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Interaction of a twisted Dirac particle with a magnetic field in high energy physics

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OUTLINE

- Twisted (vortex) particles in high energy physics. Outlook
- Dynamics of twisted particles in external electric and magnetic fields
- Relativistic quantum mechanics of a twisted Dirac particle in a nonuniform magnetic field. Tensor magnetic polarizability of a twisted electron
- Sokolov-Ternov-like effect of a radiative orbital polarization of twisted electron beams in a magnetic field
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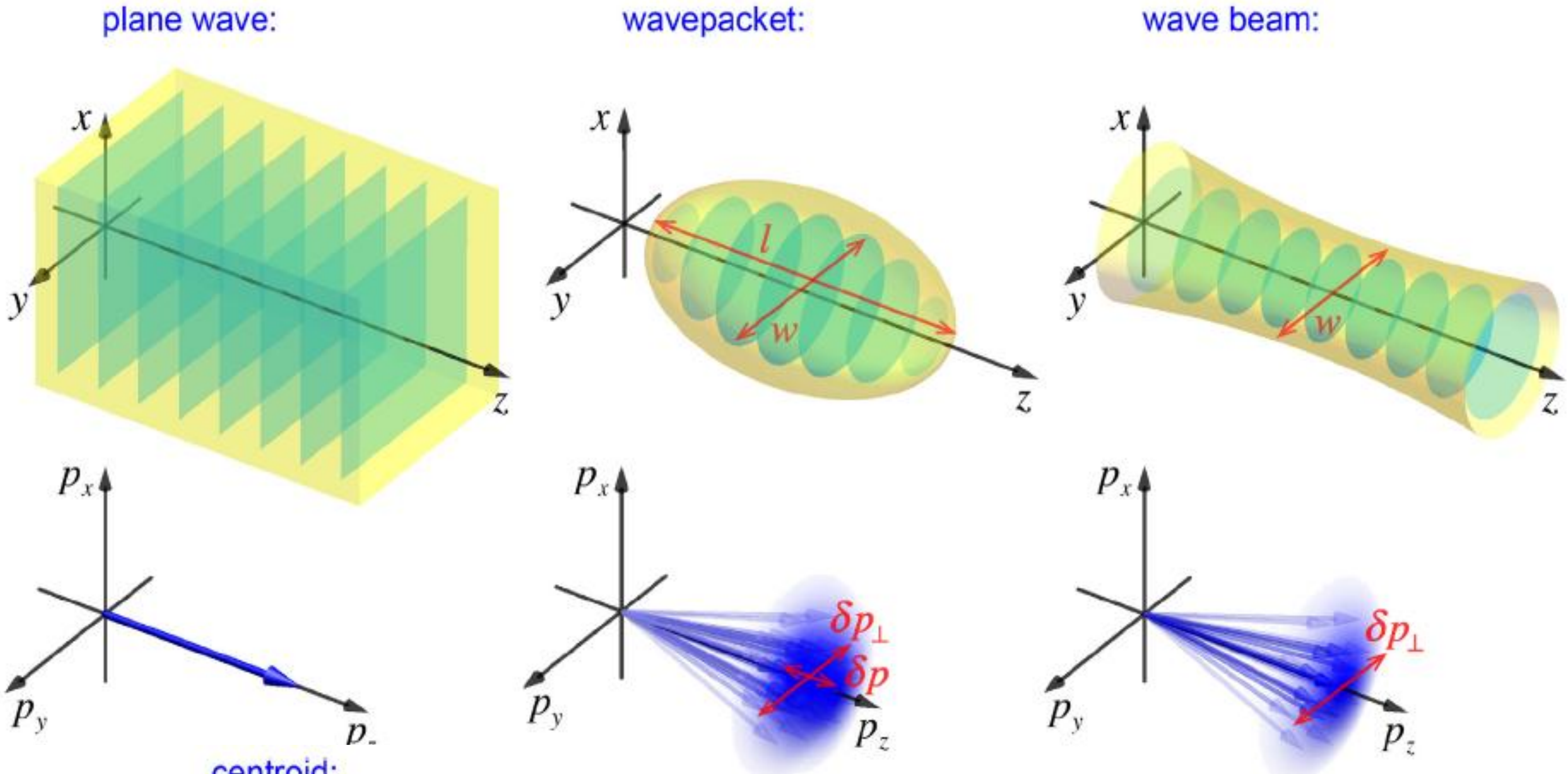
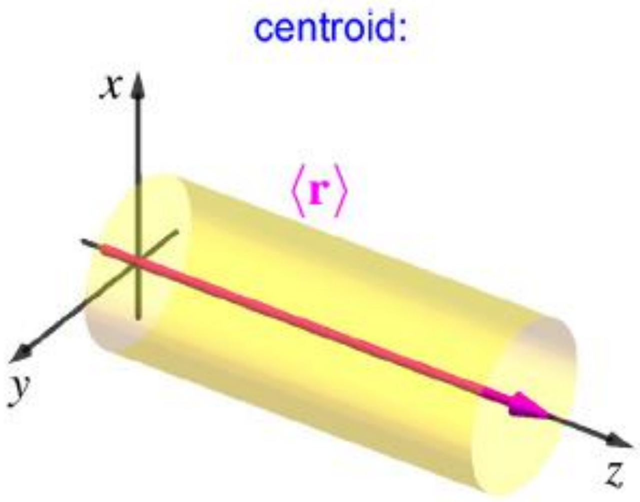
Twisted (vortex) particles in high energy physics. Outlook





