PYGMY AND GIANT DIPOLE RESONANCES IN ^{48,50}Ca AND ^{68,70}Ni

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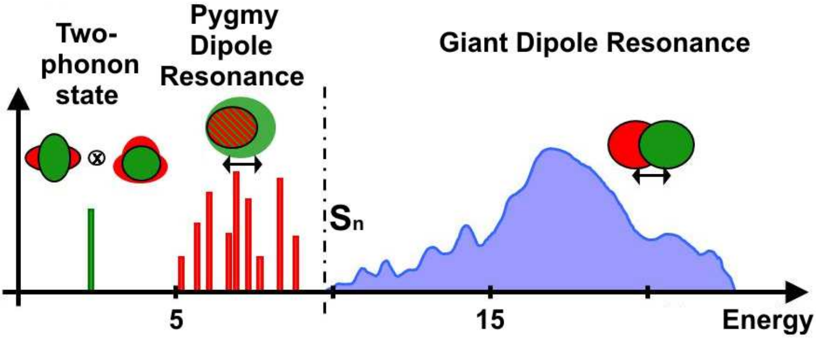
Outline

Introduction

$E1$ strength in (spherical) atomic nuclei



Courtesy: N. Pietralla

Relevance of the PDR

1. The PDR might play an important role in nuclear astrophysics. For example, the occurrence of the PDR could have a pronounced effect on neutron-capture rates in the r-process nucleosynthesis, and consequently on the calculated elemental abundance distribution.

S. Goriely, **Phys. Lett.** B436, **10 (1998)**.

2. The study of the pygmy $E1$ strength is expected to provide information on the symmetry energy term of the nuclear equation of state. This information is very relevant for the modeling of neutron stars.

C. J. Horowitz and J. Piekarewicz, **Phys. Rev. Lett.** 86, **5647 (2001)**.

3. New type of nuclear excitation: these resonances are the low-energy tail of the GDR, or if they represent a different type of excitation, or if they are generated by single-particle excitations related to the specific shell structure of nuclei with neutron excess.

N. Paar, D. Vretenar, E. Khan, G. Colò, **Rep. Prog. Phys.** 70, **691 (2007)**.

MAIN INGREDIENTS OF THE MODEL

Realization of QRPA

We employ the effective Skyrme interaction with the tensor terms in the particle-hole channel

$$V(\vec{r}_1,\vec{r}_2)^C = t_0\left(1+x_0\hat{P}_\sigma\right)\delta(\vec{r}_1-\vec{r}_2)+\frac{t_1}{2}\left(1+x_1\hat{P}_\sigma\right)\left[\delta(\vec{r}_1-\vec{r}_2)\vec{k}^2+\vec{k}'^2\delta(\vec{r}_1-\vec{r}_2)\right]$$

$$+\,t_2\left(1+x_2\hat{P}_\sigma\right)\vec{k}'\cdot\delta(\vec{r}_1-\vec{r}_2)\vec{k}+\frac{t_3}{6}\left(1+x_3\hat{P}_\sigma\right)\delta(\vec{r}_1-\vec{r}_2)\rho^\alpha\left(\frac{\vec{r}_1+\vec{r}_2}{2}\right)$$

$$+\,iW_0\,(\vec{\sigma}_1+\vec{\sigma}_2)\cdot\left[\vec{k}'\times\delta(\vec{r}_1-\vec{r}_2)\right]$$

and

$$V(\vec{r}_1,\vec{r}_2)^T = \frac{T}{2}\left\{\left[(\sigma_1\cdot\vec{k}')(\sigma_2\cdot\vec{k}')-\frac{1}{3}(\sigma_1\cdot\sigma_2)\vec{k}'^2\right]\delta(\vec{r}_1-\vec{r}_2)\right.$$

$$\left.+\,\delta(\vec{r}_1-\vec{r}_2)\left[(\sigma_1\cdot\vec{k})(\sigma_2\cdot\vec{k})-\frac{1}{3}(\sigma_1\cdot\sigma_2)\vec{k}^2\right]\right\}$$

$$+\,U\left\{(\sigma_1\cdot\vec{k}')\delta(\vec{r}_1-\vec{r}_2)(\sigma_1\cdot\vec{k})-\frac{1}{3}(\sigma_1\cdot\sigma_2)\left[\vec{k}'\delta(\vec{r}_1-\vec{r}_2)\vec{k}\right]\right\}.$$

T. H. R. Skyrme, **Nucl. Phys. 9, 615 (1959).**

D. Vautherin and D. M. Brink, **Phys. Rev. C5, 626 (1972).**

Realization of QRPA

The Hamiltonian includes the density-dependent zero-range force in the particle-particle channel.

$$V_{pair}(\vec{r}_1, \vec{r}_2) = V_0 \left(1 - \frac{\rho(r_1)}{\rho_c}\right) \delta(\vec{r}_1 - \vec{r}_2),$$

where $\rho(r_1)$ is the particle density in coordinate space, ρ_c is equal to the nuclear saturation density. The strength V_0 is a parameter fixed to reproduce the odd-even mass difference of nuclei in the studied region.

