QCD vacuum in nuclear matter?

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Motivation I

A simple holographic QCD model study has claimed that (a naive) typical scale of QCD changes in nuclei.

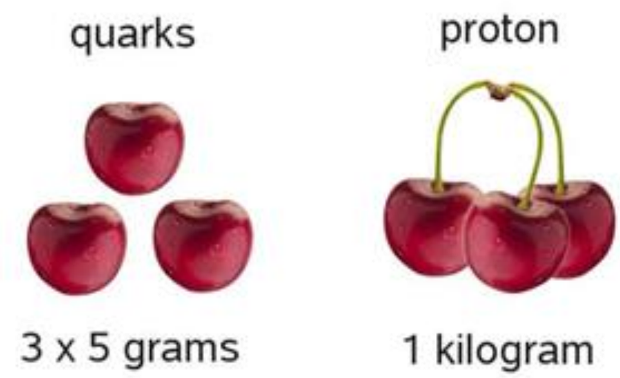
A	$1/z_m$
20	72.8 MeV
30	77.5 MeV
50	79.0 MeV
70	78.5 MeV
100	77.0 MeV

$$1/z_m \sim 320 \text{ MeV}$$

$$\rho(r) = \frac{\rho_0}{1 + e^{(r-R_{1/2})/a}}$$

K. K. Kim, YK, Y. Ko, JHEP 1010 (2010) 039

Motivation II



Origin of nucleon mass?

Can nuclear matter and nuclei do anything for this?

Nucleon mass (in the chiral limit) in the linear sigma model

$$\delta\mathcal{L} = -g_\pi \left[\left(i\bar{\psi}\gamma_5\vec{\tau}\psi \right)\vec{\pi} + \left(\bar{\psi}\psi \right)\sigma \right]$$

$$\begin{aligned} <\sigma> &= \sigma_0 = f_\pi \\ <\pi> &= 0 \\ M_N &= g_\pi\sigma_0 = g_\pi f_\pi \end{aligned}$$

Motivation III

PTEP

Prog. Theor. Exp. Phys. **2018**, 013B02 (14 pages)
DOI: 10.1093/ptep/ptx187

Stable spherically symmetric monopole field background in a pure QCD

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We consider a stationary spherically symmetric monopole-like solution with a finite energy density in a pure quantum chromodynamics (QCD). The solution can be treated as a static Wu–Yang monopole dressed in a time-dependent field corresponding to off-diagonal gluons. We have proved that such a stationary monopole field represents a background vacuum field of the QCD effective action which is stable against quantum gluon fluctuations. This resolves a long-standing problem of the existence of a stable vacuum field in QCD and opens a new avenue towards a microscopic theory of the vacuum.

Nontrivial QCD vacuum does nothing to with nuclear matter and nuclei?
Confinement washes it out?

All encrypted in LECs except chiral symmetry?

An example: Parity doublet model in dense matter

Introduce two nucleon fields that transform in a mirror way under chiral transformations:

$$\mathrm{SU}_L(2) \times \mathrm{SU}(2)_R$$

$$\begin{aligned}
\psi_{1R} &\to R\psi_{1R}, \quad \psi_{1L} \to L\psi_{1L}, \\
\psi_{2R} &\to L\psi_{2R}, \quad \psi_{2L} \to R\psi_{2L}.
\end{aligned}$$

$$\begin{aligned}
& m_0(\bar{\psi}_2\gamma_5\psi_1 - \bar{\psi}_1\gamma_5\psi_2) \\
& \quad = m_0(\bar{\psi}_{2L}\psi_{1R} - \bar{\psi}_{2R}\psi_{1L} - \bar{\psi}_{1L}\psi_{2R} + \bar{\psi}_{1R}\psi_{2L})
\end{aligned}$$

Or,

$$\mathcal{L} = i\bar{\psi}\gamma_\mu\partial^\mu\psi = i\bar{\psi}_L\gamma_\mu\partial^\mu\psi_L + i\bar{\psi}_R\gamma_\mu\partial^\mu\psi_R,$$

$$\psi_R = \frac{1}{2}\left(1+\gamma_5\right)\psi, \quad \psi_L = \frac{1}{2}\left(1-\gamma_5\right)\psi.$$

$$\psi_R \to \exp\left(\imath\frac{\theta_R^a\tau^a}{2}\right)\psi_R; \quad \psi_L \to \exp\left(\imath\frac{\theta_L^a\tau^a}{2}\right)\psi_L$$

$$\Psi = \begin{pmatrix} \Psi_+ \\ \Psi_- \end{pmatrix},$$

where the Dirac bispinors Ψ_+ and Ψ_- have positive and negative parity, respectively.

