Beam-Beam Simulations

Dmitry Shatilov

BINP, Novosibirsk

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Outline

- Beam-beam interaction
- Why do we need simulations? A couple of inspirational examples
- Tune shifts, luminosity, hour-glass
- Tracking at the IP
- Various models and problem statements
- What information can be extracted? Examples: envelope matrix, tune scans, FMA, etc.
- Space charge: problems, comparison with beam-beam
- General remarks on simulations

Beam-Beam Interaction

- The bunches fly at near-light speed and at the IP (there may be several) collide with the oncoming beam. The frequency of these collisions depends on the perimeter and the number of IPs, and is usually hundreds of kHz (for a given bunch).
- Only a small part of particles in a bunch experiences a real collision – it is for this reason that colliders are created. The fraction of colliding (and therefore retiring) particles can be estimated from the beam lifetime. For ion colliders this is hours; accordingly, in each collision less than 10^{-9} part of particles drop out of the beam.
- And what happens to the rest particles? They experience interaction with the electromagnetic field of the oncoming bunch. A characteristic feature of this interaction, called *beam-beam*, is its strong nonlinearity.
- This significantly affects the dynamics of particles and can lead to an increase in emittances (and hence to a decrease in luminosity) and appearance of long non-Gaussian tails in the transverse distribution, which increase the background in the detectors and decrease the beam lifetime.

Beam-beam effects are rightfully considered one of the main and fundamental limitations of luminosity.

Why Simulations ?

Some features of beam-beam effects:

1) This is a strong nonlinearity for most particles that are in the distribution core. In contrast to nonlinearities of magnetic lattice, which manifest themselves mainly for particles with large transverse coordinates.
2) A noticeable spread of betatron tunes appears, so the footprint (the tune range occupied by the beam) can cross strong nonlinear resonances.
3) Overlapping of resonances leads to the appearance of stochastic regions in the phase space. Perturbation theory does not work here.
4) Interference with lattice nonlinearities further complicates the task.

Analytical methods allow to make estimates and calculations for simplified models, to identify the qualitative dependence on some parameters. But it is impossible to fully describe such a complex nonlinear system analytically.

Therefore, one of the main tools for studying the beam-beam effects has long been numerical simulation. This is similar to modeling other complex effects and phenomena; it is currently widespread in many areas.

What for ?

1) For working colliders – understand what is happening. If there are problems, try different methods to improve the situation (increase luminosity, decrease the background) and give recommendations.

2) For designed colliders – the choice of collision scheme and basic parameters. Comparison of different options, identification of possible problems, finding ways to resolve or mitigate them. Parameter optimization.

3) For self-education. In simulations, it is possible to consider different limiting cases, turn on/off various effects, arbitrarily change the parameters.

   In this sense, the model is an analogue of an experimental setup, but with much greater freedom and capabilities. Many things that we know from theory become much clearer and more visual if appropriate modeling is carried out.

4) Sometimes new phenomena (e.g. new types of instabilities) were discovered (and then mitigated) in simulations.

5) Sometimes in the simulation new and very fruitful ideas were born.

Example: Crab Waist

(born in simulations)

- March 2006 – Crab Waist collision scheme was proposed by P. Raimondi for SuperB project.

  Usually it is presented in three consecutive steps: 1) large Piwinski angle, 2) decrease in β_y^* and 3) crab sextupoles (it is they who produce the crabbing of waist).
  On the first two points everything was immediately clear, but the last one caused a lot of questions. The sextuples were introduced based on simple geometric considerations, but in the simulations, Raimondi saw a large (and very positive) dynamic effect: suppression of beam-beam resonances. There were no explanations then.

- April - May 2006 – the effect was confirmed in simulations by three independent codes.

  This gave confidence that the effect is real, but there was still no explanation. Nevertheless, the new collision scheme was taken as the basis for the SuperB project and the possibility of testing this idea at the DAΦNE collider was being explored.

- End of 2006 – an explanation of the effect was obtained, at which time preparations for the DAΦNE upgrade were already in full swing.

- Beginning of 2008 – experimental confirmation at DAΦNE: Crab Waist is working, good agreement was obtained with the simulation results.

Now this idea underlies all the projects of the new e^+e^- colliders.

Coherent Beam-Beam Instability in Collision with LPA

(discovered and then mitigated in simulations)

- Spring 2016 – discovered in simulations for FCC-ee (K. Ohmi)
- End of 2016 – confirmed by independent code.
  This gave confidence that the effect is real, but there was still no explanation.
- Middle of 2017 – important dependencies were found in simulations, which allowed to develop a mitigation technique.
  We had to radically revise the parameters, change the lattice and RF voltage. If a problem had been discovered already during the experiment, it would be very expensive.
- The theory was developed in parallel, and followed the simulations.
  "The particle tracking gave guiding toward a complete theory based on the eigenmode analysis" (K. Ohmi).
- 2019 – dedicated studies were conducted at Super KEKB to find this instability, and finally it was detected.

Beam-Beam Parameters and Luminosity

The impact of the oncoming beam is equivalent to a kind of nonlinear lens, which creates a spread of betatron tunes. The maximum tune shifts, experienced by particles with small coordinates, can be described by the so-called beam-beam parameters $\xi_{x,y}$. For head-on collision:

$$\xi_{x1} = \frac{N_2 Z_1 Z_2 r_p \overbrace{(1+\beta_1\beta_2)}^{\approx 1}}{2\pi A_1 \gamma_1 \beta_1 (\beta_1+\beta_2)} \cdot \frac{\beta^*_{x1}}{\sigma^*_{x2}(\sigma^*_{x2}+\sigma^*_{y2})}$$

N is the number of particles per bunch, r_p – classical proton radius, $\beta_{1,2}$ – velocities, A – atomic number (mass/m_p). Indices $_{1,2}$ correspond to the 1st and the 2nd counter beams. ξ_{x2}, ξ_{y1}, ξ_{y2} can be obtained by replacing indices.

The actual tune shifts $\Delta\nu_{x,y} \approx \xi_{x,y}$ for $\xi_{x,y} << 1$, $\nu_{x,y}$ far from integer and half-integer resonances and without hourglass (only relevant for flat beams).

Now, for simplicity, we consider the collision of identical beams, which can consist of many (but also identical) bunches. Since the luminosity is often limited by ξ, it is convenient to express it through ξ_y:

$$L = f_0 \frac{N^2}{4\pi\sigma^*_x\sigma^*_y} = \frac{A\gamma\beta}{2eZ^3 r_p} \cdot \frac{I_{tot}\xi_y}{\beta^*_y} \cdot \underbrace{\left(1+\frac{\sigma^*_y}{\sigma^*_x}\right)}_{1 \div 2} \cdot R_{hg}$$

defined by experiment   beam-beam limit $1 \div 2$ hour-glass

Normally, the bunch population is limited by ξ (which does not depend on β^* for round beams).

