Multi-photon regime of non-linear quantum processes in short polarized electromagnetic pulses

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Content

relevance of the topic

Relevance of the topic

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Hungary	ELI
Germany	XFEL (DESY), OL+EC (HZDR Dresden)

Spontaneous vacuum rebuilding (breaking) in strong & constant electric field (Schwinger effect)

Predicted on the quality level
F. Sauter (1931).
W. Heisenberg & H. Euler (1936).

Theoretically justified by

J. Schwinger (1951).

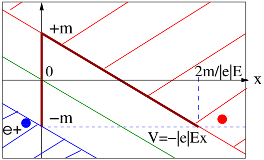
Julian Schwinger

The vacuum rebuilding is manifested in spontaneous e^+e^- pair production in strong e.m. field with production rate:

$$w \simeq (\alpha E^2/\pi^2)\exp[-\pi E_{\rm cr}/E],$$

where E is the electric field strength and $E_{\rm cr} = m^2/e$ is the critical field.

Schwinger formula



Qualitatively vacuum rebuilding may be understood as a positron tunnel penetrating through the triangle electric field barrier $w \propto \exp[-2\int_{x_1}^{x_2} dx\,|p(x)|]$ with $p^2 = (\mathcal{E} - |e|Ex)^2 - m^2$ and $x_{1,2} = \mathcal{E} \mp m/|e|E$.
Using substitution $y = \mathcal{E} - |e|E/m$ one gets

$$w \propto \mathrm{e}^{-\frac{4m^2}{eE}\int_0^1 dy\sqrt{1-y^2}} = \mathrm{e}^{-\pi\frac{m^2}{eE}} = \mathrm{e}^{-\pi\frac{E_{\rm cr}}{E}}$$

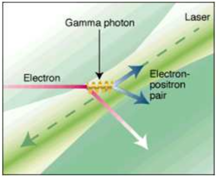
Value of the critical field:
$E_{\rm cr} = \frac{m^2}{e} = \frac{0.511^2}{1.973}\,10^{17}\mathrm{V/cm} \simeq 1.32\cdot 10^{16}\mathrm{V/cm}$ $E_{\rm atom} \sim 10^9\mathrm{V/cm}$

$E_{\rm cr}$ generated by laser ??
Required $I_{cr} = c\frac{E_{cr}^2}{\hbar c} = 4.06\cdot 10^{29}\mathrm{W/cm^2}$
present $I \simeq 10^{22}\mathrm{W/cm^2} \to 10^{24\ldots25}\mathrm{W/cm^2}$

turn to dynamically assisted e^+e^- production

E-144 SLAC experiment

SLAC (E-144) experiment D. Burke *et al.*, PRL **79** (1997)
$\gamma' + L \to e^+e^-$ - generalized Breit-Wheeler process



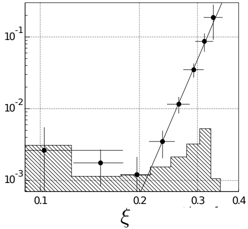
Kinematics of BW -process

$$s_{\rm thr} = (2m)^2 = (k+k')^2 = 4\omega\omega' \qquad \omega'_{\rm thr} = \frac{m^2}{\omega} = \frac{0.26\cdot 10^{12}\,({\rm eV}^2)}{2.35\,{\rm eV}} \simeq 111 \text{ GeV}$$

$$\omega_{\rm SLAC} \simeq 29 \text{ GeV} \Longrightarrow \frac{\omega_{\rm thr}}{\omega_{\rm SLAC}} \simeq 3.8$$

$$\gamma' + n\gamma \to e^+e^- \Longrightarrow n_{\rm min} \geq 4$$ *essentially multi-photon process*

positron yield



reduced e.m. field intensity ξ and beam power I

$$\xi^2 = \frac{0.56}{(\omega\,({\rm eV}))^2} \times 10^{-18}\, I\,({\rm W/cm^2})$$

$$I_{\rm SLAC} \sim 2\cdot 10^{18}{\rm W/cm^2} \Longrightarrow \xi^2 \sim 0.1$$

consequences of E-144 experiment

1. e^+e^- yield increases with pulse intensity. Expected facilities would reach $I \sim 10^{24...25}$ W/cm^2
2. Pulse duration would be small (even sub-cycle pulses with number of oscillations in a pulse < 1) The role of the pulse shape for BW e^+e^- production in case of circularly polarized pulses was analyzed

