Занятие 2. Светимость

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Luminosity

Merging particle collisions at NICA

Introduction

The goal is to Study a fundamental problem of QED - spontaneous electron-positron pair creation in supercritical Coulomb fields. Theory threshold charge number of colliding nuclei has to be $Z1 + Z2 \geq 173$, the center of mass energy has to be closed to Coulomb barrier ~ 6 – 8 MeV/u.
The required luminosity is $10^{23} - 10^{24}$ $cm^{-2}s^{-1}$.

Such energy and luminosity can be simply achieved in fixed target experiment, however in the target there is influence of orbital electrons. The possible solution is to provide the experiment at storage rings with colliding bare U nuclei beams. To avoid space charge limitation of the luminosity the circulating beam energy has to large enough.

At NICA collider this experiment can be provided with so called merging collisions.

The interaction of merging beams propagating along the same axis in one direction with different momentum and with the same masses *m* (Fig 1) was considered in

Vasily V. Parkhomchuk, Yuri K. Batygin, Takeshi Katayama, Luminosity of comoving particle collisions, Nuclear Instruments and Methods in Physics Research A 449 (2000) 140 – 146.

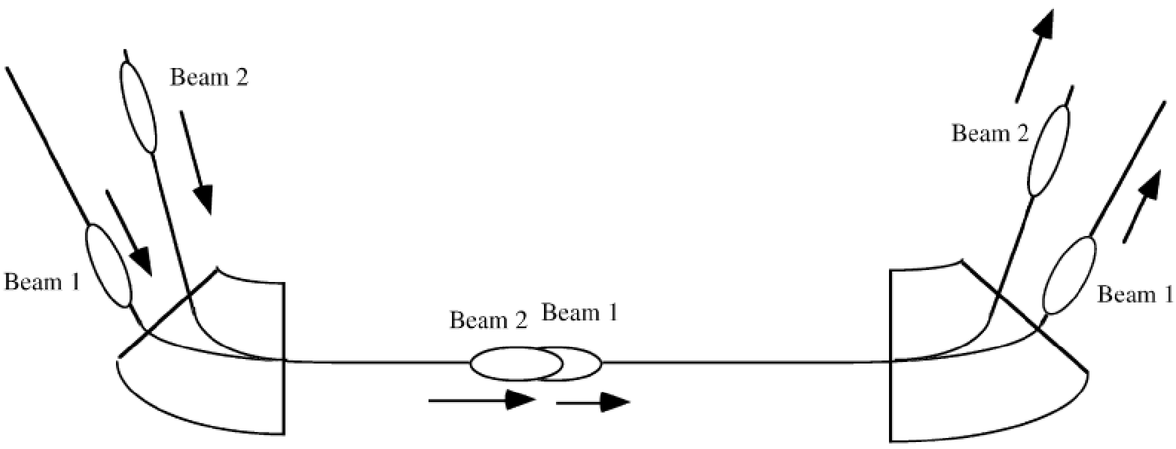


Fig. 1. Interaction region of beams with different rigidities.

In the center of mass system the total momentum is zero, due to conservation of invariant value $E^2 - p^2c^2 = inv$, the center of mass energy is

$$\begin{aligned} E_{cm} &= \sqrt{(E_1 + E_2)^2 - (p_1 + p_2)^2 c^2} \\ &= \sqrt{2(m^2c^4 + E_1 E_2 - p_1 p_2 c^2)} \\ &\approx 2mc^2 + \frac{(\Delta p)^2}{4m}. \end{aligned} \qquad (1)$$