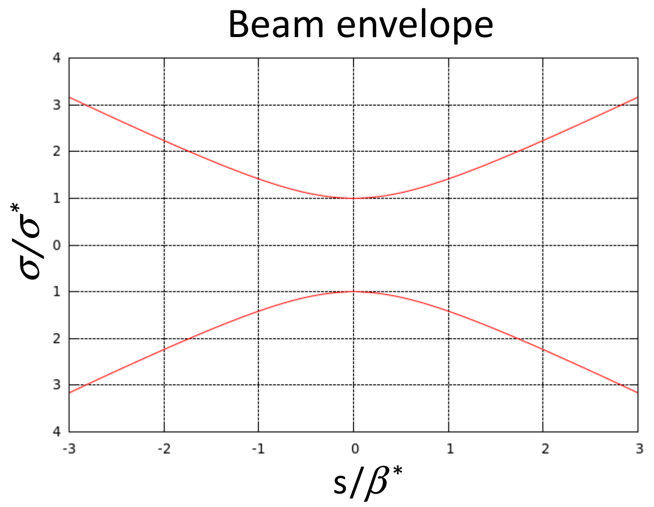
Way to increase luminosity: increase I_{tot} (number of bunches) and decrease β_y^*. In this case σ_z also should be decreased, otherwise strong hour-glass will ruin everything.

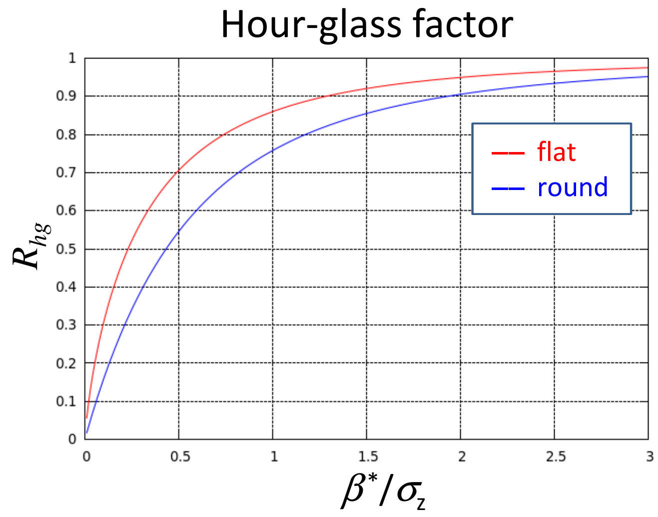
Hour-glass

$$\sigma(s) = \sigma^* \cdot \sqrt{1+\left(\frac{s}{\beta^*}\right)^2}$$

(without dispersion)

With dispersion at the IP, the shape is similar but β^* is replaced by some expression.





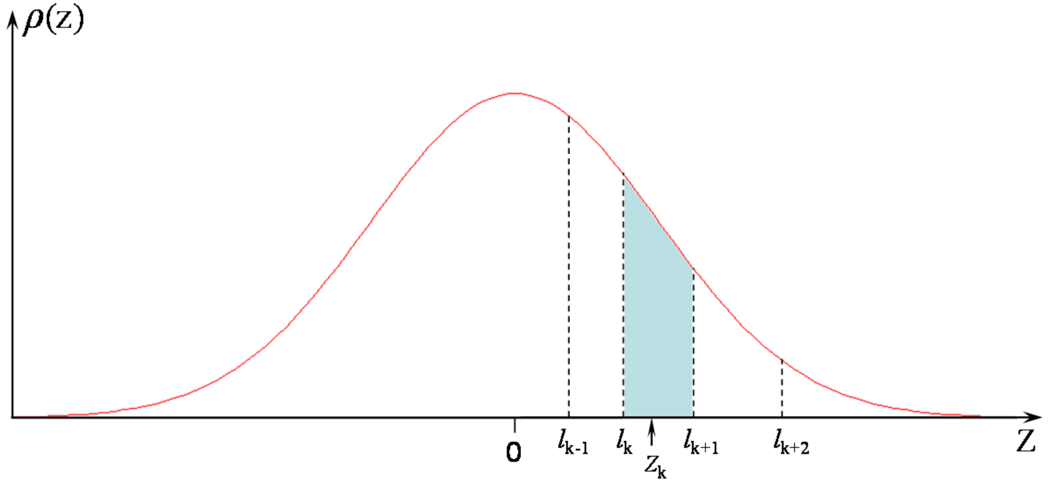
A decrease in β^* will be effective only with a decrease in σ_z, and here there are some limitations associated with impedances and collective instabilities. This probably will require a reduction in the bunch population, which means a decrease in luminosity.

Optimization is carried out individually for each collider, taking into account other restrictions. But almost everywhere, a simple condition holds: $\beta^* \geq 0.8\ \sigma_z$.

<u>The betatron phase advance along the interaction area is always large.</u> Therefore, the "collision" cannot be represented in the form of a single localized kick. Since the force acting on the particle is nonlinear and has a rather complicated form, integration of the equations of motion in the interaction region does not seem possible.

The solution is to replace the continuous action with a drift - kick sequence.

Longitudinal Slicing of the Opposite Bunch



The longitudinal distribution of the oncoming bunch is replaced by a sequence of delta functions:

$$\rho(z) = \frac{\exp(-z^2/2\sigma_z^2)}{\sqrt{2\pi}\sigma_z} \rightarrow \sum_{k=-n}^{n} w_k \delta(z - z_k)$$

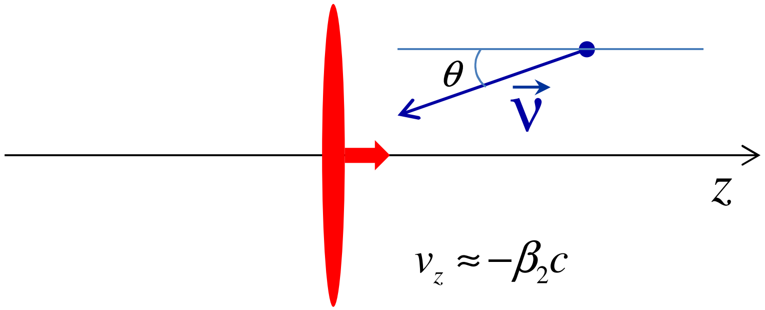
In the ultrarelativistic case, the electromagnetic field of a thin slice lies in the same plane. Therefore, a particle interacts with a slice only at the moment of intersection of its plane. And then the drift-kick scheme fits perfectly.

Slicing algorithm matters. Probably, the best one:

$$\begin{cases} \dfrac{z_k}{\sigma_z} = \dfrac{\sigma_z}{w_k} \cdot \left(\rho(l_k) - \rho(l_{k+1})\right) \\ w_k \sim \exp(-z_k^2/4\sigma_z^2) \end{cases}$$

Coordinates and weights of slices (z_k and w_k) can be found in iterations.

Usually 10-15 slices is enough. The distance between slices $\sim \sigma_z/5 >> \sigma_{x,y}$. This means that even in the non-ultrarelativistic case, the slicing model works well since the fields of slices [almost] do not interfere.



We are working in a laboratory frame, Z-axis directed along the strong bunch's velocity. The angle θ arises either due to the angular spread in the beam, or due to collision with crossing angle angle – this does not matter.

$$\vec{F} = e\cdot\left(\vec{E} + \vec{v}\times\vec{B}\right)$$ – Lorentz force

$$B_x = -\beta_1 \frac{E_y}{c},\quad B_y = \beta_1 \frac{E_x}{c},\quad B_z = 0$$

$$\Delta t \propto \frac{1}{\beta_1 - v_z/c}$$ – time-of-flight

$$\begin{cases} F_x = e\left(E_x - v_z B_y\right) = eE_x\left(1-\beta_1 v_z/c\right) \\ F_y = e\left(E_y + v_z B_x\right) = eE_y\left(1-\beta_1 v_z/c\right) \\ F_z = e\left(E_z + v_x B_y - v_y B_x\right) = e\left(E_z + \beta_1 (E_x v_x/c + E_y v_y/c)\right) \end{cases}$$

$$\frac{\partial \sigma_{x,y}}{\partial s} \neq 0$$

β_1 and β_2 are velocities of slice and particle (not beta-functions), and for simplicity we set the particle's charge Z_2 = 1

$$\Delta p_x \propto \kappa \cdot eE_x$$

$$\Delta p_y \propto \kappa \cdot eE_y$$

$$\kappa \approx \frac{1+\beta_1\beta_2}{\beta_1+\beta_2} \approx 1$$

In ultrarelativistic case (β_1 = 1): κ = 1, and does not depend on β_2 and θ.

