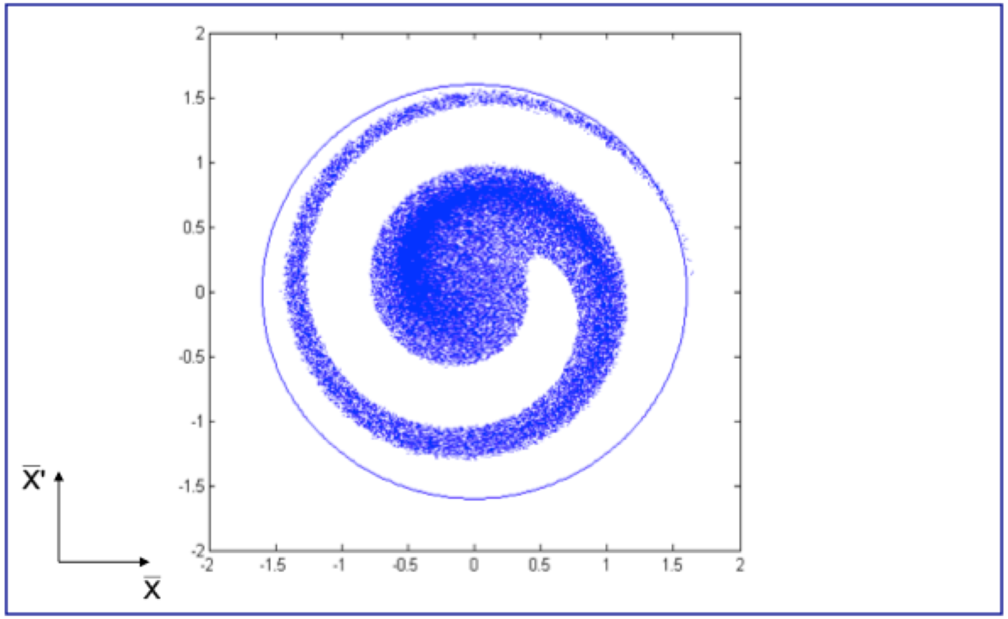
Emittance Preservation

Verena Kain

JAS, Dubna, November 2019



The importance of low emittance

- Low emittance is a key figure of merit for circular and linear colliders

$$\mathcal{L} = \frac{N_+ N_- f}{2\pi \Sigma_x \Sigma_y}$$

$$\Sigma_{x,y} = \sqrt{\sigma^{*2}_{x,y+} + \sigma^{*2}_{x,y-}}$$

  – The luminosity depends directly on the horizontal and vertical emittance
  – In case of round beams and the same emittance for both beams

$$\mathcal{L} = \frac{N_+ N_- f}{4\pi \beta^* \varepsilon}$$

- Brightness is a key figure of merit for Synchrotron Light Sources
  – High photon brightness needs low electron beam emittance

Reasons for non-conserved emittances

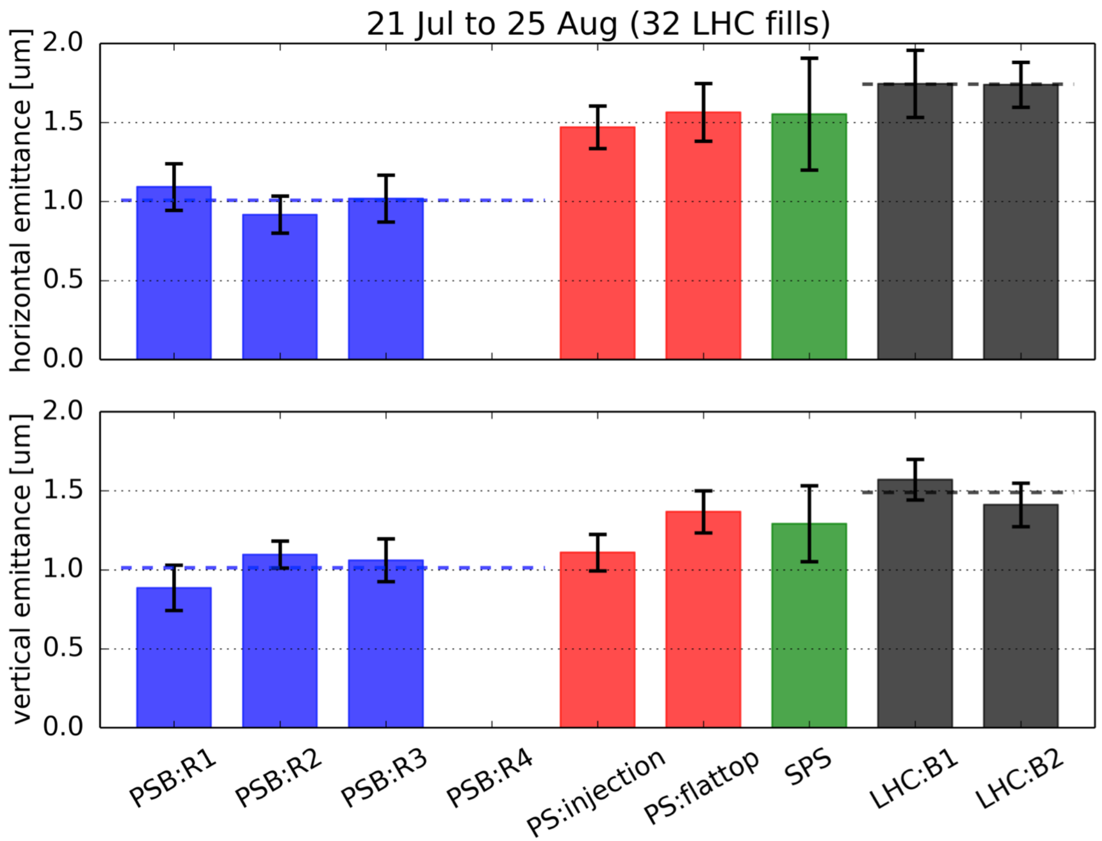
- Liouville's theorem: area (→ emittance) in phase space stays constant under conservative forces
- Some effects to decrease emittance
  - Synchrotron radiation: charged particle undergoing acceleration will radiate electromagnetic waves
    - Radiation power depends on mass of particle like $1/m^4$
    - Comparison of p^+ and e^- for the same energy

$$\frac{P_p}{P_e} = \left(\frac{m_e}{m_p}\right)^4 = 8.8 \times 10^{-14}$$

  - Stochastic or e^- cooling
- Many effects to increase emittance
  - **Intra-beam scattering, power supply noise,** crossing resonances, instabilities,...
  - Alignment errors, dispersion for e^- Linacs
  - **Mismatch at injection into synchrotrons** or linacs

Example: the LHC injector chain

- Proton beams through the LHC injector chain
  - βγ normalized emittances



Significant blow up in both planes.

~ 50 % in horizontal plane from PSB to PS.

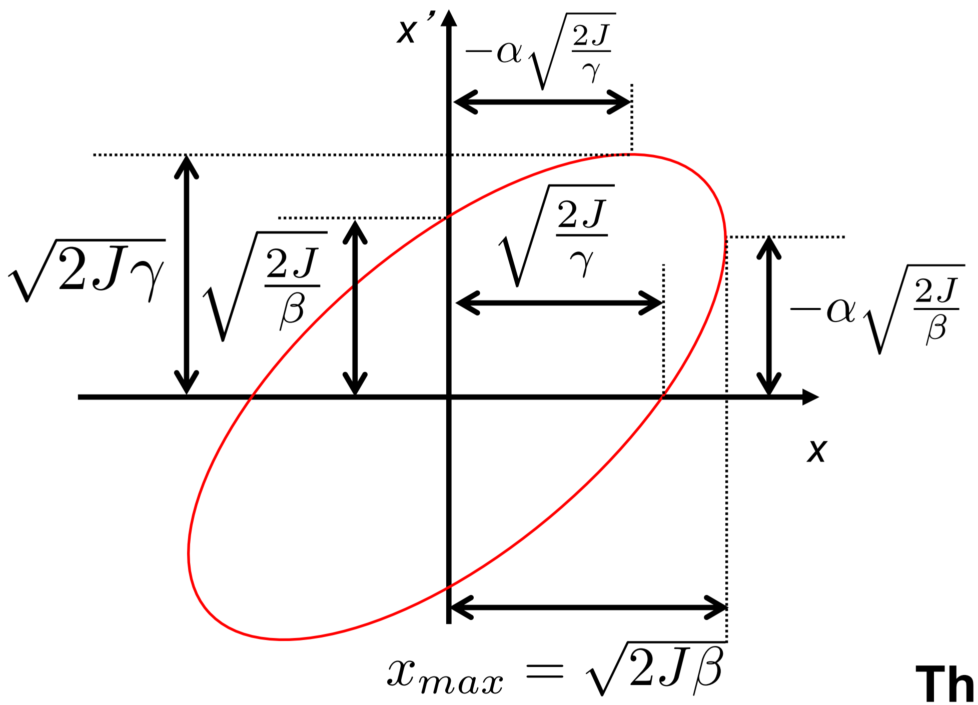
Big contribution from injection mismatch

Defining Emittance

- Defining **action-angle variables**

Cartesion coordinates $(x, x') \; (y, y') \; (z, \delta)$

phase space



Area = $2\pi J_x$

Action-angle variables:

$$2\boxed{J_x} = \gamma_x x^2 + 2\alpha_x x' x + \beta_x x'^2$$

$$\tan\boxed{\phi_x} = -\beta_x \frac{x'}{x} - \alpha_x$$

The advantage of action-angle variables: The action of a particle is constant under symplectic transport

Defining Emittance

- J_x… amplitude of the motion of a particle
  - The Cartesian variables expressed in action-angle variables

$$x = \sqrt{2\beta_x J_x}\cos\phi_x$$

$$x' = -\sqrt{\frac{2J_x}{\beta_x}}(\sin\phi_x + \alpha_x\cos\phi_x)$$

- The emittance is the average action of all particles in the beam:

$$\boxed{\varepsilon_x = \langle J_x \rangle}$$

Emittance – statistical definition

- Emittance $\equiv$ spread of distribution in phase-space

- Defined via 2nd order moments

$$\sigma = \begin{pmatrix} \langle x^2 \rangle & \langle xx' \rangle \\ \langle xx' \rangle & \langle x'^2 \rangle \end{pmatrix}$$

- **RMS emittance:**

$$\boxed{\varepsilon = \sqrt{|\sigma|} = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2}}$$

Emittance during acceleration

- What happens to the emittance if the reference momentum P_0 changes?

- Can write down transfer matrix for reference momentum change:

$$M_x = \begin{pmatrix} 1 & 0 \\ 0 & P_0/P_1 \end{pmatrix} \quad \longrightarrow \quad \epsilon_{x1} = \frac{P_0}{P_1}\epsilon_{x0}$$

- The emittance shrinks with acceleration!

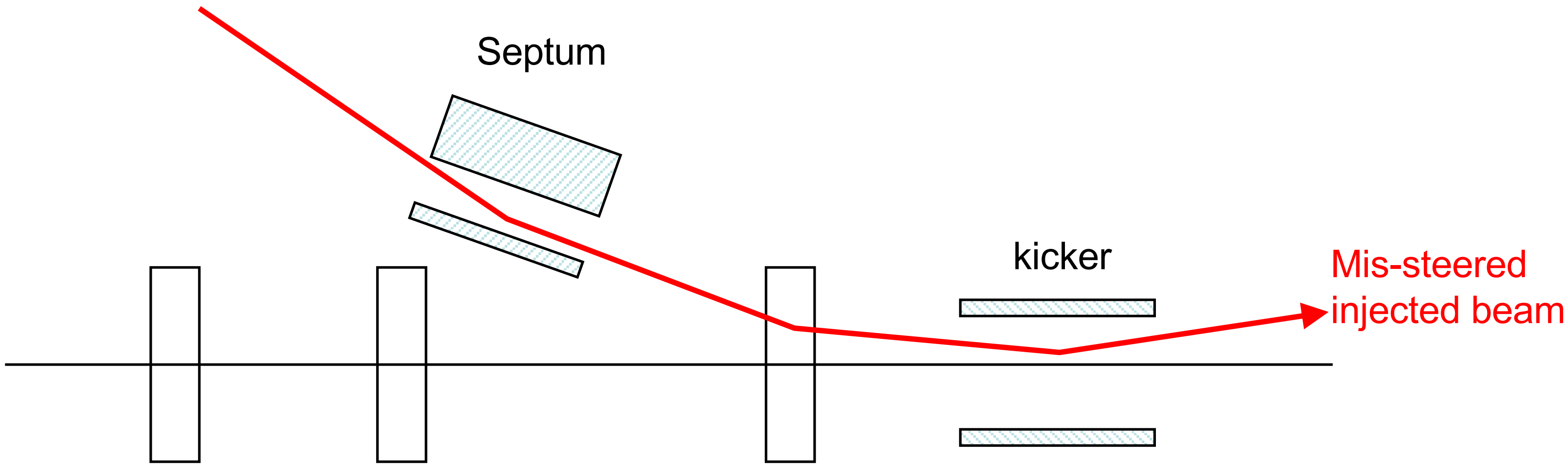
- With $P = \beta\gamma mc$ where γ, β are the relativistic parameters

- The conserved quantity is $\beta_1\gamma_1\epsilon_{x1} = \beta_0\gamma_0\epsilon_{x0}$

- It is called ***normalized emittance***.

Steering (dipole) errors

- Precise delivery of the beam is important.
  - To avoid **injection oscillations** and emittance growth in rings
  - For stability on secondary particle production targets



- Injection oscillations = if beam is not injected on the closed orbit, beam oscillates around closed orbit and eventually filaments (if not damped)

Reminder - Normalised phase space

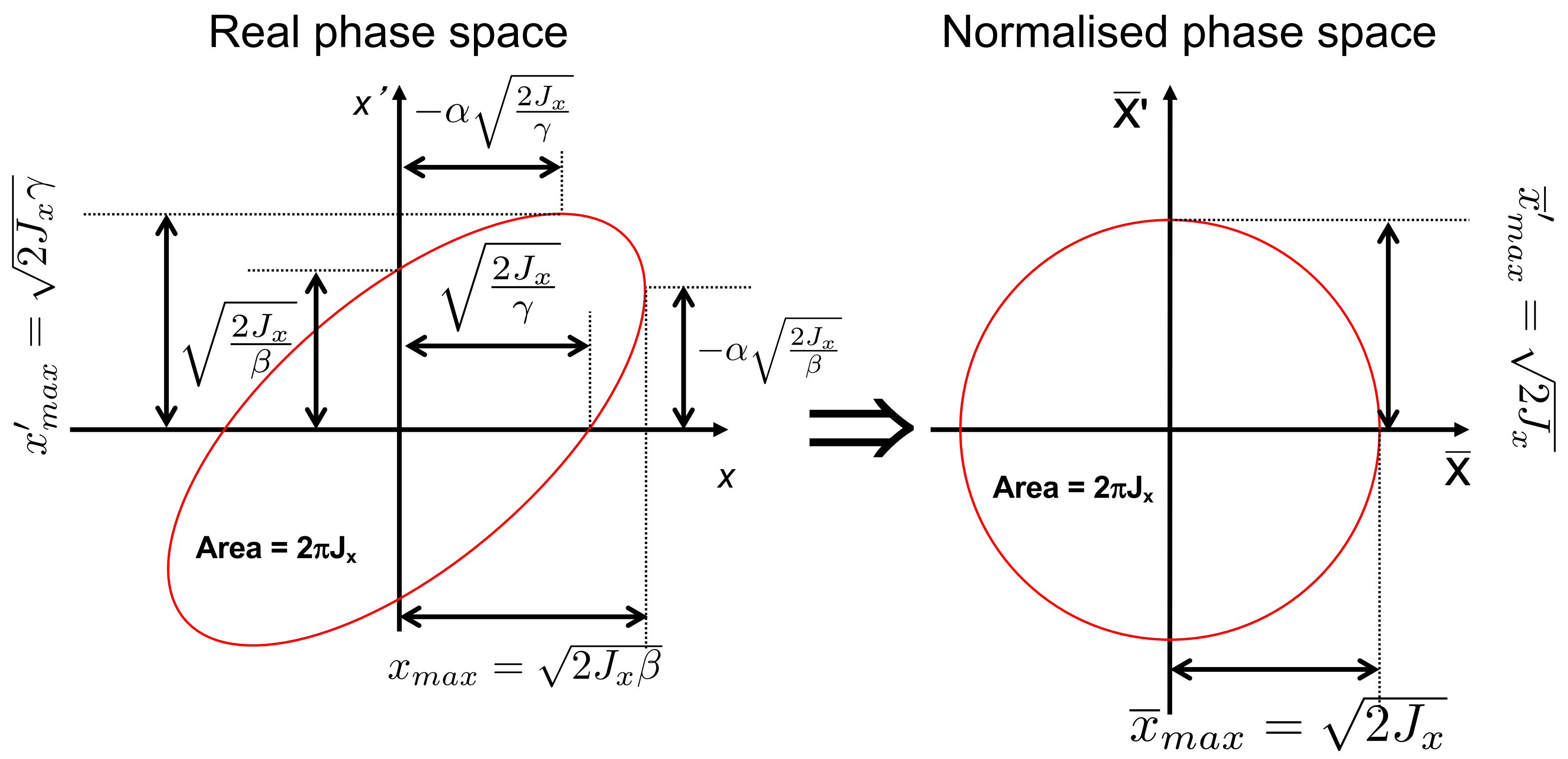
- Transform real transverse coordinates x, x' by

$$\begin{bmatrix} \overline{\mathsf{X}} \\ \overline{\mathsf{X}}' \end{bmatrix} = \mathbf{N} \cdot \begin{bmatrix} x \\ x' \end{bmatrix} = \sqrt{\frac{1}{\beta_S}} \cdot \begin{bmatrix} 1 & 0 \\ \alpha_S & \beta_S \end{bmatrix} \cdot \begin{bmatrix} x \\ x' \end{bmatrix}$$

$$\overline{\mathsf{X}} = \sqrt{\frac{1}{\beta_S}} \cdot x$$

$$\overline{\mathsf{X}}' = \sqrt{\frac{1}{\beta_S}} \cdot \alpha_S x + \sqrt{\beta_S}\, x'$$

