

BFKL evolution manifestation in jet production at colliders

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Outline:

- **High energy asymptotics of pQCD**
- **BFKLP: NLL BFKL within generalized BLM**
- **$\gamma^*\gamma^*$ - collisions at LEP2**
- **Dijets from pQCD dynamics: GLAPD vs. BFKL**
- **Summary**



Lev N. Lipatov:

■ High-energy asymptotics in QED

V. Gribov, V. Gorshkov, L. Lipatov, G. Frolov (1969-70)

■ High-energy Bjorken asymptotics in QCD: GLAPD

V. Gribov, L. Lipatov (1971-72) L. Lipatov (1974)

G. Altarelli, G. Parisi (1977) Yu. Dokshitzer (1977)

■ High-energy asymptotics in QCD: LL BFKL

V. Fadin, E. Kuraev, L. Lipatov (1975-77)

I. Balitsky, L. Lipatov (1978) L. Lipatov (1986)

■ High-energy asymptotics in QCD: NLL BFKL

V. Fadin, L. Lipatov (1989-98)

S. Brodsky, V. Fadin, V. K., L. Lipatov, G. Pivovarov (1999-02)

■ High-energy asymptotics in quantum gravity

L. Lipatov (1989)

■ High-energy QCD as an integrable theory

L. Lipatov (1993) L. Faddeev, G. Korchemsky (1994)

■ AdS/CFT (N=4 SUSY)

A. Kotikov, L. Lipatov, A. Onischenko, V. Velizhanin

■ High-order estimate in QFT (1976)

L. Lipatov (1976)



1940 - 2017

■ BFKL

High-energy asymptotics in QCD: LL BFKL

V. Fadin, E. Kuraev, L. Lipatov (1975-77)

I. Balitsky, L. Lipatov (1978) L. Lipatov (1986)

■ BFKL

High-energy asymptotics in QCD: spin-dependent evolution, GPDF

A. Bukhvostov, G. Frolov, E. Kuraev, L. Lipatov (1982-86)

■ BFKLP

High-energy asymptotics in QCD: NLL BFKL

S. Brodsky, V. Fadin, V. K., L. Lipatov, G. Pivovarov (1999-02)

High-energy asymptotics of pQCD: GLAPD and BFKL

$$s=(p_1+p_2)^2$$
$$t=(p_1-p_3)^2 \quad Q^2=-t$$

Scattering in the Standard Model (QCD) at high energies:

Large logarithms: as $\log(s)$, as $\log(Q^2)$

Bjorken limit (large-angle scattering):

$$s \sim Q^2 \gg m^2$$

$$Q^2/s = x \sim 1$$

Gribov-Lipatov-Altarelli-Parisi-Dokshitzer (GLAPD) \leftrightarrow 1-scale RG:

(as $\log(Q^2)$)ⁿ resummation

Inclusive cross section $\sim 1/Q^4$

Gribov-Regge limit (small-angle scattering):

$$s \gg Q^2 \gg m^2$$

$$Q^2/s = x \Rightarrow 0$$

Balitsky-Fadin-Kuraev-Lipatov (BFKL):

(as $\log(s)$)ⁿ resummation

Total cross section $\sim s^{(a_P-1)}$

a_P – Pomeron intercept

soft scattering data: $a_P = 1.1$

- Large-angle scattering:

QED and QCD in Bjorken limit

- **GLAPD: V. Gribov & L. Lipatov (71-72); L. Lipatov (74);
G. Altarelli & G. Parisi (77); Yu. Dokshitzer (77)**

- Small-angle scattering:

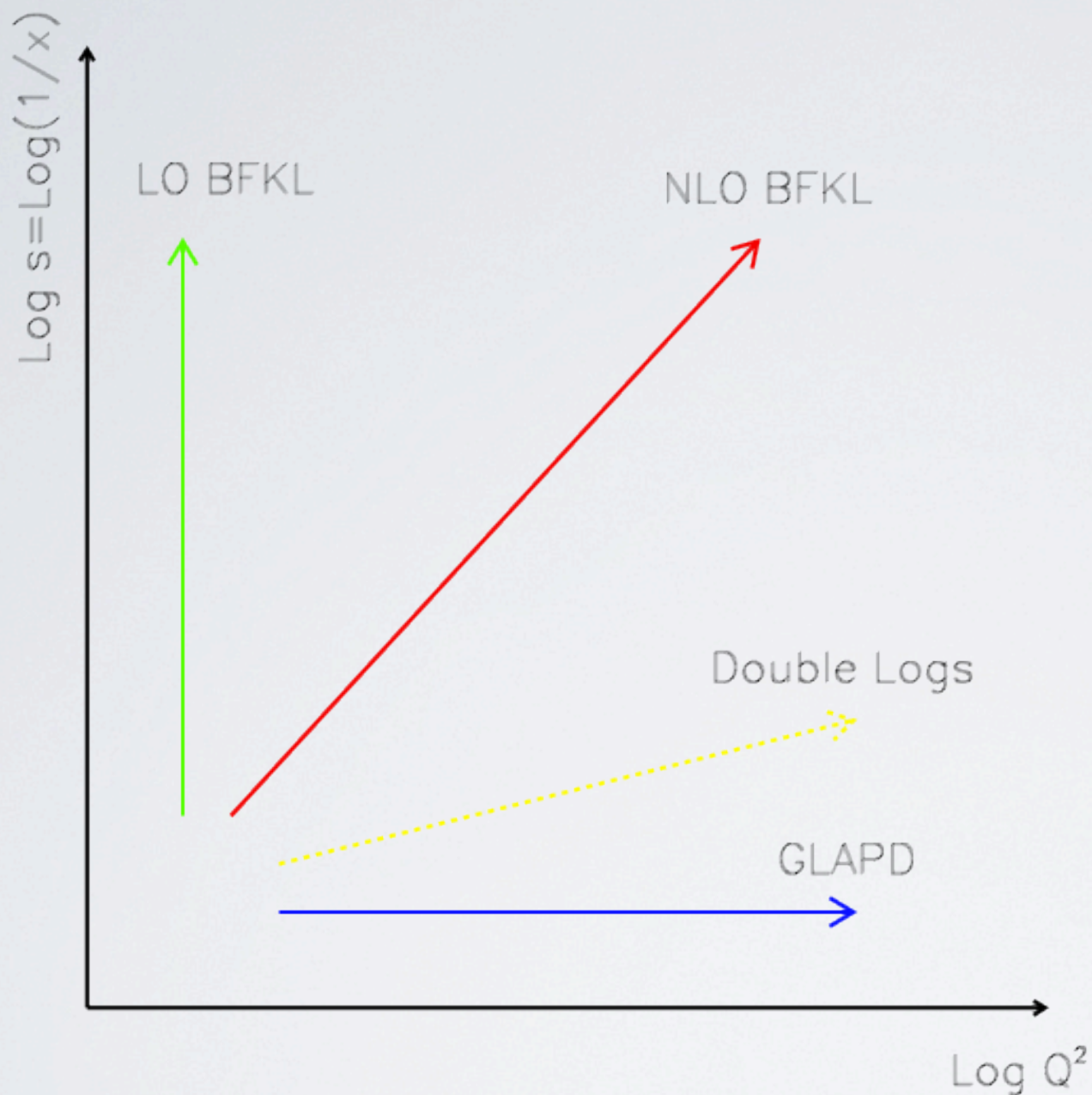
QED in Gribov-Regge limit

- **V. Gribov, V. Gorshkov, L. Lipatov & G. Frolov (67-70)
H. Cheng & T. Wu (66-70)**