Consider low energy, symmetrical case:

γ = 2 (~1 GeV per nucleon), $\beta_{1,2} \approx 0.866$, $\kappa \approx 1.01$

γ = 3 (~2 GeV per nucleon), $\beta_{1,2} \approx 0.943$, $\kappa \approx 1.002$

Transverse kick is actually the same as in the ultrarelativistic case!

Beam-Beam Kick

1) Gaussian transverse distribution

a) Round beams:

$$\frac{\Delta p_x}{p_0} = \frac{2NZ^2 r_p}{\gamma\beta A} \cdot \frac{x}{r^2}\left(1-\exp(-r^2/2\sigma^2)\right) \qquad r^2 = x^2 + y^2$$

b) Elliptical beams, $\sigma_x > \sigma_y$ (formula Bassetti – Erskine):

$$\frac{\Delta p_x}{p_0} = -\frac{NZ^2 r_p}{\gamma\beta A}\sqrt{\frac{2\pi}{\sigma_x^2-\sigma_y^2}} \cdot \mathrm{Im}\left(F(x,y)\right) \qquad \frac{\Delta p_y}{p_0} = -\frac{NZ^2 r_p}{\gamma\beta A}\sqrt{\frac{2\pi}{\sigma_x^2-\sigma_y^2}}\,\mathrm{Re}\left(F(x,y)\right)$$

$$F(x,y) = w\left(\frac{x+iy}{\sqrt{2(\sigma_x^2-\sigma_y^2)}}\right) - \exp\left(-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2}\right) \cdot w\left(\frac{x\dfrac{\sigma_y}{\sigma_x} + iy\dfrac{\sigma_x}{\sigma_y}}{\sqrt{2(\sigma_x^2-\sigma_y^2)}}\right)$$

See [K. Hirata et al., KEK 92-117, 1992] for E_z and Δp_z calculation.

2) Non-Gaussian transverse distribution

In order for the transformation to be symplectic, one must first find the potential of the counter bunch. In most cases, the Poisson equation is solved on a grid (mesh), and this is time-consuming…

Disruption Parameter

In a collision, both beams act on each other. Is it necessary to take into account changes in the distribution function of the beams during the collision?

- Beam-beam kicks depend on the distribution of transverse coordinates of the oncoming beam, and does not depend (except E_z) on the distribution of transverse momenta.
- The kicks change the transverse momenta, not the coordinates. However, during the interaction, $\Delta p_{x,y}$ will have time to transfer into Δx, Δy.
- The relative change in the transverse coordinates in one collision is described by the disruption parameter (note that $\xi_{x,y}$ refers to one IP):

$$D_{x,y} = 4\pi\xi_{x,y}\frac{\sigma_z}{\beta^*_{x,y}}$$

- Small disruption ($D_{x,y}$ << 1) means that the distribution of transverse coordinates remains unchanged. This is always true for ion colliders.
- Examples of $D_{x,y}$ >> 1: linear colliders (ILC, CLIC).

Simulation Models

Consideration of the opposite bunch

1) **Weak-strong**

   The opposite (strong) bunch is not affected during long-term (many turns) tracking. It is simple and fast. It is always recommended to start with this approach.

2) **Strong-strong**

   Both bunches are affected and updated either every turn (small $D_{x,y}$) or during every collision. The latter is the most complex and time-consuming, but we must use it when $D_{x,y} >> 1$.
   Simplified variant (to avoid solving Poisson equation): account the barycenter of each slice (transverse modes) and fit the transverse distribution to Gaussian, so only $\sigma_{x,y}$ are updated.

3) **Quasi-strong-strong**

   Swap the "weak" and the "strong" bunches every n-th turn, and thus update the strong bunch. It is much faster than strong-strong approach, but can be used only for $D_{x,y} << 1$.

Particle tracking between IP(s)

1) **Linear lattice (constant transport matrix)**

   It is simple, fast and most flexible. If beam-beam is considered as the major nonlinearity, it is recommended to start with this approach. Perhaps some damping and noise can be applied too.

2) **Realistic lattice**

   This is more time-consuming, but accounts chromaticity, DA and energy acceptance, interference between beam-beam and lattice-driven resonances.

Note: Crab Waist was discovered in weak-strong simulations, coherent beam-beam instability – in strong-strong, and then confirmed in quasi-strong-strong simulations. In all cases – linear lattice between IPs.

Transport and Envelope Matrices

One-turn transport matrix

We only need to track one particle through the ring 7 times to get this matrix.

- Eigenvalues give us the tunes.
- Eigenvectors give us the beta- and alpha-functions, betatron coupling, dispersion.
- Do it with and w/o beam-beam – get the real tune shifts (not equal to $\xi_{x,y}$!) and dynamic beta.
- Do it for off-energy particle – get the chromaticity of betatron tunes and beta-functions.

This technique is very useful for tracking in nonlinear lattice, which was most likely taken from another program (e.g. MADX). First, we need to check that the "export" of lattice was correct.

Envelope matrix

After long-term tracking (many particles, many turns) we get the equilibrium orbit and the envelope matrix:

$$O_i = \langle x_i \rangle, \quad \Sigma_{ij} = \langle x_i x_j \rangle, \quad i,\, j = 1 \ldots 6$$

- From Σ matrix we get X-Y tilt and crabbing (X-Z and/or Y-Z tilt).
- From $\Sigma \cdot S$ matrix we get the emittances (eigenvalues) and all lattice functions (eigenvectors). Note that now it corresponds to the whole beam, assumed to be 6D Gaussian.

Scan of Betatron Tunes

- Performing a simulation for only one set of parameters is not very efficient. We need to understand some dependencies, and so we should try different input parameters. First thing that comes to mind – scan of betatron tunes.
- If the range of tunes is large enough, this can be done only in a weak-strong model, linear lattice.

  In nonlinear lattice, a significant change in betatron tunes will require reconfiguration of the sextupoles, otherwise this will affect DA. This is a non-trivial and lengthy process that cannot be automated.

Output:

1) Luminosity.

   When particle crosses a slice of the opposite bunch, we know the slice's density at that point, which gives us the luminosity of this elementary collision. Averaging over all the slices, particles and turns, we get the luminosity per one particle, one collision. Hour-glass, crossing angle (if any), etc. are already taken into account here.