A.T., B. Kämpfer, A. Hosaka et al. PRL, **108***, 240406 (2012), PRA* **87***, 042106 (2013), Phys.Part.Nucl.,***47***, 456 (2016)*

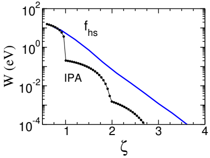
Probability of e^+e^- production as a function of subthreshold parameter $\zeta = \frac{4m^2}{s}$

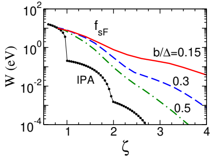
one − parameter shape

$$f_{\rm hs}(\phi) = \frac{1}{\cosh[\phi/\Delta]}$$

two − parameter shape

$$f_{\rm sF}(\phi) = \frac{\cosh[\Delta/b]+1}{\cosh[\Delta/b]+\cosh[\phi/b]} .$$





where Δ is the pulse duration, b ramping parameter (thickness). The threshold parameter $\zeta = \frac{4m^2}{s}$. Region $\zeta > 1$ $(s < 4m^2)$. corresponds to the subthreshold (multi-photon) regime; *IPA* means infinite pulse (plane wave) approximation (*Ritus & Nikishov (1962)*).

Selected topics of the theory of non-linear quantum processes were considered in Refs.

A.T., B.Kämpfer et al. PRD **98** *(2018),* **93** *(2016)* **83** *(2011); J.Phys.Plasma* **82** *(2016), PEPAN* **47** *(2016), EPJD* **68** *(2014), and some others.*
For details see arXive: 1907.00643

Open questions:

Simultaneous analysis of Breit-Wheeler and crossed Compton scattering processes.

The effect of beam polarization on observables.

Background e.m. field

Below we suppose the external electric field (laser pulse) is determined by the electromagnetic (e.m.) four-potential in the axial gauge $A = (0, A)$ as

$$E = -\frac{\partial A}{\partial t},$$

e.m. potential

Circular pol.

$$A^{(circ)}(\phi) = f(\phi)\ [a_1 \cos(\phi + \phi_{CEP}) + a_2 \sin(\phi + \phi_{CEP})]$$

Lin. pol.

$$A^{(lin)}(\phi) = f(\phi)\, a_1 \cos(\phi + \phi_{CEP})\ ,$$

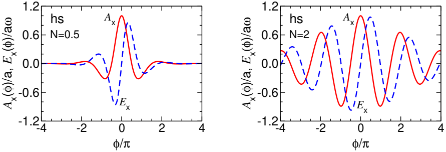
$\phi = k \cdot x$ is the invariant phase, $f(\phi)$ is the envelope function, ϕ_{CEP} is the carrier envelope phase

the hyperbolic secant

$$f(\phi) = \frac{1}{\cosh(\frac{\phi}{\Delta})}$$

$$\Delta = \pi N$$

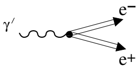
pulse width $= 2\Delta = 2\pi \cdot N$



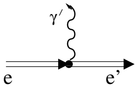
Furry picture

The interaction of charged particles with background field is considered in Furry picture

BW process (originally it is $\gamma' + \gamma \to e^+ + e^-$)



Coulomb scattering (originally it is $\gamma' + e \to \gamma' + e'$)



Volkov solution

$$\psi_p(\phi) = \left[1 + \frac{e(\gamma\cdot k)(\gamma\cdot A)}{2(k\cdot p)}\right]\frac{u_{p'}}{\sqrt{2p_0}}\mathrm{e}^{-ip\cdot x}\exp\left[-i\int\limits_{-\infty}^{\phi}\left(\frac{e(p\cdot A)}{(k\cdot p)} - \frac{e^2A^2}{2(k\cdot p)}\right)d\phi'\right]$$

$$S_{fi} = -ie\int d^4x\langle f|\gamma\cdot\varepsilon(k')|i\rangle\,\frac{\mathrm{e}^{-ik\cdot x}}{\sqrt{2\omega'}}$$

$$M_{fi}(kx) = \int\limits_{-\infty}^{\infty} d\ell\mathrm{e}^{-i\ell kx}\,M_{fi}(\ell) \Rightarrow S_{fi} = \int\limits_{\ell_{\min}}^{\infty} d\ell M_{fi}(\ell)\,\delta^4(k' + \ell k - p_{e^-} - p_{e^+})$$

$\ell\,\omega$ *is the energy of the pulse involved into the process*

Cross section of BW e^+e^- pair production

$$\frac{d\sigma^{(i)}}{d\phi_e} = \frac{\alpha^2\,\zeta}{4m^2\xi^2 N_0^{(i)}} \int\limits_{\ell_{\min}}^{\infty} d\ell \int\limits_{-1}^{1} v\, d\cos\theta_e\, w^{(i)}(\ell)$$

important: $\ell_{\min} = \zeta = \frac{4m^2}{s}$; v is the electron velocity, θ_e and ϕ_e are polar and angular vectors of electron, respectively, index $i = circ.$ or $lin.$ pol.