Reminder - Normalised phase space

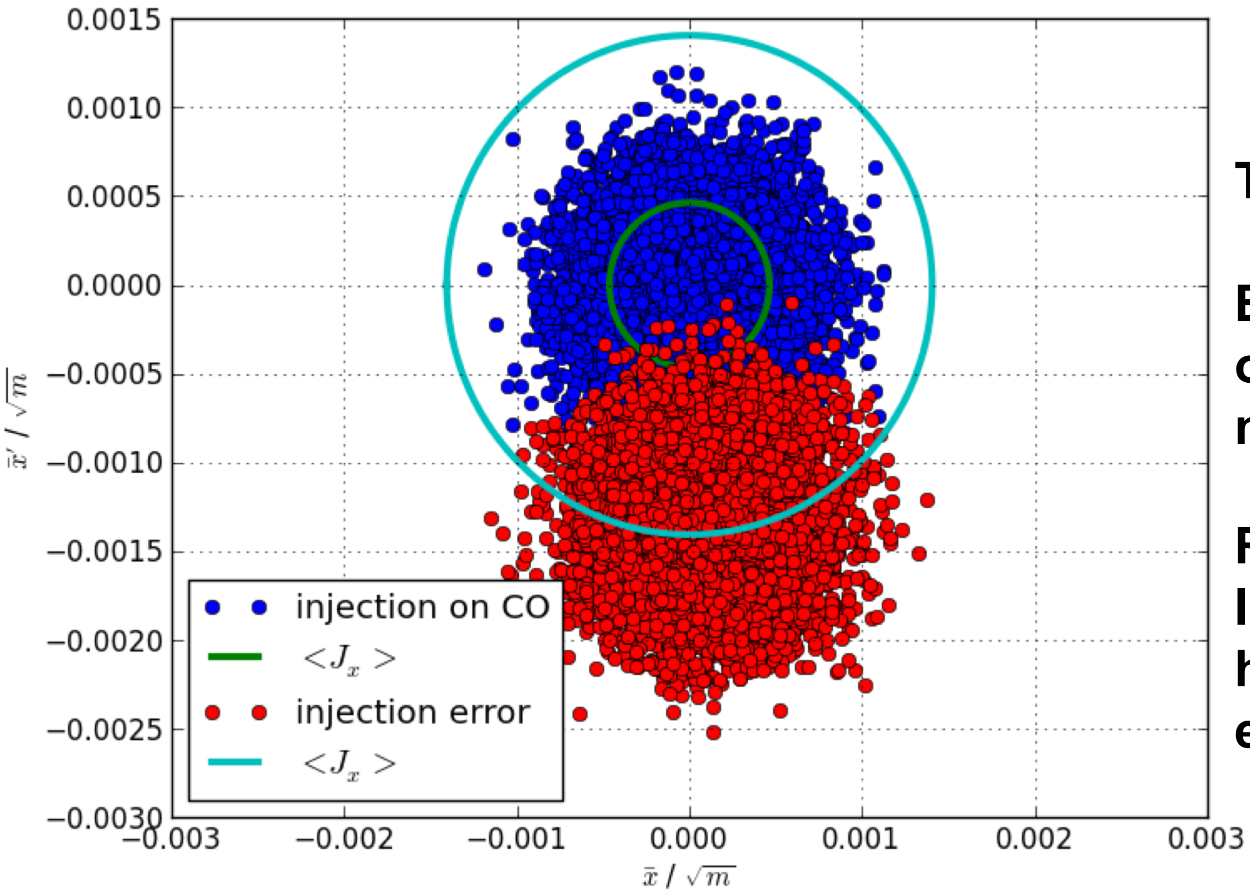


$$2J_x = \gamma \cdot x^2 + 2\alpha \cdot x \cdot x' + \beta x'^2$$

$$2J_x = \overline{x}^2 + \overline{x}'^2$$

Steering error – linear machine

- What will happen to particle distribution and hence emittance?



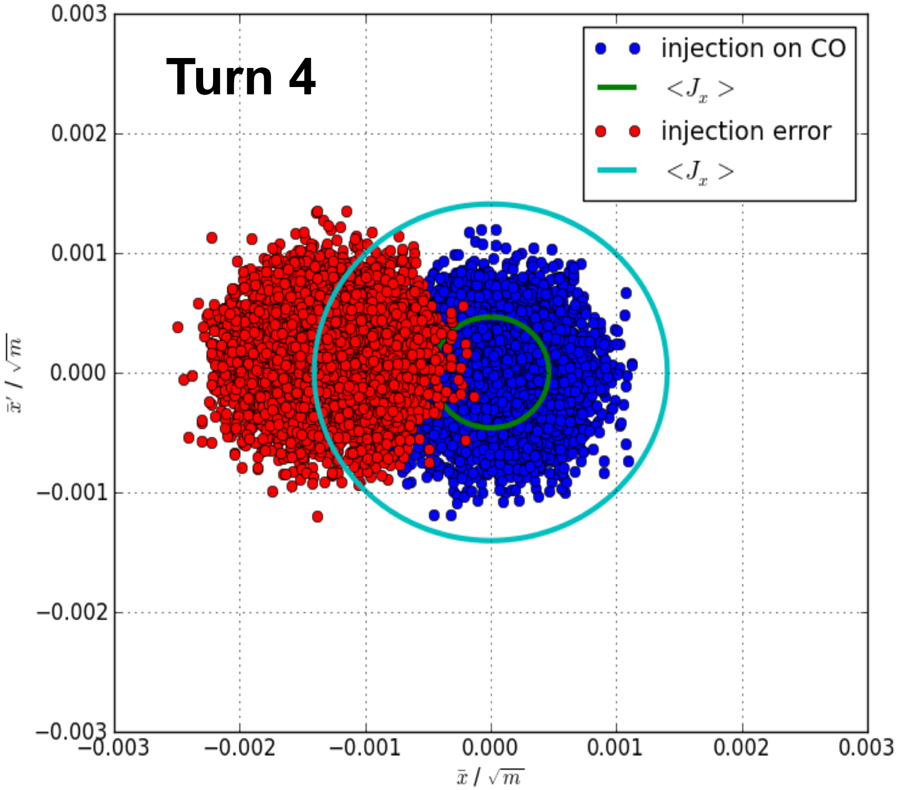
Turn 1:

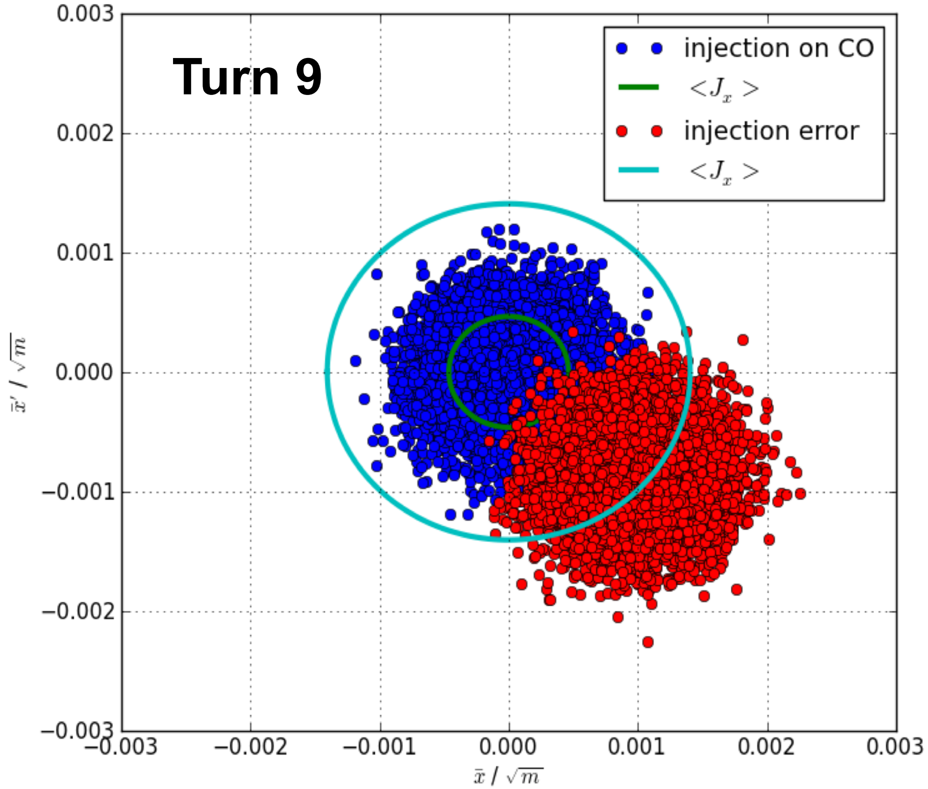
Blue distribution: on axis injection – no error

Red distribution: Injection with horizontal injection error: mainly in x'

Steering error – linear machine

- What will happen to particle distribution and hence emittance?

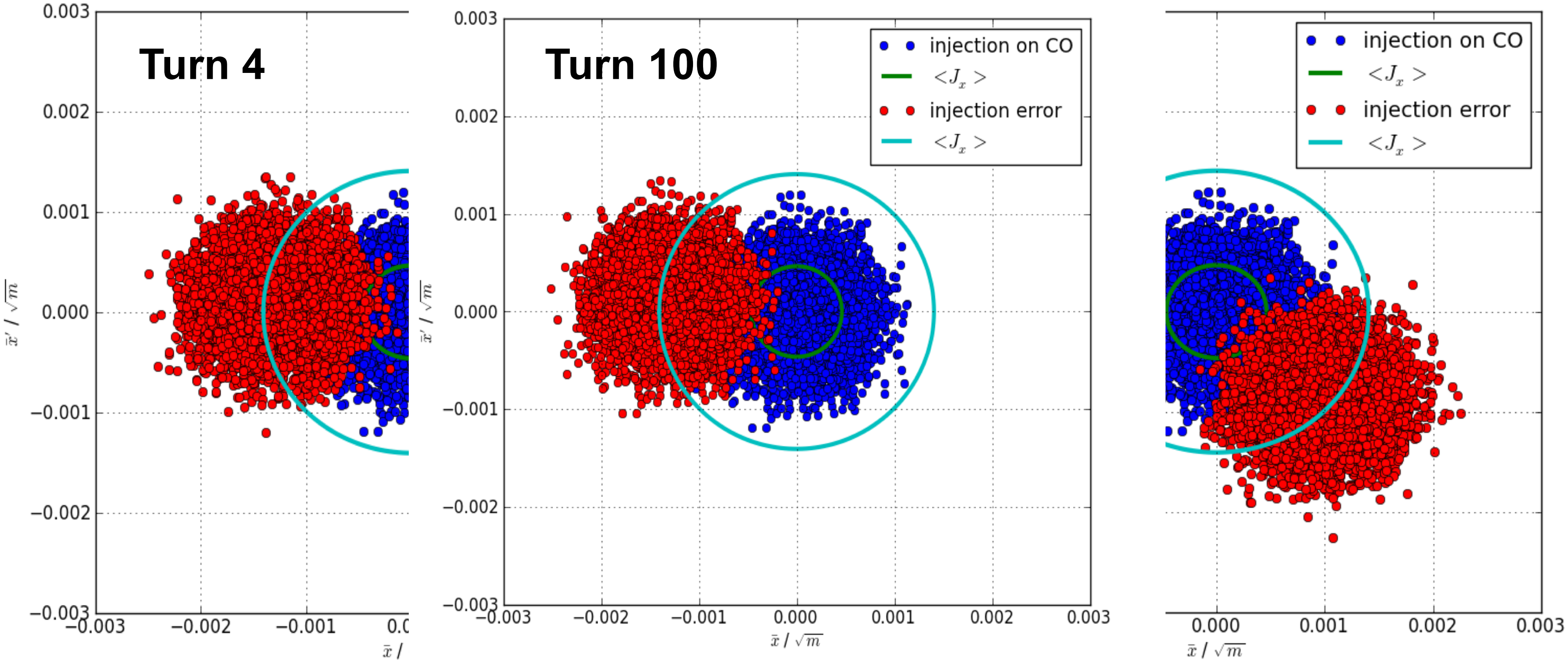




- The beam will keep oscillating. The centroid will keep oscillating.

Steering error – linear machine

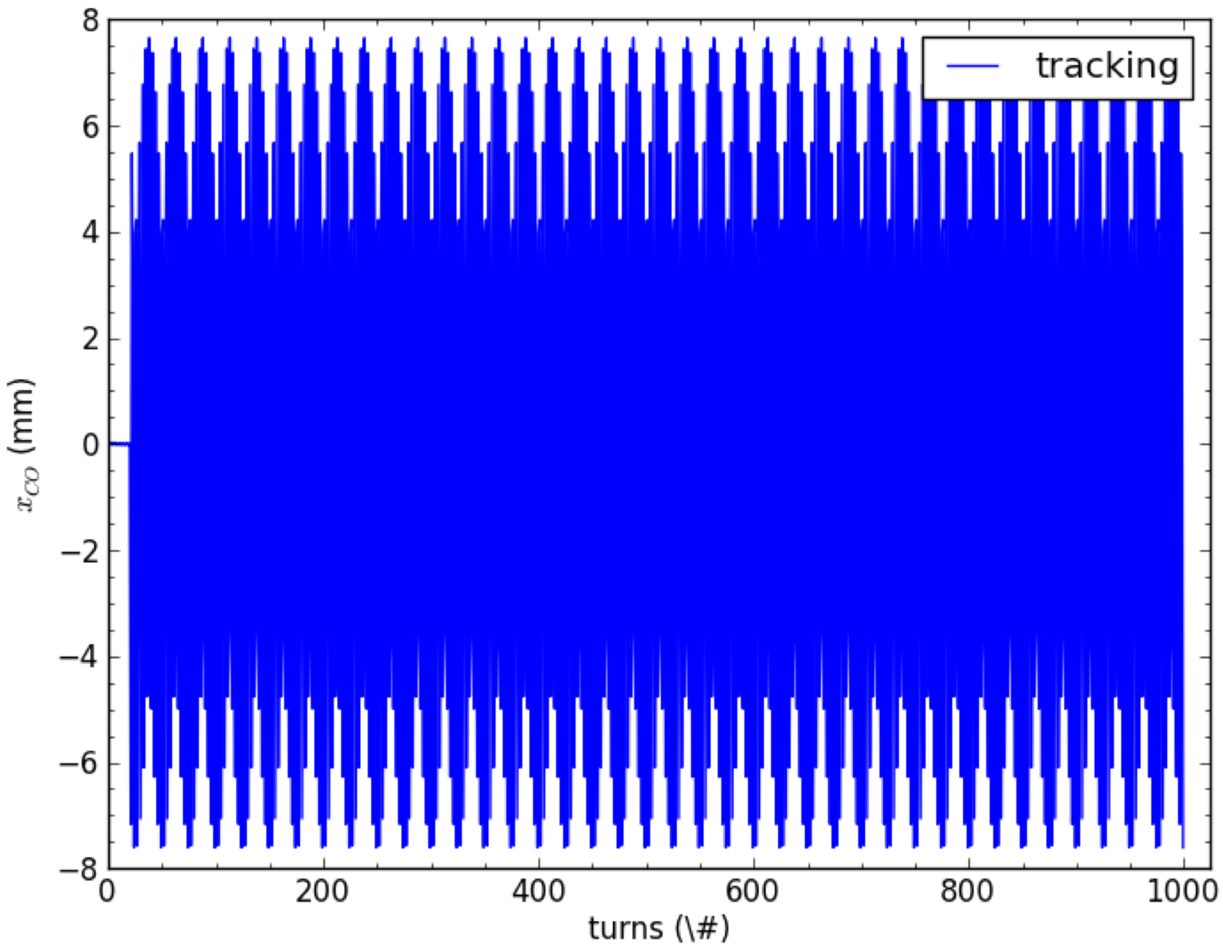
- What will happen to particle distribution and hence emittance?



