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# Role of the $h_1(1800)$ and $f_1(1285)$ states in the $J/\psi$ decays

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# Outline

## Introduction

$K^* \bar{K}^*$  and  $K^* \bar{K}$  interactions in the Chiral Unitary Approach

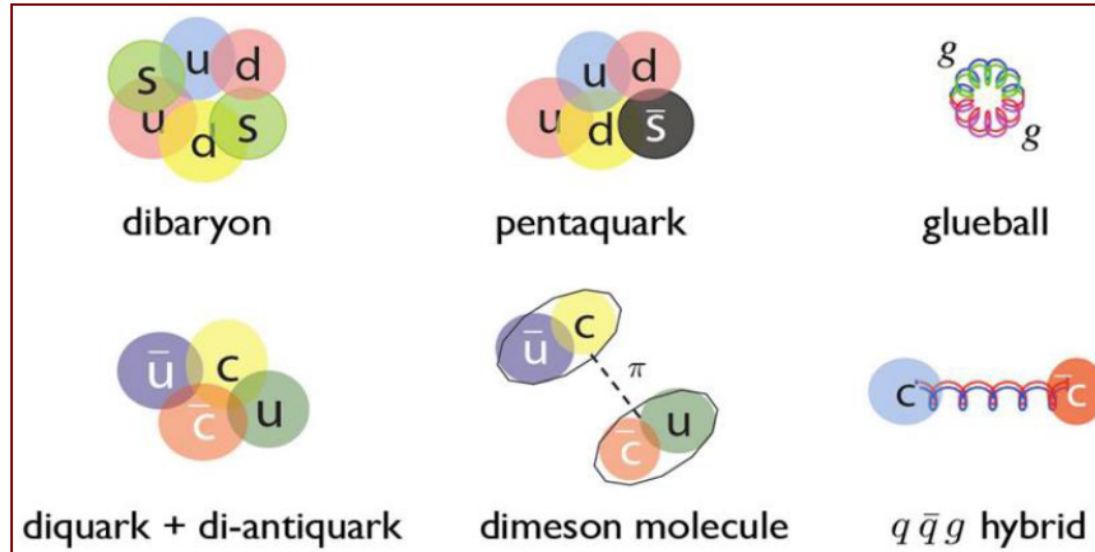
Signature of an  $h_1$  state in the  $J/\psi \rightarrow \eta h_1(1800) \rightarrow \eta K^{*0} \bar{K}^{*0}$  decay

The  $J/\psi \rightarrow \phi \bar{K} K^*$  decay and the  $f_1(1285)$  state

## Summary

# Hadrons: normal & exotic

- Quark model: hadrons are composed from 2 (meson) quarks or 3 (baryon) quarks



- QCD does not forbid hadrons with  $N_{\text{quarks}} \neq 2, 3$ 
  - Glueball:  $N_{\text{quarks}} = 0$  (gg, ggg, ...)
  - Hybrid:  $N_{\text{quarks}} = 2$  (or more) + excited gluon
  - Multiquark state:  $N_{\text{quarks}} > 3$
  - Molecule: bound state of more than 2 hadrons
  - ...

From Cheng-Ping Shen.

# Hadronic molecules



$$t = v + vgv + vgvgv + \dots = v + vgt \quad (g : \text{two body loop function})$$

$$= [1 - vg]^{-1} v$$

$v$  : from chiral Lagrangian

Example  $\Lambda(1405)$  as  $K\bar{b}N$  molecular state

$P_c(4312)$

$P_c(4440)$

$P_c(4457)$

?

## Hadronic molecules

# Vector-Vector interactions

$$\mathcal{L}_{III} = -\frac{1}{4}\langle V_{\mu\nu} V^{\mu\nu} \rangle, \quad V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu - ig[V_\mu, V_\nu],$$

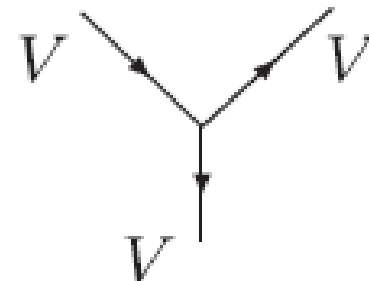
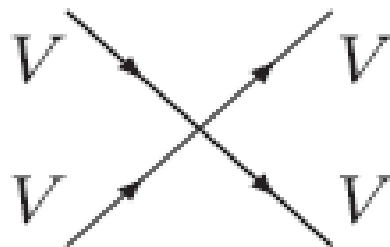
$$V_\mu = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}_\mu.$$



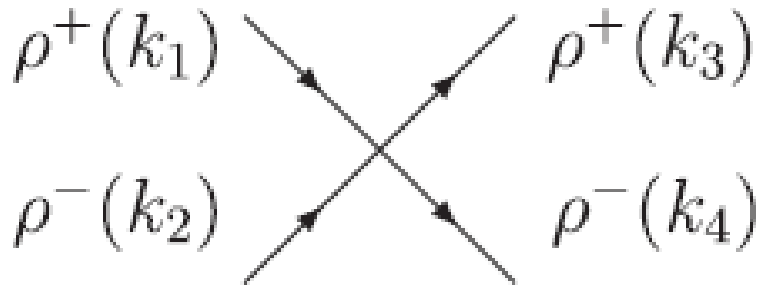
$$\mathcal{L}_{III}^{(c)} = \frac{g^2}{2}\langle V_\mu V_\nu V^\mu V^\nu - V_\nu V_\mu V^\mu V^\nu \rangle,$$



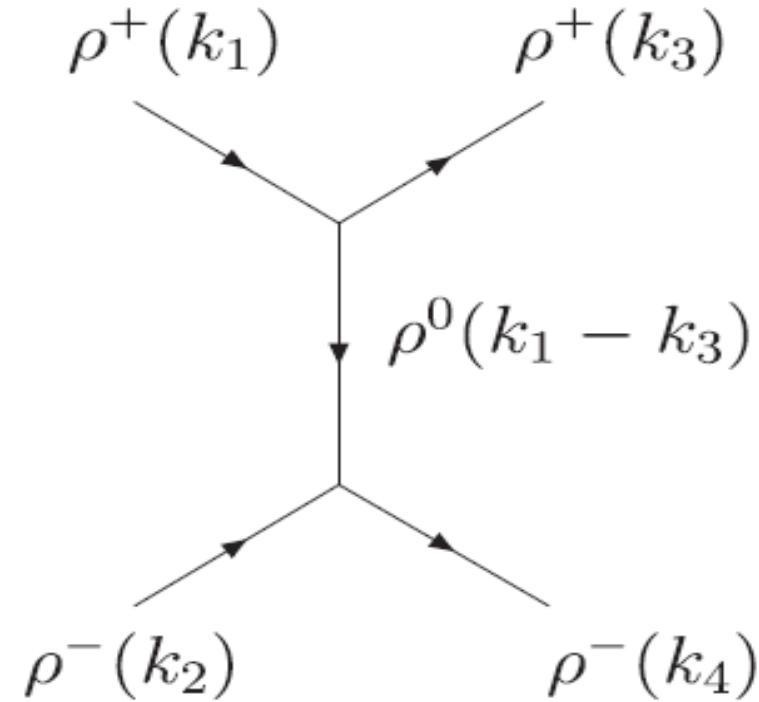
$$\mathcal{L}_{III}^{(3V)} = ig\langle (\partial_\mu V_\nu - \partial_\nu V_\mu) V^\mu V^\nu \rangle,$$



# Vector-Vector scattering amplitudes



Contact term of the  $\rho\rho$  interaction.