Normalization factors read

$$N_0^{(circ)} = \frac{1}{2\pi} \int\limits_{-\infty}^{\infty} d\phi \left(f^2(\phi) + f'^2(\phi)\right), \qquad N_0^{(lin)} = \frac{1}{2\pi} \int\limits_{-\infty}^{\infty} d\phi \left(f^2(\phi) + f'^2(\phi)\right) \cos^2\phi$$

$w_\ell^{(i)}$ is the partial probability

Partial probabilities and basis functions

Circular pol.

$$w^{(circ)}(\ell) = 2|\widetilde{Y}_\ell(z)|^2 + \xi^2(2u-1) \times \left(|Y_{\ell-1}(z)|^2 + |Y_{\ell+1}(z)|^2 - 2\mathrm{Re}\,(\widetilde{Y}_\ell(z)X^*_\ell(z))\right)$$

with

$$Y_\ell(z) = \frac{1}{2\pi}\mathrm{e}^{-i\ell(\phi_e-\phi_{CEP})}\int\limits_{-\infty}^{\infty} d\phi\, f(\phi)\,\mathrm{e}^{i\ell\phi - i\mathcal{P}^{(circ)}(\phi)},$$

$$X_\ell(z) = \frac{1}{2\pi}\mathrm{e}^{-i\ell(\phi_e-\phi_{CEP})}\int\limits_{-\infty}^{\infty} d\phi\, f^2(\phi)\,\mathrm{e}^{i\ell\phi - i\mathcal{P}^{(circ)}(\phi)},$$

$$\widetilde{Y}_\ell(z) = \frac{z}{2l}\left(Y_{\ell+1}(z) + Y_{\ell-1}(z)\right) - \xi^2\frac{u}{u_l}X_\ell(z),$$

$$\mathcal{P}^{(circ)}(\phi) = z\int\limits_{-\infty}^{\phi} d\phi'\ \cos(\phi' - \phi_e + \phi_{CEP})f(\phi') - \xi^2\frac{\ell u}{u_\ell}\int\limits_{-\infty}^{\phi} d\phi'\ f^2(\phi').$$

Linear pol.

$$\frac{1}{2}\,w(\ell)^{(lin)} = |\widetilde{A}_0(\ell)|^2 + \xi^2(2u-1) \times \left(|\widetilde{A}_1(\ell)|^2 - \mathrm{Re}[\widetilde{A}_0(\ell)\widetilde{A}_2(\ell)]\right)$$

with

$$\widetilde{A}_m(\ell) = \frac{1}{2\pi}\int\limits_{-\infty}^{\infty} d\phi\,\cos^m(\phi+\tilde{\phi})\,\mathrm{e}^{i\ell\phi - i\mathcal{P}^{(lin)}(\phi)}$$

$$\mathcal{P}^{(lin)}(\phi) = \tilde{\alpha}(\phi) - \tilde{\beta}(\phi),$$

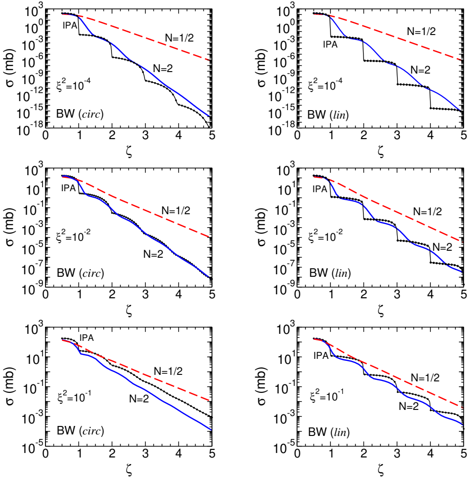
$$\tilde{\alpha}(\phi) = z\cos\phi_e\int\limits_{-\infty}^{\phi} d\phi' f(\phi')\cos(\phi' + \phi_{CEP}),$$

$$\tilde{\beta}(\phi) = \frac{u\ell\xi^2}{u_\ell}\int\limits_{-\infty}^{\phi} d\phi' f^2(\phi')\cos^2(\phi' + \phi_{CEP}).$$

$$u = (kk')/4(kp)(kp'),\ u_\ell = \ell/\zeta,\ z = 2\ell\xi((u/u_\ell)(1-(u/u_\ell))^{1/2}$$

