of the 240 models, only 16 are shown to satisfy all these constraints. Additional, more microscopic, constraints on the density dependence of the neutron and proton effective mass β-equilibrium matter, Landau parameters of symmetric and pure neutron nuclear matter, and observational data on high- and low-mass cold neutron stars further reduce this number to 5, a very small group of recommended Skyrme parametrizations to be used in future applications of the Skyrme interaction of nuclear-matter-related observables. Full information on partial fulfillment of individual constraints by all Skyrme models considered is given. The results are discussed in terms of the physical interpretation of the Skyrme interaction and the validity of its use in mean-field models. Future work on application of the Skyrme forces, selected on the basis of variables of nuclear matter, in the Hartree-Fock calculation of properties of finite nuclei, is outlined.

SKYRME EDF

M. Dutra et al, PRC 85, 035201 (2012)

$$
\begin{aligned}
E = \frac{\mathcal{E}}{\rho} = {} & \frac{3\hbar^2}{10M}\left(\frac{3\pi^2}{2}\right)^{2/3}\rho^{2/3}H_{5/3} + \frac{t_0}{8}\rho[2(x_0+2)-(2x_0+1)H_2] + \frac{1}{48}\sum_{i=1}^{3} t_{3i}\rho^{\sigma_i+1}[2(x_{3i}+2)-(2x_{3i}+1)H_2] \\
& + \frac{3}{40}\left(\frac{3\pi^2}{2}\right)^{2/3}\rho^{5/3}(aH_{5/3}+bH_{8/3}) + \frac{3}{40}\left(\frac{3\pi^2}{2}\right)^{2/3}\rho^{5/3+\delta}\left[t_4(x_4+2)H_{5/3} - t_4\left(x_4+\frac{1}{2}\right)H_{8/3}\right] \\
& + \frac{3}{40}\left(\frac{3\pi^2}{2}\right)^{2/3}\rho^{5/3+\gamma}\left[t_5(x_5+2)H_{5/3} + t_5\left(x_5+\frac{1}{2}\right)H_{8/3}\right],
\end{aligned}
$$

$$
\begin{aligned}
V(\boldsymbol{r}_1,\boldsymbol{r}_2) = {} & t_0\,(1+x_0P_\sigma)\,\delta(\boldsymbol{r}) && \text{central term} \\
& +\frac{1}{2}t_1\,(1+x_1P_\sigma)\left[\boldsymbol{P}'^2\delta(\boldsymbol{r})+\delta(\boldsymbol{r})\boldsymbol{P}^2\right] && \\
& +t_2\,(1+x_2P_\sigma)\,\boldsymbol{P}'\cdot\delta(\boldsymbol{r})\boldsymbol{P} && \text{non-local terms} \\
& +\frac{1}{6}t_3\,(1+x_3P_\sigma)\left[\rho\,(\boldsymbol{R})\right]^\sigma\delta(\boldsymbol{r}) && \text{density-dependent term} \\
& +iW_0\boldsymbol{\sigma}\cdot\left[\boldsymbol{P}'\times\delta(\boldsymbol{r})\boldsymbol{P}\right] && \text{spin–orbit term .}
\end{aligned}
$$

Chananat, Bonche, Haensel, Meyer, Schaeffer, NPA 627, 710 (1997)

GENERALIZED SK

B. Cochet, K. Bennaceur, P. Bonche, T. Duguet, J. Meyer, NPA 731, 34 (2004)
B. Cochet, K. Bennaceur, J. Meyer, P. Bonche, T. Duguet, IJMPE 13, 187 (2004)
B.K. Agrawal, S.K. Dhiman, R. Kumar, PRC 85, 035201 (2012)

$$
\begin{aligned}
V(\vec{r}_1,\vec{r}_2) = {} & t_0(1+x_0P_\sigma)\delta(\vec{r}) \\
& + \frac{1}{2}t_1(1+x_1P_\sigma)[\delta(\vec{r})\vec{P}'^2 + \vec{P}^2\delta(\vec{r})] \\
& + t_2(1+x_2P_\sigma)\vec{P}'\cdot\delta(\vec{r})\vec{P} \\
& + \sum_i t_{3i}\rho^{\alpha_i}(1+x_{3i}P_\sigma)\delta(\vec{r}) \\
& + iW_0\vec{\sigma}\cdot[\vec{P}'\times\delta(\vec{r})\vec{P}],
\end{aligned}
$$

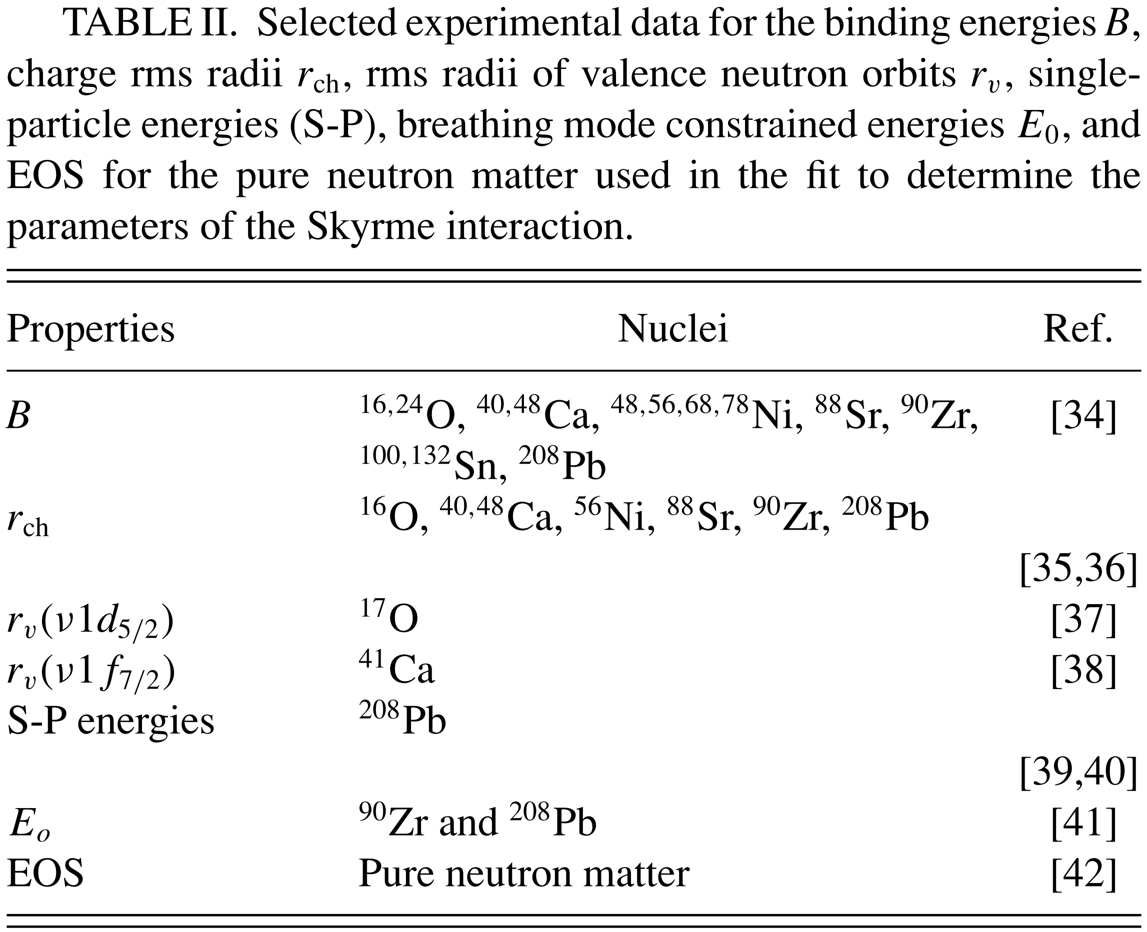
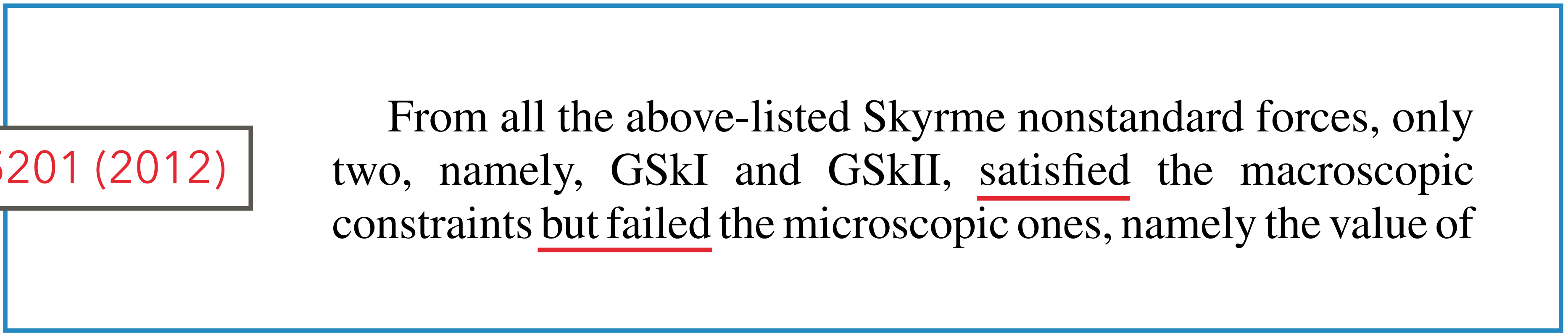


TABLE II. Selected experimental data for the binding energies B, charge rms radii r_{ch}, rms radii of valence neutron orbits r_v, single-particle energies (S-P), breathing mode constrained energies E_0, and EOS for the pure neutron matter used in the fit to determine the parameters of the Skyrme interaction.