- The beam will keep oscillating. The centroid will keep oscillating.

Injection Oscillations

- The motion of the centroid of the particle distribution over time
- Measured in a beam position monitor
  - Measures mean of particle distribution



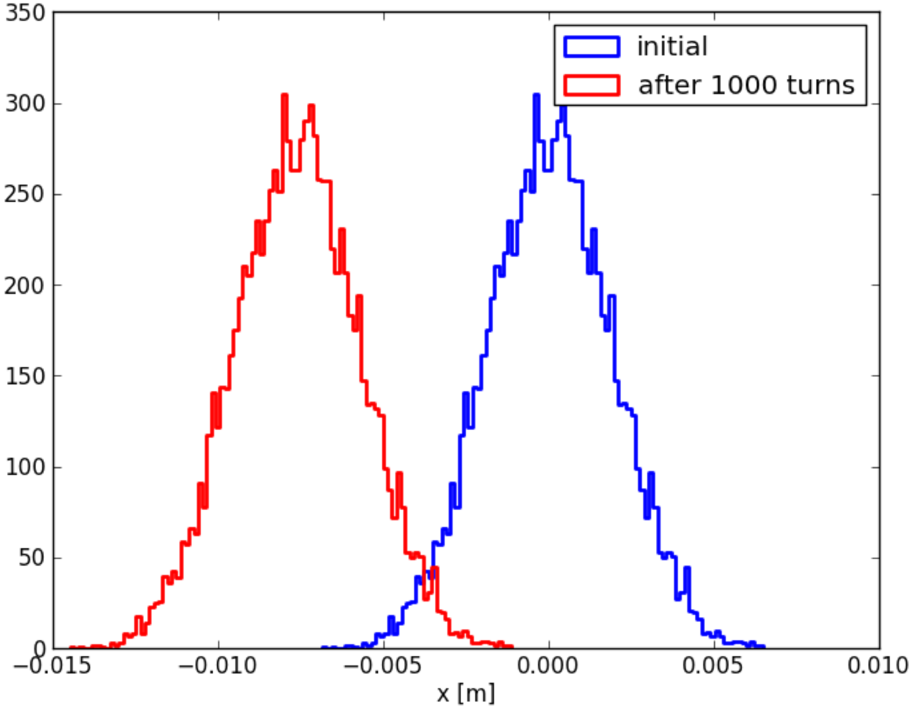
Betatron oscillations.

Undamped.

Beam will keep oscillating.

Steering error – linear machine

- Turn-by-turn profile monitor: initial and after 1000 turns
  - Measures distribution in e.g. horizontal plane

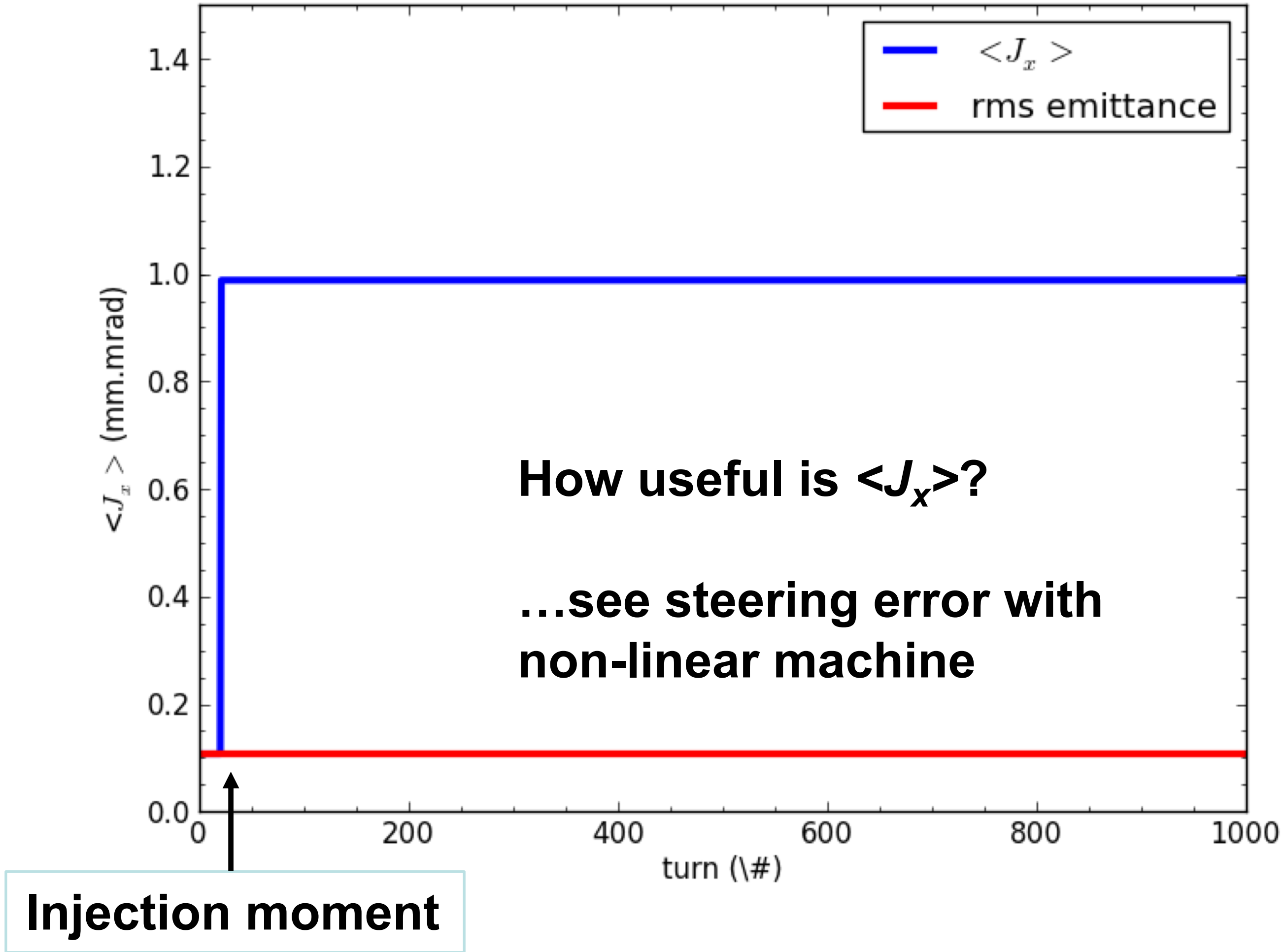


The same beam size, but mean position is not constant

- Now what happens with emittance definition and $\langle J_x \rangle$?
  - Mean amplitude in phase-space

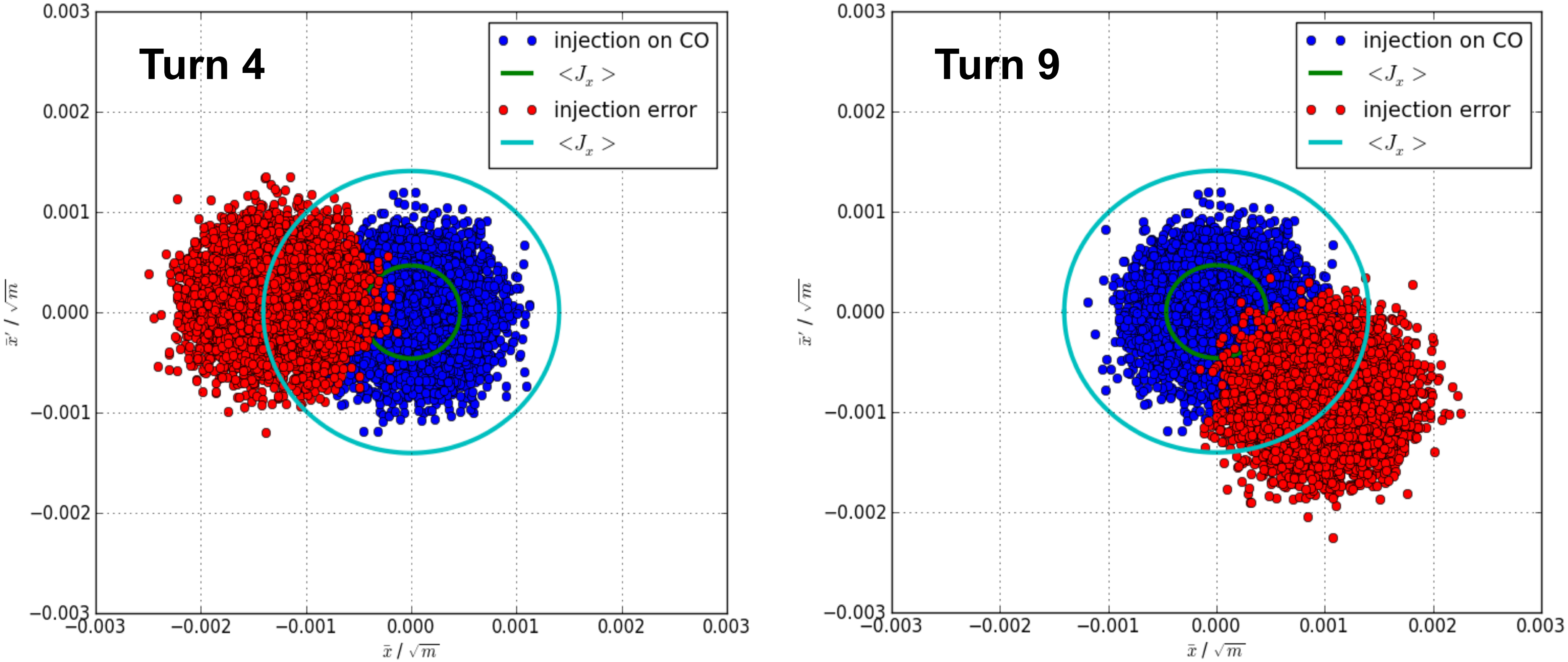
Steering error – linear machine

- How does $<J_x>$ behave for steering error in linear machine?
- And what about the rms definition?



Steering error – non-linear machine

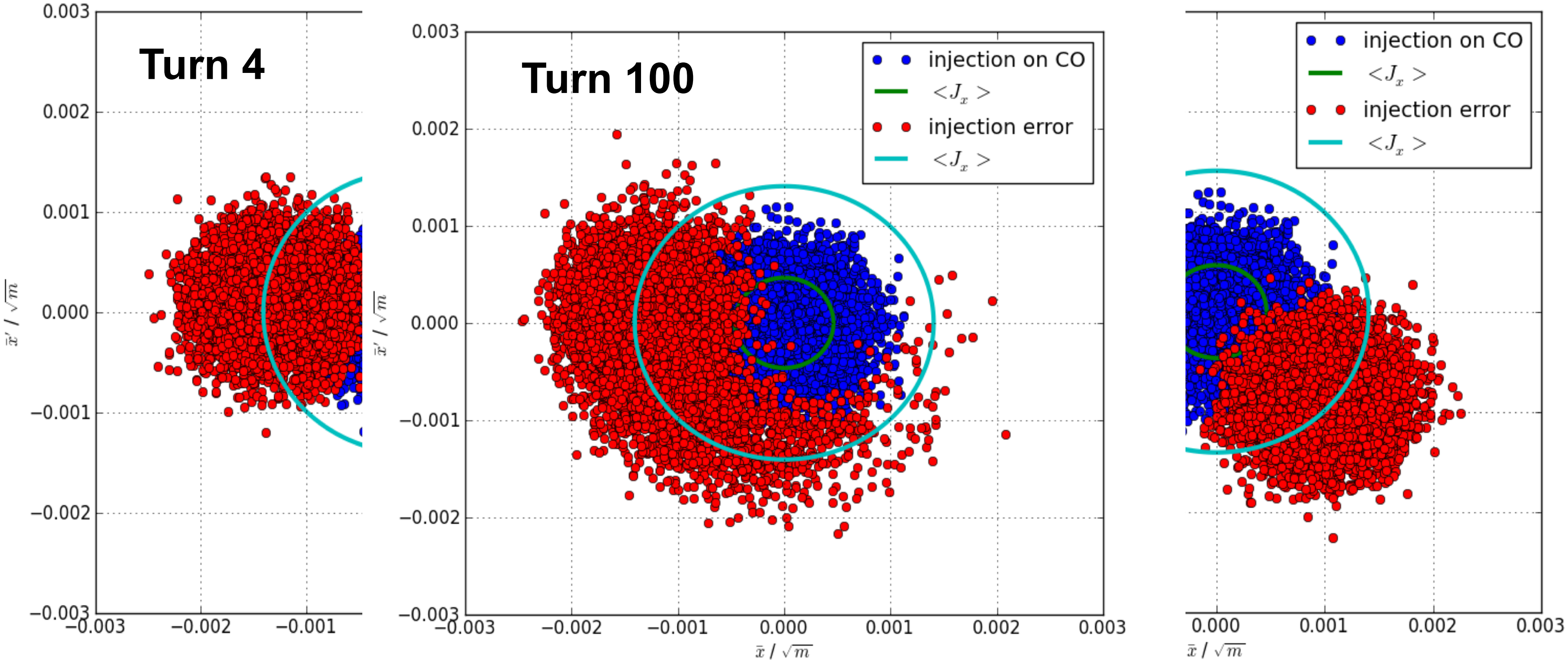
- What will happen to particle distribution and hence emittance?



- The beam is filamenting….

Steering error – non-linear machine

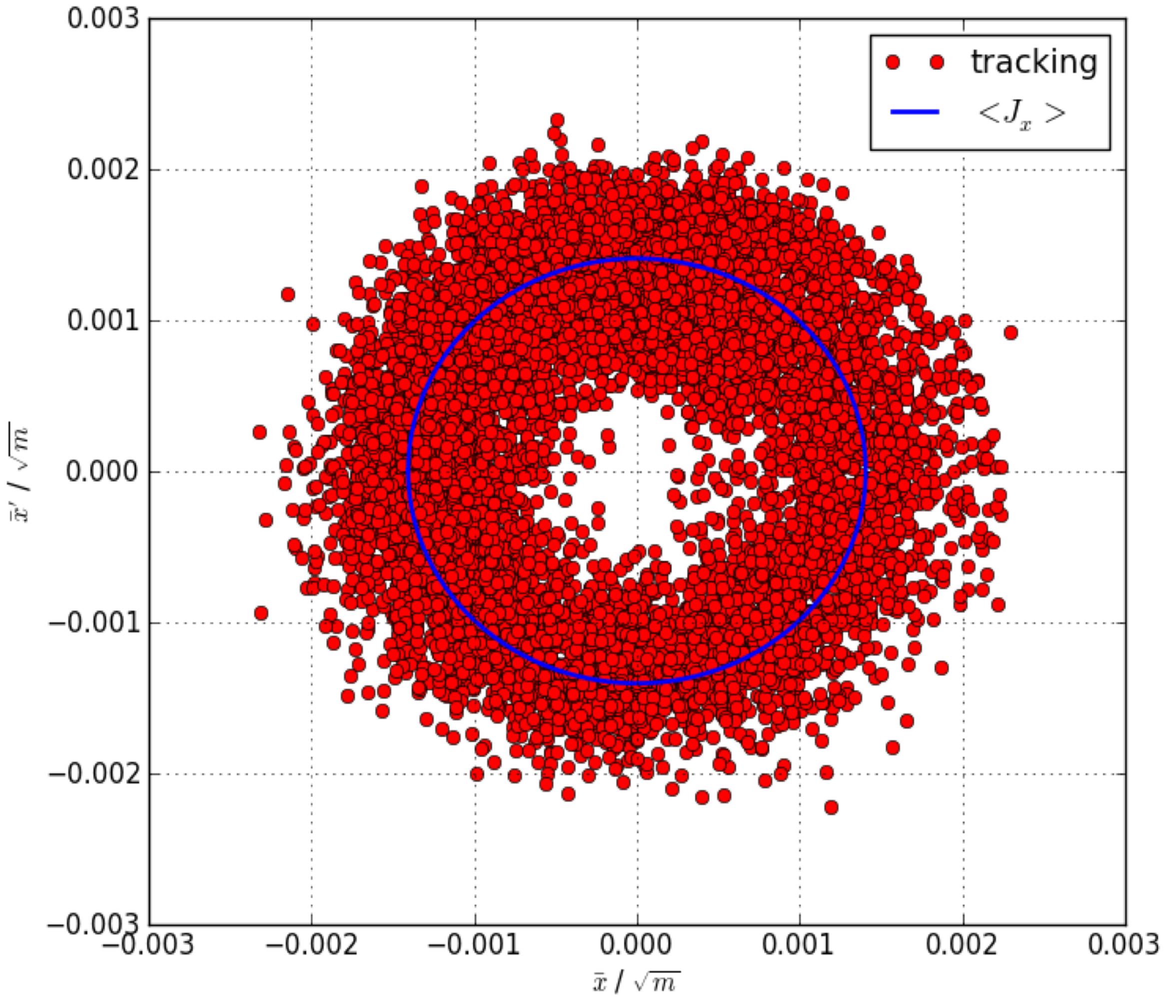
- What will happen to particle distribution and hence emittance?