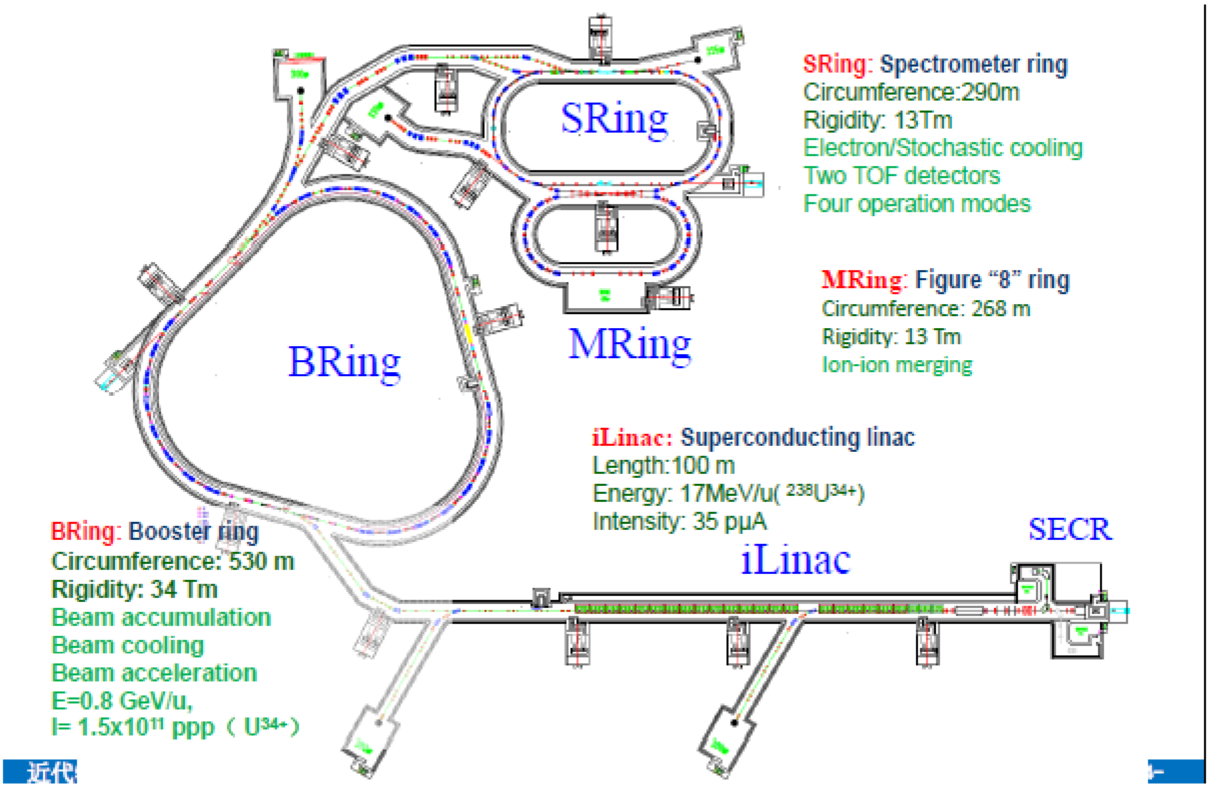
The Luminosity

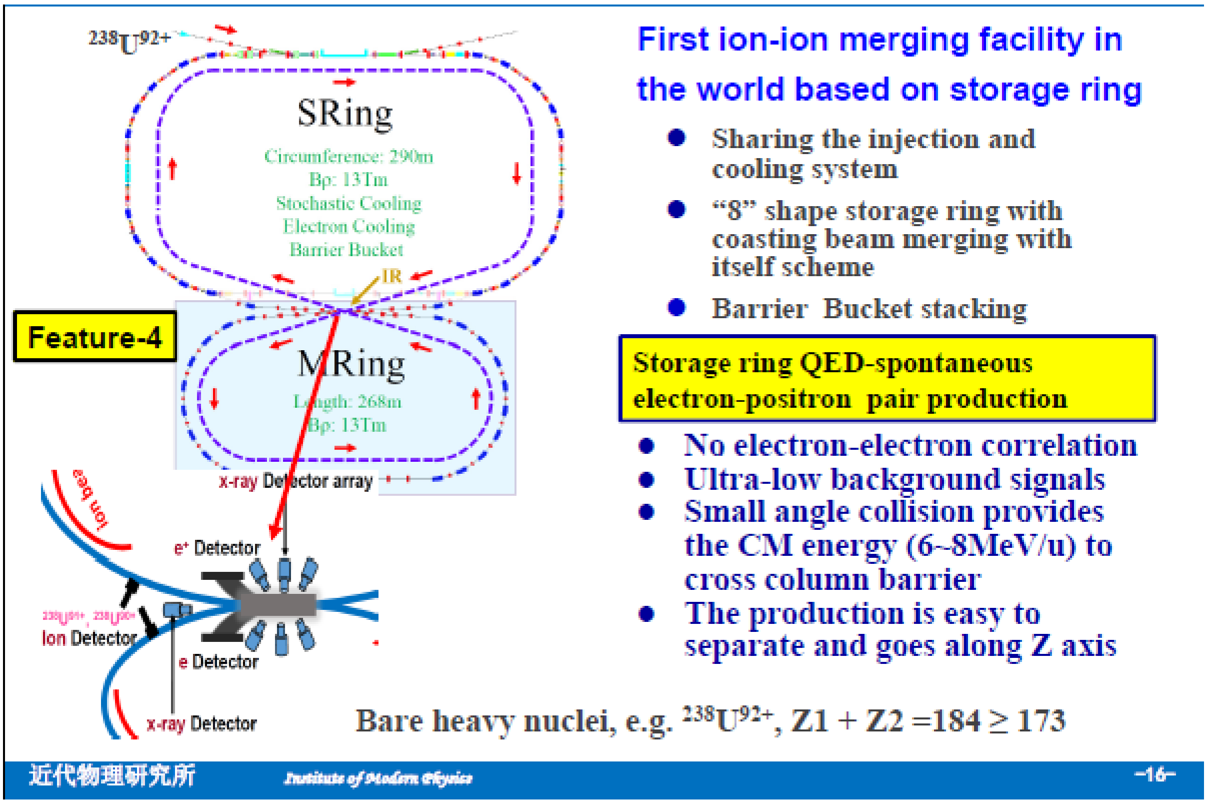
$$L_o = \frac{N_b f_o N_1 N_2}{2\pi\sqrt{\sigma_{x1}^2 + \sigma_{x2}^2}\sqrt{\sigma_{y1}^2 + \sigma_{y2}^2}} \qquad (2)$$

which is the same as luminosity of head-on collisions. Equivalence of luminosity for comoving and head-on collisions is explained by the fact that in both cases that since the two beams completely move through each other, the same number of collisions occur during the same time of revolution in the ring. Therefore, the luminosity of on-axis collisions does not depend on relative particle velocities. To interact, the bunches of both beams have to pass through each other completely.

The disadvantage – long strait section where one needs to provide focusing of the beams at different magnetic rigidities. Realization of this scheme at NICA collider requires sufficient modification of the optic structure.

In 2015 at IMP (Institute for Modern Physics, China) the merging collisions at small crossing angle was proposed (Fig. 2)


SRing: Spectrometer ring
Circumference:290m
Rigidity: 13Tm
Electron/Stochastic cooling
Two TOF detectors
Four operation modes
SRing
MRing: Figure "8" ring
Circumference: 268 m
Rigidity: 13 Tm
Ion-ion merging
MRing
BRing
iLinac: Superconducting linac
Length:100 m
Energy: 17MeV/u(238U34+)
Intensity: 35 pμA
SECR
iLinac
BRing: Booster ring
Circumference: 530 m
Rigidity: 34 Tm
Beam accumulation
Beam cooling
Beam acceleration
E=0.8 GeV/u,
I= 1.5x10¹¹ ppp（U34+）
近代