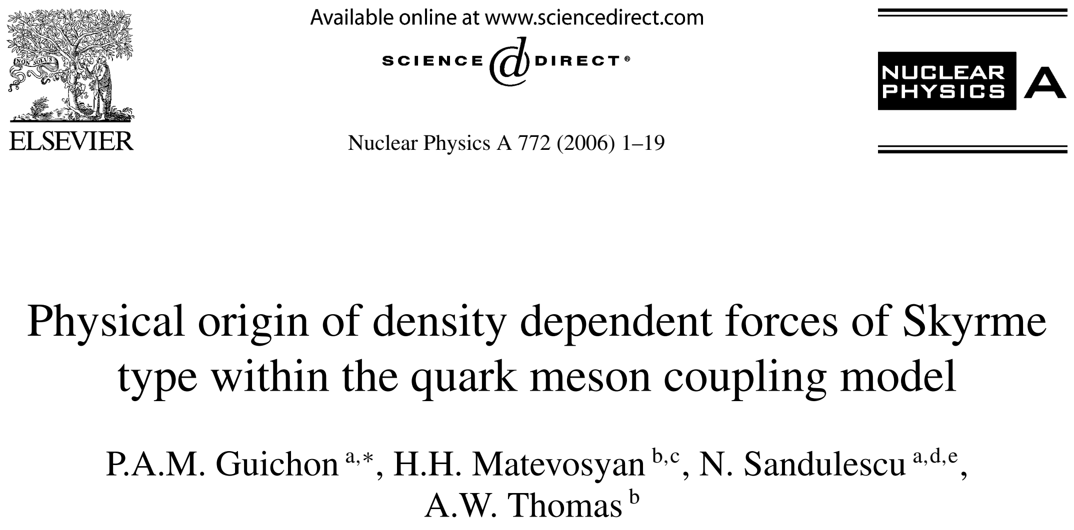
Properties	Nuclei	Ref.
B	^{16,24}O, ^{40,48}Ca, ^{48,56,68,78}Ni, ^{88}Sr, ^{90}Zr, ^{100,132}Sn, ^{208}Pb	[34]
r_{ch}	^{16}O, ^{40,48}Ca, ^{56}Ni, ^{88}Sr, ^{90}Zr, ^{208}Pb	
		[35,36]
$r_v(\nu 1d_{5/2})$	^{17}O	[37]
$r_v(\nu 1f_{7/2})$	^{41}Ca	[38]
S-P energies	^{208}Pb	
		[39,40]
E_o	^{90}Zr and ^{208}Pb	[41]
EOS	Pure neutron matter	[42]



From all the above-listed Skyrme nonstandard forces, only two, namely, GSkI and GSkII, satisfied the macroscopic constraints but failed the microscopic ones, namely the value of

M. Dutra et al, PRC 85, 035201 (2012)

SKYRME EDF AND QUARK–MESON COUPLING MODEL



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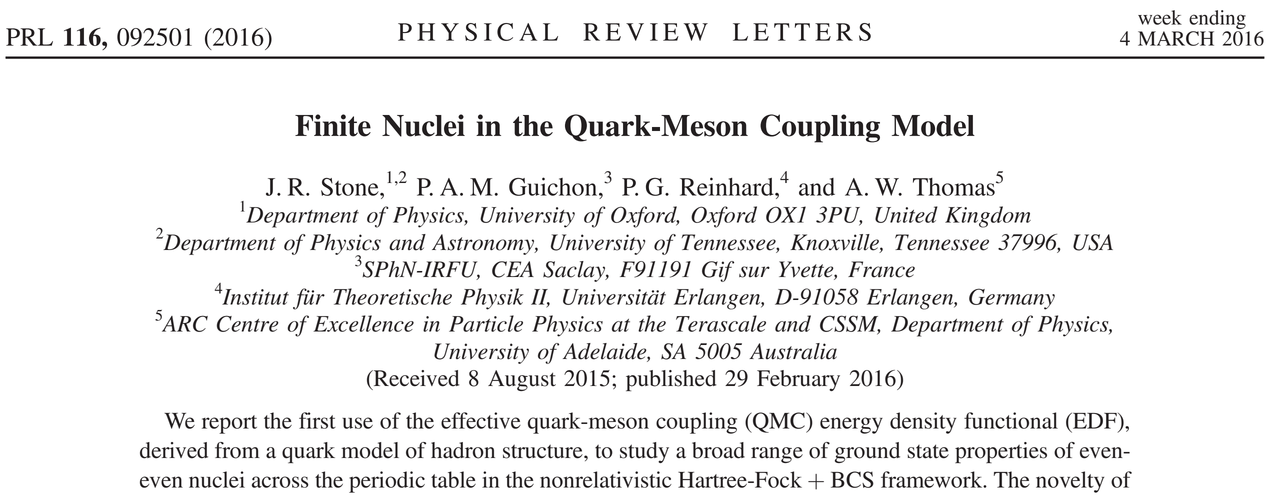
Nuclear Physics A 772 (2006) 1–19
NUCLEAR PHYSICS A

Physical origin of density dependent forces of Skyrme type within the quark meson coupling model

P.A.M. Guichon [a,*], H.H. Matevosyan [b,c], N. Sandulescu [a,d,e], A.W. Thomas [b]



PRL **116,** 092501 (2016) PHYSICAL REVIEW LETTERS week ending 4 MARCH 2016

Finite Nuclei in the Quark-Meson Coupling Model

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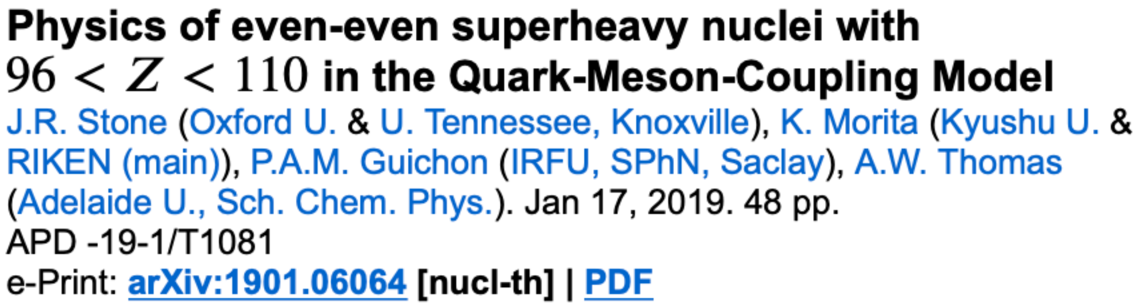
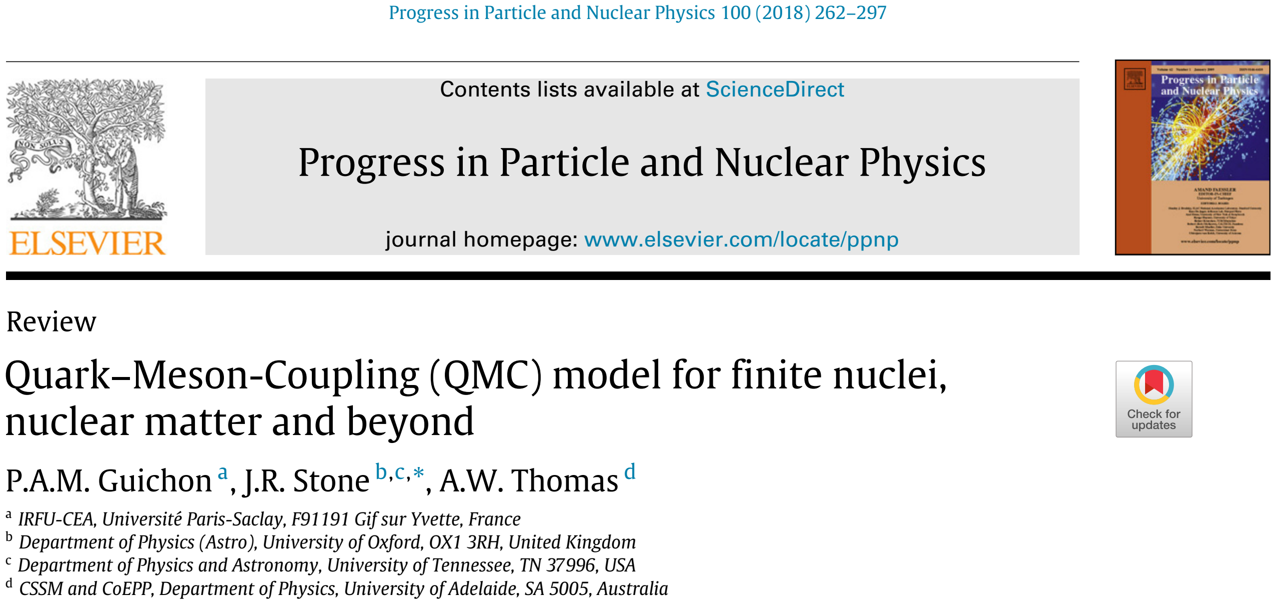
[3]*SPhN-IRFU, CEA Saclay, F91191 Gif sur Yvette, France*
[4]*Institut für Theoretische Physik II, Universität Erlangen, D-91058 Erlangen, Germany*
[5]*ARC Centre of Excellence in Particle Physics at the Terascale and CSSM, Department of Physics, University of Adelaide, SA 5005 Australia*
(Received 8 August 2015; published 29 February 2016)

We report the first use of the effective quark-meson coupling (QMC) energy density functional (EDF), derived from a quark model of hadron structure, to study a broad range of ground state properties of even-even nuclei across the periodic table in the nonrelativistic Hartree-Fock + BCS framework. The novelty of

106 nuclei were studied.



Physics of even-even superheavy nuclei with $96 < Z < 110$ in the Quark-Meson-Coupling Model
J.R. Stone (Oxford U. & U. Tennessee, Knoxville), K. Morita (Kyushu U. & RIKEN (main)), P.A.M. Guichon (IRFU, SPhN, Saclay), A.W. Thomas (Adelaide U., Sch. Chem. Phys.). Jan 17, 2019. 48 pp.
APD -19-1/T1081
e-Print: arXiv:1901.06064 [nucl-th] | PDF



Progress in Particle and Nuclear Physics 100 (2018) 262–297

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Review

Quark–Meson-Coupling (QMC) model for finite nuclei, nuclear matter and beyond

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J.R. Stone et al., PRL116, 092501 (2016)

$$\langle H(\vec{r}\,)\rangle = \rho M + \frac{\tau}{2M} + \mathcal{H}_0 + \mathcal{H}_3 + \mathcal{H}_{\rm eff} + \mathcal{H}_{\rm fin} + \mathcal{H}_{\rm so},$$

where

$$\mathcal{H}_0 + \mathcal{H}_3 = \rho^2\left[\frac{-3G_\rho}{32} + \frac{G_\sigma}{8(1+d\rho G_\sigma)^3} - \frac{G_\sigma}{2(1+d\rho G_\sigma)} + \frac{3G_\omega}{8}\right] + (\rho_n - \rho_p)^2\left[\frac{5G_\rho}{32} + \frac{G_\sigma}{8(1+d\rho G_\sigma)^3} - \frac{G_\omega}{8}\right],$$

$$\mathcal{H}_{\rm eff} = \left[\left(\frac{G_\rho}{8m_\rho{}^2} - \frac{G_\sigma}{2m_\sigma{}^2} + \frac{G_\omega}{2m_\omega{}^2} + \frac{G_\sigma}{4M_N{}^2}\right)\rho_n + \left(\frac{G_\rho}{4m_\rho^2} + \frac{G_\sigma}{2M_N^2}\right)\rho_p\right]\tau_n + p \leftrightarrow n,$$

$$\mathcal{H}_{\rm fin} = \left[\left(\frac{3G_\rho}{32m_\rho{}^2} - \frac{3G_\sigma}{8m_\sigma{}^2} + \frac{3G_\omega}{8m_\omega{}^2} - \frac{G_\sigma}{8M_N{}^2}\right)\rho_n + \left(\frac{-3G_\rho}{16m_\rho{}^2} - \frac{G_\sigma}{2m_\sigma{}^2} + \frac{G_\omega}{2m_\omega{}^2} - \frac{G_\sigma}{4M_N{}^2}\right)\rho_p\right]\nabla^2(\rho_n) + p \leftrightarrow n,$$

$$\mathcal{H}_{\rm so} = \nabla\cdot J_n\left[\left(\frac{-3G_\sigma}{8M_N{}^2} - \frac{3G_\omega(-1+2\mu_s)}{8M_N{}^2} - \frac{3G_\rho(-1+2\mu_v)}{32M_N{}^2}\right)\rho_n + \left(\frac{-G_\sigma}{4M_N{}^2} + \frac{G_\omega(1-2\mu_s)}{4M_N{}^2}\right)\rho_p\right] + p \leftrightarrow n.$$