- The beam is filamenting….

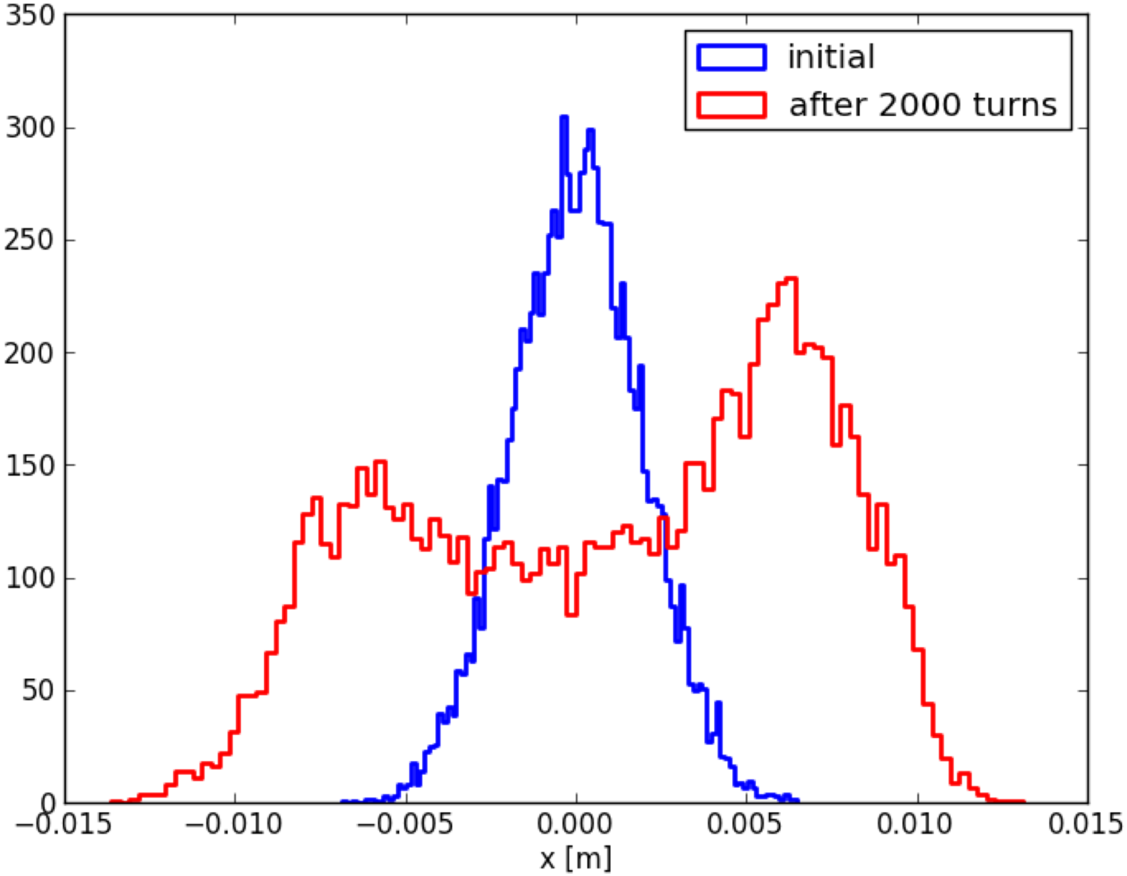
Steering error – non-linear machine

- Phase-space after an even longer time



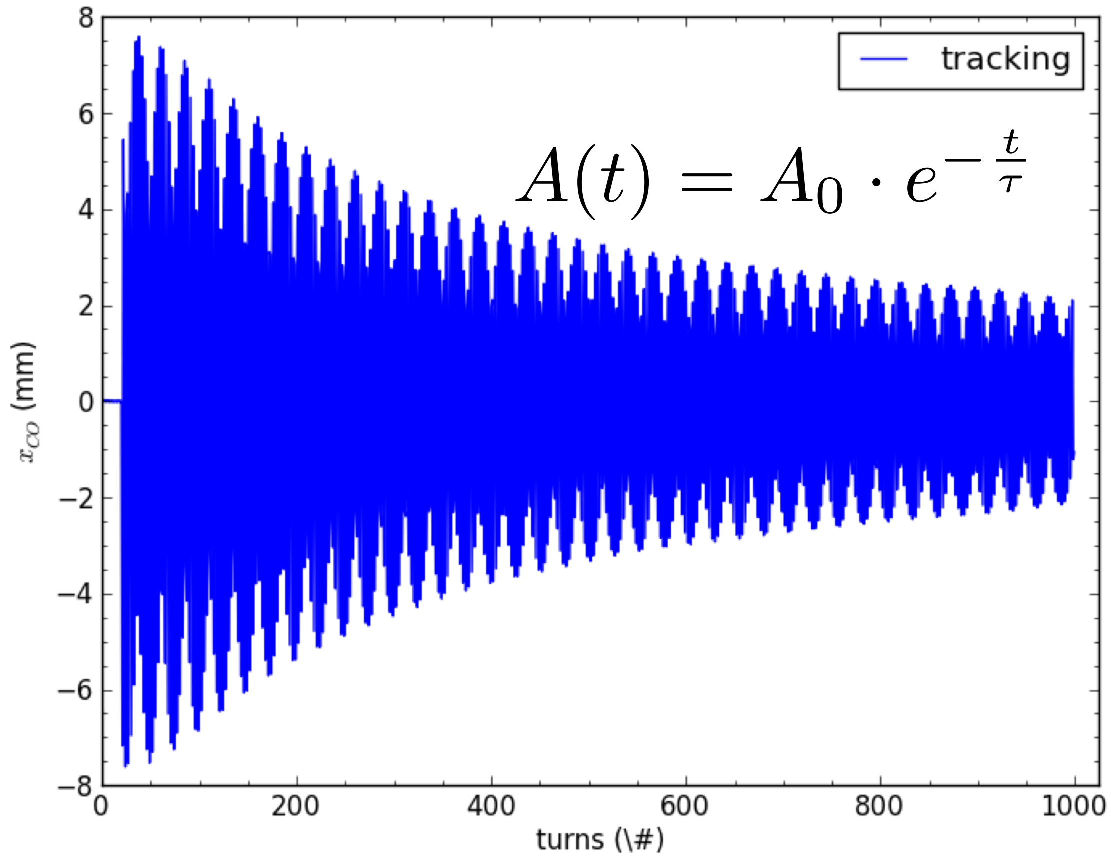
Steering error – non-linear machine

- Generation of non-Gaussian distributions:
  - Non-Gaussian tails



Injection oscillations

- Oscillation of centroid decays in amplitude
- **Time constant of exponential decay: filamentation time τ**



Injection oscillations

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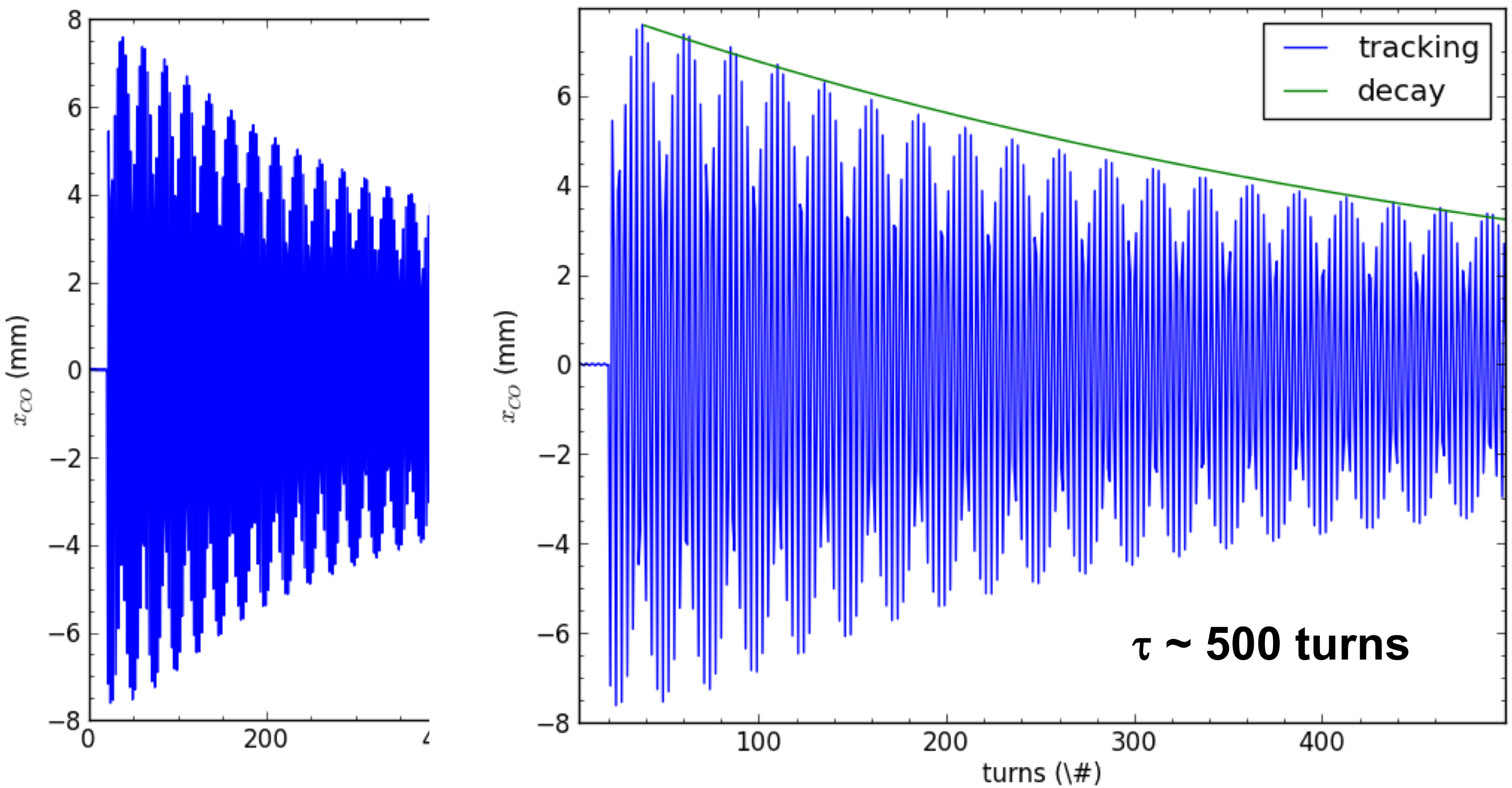
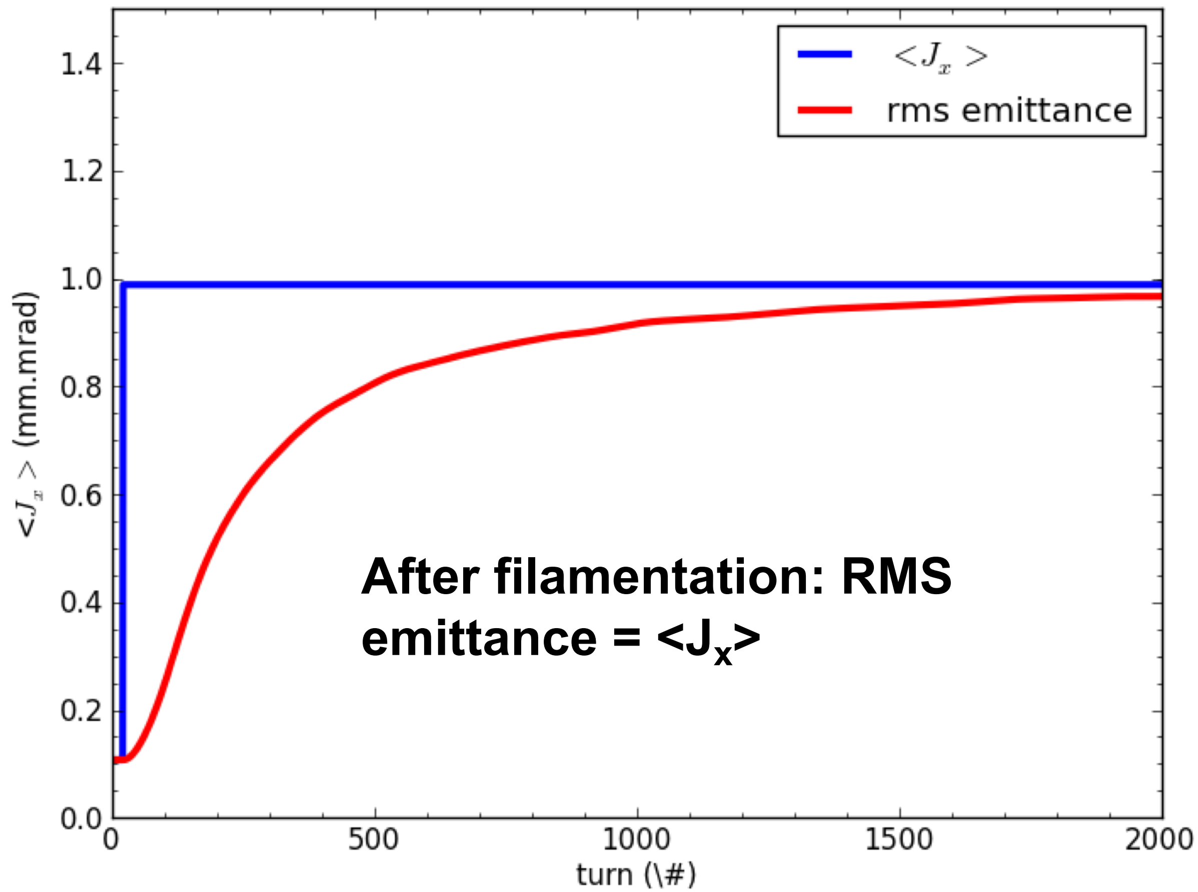


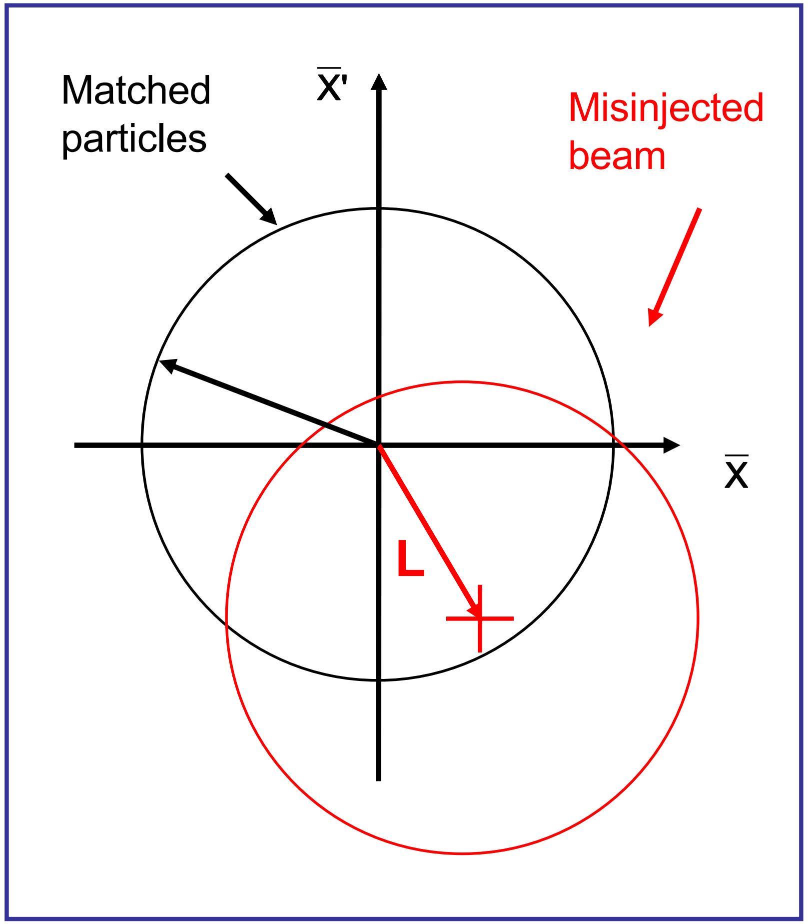
Steering error – non-linear machine

- How does $<J_x>$ behave for steering error in non-linear machine?
- And what about the rms emittance