Twisted (vortex) particles possess intrinsic orbital angular momenta.

Wave beams/packets are localized with respect to two/three dimensions and are described by two/three discrete transverse quantum numbers.

The wave function of free twisted particles with an intrinsic orbital angular momentum (OAM) in

cylindrical coordinates $\mathbf{r}(\rho, \phi, z)$:

$$\psi_l(\mathbf{r}, t) = u(\rho, z)\exp(il\phi)\exp(ik_z z)\exp(-i\omega t).$$

$u(\rho, z)$: **the Laguerre-Gauss function;**
the Bessel function of the first kind

Laguerre-Gauss beam:

$$\Psi = \mathcal{A}\exp(i\Phi),$$

$$\mathcal{A} = \frac{C_{nl}}{w(z)}\left(\frac{\sqrt{2}r}{w(z)}\right)^{|l|} L_n^{|l|}\left(\frac{2r^2}{w^2(z)}\right)\exp\left(-\frac{r^2}{w^2(z)}\right),$$

$$\Phi = l\phi + \frac{kr^2}{2R(z)} - (2n + |l| + 1)\varphi(z),$$

$$C_{nl} = \sqrt{\frac{2n!}{\pi(n+|l|)!}}, \quad w(z) = w_0\sqrt{1+\frac{4z^2}{k^2 w_0^4}},$$

$$R(z) = z + \frac{k^2 w_0^4}{4z}, \quad \varphi(z) = \arctan\left(\frac{2z}{k w_0^2}\right),$$

$$\int \Psi^\dagger \Psi r dr d\phi = 1,$$

where the real functions $\mathcal{A}$ and Φ define the amplitude and phase of the beam, k is the beam wavenumber, w_0 is the minimum beam width, $L_n^{|l|}$ is the generalized Laguerre polynomial,

Bessel beam:

$$\psi_\ell^B \propto J_{|\ell|}(\kappa r)\exp[i(\ell\varphi + k_z z)],$$

where J_ℓ is the Bessel function of the first kind, $\ell = 0, \pm 1, \pm 2, \ldots$ is an integer number (azimuthal quantum number), $k_z = p_z/\hbar$ is the longitudinal wave number, and $\kappa = p_\perp/\hbar$ is the transverse (radial) wave number. Solutions (2.5) satisfy

The Bessel beams represent the simplest theoretical example of vortex beams. Despite the probability density of Bessel modes decaying as $|\psi_\ell^B| \sim 1/r$ when $r \to \infty$, these solutions are not properly localized in the transverse dimensions. Indeed, the integral $\int_0^\infty |\psi_\ell^B|^2 r\, dr$ diverges, and the function cannot be normalized with respect to the transverse dimensions.[2] The delocalized nature of Bessel beams is reflected in the absence of diffraction and a single transverse quantum number ℓ (instead of two transverse quantum indices in the properly-localized modes).