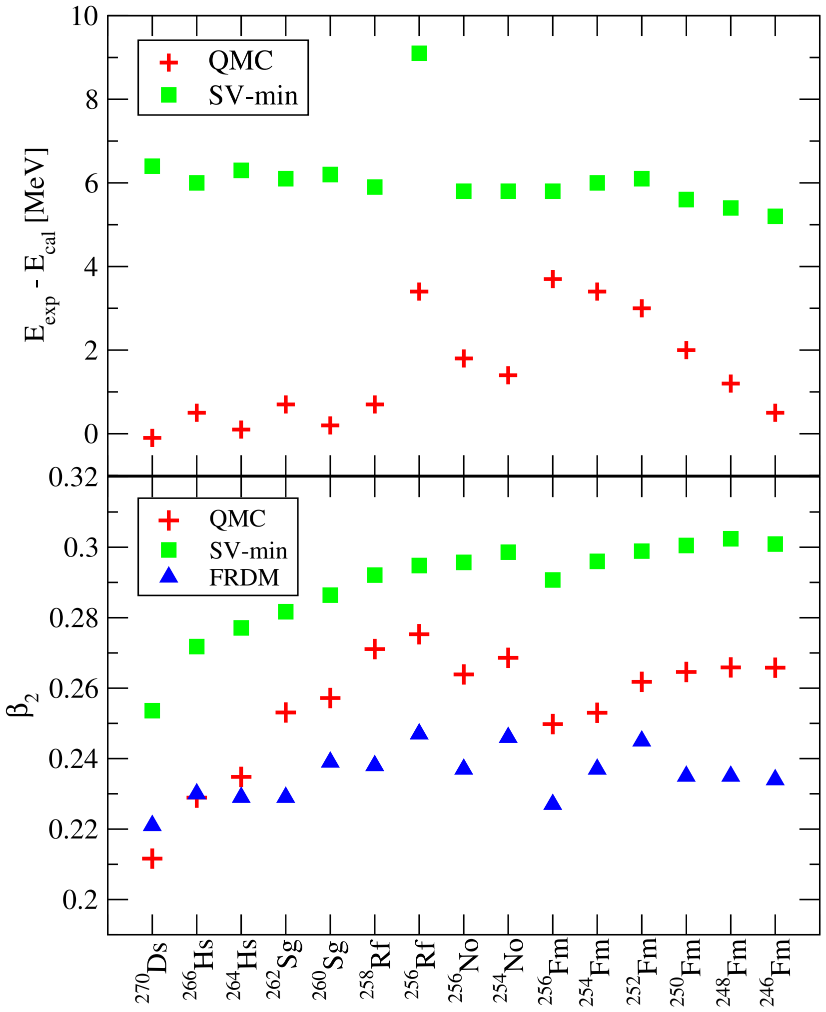


FIG. 1. Difference between calculated and experimental ground state binding energies of SHE as obtained with QMC and SV-min EDFs (top panel). β_2 are shown in the bottom panel, which also includes FRDM [20] predictions.

SV-min: Klüpfel et al., PRC79, 034310 (2009)

Table 2
Comparison of the SkM* parameters with the QMC predictions for several values of m_σ

m_σ (MeV)	t_0 (fm^2)	t_1 (fm^4)	t_2 (fm^4)	t_3 (fm$^{5/2}$)	x_0	W_0 (fm^4)	Deviation
600	−12.72	2.64	−1.12	74.25	0.17	0.6	33%
650	−12.48	2.21	−0.77	71.73	0.13	0.56	18%
700	−12.31	1.88	−0.49	69.8	0.1	0.53	18%
750	−12.18	1.62	−0.28	68.28	0.08	0.51	38%
SkM*	−13.4	2.08	−0.68	79	0.09	0.66	0%

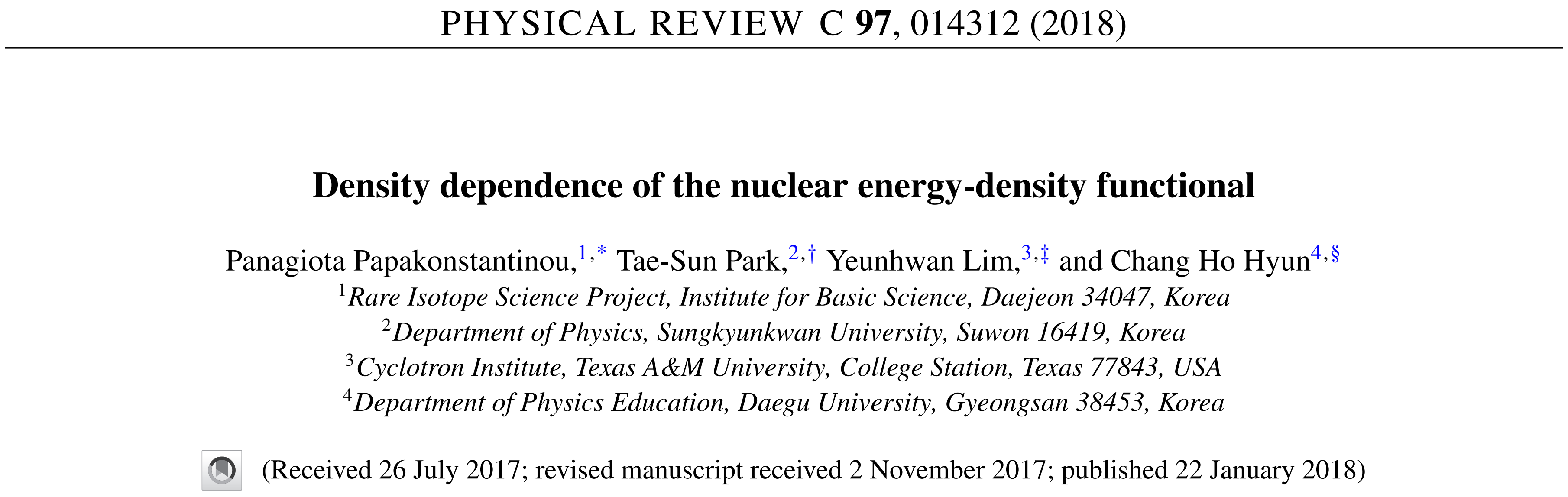
240 + 1 ?

240 + 1 ?

Unified phenomenological model for nuclear matter and nuclei

From nuclear matter to nuclei

POWER EXPANSION OF NUCLEAR EDF



PHYSICAL REVIEW C **97**, 014312 (2018)

Density dependence of the nuclear energy-density functional

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$$\mathcal{E}(\rho,\delta) = \frac{E(\rho,\delta)}{A} = \mathcal{T}(\rho,\delta) + \sum_{i=0}^{3} c_i(\delta)\rho^{1+i/3}$$

$$c_k(\delta) = \alpha_k + \delta^2 \beta_k$$

$$\delta = (\rho_n - \rho_p)/\rho$$

$$\mathcal{E} = \mathcal{T} + \sum_{i=0}^{3} \mathcal{E}_i + \mathcal{E}_{\ln},$$

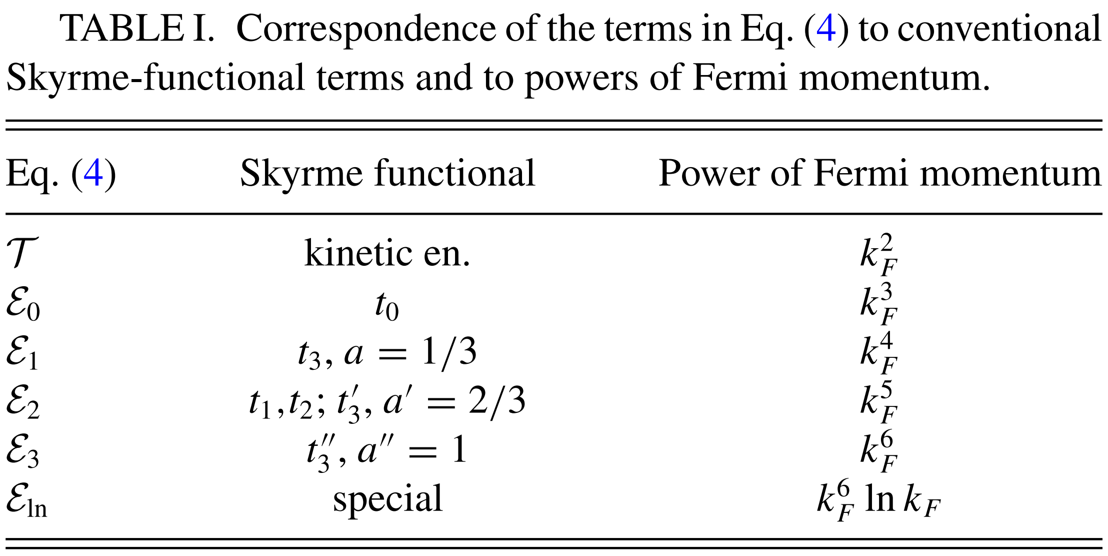
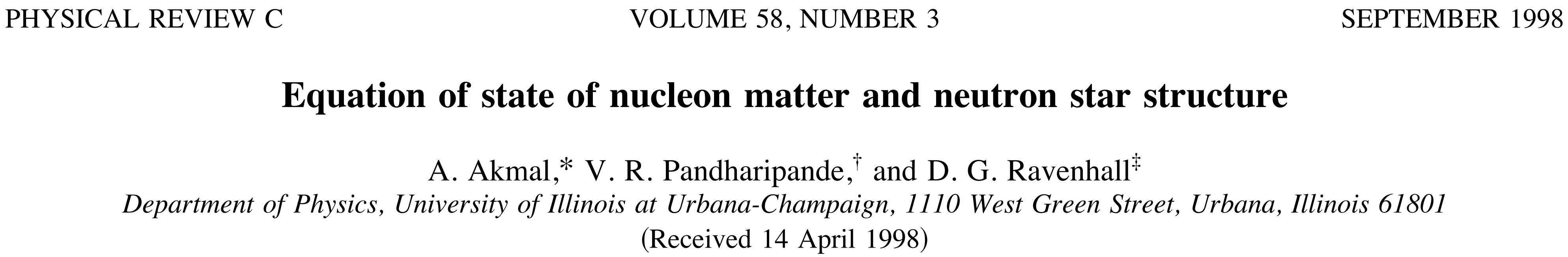


TABLE I. Correspondence of the terms in Eq. (4) to conventional Skyrme-functional terms and to powers of Fermi momentum.