Vector exchange diagram for  $\rho^+\rho^- \rightarrow \rho^+\rho^-$ .

Potential (Kernel)  $V$

$G$ : two vectors loop function

$$T = (1 - VG)^{-1}V$$

$$G = i \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2 - M_{V_1}^2 + i\epsilon} \frac{1}{(P - q)^2 - M_{V_2}^2 + i\epsilon}$$

An  $h_1$  state in  $K^* \bar{K}^*$  system

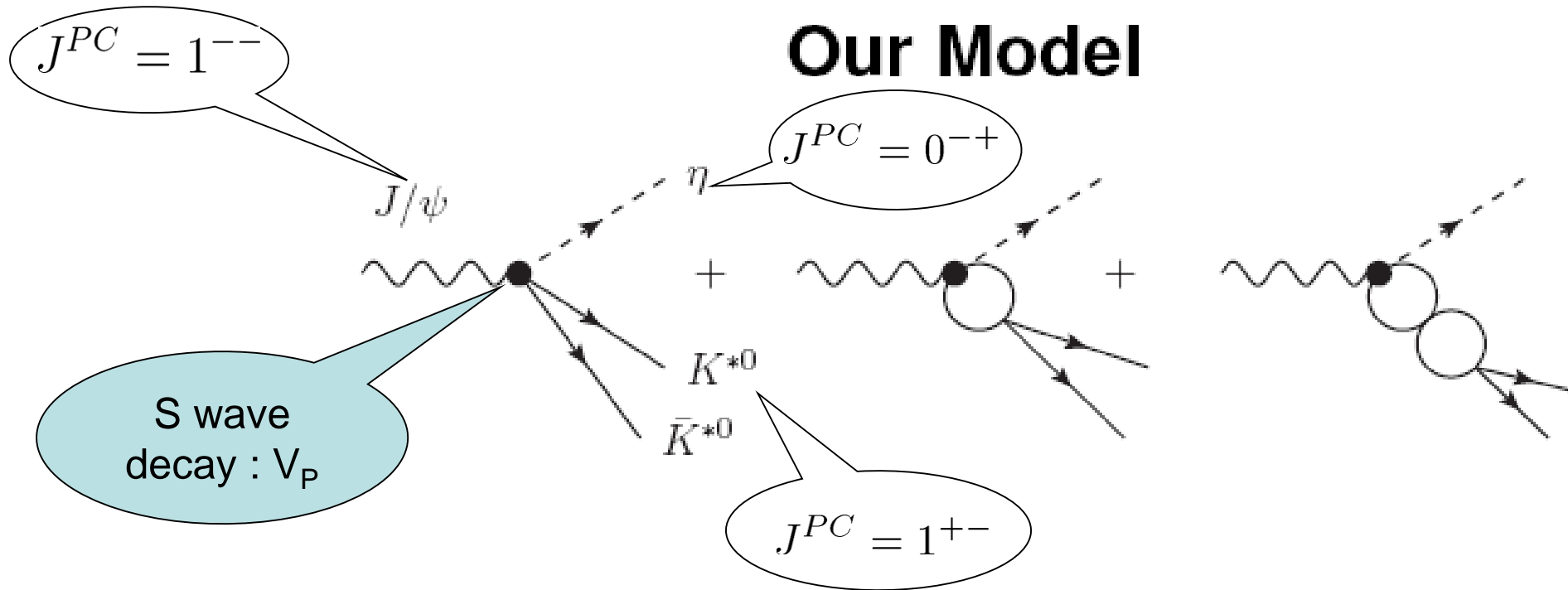
$$h_1 : I^G(J^{PC}) = 0^-(1^{+-})$$

The pseudoscalar--vector channels are allowed, but their thresholds are far away. They can contribute to the width, but have little effect in the energy of the interacting  $VV$  components.

It cannot couple to other vector--vector because of Charge parity, which makes its observation difficult.

$$\text{Pole position : } (1802, -i39) \text{ MeV}$$

$$\text{Coupling : } g_{K^* \bar{K}^*}^R = (8034, -i2542) \text{ MeV}$$



$$t_P = V_P \left( 1 + \tilde{G}(M_{\text{inv}}^2) t(M_{\text{inv}}^2) \right) = V_P \frac{t(M_{\text{inv}}^2)}{v(M_{\text{inv}}^2)}$$

$$t = v + v\tilde{G}t = v(1 + \tilde{G}t) = (1 - v\tilde{G})^{-1}v$$

$$v = g^2 \left( 9 + b \left( 1 - \frac{3M_{\text{inv}}^2}{4m_{K^*}^2} \right) \right)$$

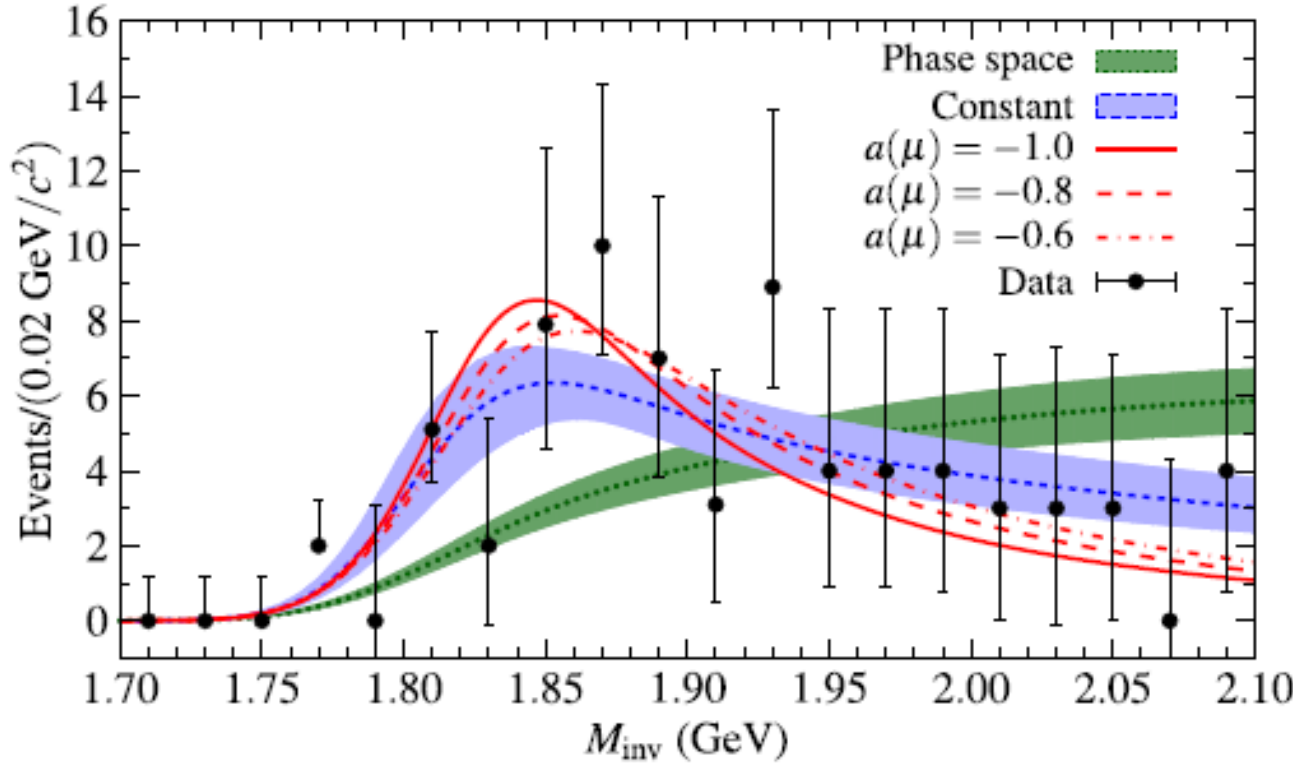
L.S. Geng *et al.*, Phys. Rev. D **79**, 074009 (2009).