QCD in Gribov-Regge limit

- **BFKL: V. Fadin, E. Kuraev & L. Lipatov (75-78)
I. Balitsky & L. Lipatov (78)**

Asymptotics of pQCD: x-section



Bjorken limit (GLAPD):

$$s \sim Q^2 \gg m^2$$

$$Q^2/s = x \sim 1$$

Large-angle (large-x) scattering

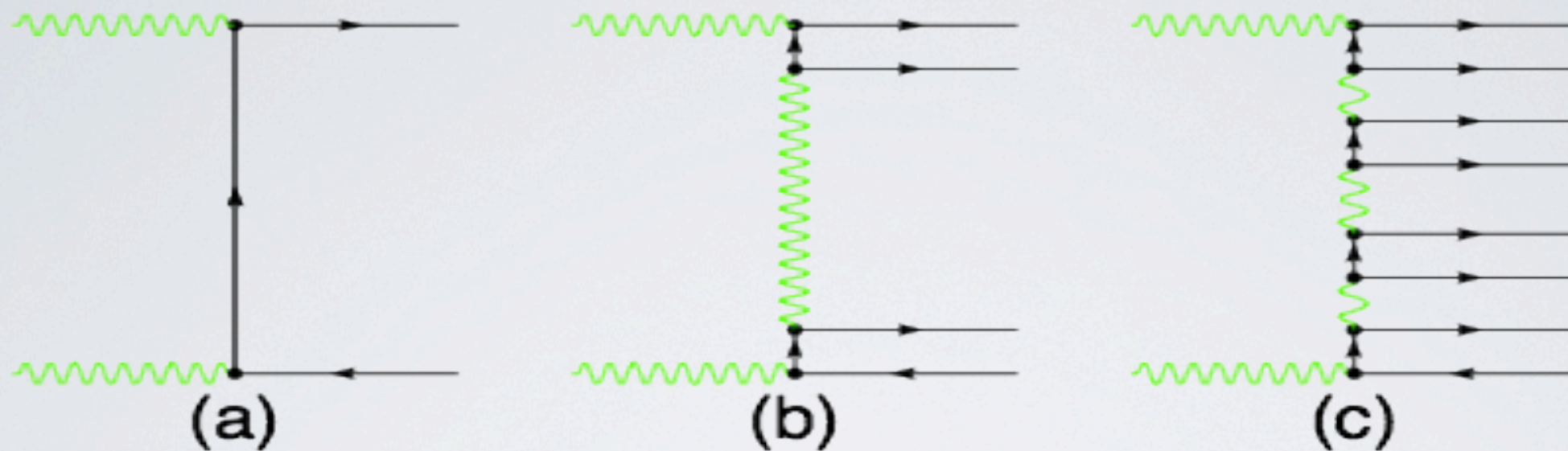
Gribov-Regge limit (BFKL):

$$s \gg Q^2 \gg m^2$$

$$Q^2/s = x \rightarrow 0$$

Small-angle (small-x) scattering

Asymptotics of QED cross sections



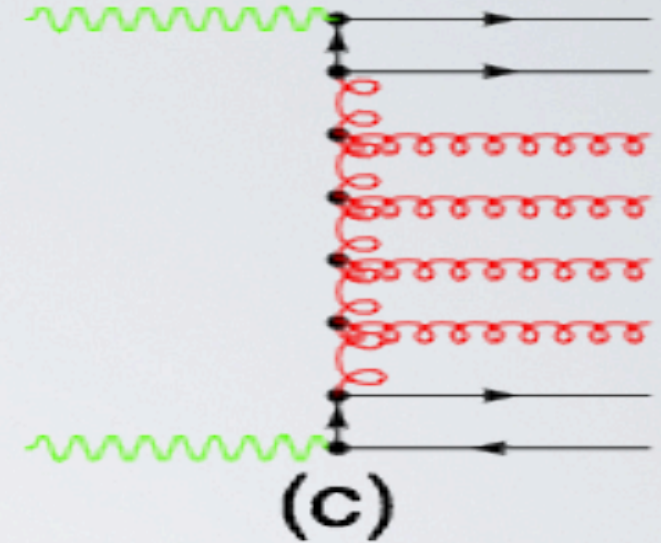
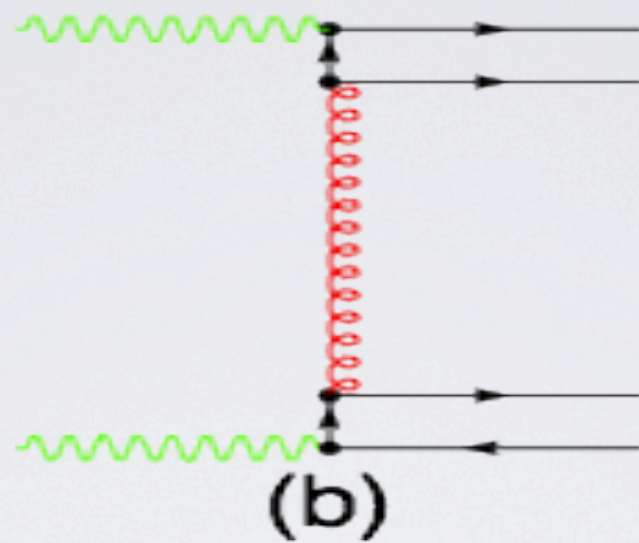
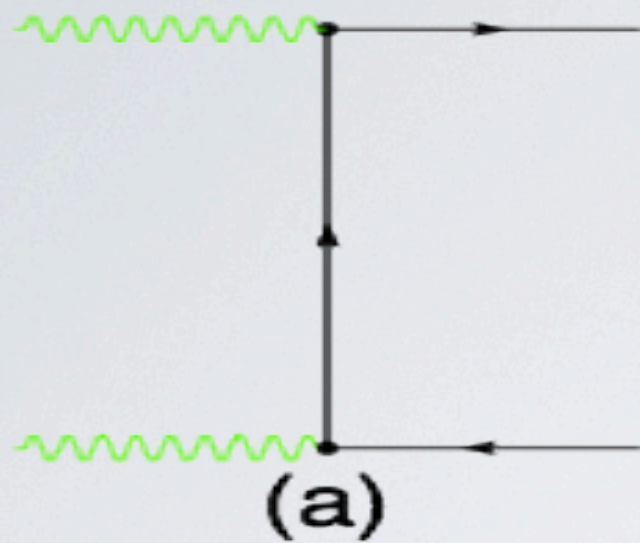
$$\sigma \sim (a_{\text{QED}})^2 \log(s)/s \quad \sigma \sim (a_{\text{QED}})^4 \text{const}(s)$$

All orders: V.N. Gribov, L.N. Lipatov, G.V. Frolov & V.G. Gorshkov (69-71)
H. Cheng & T.T. Wu (69-70)

Cross section at $s \rightarrow \infty$: $\sim (a_{\text{QED}})^4 (S/S_0)^{(a_P-1)}$

$$a_P = 1 + C (a_{\text{QED}})^2 \approx 1.002$$

Asymptotics of QCD cross sections: $\gamma\gamma$



$$\sigma \sim (a_{\text{QED}})^2 \log(s)/s \quad \sigma \sim (a_{\text{QED}})^2 (a_s)^2 \text{const}(s)$$

All orders: LL BFKL

Cross section at $s \rightarrow \infty$: $\sim (a_{\text{QED}})^2 (a_s)^2 (S/S_0)^{(a_P-1)}$

$a_P = 1 + C(a_s) \approx 1.5$ LL BFKL S. Brodsky & F. Hautmann (96)

**$a_P = 1 + C(a_s) \approx 1.2$ NLL BFKL S. Brodsky, V Fadin, VK,
L. Lipatov, G. Pivovarov (2001-02)**

Ultrahigh energies: all particles behave as hadrons!