Eq. (4)	Skyrme functional	Power of Fermi momentum
$\mathcal{T}$	kinetic en.	k_F^2
$\mathcal{E}_0$	t_0	k_F^3
$\mathcal{E}_1$	t_3, $a = 1/3$	k_F^4
$\mathcal{E}_2$	t_1, t_2; t_3', $a' = 2/3$	k_F^5
$\mathcal{E}_3$	t_3'', $a'' = 1$	k_F^6
$\mathcal{E}_{\ln}$	special	$k_F^6 \ln k_F$

KIDS MODEL (KOREA-IBS-DAEGU-SKKU)

- Symmetric Nuclear Matter (SNM): saturation density, binding energy and compressibility at saturation density
- Pure Neutron Matter (PNM): Pseudo-data (APR)



PHYSICAL REVIEW C VOLUME 58, NUMBER 3 SEPTEMBER 1998

Equation of state of nucleon matter and neutron star structure

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(Received 14 April 1998)

KIDS RESULTS

P. Papakonstantinou, T.-S. Park, Y. Lim, C.H. Hyun, PRC 85, 035201 (2012)

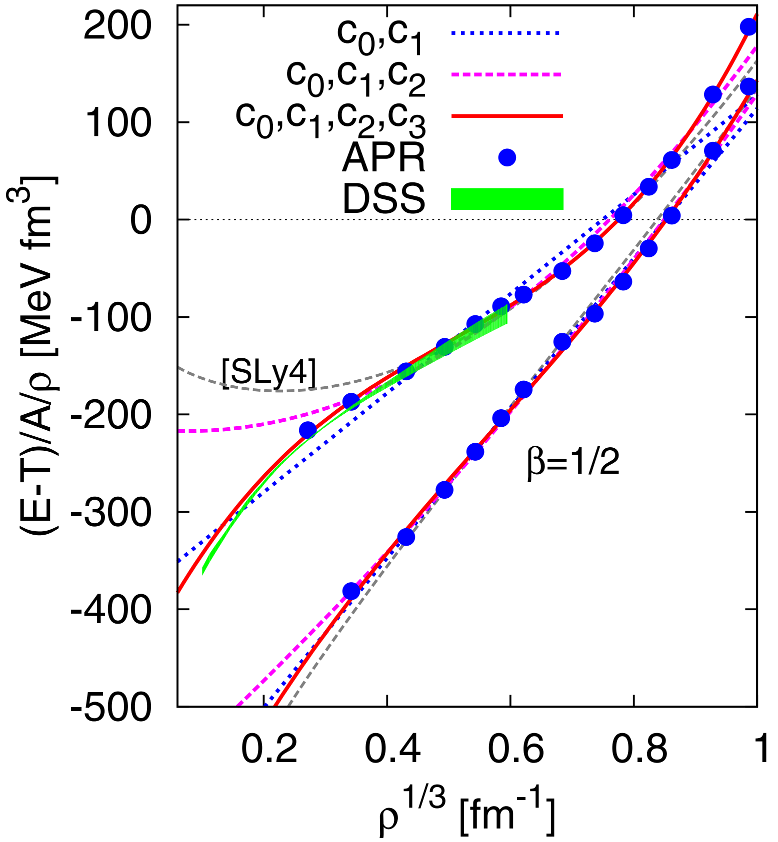


FIG. 1. Representative results ($\beta = 1/2$) of fits with two, three, or four parameters, as indicated, along with the APR pseudodata, the DSS results from chiral EFT and the SLy4 functional. Shown is the potential energy per particle divided by the density as a function of $\rho^{1/3}$, in SNM and PNM.

$$|\mathcal{E}_0| > |\mathcal{E}_1| > |\mathcal{E}_2| > |\mathcal{E}_3|$$

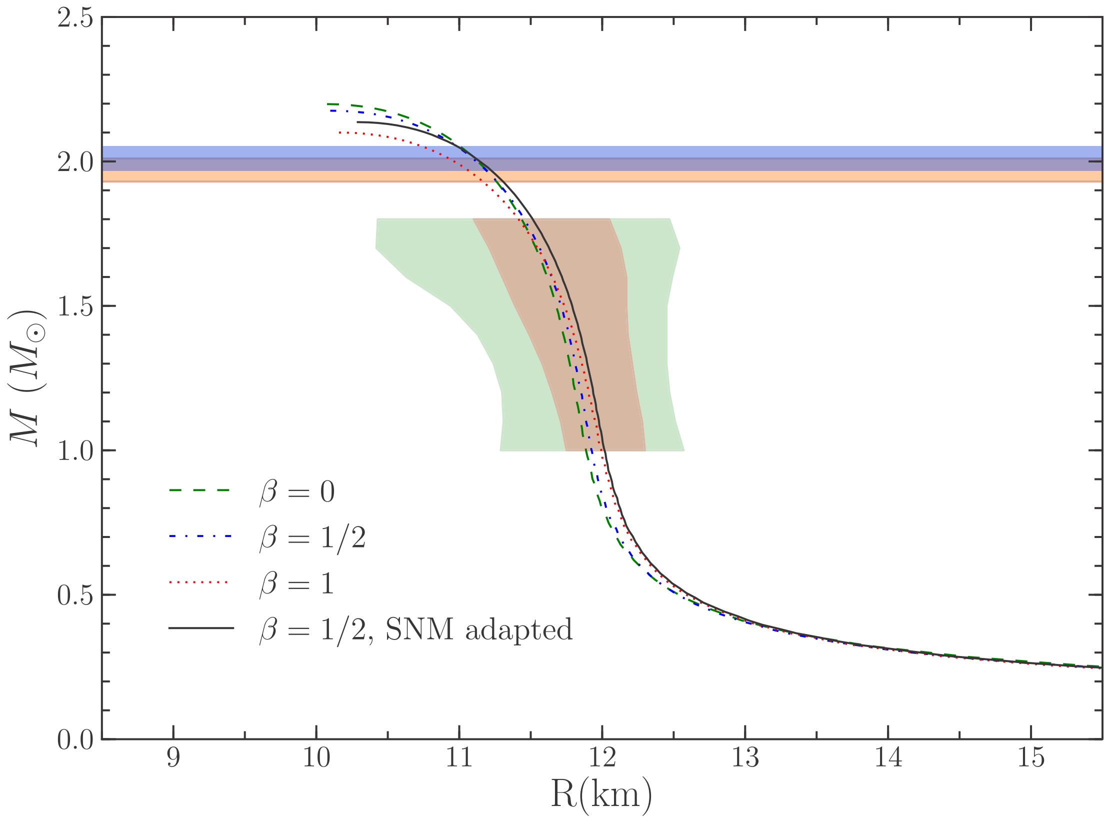


FIG. 3. Mass and radius relation of neutron stars for the models of Table III. The two horizontal bands represent the minimum of maximum mass of neutron stars [3,4]. The central region shows the allowed area of mass and radius of neutron stars analyzed from x-ray burst data [49]. The corresponding values for the central baryon density and the maximum mass (ϱ_c, M) in units of $(\mathrm{fm}^{-3}, M_\odot)$ are $(1.135, 2.20)$ $(\beta = 0)$, $(1.140, 2.18)$ $(\beta = 1/2)$, $(1.165, 2.10)$ $(\beta = 1)$, $(1.135, 2.14)$ ($\beta = 1/2$, adapted to the saturation point of SNM).

EXTENSION TO NUCLEAR PROPERTIES

β	Matter	c_0	c_1	c_2	c_3	ϱ_0	$\mathcal{E}_0$	K_∞
							J	L
0	SNM	−863.36	1945.05	−2060.20	1129.96	0.178	−15.4	215
	PNM	−483.96	1433.54	−2119.68	1385.22		34.2	55.9
$\frac{1}{2}$	SNM	−753.98	1389.20	−1171.03	678.87	0.177	−15.8	234
	PNM	−451.91	1254.32	−1812.62	1221.33		34.4	56.0
1	SNM	−613.13	620.22	154.72	−46.05	0.171	−16.1	247
	PNM	−408.56	991.76	−1323.81	937.96		34.0	54.9
ad-1	SNM	−648.72	676.25	200.92	−98.73	0.160	−16.0	240
	PNM	−451.91	1254.32	−1812.62	1221.33		32.8	47.9
ad-2	SNM	−664.52	763.55	40.13	0.00	0.160	−16.0	240
	PNM	−411.13	1007.78	−1354.64	956.47		33.5	50.5

KIDS parameters

We need
3 terms for SNM,
4 terms for PNM

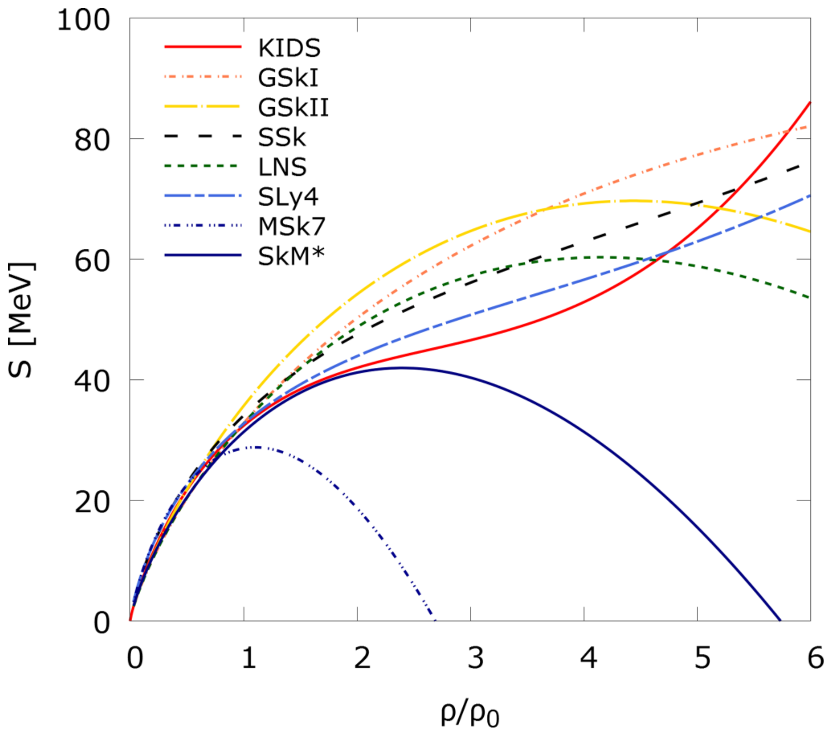
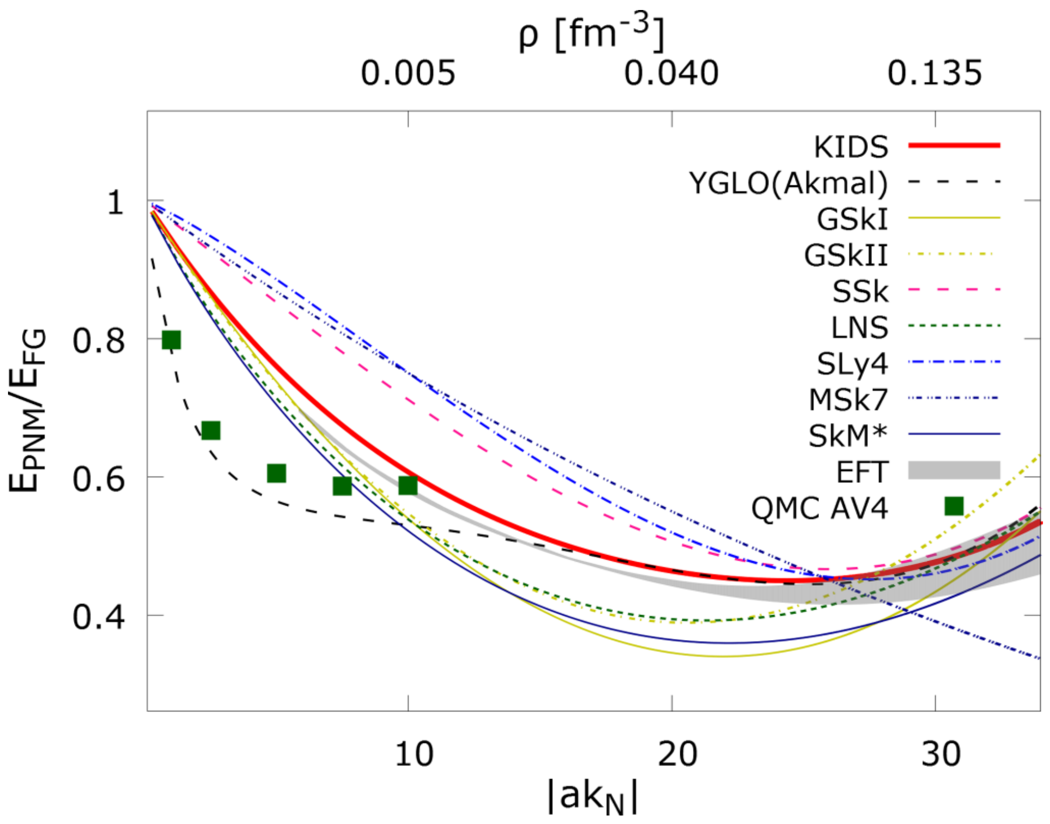


FIG. 1. Nuclear symmetry energy. The KIDS-ad2 equation of state employed in this paper is compared with other available models which have been fitted to nuclear data under various protocols.