$$\Psi_R = \frac{1}{\sqrt{2}}\left(\Psi_+ + \Psi_-\right); \quad \Psi_L = \frac{1}{\sqrt{2}}\left(\Psi_+ - \Psi_-\right),$$

$$\begin{aligned}\mathcal{L} &= i\bar{\Psi}\gamma^\mu\partial_\mu\Psi - m\bar{\Psi}\Psi \\ &= i\bar{\Psi}_+\gamma^\mu\partial_\mu\Psi_+ + i\bar{\Psi}_-\gamma^\mu\partial_\mu\Psi_- - m\bar{\Psi}_+\Psi_+ - m\bar{\Psi}_-\Psi_-.\end{aligned}$$

$$\mathcal{L} = \bar{\psi}_1 i\partial\!\!\!/ \psi_1 + \bar{\psi}_2 i\partial\!\!\!/ \psi_2 + m_0(\bar{\psi}_2\gamma_5\psi_1 - \bar{\psi}_1\gamma_5\psi_2)$$
$$+ a\bar{\psi}_1(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})\psi_1 + b\bar{\psi}_2(\sigma - i\gamma_5\vec{\tau}\cdot\vec{\pi})\psi_2$$

$$m_{N\pm} = \frac{1}{2}\left(\sqrt{(a+b)^2\sigma^2 + 4m_0^2} \mp (a-b)\sigma\right)$$

The state N+ is the nucleon N(938). while N- is its parity partner conventionally identified with N(1500).

the decay width $\Gamma_{N\pi}$ for $N^*(1535) \to N + \pi$, $m_0 = 270$ MeV

“Linear sigma model with parity doubling,” C. E. DeTar and T. Kunihiro, Phys. Rev. D **39**, 2805 (1989)

Cold, dense nuclear matter in a SU(2) parity doublet model

$$
\begin{aligned}
\mathcal{L} \;=\; & \bar{\psi}_1 i\partial\!\!\!/ \psi_1 + \bar{\psi}_2 i\partial\!\!\!/ \psi_2 + m_0(\bar{\psi}_2\gamma_5\psi_1 - \bar{\psi}_1\gamma_5\psi_2) \\
& + a\bar{\psi}_1(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})\psi_1 + b\bar{\psi}_2(\sigma - i\gamma_5\vec{\tau}\cdot\vec{\pi})\psi_2 \\
& - g_\omega\bar{\psi}_1\gamma_\mu\omega^\mu\psi_1 - g_\omega\bar{\psi}_2\gamma_\mu\omega^\mu\psi_2 + \mathcal{L}_M,
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_M \;=\; & \frac{1}{2}\partial_\mu\sigma^\mu\partial^\mu\sigma_\mu + \frac{1}{2}\partial_\mu\vec{\pi}^\mu\partial^\mu\vec{\pi}_\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\
& + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + g_4^4(\omega_\mu\omega^\mu)^2 \\
& + \frac{1}{2}\bar{\mu}^2(\sigma^2 + \vec{\pi}^2) - \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2)^2 + \epsilon\sigma,
\end{aligned}
$$

D. Zschiesche, L. Tolos, Jurgen Schaffner-Bielich, Robert D. Pisarski, Phys.Rev. C75 (2007) 055202

PHYSICAL REVIEW C **92**, 025201 (2015)

Asymmetric nuclear matter in a parity doublet model with hidden local symmetry

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(Received 11 May 2015; published 3 August 2015)

We construct a model to describe dense hadronic matter at zero and finite temperatures, based on the parity doublet model of DeTar and Kunihiro [C. E. DeTar and T. Kunihiro, Phys. Rev. D **39**, 2805 (1989)], including the isosinglet scalar meson σ as well as ρ and ω mesons. We show that, by including a six-point interaction of the σ meson, the model reasonably reproduces the properties of normal nuclear matter with the chiral invariant nucleon mass m_0 in the range from 500 to 900 MeV. Furthermore, we study the phase diagram based on the model, which shows that the value of the chiral condensate drops at the liquid-gas phase transition point and at the chiral phase transition point. We also study asymmetric nuclear matter and find that the first-order phase transition for the liquid-gas phase transition disappears in asymmetric matter and that the critical density for the chiral phase transition at nonzero density becomes smaller for larger asymmetry.