Twisted electrons and their interactions with external fields and matter are considered in detail in the following recent reviews:

K. Y. Bliokh, I. P. Ivanov, G. Guzzinati, L. Clark, R. Van Boxem, A. Beche, R. Juchtmans, M. A. Alonso, P. Schattschneider, F. Nori, and J. Verbeeck, **Theory and applications of free-electron vortex states, Phys. Rep. 690,** 1 (2017).

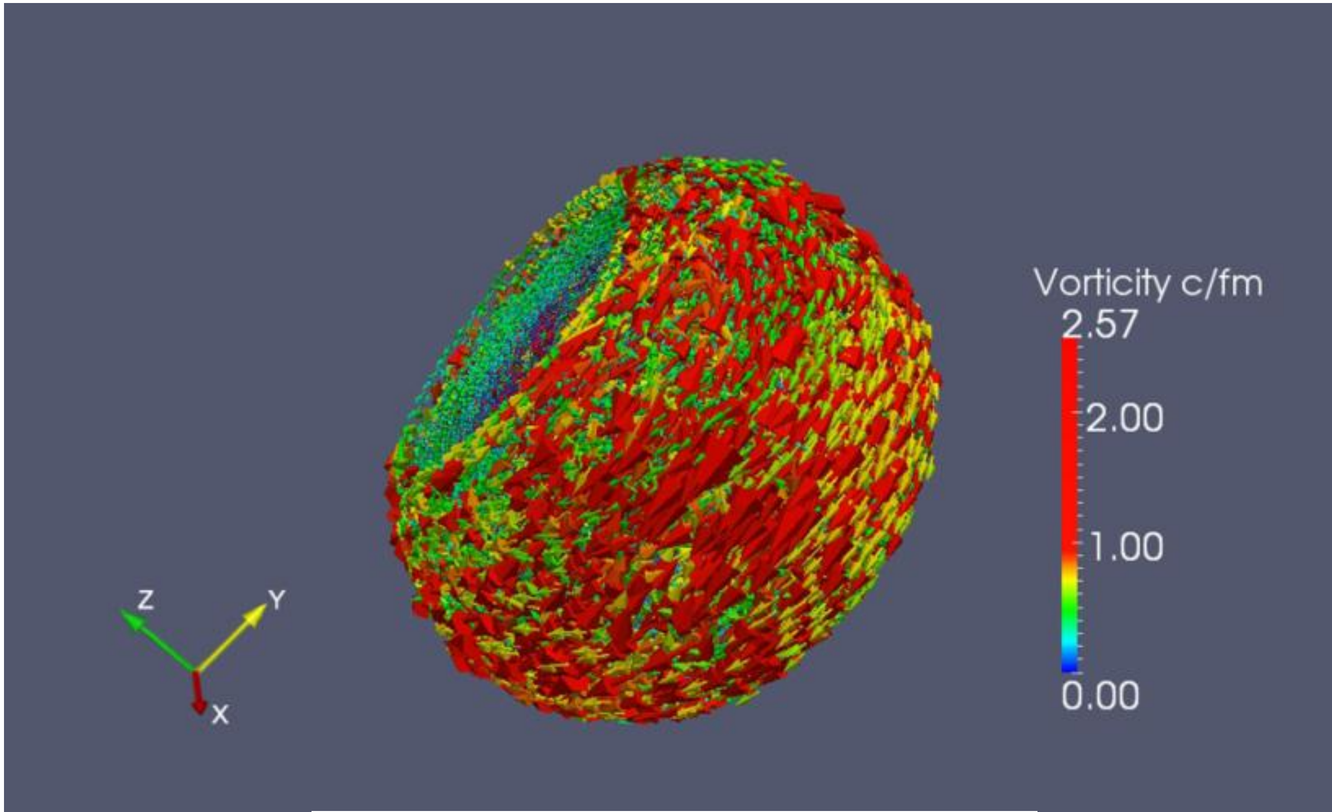
S. M. Lloyd,M. Babiker, G. Thirunavukkarasu, and J. Yuan, Electron vortices: Beams with orbital angular momentum, Rev. Mod. Phys. 89, 035004 (2017).

