Estimation of electron cooling rate.

Case study 6.

Problem

To estimate of the initial cooling time of protons with energy E_p=1 GeV at cooling by the electron cooling system with following parameters:

Electron current J_e=1 A, electron beam radius a_e=1 cm, the longitudinal magnetic field in the cooling section is B_{cool}=2 kG, the length of cooling section 3 m, waviness of magnetic force line is $\delta B_\perp/Bcool=10^{-4}$, the temperature of electron beam at cathode T_e=0.1 eV, magnetic field on cathode surface B_{cath}=500 G.

Parameters of proton beam: radius of proton beam in the cooling section a_i=0.3 cm, longitudinal momentum spread $\delta p_{||}/p_0=2\cdot 10^{-4}$, beta function in the cooling section $\beta_x=\beta_y$ =10 m, perimeter is storage ring is Π=200 m.

Parkhomchuk's equation is useful for simple estimation of electron cooling

$$\vec{F} = -\frac{4\, n_e\, e^4}{m_e} \frac{\vec{v}_i}{(v_i^2 + v_{eff}^2)^{3/2}} Ln\left(\frac{\rho_{\max} + \rho_{\min} + \rho_L}{\rho_{\min} + \rho_L} \right)$$

Friction force in co-moving reference system (it is useful because it doesn't depend from energy)

$\rho_L = mv_{Te}/eB$ - Larmour radius

$\rho_{\max} = v_i \tau$ - maximal impact parameter (simple version)

τ - flight time through cooling section (interaction time)

$\rho_{\min} = \dfrac{e^2}{mv_i^2}$ - minimal impact parameter

v_{eff} - all parasitic velocities of electrons respectively ions

$$v_{eff}^2 = v_{\Delta\Theta}^2 + v_{E\times B}^2 + v_{\mathrm{II}e}^2$$

$T_{IIe} \approx e^2 n_e^{1/3}$ - spread of longitudinal velocities of electrons

$v_{\Delta\Theta} = \gamma\beta c \sqrt{\langle \Delta B^2 \rangle}$ - waviness of magnetic force line

$v_{E\times B} = c\dfrac{E_{sp\_charge}}{B}$ - drift of electrons induced by space charge

- Problem 1. Estimate the relativistic parameters β, γ of proton beam and energy of the electron beam E_e.

3 min

statement of problem: to estimate of the initial cooling time of protons with energy E_p=1 GeV at cooling by the electron cooling system

- <u>Problem 1</u>. Estimate the relativistic parameters β, γ of proton beam and energy of the electron beam E_e.

3 min

<u>statement of problem</u>: to estimate of the initial cooling time of protons with energy E_p=1 GeV at cooling by the electron cooling system

Answer:

$$Ep := 1 \cdot 10^9$$

$$\gamma := 1 + \frac{Ep}{909\,10^6} \qquad \beta := \sqrt{1 - \frac{1}{\gamma^2}}$$

$$Ee := (\gamma - 1) \cdot 0.511\,10^6 \qquad Ee = 5.622 \times 10^5$$

$$\gamma = 2.1 \qquad \beta = 0.879$$

$$\vec{F} = -\frac{4\, n_e\, e^4}{m_e} \frac{\vec{v}_i}{(v_i^2 + v_{eff}^2)^{3/2}} Ln\left(\frac{\rho_{\max} + \rho_{\min} + \rho_L}{\rho_{\min} + \rho_L} \right)$$

- Problem 2. Estimate the density $\mathbf{n_e}$ of electron beam in co-moving reference system.

statement of problem: electron current J_e=1 A, electron beam radius a_e=1 cm

5 min

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5 min

Answer:

$$ne := \frac{Je}{\pi \cdot ae^2 \cdot \gamma \cdot \beta \cdot c \cdot ee}$$

$$ne = 3.591 \times 10^7$$

$$c := 3 \cdot 10^{10}$$

$$ee := 1.6 \cdot 10^{-19}$$

$$\vec{F} = -\frac{4\, n_e\, e^4}{m_e} \frac{\vec{v}_i}{(v_i^2 + v_{eff}^2)^{3/2}} Ln\left(\frac{\rho_{\max} + \rho_{\min} + \rho_L}{\rho_{\min} + \rho_L} \right)$$

- Problem 3. Estimate the ion velocity $\mathbf{v_i}$ of proton in co-moving reference system (longitudinal $\mathbf{v_{li}}$, transverse $\mathbf{v_{ti}}$ and total).