Y. Motohiro, M. Harada, YK, Erratum: Phys.Rev. C95 (2017) 059903

$$
\begin{aligned}
\mathcal{L} = {} & \bar{\psi}_1 i\partial\!\!\!/ \psi_1 + \bar{\psi}_2 i\partial\!\!\!/ \psi_2 + m_0(\bar{\psi}_2\gamma_5\psi_1 - \bar{\psi}_1\gamma_5\psi_2) \\
& + g_1\bar{\psi}_1(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})\psi_1 + g_2\bar{\psi}_2(\sigma - i\gamma_5\vec{\tau}\cdot\vec{\pi})\psi_2 \\
& - g_{\omega NN}\bar{\psi}_1\gamma_\mu\omega^\mu\psi_1 - g_{\omega NN}\bar{\psi}_2\gamma_\mu\omega^\mu\psi_2 \\
& - g_{\rho NN}\bar{\psi}_1\gamma_\mu\vec{\rho}^\mu\cdot\vec{\tau}\psi_1 - g_{\rho NN}\bar{\psi}_2\gamma_\mu\vec{\rho}^\mu\cdot\vec{\tau}\psi_2 \\
& - e\bar{\psi}_1\gamma^\mu A_\mu\frac{1-\tau_3}{2}\psi_1 - e\bar{\psi}_2\gamma^\mu A_\mu\frac{1-\tau_3}{2}\psi_2 + \mathcal{L}_M\,,
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_M = {} & \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi} \\
& - \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} - \frac{1}{4}\vec{R}_{\mu\nu}\cdot\vec{R}^{\mu\nu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\
& + \frac{\bar{\mu}^2}{2}(\sigma^2+\vec{\pi}^2) - \frac{\lambda}{4}(\sigma^2+\vec{\pi}^2)^2 + \frac{\lambda_6}{6}(\sigma^2+\vec{\pi}^2)^3 + \epsilon\sigma \\
& + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \frac{1}{2}m_\rho^2\vec{\rho}_\mu\cdot\vec{\rho}^\mu
\end{aligned}
$$

To fix the parameters in our model with fixed m_0

TABLE I. The inputs from free space (in MeV).

m_+	m_-	m_ω	m_ρ	f_π	m_π
939	1535	783	776	93	138

$$\frac{E}{A} - m_N = -16 \text{ MeV}\,, \quad n_0 = 0.16 \text{ fm}^{-3},$$
$$K = 240 \pm 40 \text{ MeV}\,, \quad E_{\text{sym}} = 31 \text{ MeV}\,.$$

Unfortunately, we don't find so far any nuclear matter properties that are sensitive to the value of the chiral invariant mass.

Delta matter in a parity doublet model (within MFA)

Yusuke Takeda, YK, Masayasu Harada, Phys. Rev. C97 (2018) 065202

* In symmetric matter, Delta enters into matter at (1-4) times the saturation density. The stable Δ-nucleon matter is realized around 1.5 times the saturation density, and the phase transition from nuclear matter to Δ-nucleon matter is of first order for small value of the chiral invariant mass of Delta.

* In asymmetric matter, the phase transition from the nuclear matter to the stable Δ-nucleon matter can be of the second order for most parameter region. The onset density is smaller than that in symmetric matter.

* In symmetric dense matter, larger chiral invariant nucleon mass tends to lower the transition density to the stable N-Δ phase.

* Partial restoration of chiral symmetry is enhanced by Delta matter.