H. Larocque, I. Kaminer, V. Grillo, G. Leuchs, M. J. Padgett, R. W. Boyd, M. Segev, E. Karimi, `Twisted' electrons, Contemp. Phys. 59, 126 (2018).

An analysis of experimental data on heavy ion collisions unambiguously shows an existence of the vorticity of a strongly interacting nuclear fluid. The formation of specific toroidal structures of vorticity field (vortex sheets) takes place.

Baznat, Gudima, Sorin, Teryaev, Phys. Rev. C 88, 061901(R) (2013).
Teryaev, Usubov, Phys. Rev. C 92, 014906 (2015).
Baznat, Gudima, Sorin, Teryaev, Phys. Rev. C 93, 031902(R) (2016).

In principle, quarks and gluons can possess not only extrinsic but also intrinsic OAMs (can be vortex)



The vortex sheet

M. Katoh, M. Fujimoto, H. Kawaguchi, K. Tsuchiya, K. Ohmi, T. Kaneyasu, Y. Taira, M. Hosaka, A. Mochihashi, and Y. Takashima, Angular Momentum of Twisted Radiation from an Electron in Spiral Motion, Phys. Rev. Lett. 118, 094801 (2017).

``This work indicates that twisted photons are naturally emitted by free electrons and are more ubiquitous in laboratories and in nature than ever thought.”

S. V. Abdrashitov, O. V. Bogdanov, P. O. Kazinski, and T. A. Tukhfatullin, Orbital angular momentum of channeling radiation from relativistic electrons in thin Si crystal, Phys. Lett. A 382, 3141 (2018).

``We obtain that the average OAM of channeling radiation in this case is approximately 1÷6ħ per photon with the photon energies about 1÷2 MeV.”

V. Epp, J. Janz, and M. Zotova, Angular momentum of radiation at axial channeling, Nucl. Instrum. Methods Phys. Res., Sect. B 436, 78 (2018).

``It is shown that high energy particles channeled in the presence of magnetic field are effective source of vortex radiation in the X-ray and gamma-range of photons energies. The magnetic field favours additional "twisting" of the channeled particles in a certain direction."

M. Katoh et al. Helical Phase Structure of Radiation from an Electron in Circular Motion, Sci. Rep. 7, 6130 (2017).

``We have shown that the harmonic components of an electromagnetic field radiated by electrons in circular motion naturally have a helical phase structure, which suggests the presence of orbital angular momentum. We demonstrated this experimentally by observing helical undulator radiation."

Recent theoretical results:

O. V. Bogdanov, P. O. Kazinski, and G. Yu. Lazarenko, Probability of radiation of twisted photons by classical currents, Phys. Rev. A 97, 033837 (2018).

O. V. Bogdanov, P. O. Kazinski, and G. Yu. Lazarenko, Semiclassical probability of radiation of twisted photons in the ultrarelativistic limit, Phys. Rev. D 99, 116016 (2019).

Magnetic fields created in noncentral heavy-ion collisions are very strong, $eB_y \sim m_\pi^2$ for the RHIC energies:

V. Skokov, A. Y. Illarionov, and V. Toneev, Estimate of the magnetic field strength in heavy-ion collisions, Int. J. Mod. Phys. A 24, 5925 (2009).

In noncentral collisions, the nuclear matter rotates with a large angular velocity and a strong magnetic field appears. As a result, a probability of radiation of twisted photons can be large. In our opinion, a probability of creation of massive twisted particles can also be considerable. Their interactions, especially with a magnetic field, should be studied.