H. Gil, P. Papakonstantinou, C.H. Hyun, Y. Oh, PRC 99, 064319 (2019)

EXTENSION TO NUCLEAR PROPERTIES

EDF FOR NUCLEAR MATTER ⇒ **SKYRME FUNCTIONAL**

$$\mathcal{E}(\rho,\delta) = \frac{E(\rho,\delta)}{A} = \mathcal{T}(\rho,\delta) + \sum_{i=0}^{3} c_i(\delta)\rho^{1+i/3}$$

$$c_k(\delta) = \alpha_k + \delta^2 \beta_k$$

$$\begin{aligned} v_{ij} = {} & (t_0 + y_0 P_\sigma)\delta(\mathbf{r}_i - \mathbf{r}_j) \\ & + \frac{1}{2}(t_1 + y_1 P_\sigma)[\delta(\mathbf{r}_i - \mathbf{r}_j)\mathbf{k}^2 + \mathbf{k}'^2\delta(\mathbf{r}_i - \mathbf{r}_j)] \\ & + (t_2 + y_2 P_\sigma)\mathbf{k}' \cdot \delta(\mathbf{r}_i - \mathbf{r}_j)\mathbf{k} \\ & + \frac{1}{6}\sum_{n=1}^{N-1}(t_{3n} + y_{3n} P_\sigma)\rho^{n/3}\delta(\mathbf{r}_i - \mathbf{r}_j) \\ & + iW_0\, \mathbf{k}' \times \delta(\mathbf{r}_i - \mathbf{r}_j)\,\mathbf{k} \cdot (\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j), \end{aligned}$$

$$\boxed{\begin{aligned} \mathscr{E} = {} & \frac{\hbar^2}{2m}\tau + \frac{3}{8}t_0\rho - \frac{1}{8}(t_0 + 2y_0)\rho\delta^2 + \frac{1}{16}\sum_{n=1}^{N-1} t_{3n}\rho^{1+n/3} \\ & - \frac{1}{48}\sum_{n=1}^{N-1}(t_{3n} + 2y_{3n})\rho^{1+n/3}\delta^2 + \frac{1}{64}(9t_1 - 5t_2 - 4y_2) \\ & \times \frac{(\nabla\rho)^2}{\rho} - \frac{1}{64}(3t_1 + 6y_1 + t_2 + 2y_2)\frac{(\nabla\rho\delta)^2}{\rho} \\ & + \frac{1}{8}(2t_1 + y_1 + 2t_2 + y_2)\tau - \frac{1}{8}(t_1 + 2y_1 - t_2 - 2y_2) \\ & \times \sum_q \frac{\rho_q\tau_q}{\rho} + \frac{1}{2}W_0\left(\frac{\mathbf{J}\cdot\nabla\rho}{\rho} + \sum_q \frac{\mathbf{J}_q\cdot\nabla\rho_q}{\rho}\right), \qquad (15) \end{aligned}}$$

Unified phenomenological model for nuclear matter and nucleus?

EXTENSION TO NUCLEAR PROPERTIES

$$
\begin{aligned}
t_0 &= \frac{8}{3}c_0(0), \quad y_0 = \frac{8}{3}c_0(0) - 4c_0(1), \\
t_{3n} &= 16c_n(0)\,, \quad y_{3n} = 16c_n(0) - 24c_n(1), \quad (n \neq 2), \\
t_{32} &= 16c_2(0) - \frac{3}{5}\left(\frac{3}{2}\pi^2\right)^{2/3}\theta_s \\
&\equiv 16c_2(0)(1-\zeta), \\
y_{32} &= 16c_2(0) - 24c_2(1) + \frac{3}{5}(3\pi^2)^{2/3}\left(3\theta_\mu - \frac{\theta_s}{2^{2/3}}\right) \\
&\equiv [16c_2(0) - 24c_2(1)](1-\zeta'),
\end{aligned}
$$

$$
\begin{aligned}
\theta_s &\equiv 3t_1 + 5t_2 + 4y_2 = \frac{5}{3}\left(\frac{3\pi^2}{2}\right)^{-2/3} 16c_2(0)\zeta\,, \\
\theta_\mu &\equiv t_1 + 3t_2 - y_1 + 3y_2 \\
&= \frac{\theta_s}{3\cdot 2^{2/3}} - \frac{5}{9}(3\pi^2)^{-2/3}[16c_2(0) - 24c_2(1)]\zeta'.
\end{aligned}
$$

Most parameters, $t_0, y_0, t_{31}, y_{31}, t_{33}, y_{33}$, are uniquely determined.

c_2 term has two components

- density-dependent terms t_{32}, y_{32}
- momentum-dependent terms t_1, t_2, y_1, y_2

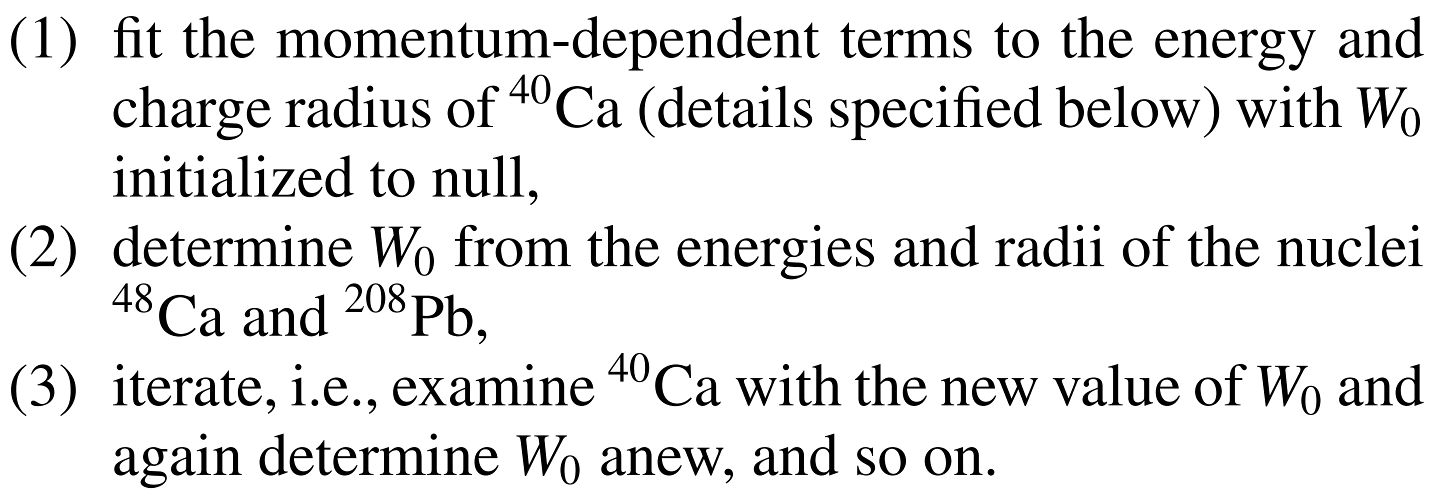
Isoscalar and isovector effective masses

$$
\begin{aligned}
\mu_s^{-1} &\equiv (m^*_{\rm IS}/m)^{-1} = 1 + \frac{m}{8\hbar^2}\rho\theta_s, \\
\mu_v^{-1} &\equiv (m^*_{\rm IV}/m)^{-1} = 1 + \frac{m}{4\hbar^2}\rho(\theta_s - \theta_\mu),
\end{aligned}
$$

H. Gil, Y.-M. Kim, C.H. Hyun, P. Papakonstantinou, Y. Oh, PRC 100 (2019)

PARAMETERS DETERMINATION

- Unknown coefficients are determined as follows.



(1) fit the momentum-dependent terms to the energy and charge radius of ^{40}Ca (details specified below) with W_0 initialized to null,
(2) determine W_0 from the energies and radii of the nuclei ^{48}Ca and ^{208}Pb,
(3) iterate, i.e., examine ^{40}Ca with the new value of W_0 and again determine W_0 anew, and so on.

- Check the dependence on the effective masses.

TABLE I. The Skyrme-type parameters, deviations of the calculated energies, and charge radii defined by Eq. (8), and the predictions for ^{60}Ca, for the models indicated on the leftmost column. The corresponding powers of the density-dependent couplings (a_1, a_2, a_3) are as in Eq. (1) except for SLy4 where there is only one density-dependent term with $a_1 = 1/6$. The errors D_E and D_R in the cases of GSkI and SLy4 are calculated based only on the values reported in the respective original publications [32,37].