Ju-Jun Xie *et al.*, Phys. Lett. B **728**, 319 (2014).

The constant  $b$  is determined by the masses of the vector mesons and its value turns out to be  $b = 6.8$ .



$$J/\psi \rightarrow \eta K^{*0} \bar{K}^{*0} \text{ decay} \quad \frac{d\Gamma}{dM_{\text{inv}}} = \frac{C}{|v(M_{\text{inv}}^2)|^2} \frac{p_1 \tilde{p}_2}{M_{J/\psi}} |t(M_{\text{inv}}^2)|^2$$



$$p_1 = \frac{\lambda^{1/2}(M_{J/\psi}^2, m_\eta^2, M_{\text{inv}}^2)}{2M_{J/\psi}},$$

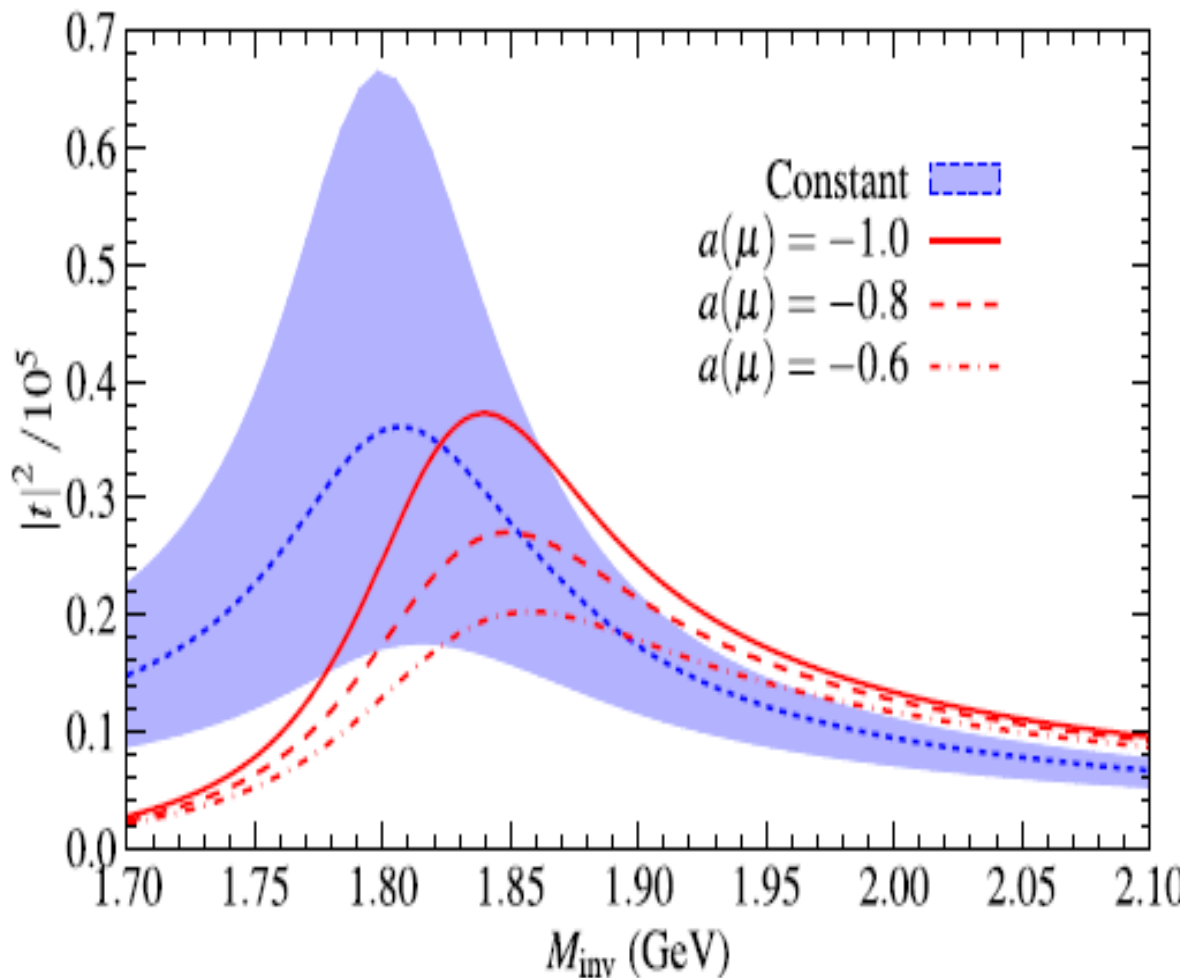
$$p_2 = \frac{\lambda^{1/2}(M_{\text{inv}}^2, m_1^2, m_2^2)}{2M_{\text{inv}}}.$$

M. Ablikim *et al.* (BES Collaboration), PLB685, 27 (2010).

Values of some of the parameters used or determined in this work.

Potential	$C$ ( $\text{GeV}^{-1}$ )	$a_\mu$	$v/g^2$	$\chi^2/\text{d.o.f.}$
Constant	$42 \pm 6$	-0.8	$-6.2 \pm 1.2$	0.45
Hidden gauge	$42 \pm 6$	-1.0	Eq. (8)	0.56
Hidden gauge	$53 \pm 7$	-0.8	Eq. (8)	0.47
Hidden gauge	$67 \pm 9$	-0.6	Eq. (8)	0.42

The modulus squared  $|t|^2$  for  $K^* \bar{K}^* \rightarrow K^* \bar{K}^*$



$$t = v + v\tilde{G}t = v(1 + \tilde{G}t) = (1 - v\tilde{G})^{-1}v = (v^{-1} - \tilde{G})^{-1}$$

$$v = \left( 9 + b \left( 1 - \frac{3M_{\text{inv}}^2}{4m_{K^*}^2} \right) \right) g^2$$

$$\tilde{G}(s) = \int_{m_-^2}^{m_+^2} dm_1^2 dm_2^2 \omega(m_1^2) \omega(m_2^2) G(s, m_1^2, m_2^2),$$

$$16\pi^2 G(s, m_1^2, m_2^2)$$

$$= a(\mu) + \log \frac{m_1 m_2}{\mu^2} + \frac{\Delta}{2s} \log \frac{m_2^2}{m_1^2} + \frac{v}{2s} \left( \log \frac{s - \Delta + v}{-s + \Delta + v} + \log \frac{s + \Delta + v}{-s - \Delta + v} \right),$$

$$\Delta = m_2^2 - m_1^2, \quad v = \lambda^{1/2}(s, m_1^2, m_2^2),$$

$$\omega(m_1^2) = \frac{1}{\mathcal{N}} \text{Im} \frac{1}{m_1^2 - m_{K^*}^2 + i\Gamma(m_1^2)m_1},$$