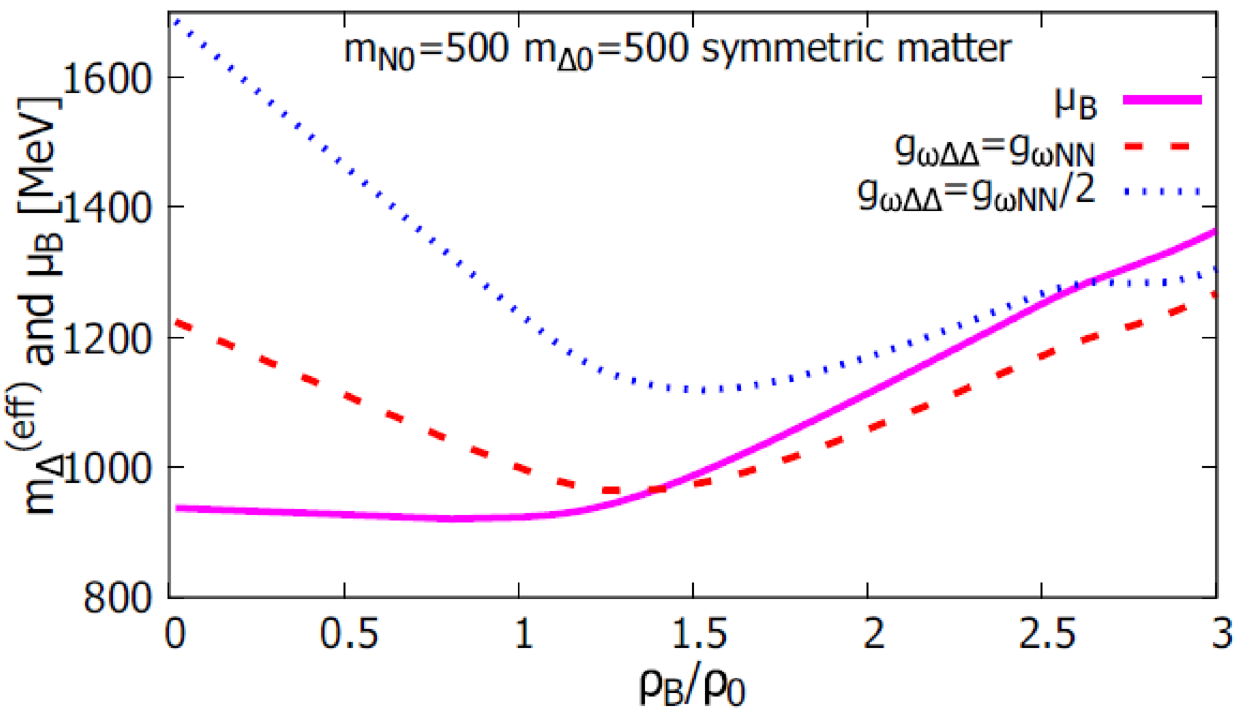


FIG. 3. Density dependence of the effective masses of Δ for $g_{\omega\Delta\Delta} = g_{\omega NN}$ (red dashed curve) and $g_{\omega\Delta\Delta} = g_{\omega NN}/2$ (blue dotted curve) with fixed values of $m_{N0} = m_{\Delta 0} = 500\,\mathrm{MeV}$ in symmetric nuclear matter. The pink solid curve shows the density dependence of the baryon chemical potential μ_B.

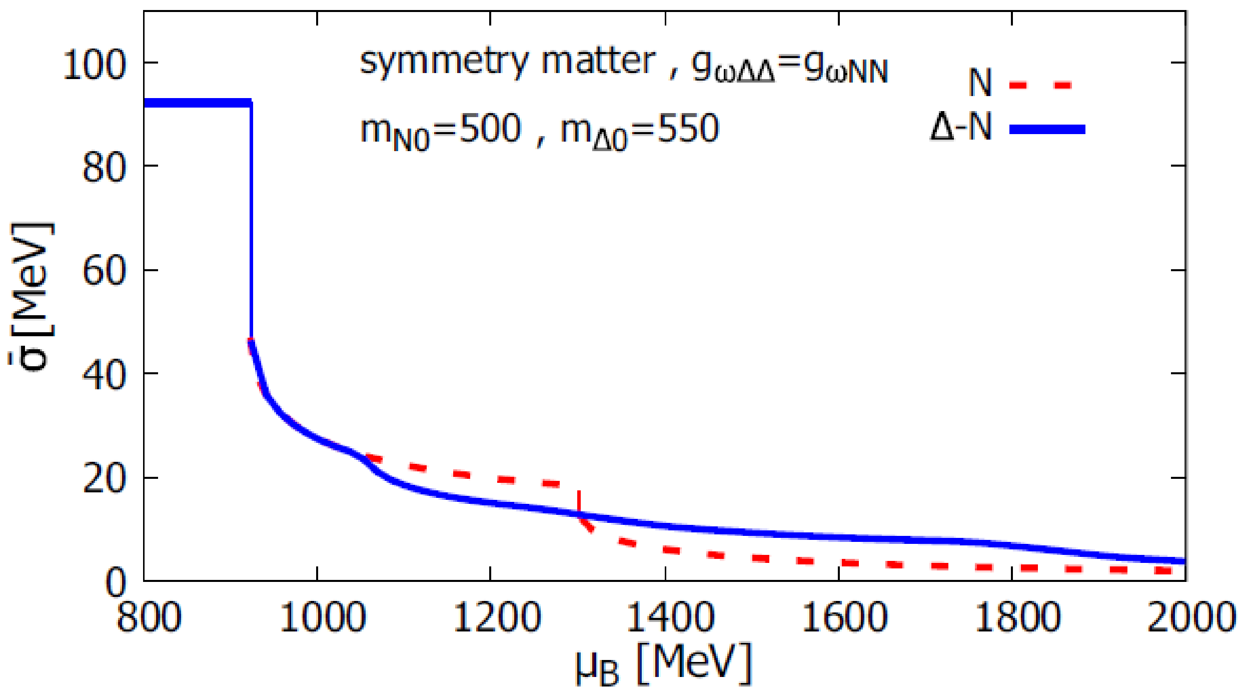


FIG. 13. Chemical potential dependence of the chiral condensate $\bar{\sigma}$ (blue solid curve). The red dashed curve shows the one with assuming no Δ in matter. Horizontal axis shows the baryon number chemical potential in unit of MeV, while vertical axis shows the value of the chiral condensate $\bar{\sigma}$ in unit of MeV. The parameters are chosen as $m_{N0} = 500\,\text{MeV}$, $m_{\Delta 0} = 550\,\text{MeV}$, and $g_{\omega\Delta\Delta} = g_{\omega NN}$.