A. P. Severyukhin, V. V. Voronov, N. V. Giai, **Phys. Rev.** C77, **024322 (2008)**.

The starting point of the method is the HF-BCS calculations of the ground state, where spherical symmetry is assumed for the ground states. The continuous part of the single-particle spectrum is discretized by diagonalizing the HF Hamiltonian on a harmonic oscillator basis.

J. P. Blaizot and D. Gogny, **Nucl. Phys.** A284, **429 (1977)**.

Realization of QRPA

We introduce the phonon creation operators

$$Q^{+}_{\lambda\mu i} = \frac{1}{2}\sum_{jj'}\left(X^{\lambda i}_{jj'}\, A^{+}(jj';\lambda\mu) - (-1)^{\lambda-\mu}\, Y^{\lambda i}_{jj'}\, A(jj';\lambda-\mu)\right),$$

$$A^{+}(jj';\lambda\mu) = \sum_{mm'} C^{\lambda\mu}_{jm\,j'm'}\alpha^{+}_{jm}\alpha^{+}_{j'm'}.$$

The index λ denotes total angular momentum and μ is its z-projection in the laboratory system. One assumes that the ground state is the QRPA phonon vacuum $|0\rangle$ and one-phonon excited states are $Q^{+}_{\lambda\mu i}|0\rangle$ with the normalization condition

$$\langle 0|\left[Q_{\lambda\mu i}, Q^{+}_{\lambda\mu i'}\right]|0\rangle = \delta_{ii'}.$$

Making use of the linearized equation-of-motion approach one can get the QRPA equations

$$\begin{pmatrix} \mathcal{A} & \mathcal{B} \\ -\mathcal{B} & -\mathcal{A} \end{pmatrix}\begin{pmatrix} X \\ Y \end{pmatrix} = \omega \begin{pmatrix} X \\ Y \end{pmatrix}.$$

Solutions of this set of linear equations yield the one-phonon energies ω and the amplitudes X, Y of the excited states.

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P. Ring and P. Schuck, **The Nuclear Many Body Problem (Springer, Berlin 1980).**

Phonon-phonon coupling (**PPC**)

To take into account the effects of the phonon-phonon coupling (PPC) in the simplest case one can write the wave functions of excited states as

$$\Psi_\nu(JM) = \left[\sum_i R_i(J\nu) Q^+_{JMi} + \sum_{\lambda_1 i_1 \lambda_2 i_2} P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu) \left[Q^+_{\lambda_1 \mu_1 i_1} Q^+_{\lambda_2 \mu_2 i_2}\right]_{JM}\right] |0\rangle$$

with the normalization condition

$$\sum_i R_i^2(J\nu) + 2 \sum_{\lambda_1 i_1 \lambda_2 i_2} \left[P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu)\right]^2 = 1.$$

V. G. Soloviev, **Theory of Atomic Nuclei: Quasiparticles and Phonons (Inst. of Phys., Bristol 1992).**

Phonon-phonon coupling (PPC)

Using the variational principle in the form

$$\delta\Big(\langle\Psi_\nu(JM)|\mathcal{H}|\Psi_\nu(JM)\rangle - E_\nu\left[\langle\Psi_\nu(JM)|\Psi_\nu(JM)\rangle - 1\right]\Big) = 0,$$

one obtains a set of linear equations for the unknown amplitudes $R_i(J\nu)$ and $P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu)$:

$$(\omega_{Ji} - E_\nu)R_i(J\nu) + \sum_{\lambda_1 i_1 \lambda_2 i_2} U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji) P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu) = 0;$$

$$\sum_i U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji) R_i(J\nu) + 2(\omega_{\lambda_1 i_1} + \omega_{\lambda_2 i_2} - E_\nu) P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu) = 0.$$

$U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji)$ is the matrix element coupling one- and two-phonon configurations:

$$U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji) = \langle 0|Q_{Ji}\mathcal{H}\left[Q^+_{\lambda_1 i_1} Q^+_{\lambda_2 i_2}\right]_J|0\rangle.$$

These equations have the same form as the QPM equations, but the single-particle spectrum and the parameters of the residual interaction are calculated with the Skyrme forces.