The total cross sections as a function of threshold parameter ζ

Circular pol. *Linear pol.*



Comparison with asymptotic prediction for $\xi^2 \gg 1$

At finite ξ^2 the cross sections decrease almost exponentially with increasing ζ:

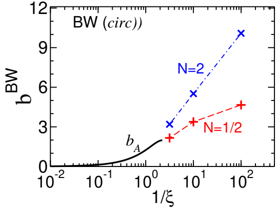
$$\sigma^{(i)} \propto \exp[-b^{(i)}\,\zeta],$$

where the slopes $b^{(i)}$ depend on the pulse duration and field intensity ξ^2.

Asymptotic expressions: *IPA(Ritus (1979)), FPA (A.T., Kämpfer et al. (2013))*

$$\sigma = \left(\frac{3}{2}\right)^{\frac{3}{2}} \frac{\pi\alpha^2}{2m^2\xi}\, d(\xi,\zeta)\, \exp[-\frac{4}{3}\frac{\zeta}{\xi}\left(1-\frac{1}{15\xi^2}\right)]$$

$$d(\xi,\zeta) = 1 + \frac{\xi}{6\zeta}\left(1+\frac{\xi}{8\zeta}\right) + \mathcal{O}(\frac{\xi}{\zeta})\,.$$



Comparison of static Schwinger mechanism and dynamically assisted BW process

$$W_{\rm Sch.} \propto \exp[-\pi \frac{E_{\rm cr}}{E}]$$

$$W_{\rm BW}^{A} \propto \exp[-\frac{4}{3}\frac{\zeta}{\xi}]$$

Using identities $\zeta = \frac{4m^2}{s_1} = \frac{4m^2}{4\omega\omega'}$, one gets $E = \omega a$, $E_{\rm cr} = \frac{m^2}{e}$, and $\xi = \frac{ma}{E_{\rm cr}} = \frac{m}{\omega}\frac{E}{E_{\rm cr}}$,

$$W_{\rm BW}^{A} \propto \exp[-\frac{4}{3}\frac{m}{\omega'}\frac{E_{\rm cr}}{E}] \, .$$

Taking into account that (for example in SLAC(E-144)) experiment $\omega = 2.35$ eV, $\omega' = 111$ GeV, one obtains as following

$$W_{\rm BW}^{A} \propto \exp[-6.14 \times 10^{-6}\frac{E_{\rm cr}}{E}] \, .$$

So, the large ratio $E_{\rm cr}/E$ is compensated by the dynamical factor m/ω'.

Azimuthal angle distributions

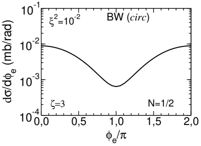
Circ. pol.

i. Infinite pulse (IPA).
Background field is symmetric with respect to rotation in the azimuthal plane which leads to isotopic distributions.

ii. The finite pulse (FPA).
The azimuthal angle symmetry is broken. Basis functions $Y_(\ell)$, (X_ℓ) $\propto \exp[i\Psi](\phi)$ with

$$\Psi(\phi) = \ell\phi - z\cos\phi_e \int\limits_{-\infty}^{\phi} d\phi'\, f(\phi')\cos\phi',$$

which leads to maxima at the points $\phi_e = 0,\ 2\pi$, and minima at $\phi_e = \pi$.



Lin. pol.

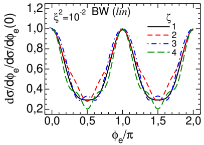
i. Infinite pulse (IPA).
The direction of the beam polarization vector determine the angular distributions

$$A(n) \sim \frac{1}{2\pi}\int\limits_{-\pi}^{\pi} d\phi\, e^{in\phi - iz\cos\phi_e \sin(\phi) + \beta\sin(2\phi)}$$

$$\propto \sum_k \frac{(-z\cos\phi_{e'})^k}{2^k k!}\int\limits_{-\pi}^{\pi} d\phi\, e^{in\phi}\left(e^{i\phi} - e^{-i\phi}\right)^k.$$

$$\frac{d\sigma^{(lin)}}{d\phi_e} \propto \cos^{2n_{\min}}\phi_e$$

which leads to maxima at the points $\phi_e = 0,\ \pi$, and 2π and minima at $\phi_e = \pi/4$ and $3\pi/4$.