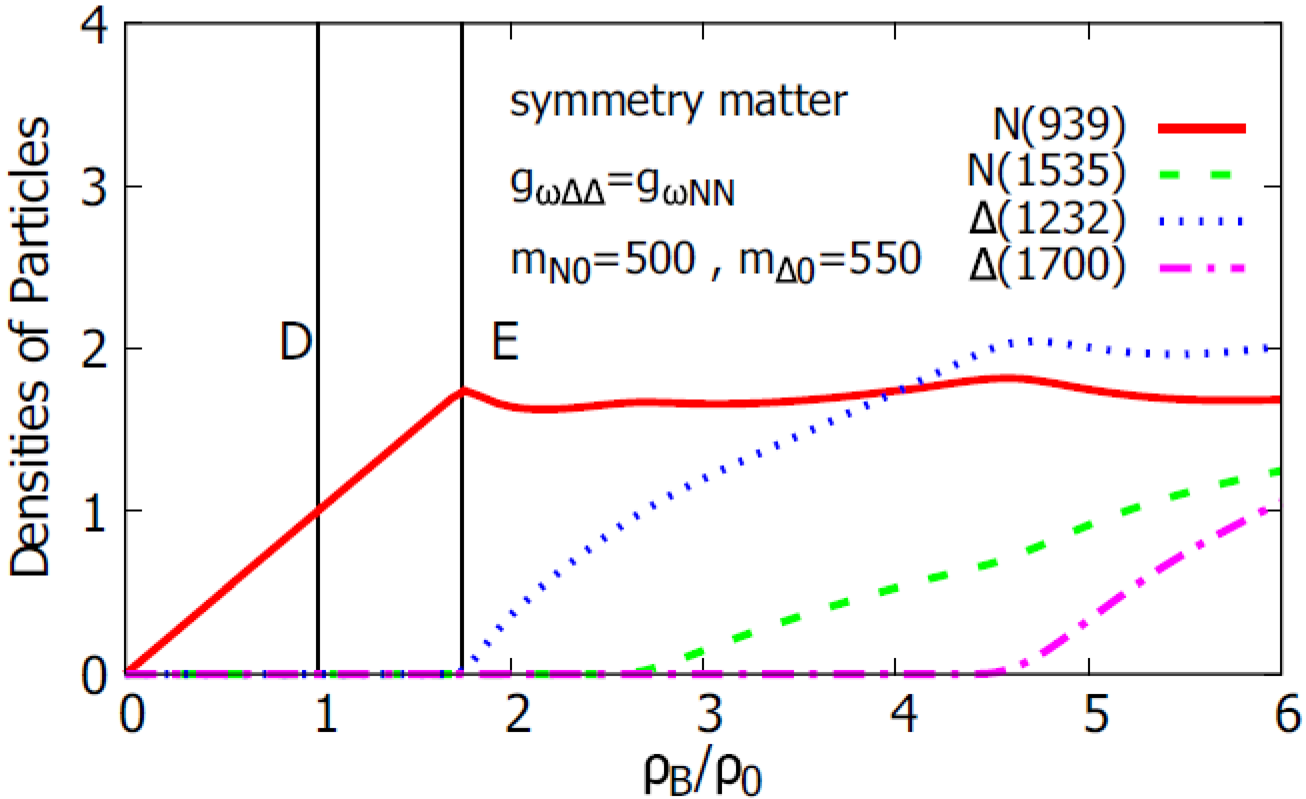


FIG. 7. Densities of $N(939)$, $\Delta(1232)$, and $N^*(1535)$. Horizontal axis shows the baryon number density scaled by normal nuclear matter density ρ_0. Vertical axis shows densities of $N(939)$ (red solid curve), $\Delta(1232)$ (blue dotted curve), $N^*(1535)$ (green dashed curve), and $\Delta(1700)$ (pink dot-dashed curve) scaled by ρ_0.

Parity doublet model in relativistic mean field theory

- ✓ Spherical code was provided by Jie Meng (Peking Univ.).
- ✓ Main difference is the behavior of sigma mean field.
- ✓ Revised the code to incorporate the difference.
- ✓ With no Delta baryons

Ik Jae Shin, Won-Gi Paeng, Masayasu Harada, YK, 1805.03402 in nucl-th

The equations of motion (EoM) for the stationary mean fields $\tilde{\sigma}$, ω_0, ρ_0^3 and A_0 read

$$
\begin{aligned}
\Big(-\vec{\nabla}^2 + m_\sigma^2\Big)\langle\tilde{\sigma}(\vec{x})\rangle &= -\bar{N}(\vec{x})N(\vec{x})\left.\frac{\partial\, m_N(\tilde{\sigma})}{\partial\tilde{\sigma}}\right|_{\tilde{\sigma}=\langle\tilde{\sigma}(\vec{x})\rangle} \\
&\quad + \left(-3f_\pi\lambda + 10f_\pi^3\lambda_6\right)\langle\tilde{\sigma}(\vec{x})\rangle^2 \\
&\quad + \left(-\lambda + 10f_\pi^2\lambda_6\right)\langle\tilde{\sigma}(\vec{x})\rangle^3 \\
&\quad + 5f_\pi\lambda_6\langle\tilde{\sigma}(\vec{x})\rangle^4 + \lambda_6\langle\tilde{\sigma}(\vec{x})\rangle^5 \\
\Big(-\vec{\nabla}^2 + m_\omega^2\Big)\langle\omega_0(\vec{x})\rangle &= g_{\omega NN}N^\dagger(\vec{x})N(\vec{x})\,, \\
\Big(-\vec{\nabla}^2 + m_\rho^2\Big)\langle\rho_0^3(\vec{x})\rangle &= g_{\rho NN}N^\dagger(\vec{x})\tau^3N(\vec{x})\,, \\
-\vec{\nabla}^2\langle A_0(\vec{x})\rangle &= eN^\dagger(\vec{x})\frac{1-\tau_3}{2}N(\vec{x})\,.
\end{aligned}
$$