Leading Log (LL) BFKL: problems

LL BFKL: designed for infinite collision energies

LL BFKL problems (at finite energies):

- **fixed (non-running) coupling α_s**
- **energy-momentum conservation**
- **transverse momentum conservation**

Cross section in LL BFKL:

$$\sigma = \sigma_0 (S/S_0)^{(\alpha_P - 1)}$$

$$\alpha_P = 1 + C \alpha_s \approx 1.5 - 1.6$$

Data: $\alpha_P \approx 1.2 - 1.3$

BFKL: next-to-leading logs (NLL)

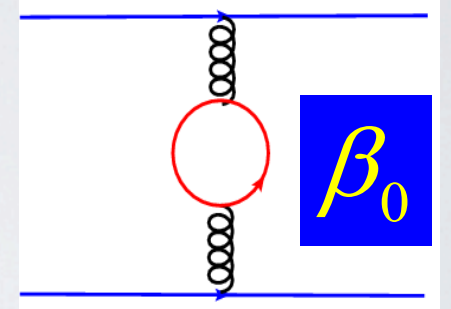
V.S. Fadin & L.N. Lipatov (89-98)

C. Camici & M. Ciafaloni (96-98)

next-to-leading log approximation (NLL) BFKL
MSbar-renormalization scheme: large corrections

S. Brodsky, P. Lepage & P. Mackenzie - BLM (1983)

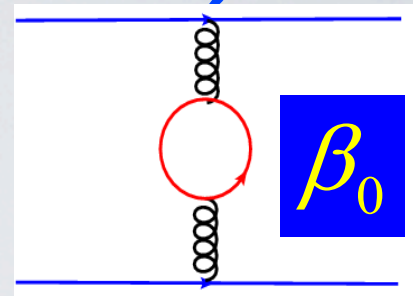
**Resummation of running coupling within
the standard BLM is not possible for NLL BFKL:
BLM approach valid only for Abelian case**



NLL BFKL in BFKLP: generalized BLM

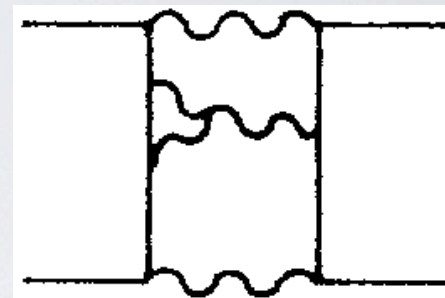
S. Brodsky, P. Lepage & P. Mackenzie (83) BLM approach for NLO

- QCD – asymptotically conformal
- non-conformal corrections (running coupling corrections) are resummed into optimal scale



Standard BLM approach for does not work (!) for:

- NLL BFKL in \overline{MS} scheme
- $\epsilon \rightarrow ggg$ decay in NLO in \overline{MS} scheme



BLM resummation depends on non-Abelian structure in LO

BLM generalized on non-Abelian case:

S.J. Brodsky, V.S. Fadin, VK, L.N. Lipatov, G.B. Pivovarov (98-99) BFKLP

BFKLP: NLL BFKL within generalized BLM - 1

Direct BLM application does not work in non-Abelian case(!):

- **NLL BFKL in \overline{MS} scheme**
- **Upsilon \rightarrow ggg decay in NLO in \overline{MS} scheme**

\overline{MS} -scheme: nonphysical RG scheme (!)

numerically close to V-scheme (heavy quark potential) – Abelian in LO

physical RG scheme: MOM scheme (gauge dependent)

- **NLL BFKL in non-Abelian in LO**
- **Upsilon \rightarrow ggg decay in non-Abelian in LO**

one can use MOM-scheme based on ggg-vertex non-Abelian in LO

BLM generalized on non-Abelian case:

S.J. Brodsky, V.S. Fadin, VK, L.N. Lipatov, G.B. Pivovarov(98-99) BFKLP

BFKLP: NLL BFKL + resummation of running coupling as

BLM resummation depends on non-Abelian structure in LO

BFKLP: NLL BFKL within generalized BLM - 2

$$\omega_{\overline{MS}}(Q_1^2, \nu) = \int d^2 Q_2 K_{\overline{MS}}(\mathbf{Q}_1, \mathbf{Q}_2) \left(\frac{Q_2^2}{Q_1^2} \right)^{-\frac{1}{2} + i\nu} \quad \sigma \sim S^{\alpha_{IP} - 1} = S^{\omega^{\max}}$$

$$= N_C \chi_L(\nu) \frac{\alpha_{\overline{MS}}(Q_1^2)}{\pi} \left[1 + r_{\overline{MS}}(\nu) \frac{\alpha_{\overline{MS}}(Q_1^2)}{\pi} \right],$$

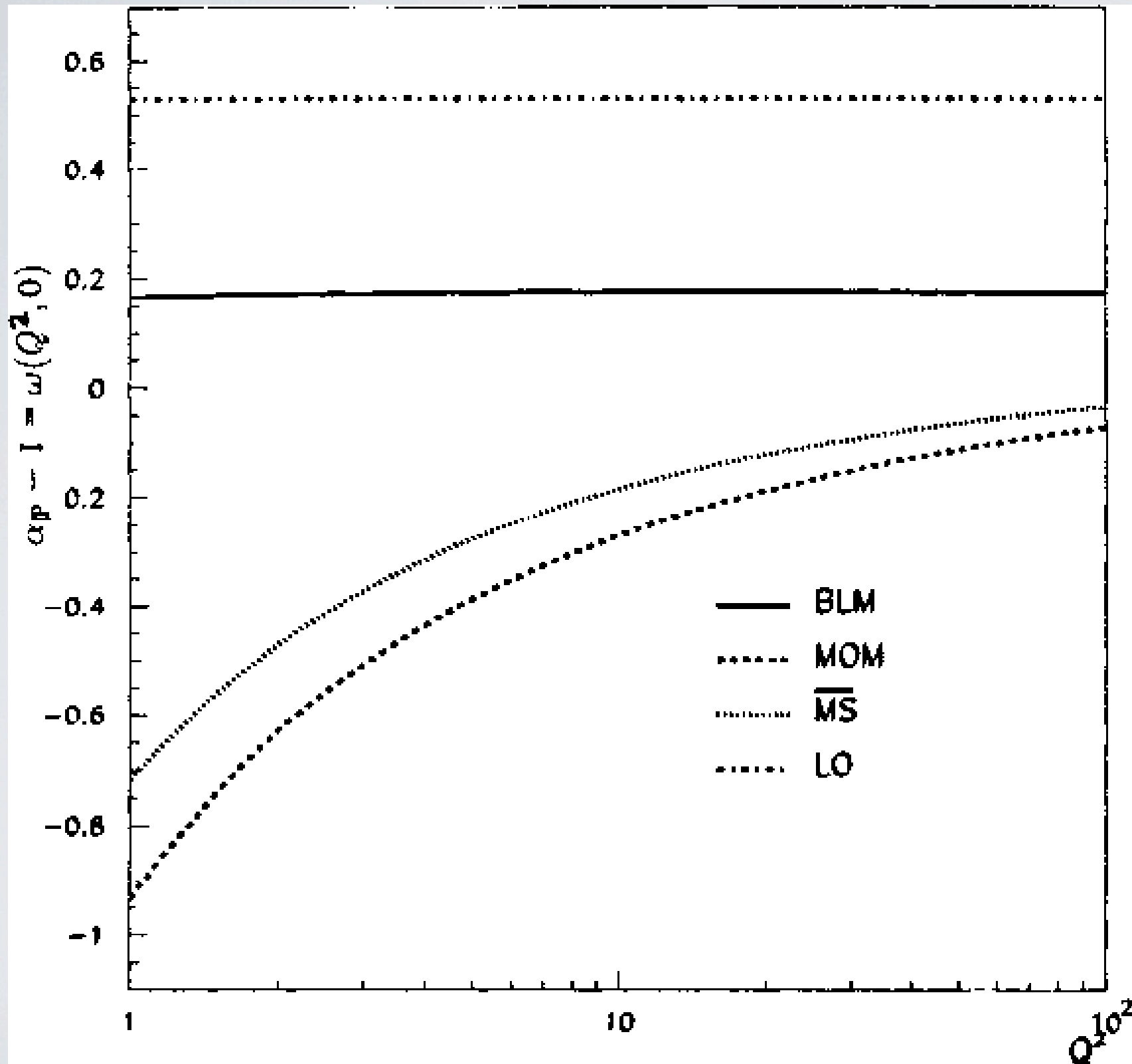
$$\chi_L(\nu) = 2\psi(1) - \psi(1/2 + i\nu) - \psi(1/2 - i\nu)$$