238U92+
SRing
Circumference: 290m
Bρ: 13Tm
Stochastic Cooling
Electron Cooling
Barrier Bucket
IR
Feature-4
MRing
Length: 268m
Bρ: 13Tm
x-ray Detector array
Ion beam
e+ Detector
Ion Detector
e Detector
x-ray Detector
First ion-ion merging facility in the world based on storage ring
Sharing the injection and cooling system
"8" shape storage ring with coasting beam merging with itself scheme
Barrier Bucket stacking
Storage ring QED-spontaneous electron-positron pair production
No electron-electron correlation
Ultra-low background signals
Small angle collision provides the CM energy (6~8MeV/u) to cross column barrier
The production is easy to separate and goes along Z axis
Bare heavy nuclei, e.g. 238U92+, Z1 + Z2 =184 ≥ 173
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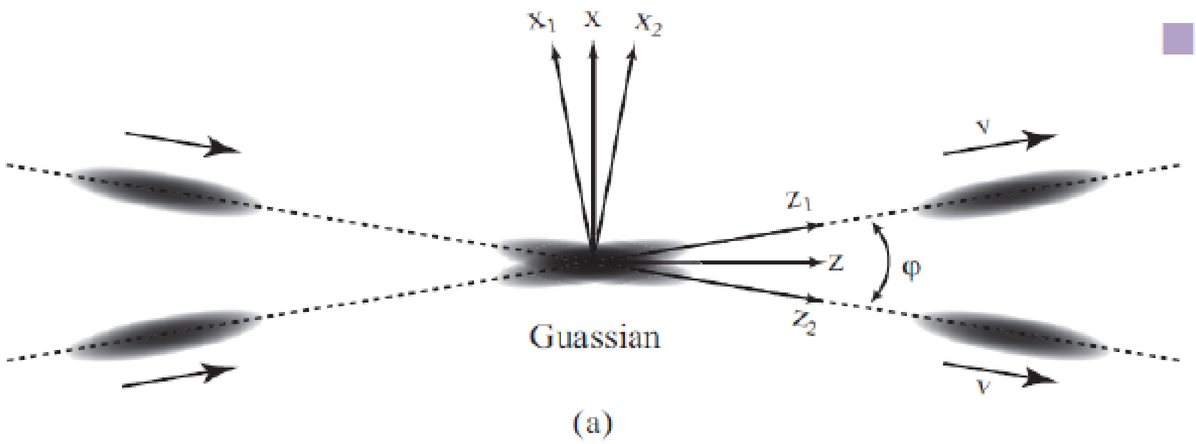

Fig. 2. Schematics of the merging collisions of the beams at the same momentum at crossing angle θ.

Such a scheme can be realized at NICA collider by modification of the interaction region.

Tasks

Task 1:

Calculate the crossing angle value, corresponding to the kinetic energy in the center of mass frame in the range of 6 – 8 MeV/n for the colliding beams at energies in the range 3 – 4.5 GeV/u.

Task 2:

How the luminosity will be changed in the space charge dominated regime if the gold nuclei will be replaced by uranium ones?

Task 3:

Estimate level of luminosity for merging collisions at NICA beam parameters.

Solutions:

Task 1.

The center of mass energy

$$\sqrt{s} = \sqrt{(E_1+E_2)^2 - (\vec{p}_1+\vec{p}_2)^2 c^2}$$

at $E_1 = E_2 = E$, and $|\vec{p}_1| = |\vec{p}_2| = p$

is equal to

$$\sqrt{(2E)^2 - \left(2p\cos\left(\frac{\theta}{2}\right)\right)^2 c^2}$$

Introducing $\varphi = \frac{\theta}{2}$ it can be expressed as

$$\sqrt{(2E)^2 - (2p)^2(1-\sin^2\varphi)c^2} ,$$

which using the invariant it is transformed to

$$\sqrt{(2mc^2)^2 + (2p)^2 c^2 \sin^2\varphi} ,$$

or

$$2mc^2 \sqrt{1 + \frac{(2p)^2 c^2 \sin^2\varphi}{(2mc^2)^2}} .$$

At small angle $\sin\varphi \approx \varphi$.