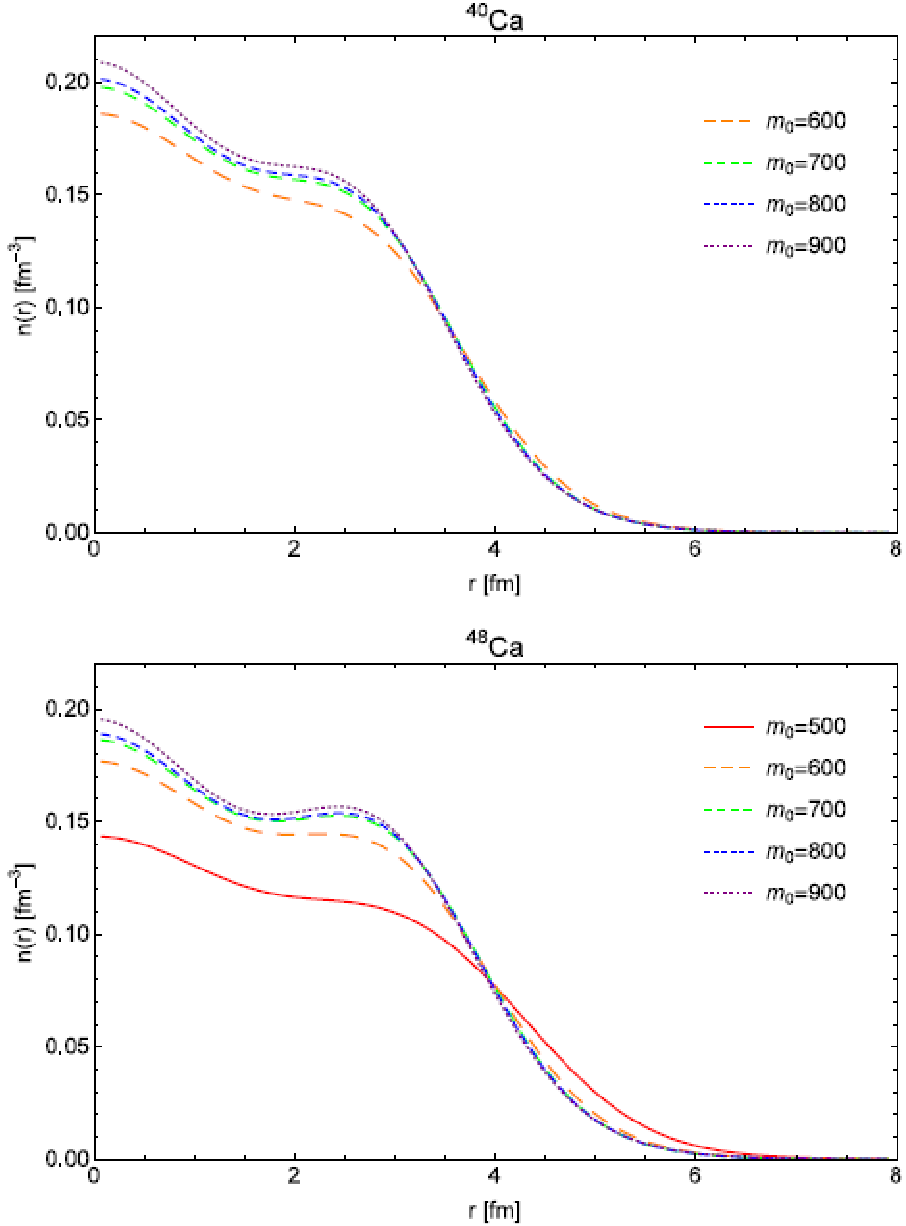


FIG. 1. (Color online) Nucleon density profile in ^{40}Ca and ^{48}Ca calculated with the parameter set 1.

	BE (MeV)			R_C (fm)	
	PDM	PC-PK1	Exp.	PDM	Exp.
^{16}O	8.04	7.96	7.98	2.76	2.70
^{24}O	7.06	7.12	7.04	2.82	–
^{32}Mg	7.83	7.91	7.80	3.14	–
^{38}Si	7.59	7.90	7.89	3.28	–
^{38}Ar	8.51	8.62	8.61	3.39	3.40
^{40}Ca	8.57	8.58	8.55	3.46	3.48
^{48}Ca	8.42	8.65	8.67	3.52	3.48
^{42}Ti	8.16	8.32	8.26	3.58	–
^{58}Ni	8.12	8.69	8.73	3.84	3.78
^{72}Kr	8.22	8.31	8.43	4.11	4.16
^{208}Pb	7.86	7.87	7.87	5.53	5.50

We observed that our results, especially the binding energies, are closest to the experiments when we take m_0 = 700 MeV.

!