Calculate blow-up from steering error

- Consider a collection of particles
- The beam can be injected with a error in angle and position.
- For an injection error Δa (in units of sigma = $\sqrt{\beta\varepsilon}$) the mis-injected beam is offset in normalised phase space by $L = \Delta a\sqrt{\varepsilon}$



Blow-up from steering error

- The new particle coordinates in normalised phase space are

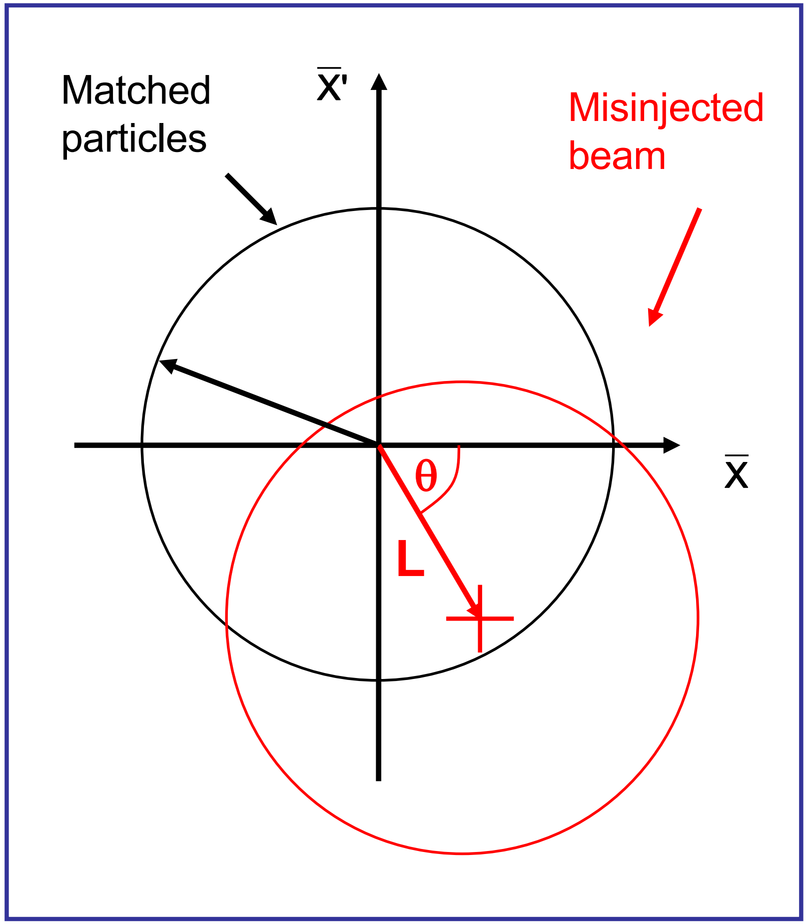
$$\overline{x}_{new} = \overline{x}_0 + L\cos\theta$$

$$\overline{x}'_{new} = \overline{x}'_0 + L\sin\theta$$

- From before we know…

$$2J_x = \overline{x}^2 + \overline{x}'^2$$

$$\varepsilon_x = \langle J_x \rangle$$



Blow-up from steering error

- So if we plug in the new coordinates….

$$\begin{aligned} 2J_{new} &= \overline{x}^2_{new} + \overline{x}'^2_{new} = (\overline{x}_0 + L\cos\theta)^2 + (\overline{x}'_0 + L\sin\theta)^2 \\ &= \overline{x}^2_0 + \overline{x}'^2_0 + 2L(\overline{x}_0\cos\theta + \overline{x}'_0\sin\theta) + L^2 \end{aligned}$$

$$\begin{aligned} 2\langle J_{new}\rangle &= \langle\overline{x}^2_0\rangle + \langle\overline{x}'^2_0\rangle + \langle 2L(\overline{x}_0\cos\theta + \overline{x}'_0\sin\theta)\rangle + L^2 \\ &= 2\varepsilon_0 + 2L(\underbrace{\cancel{\langle\overline{x}_0\cos\theta\rangle}}_{0} + \underbrace{\cancel{\langle\overline{x}'_0\sin\theta\rangle}}_{0}) + L^2 \\ &= 2\varepsilon_0 + L^2 \end{aligned}$$

- Giving for the emittance increase

$$\boxed{\begin{aligned} \varepsilon_{new} &= \langle J_{new}\rangle = \varepsilon_0 + L^2/2 \\ &= \varepsilon_0(1 + \Delta a^2/2) \end{aligned}}$$

Blow-up from steering error

$$\frac{\varepsilon}{\varepsilon_0} = 1 + \frac{1}{2}\frac{\Delta x^2 + (\beta\Delta x' + \alpha\Delta x)^2}{\beta\varepsilon_0}$$

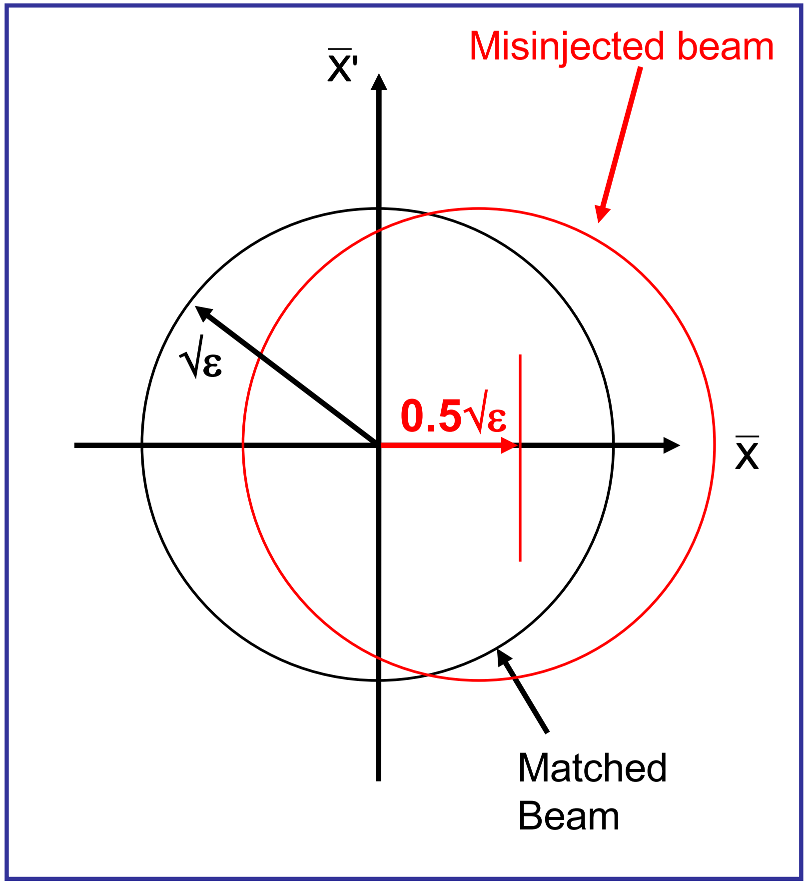
A numerical example….

Consider an offset Δa of 0.5 sigma for injected beam

$$\varepsilon_{new} = \varepsilon_0\left(1 + \Delta \mathbf{a}^2 / 2\right)$$

$$= 1.125\varepsilon_0$$

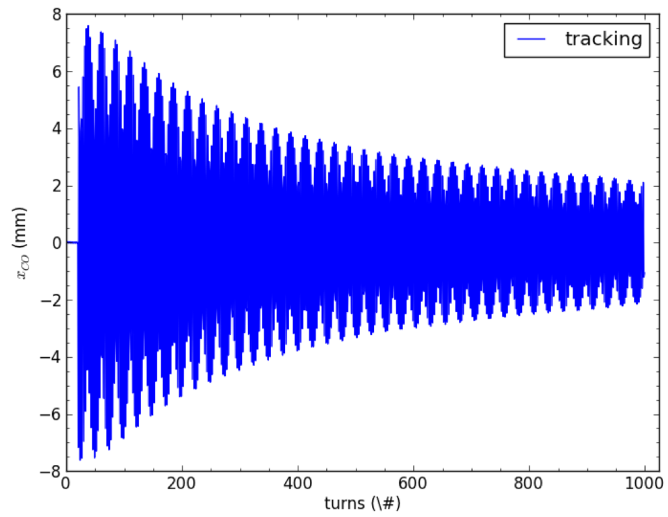
For nominal LHC beam:
ε_{norm} = 3.5 μm
allowed growth through LHC cycle ~ 10 %



How to correct injection oscillations?

- Injection oscillations:

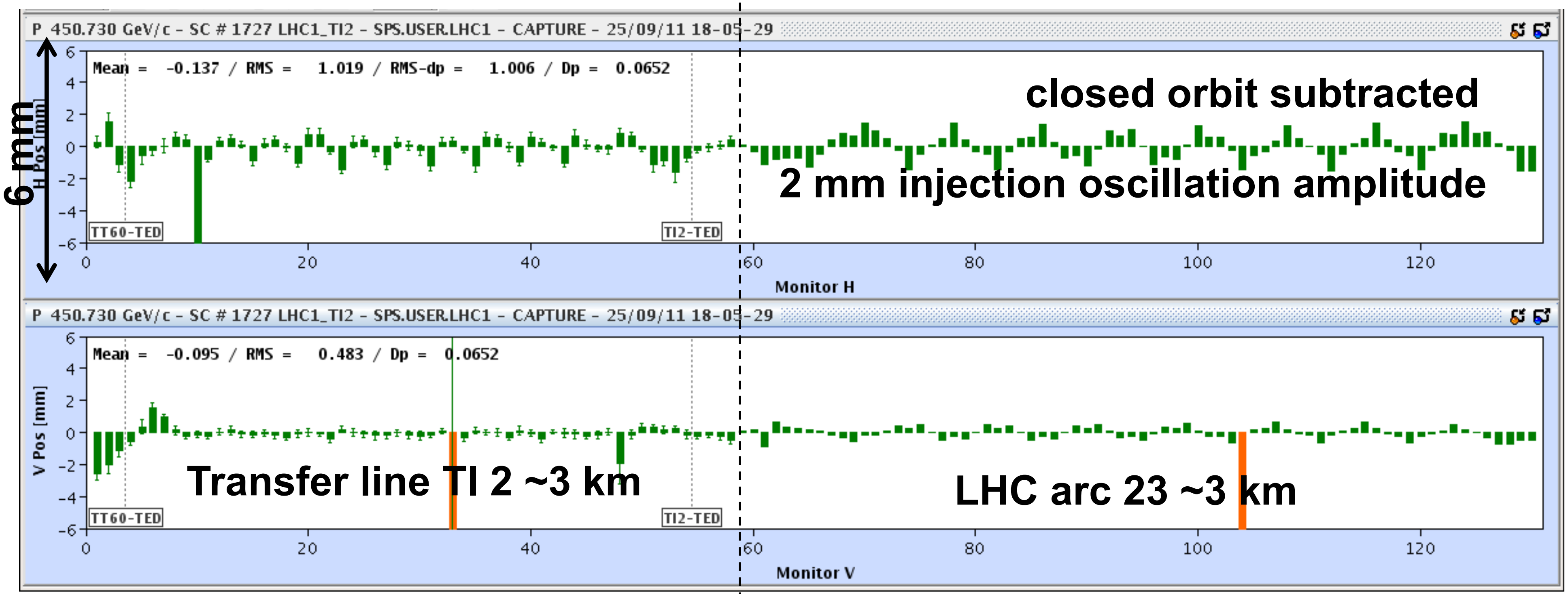
Beam position measured at one BPM over many turns



- Instead of looking at one BPM over many turns, look at first turn for many BPMs
  - i.e. difference of first turn and closed orbit.
  - Treat the first turn of circular machine like transfer line for correction
  - Other possibility is measure first and second turn and minimize the difference between in algorithm

Example: LHC injection of beam 1

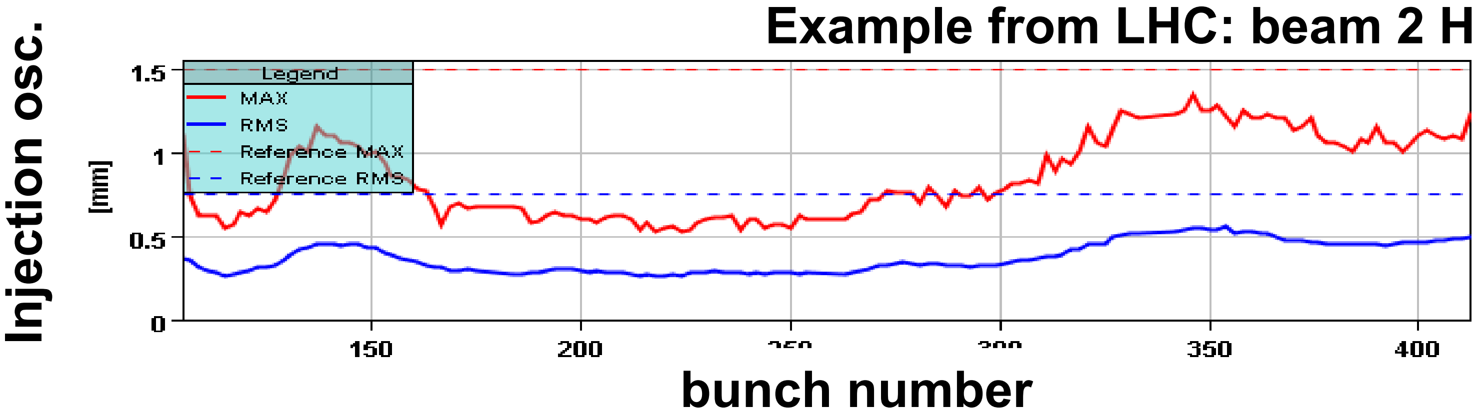
- Injection oscillation display from the LHC control room.
- The first 3 km of the LHC treated like extension of transfer line
- Only correctors in transfer line are used for correction



Injection point in LHC IR2

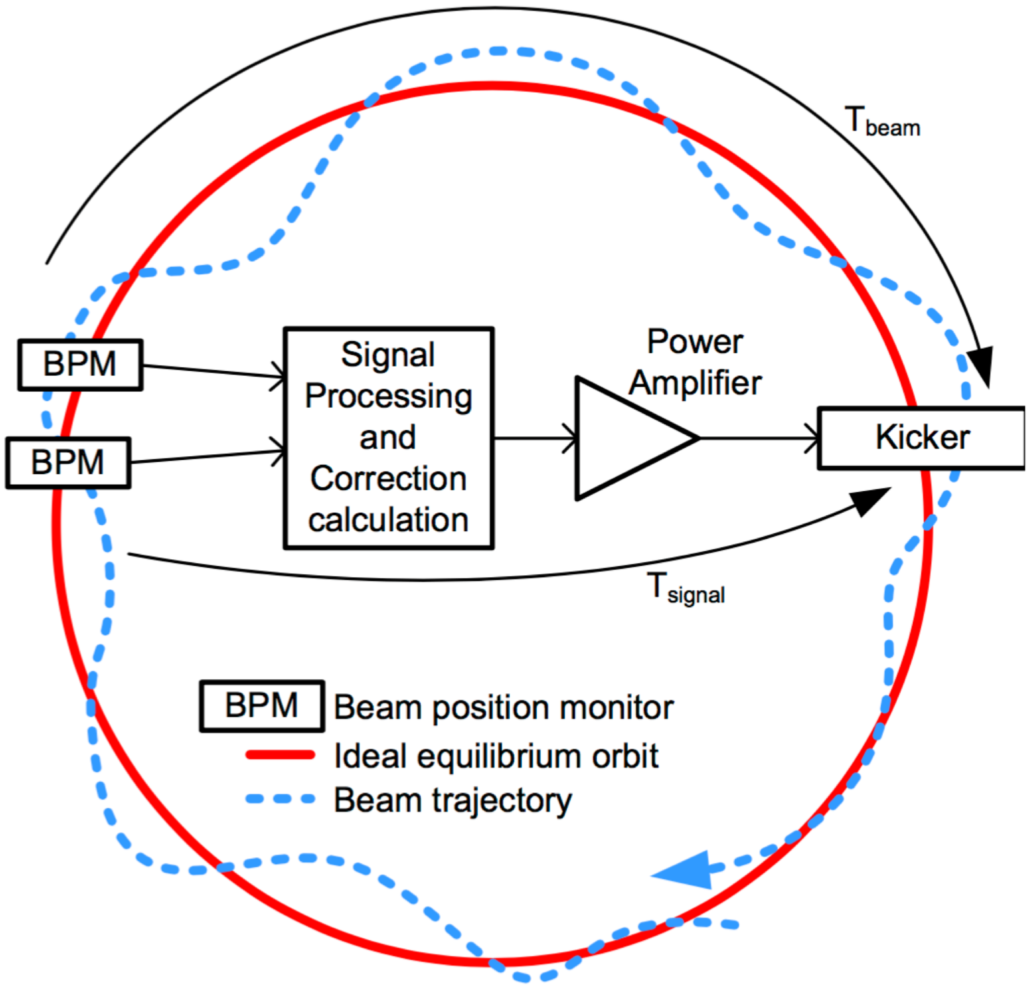
How to correct injection oscillations?