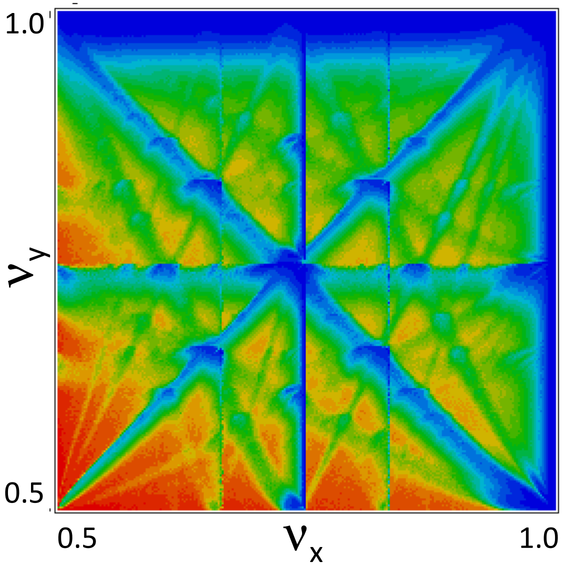
2) rms beam sizes. Note: long non-Gaussian tails (if any) strongly affect rms.

3) Maximum achieved betatron amplitudes.

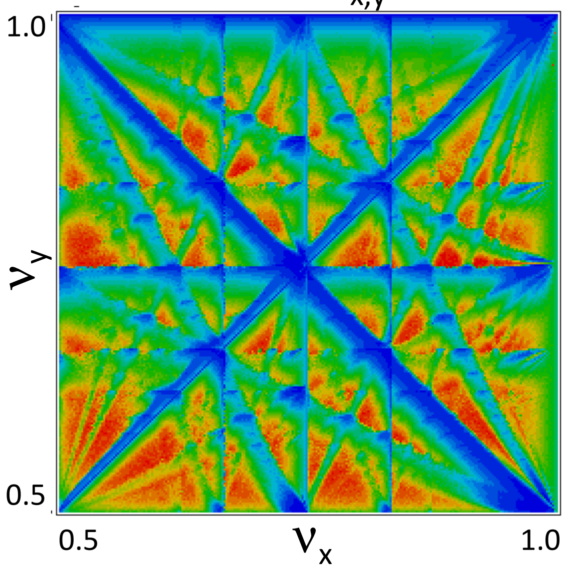
In what follows, we consider e+e- collider with flat beams. At each working point, tracking was performed for a few damping times to find the equilibrium. Colors: red – good (high luminosity, small rms sizes). Resonances are seen as blue lines.

Collision of Short Bunches (flat beams)

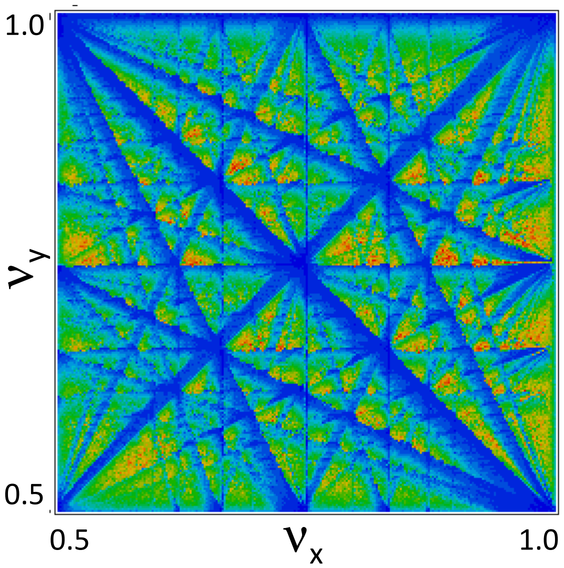
Luminosity



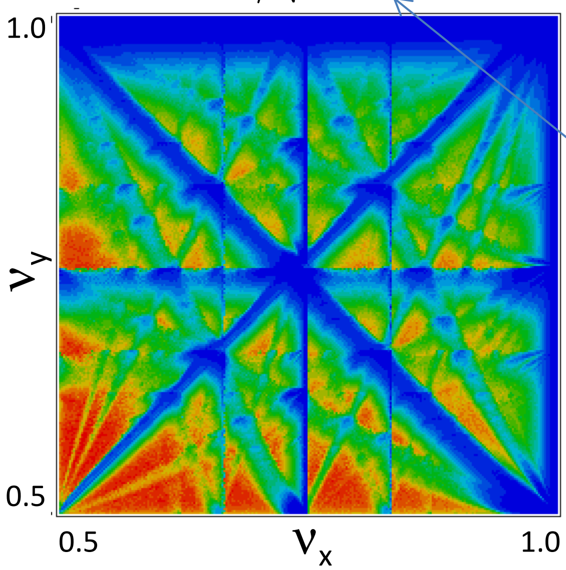
rms $\sigma_{x,y}$



Max. ampl.



$L/\sqrt{\sigma_x \sigma_y}$



The width of resonance is proportional to $(\partial\nu/\partial A)^{-1}$, A is the normalized betatron amplitude.

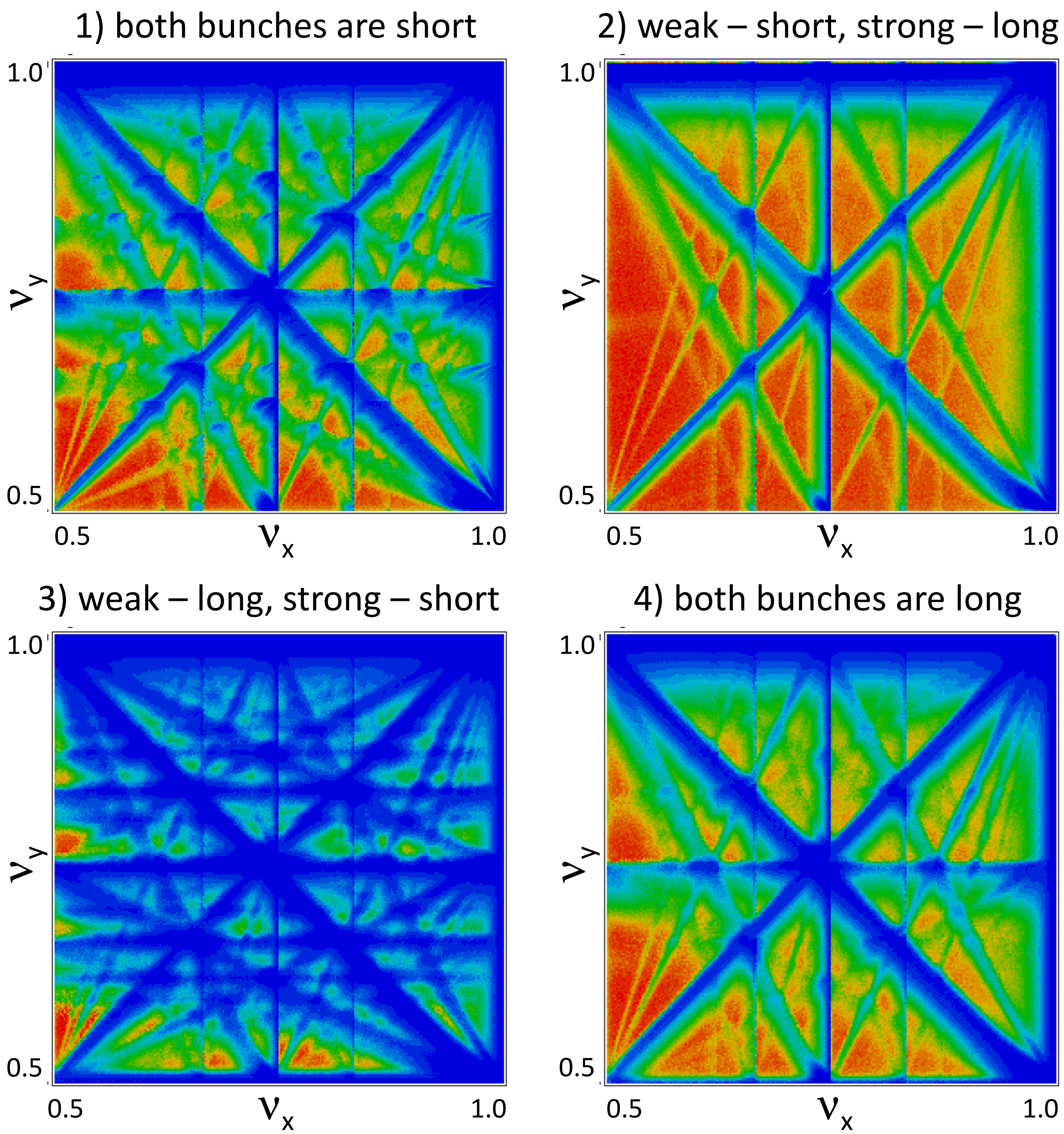
At large betatron amplitudes, the tune dependence on A is weaker, therefore resonances are wider.