5 min

statement of problem: radius of proton beam in the cooling section a_i=0.3 cm, longitudinal momentum spread $\delta p_{//}/p_0 = 2\cdot 10^{-4}$, beta function in the cooling section $\beta_x = \beta_y = 10$ m.

$$\vec{F} = -\frac{4\, n_e\, e^4}{m_e} \frac{\vec{v}_i}{(v_i^2 + v_{eff}^2)^{3/2}} Ln\left(\frac{\rho_{\max} + \rho_{\min} + \rho_L}{\rho_{\min} + \rho_L} \right)$$

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Answer:

$\theta := \frac{ai}{\beta x}$ $\qquad \theta = 3\times 10^{-4}$ $\qquad Vti := \gamma\cdot\beta\cdot\theta\cdot c$ $\qquad Vti = 1.662\times 10^{7}$ $\qquad c := 3\cdot 10^{10}$

$\delta p := 2\cdot 10^{-4}$ $\qquad Vli := \beta\cdot c\cdot\delta p$ $\qquad Vi := \sqrt{Vti^2 + Vli^2}$ $\qquad Vli = 5.276\times 10^{6}$

$Vi = 1.744\times 10^{7}$

$$\vec{F} = -\frac{4\, n_e\, e^4}{m_e} \frac{\vec{v}_i}{(v_i^2 + v_{eff}^2)^{3/2}} Ln\left(\frac{\rho_{\max} + \rho_{\min} + \rho_L}{\rho_{\min} + \rho_L} \right)$$

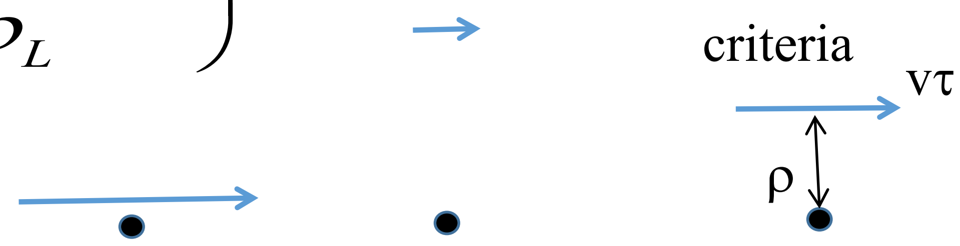
$\rho_{\max} = v_i \tau$ - maximal impact parameter (simple version)

τ - flight time through cooling section (interaction time)

Why estimation is so unpunctual because the LOG approximation. The function Ln is insensitive to error.

no collision

criteria



classical collision (particle move about from -∞ to + ∞)

ρ~vτ

- Problem 4. Estimate the maximum impact parameter of ion ρ_{max} at Coulomb interaction of ion and electron in co-moving reference system.

statement of problem: the length of cooling section 3 m

5 min

$$\vec{F} = -\frac{4\, n_e\, e^4}{m_e} \frac{\vec{v}_i}{(v_i^2 + v_{eff}^2)^{3/2}} Ln\left(\frac{\rho_{max} + \rho_{min} + \rho_L}{\rho_{min} + \rho_L}\right)$$