Dynamics of twisted particles in external electric and magnetic fields

We base the explanations on our recent publications.

A. J. Silenko, Pengming Zhang and Liping Zou, Manipulating Twisted Electron Beams, Phys. Rev. Lett. 119, 243903 (2017).
A. J. Silenko, Pengming Zhang and Liping Zou, Relativistic Quantum Dynamics of Twisted Electron Beams in Arbitrary Electric and Magnetic Fields, Phys. Rev. Lett. 121, 043202 (2018).

The classical Hamiltonian is given by

$$H = -\frac{e}{2mc}\left[\boldsymbol{B}\cdot\boldsymbol{L}^{(0)} - \frac{\gamma}{\gamma+1}(\boldsymbol{\beta}\cdot\boldsymbol{B})(\boldsymbol{\beta}\cdot\boldsymbol{L}^{(0)}) - (\boldsymbol{\beta}\times\boldsymbol{E})\cdot\boldsymbol{L}^{(0)}\right]$$
$$= -\frac{e}{2mc\gamma}[\boldsymbol{B}\cdot\boldsymbol{L} - (\boldsymbol{\beta}\times\boldsymbol{E})\cdot\boldsymbol{L}].$$

The Foldy-Wouthuysen (FW) representation in relativistic quantum mechanics is equivalent to the Schrödinger representation in nonrelativistic quantum mechanics. The corresponding relativistic FW Hamiltonian is similar:

$$\mathcal{H}_{\mathrm{FW}} = \beta\epsilon + e\Phi - \beta\frac{e}{4}\left[\frac{1}{\epsilon}\boldsymbol{L}\cdot\boldsymbol{B}(\boldsymbol{R}) + \boldsymbol{B}(\boldsymbol{R})\cdot\boldsymbol{L}\frac{1}{\epsilon}\right]$$

$$+\frac{e}{4}\left\{\frac{1}{\epsilon^2}\boldsymbol{L}\cdot[\boldsymbol{\pi}'\times\boldsymbol{E}(\boldsymbol{R})] - [\boldsymbol{E}(\boldsymbol{R})\times\boldsymbol{\pi}']\cdot\boldsymbol{L}\frac{1}{\epsilon^2}\right\}.$$

In this equation, spin effects are disregarded because they can be neglected on the condition that $L \gg 1$. The term $e\Phi$ does not include the interaction of the intrinsic OAM with the electric field.

The equation of motion of the intrinsic OAM has the form

$$\frac{d\boldsymbol{L}}{dt} = i[\mathcal{H}_{\mathrm{FW}}, \boldsymbol{L}] = \frac{1}{2}(\boldsymbol{\Omega}\times\boldsymbol{L} - \boldsymbol{L}\times\boldsymbol{\Omega}),$$

$$\boldsymbol{\Omega} = -\beta\frac{e}{4}\left\{\frac{1}{\epsilon}, \boldsymbol{B}(\boldsymbol{R})\right\} + \frac{e}{4}\left[\frac{1}{\epsilon^2}\boldsymbol{\pi}'\times\boldsymbol{E}(\boldsymbol{R}) - \boldsymbol{E}(\boldsymbol{R})\times\boldsymbol{\pi}'\frac{1}{\epsilon^2}\right].$$

In the classical limit,

$$\Omega = -\frac{e}{2mc\gamma}[B - \beta \times E].$$

The corresponding equation of the spin precession is very different.

The results obtained allow us to develop methods for the manipulation of electron vortex beams.