Model, or: K_0 (μ_s, μ_v)	t_0 y_0	t_1 y_1	t_2 y_2	t_{31} y_{31}	t_{32} y_{32}	t_{33} y_{33}	W_0	D_E (%) D_R (%)	^{60}Ca: $\frac{E}{A}$ (MeV) R_c (fm)
KIDS0	−1772.04	275.72	−161.50	12216.73	571.07	0	108.35	0.32	7.6561
	−127.52	0	0	−11969.99	29485.49	−22955		0.56	3.6465
240 MeV	−1772.04	270.52	−355.95	12216.73	642.12	0	97.61	0.38	7.6993
(1.0, 0.82)	−127.52	156.90	242.04	−11969.99	29224.07	−22955		0.56	3.6416
240 MeV	−1772.04	448.99	−279.45	12216.73	−2572.65	0	135.24	0.26	7.6464
(0.7,0.82)	−127.52	−345.72	234.74	−11969.99	41318.69	−22955		0.52	3.6420
240 MeV	−1772.04	315.97	−527.58	12216.73	−191.34	0	107.58	0.38	7.6933
(0.9,1.00)	−127.52	−56.87	480.10	−11969.99	36289.12	−22955		0.57	3.6370
220 MeV	−1938.71	281.04	−479.05	15900.76	−2750.91	0	88.96	0.52	7.7701
(1.0,0.82)	−294.19	236.07	388.03	−8285.96	25831.04	−22955		0.94	3.6524
220 MeV	−1938.71	466.23	−439.68	15900.76	−5965.68	0	133.36	0.44	7.6807
(0.7,0.82)	−294.19	−247.10	422.10	−8285.96	37925.67	−22955		0.82	3.6663
GSkI [32]	−1855.45	397.23	264.63	13858.00	−2694.06	−319.87	169.57	0.16	7.6294
	−219.02	−698.59	−478.13	1747.29	3200.69	146.94		0.50	3.6640
SLy4 [37]	−2488.91	486.82	−546.39	13777.00			122.69	0.33	7.7030
	−2075.75	−167.37	546.39	18654.06				0.91	3.6734

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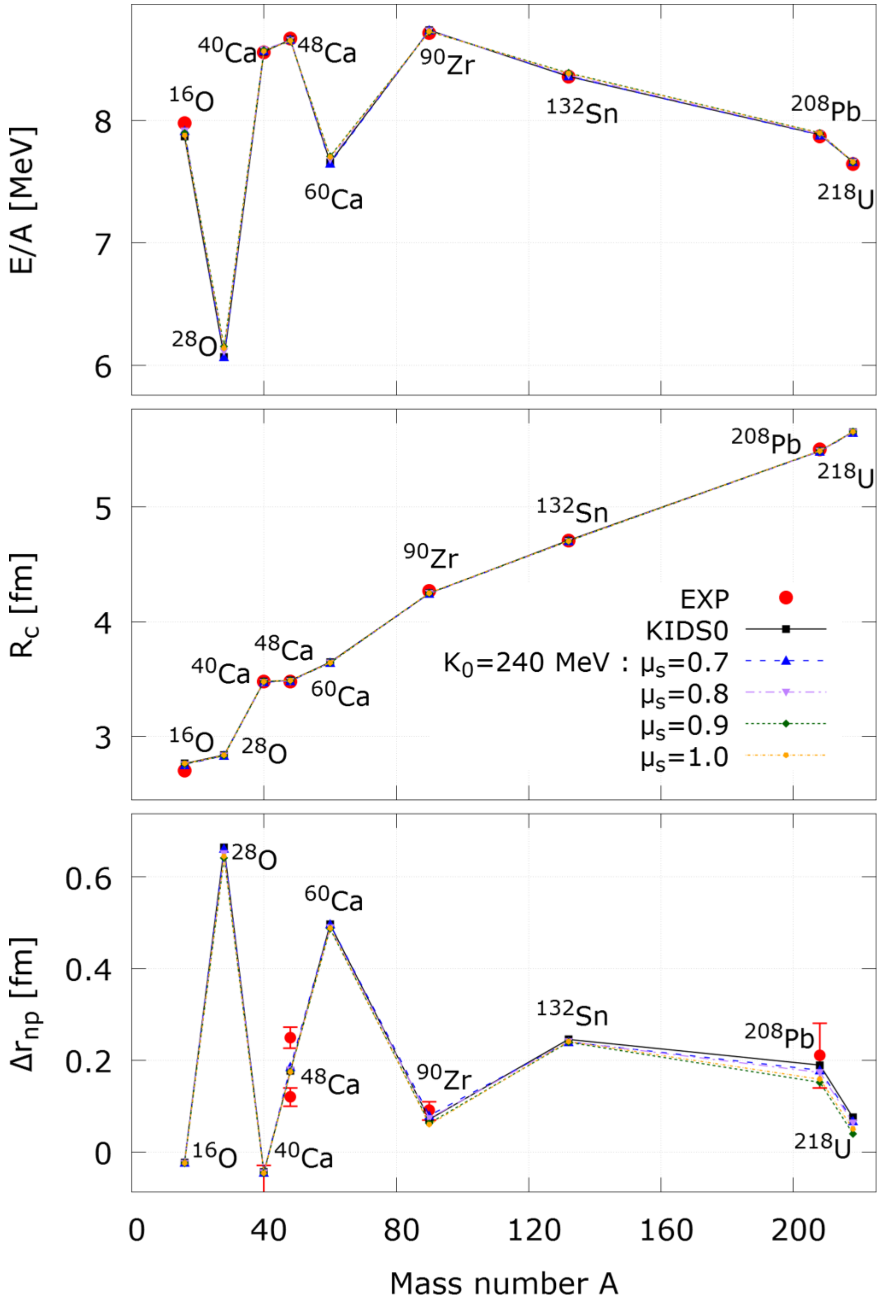

FIG. 3. Results for binding energy per nucleon E/A, charge radius R_c, and neutron-skin thickness Δr_{np}. All results for ^{16}O, ^{28}O, ^{60}Ca, ^{90}Zr, ^{132}Sn, and ^{218}U and all neutron-skin results are predictions. Data of energy per particle and charge radius are taken from the National Nuclear Data Center and Ref. [38], whereas neutron-skin data from Refs. [39–41].

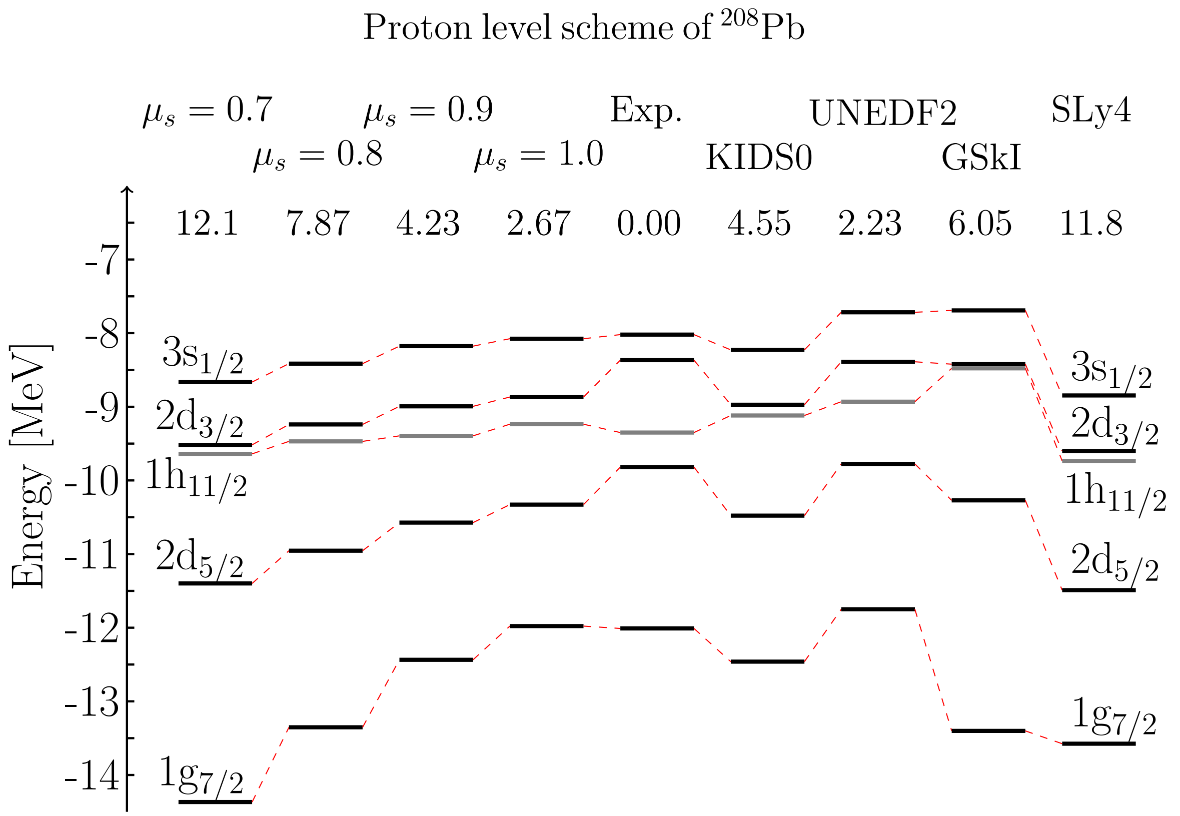

FIG. 4. Energies of occupied proton levels of ^{208}Pb from various models compared with empirical removal energies [44]. The KIDS results for $K_0 = 240$ MeV, $\mu_v = 0.82$, and varying μ_s are on the left of the experimental levels. The number under each model name is the deviation D of Eq. (9), in percentage points, for the levels shown underneath.

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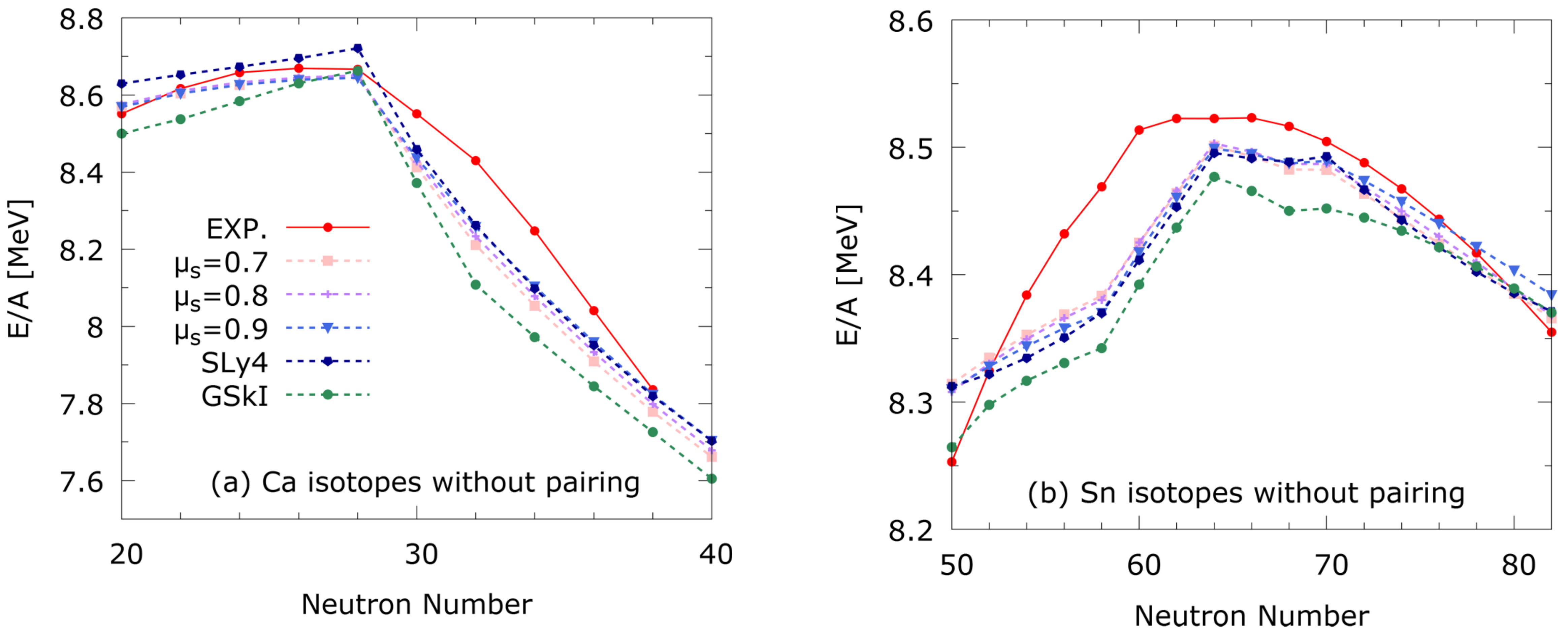

FIG. 5. Energy per particle of (a) Ca and (b) Sn isotopes without considering pairing correlations. The KIDS results for $K_0 = 240$ MeV, $\mu_v = 0.82$, and varying μ_s are shown along with those from representative Skyrme functionals (SLy4, GSkI) and experimental data.