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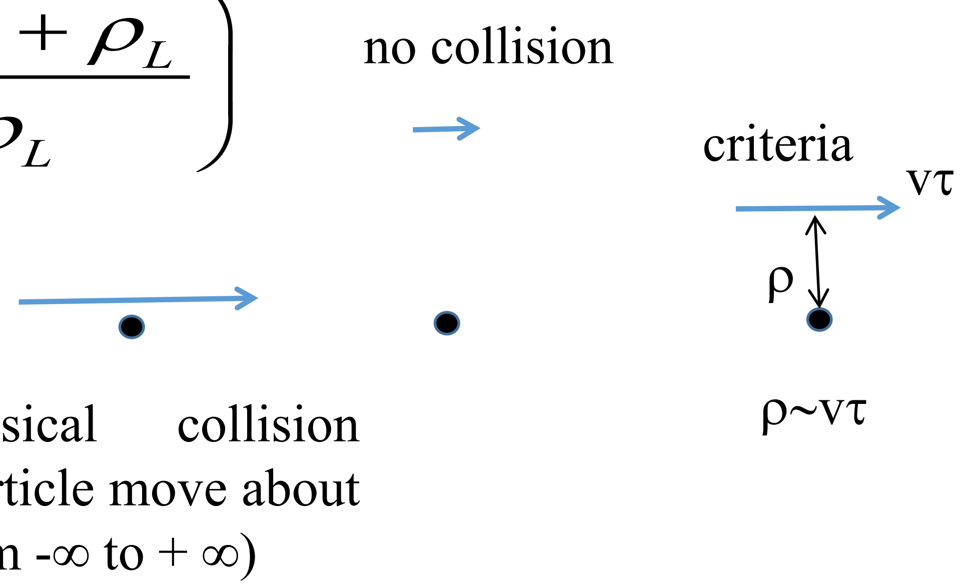

- Problem 4. Estimate the maximum impact parameter of ion $\boldsymbol{\rho_{max}}$ at Coulomb interaction of ion and electron in co-moving reference system.

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5 min

Answer from task 3

Answer:

$$\tau := \frac{Lcool}{\gamma \cdot \beta \cdot c} \qquad \tau = 5.415 \times 10^{-9} \qquad Vti = 1.662 \times 10^{7}$$

$$Vi := \sqrt{Vti^2 + Vli^2} \qquad \rho max := Vi \cdot \tau \qquad \rho max = 0.094 \qquad Vli = 5.276 \times 10^{6}$$

$$\vec{F} = -\frac{4\, n_e\, e^4}{m_e} \frac{\vec{v}_i}{(v_i^2 + v_{eff}^2)^{3/2}} Ln\left(\frac{\rho_{\max} + \rho_{\min} + \rho_L}{\rho_{\min} + \rho_L} \right)$$

$$\rho_{\min} = \frac{e^2}{m v_i^2}$$ - minimal impact parameter

- Problem 5. Estimate the minimum impact parameter ion $\boldsymbol{\rho_{min}}$ at Coulomb interaction of ion and electron in co-moving reference system.

5 min

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- Problem 5. Estimate the minimum impact parameter ion $\boldsymbol{\rho_{min}}$ at Coulomb interaction of ion and electron in co-moving reference system.

5 min

Answer:

$$\rho min := re \cdot \frac{c^2}{Vi^2}$$

$$\rho min = 8.287 \times 10^{-7}$$

$$\rho_{\min} = r_e \frac{c^2}{v_i^2}$$

$$r_e = \frac{e^2}{m_e c^2}$$

$$re := 2.8 \cdot 10^{-13}$$

$$\vec{F} = -\frac{4\, n_e\, e^4}{m_e} \frac{\vec{v}_i}{(v_i^2 + v_{eff}^2)^{3/2}} Ln\left(\frac{\rho_{\max} + \rho_{\min} + \rho_L}{\rho_{\min} + \rho_L} \right)$$

- Problem 6. Estimate the Larmour radius of electron ρ_L in co-moving reference system.

statement of problem: the longitudinal magnetic field in the cooling section is B_{cool}=2 kG, magnetic field on cathode surface B_{cath}=500 G, the temperature of electron beam at cathode Te=0.1 eV,

5 min

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Answer:

$$\rho L := \sqrt{\frac{Bcath}{Bcool}} \cdot \frac{0.511 \cdot 10^6}{300 \cdot Bcath} \cdot \sqrt{\frac{2 \cdot Te}{0.511 \cdot 10^6}} \qquad \rho L = 1.066 \times 10^{-3} \quad cm$$