A. P. Severyukhin, V. V. Voronov, N. V. Giai, Eur. Phys. J. A22, 397 (2004).

A. P. Severyukhin, N. N. Arsenyev, N. Pietralla, V. Werner, Phys. Rev. C90, 011306(R) (2014).

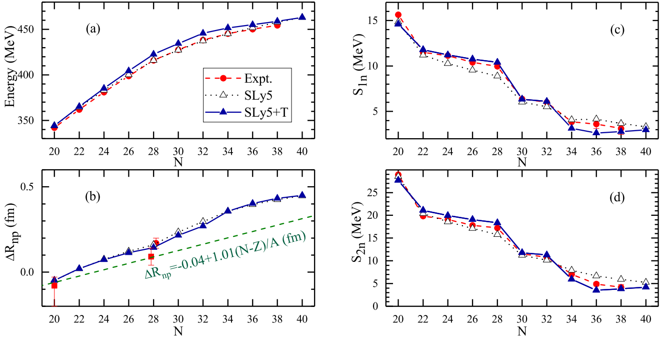
RESULTS AND DISCUSSION

Details of calculations SLy5 vs SLy5+T

We use the Skyrme interactions SLy5 and SLy5+T. The SLy5+T involve the tensor terms added without refitting the parameters of the central interaction (the tensor interaction parameters are α_T=-170 MeV·fm^5 and β_T=100 MeV·fm^5). The pairing strength V_0=-270 MeV·fm^3 is fitted to reproduce the experimental neutron pairing energies near ^{50}Ca.

***E. Chabanat et al.*, Nucl. Phys. A635, 231 (1998).**

***G. Colò et al.*, Phys. Lett. B646, 227 (2007).**



***N. N. Arsenyev et al.*, Phys. Rev. C95, 054312 (2017).**

***M. Wang et al.*, Chin. Phys. C41, 030003 (2017).**

***J. Birkhan et al.*, PRL 118, 252501 (2017).**

***A. Trzcińska et al.*, PRL 87, 082501 (2001).**

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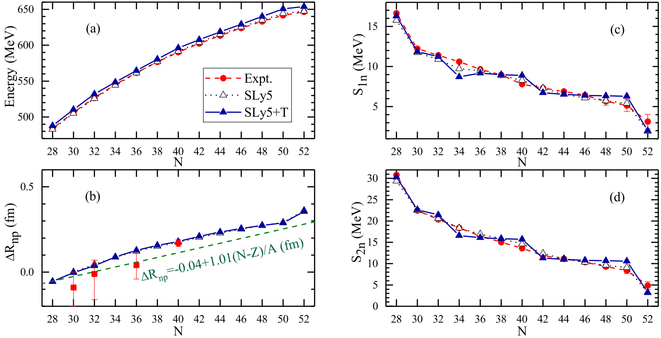
***G. Colò et al.*, Phys. Lett. B646, 227 (2007).**



***N. N. Arsenyev et al.*, in preparation.**

***M. Wang et al.*, Chin. Phys. C41, 030003 (2017).**

***D. M. Rossi et al.*, PRL 111, 242503 (2013).**

***A. Trzcińska et al.*, PRL 87, 082501 (2001).**

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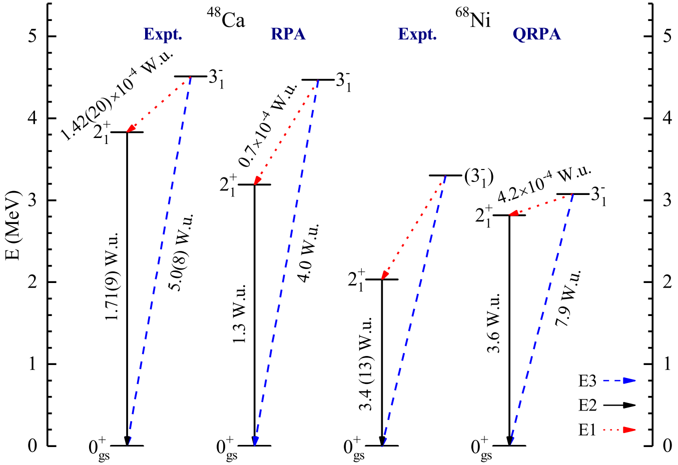
Properties of $[2^+_1]_{\rm QRPA}$ and $[3^-_1]_{\rm QRPA}$ states SLy5



