Application of the Prony least squares method for fitting signal waveforms measured by sampling ADC

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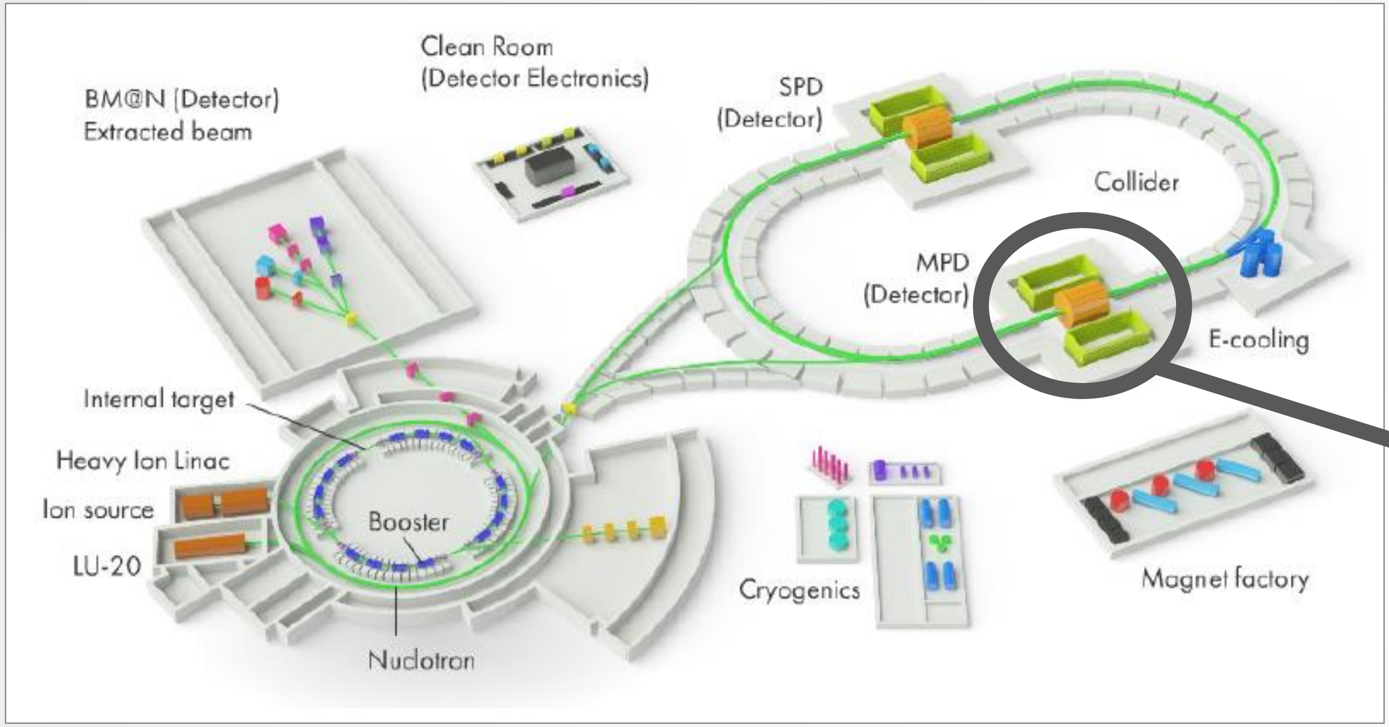
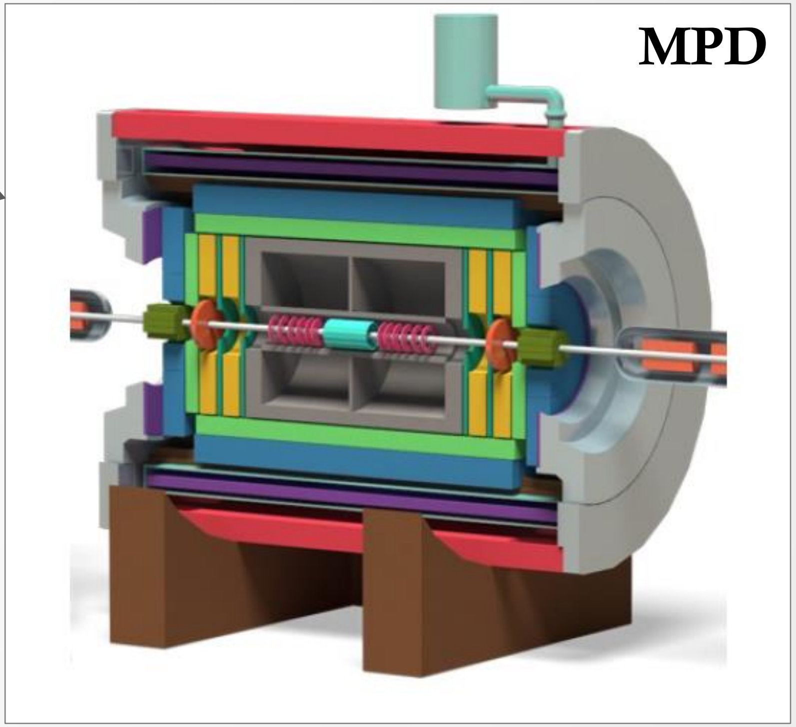
Outline