Azimuthal angle ϕ_e and ϕ_{CEP}

Circ. pol.

The phase factor is modified

$$\Psi(\phi) = \ell\phi - z\cos(\phi_e - \phi_{CEP}) \int_{-\infty}^{\phi} d\phi' \, f(\phi') \cos\phi',$$

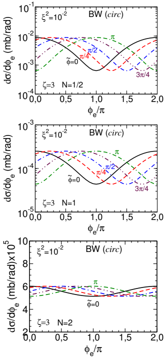
which leads to maxima at the points $\phi_e - \phi_{CEP} = 0,\ 2\pi$,

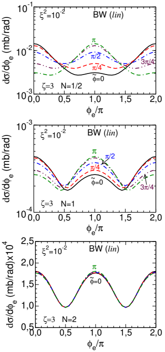
and minima at $\phi_e - \phi_{CEP} = \pi$.

Lin. pol.

The phase factor in basis functions $\widetilde{A}_\ell$

$$e^{i(\ell - z\cos\phi_e \cos\phi_{CEP}\, f(0))\phi}\ ,$$





The cross section

$$\sigma^{(i)}(\kappa) = \int\limits_{\ell_{\min}}^{\infty} d\ell \int\limits_{-1}^{1} d\cos\theta' \frac{d\sigma_\ell^{(i)}}{d\cos\theta'} \quad \text{with} \quad \frac{d\sigma_\ell^{(i)}}{d\cos\theta'} = \frac{\alpha^2}{\xi^2 (p\cdot k) N_0^{(i)}} \frac{\omega'^2}{k\cdot p} \int\limits_0^{2\pi} d\phi_{e'}\, w^{(i)}(\ell)$$

where $\ell_{\min} = \kappa$ is the dynamic parameter and ω' is the frequency of the outgoing photon

$$\omega'_\ell = \frac{\ell\,\omega(E + |\mathbf{p}|)}{E + |\mathbf{p}|\cos\theta' + \ell\omega(1-\cos\theta')} \,.$$

Partial probabilities

Circular pol.

$$w^{(circ)}(\ell) = -2|\widetilde{Y}_\ell(z)|^2 + \xi^2\left(1 + \frac{u^2}{2(1+u)}\right) \times \left(|Y_{\ell-1}(z)|^2 + |Y_{\ell+1}(z)|^2 - 2\mathrm{Re}\,(\widetilde{Y}_\ell(z) X_\ell^*(z))\right).$$

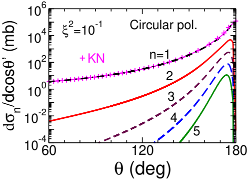
$u = (kk')/(kp')$, $u_\ell = \ell(kp)/m^2$

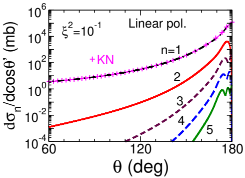
$z = 2\ell\xi((u/u_\ell)(1-(u/u_\ell))^{1/2}$

Linear pol.

$$\frac{1}{2}\, w^{(lin)}(\ell) = -|\widetilde{A}_0(\ell)|^2 + \xi^2\left(1 + \frac{u^2}{2(1+u)}\right) \times \left(|\widetilde{A}_1(\ell)|^2 - \mathrm{Re}\widetilde{A}_0(\ell)\widetilde{A}_2(\ell)\right),$$

Differential cross section $\frac{d\sigma_n^{(i)}}{d\cos\theta'}$ in *IPA*





for $E_e = 4$ MeV and $\omega = 1.55$ eV cross section has a maximum at $\theta' = theta'_0 \simeq 175^o$

$$\frac{\omega'(\ell = \kappa, \theta'_0)}{\omega'(\ell = 1, \theta_0)} = \kappa \, \frac{1}{1 + \delta(\kappa - 1)} ,$$

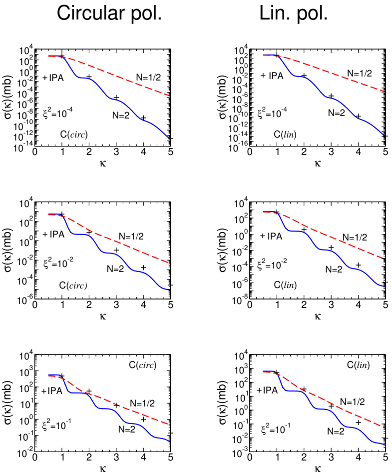
where for the chosen kinematics, $\varepsilon \simeq 3.85 \times 10^{-7}$ and $\delta \simeq 6.46 \times 10^{-5}$, which leads to an approximate equality

$$\omega'(\ell = \kappa) \cong \kappa\omega'(\ell = 1)$$

that holds with an accuracy of $(0.65 \cdots 3.9) \times 10^{-4}$ for $\kappa = 2 \cdots 7$.