$$r_{\overline{MS}}(\nu) = r_{\overline{MS}}^{\beta}(\nu) + r_{\overline{MS}}^{\text{conf}}(\nu)$$

$$r_{\overline{MS}}^{\beta}(\nu) = -\frac{\beta_0}{4} \left[\frac{1}{2} \chi_L(\nu) - \frac{5}{3} \right]$$

$$r_{\overline{MS}}^{\text{conf}}(\nu) = -\frac{N_C}{4\chi_L(\nu)} \left[\frac{\pi^2 \sinh(\pi\nu)}{2\nu \cosh^2(\pi\nu)} \left(3 + \left(1 + \frac{N_F}{N_C^3} \right) \frac{11 + 12\nu^2}{16(1 + \nu^2)} \right) - \chi_L''(\nu) + \frac{\pi^2 - 4}{3} \chi_L(\nu) - \frac{\pi^3}{\cosh(\pi\nu)} - 6\zeta(3) + 4\varphi(\nu) \right]$$

BFKLP: NLL BFKL within generalized BLM - 3



$$\sigma \sim S^{\alpha_{IP} - 1} = S^{\omega^{\max}}$$

BFKLP: NLL BFKL within generalized BLM - 4

V.S. Fadin & L.N. Lipatov (89-98)

C. Camici & M. Ciafaloni (96-98)

**next-to-leading log approximation (NLL) BFKL
MSbar-renormalization scheme: large corrections**

S.J. Brodsky, V.S. Fadin, VK, L.N. Lipatov, G.B. Pivovarov (98-99) **BFKLP**

D. Colferai, M. Ciafaloni & G. Salam (99) ...

**BFKLP: NLL BFKL + resummation of running coupling α_s
in physical renormalization scheme**

BFKLP: Conformal BFKL kernel in NLL \rightarrow SUSY N=4

Pomeron intercept: $a_P = 1.2 - 1.3$

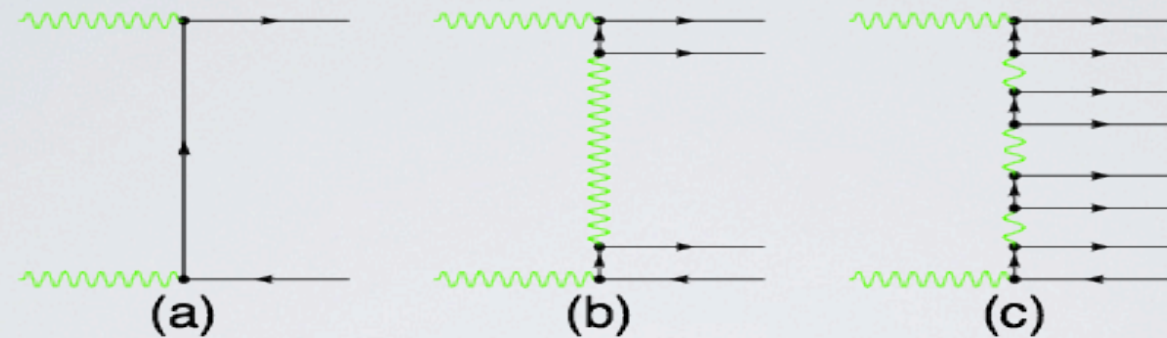
Cross section: $\sigma_0 (S/S_0)^{(a_P-1)}$ $a_P = 1 + C \alpha_s$

L.N. Lipatov, A.V. Kotikov et al. (2000-06)

SUSY N=4 BFKL-Pomeron

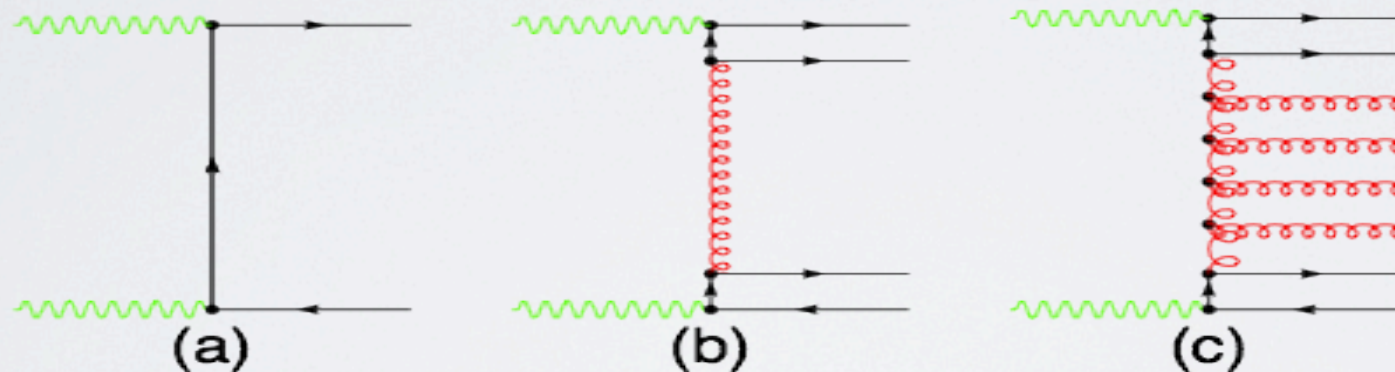
Anomalous dimensions: test of AdS/CFT

Asymptotics of QED cross sections



V.N. Gribov, L.N. Lipatov, G.V. Frolov & V.G. Gorshkov (69-71)
Cheng & T.T. Wu (69-71)