$p_\perp, R_L$ *- is preserved at acceleration, but it is not preserved at changing longitudinal magnetic field*

$$\frac{p_\perp^2}{B} = const$$ *conservation of the adiabatic invariant*

$$\frac{p_\perp v_\perp}{R_L} = \frac{e}{c} v_\perp B$$ *Lorenz force is equal to centrifugal force*

$$p_\perp = m_e v_{\perp e} = m_e c \sqrt{\frac{2T_e}{m_e c^2}}$$ *- transverse impulse on cathode*

$$\vec{F} = -\frac{4\, n_e\, e^4}{m_e} \frac{\vec{v}_i}{(v_i^2 + v_{eff}^2)^{3/2}} Ln\left(\frac{\rho_{\max} + \rho_{\min} + \rho_L}{\rho_{\min} + \rho_L} \right)$$

- Problem 7. Estimate parasitic velocities of electrons respectively ions in co-moving reference system.

$$v_{eff}^2 = v_{\Delta\Theta}^2 + v_{E\times B}^2 + v_{\text{IIe}}^2$$

$$v_{\Delta\Theta} = \gamma\beta c \sqrt{\left\langle \Delta B^2 \right\rangle}$$ - waviness of magnetic force line

$$v_{E\times B} = c\,\frac{E_{sp\_ch\arg e}}{B}$$

$$T_{IIe} \approx e^2 n_e^{1/3}$$

$$v_{IIe} \approx \sqrt{2e^2 n_e^{1/3} / m_e}$$ - spread of longitudinal velocities of electrons

10 min

statement of problem: electron current J_e=1 A, electron beam radius a_e=1 cm, the longitudinal magnetic field in the cooling section is B_{cool}=2 kG, waviness of magnetic force line is $\delta B_\perp/Bcool=10^{-4}$

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Answer:

$\delta B := 10^{-4}$ $\quad VB := \gamma\cdot\beta\cdot c\cdot\delta B$ $\quad VB = 5.54\times 10^{6}$

$Vdr := c\cdot\frac{2\cdot\pi\cdot qe\cdot ne\cdot ae}{Bcool}$ $\quad qe = 4.8\times 10^{-10}$ $\quad Vdr = 1.624\times 10^{6}$

$Ve := c\cdot\sqrt{2re\,ne^{0.333}}$ $\quad re = 2.8\times 10^{-13}$ $\quad Ve = 4.066\times 10^{5}$

$\sqrt{Vdr^2 + VB^2 + Ve^2} = 5.788\times 10^{6}$

$$\vec{F} = -\frac{4\, n_e\, e^4}{m_e} \frac{\vec{v}_i}{(v_i^2 + v_{eff}^2)^{3/2}} Ln\left(\frac{\rho_{\max} + \rho_{\min} + \rho_L}{\rho_{\min} + \rho_L} \right)$$

- Problem 8. Now it is possible to collect all parameters together.

$$\lambda l_{cool}^{-1} = \frac{\gamma\beta m_p c\left(a_i/\beta_x\right)}{F_t \tau f_0} \qquad \lambda t_{cool}^{-1} = \frac{\gamma\beta m_p c\left(\delta p_{II}/p_0\right)}{\gamma F_l \tau f_0}$$

5 min

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Answer: cooling time is about 40 s for both direction

5 min

$$Ft := \frac{4 \cdot qe^4 \cdot ne}{me} \cdot \frac{Vti}{\left(\sqrt{Vti^2 + Vli^2 + Vdr^2 + VB^2 + Ve^2}\right)^3} \cdot Lnc \qquad \left(\frac{Ft \cdot \tau}{\gamma \cdot \beta \cdot mp \cdot c} \cdot f0\right)^{-1} \cdot \theta = 38.55$$

$$Fl := \frac{4 \cdot qe^4 \cdot ne}{me} \cdot \frac{Vli}{\left(\sqrt{Vti^2 + Vli^2 + Vdr^2 + VB^2}\right)^3} \cdot Lnc \qquad \left(\frac{\gamma \cdot Fl \cdot \tau}{\gamma \cdot \beta \cdot mp \cdot c} \cdot f0\right)^{-1} \cdot \delta p = 38.522$$