H. Gil, P. Papakonstantinou, C.H. Hyun, Y. Oh, PRC 99, 064319 (2019)

CONVERGENCE IN THE CASE OF NUCLEAR PROPERTIES

$$\mathscr{E}(\rho,\delta) = \mathscr{T}(\rho,\delta) + \sum_{i=0}^{N-1} c_i(\delta)\rho^{1+i/3},$$

Energy per particle in homogeneous matter

$$\mathcal{S}(\rho) = \frac{1}{2}\frac{\partial^2}{\partial\delta^2}\mathscr{E}(\rho,\delta)\bigg|_{\delta=0} = \mathscr{T}_{\rm sym}(\rho) + \sum_{i=0}^{N-1} \beta_i\rho^{1+i/3},$$

Nuclear symmetry energy

EoS Parameters

$$\begin{aligned}\mathscr{E}(\rho,0) &= E_0 + \tfrac{1}{2}K_0x^2 + \tfrac{1}{6}Q_0x^3 + O(x^4),\\ \mathcal{S}(\rho) &= J + Lx + \tfrac{1}{2}K_{\rm sym}x^2 + \tfrac{1}{6}Q_{\rm sym}x^3 + \tfrac{1}{24}R_{\rm sym}x^4\\ &\quad + O(x^5),\end{aligned}$$

$$x = (\rho - \rho_0)/(3\rho_0)$$

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CONVERGENCE IN THE CASE OF NUCLEAR PROPERTIES

SNM

ρ_0 Saturation density

$$K_0 \equiv 9\rho_0^2 \frac{d^2}{d\rho^2} \frac{\mathscr{E}(\rho,0)}{\rho}\bigg|_{\rho=\rho_0},$$

Compression modulus

E_0 Energy per particle at saturation density

$$Q_0 \equiv 27\rho_0^3 \frac{d^3}{d\rho^3} \mathscr{E}(\rho,0)\bigg|_{\rho=\rho_0}.$$

Skewness coefficient

PNM

$J = S(\rho_0)$ Symmetry energy at saturation density

$$L \equiv 3\rho_0 \frac{d}{d\rho} \mathcal{S}(\rho)\bigg|_{\rho=\rho_0},$$

Slope

$$Q_{\rm sym} \equiv 27\rho_0^3 \frac{d^3}{d\rho^3} \mathcal{S}(\rho)\bigg|_{\rho=\rho_0},$$

Skewness coefficient

$$K_{\rm sym} \equiv 9\rho_0^2 \frac{d^2}{d\rho^2} \frac{\mathcal{S}(\rho)}{\rho}\bigg|_{\rho=\rho_0}.$$

Curvature

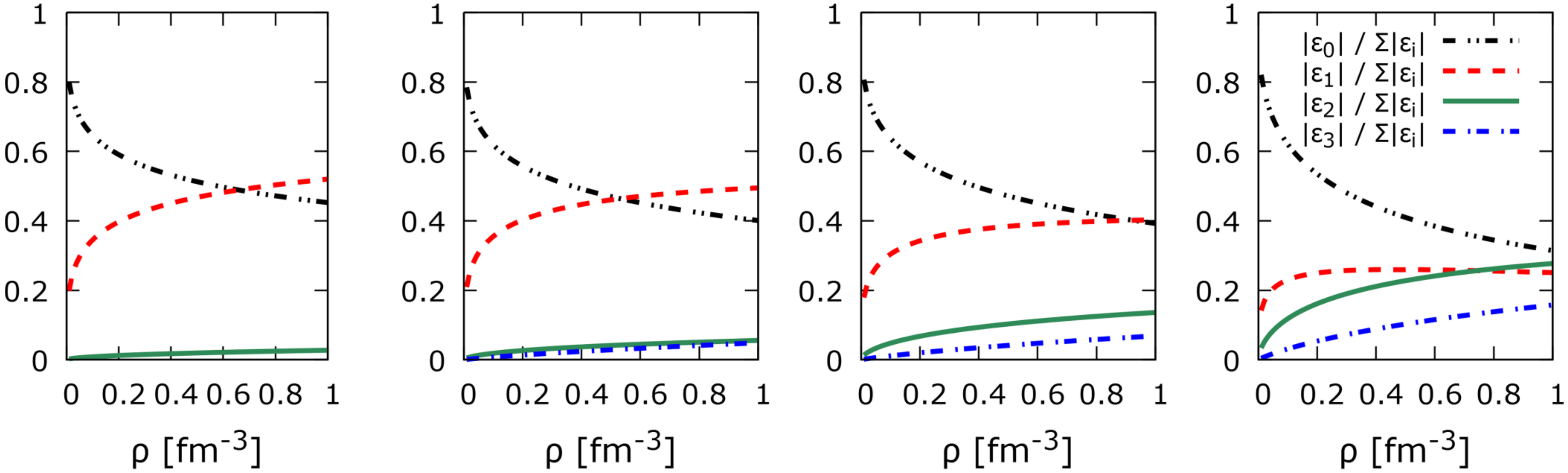
$$R_{\rm sym} \equiv 81\rho_0^4 \frac{d^4}{d\rho^4} \mathcal{S}(\rho)\bigg|_{\rho=\rho_0}.$$

Kurtosis

SNM

TABLE I. Fitted values of parameters α_i in units of MeV fm^{3+i}. Model S3b with $N = 3$ the EoS parameters are fixed assuming $\alpha_3 = 0$ with $\rho_0 = 0.16\,\mathrm{fm}^{-3}$, $E_0 = -16.0\,\mathrm{MeV}$, and $K_0 = 240.0\,\mathrm{MeV}$ with β_i of KIDS-ad2. Models S4a, S4b, and S4c correspond to $Q_0 = -360$, -390, and $-420\,\mathrm{MeV}$, respectively. For S3b, we obtain $Q_0 = -372.65\,\mathrm{MeV}$. The EoS of PNM is fixed by the baseline parameters shown at the bottom, which corresponds to KIDS-ad2.

Model	N	α_0	α_1	α_2	α_3
S3b	3	−664.52	763.55	40.13	0
S4a	4	−677.69	836.34	−93.95	82.33
S4b	4	−646.44	663.65	224.15	−112.99
S4c	4	−615.19	490.96	542.24	−308.30
PNM	N	$\alpha_0 + \beta_0$	$\alpha_1 + \beta_1$	$\alpha_2 + \beta_2$	$\alpha_3 + \beta_3$
KIDS-ad2	4	−411.13	1007.78	−1354.64	956.47



(a) S3b (b) S4a (c) S4b (d) S4c

FIG. 1. Relative magnitude of each interaction potential for symmetric matter for model (a) S3b, (b) S4a, (c) S4b, and (d) S4c.

H. Gil, Y.-M. Kim, C.H. Hyun, P. Papakonstantinou, Y. Oh, PRC 100 (2019)

PNM

TABLE IV. Values of $c_i(1)$ fitted to APR EoS of PNM. The unit of c_i is MeV fm^{3+i} and the units of J, L, $K_{\rm sym}$, $Q_{\rm sym}$, and $R_{\rm sym}$ are MeV.

Model	N	$c_0(1)$	$c_1(1)$	$c_2(1)$	$c_3(1)$	$c_4(1)$	$c_5(1)$	χ_n^2	J	L	$K_{\rm sym}$	$Q_{\rm sym}$	$R_{\rm sym}$
P3	3	−266.72	133.50	281.38	–	–	–	5.3×10^{-4}	32.6	53.5	−129.7	422.3	−2421.8
P4	4	−407.94	990.09	−1321.86	937.14	–	–	1.4×10^{-4}	32.8	49.2	−156.3	583.1	−2469.7
P5	5	−224.16	−479.28	2814.48	−3963.71	2075.79	–	6.3×10^{-5}	33.0	51.4	−166.8	461.4	−1388.4
P6a	6	−224.81	−473.46	2795.50	−3935.18	2056.11	4.94	6.3×10^{-5}	33.0	51.4	−166.8	461.6	−1391.7
P6b	6	−283.99	110.63	604.05	−10.59	−1312.44	1117.76	6.4×10^{-5}	33.0	51.5	−163.8	450.0	−1545.9
P6c	6	−313.98	400.88	−463.41	1864.00	−2891.61	1630.37	6.5×10^{-5}	33.0	51.5	−162.3	446.6	−1631.2

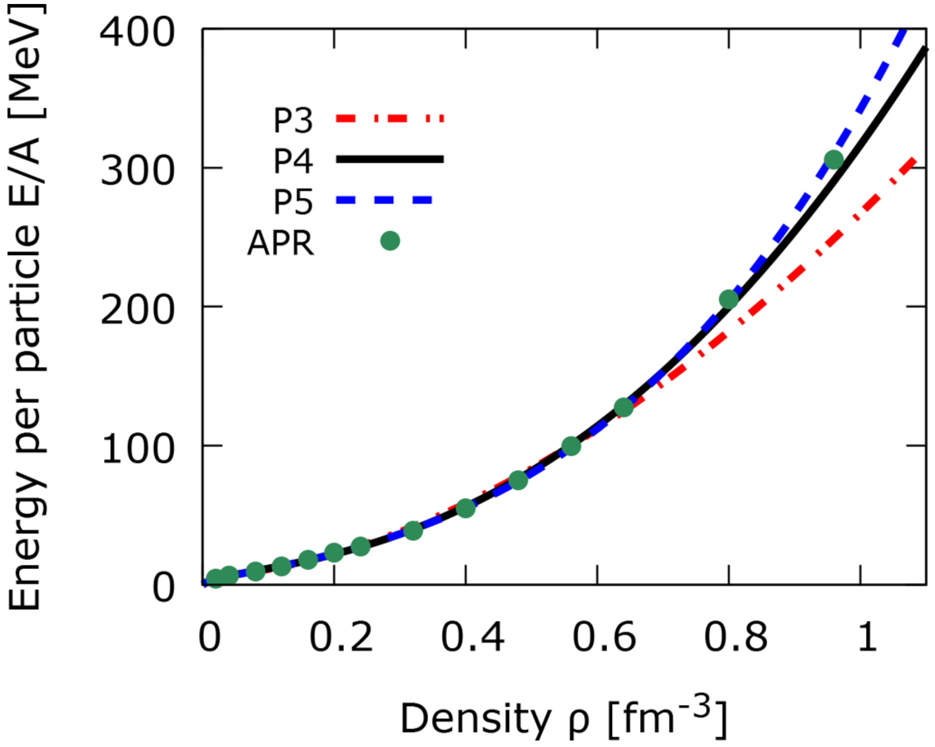


FIG. 5. Energy per particle of pure neutron matter with models P3, P4, and P5 presented in Table IV. Here, the symmetric EoS parameters α_i are fixed as model S3b in Table I.