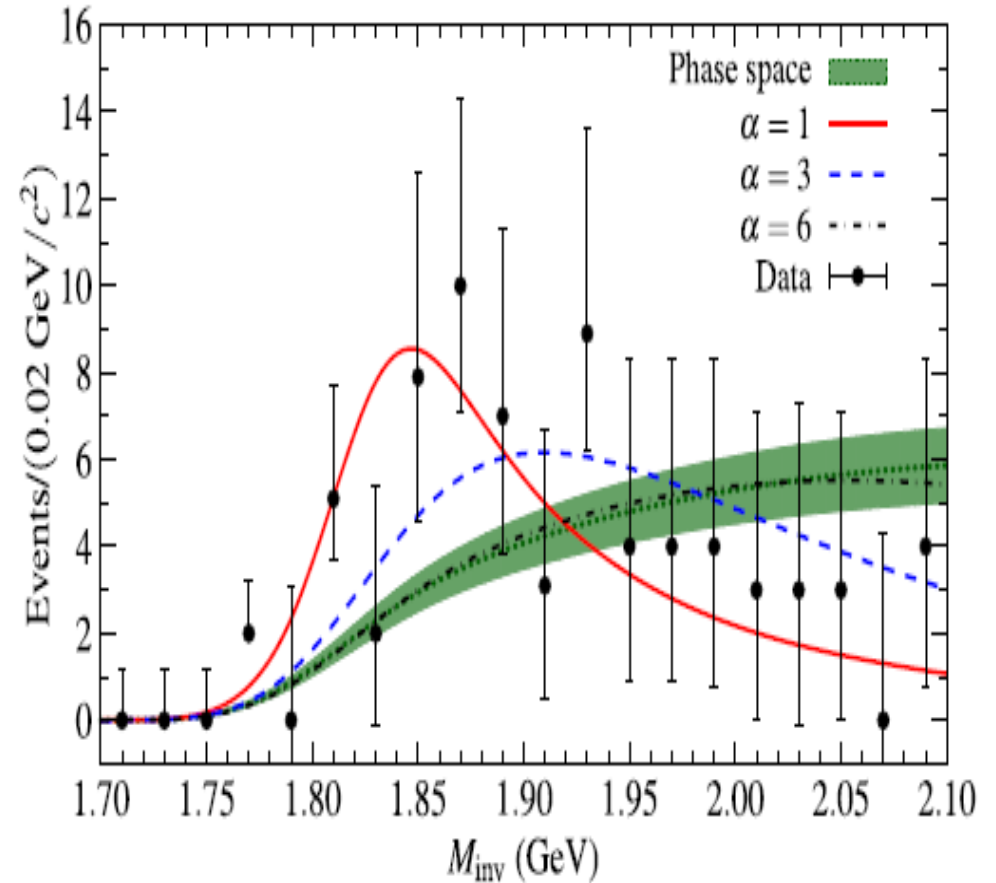
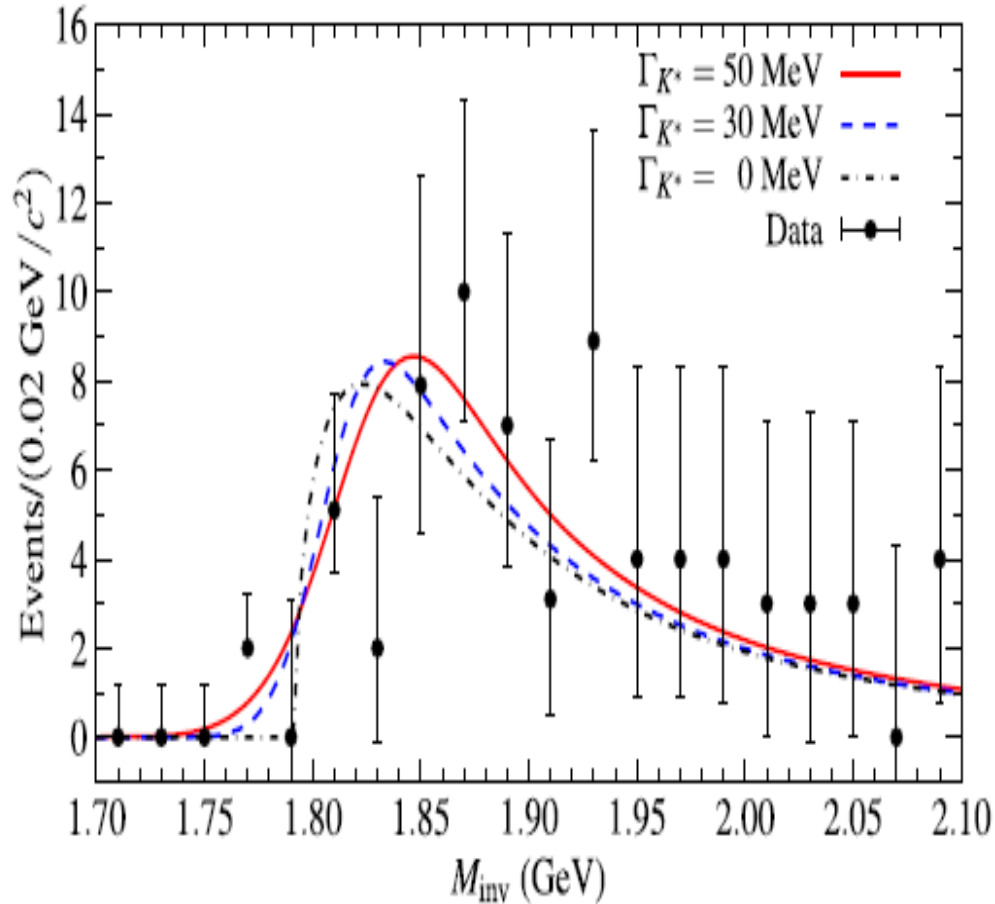
$$\mathcal{N} = \int_{m_-^2}^{m_+^2} dm_1^2 \text{Im} \frac{1}{m_1^2 - m_{K^*}^2 + i\Gamma(m_1^2)m_1},$$

$$\Gamma(m_1^2) = \Gamma_{K^*} \frac{p^3(m_1^2)}{p^3(m_{K^*}^2)}, \quad p(m_1^2) = \frac{\lambda^{1/2}(m_1^2, m_\pi^2, m_K^2)}{2m_1}.$$

$$s = M_{\text{inv}}^2 \quad m_\pm = m_{K^*} \pm 2\Gamma_{K^*}$$

$$\Gamma_{K^*} = 50 \text{ MeV}$$

# More check



$$a(\mu) = -1 \quad g^2 \rightarrow g^2/\alpha$$

Ju-Jun Xie, M. Albaladejo, and E. Oset, PLB728, 319 (2014).