1. **Separations of beams with opposite directions of the OAM (in a nonuniform longitudinal magnetic field).**
2. **Freezing the intrinsic OAM in electromagnetic fields**

$$B = \left(\frac{2}{\beta^2} - 1\right)\beta \times E.$$

The device freezes the OAM relative to the momentum direction and rotates the beam direction with the angular velocity

$$\Omega_L = \omega = -\frac{eB}{mc\gamma(\gamma^2 + 1)}.$$

3. Flipping the intrinsic OAM. The OAM flip can be forced by a longitudinal magnetic field oscillating with the resonance frequency.
4. Rotation of the intrinsic OAM in crossed electric and magnetic fields. For this purpose, crossed electric and magnetic fields ($\boldsymbol{E}\perp\boldsymbol{B}\perp\boldsymbol{\beta}$) satisfying the relation $\boldsymbol{E}=-\boldsymbol{\beta}\times\boldsymbol{B}$ can be used. Such fields characterizing the Wien filter do not affect a beam trajectory. We suppose the fields ***E*** and ***B*** to be uniform. In the considered case, the classical limit of the relativistic equation for the angular velocity of precession of the intrinsic OAM is given by

$$\boldsymbol{\Omega}^{(W)} = -\frac{e\left(m^2+\mathfrak{p}^2\right)}{2\epsilon^3}\boldsymbol{B}.$$

Here $\mathfrak{p}$ is the momentum characterizing the internal motion inside of the centroid and $\varepsilon=m\gamma$.

Relativistic quantum mechanics of a twisted Dirac particle in a nonuniform magnetic field. Tensor magnetic polarizability of a twisted electron

The general relativistic quantum-mechanical description is presented in the following paper:

A. J. Silenko, Pengming Zhang and Liping Zou, Electric Quadrupole Moment and the Tensor Magnetic Polarizability of Twisted Electrons and a Potential for their Measurements, Phys. Rev. Lett. 122, 063201 (2019).

The exact relativistic Hamiltonian in the FW representation (the FW Hamiltonian) for a Dirac particle in a magnetic field is given by (Case, 1954; Tsai, 1973)

$$\mathcal{H}_{FW} = \beta\sqrt{m^2 + \boldsymbol{\pi}^2 - e\boldsymbol{\Sigma}\cdot\boldsymbol{B}},$$

where $\boldsymbol{\pi} = \boldsymbol{p} - e\boldsymbol{A}$ is the kinetic momentum, $\boldsymbol{B} = \nabla \times \boldsymbol{A}$ is the magnetic induction, and β and $\boldsymbol{\Sigma}$ are the Dirac matrices. This Hamiltonian is valid for a twisted and a untwisted particle. The spin angular momentum operator is equal to $\boldsymbol{s} = \hbar\boldsymbol{\Sigma}/2$. The magnetic field is, in general, nonuniform.

It is necessary to take into account that a twisted electron is a charged centroid (Bliokh et al., 2007; 2017). To describe observable quantum-mechanical effects, we need to present the Hamiltonian in terms of the centroid parameters. The centroid as a

whole is characterized by the center-of-charge radius vector $\boldsymbol{R}$ and by the kinetic momentum $\boldsymbol{\pi}' = \boldsymbol{P} - e\boldsymbol{A}(\boldsymbol{R})$, where $\boldsymbol{P} = -i\hbar\partial/(\partial\boldsymbol{R})$. The intrinsic motion is defined by the kinetic momentum $\boldsymbol{\pi}'' = \mathbf{p} - e[\boldsymbol{A}(\boldsymbol{r}) - \boldsymbol{A}(\boldsymbol{R})]$. Here $\mathbf{p} = -i\hbar\partial/(\partial\mathbf{r})$, $\mathbf{r} = \boldsymbol{r} - \boldsymbol{R}$, $\boldsymbol{\pi}' + \boldsymbol{\pi}'' = \boldsymbol{\pi}$, $\boldsymbol{P} + \mathbf{p} = \boldsymbol{p}$. Since

$$\boldsymbol{A}(\boldsymbol{r}) = \boldsymbol{A}(\boldsymbol{R}) + \frac{1}{2}\boldsymbol{B}(\boldsymbol{R}) \times \mathbf{r},$$

the operator $\boldsymbol{\pi}^2$ takes the form

$$\boldsymbol{\pi}^2 = \boldsymbol{\pi}'^2 + \mathbf{p}^2 - \frac{e}{2}[\boldsymbol{L}\cdot\boldsymbol{B}(\boldsymbol{R}) + \boldsymbol{B}(\boldsymbol{R})\cdot\boldsymbol{L}] + \frac{e^2}{4}[\boldsymbol{B}(\boldsymbol{R}) \times \mathbf{r}]^2,$$