TABLE VI. Same as Table III but for P3, P4, and P5. Note that $t_{33} = t_{34} = 0$ as we use S3b for $\alpha_i = c_i(0)$.

Parameter	P3	P4	P5
t_0 (MeV fm^3)	−1772.04	−1772.04	−1772.04
y_0 (MeV fm^3)	−705.16	−140.27	−875.42
t_1 (MeV fm^5)	247.33	275.83	269.90
t_2 (MeV fm^5)	−173.00	−161.48	−163.95
t_{31} (10^4 MeV fm^4)	12216.73	12216.73	12216.73
y_{31} (10^4 MeV fm^4)	9012.81	−11545.41	23719.36
t_{32} (MeV fm^5)	1087.14	569.38	678.46
y_{32} (10^4 MeV fm^5)	−10346.18	28700.54	−70692.70
y_{33} (10^4 MeV fm^6)	–	−22491.36	95128.93
y_{34} (10^4 MeV fm^7)	–	–	−49818.87
ζ	−0.6931	0.1133	−0.0566
W_0 (MeV fm^5)	104.12	108.46	108.25

TABLE VII. Same as Table II but for P3, P4, and P5. The SNM parameters are fixed to the values of model S3b in Table I. The experimental data are from Refs. [16,17].

Nuclei	Energy per particle (MeV)				Charge radius (fm)			
	Expt.	P3	P4	P5	Expt.	P3	P4	P5
^{40}Ca	8.5513*	8.5573	8.5564	8.5561	3.4776*	3.4785	3.4781	3.4782
		(0.070%)	(0.059%)	(0.056%)		(0.026%)	(0.014%)	(0.015%)
^{48}Ca	8.6667*	8.6556	8.6565	8.6581	3.4771*	3.4891	3.4867	3.4870
		(0.129%)	(0.118%)	(0.099%)		(0.345%)	(0.277%)	(0.285%)
^{208}Pb	7.8675*	7.8849	7.8806	7.8793	5.5012*	5.4934	5.4886	5.4891
		(0.222%)	(0.167%)	(0.151%)		(0.141%)	(0.228%)	(0.221%)
^{16}O	7.9762	7.8641	7.8683	7.8669	2.6991	2.7634	2.7618	2.7621
		(1.405%)	(1.353%)	(1.371%)		(2.382%)	(2.322%)	(2.335%)
^{28}O	–	6.0705	6.0628	6.0585	–	2.8435	2.8371	2.8396
^{60}Ca	–	7.6659	7.6548	7.6513	–	3.6511	3.6465	3.6478
^{90}Zr	8.7100	8.7336	8.7330	8.7344	4.2694	4.2489	4.2476	4.2476
		(0.272%)	(0.264%)	(0.280%)		(0.480%)	(0.510%)	(0.511%)
^{132}Sn	8.3549	8.3592	8.3559	8.3549	4.7093	4.7133	4.7088	4.7090
		(0.052%)	(0.013%)	(0.001%)		(0.085%)	(0.010%)	(0.006%)

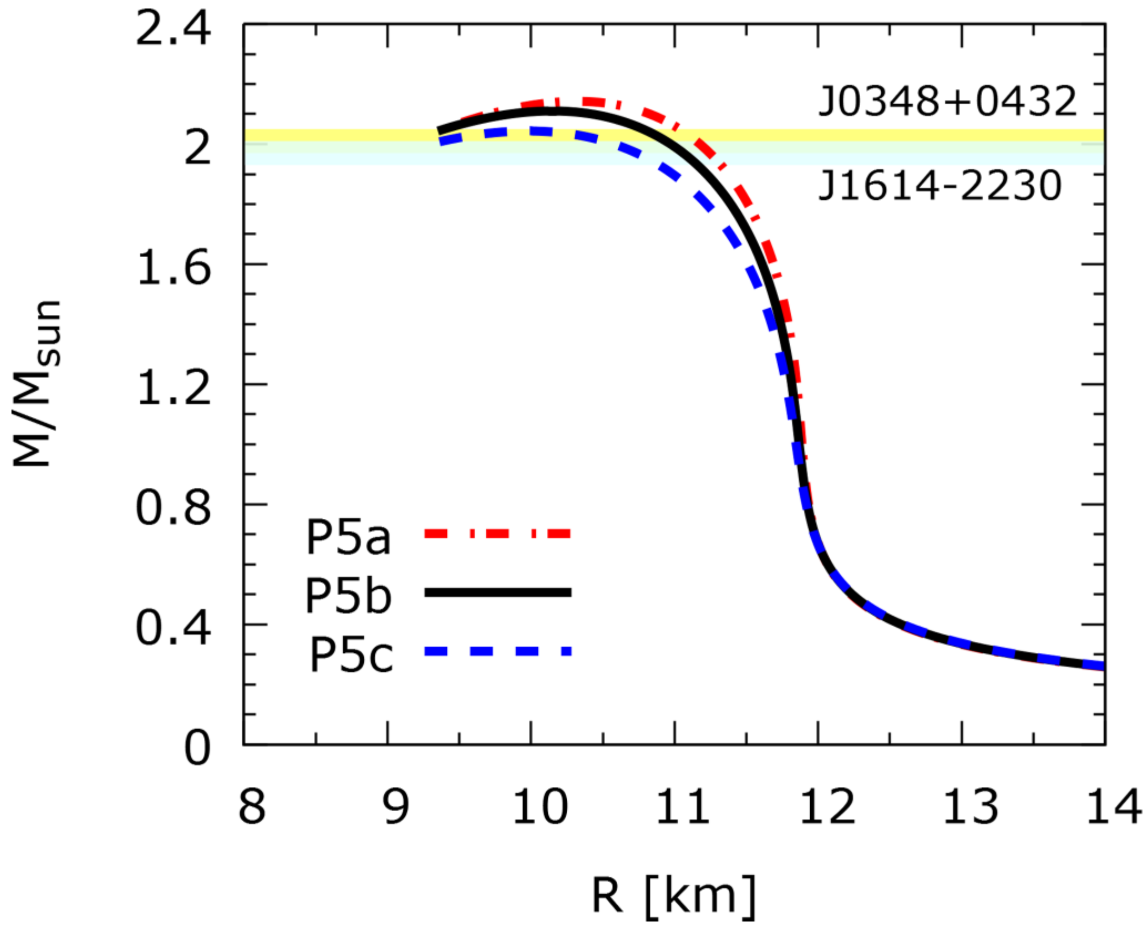


FIG. 8. Neutron star mass-radius relations: Results corresponding to the parameter sets P5a, P5b, and P5c.

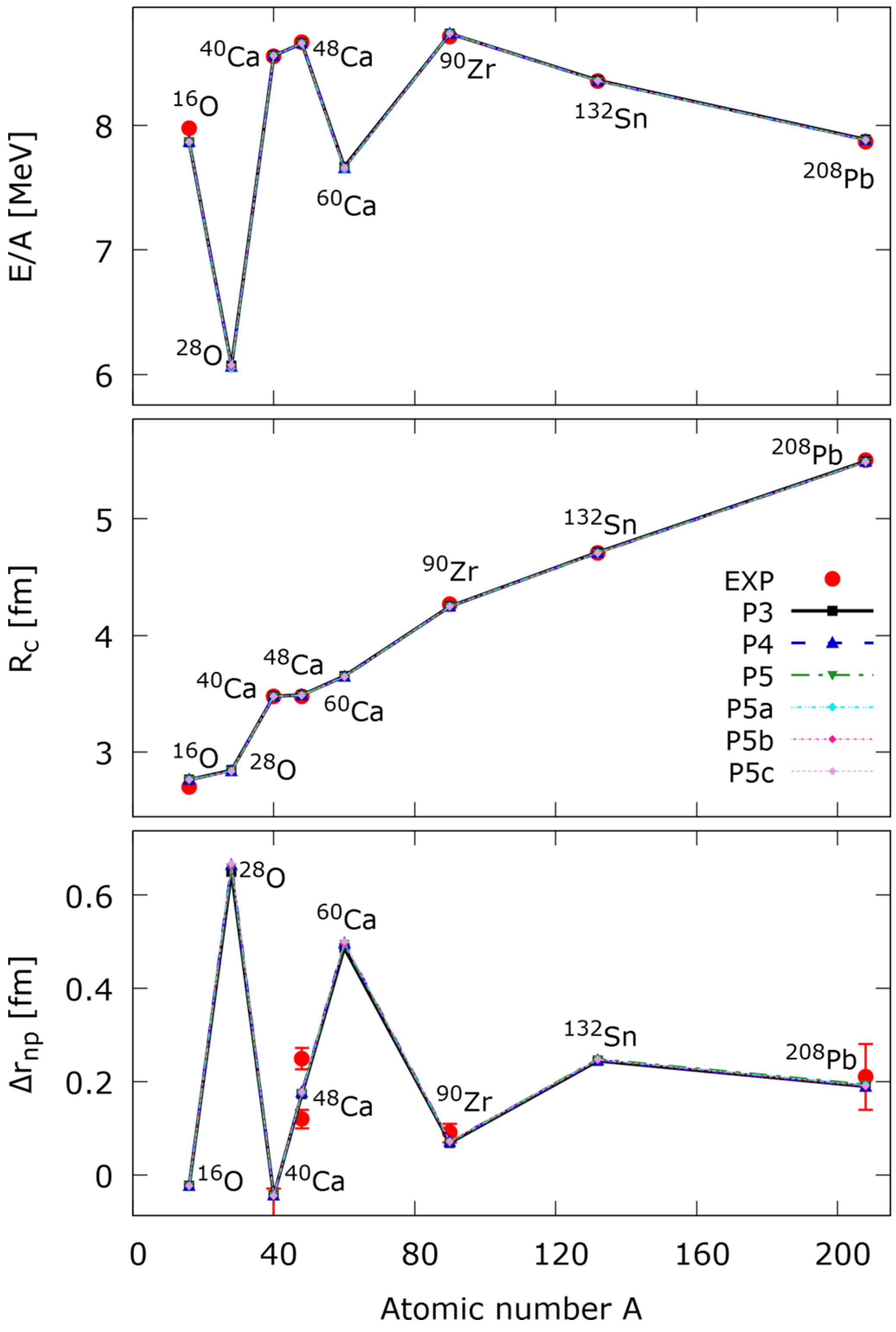


FIG. 10. Results for E/A, R_c, and neutron skin thickness Δr_{np}. Neutron skin thickness data are from Refs. [24–26].

CONCLUSIONS & OUTLOOK

- We developed a generalized Skyrme functional for nucleus: Nuclear Matter → Nucleus
- Power series expansion of nuclear EDF in the Fermi momentum.
- Independence of nuclear bulk properties on effective masses
- Neutron star mass-radius relation
- Other nucleus; nuclear chart; drip lines; pairing; deformation, response to external perturbations and much more

M. Dutra et al, PRC 85, 035201 (2012)

240 → 16 → 5

$\Downarrow$

241 → 17 → 6 ?!

MORE WORKS TO BE DONE!

CONCLUSIONS & OUTLOOK

- To develop more realistic theories on the nuclear α decay.
  - simple potential models
  - based on EDF
- Other elements
  - deformation
  - direct calculation using α cluster models
  - other theoretical framework