⇩

Beam tails are more sensitive to high-order resonances.

A "good" working point should be good for all tests. In the next slide we present the scans for this function, which somehow accounts both the luminosity and the beam tails.

Dependence on the Bunch Length



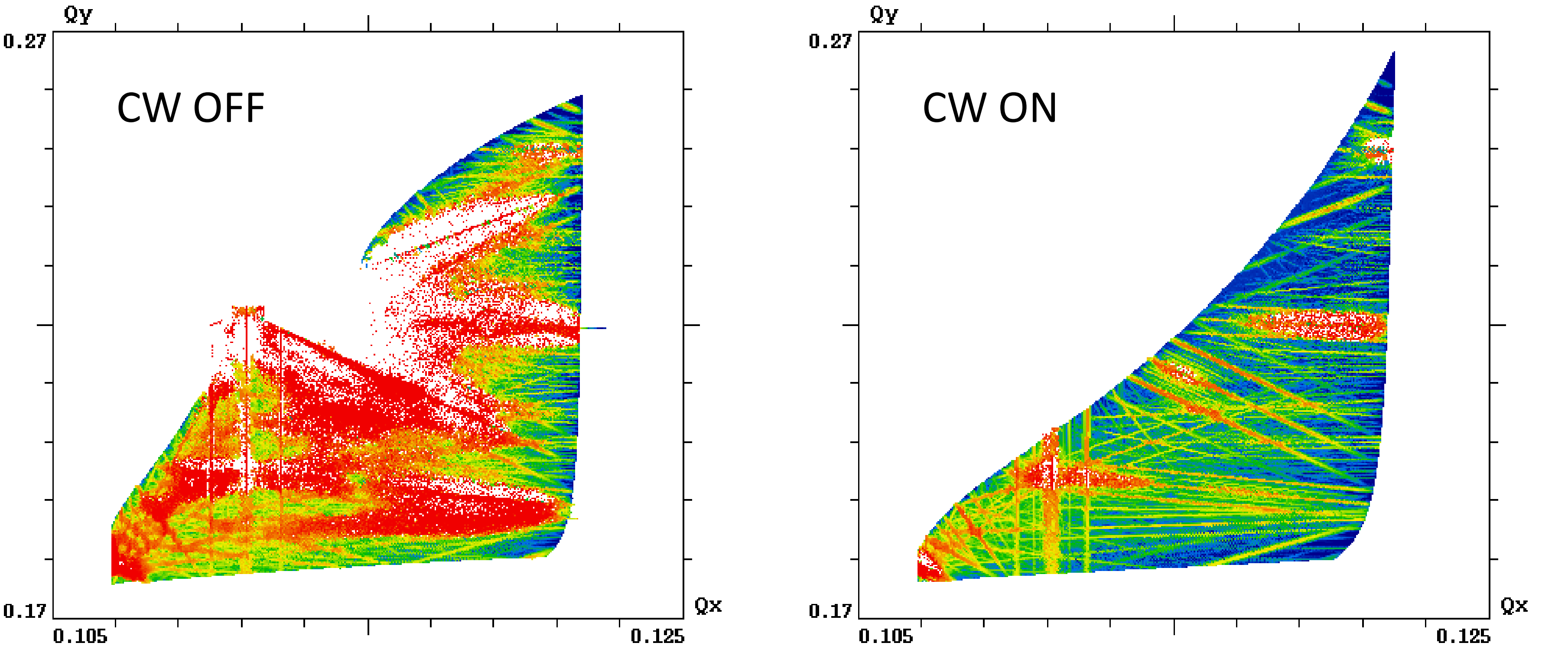
Here "short" means $\sigma_z = 0$, and "long" means $\sigma_z = \beta_y^*$

1) There is no averaging of the betatron phase and no synchro-betatron resonances.
2) Betatron phase averaging during collision makes the resonances weaker.
3) Synchrotron motion of the "weak" particles produces modulation of the betatron phase at the moment of collision. This greatly enhances synchro-betatron resonances.
4) The two aforementioned effects almost compensate each other.

Frequency Map Analysis

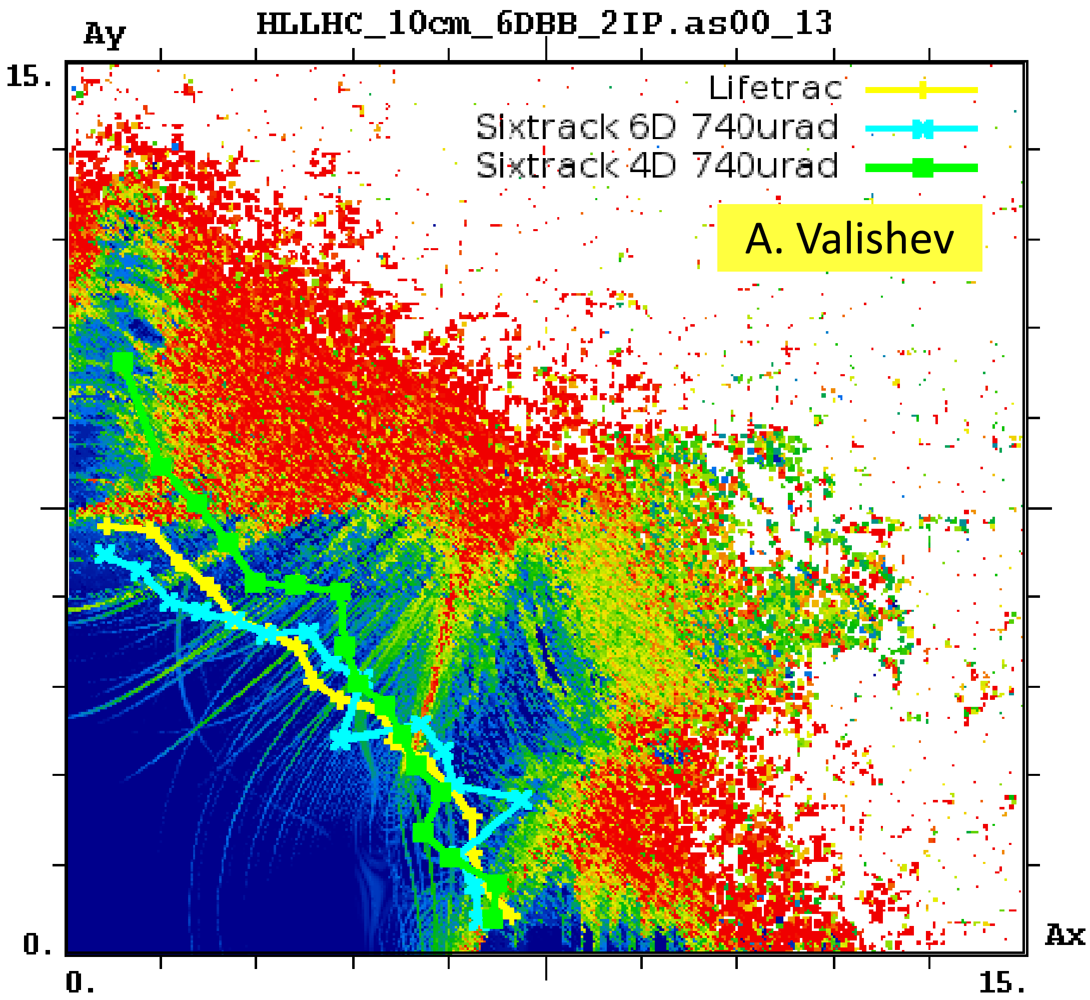
Is it possible to see the working resonances without changing tunes? Yes!