TABLE VIII. The neutron(ν) and proton(π) spin-orbit splittings of ^{40}Ca and ^{48}Ca

State	^{40}Ca		^{48}Ca	
	PDM	Exp.	PDM	Exp.
$\nu 1d$	1.52	6.75	1.28	5.30
$\nu 1f$			1.75	8.01
$\nu 2p$	0.48	2.00	0.44	1.67
$\pi 1d$	1.52	5.94	1.33	5.01
$\pi 2p$			0.45	2.14

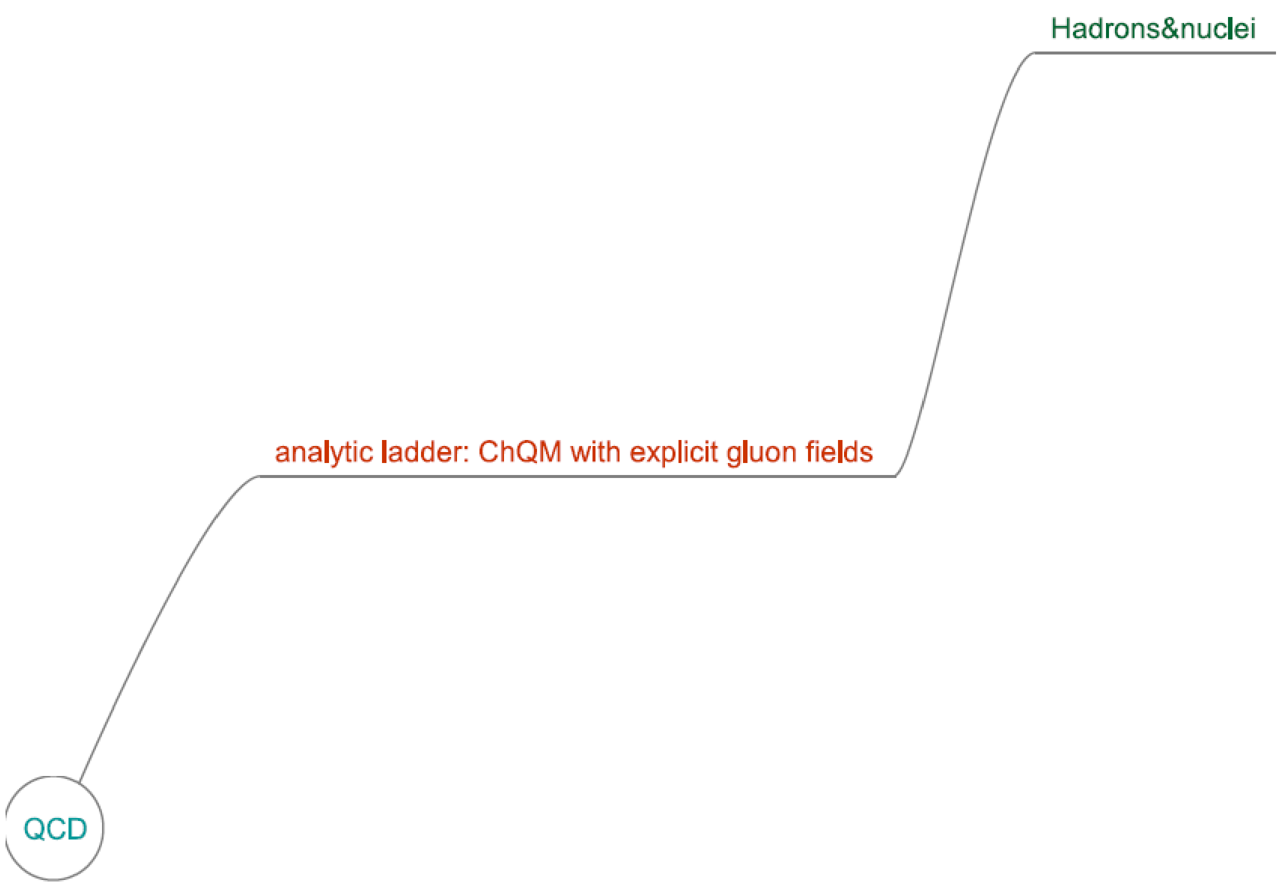
PHYSICAL REVIEW C VOLUME 22, NUMBER 5 NOVEMBER 1980

Nuclear spin-orbit splitting from an intermediate Δ excitation

Kohichi Ohta,* Tokuo Terasawa, and Mitsuru Tohyama
Institute of Physics, University of Tokyo, Komaba, Meguro-ku, Tokyo 153, Japan
(Received 2 July 1980)

The strength of the single particle spin-orbit potential is calculated from the two pion exchange box diagrams involving an intermediate $\Delta(1232)$ resonance excitation by taking account of the exclusion principle for the intermediate nucleon states. The effect of the ρ meson is also considered. The predicted strength is found to account for a substantial part of the empirical spin-orbit splittings.

Discussion: QCD vacuum characterized by non-trivial gluon background field and dense matter?