In order to isolate multi-photon events, one has to install a detector at fixed polar angle θ' and register only such photons with the frequencies higher than ω'_κ (or $\omega' \geq \omega'_\kappa$) with $\kappa > 1$.

The total cross sections



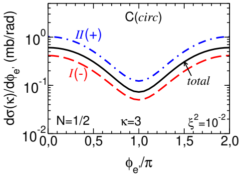
$$\sigma(\kappa) \propto \exp[-b^C(\xi, N)\,\kappa]$$

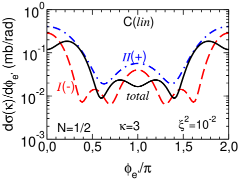
Azimuthal angle distributions

Destructive interference in partial probabilities

$$w^{(circ)}(\ell) = -2|\widetilde{Y}_\ell(z)|^2 + \xi^2 \left(1 + \frac{u^2}{2(1+u)}\right) \times \left(|Y_{\ell-1}(z)|^2 + |Y_{\ell+1}(z)|^2 - 2\mathrm{Re}\left(\widetilde{Y}_\ell(z) X_\ell^*(z)\right)\right).$$

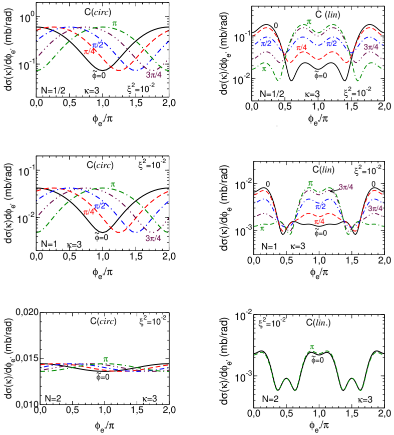
$$\frac{1}{2}\, w^{(lin)}(\ell) = -|\widetilde{A}_0(\ell)|^2 + \xi^2 \left(1 + \frac{u^2}{2(1+u)}\right) \times \left(|\widetilde{A}_1(\ell)|^2 - \mathrm{Re}\widetilde{A}_0(\ell)\widetilde{A}_2(\ell)\right),$$





Interplay of azimuthal angle $\phi_{e'}$ and ϕ_{CEP}

Circular pol. | Lin. pol.



Interplay of destructive interference of different terms plus phase factors

Circular polarization:

$$e^{i(\ell - z\cos(\phi_e - \phi_{CEP}))\phi}$$

Linear polarization

$$e^{i(\ell - z\cos\phi_e\cos\phi_{CEP})\phi}$$

1. The total cross sections manifest exponential behavior
$\sigma^{BW} \propto \exp[-b^{BW}\zeta], \quad \sigma^{C} \propto \exp[-b^{C}\kappa]$

$$\zeta = \frac{4m^2}{s} \qquad\qquad \kappa = \frac{\omega'_\kappa}{\omega'_1}$$

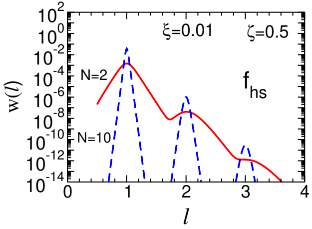
2. The slopes depend on pulse duration, field intensity, etc.

3. The exponential dependence for dynamically assisted BW process is quite different from that predicted by the Schwinger formula.

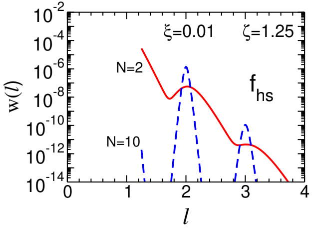
4. The azimuthal angle distributions are different for Breit-Wheeler and Compton processes for circular and linear polarizations.

5. This difference is particularly large for the interplay of azimuthal angle of outgoing fermion and ϕ_{CEP}.

Reason of enhancement



above threshold $\zeta < 1$



subthreshold $\zeta > 1$