FMA (introduced by J. Laskar in 1990) calculates the so-called diffusion index, which is used to distinguish between regular and stochastic motion. Historically, regular trajectories are colored in blue, and stochastic – in red.



Beam-beam footprints for DAΦNE working in the Crab Waist collision scheme. The only difference is the strength of crab sextupoles. It is clearly seen how the betatron coupling resonances are suppressed by the crab sextupoles.

Dynamic Aperture for LHC



Nonlinear lattice + 2 IPs + many PCs (long range beam-beam effects).

Plot in the plane of normalized betatron amplitudes

Three color lines – the border of DA (particles did not survive after 10^6 turns) obtained by two independent codes.

FMA : each point corresponds to a particle trajectory ~16k turns. White color – particles did not survive after 16k turns.

As seen, the DA corresponds to the border of stochastic region, and it can be determined by FMA with only 16k turns.

Space Charge

- The Laslett tune shifts $\Delta\nu_{x,y} \propto 1/\gamma^3$, and at low energies they become larger than $\xi_{x,y}$. So we have to include the space charge effect in the tracking code.
- The standard simulation method is that many special SC elements are placed on the ring, where particles receive 3D kicks depending on their coordinates. The number of SC elements can be estimated similar to the number of slices for beam-beam: betatron phase advance between them should be small.
- The potential of space charge is similar to beam-beam, but the 3D kicks cannot be expressed through known functions. We need some good approximation of the potential, and this is a non-trivial task...
- In what follows, we consider a <u>test</u> collider based on NICA lattice. 200 SC elements are placed equidistantly, but tracking between them is made by linear transformations (to avoid interference with the lattice nonlinearities). In addition, the tunes were changed for some reason.

Space Charge vs. Beam-Beam

Some interesting questions:

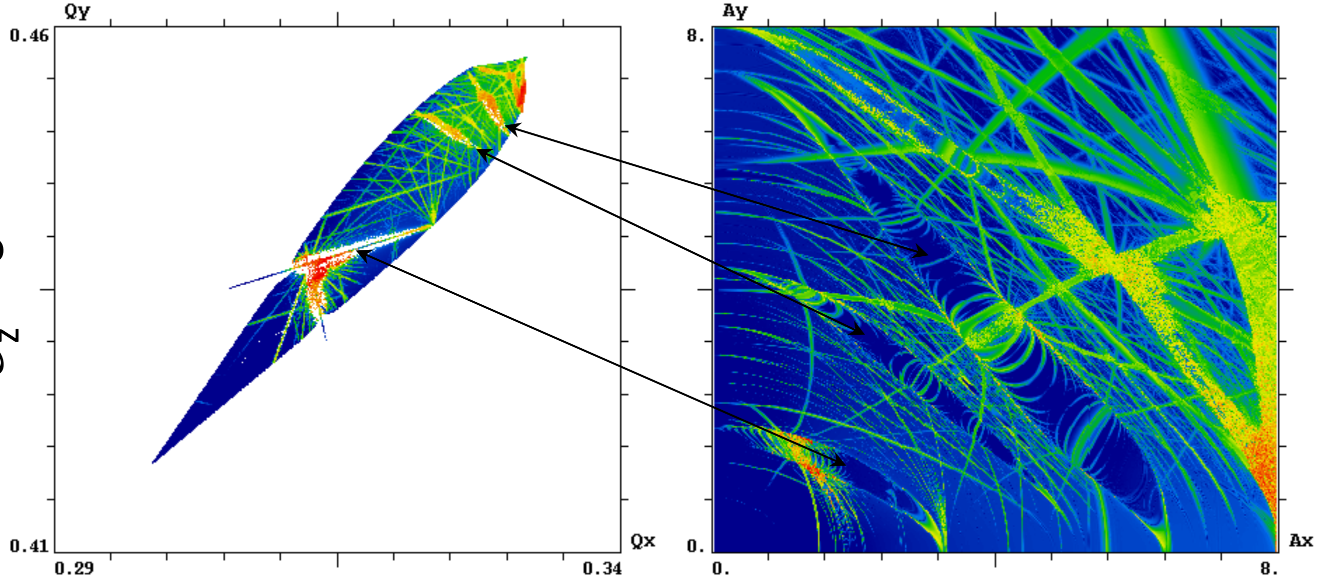
1) The beam-beam kick is somehow localized, while the space charge is distributed over the whole ring. How about the betatron phase averaging: is it better for SC?
2) The dependence on synchrotron oscillations is also different. Will tune shifts from beam-beam and space charge be equivalent?
3) In electron-ion collider, the tune shifts from beam-beam and space charge have the opposite signs. Can they compensate each other?

To answer, we compare two different cases:

1. Space charge without beam-beam.
2. Beam-beam without space charge, while the population of the opposite bunch is increased so that the tune shifts are the same.