- What if there are shot-by-shot changes or bunch-by-bunch changes of the injection steering errors?
- Previous method: remove only static errors
- What if there are bunch-by-bunch differences in injected train of injection oscillations?



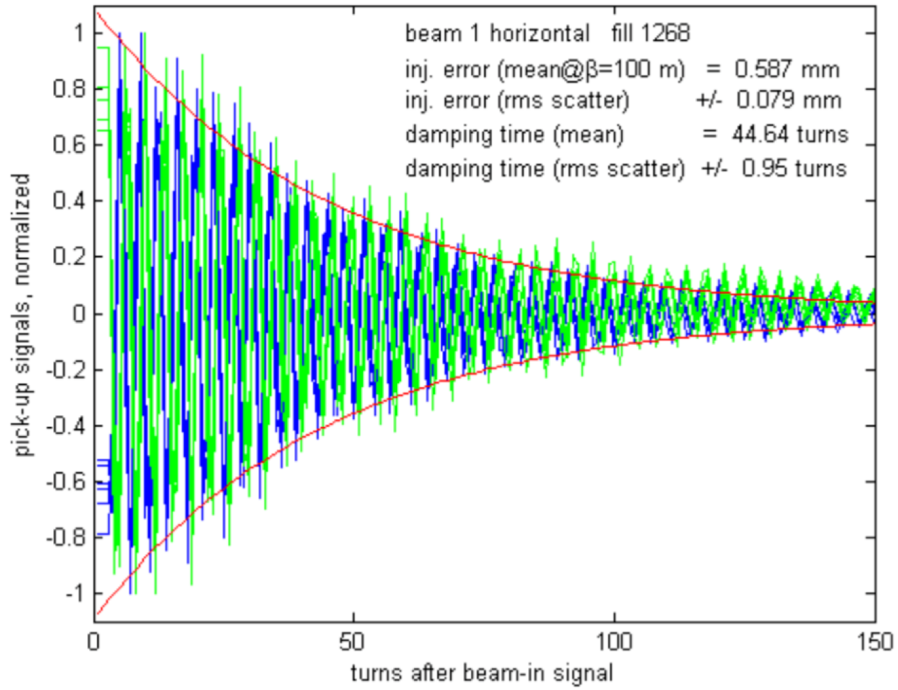
- → **transverse feedback (damper)**
  - **Sufficient bandwidth to deal with bunch-by-bunch differences**
- **Damping time has to be faster than filamentation time**

Transverse feedback system



$$T_{signal} = T_{beam} + n\,T_{rev}$$

LHC injection oscillation damping

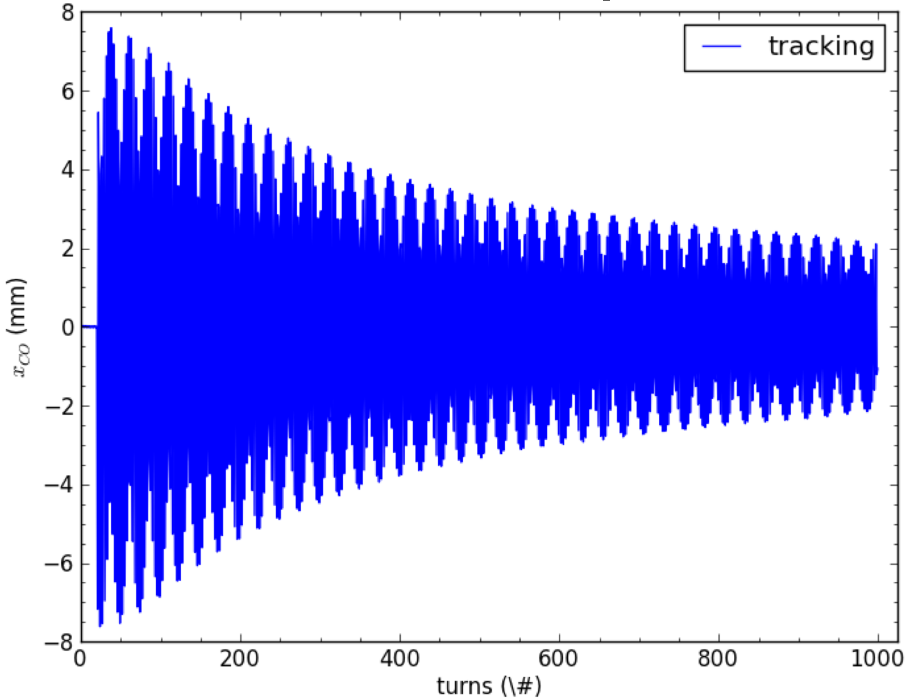


Steering error - damper

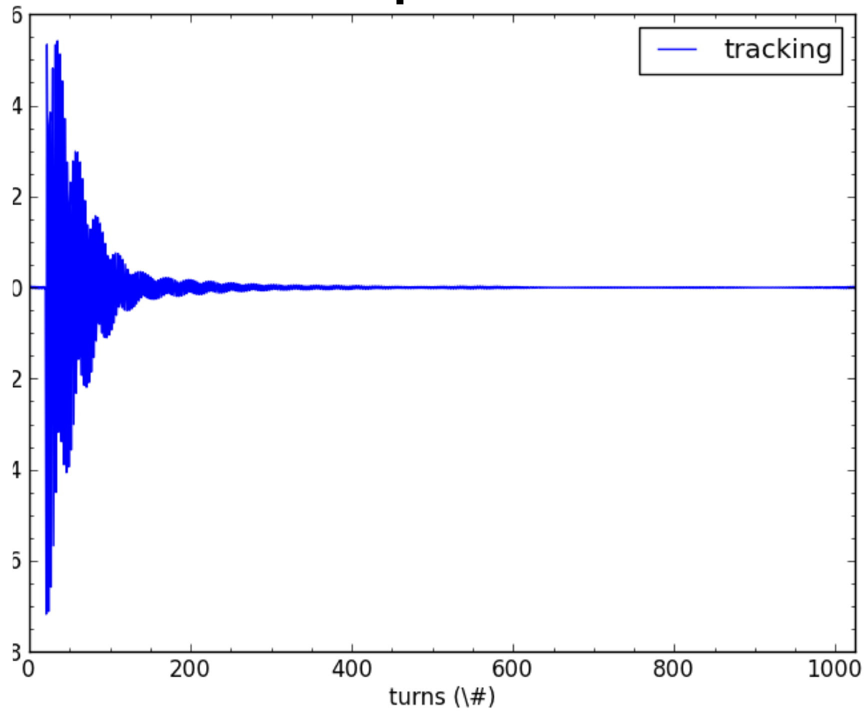
- Damper in simulation: injection oscillations damped faster than through filamentation

Same injection error

Without damper

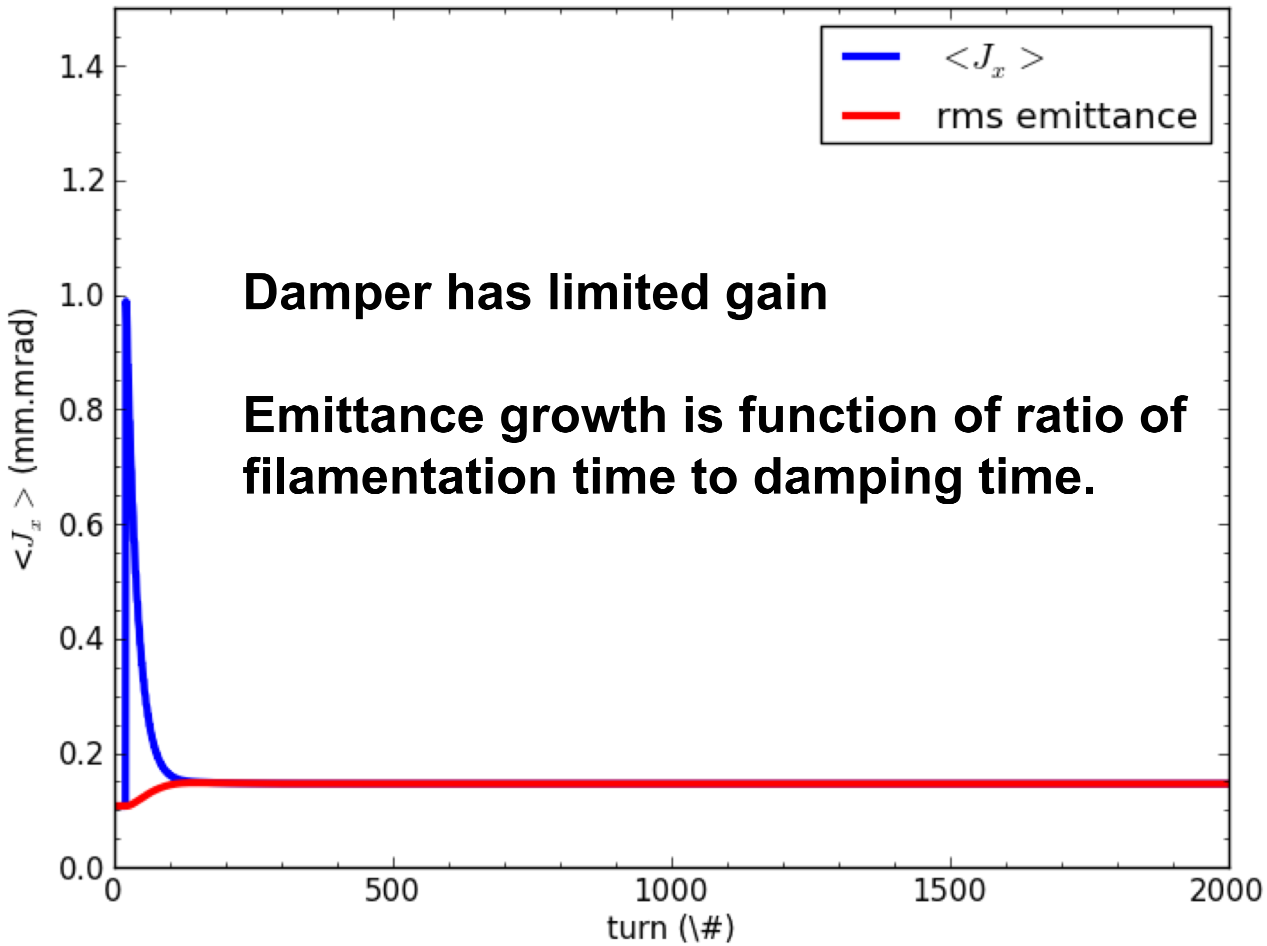


With damper



Steering error - damper

- And what about the emittance?



Steering error -damper

- Emittance growth with damper for damping time τ_d

Damper has limited gain

Emittance growth is function of ratio of filamentation time to damping time.

$$\frac{\varepsilon}{\varepsilon_0} = 1 + \frac{1}{2}\frac{\Delta x^2 + (\beta \Delta x' + \alpha \Delta x)^2}{\beta \varepsilon_0}\left(\frac{1}{1+\tau_{DC}/\tau_d}\right)^2$$

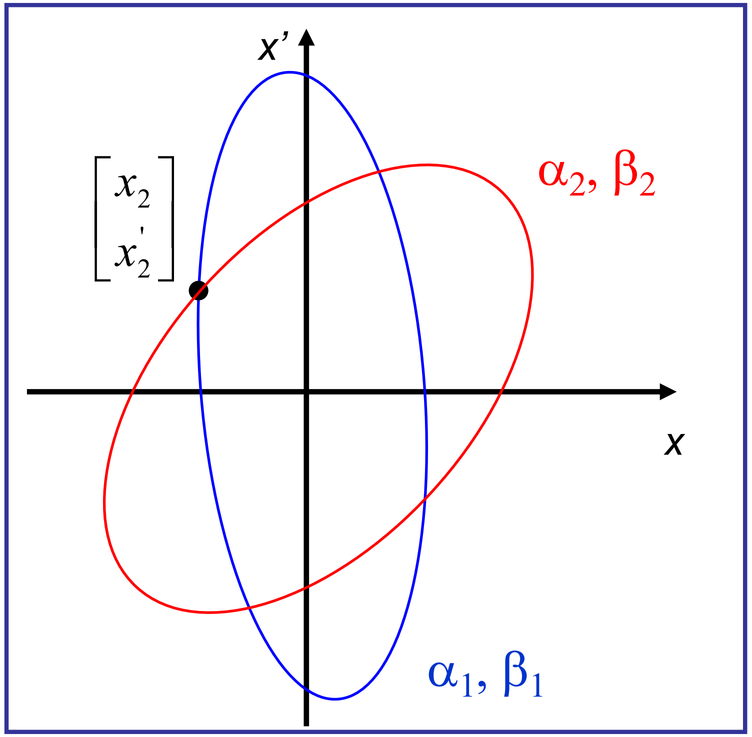
L. Vos, **Transverse emittance blow-up from double errors in proton machines**, CERN, 1998

Blow-up from betatron mismatch

- Optical errors occur in transfer line and ring, such that the beam can be injected with a mismatch.

- The shape of the injected beam corresponds to different α, β than the closed solution of the ring.

- At the moment of the injection the area in phase space might be the same



real phase-space

- Filamentation will produce an emittance increase.

Blow-up from betatron mismatch

The coordinates of the ellipse: betatron oscillation

$$x_2 = \sqrt{2\beta_2 J_x}\cos\phi \qquad x'_2 = -\sqrt{\frac{2J_x}{\beta_2}}(\sin\phi + \alpha_2\cos\phi)$$

applying the normalising transformation to the matched space

$$\begin{bmatrix} \overline{\mathrm{X}}_2 \\ \overline{\mathrm{X}}'_2 \end{bmatrix} = \sqrt{\frac{1}{\beta_1}} \cdot \begin{bmatrix} 1 & 0 \\ \alpha_1 & \beta_1 \end{bmatrix} \cdot \begin{bmatrix} x_2 \\ x'_2 \end{bmatrix}$$

an ellipse is obtained in normalised phase space

$$2J_x = \overline{x}_2^2\left[\frac{\beta_1}{\beta_2} + \frac{\beta_2}{\beta_1}\left(\alpha_1 - \alpha_2\frac{\beta_1}{\beta_2}\right)^2\right] + \overline{x'}_2^2\frac{\beta_2}{\beta_1} - 2\overline{x}_2\overline{x}'_2\left[\frac{\beta_2}{\beta_1}\left(\alpha_1 - \alpha_2\frac{\beta_1}{\beta_2}\right)\right]$$

characterised by γ_{new}, β_{new} *and* α_{new}, where

$$\alpha_{new} = \frac{-\beta_2}{\beta_1}\left(\alpha_1 - \alpha_2\frac{\beta_1}{\beta_2}\right), \qquad \beta_{new} = \frac{\beta_2}{\beta_1}, \quad \gamma_{new} = \frac{\beta_1}{\beta_2} + \frac{\beta_2}{\beta_1}\left(\alpha_1 - \alpha_2\frac{\beta_1}{\beta_2}\right)^2$$

Blow-up from betatron mismatch

The coordinates of the ellipse: betatron oscilation

$$x_2 = \sqrt{2\beta_2 J_x} \cos\phi \qquad x_2' = -\sqrt{\frac{2J_x}{\beta_2}}(\sin\phi + \alpha_2 \cos\phi)$$

applying the normalising transformation to the matched space

$$\begin{bmatrix} \overline{X}_2 \\ \overline{X}'_2 \end{bmatrix} = \sqrt{\frac{1}{\beta_1}} \cdot \begin{bmatrix} 1 & 0 \\ \alpha_1 & \beta_1 \end{bmatrix} \cdot \begin{bmatrix} x_2 \\ x'_2 \end{bmatrix}$$

Remember:

$$2J_x = \gamma \cdot x^2 + 2\alpha \cdot x \cdot x' + \beta x'^2$$

an ellipse is obtained in normalised phase space

$$2J_x = \overline{x}_2^2\left[\frac{\beta_1}{\beta_2} + \frac{\beta_2}{\beta_1}\left(\alpha_1 - \alpha_2\frac{\beta_1}{\beta_2}\right)^2\right] + \overline{x'}_2^2\frac{\beta_2}{\beta_1} - 2\overline{x}_2\overline{x}'_2\left[\frac{\beta_2}{\beta_1}\left(\alpha_1 - \alpha_2\frac{\beta_1}{\beta_2}\right)\right]$$

characterised by γ_{new}, β_{new} *and* α_{new}, where

$$\alpha_{new} = \frac{-\beta_2}{\beta_1}\left(\alpha_1 - \alpha_2\frac{\beta_1}{\beta_2}\right), \qquad \beta_{new} = \frac{\beta_2}{\beta_1}, \quad \gamma_{new} = \frac{\beta_1}{\beta_2} + \frac{\beta_2}{\beta_1}\left(\alpha_1 - \alpha_2\frac{\beta_1}{\beta_2}\right)^2$$