- ❖ MPD experiment and FHCal
- ❖ Why do we need waveform fitting procedure
- ❖ Prony LS method
- ❖ Fit quality assessment
- ❖ New muon calibration approach

MPD experiment at NICA





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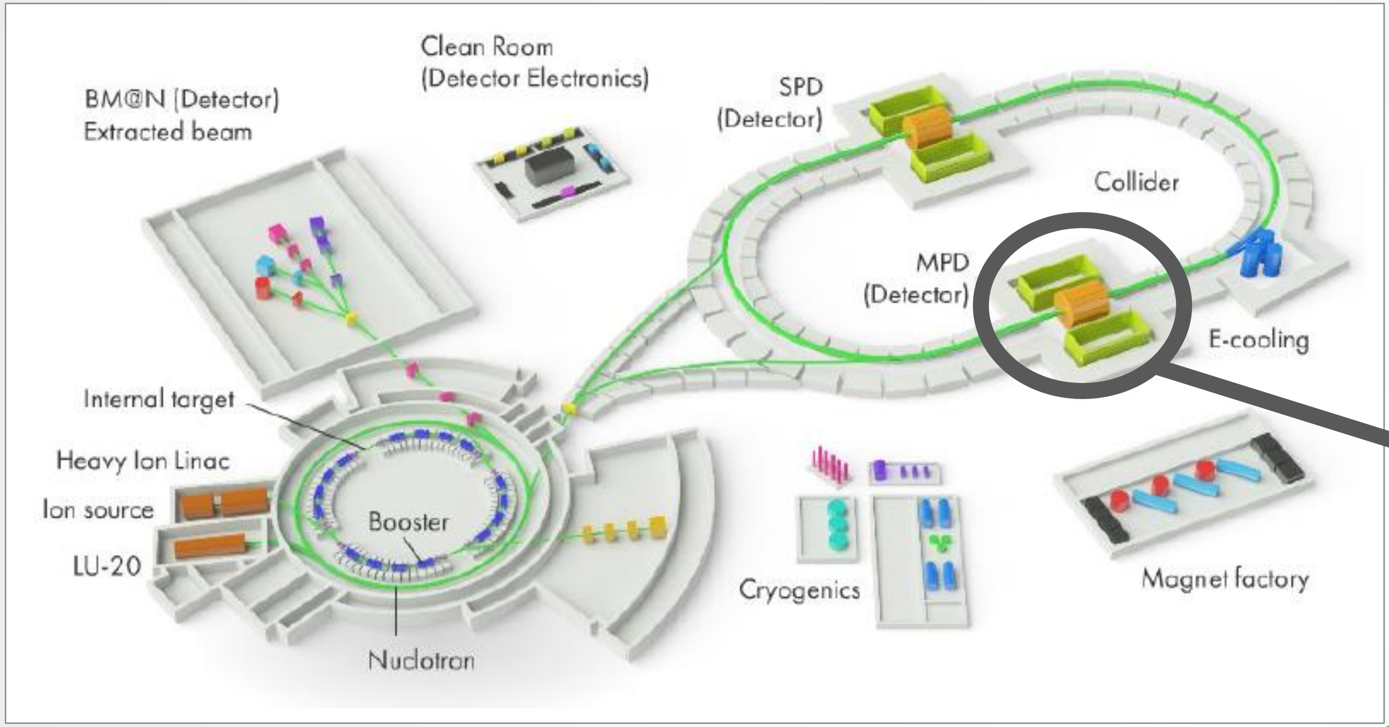
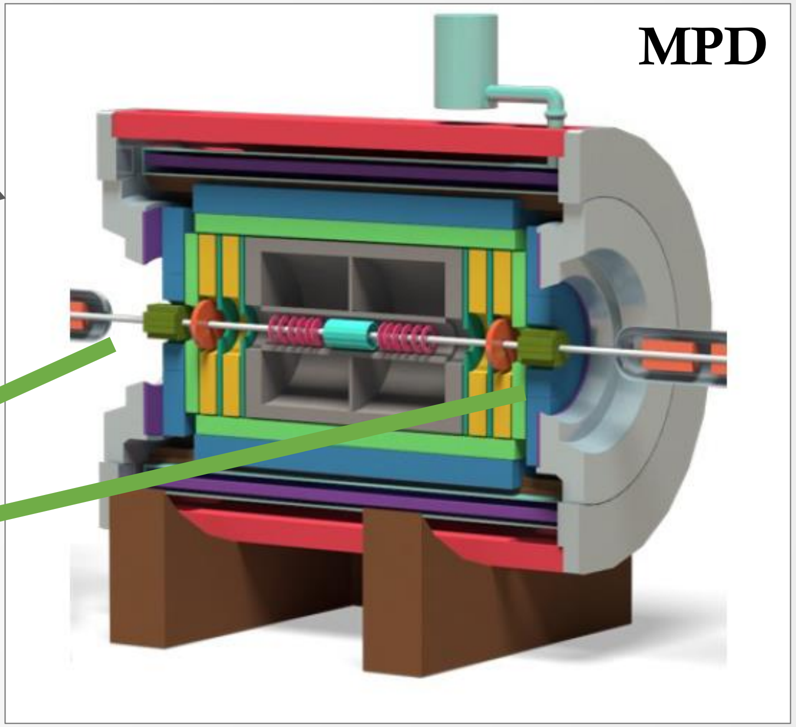
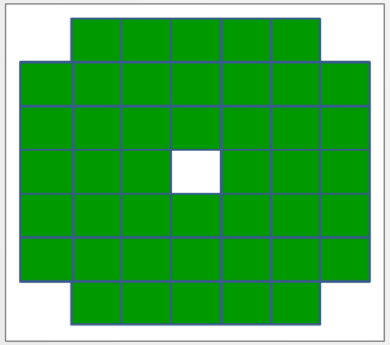