Asymptotics of QCD cross sections



LL BFKL

J. Bartels et al (96), S.J. Brodsky & Hautmann (97)

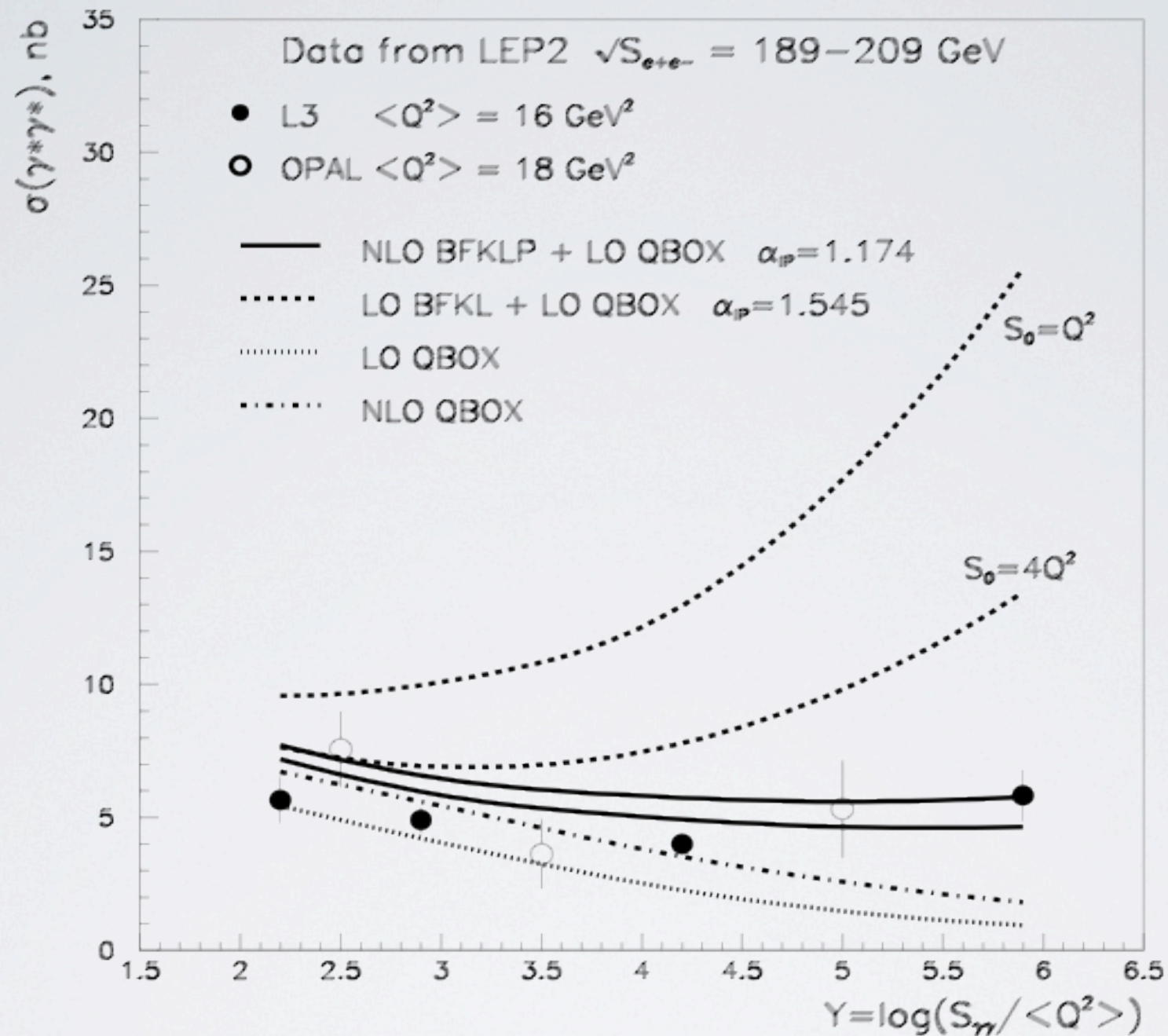
NLL BFKL (with LO impact factors)

S.J. Brodsky, V.S. Fadin, VK, L.N. Lipatov & G.B. Pivovarov (2001-02)

NLO impact factors and full NLL BFKL (in progress):

I. Balitsky, J.Chirilli, J. Bartels et al., A. Papa, D. Ivanov et al.

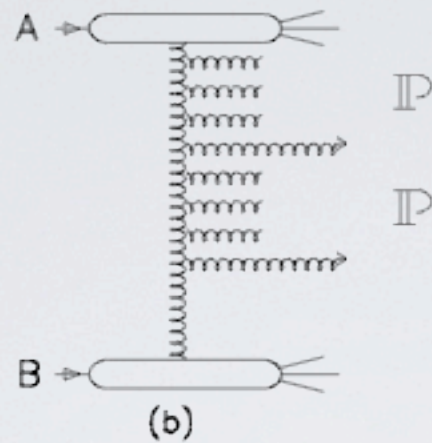
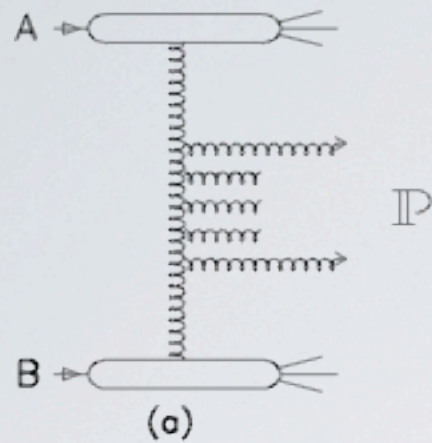
Highly virtual photon scattering at LEP-2



S.J Brodsky, VK, L.N. Lipatov, V.S. Fadin & G.B. Pivovarov (2002)
BFKLP: NLL BFKL + generalized BLM (LO impact factors)

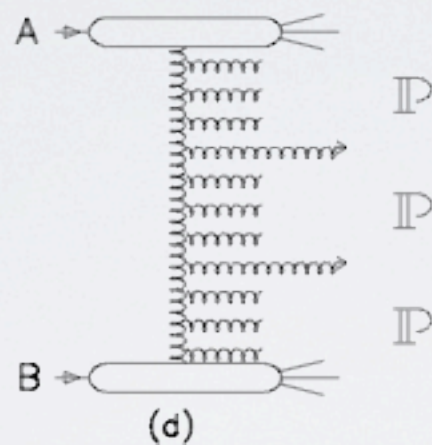
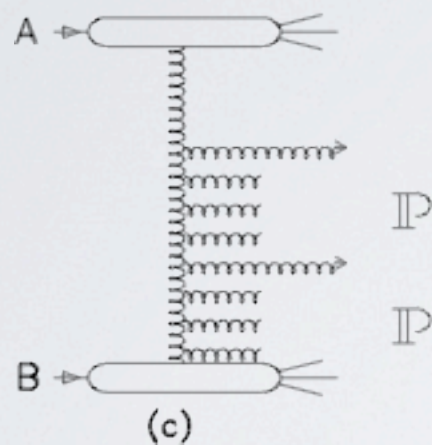
LL BFKL: ruled out

BFKL: dijet processes



Jet production

**GLAPD: ordering on κT
y – no ordering**



**BFKL: ordering on y
 κT – no ordering**

A. Mueller & H. Navelet, Nucl. Phys. (87)

Most forward/backward (Mueller-Navelet) dijets: x-section $\sim \exp(|\Delta|y)$

V.T. Kim & G.B. Pivovarov, Phys. Rev. (96)

Inclusive dijets

J.C. Collins, R.K. Ellis (91), S. Catani et al (91)

E.M.Levin, M.G.Ryskin, Yu.M.Shabelsky, A.G.Shuvaev (91)

kT-factorization

Dijet K-factor: 2-parton scattering

K-factor = x-section / Born x-section

GLAPD: x-section $\rightarrow C_1 \alpha_s^2 + C_2 \alpha_s^3 + \dots$
Born x-section $\rightarrow C_1 \alpha_s^2$

K-factor = $(1 + C_2 / C_1 \alpha_s + C_3 / C_1 \alpha_s^2 + \dots)$

Mueller-Navelet (87):

BFKL \rightarrow enhanced $(\alpha_s \Delta y)$ -terms
x-section $\rightarrow B_1 \alpha_s^2 \Delta y + B_2 \alpha_s^3 \Delta y^2 + \dots$
Born x-section $\rightarrow B_1 \alpha_s^2 \Delta y$

K-factor_MN $\rightarrow \exp(\alpha_s \Delta y)$

$\Delta y = |y_1 - y_2| \sim \log(1/x)$

Dijet K-factor: not measurable

K-factor = x-section / Born x-section

Born x-section: no real and no virtual corrections

only a theoretical quantity - > not measurable (!)