where $\boldsymbol{R}$ and $\boldsymbol{\pi}'$ are the center-of-charge radius vector and the kinetic momentum of the centroid as a whole, $\mathbf{r} = \boldsymbol{r} - \boldsymbol{R}$ and $\mathbf{p} = -i\hbar\partial/(\partial\mathbf{r})$ are internal canonical variables, and $\boldsymbol{L} \equiv \mathbf{r} \times \mathbf{p}$ is the intrinsic OAM [20]. The operator $\boldsymbol{\pi}'\cdot\boldsymbol{\pi}'' + \boldsymbol{\pi}''\cdot\boldsymbol{\pi}'$ has zero expectation values for any eigenstates and can be omitted. The square root in the

FW Hamiltonian can be approximately extracted.

$$\mathcal{H}_{FW} = \beta\epsilon - \beta\frac{e}{4}\left(\frac{1}{\epsilon}\Lambda\cdot \boldsymbol{B}(\boldsymbol{R}) + \boldsymbol{B}(\boldsymbol{R})\cdot\boldsymbol{\Lambda}\frac{1}{\epsilon}\right) + \beta\frac{e^2}{16}\left(\left\{\frac{1}{\epsilon}, [\boldsymbol{B}(\boldsymbol{R})\times\mathbf{r}]^2\right\} - \left\{\frac{1}{\epsilon^3}, [\boldsymbol{B}(\boldsymbol{R})]^2\right\}\right)$$
$$-\beta\frac{e^2}{16}\left(\frac{1}{\epsilon^3}[\boldsymbol{L}\cdot\boldsymbol{B}(\boldsymbol{R}) + \boldsymbol{B}(\boldsymbol{R})\cdot\boldsymbol{L}][\boldsymbol{\Sigma}\cdot\boldsymbol{B}(\boldsymbol{R})] + [\boldsymbol{\Sigma}\cdot\boldsymbol{B}(\boldsymbol{R})][\boldsymbol{L}\cdot\boldsymbol{B}(\boldsymbol{R}) + \boldsymbol{B}(\boldsymbol{R})\cdot\boldsymbol{L}]\frac{1}{\epsilon^3}\right)$$
$$-\beta\frac{e^2}{64}\left\{\frac{1}{\epsilon^3}, [\boldsymbol{L}\cdot\boldsymbol{B}(\boldsymbol{R}) + \boldsymbol{B}(\boldsymbol{R})\cdot\boldsymbol{L}]^2\right\},$$

$$\epsilon = \sqrt{m^2 + \boldsymbol{\pi}'^2 + \mathfrak{p}^2}.\ \boldsymbol{\Lambda} = \boldsymbol{L} + \boldsymbol{\Sigma}.$$

Term defining the tensor magnetic polarizability (TMP)

The TMP caused by the intrinsic OAM is given by

$$W = -\beta\beta_T(\boldsymbol{L}\cdot\boldsymbol{B})^2, \quad \beta_T = \frac{e^2\hbar^2}{8m^3} = 5.25\times 10^4\ \mathrm{fm}^3.$$

For pointlike $W^{\pm}$ bosons $\beta_T(W) \sim 10^{-11}$ fm^3. For the deuteron, the theoretical estimation is $\beta_T(d)$ =0.195 fm^3.

The evolution of the intrinsic OAM does not reduce to its precession.