# $f_1(1285)$

 $I^G(J^{PC}) = 0^+(1^{++})$ 

## $f_1(1285)$ DECAY MODES

	Mode	Fraction ( $\Gamma_i/\Gamma$ )	Scale factor/ Confidence level
$\Gamma_1$	$4\pi$	$(33.5^{+2.0}_{-1.8})\%$	S=1.3
$\Gamma_2$	$\pi^0\pi^0\pi^+\pi^-$	$(22.3^{+1.3}_{-1.2})\%$	S=1.3
$\Gamma_3$	$2\pi^+2\pi^-$	$(11.2^{+0.7}_{-0.6})\%$	S=1.3
$\Gamma_4$	$\rho^0\pi^+\pi^-$	$(11.2^{+0.7}_{-0.6})\%$	S=1.3
$\Gamma_5$	$\rho^0\rho^0$	seen	
$\Gamma_6$	$4\pi^0$	$< 7 \times 10^{-4}$	CL=90%
$\Gamma_7$	$\eta\pi^+\pi^-$	$(35 \pm 15)\%$	
$\Gamma_8$	$\eta\pi\pi$	$(52.0^{+1.8}_{-2.1})\%$	S=1.2
$\Gamma_9$	$a_0(980)\pi$ [ignoring $a_0(980) \rightarrow K\bar{K}$ ]	$(38 \pm 4)\%$	
$\Gamma_{10}$	$\eta\pi\pi$ [excluding $a_0(980)\pi$ ]	$(14 \pm 4)\%$	
$\Gamma_{11}$	$K\bar{K}\pi$	$(9.1 \pm 0.4)\%$	S=1.1
$\Gamma_{12}$	$K\bar{K}^*(892)$	not seen	
$\Gamma_{13}$	$\pi^+\pi^-\pi^0$	$(3.0 \pm 0.9) \times 10^{-3}$	
$\Gamma_{14}$	$\rho^\pm\pi^\mp$	$< 3.1 \times 10^{-3}$	CL=95%
$\Gamma_{15}$	$\gamma\rho^0$	$(5.3 \pm 1.2)\%$	S=2.9
$\Gamma_{16}$	$\phi\gamma$	$(7.5 \pm 2.7) \times 10^{-4}$	
$\Gamma_{17}$	$\gamma\gamma^*$		

$$f_1(1285) \quad J^{PC} = 0^+(1^{++})$$

$$\mathcal{L}_{VVPP} = -\frac{1}{4f^2} \text{Tr} ([V^\mu, \partial^\nu V_\mu][P, \partial_\nu P])$$

$$t = \frac{v}{1 - vG}, \quad \text{Pole position : } (1282, -i0) \text{ MeV}$$

$$\text{Coupling : } g_{K^* \bar{K}}^R = (7555, -i0) \text{ MeV}$$

where  $v$  is the  $\bar{K}K^* \rightarrow \bar{K}K^*$  transition potential and  $G$  is the loop function for the propagators of the  $\bar{K}$  and  $K^*$

L. Roca, and E. Oset, “Low lying axial-vector mesons as dynamically generated resonances”, Phys. Rev. D 72, 014002 (2005).

F. Aceti, J.M. Dias and E. Oset, “ $f_1(1285)$  decays into  $a_0(980)\pi_0$ ,  $f_0(980)\pi_0$  and isospin breaking,” Eur. Phys. J. A 51, 48 (2015).

R. Molina, M. Döring and E. Oset, “Determination of the compositeness of resonances from decays: the case of the  $B_0s \rightarrow J/\psi f_1(1285)$ ,” Phys. Rev. D 93, 114004 (2016).

# $f_1(1285)$ in the unitized chiral perturbation theory

TABLE II. Pole positions and couplings in the  $(S, I) = (0, 0)$  channel.

	$h_1(1170)$		$h_1(1380)$		$f_1(1285)$	
	LO	NLO	LO	NLO	LO	NLO
$\sqrt{s}$	$918 - i17$	$925 - i29$	$1244 - i7$	$1257 - i0$	$1286 - i0$	$1289 - i0$
$\frac{1}{\sqrt{2}}(\bar{K}^*K + K^*\bar{K})$	...	...	...	...	$7219 + i0$	$7884 + i0$
$\phi\eta$	$-46 + i13$	$69 - i102$	$-3309 + i47$	$-5963 - i38$	...	...
$\omega\eta$	$-24 + i28$	$711 - i427$	$3019 - i22$	$2642 - i47$	...	...
$\rho\pi$	$3452 - i1681$	$3576 - i1909$	$650 - i961$	$134 - i233$	...	...
$\frac{1}{\sqrt{2}}(\bar{K}^*K - K^*\bar{K})$	$-784 + i499$	$-1488 + i757$	$6137 + i183$	$6435 + i35$	...	...

PHYSICAL REVIEW D **90**, 014020 (2014)

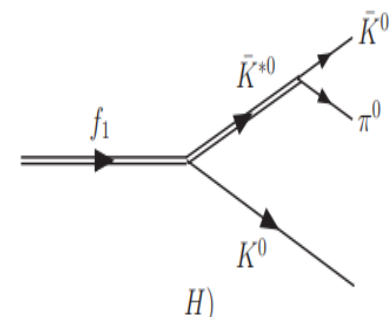
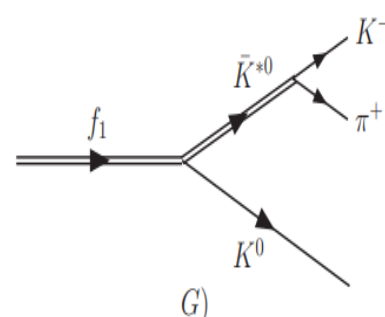
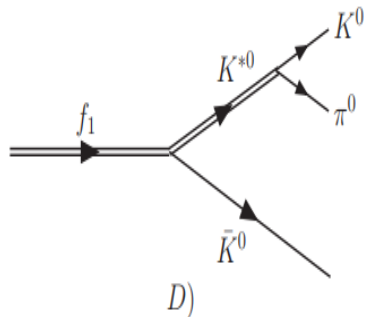
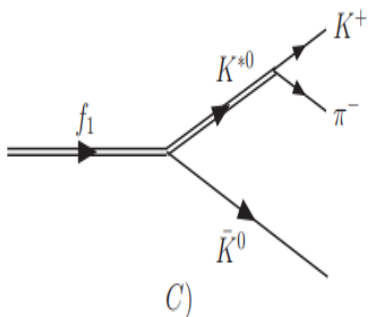
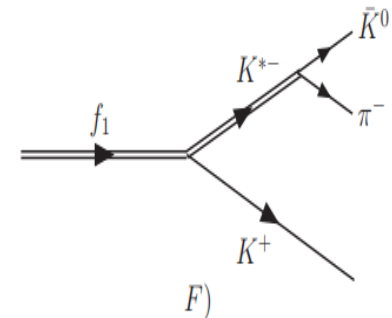
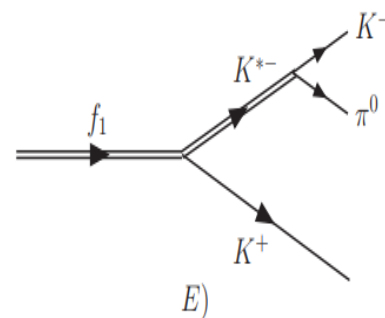
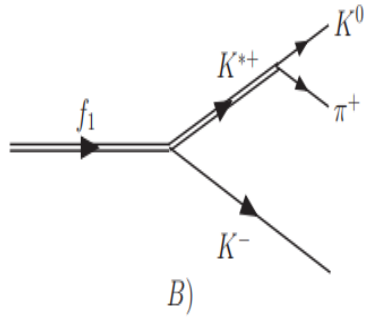
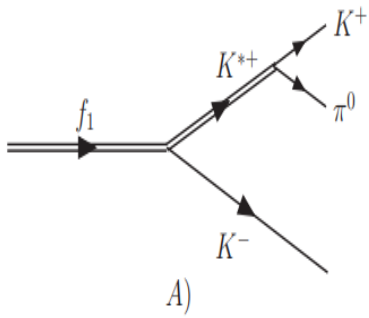
## Pseudoscalar meson and vector meson interactions and dynamically generated axial-vector mesons