**Experiment: one cannot forbid virtual corrections
by kinematical conditions**

**Exclusive dijet x-section: always contains virtual
corrections**

VK & G. Pivovarov:

**Using dijets with extra jet veto
instead of Born dijets**

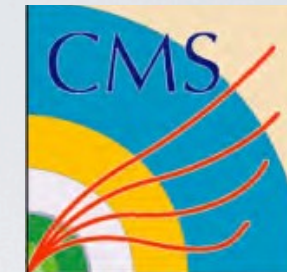
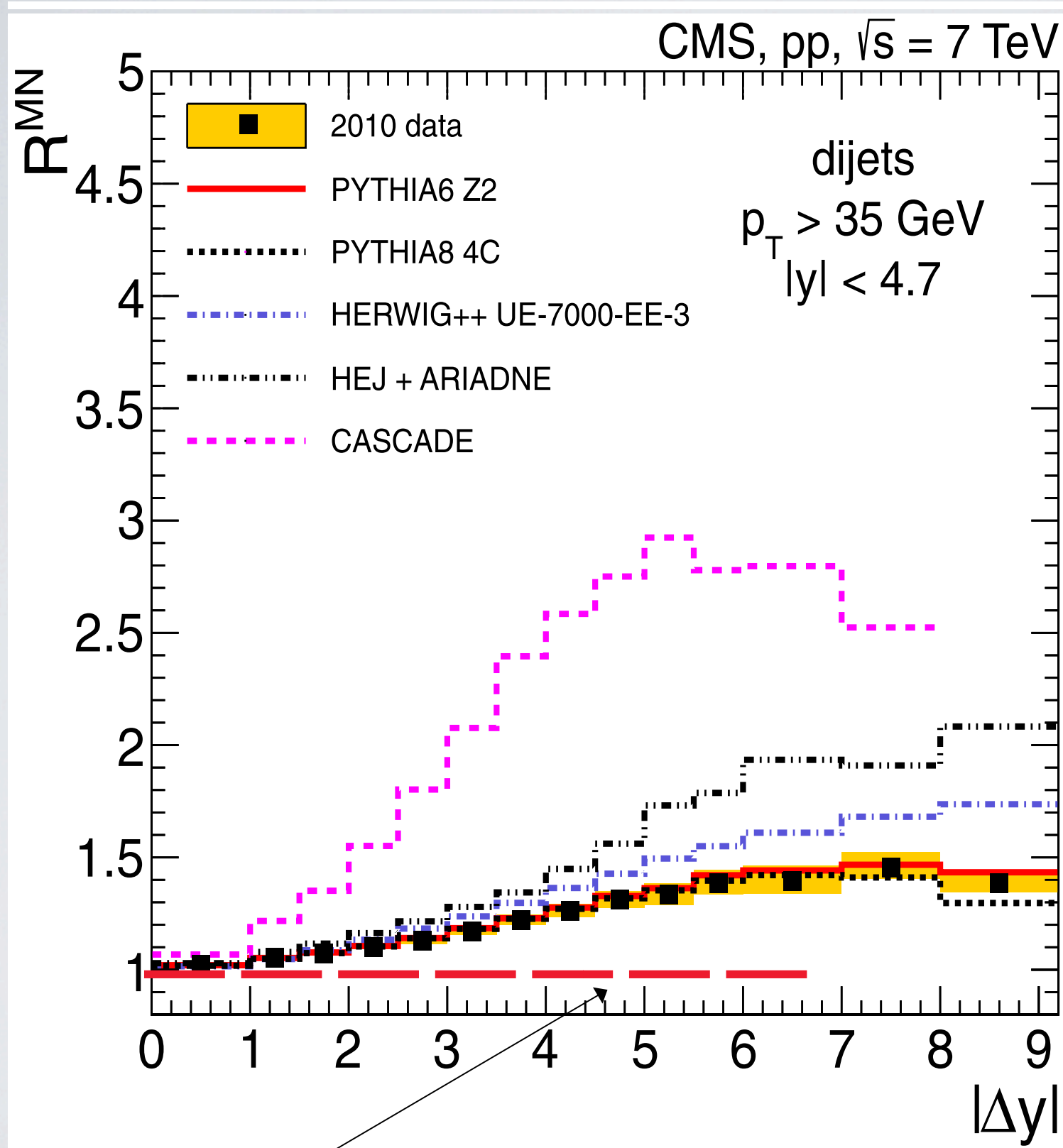
Forward dijets at Tevatron and LHC

Tevatron : D0 -> $|\Delta y| < 6$ $p_{Tmin} = 20$ GeV
- azimuthal decorr. (1997)
- 1800/630 GeV x-section ratio (2001)

LHC: ATLAS -> $|\Delta y| < 6$ 70 GeV $< p_T < 90$ GeV
- (inverse) “K-factor” (2011)

LHC: CMS -> $|\Delta y| < 9.4$ $p_{Tmin} = 35$ GeV
- “K-factor” (2012)
- azimuthal angle decorr. EJP C (2016)

CMS: dijet “K-factor”



EPJ C 72 (2012) 2216
7 TeV, $p_{T_min} = 35$ GeV
 $\Delta y = | | < 9.4$

GLAPD (no AO and CC)

Forward dijets at LHC:

Color coherence and AO effects

GLAPD: strong k_T -ordering & no rapidity ordering

BFKL: strong rapidity ordering & no k_T -ordering

Color coherence effects \Rightarrow rapidity ordering

Polar angle ordering (AO):

jet cone veto for larger cone angles \Rightarrow rapidity ordering

Pythia 6 and 8: GLAPD + AO (AO cannot be fully switched off!)

Herwig++: GLAPD + color coherence (CC cannot be switched off)

**No pure GLAPD MC generators (!) available
at present: Pythia and Herwig generators contain $|\Delta y|$ -effects**

**small CC and AO $|\Delta y|$ -effects in GLAPD-regime
can be large in BFKL-regime at large $|\Delta y|$**

Forward dijets at LHC

GLAPD generators Pythia 6 and 8 (with AO) are consistent with CMS dijet “K-factor” data rather well:

- 1) no sizeable BFKL effects at present energies?**
- 2) BFKL effects partially cancels out in dijet ratio?**

**in the latter case: “K-factor” with extra jet veto
can be more sensitive BFKL effects**

2-jet “exclusive” events: impose an extra jet veto $p_{T\text{veto}} < p_{T\text{min}}$

Forward dijets: azimuthal angle decorrelations

Cosines

V. Del Duca & C. Schmidt (94)

J. Stirling (94)

V. K. & G. Pivovarov (96)

Conformal properties of BFKL:

Cosine ratios → **GLAPD cancellation**

→ **more sensitive to BFKL (!)**

A. Sabio Vera et al (2011)

Forward dijets: azimuthal decorrelations

$$\frac{1}{\sigma} \frac{d\sigma}{d(\Delta\phi)}(\Delta y, p_{T\min}) = \frac{1}{2\pi} \left[1 + 2 \sum_{n=1}^{\infty} C_n(\Delta y, p_{T\min}) \cdot \cos(n(\pi - \Delta\phi)) \right]$$

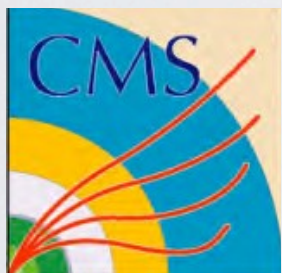
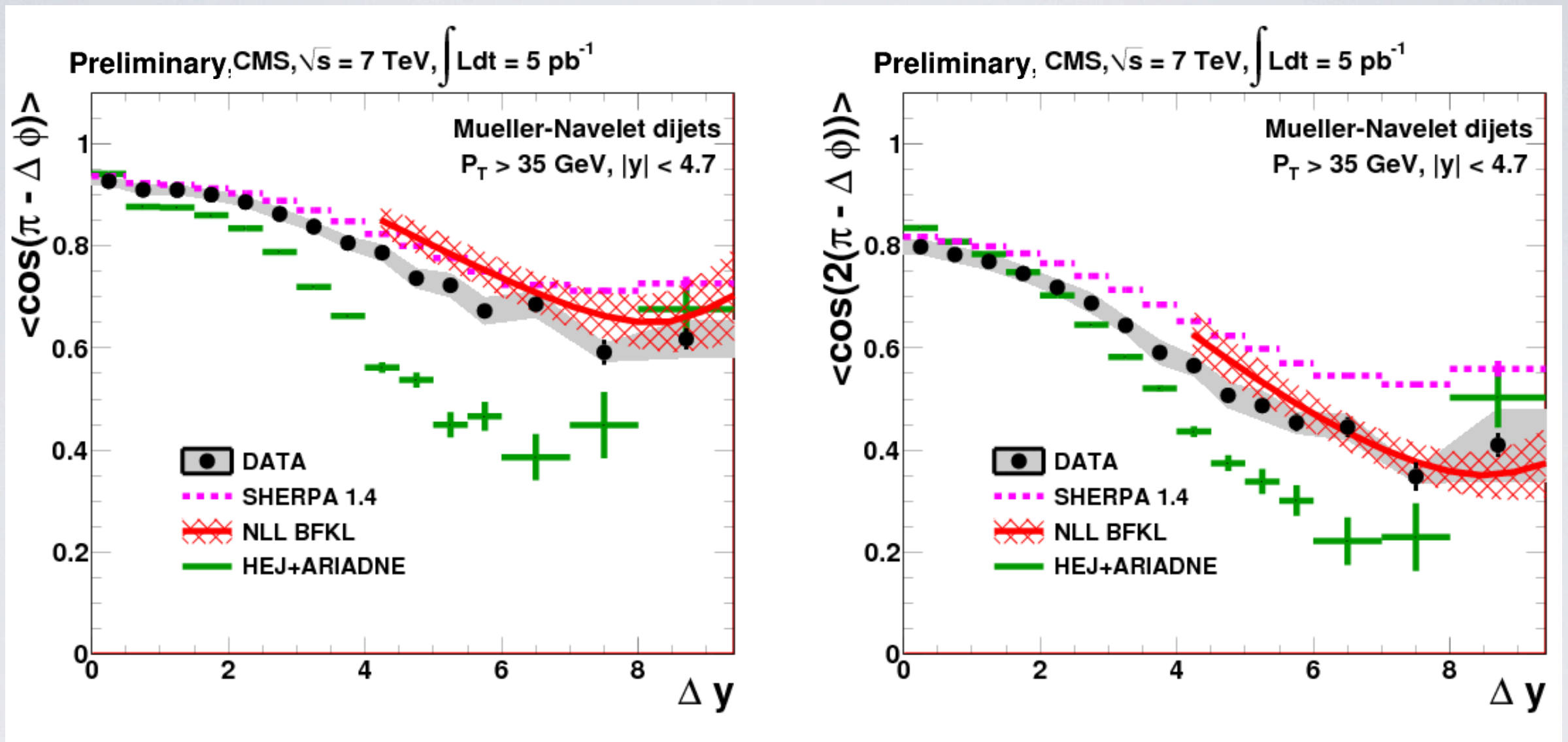
$$C_n(\Delta y, p_{T\min}) = \langle \cos(n(\pi - \Delta\phi)) \rangle, \text{ where } \Delta\phi = \phi_1 - \phi_2$$