Blow-up from betatron mismatch

From the general ellipse properties, see [4]

$$a = \frac{A}{\sqrt{2}}\left(\sqrt{H+1} + \sqrt{H-1}\right), \qquad b = \frac{A}{\sqrt{2}}\left(\sqrt{H+1} - \sqrt{H-1}\right)$$

$$A = \sqrt{2J}$$

where

$$H = \frac{1}{2}\left(\gamma_{new} + \beta_{new}\right)$$

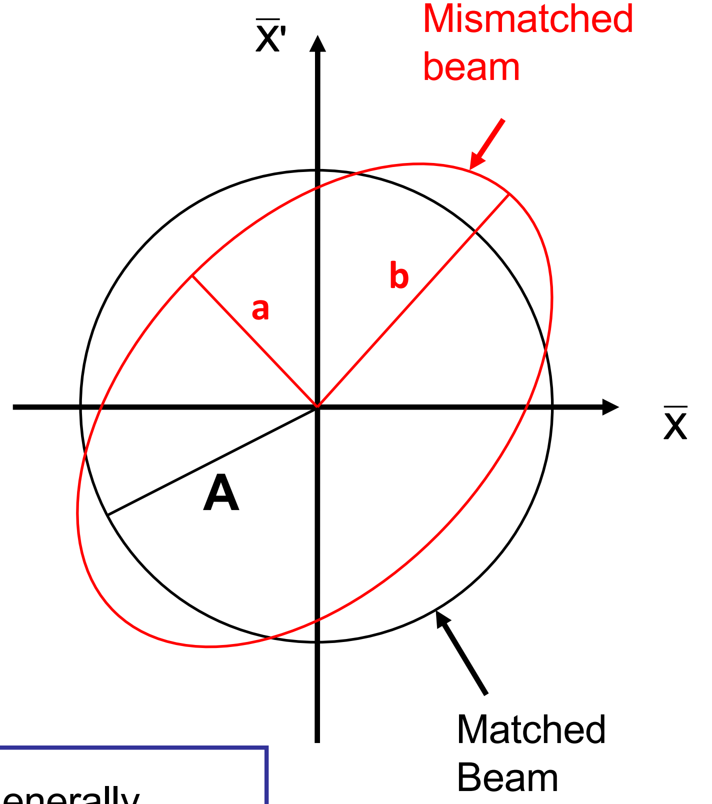
$$= \frac{1}{2}\left(\frac{\beta_1}{\beta_2} + \frac{\beta_2}{\beta_1}\left(\alpha_1 - \alpha_2\frac{\beta_1}{\beta_2}\right)^2 + \frac{\beta_2}{\beta_1}\right)$$

giving

$$\lambda = \frac{1}{\sqrt{2}}\left(\sqrt{H+1} + \sqrt{H-1}\right), \qquad \frac{1}{\lambda} = \frac{1}{\sqrt{2}}\left(\sqrt{H+1} - \sqrt{H-1}\right)$$

$$\overline{x}_{new} = \lambda \cdot A\sin(\phi + \phi_1)$$

$$\overline{x}'_{new} = \frac{1}{\lambda} \cdot A\cos(\phi + \phi_1)$$



generally

$$a = A/\lambda$$
$$b = A \cdot \lambda$$

Blow-up from betatron mismatch

We can evaluate the square of the distance of a particle from the origin as

$$2J_{new} = \overline{x}_{new}^2 + \overline{x}_{new}'^2 = \lambda^2 \cdot 2J_0 \sin^2(\phi + \phi_1) + \tfrac{1}{\lambda^2} 2J_0 \cos^2(\phi + \phi_1)$$

The new emittance is the average over all phases

$$\begin{aligned} \varepsilon_{new} = \langle J_{new} \rangle &= \tfrac{1}{2}\left(\lambda^2 \langle 2J_0 \sin^2(\phi + \phi_1)\rangle + \tfrac{1}{\lambda^2}\langle 2J_0 \cos^2(\phi + \phi_1)\rangle\right) \\ &= \langle J_0 \rangle \left(\lambda^2 \langle \cancel{\sin^2(\phi + \phi_1)} \rangle + \tfrac{1}{\lambda^2}\langle \cancel{\cos^2(\phi + \phi_1)} \rangle\right) \\ &= \tfrac{1}{2}\varepsilon_0\left(\lambda^2 + \tfrac{1}{\lambda^2}\right) \end{aligned}$$

0.5 0.5

If we' re feeling diligent, we can substitute back for λ to give

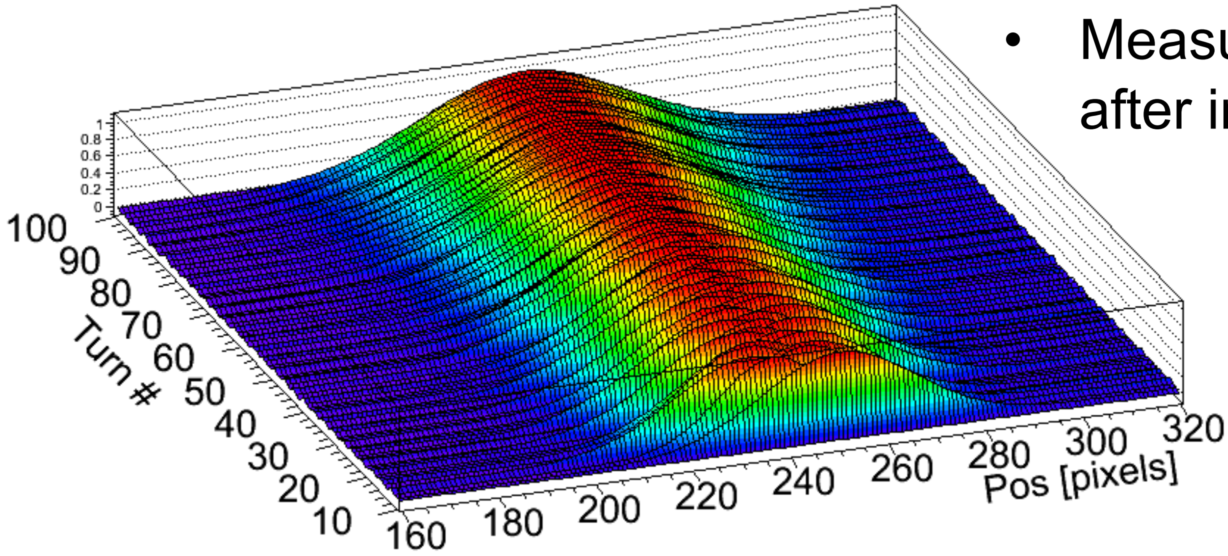
$$\varepsilon_{new} = \frac{1}{2}\varepsilon_0\left(\lambda^2 + \frac{1}{\lambda^2}\right) = H\varepsilon_0 = \frac{1}{2}\varepsilon_0\left(\frac{\beta_1}{\beta_2} + \frac{\beta_2}{\beta_1}\left(\alpha_1 - \alpha_2\frac{\beta_1}{\beta_2}\right)^2 + \frac{\beta_2}{\beta_1}\right)$$

where subscript 1 refers to matched ellipse, 2 to mismatched ellipse.

How to measure oscillating width of distribution?

MATCHING SCREEN

- 1 OTR screen or SEM grid in the circular machine
- Measure turn-by-turn profile after injection



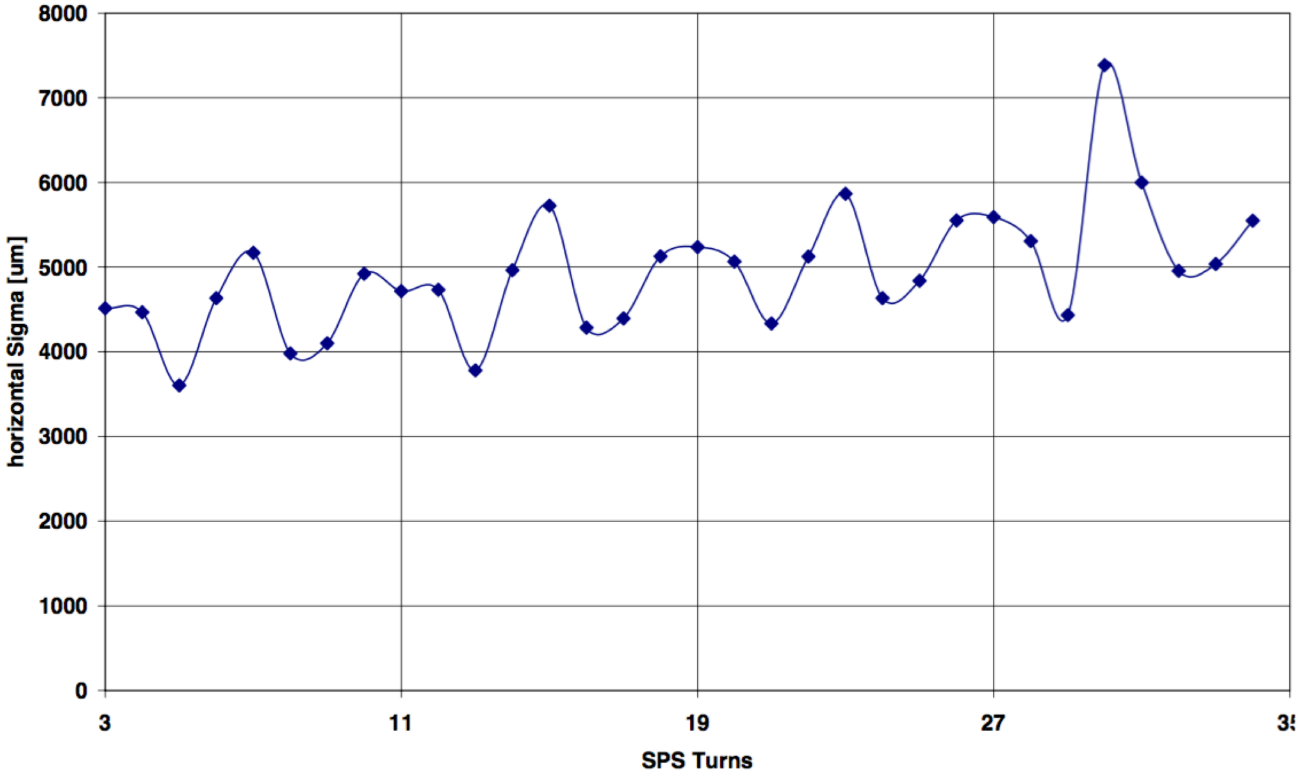
Profiles at matching monitor after injection with steering error.

Requires radiation hard fast cameras

Another limitation: only low intensity

Example of betatron mismatch measurement

- Measurement at injection into the SPS with matching monitor

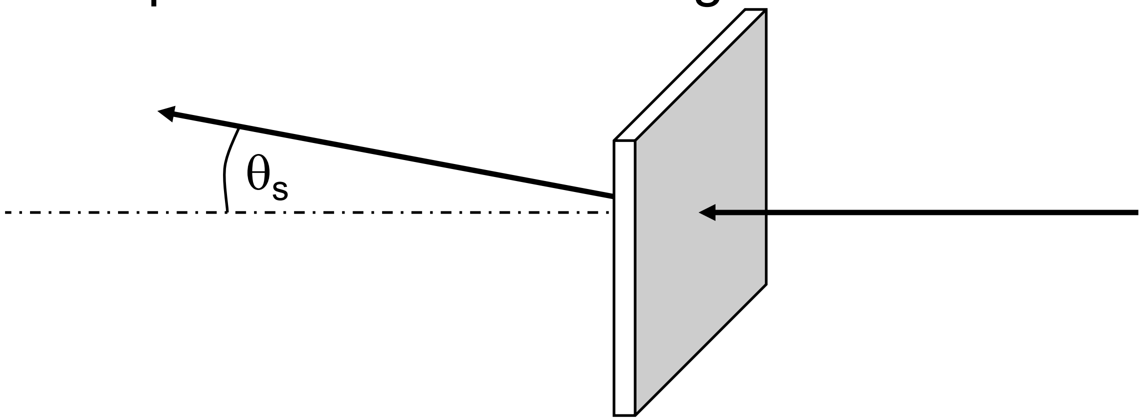


Uncorrected measured horizontal beam size versus number of turns in the SPS. The oscillation indicates mismatch, the positive slope blow-up is due to the foil

G. Arduini et al., Mismatch Measurement and Correction Tools for the PS-SPS Transfer of the 26 GeV/c LHC Beam, 1999

Blow-up from thin scatterer

- Scattering elements are sometimes required in the beam
  - Thin beam screens (Al_2O_3,Ti) used to generate profiles.
  - Metal windows also used to separate vacuum of transfer lines from vacuum in circular machines.
  - Foils are used to strip electrons to change charge state
- The emittance of the beam increases when it passes through, due to multiple Coulomb scattering.



rms angle increase: $\sqrt{\langle \theta_s^2 \rangle}[mrad] = \frac{14.1}{\beta_c p[MeV/c]} Z_{inc} \sqrt{\frac{L}{L_{rad}}} \left(1 + 0.11 \cdot \log_{10} \frac{L}{L_{rad}}\right)$

β_c = v/c, p = momentum, Z_{inc} = particle charge /e, L = target length, L_{rad} = radiation length

Blow-up from thin scatterer

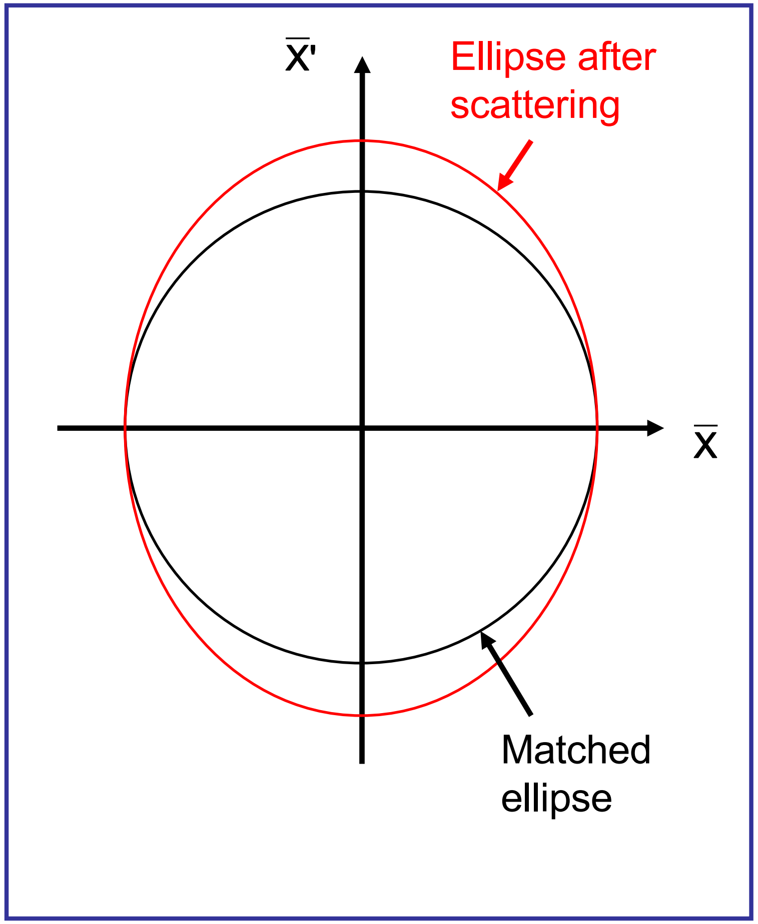
Each particles gets a random angle change θ_s but there is no effect on the positions at the scatterer

$$\overline{x}_{new} = \overline{x}_0$$

$$\overline{x}'_{new} = \overline{x}'_0 + \sqrt{\beta}\Theta_s$$

After filamentation the particles have different amplitudes and the beam has a larger emittance