N. N. Arsenyev, A. P. Severyukhin, V. V. Voronov, N. V. Giai, **Phys. Part. Nucl. 50, 528 (2019).**

http://www.nndc.bnl.gov/ensdf/ [14 January (2019)]

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Details of calculations

The dipole transition probabilities can be expressed as

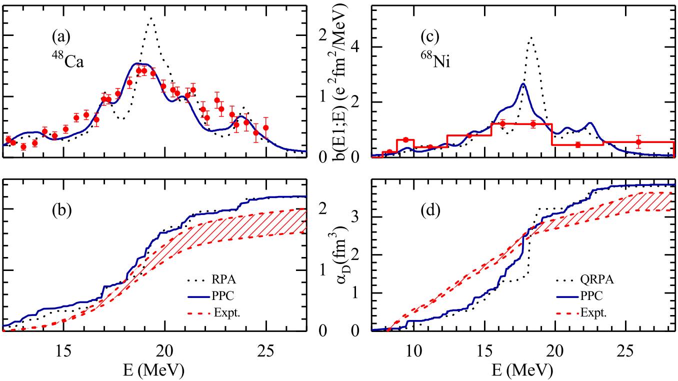
$$B(E1; 0^+_{gs} \to 1^-_i) = \left| e^{(\mathrm{n})}_{eff} \langle i | \hat{M}^{(\mathrm{n})} | 0 \rangle + e^{(\mathrm{p})}_{eff} \langle i | \hat{M}^{(\mathrm{p})} | 0 \rangle \right|^2 ,$$

where $\hat{M}^{(\mathrm{p})} = \sum_i^Z r_i Y_{1\mu}(\hat{r}_i)$ and $\hat{M}^{(\mathrm{n})} = \sum_i^N r_i Y_{1\mu}(\hat{r}_i)$. The spurious 1^- state is excluded from the excitation spectra by introduction of the effective neutron $e^{(\mathrm{n})}_{eff} = -Z/A\, e$ and proton $e^{(\mathrm{p})}_{eff} = N/A\, e$ charges.

A. Bohr and B. Mottelson, **Nuclear Structure Vol. II (Benjamin, New York 1975).**

To construct the wave functions of the 1^- states, in the present study we take into account all two-phonon terms that are constructed from the phonons with multipolarities $\lambda \leq 5$. All dipole excitations with energies below 35 MeV and 15 most collective phonons of the other multipolarities are included in the wave function. We have checked that extending the configuration space plays a minor role in our calculations.

$E1$ strength distributions of ^{48}Ca and ^{68}Ni SLy5



$$\alpha_{\mathrm{D}} = \frac{\hbar c}{2\pi^2 e^2} \int \frac{\sigma_\gamma(\omega)}{\omega^2} d\omega$$

N. N. Arsenyev, A. P. Severyukhin, V. V. Voronov, N. V. Giai, Phys. Part. Nucl. 50, 528 (2019).

J. Birkhan et al., PRL 118, 252501 (2017).

D. M. Rossi et al., PRL 111, 242503 (2013).

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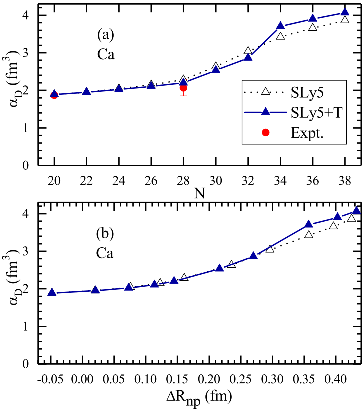
The dipole polarizability α_D SLy5 vs SLy5+T



Inclusion of the tensor components does not change the value of α_D.