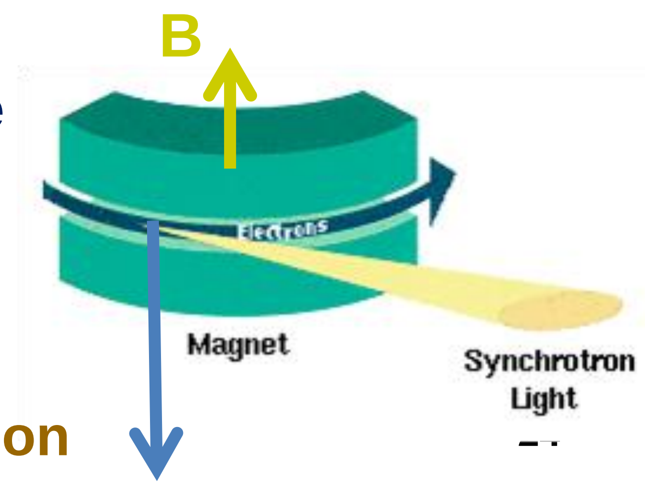
Sokolov-Ternov-like effect of a radiative orbital polarization of twisted electron beams in a magnetic field

The well-known effect is the radiative spin polarization of electron or positron beams in storage rings caused by the synchrotron radiation (Sokolov-Ternov effect). The radiative spin polarization acquired by unpolarized electrons is opposite to the direction of the main magnetic field. The reason for the effect is a dependence of spin-flip transitions from the initial particle polarization. It follows from the previously obtained results (Ivanov, 2012; Ivanov et al., 2016) that quantum-electrodynamics effects are rather similar for twisted and untwisted particles. We can note the evident similarity between interactions of the spin and the intrinsic OAM with the magnetic field. In particular, energies of stationary states depend on projections of the spin and the intrinsic OAM on the field direction. This similarity validates the existence of the effect of the radiative orbital polarization. As well as the radiative spin polarization, the corresponding orbital polarization acquired by unpolarized twisted electrons should be opposite to the direction of the main magnetic field.

The effect is conditioned by transitions with a change of a projection of the intrinsic OAM. The probability of such transitions is large enough if the electron energy is not too small. Similarly to the spin polarization, the orbital one is observable when electrons are accelerated up to the energy of the order of 1 GeV. The acceleration can depolarize twisted electrons but cannot vanish *L*. During the process of the radiative polarization, the average energy of the electrons should be kept unchanged.

A. J. Silenko, Pengming Zhang and Liping Zou, Relativistic Quantum Dynamics of Twisted Electron Beams in Arbitrary Electric and Magnetic Fields, Phys. Rev. Lett. 121, 043202 (2018).

New Sokolov-Ternov-like effect of a radiative OAM polarization of electron or positron beams in storage rings caused by the synchrotron radiation is predicted.



It is important that the orbital polarization of twisted electrons carrying an intrinsic orbital angular momentum is not influenced by field perturbations in arbitrary magnetic fields. This property means an existence of the Siberian snake-like behavior for an orbital polarization of a beam of twisted electrons in cyclotrons with the main magnetic field and magnetic focusing. As a result, the acceleration of twisted electron beams in cyclotrons necessary for their applications in high-energy-physics experiments considerably simplifies.

$$\frac{d\boldsymbol{L}}{dt} = \boldsymbol{\Omega} \times \boldsymbol{L}, \qquad \boldsymbol{\Omega} = -\frac{e}{2mc\gamma}\boldsymbol{B},$$

$$\frac{d\boldsymbol{N}}{dt} = \boldsymbol{\omega} \times \boldsymbol{N}, \qquad \boldsymbol{\omega} = -\frac{e}{mc\gamma}\boldsymbol{B} = 2\boldsymbol{\Omega}.$$

A. J. Silenko and O. V. Teryaev, Siberian Snake-Like Behavior for an Orbital Polarization of a Beam of Twisted (Vortex) Electrons, Phys. Part. Nucl. Lett. 16, 77 (2019).

Summary

- Main distinctive features of twisted (vortex) particles have been discussed. An importance of twisted states in high energy physics has been shown
- General equation defining dynamics of twisted electrons in external electric and magnetic fields has been derived. Methods for the manipulation of twisted electron beams have been elaborated
- Relativistic quantum mechanics of a twisted Dirac particle in a nonuniform magnetic field has been constructed. Tensor magnetic polarizability of a twisted electron has been calculated
- Sokolov-Ternov-like effect of a radiative orbital polarization of twisted electron beams in a magnetic field has been predicted


Thank you for your attention