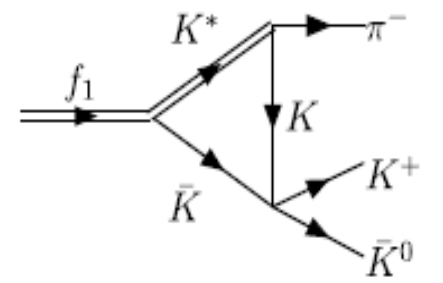
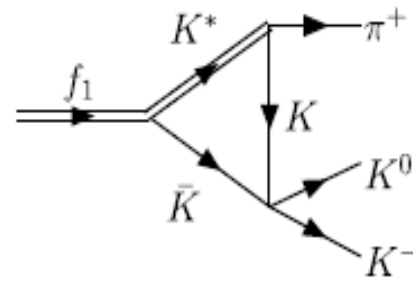
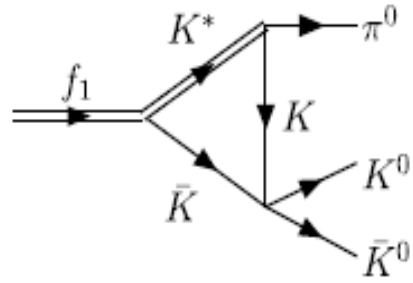
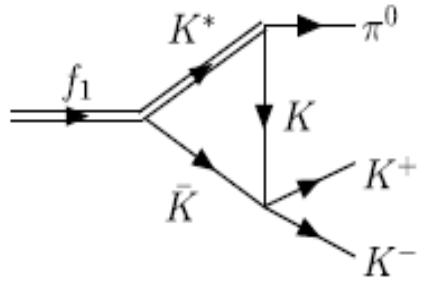
Yu Zhou, Xiu-Lei Ren, Hua-Xing Chen, and Li-Sheng Geng\*

# $f_1(1285) \rightarrow \pi K \bar{K}$ decay

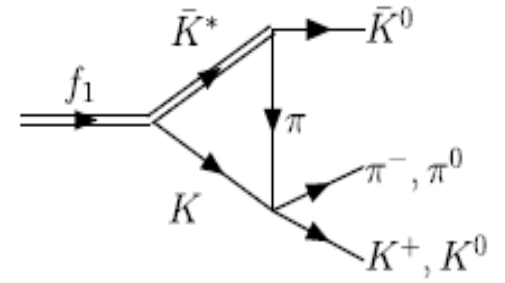
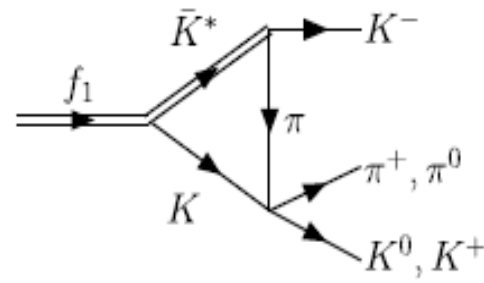
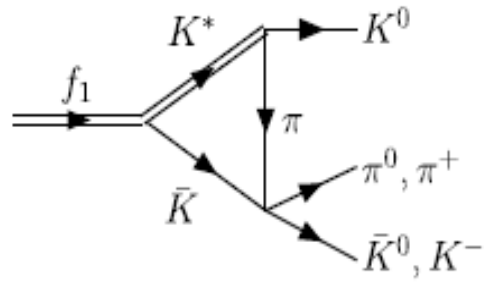
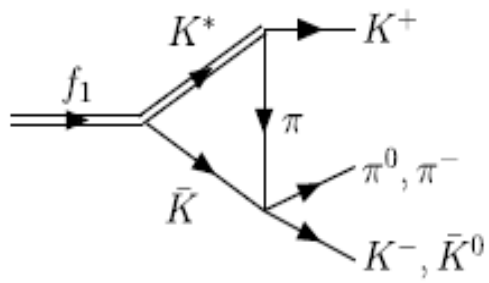
$$\frac{1}{\sqrt{2}}(K^* \bar{K} - \bar{K}^* K) = -\frac{1}{2}(K^{*+} K^- + K^{*0} \bar{K}^0 - K^{*-} K^+ - \bar{K}^{*0} K^0) .$$

We take the convention  $CK^* = -\bar{K}^*$ ,





A)



B)

$$\Gamma = 6 \frac{1}{64\pi^3 M_{f_1}} \int \int d\omega_{K^+} d\omega_{K^-} \sum |M|^2 \times \theta(1 - \cos^2 \theta_{K\bar{K}}) \theta(M_{f_1} - \omega_{K^+} - \omega_{K^-} - m_\pi), \quad (24)$$

where  $M$  is the full amplitude of the process  $f_1(1285) \rightarrow \pi^0 K^+ K^-$  including the FSI's,

$$M = M_{\text{tree}} + M_{\text{FSI}}^{K\bar{K}} + M_{\text{FSI}}^{\pi K}, \quad (25)$$

**Theory**

$$\text{Br}[f_1(1285) \rightarrow \pi K \bar{K}] = (7.2 \sim 7.8)\%$$

**Experiment**

$$\text{Br}[f_1(1285) \rightarrow \pi K \bar{K}] = (9.0 \pm 0.4)\%$$

F. Aceti, J.J. Xie and E. Oset, "The K-bar K pi decay of the f1(1285) and its nature as a K\* K-bar molecule," Phys. Lett. B750, 609 (2015).