$$\varepsilon = \langle J_{new} \rangle$$



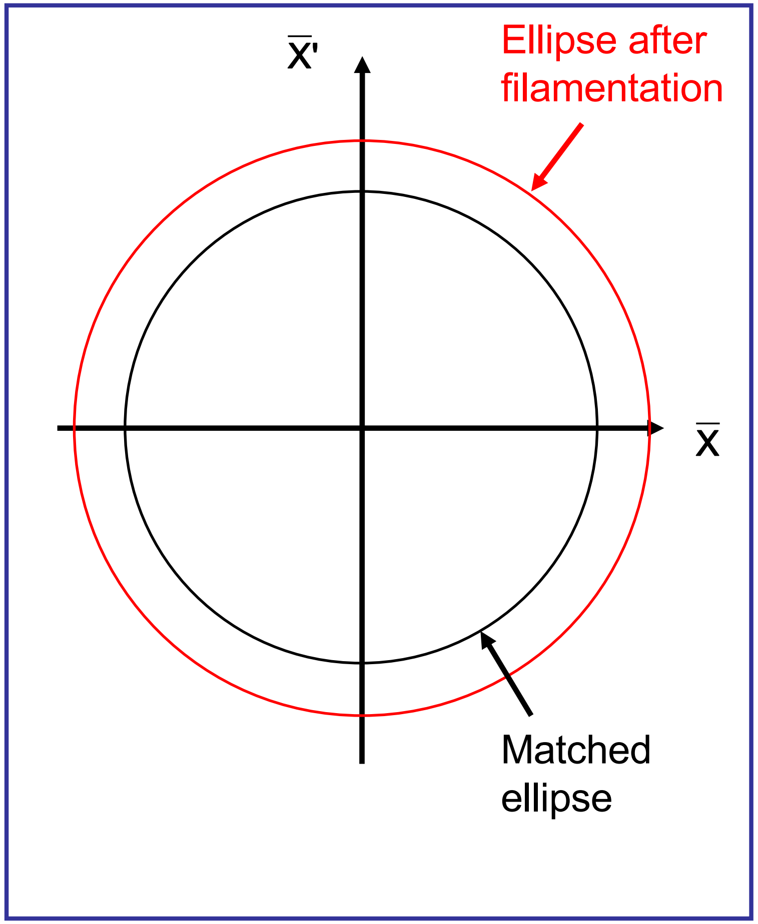
Blow-up from thin scatterer

$$
\begin{aligned}
2J_{new} &= \overline{x}_{new}^2 + \overline{x}_{new}'^2 \\
&= \overline{x}_0^2 + (\overline{x}_0' + \sqrt{\beta}\Theta_s)^2 \\
&= \overline{x}_0^2 + \overline{x}_0'^2 + 2\sqrt{\beta}(\overline{x}_0'\Theta_s) + \beta\Theta_s^2
\end{aligned}
$$

uncorrelated

$$
\begin{aligned}
2\langle J_{new}\rangle &= \langle \overline{x}_0^2\rangle + \langle \overline{x}_0'^2\rangle + 2\sqrt{\beta}\langle \overline{x}_0'\Theta_s\rangle + \beta\langle\Theta_s^2\rangle \\
&= 2\varepsilon_0 + 2\sqrt{\beta}\underbrace{\cancel{\langle \overline{x}_0'\rangle}}_{0}\langle\Theta_s\rangle + \beta\langle\Theta_s^2\rangle \\
&= 2\varepsilon_0 + \beta\langle\Theta_s^2\rangle
\end{aligned}
$$

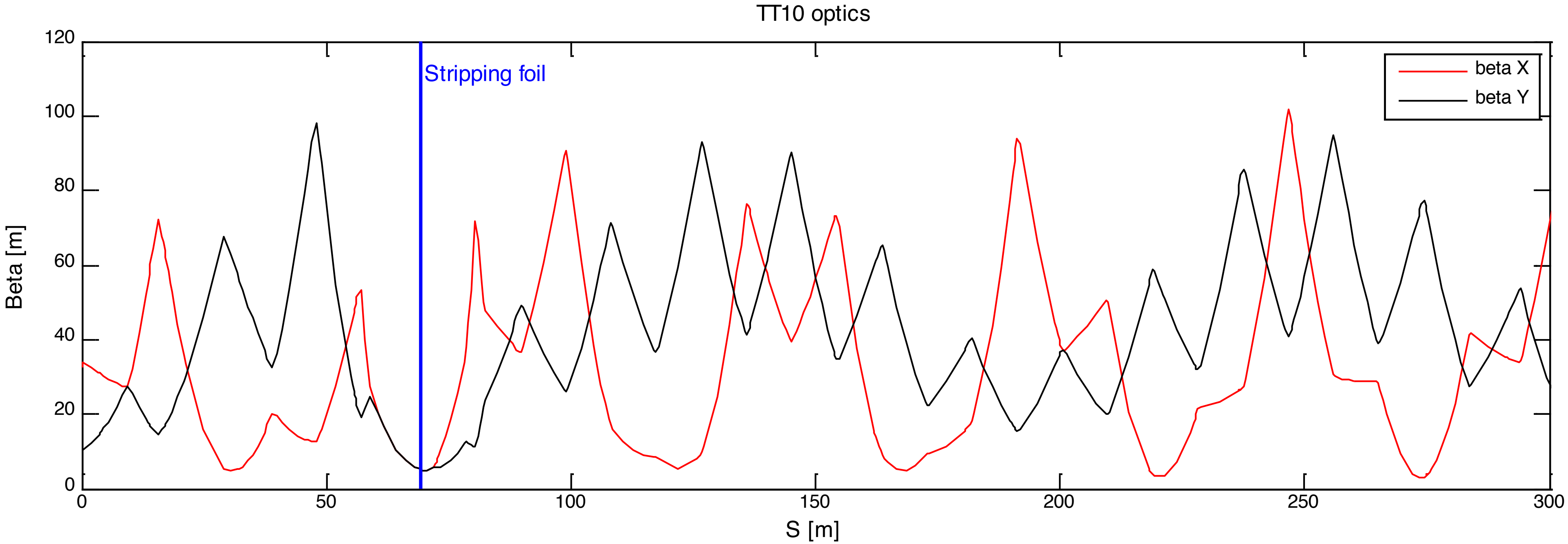
$$
\boxed{\varepsilon_{new} = \varepsilon_0 + \frac{\beta}{2}\langle\theta_s^2\rangle}
$$



Need to keep β small to minimise blow-up (small β means large spread in angles in beam distribution, so additional angle has small effect on distn.)

Blow-up from charge stripping foil

- For LHC heavy ions, Pb^{54+} is stripped to Pb^{82+} at 4.25GeV/u using a 0.8mm thick Al foil, in the PS to SPS line
- $\Delta\varepsilon$ is minimised with low-β insertion (β_{xy} ~5 m) in the transfer line
- Emittance increase expected is about 8%



Other mismatch effects at injection

- Dispersion mismatch

$$\frac{\varepsilon}{\varepsilon_0} = 1 + \frac{1}{2}\frac{\Delta D^2 + (\beta \Delta D' + \alpha \Delta D)^2}{\beta \varepsilon_0}\left(\frac{\Delta p}{p}\right)^2$$

- Energy error

$$\frac{\varepsilon}{\varepsilon_0} = 1 + \frac{1}{2}\frac{D^2}{\beta \varepsilon_0}\left(\frac{\Delta p}{p}\right)^2$$

- Geometrical mismatch: tilt angle Θ between beam reference systems at injection point: e.g. horizontal plane

$$\frac{\varepsilon_x}{\varepsilon_{x0}} = 1 + \frac{1}{2}(\beta_x \gamma_y + \beta_y \gamma_x - 2\alpha_x \alpha_y - 2)\sin^2\Theta$$

Scattering on residual gas

- What about the vacuum requirements in a storage ring to contain emittance blow-up?
- Use considerations from blow-up on thin scatterer →

$$\boxed{\varepsilon_{new} = \varepsilon_0 + \frac{\beta}{2}\left\langle \theta_s^2 \right\rangle}$$

- RMS scattering angle increase was

$$\sqrt{\left\langle \theta_s^2 \right\rangle}[mrad] = \frac{14.1}{\beta_c p[MeV/c]} Z_{inc} \sqrt{\frac{L}{L_{rad}}} \left(1 + \cancel{0.11 \cdot \log_{10} \frac{L}{L_{rad}}} \right)$$

Neglect this

- → Need L and L_{rad}
- Traversed length is straight forward: $L = \beta_c c t$

Scattering on residual gas

- L_{rad} for gas depends on the pressure
- Example: pure nitrogen (N_2)

$$L_{rad} = \frac{327[m]}{P[Torr]/760}$$

- For momentum (proton mass m_{p0}, A mass number) :

$$p = m_0 \cdot \gamma \cdot \beta_c \cdot c = m_{p0} \cdot A_{inc} \cdot \gamma \cdot \beta_c \cdot c$$

$$\boxed{\Delta\varepsilon_{x,y} \approx 0.14 \frac{Z_{inc}^2}{A_{inc}^2} \overline{\beta}_{x,y} [\mathrm{m}] \frac{\mathrm{P[Torr]t[s]}}{\beta_{\mathrm{c}}^3 \gamma^2}}$$

- Residual atmosphere with different gas components of partial pressures P_i, define N_2 equivalent pressure for Coulomb scattering

$$P_{N_2 equ} = \sum P_i \frac{L_{rad,N_2}}{L_{rad,i}}$$

*W. Hardt, **A few simple expressions for checking vacuum requirements in proton synchrotrons**, internal report CERN ISR-300/GS/68-11.*

Scattering on residual gas

- In case of changing β_c, γ_c due to acceleration need to integrate to get total emittance growth

- For different radiation length see:

Particle Data Group, **Review of particle physics**, Eur. Phys. J. 3 (1998) 144. (**Chapter 23, Passage of particles through matter)**

Power supply ripples

- Idea simply: what is the rms kick one gets due to dipole **field error** and use this in the formula already established for thin scatterer
  - Kicks add up statistically in case of true (i.e. "white") noise: after n turns:

    $n = f_{rev} \cdot t$

$$\Delta\varepsilon_{x,y} = \frac{1}{2}\beta_{x,y}\theta_{rms}^2 f_{rev} t$$

- Quadrupoles: A. Chao and D. Douglas**, "Preliminary estimate of emittance growth due to position jitter and magnet strength noise in quadrupole and sextupole magnets,"** SSC Laboratory preprint No.SSC-N-34(1985).
- V. Lebedev, V. Parkhomchuk, V. Shiltsev, G. Stupakov**, Emittance Growth due to Noise and its Suppression with the Feedback System in Large Hadron Colliders**, SSCL-Preprint-188, March 1993
- If frequency spectrum is not constant, only noise at frequencies at the tune sidebands have an effect:

$$f = (m \pm Q) f_{rev}$$

  - For the lowest sideband: ~ 1 - 100 kHz (depending on the size of the machine)

Power supply ripples

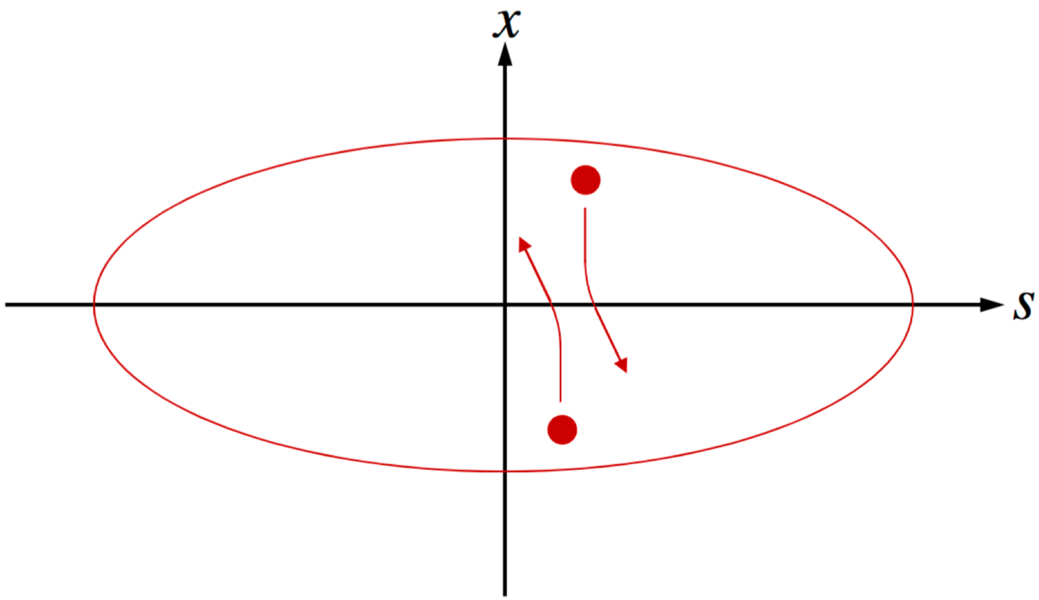
- The noise of the power supplies is not equivalent to the noise the beam sees.
  - Need to know the transfer function $H(\omega)$. Magnet and connections, vacuum chambers act like filter.
  - Assuming only magnet inductance L: filter factor

$$F \propto \frac{1}{(\omega L)^2}$$

  - Thus luckily the dangerous frequencies are very often strongly suppressed.

Intra-beam scattering

- Intra-beam scattering formulated by Piwinski (1974), Bjorken and Mtingwa (1983)
- Particles within a bunch collide while doing their betatron and synchrotron oscillations → redistribution of the momenta → change of emittances.
  - → Increase of energy spread
  - If transfer from transverse to longitudinal momentum at location with non-zero dispersion → transverse emittance increase



Intra-beam scattering

Determine rise times or damping times of emittances following coulomb scattering within bunch:

Calculations become quite involved, the methodology for the derivation of formulae is however straight forward:

1) Transformation of momenta of two colliding particles into their centre-of-mass system
2) Calculate changes of momenta due to collision (scattering angles ψ, ϕ) and transform back to storage ring frame
3) Calculate change of oscillation amplitudes at location of collision with dispersion → $\Delta\varepsilon_{x,y,z}$
4) Average over all scattering angles ψ, ϕ assuming distribution according to Rutherford scattering (impact parameters from nucleus to beam radius)
5) Average over all particles assume Gaussian distribution in position and momenta
6) Average over all lattice elements

Intra-beam scattering

- Below transition: equilibrium emittance in all three planes
- Above transition: emittances infinitely grow

Calculate growth rates:

$$\frac{1}{T_i} = \frac{1}{2\varepsilon_i}\frac{d\varepsilon}{dt}$$

$$\frac{1}{T_z} = \frac{1}{\sigma_\delta}\frac{d\sigma_\delta}{dt}$$

High energy approximation: a,b <<1 [K.L.F Bane, 2002]

$$\frac{1}{T_x} = \frac{r_0^2 c N_b}{64\gamma_0\pi^2\varepsilon_x^*\varepsilon_y^*\varepsilon_z^*}\left\langle f\left(\frac{1}{a}, \frac{b}{a}, \frac{q}{a}\right) + \frac{\eta_x^2\sigma_H^2}{\beta_x\varepsilon_x} f(a,b,q)\right\rangle$$

$$\frac{1}{T_y} = \frac{r_0^2 c N_b}{64\gamma_0\pi^2\varepsilon_x^*\varepsilon_y^*\varepsilon_z^*}\left\langle f\left(\frac{1}{b}, \frac{a}{b}, \frac{q}{b}\right) + \frac{\eta_y^2\sigma_H^2}{\beta_y\varepsilon_y} f(a,b,q)\right\rangle$$

Intra-beam scattering

With:

$$a = \frac{\sigma_H}{\gamma_0}\sqrt{\frac{\beta_x}{\varepsilon_x}}$$

$$b = \frac{\sigma_H}{\gamma_0}\sqrt{\frac{\beta_y}{\varepsilon_y}}$$

$$\frac{1}{\sigma_H^2} = \frac{1}{\sigma_\delta^2} + \frac{\mathcal{H}_x}{\varepsilon_x} + \frac{\mathcal{H}_y}{\varepsilon_y}$$

$$(log)_P = 2\ln\left(\frac{q}{2}\left(\frac{1}{a}+\frac{1}{b}\right)\right) - 0.577$$

$$f(a,b,q) = 4\pi(log)_P I_P(a,b)$$

$$I_P(a,b) = 2\int_0^1 du \frac{1-3u^2}{\sqrt{(a^2+(1-a^2)u^2)(b^2+(1-b^2)u^2)}}$$

Intra-beam scattering

- In practice calculate growth iteratively, many codes available
- CERN: analytical calculations in MADX based on Bjorken-Mtingwa formalism.
  1. http://madx.web.cern.ch/madx/webguide/manual.html#Ch28
  2. https://cds.cern.ch/record/1445924/files/CERN-ATS-2012-066.pdf

  In a similar way, the Bjorken-Mtingwa formalism is also implemented in ZAP, SAD, Elegant, OPA.
- CERN: multi-particle Monte Carlo simulation code by M. Martini and A. Vivoli, based on the Monte Carlo Code MOCAC.
  - Track particles and apply intrabeam Coulomb scattering

  1. http://cds.cern.ch/record/1240834/files/sLHC-PROJECT-REPORT-0032.pdf?version=1
  2. https://twiki.cern.ch/twiki/bin/view/ABPComputing/SIRE
  3. https://indico.cern.ch/event/647301/contributions/2630198/attachments/1489047/2313796/ABPCWGpres.pdf

Thanks for your attention

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