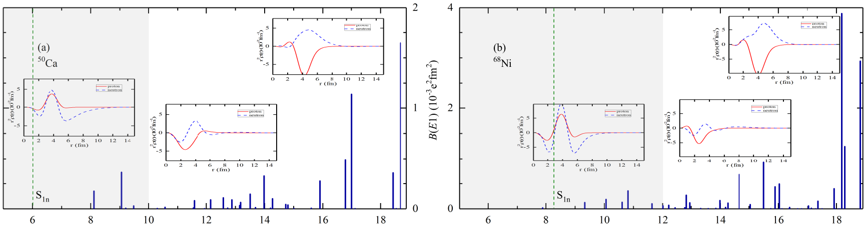
	$\alpha_D(^{48}Ca)$
SLy5:	2.28
SLy5+T:	2.20
Piekarewicz:	2.306(89)
Hagen:	2.19÷2.60
Expt.:	2.07(22)

N. N. Arsenyev, A. P. Severyukhin, V. V. Voronov, N. V. Giai, Phys. Rev. C95, 054312 (2017).

J. Piekarewicz et al., Phys. Rev. C85, 041302(R) (2012).

G. Hagen et al., Nature Phys. 12, 186 (2016).

$E1$ strength distributions of ^{50}Ca and ^{68}Ni SLy5



N. N. Arsenyev, A. P. Severyukhin, V. V. Voronov, N. V. Giai, in preparation.

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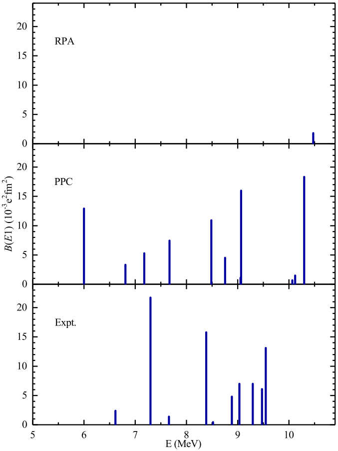
Low-energy 1^- distributions: ^{48}Ca SLy5



The dominant contribution in the wave function of the 1^- states comes from the two-phonon configurations ($>$ 60%). These states originate from the fragmentation of the RPA states above 10 MeV.

	$\sum B(E1;\uparrow)$	$\sum EB(E1;\uparrow)$
SLy5:		
RPA	0.00	0.00
PPC	0.06	0.50
Kamerdzhiev:	0.071	0.509
Egorova:	0.10	0.95
Expt.:	0.0687(75)	0.570(62)

N. N. Arsenyev, A. P. Severyukhin, V. V. Voronov, N. V. Giai, Phys. Rev. C95, 054312 (2017).

T. Hartmann et al. PRL 93, 192501 (2004).

I. A. Egorova and E. Litvinova, Phys. Rev. C94, 034322 (2016).

Properties of the PDR SLy5

Nucleus	$\sum B(E1;\uparrow)$ (e^2fm^2)			f_{PDR} (%)		
	(Q)RPA	PPC	Expt.	(Q)RPA	PPC	Expt.
^{48}Ca	0.00	0.06	0.08(1)	0.00	0.28	0.33(4)
^{50}Ca	0.57	0.58	–	2.81	2.82	–
^{68}Ni	0.97	1.10	–	4.20	4.61	5.0(15)
^{70}Ni	1.06	1.15	3.26(54)	4.54	4.62	6.3(11)

$$f_{\rm PDR} = \frac{\sum_{k}^{\leq 10(12)\ MeV} E_{1_k^-} B(E1; 0_{gs}^+ \to 1_k^-)}{14.8NZ/A\ e^2{\rm fm}^2{\rm MeV}}$$

***N. N. Arsenyev, A. P. Severyukhin, V. V. Voronov, N. V. Giai*, in preparation.**

***J. Birkhan et al.*, PRL 118, 252501 (2017).**

***D. M. Rossi et al.*, PRL 111, 242503 (2013).**

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Conclusions

Starting from the Skyrme mean-field calculations, the properties of the electric dipole strength in neutron-rich Ca and Ni isotopes are studied by taking into account the coupling between one- and two-phonons terms in the wave functions of excited states.

The electric dipole polarizability α_{D} is a particularly important observable, as it can be measured in finite nuclei and it provides important information on the neutron skin thickness that can be extracted. It is shown that the PPC and the tensor components have small influence on the dipole polarizability.

For ^{48}Ca and ^{68}Ni, the PPC effect have a damping and smoothing action which yields a GDR strength distribution close to the experimental one in shape and magnitude. We find the strong increase of the summed $E1$ strength below 10 MeV [$\sum B(E1;\uparrow)$], with increasing neutron number from ^{48}Ca until ^{50}Ca. The comparison of the ^{68}Ni and ^{70}Ni summed strength values, in particular if integrated in the region below 12 MeV, shows an insignificant increase with neutron number.

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THE END