# Invariant mass distributions

$$\frac{d\Gamma}{dM_{K^+K^-}} = \frac{M_{K^+K^-}}{64\pi^3 M_{f_1}^2} \int d\omega_{K^+} \overline{\sum} |M|^2 \theta(1 - \cos^2 \theta_{K\bar{K}}) \times \theta(M_{f_1} - \omega_{K^+} - \omega_{K^-} - m_\pi) \theta(\omega_{K^-} - m_K), (31)$$

$$\frac{d\Gamma}{dM_{\pi^0 K^+}} = \frac{M_{\pi^0 K^+}}{64\pi^3 M_{f_1}^2} \int d\omega_{K^+} \overline{\sum} |M|^2 \theta(1 - \cos^2 \theta_{K\bar{K}}) \times \theta(M_{f_1} - \omega_{K^+} - \omega_{K^-} - m_\pi) \theta(\omega_{K^-} - m_K), (32)$$

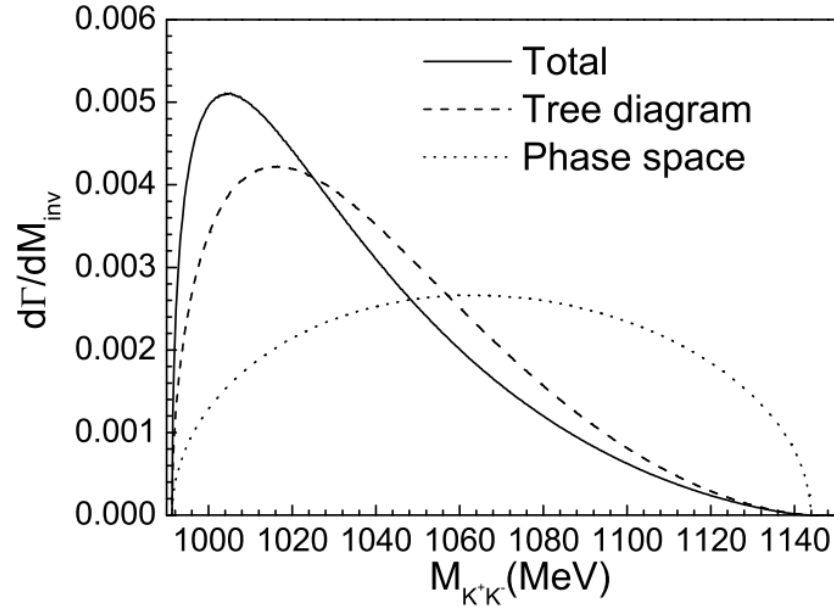


FIG. 3: The mass distribution  $\frac{d\Gamma}{dM_{K^+K^-}}$  for  $f_1(1285) \rightarrow \pi^0 K^+ K^-$  as a function of the invariant mass of the  $K^+ K^-$  system.

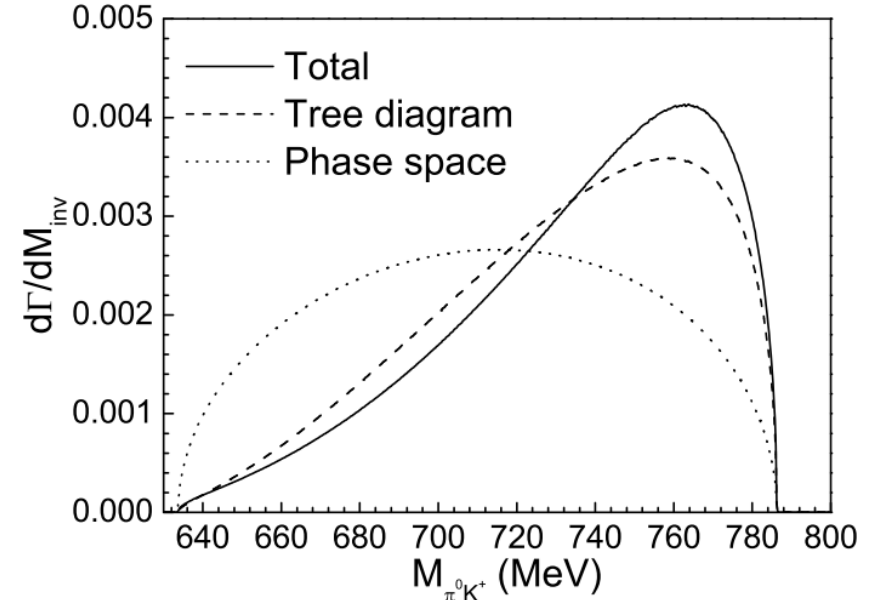
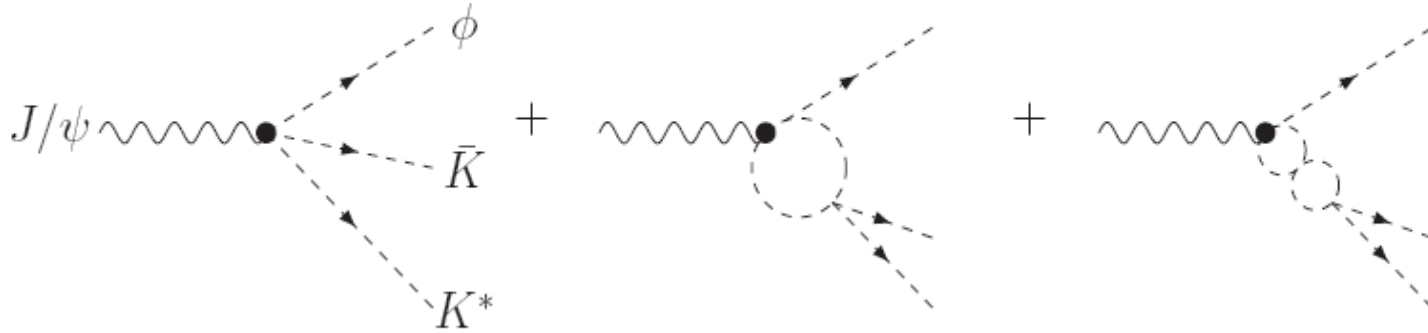


FIG. 4: The mass distribution  $\frac{d\Gamma}{dM_{\pi^0 K^+}}$  for  $f_1(1285) \rightarrow \pi^0 K^+ K^-$  as a function of the invariant mass of the  $\pi^0 K^+$  system.

F. Aceti, J.J. Xie and E. Oset, "The  $K\bar{K}\pi$  decay of the  $f_1(1285)$  and its nature as a  $K^* K\bar{K}$  molecule," Phys. Lett. B750, 609 (2015).

$J/\psi \rightarrow \phi \bar{K} K^*$  and  $J/\psi \rightarrow \phi f_1(1285)$  decays



$$T_{J/\psi \rightarrow \phi \bar{K} K^*} = V_p C_s \left[ 1 + G(M_{\text{inv}}^2) t(M_{\text{inv}}^2) \right]$$

$$= V_p C_s \frac{t(M_{\text{inv}})}{v(M_{\text{inv}})},$$

$$C_s = \epsilon_{ijk} \epsilon_i(J/\psi) \epsilon_j(\phi) \epsilon_k(K^*).$$

$$\overline{\sum \sum} C_s^2 = \frac{2}{3} \left( 3 + \frac{p_\phi^2}{m_\phi^2} + \frac{p_{K^*}^2}{m_{K^*}^2} \right), \quad (4)$$

$$\frac{d\Gamma_{J/\psi \rightarrow \phi \bar{K} K^*}}{dM_{\text{inv}}} = \frac{V_p^2}{(2\pi)^3} \frac{M_{\text{inv}}}{8M_{J/\psi}^3} \left| \frac{t(M_{\text{inv}})}{v(M_{\text{inv}})} \right|^2$$