CHIRAL QUARKS AND THE NON-RELATIVISTIC QUARK MODEL

Aneesh MANOHAR and Howard GEORGI

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The success of the NR QM?

First of all the quarks should be massive, having constituent mass.

So, we assume that the bulk of the light quark mass is the effect of chiral symmetry breaking.

The leading contribution to the baryon mass is just sum of the constituent quark masses.

$$\mathcal{L} = \bar{\psi}\left(i\not{D} + \not{V}\right)\psi + g_A \bar{\psi}\not{A}\gamma_5\psi - m\bar{\psi}\psi$$

$$+ \tfrac{1}{4}f^2 \,\mathrm{tr}\, \partial^\mu \Sigma^\dagger \partial_\mu \Sigma - \tfrac{1}{2}\,\mathrm{tr}\, F_{\mu\nu}F^{\mu\nu} + \cdots,$$

where

$$V_\mu = \tfrac{1}{2}\left(\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger\right),$$

$$A_\mu = \tfrac{1}{2} i\left(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger\right),$$

$$m \simeq g^2 \frac{\langle \bar{\psi}\psi \rangle}{q^2},$$

Background gluon field?

Volume 71B, number 1 PHYSICS LETTERS 7 November 1977

INFRARED INSTABILITY OF THE VACUUM
STATE OF GAUGE THEORIES AND ASYMPTOTIC FREEDOM

G.K. SAVVIDY

$$A_\mu^a = -\tfrac{1}{2} F_{\mu\nu}^{\mathrm{YM}} x_\nu n^a,$$

$$\mathcal{L}^{(1)} = -\frac{11(gH)^2}{48\pi^2}\left[\ln\frac{H}{\mu^2} - \frac{1}{2}\right].$$

Thus the energy density looks like

$$\epsilon = \frac{H^2}{2} + \frac{11(gH)^2}{48\pi^2}\left[\ln\frac{H}{\mu^2} - \frac{1}{2}\right]$$

with its new minimum beyond the point $H = 0$.

AN UNSTABLE YANG-MILLS FIELD MODE

N.K. NIELSEN and P. OLESEN
NORDITA and the Niels Bohr Institute, DK-2100 Copenhagen Ø, Denmark

Nuclear Physics B144 (1978) 376–396

$$A_1 = A_3 = A_4 = 0 \text{ and } A_2 = Hx_1 .$$

$$\mathrm{Re}\,\epsilon = \tfrac{1}{2}H^2 + \frac{11}{48\pi^2}\, e^2 H^2 \left(\ln \frac{eH}{\mu^2} - \frac{1}{2}\right) .$$

This expression has a minimum away from the point $H = 0$, namely (with eH by definition positive)

$$eH_{\min} = \mu^2 \mathrm{e}^{-24\pi^2/11e^2} .$$

$$\mathrm{Im}\,\epsilon = \frac{-1}{8\pi}\, e^2 H^2 .$$

$$n = 0 \text{ and } S_3 = +1$$

$$W_{\text{vector}} = c \int_{-\infty}^{+\infty} \mathrm{d}k_3 \sum_{n=0}^{\infty} \{\sqrt{2eH(n+\tfrac{3}{2}) + k_3^2} + \sqrt{2eH(n-\tfrac{1}{2}) + k_3^2}\} .$$

IMPROVED QCD VACUUM
FOR GAUGE GROUPS SU(3) AND SU(4)

H. FLYVBJERG

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Nuclear Physics B176 (1980) 379–396

$$\mathcal{H} = \sum_{\alpha>0} H^{(\alpha)^2} \left\{ \frac{1}{3g^2} - \frac{11}{96\pi^2} + \frac{11}{48\pi^2} \log \frac{H^{(\alpha)}}{\mu_0^2} - \frac{i}{8\pi} \right\} .$$

A QUANTUM LIQUID MODEL FOR THE QCD VACUUM

Gauge and rotational invariance of domained and quantized homogeneous color fields

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Received 27 June 1979

We show that domains are formed in a homogeneous SU(2) color magnetic field. Due to quantum fluctuations the domains have fluid properties. It is then argued that, quantum mechanically, superpositions of such domains must be considered. The resulting state is gauge and rotational invariant, in spite of the fact that the original color magnetic field breaks these invariances. We point out that in our model for the QCD vacuum, color magnetic monopoles are not confined.

Nuclear Physics B160 (1979) 380–396

A happy coincidence?

$$\frac{g^2}{4\pi^2} = 0.16\,, \qquad\qquad \frac{g^2}{4\pi} = 0.50\,.$$

We thus see that the relevant coupling is reasonably small,

The value of α_s in the effective theory can be computed by looking at the ratio of colour and electromagnetic hyperfine splittings of the baryon spectrum

$$\alpha_s \simeq 0.28$$

- **Since the chiral quark model has no confinement yet, I think, still I have a chance to introduce non-trivial background gluon field such as a constant chromo-magnetic field in the model to connect QCD vacuum analytically with phenomenology.**