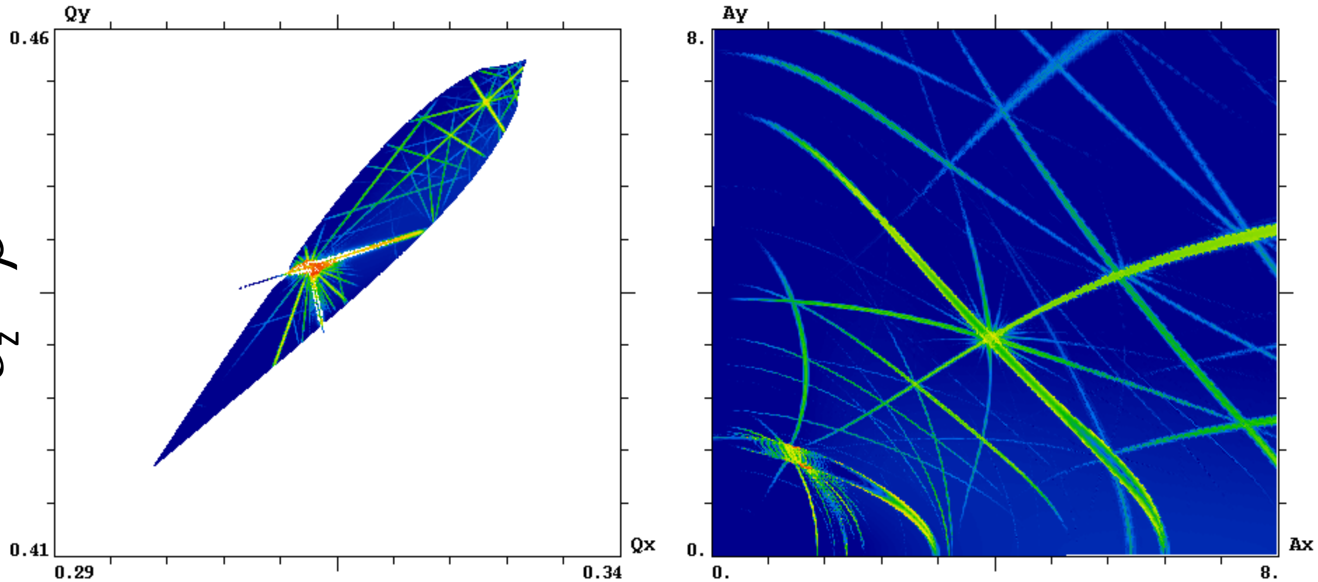
Beam-Beam for A_s=0: Dependence on σ_z

$\sigma_z = 0$



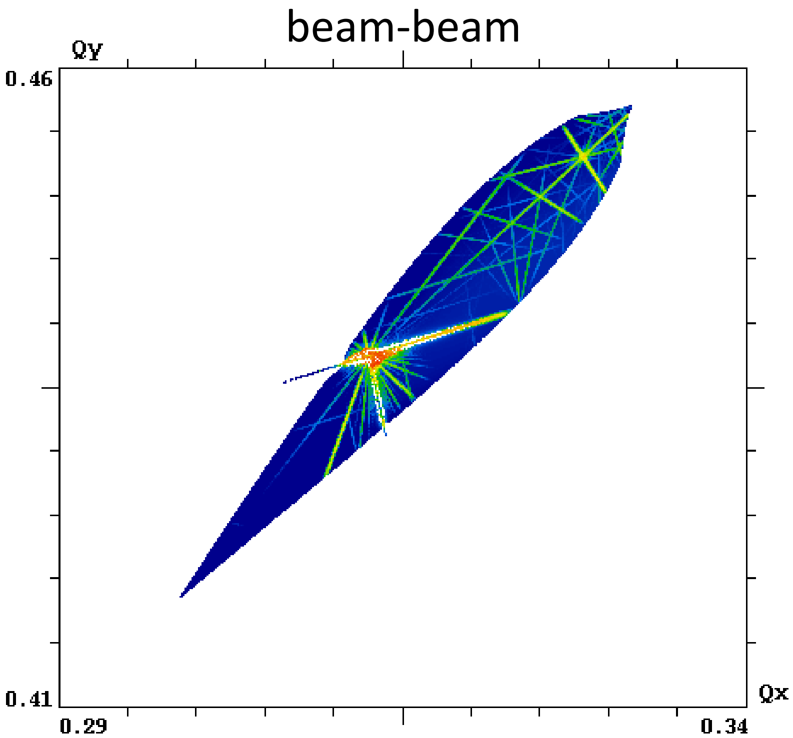
Single kick: no betatron phase averaging.

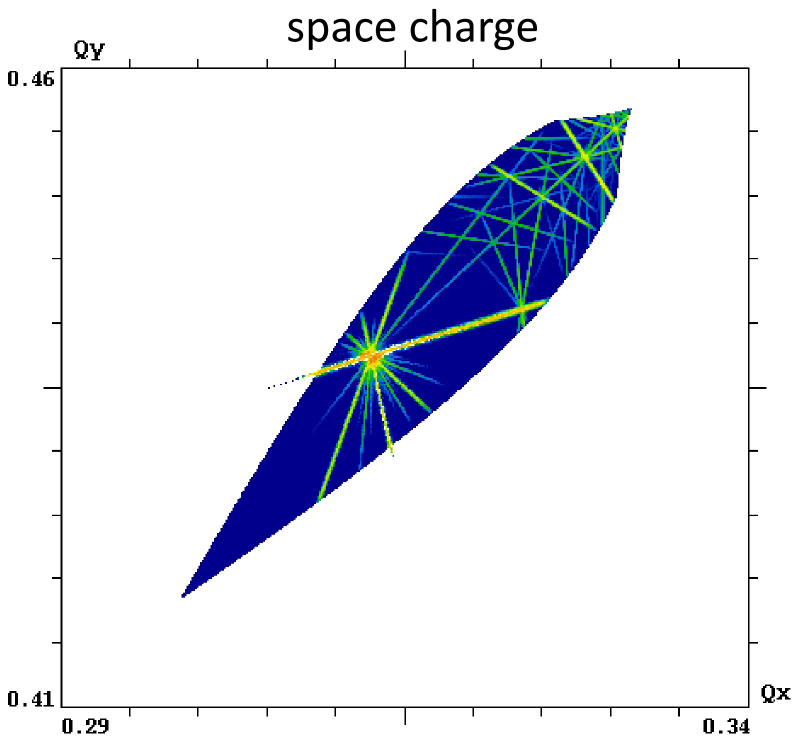
$\sigma_z = \beta^*$



Smoothed kick: betatron phase averaging.

Space Charge vs. Beam-Beam: $A_s = 0$



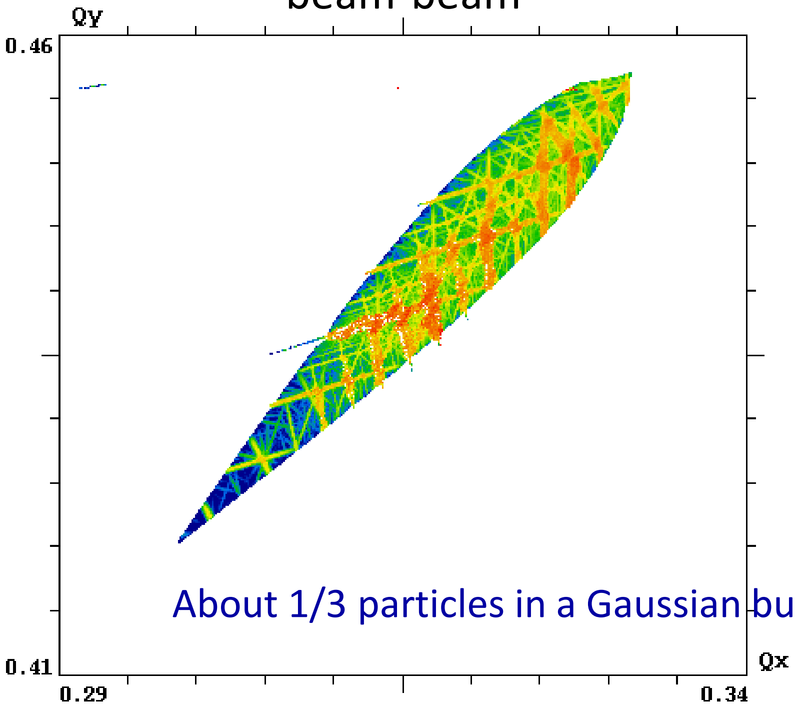


The shape of the footprints is slightly different, since the beam is not round at the locations of SC kicks. But the strengths of resonances are almost the same.