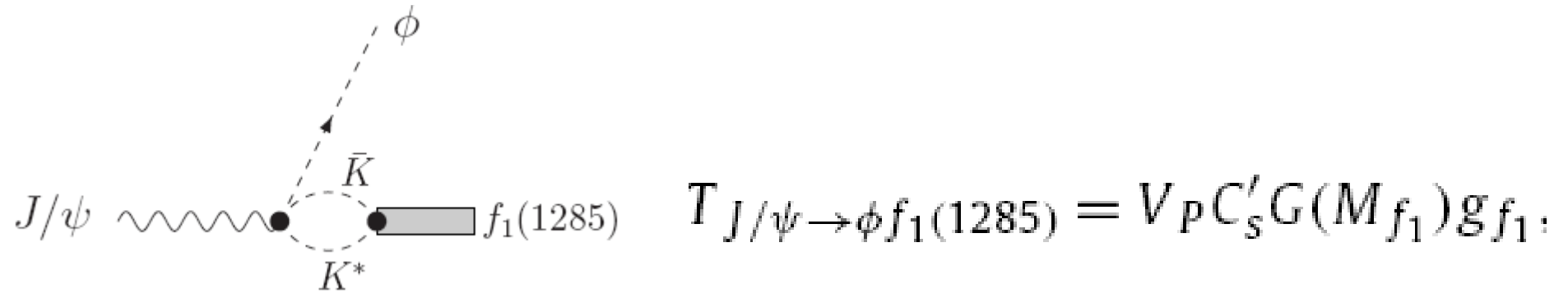
$$\times \int_{M_{\phi \bar{K}}^{\text{min}}}^{M_{\phi \bar{K}}^{\text{max}}} \overline{\sum \sum} C_s^2 M_{\phi \bar{K}} dM_{\phi \bar{K}}.$$

where  $p_\phi$  and  $p_{K^*}$  are the  $\phi$  and  $K^*$  momenta in the  $J/\psi$  rest frame, respectively,

$$p_\phi = \frac{\lambda^{1/2}(M_{J/\psi}^2, m_\phi^2, M_{\text{inv}}^2)}{2M_{J/\psi}}, \quad (5)$$

$$p_{K^*} = \frac{\lambda^{1/2}(M_{J/\psi}^2, m_{K^*}^2, M_{\phi \bar{K}}^2)}{2M_{J/\psi}}, \quad (6)$$

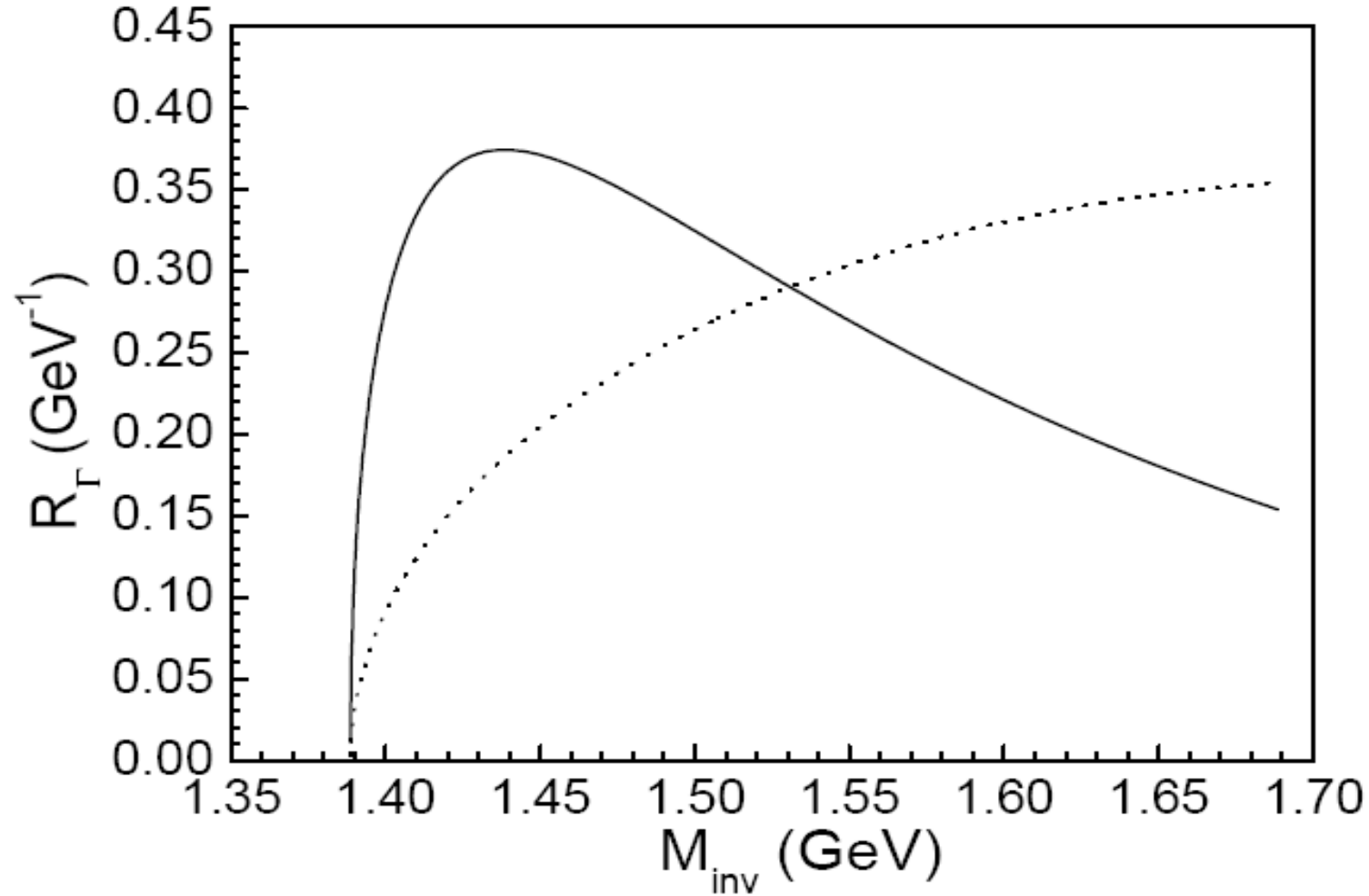
$J/\psi \rightarrow \phi \bar{K} K^*$  and  $J/\psi \rightarrow \phi f_1(1285)$  decays



$$\Gamma_{J/\psi \rightarrow \phi f_1(1285)} = \frac{V_P^2 G^2(M_{f_1}) g_{f_1}^2 p'_\phi}{8\pi M_{J/\psi}^2} \overline{\sum \sum} C_s'^2, \quad C'_s = \epsilon_{ijk} \epsilon_i(J/\psi) \epsilon_j(\phi) \epsilon_k(f_1).$$

$$\overline{\sum \sum} C_s'^2 = \frac{2}{3} \left( 3 + \frac{p'^2_\phi}{M_{f_1}^2} + \frac{p'^2_\phi}{m_\phi^2} \right), \quad p'_\phi = \frac{\lambda^{1/2}(M_{J/\psi}^2, m_\phi^2, M_{f_1}^2)}{2M_{J/\psi}}.$$

$$R_\Gamma = \frac{d\Gamma_{J/\psi \rightarrow \phi \bar{K} K^*} / dM_{\text{inv}}}{\Gamma_{J/\psi \rightarrow \phi f_1(1285)}}.$$



The dotted curve stands for the phase space.

# Summary

- The  $h_1(1800)$  and  $f_1(1285)$  are dynamically generated states from the vector-vector and pseudoscalar-vector interactions.
- More experimental measurements of the  $J/\psi$  decays can be used to study the  $h_1(1800)$  and  $f_1(1285)$  states.

*Thank you very much for  
your attention!*