V. del Duca & C. Schmidt (94-95)
Strling (94)

V. K. & G. Pivovarov (96-98)

A. Sabio Vera et al (2007-11)

Dijets: $\langle \cos \rangle$ vs NLL BFKL @BFKLP

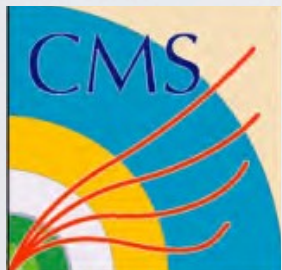
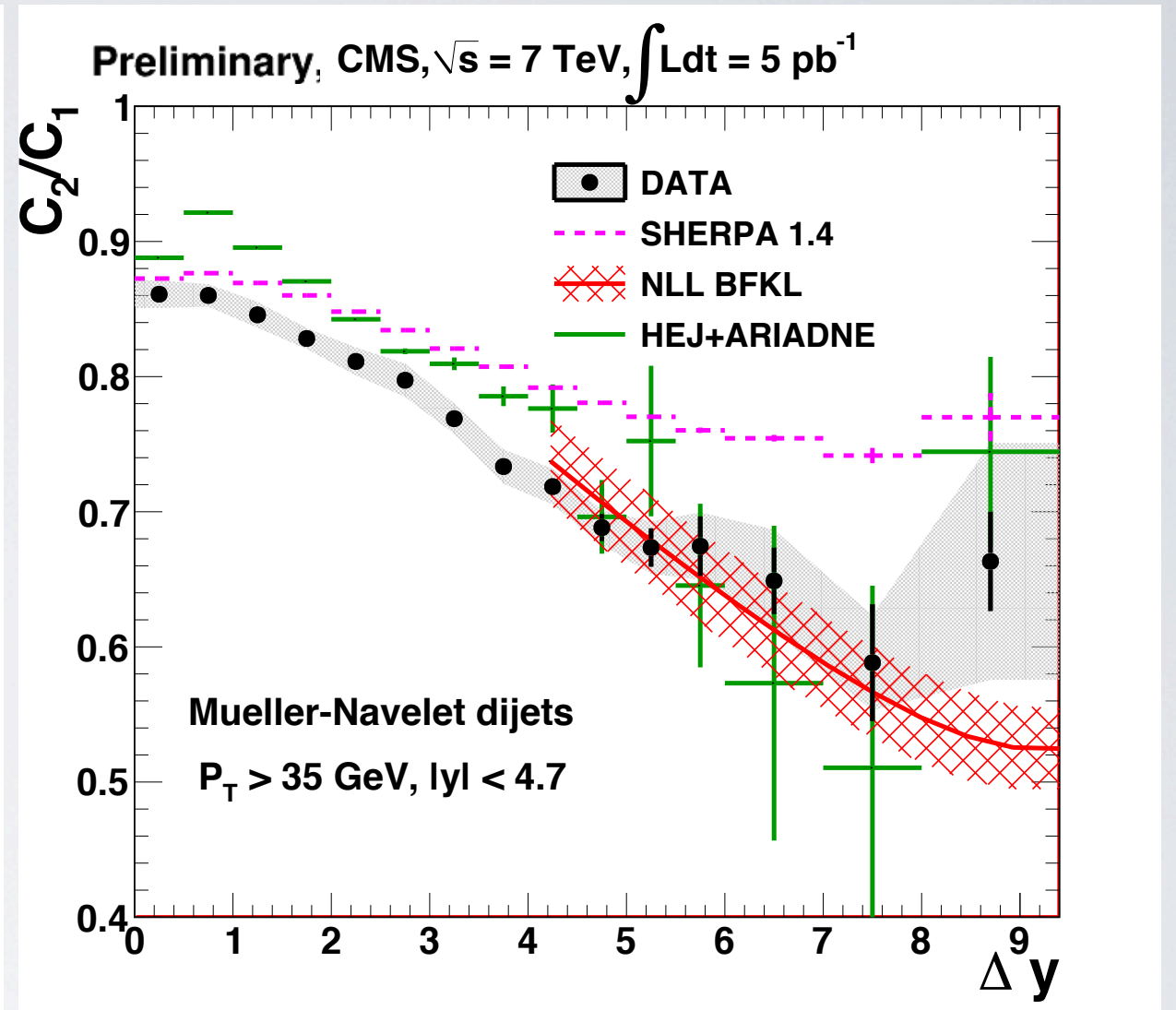
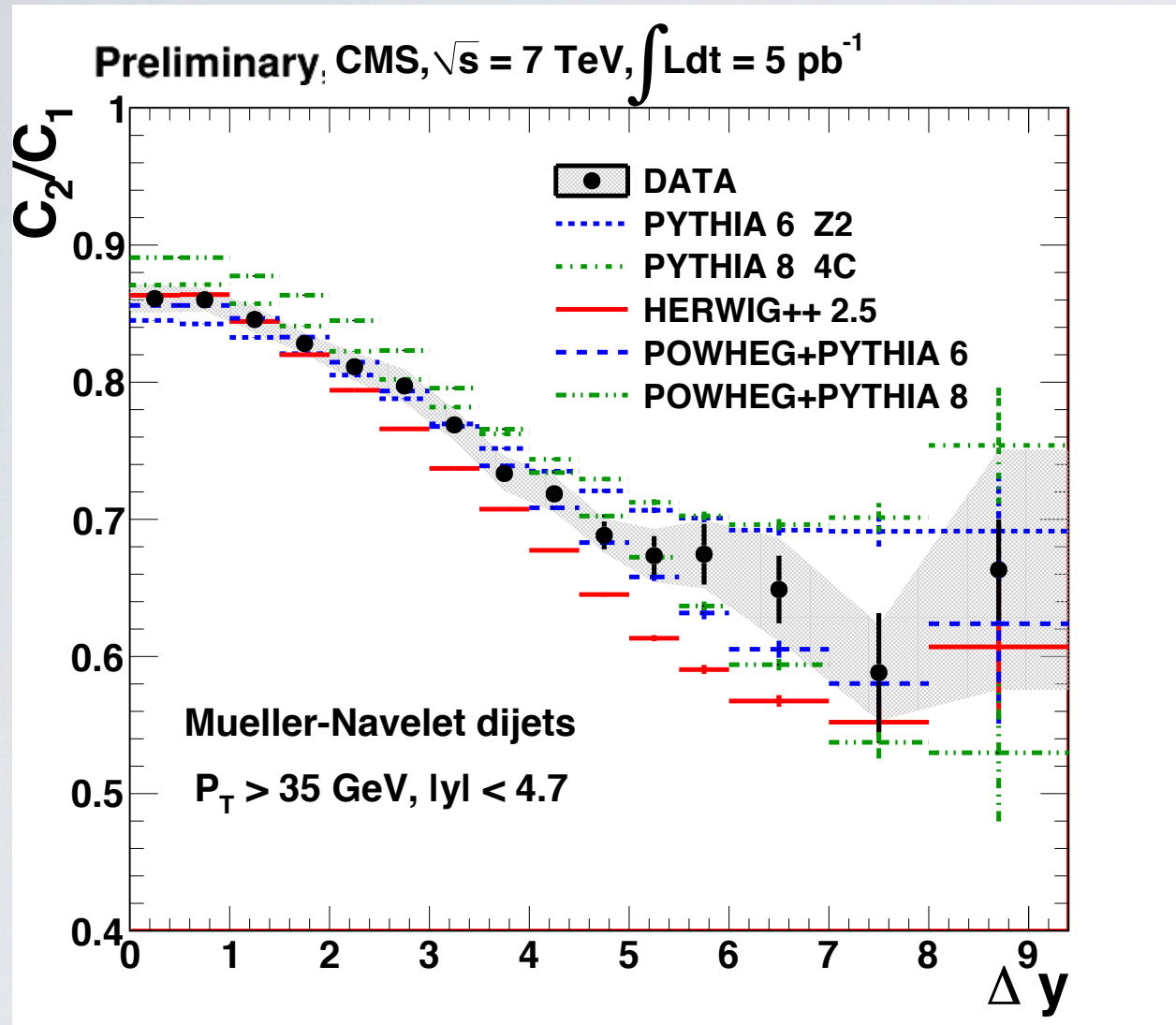


CMS: $\langle \cos \rangle$ and $\langle \cos^2 \rangle$
JHEP 08 (2016) 139
7 TeV, $p_{T_min} = 35$ GeV
 $\Delta y = | | < 9.4$

NLL BFKL @BFKLP

B. Ducloue, L. Szymanowski & S. Wallon (2014)

Dijets: $\langle \cos^2 \rangle / \langle \cos \rangle$ vs NLL BFKL @ BFKLP



CMS: $\langle \cos \rangle$ ratio -> indication on BFKL?
JHEP 08 (2016) 139
7 TeV, $p_{T_min} = 35 \text{ GeV}$
 $\Delta y < 9.4$

NLL BFKL @BFKLP

B. Ducloue, L. Szymanowski & S. Wallon (2014)

BFKL search:

other observables with jets

- **dijet “K-factor” with veto on extra jets**
VK, G. Pivovarov et al. (2008)

- **number of produced jets**
H. Jung et al. (2012)

- **dijets with rapidity gaps**
A. Mueller & W.-K. Tang (1992)
B. Peschanski, C. Royon et al. (2007-09)

Summary – 1

$\gamma^*\gamma^*$ - collisions at LEP2

**NLL BFKL improved by BFKLP (generalized BLM)
with LO impact factors (2001-02):**

Indication on BFKL evolution

Outlooks:

- **Full NLL BFKL $\gamma^*\gamma^*$ - collisions calculations**
- **$\gamma^*\gamma^*$ - collisions at LHC(?)**
- **Future linear colliders**

Summary - 2:

- **Forward dijet “K-factor” by CMS at 7 TeV :**
moderate rise with increasing $|\Delta y|$

- **however: pure GLAPD -> const?**

Indication on BFKL evolution at LHC

- **Azimuthal angle decorrelations (AAD) of CMS dijets:**

- **agreement with NLL BFKL improved by BFKLP (generalized BLM)**

- **partial agreement with GLAPD generators (Pythia, Herwig)**

Indication on BFKL evolution at LHC

Other observables:

- **dijet “K-factor” with extra jet veto, number of extra jets, ...**
- **dijets with rapidity gaps, ... ?**

Upcoming LHC Run 2 data at 13 TeV ?!