At the condition $\frac{(2p)^2 c^2 \varphi^2}{(2mc^2)^2} << 1$, the square root can be expressed as

$$\sqrt{1 + \frac{(2p)^2 c^2 \varphi^2}{(2mc^2)^2}} \approx 1 + \frac{1}{2}\frac{(2p)^2 c^2 \varphi^2}{(2mc^2)^2} .$$

And finally for the kinetic energy in the center of mass system we have.

$$\sqrt{s} - 2mc^2 \approx \frac{1}{2}\frac{(2p)^2 c^2 \varphi^2}{2mc^2} .$$

Or, using the momentum expression

$$pc = mc^2 \beta\gamma ,$$

One can write

$\sqrt{s} - 2mc^2 \approx mc^2\beta^2\gamma^2\varphi^2$.

Minimum of the kinetic energy in the center of mass system (6 MeV/u) has to correspond to minimum of the kinetic energy of the circulating beam.

Kinetic energy of the circulating beams is 3 Gev/u, kinetic energy in center of mass system is to be 6 MeV/u:

$\beta \sim 1$, $\gamma \sim 4$, $mc^2 \sim 10^9$ eV

$$\varphi^2 \approx \frac{6 \cdot 10^6}{10^9 \cdot 16}$$

$$\varphi \approx 2 \cdot 10^{-2}$$

$$\theta \approx 40\ mrad$$

Lets check: is this crossing angle permits to achieve the kinetic energy in the center of mass system larger than 8 MeV/u at maximum kinetic energy of the circulating beam?

Kinetic energy of the circulating beams is 4.5 Gev/u, crossing angle is 40 mrad, kinetic energy in center of mass system is:

$\beta \sim 1$, $\gamma \sim 5.5$

$\sqrt{s} - 2mc^2 \approx 10^9 \cdot 30 \cdot 4 \cdot 10^{-4} \sim$ 12 MeV/u.

The answer:

The crossing angle of about 40 mrad permits to cover the total required energy range.

Task 2.

In the space charge dominated regime the NICA luminosity can be estimated as

$$L = \left(\frac{A}{Z^2 r_p}\right)^2 \frac{\varepsilon}{\beta^*} \frac{8\pi^2\sigma_s^2 c}{C^2 l_{bb}} \gamma^6 \beta^5 f\left(\frac{\sigma_s}{\beta^*}\right) \Delta Q^2$$

At all equal parameters the luminosity is scaled with the nuclei parameters as $\frac{A^2}{Z^4}$.

Gold: A = 197, Z = 79, $\frac{A^2}{Z^4}$ = 9.96E-4

Uranium A = 238, Z = 92 $\frac{A^2}{Z^4}$ = 7.91E-4

The answer:

The luminosity for Uranium beams will be less than for Gold by 1.26 times.

Task 3.

For estimation one can use the luminosity formula:

$$L_o = \frac{N_b f_o N_1 N_2}{2\pi\sqrt{\sigma_{x1}^2 + \sigma_{x2}^2}\sqrt{\sigma_{y1}^2 + \sigma_{y2}^2}}$$

At NICA conditions $\sigma_{x1} = \sigma_{x2} = \sqrt{\varepsilon\beta^*} = \sqrt{10^{-6} \cdot 0.6} \approx 8 \cdot 10^{-4}\ m$. The same value is the beam size in vertical direction. However, due to geometry of the merging collisions, instead of the vertical size we need to substitute the bunch length σ_s, which is equal 0.6 m at the NICA.

So the luminosity of the merging collisions will be about $1.3 \cdot 10^{-3}$ from the head-on one. The luminosity in the head-on collisions is expected to be 10^{27} cm^{-2}s^{-1}, so for the merging collision one can expect the luminosity value of $1.3 \cdot 10^{24}$ cm^{-2}s^{-1}.

The answer:

The luminosity is of the level of 10^{24} cm^{-2}s^{-1}.