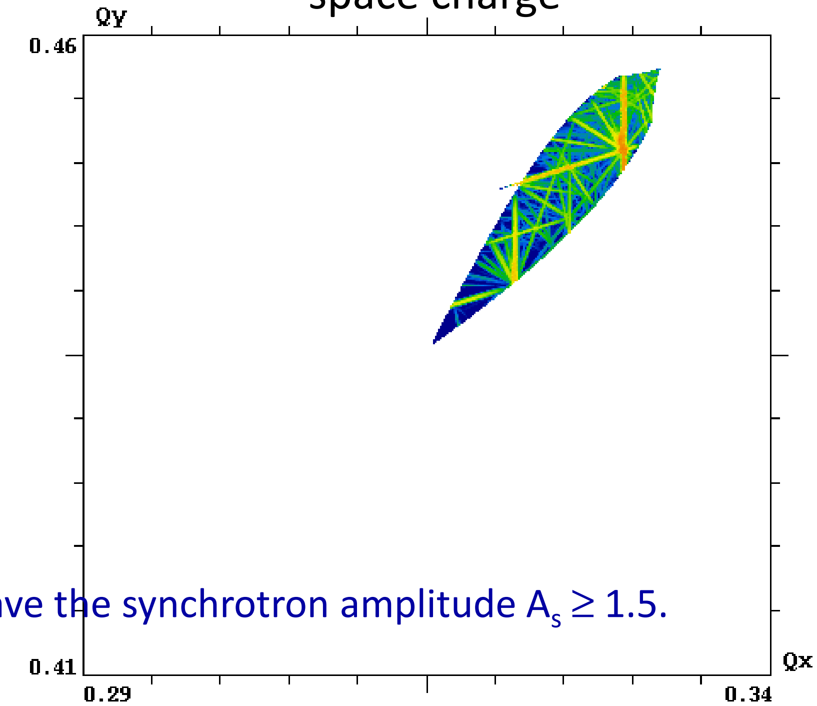
This means that the betatron phase averaging for beam-beam is quite good – no worse than for space charge.

Space Charge vs. Beam-Beam: $A_s = 1.5$

beam-beam



space charge



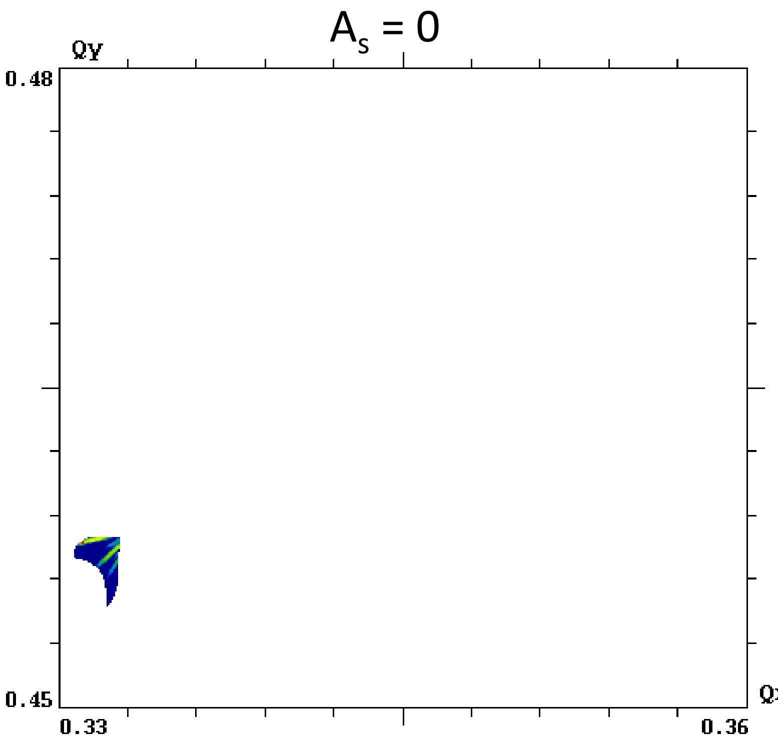
About 1/3 particles in a Gaussian bunch have the synchrotron amplitude $A_s \geq 1.5$.

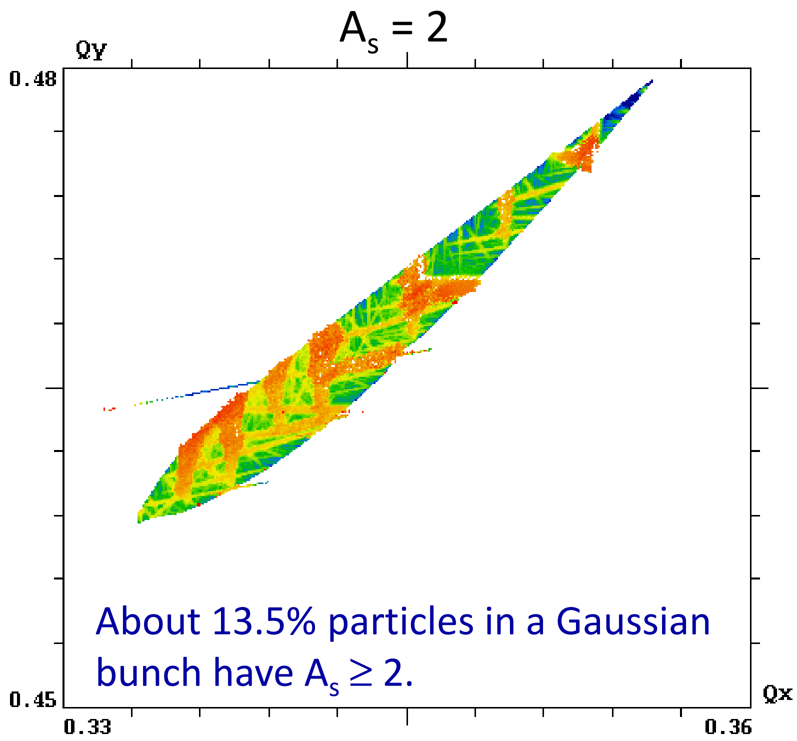
Space charge: for particles with large synchrotron oscillations, the effective charge density decreases, together with the Laslett tune shifts. The strength of transverse kicks is modulated, so the synchro-betatron resonances appear.

In contrast to this, the strength of beam-beam kick remains unchanged (the same tune shifts) and synchro-betatron resonances are excited by the betatron phase modulation, which is a stronger effect compared to the modulation of kick's strength.

The most harmful are synchro-betatron beam-beam resonances. Beam-beam and Laslett tune shifts are not equivalent!

Space Charge vs. Beam-Beam: Compensation





Here we consider the electron-ion collision, so the beam-beam tune shift is positive, and its absolute value is made equal to that of space charge.

Compensation can be achieved only for a small part of the beam, namely, for particles with small synchrotron amplitudes.

Many tried to come up with a scheme for beam-beam compensation, no one succeeded…

General Remarks on Simulations

- All codes are developing, new functions and capabilities are added to them – as a rule, in response to requests from the experiment, or when designing new colliders. It is important to be involved in a "living" project in order to create a good code.
- There must be several independent codes. As a rule, they differ in functionality and can be sharpened for different tasks, but there is always a significant area of overlap. And here it is very useful to carry out cross-checks of the simulation results.
- It is necessary all the time to compare the results with theory, analytical estimates and experimental data. Only then will confidence in the results appear.

Try to develop you own code and have fun!