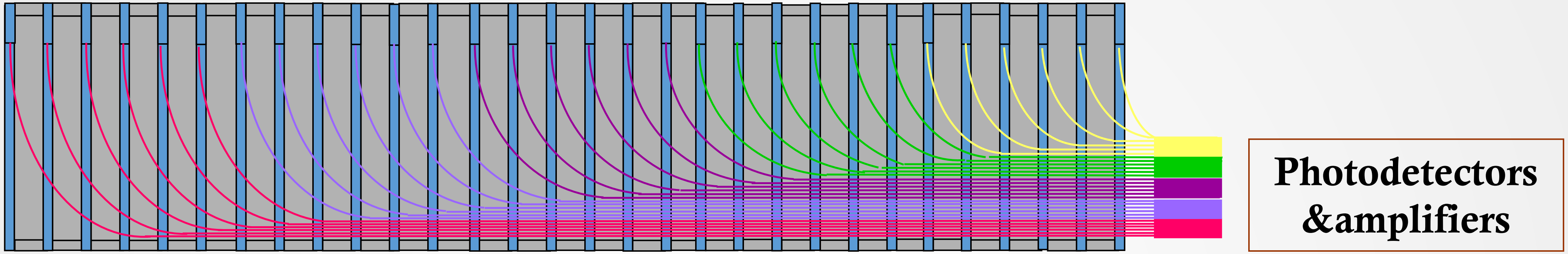


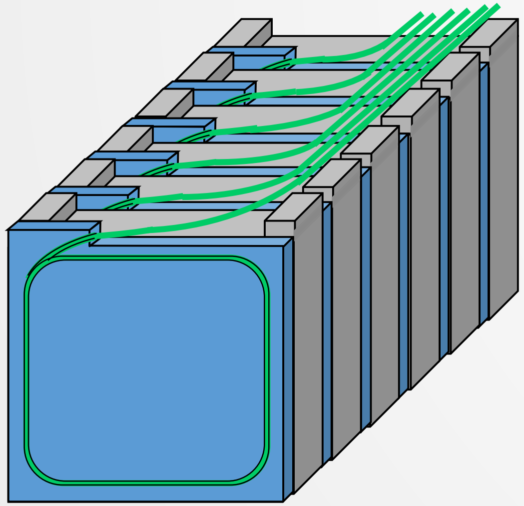
FHCal

- ❖ Centrality
- ❖ Reaction plane orientation

Each arm:
44 modules, Beam hole, Weight ~9 tons.

Structure of calorimeter module

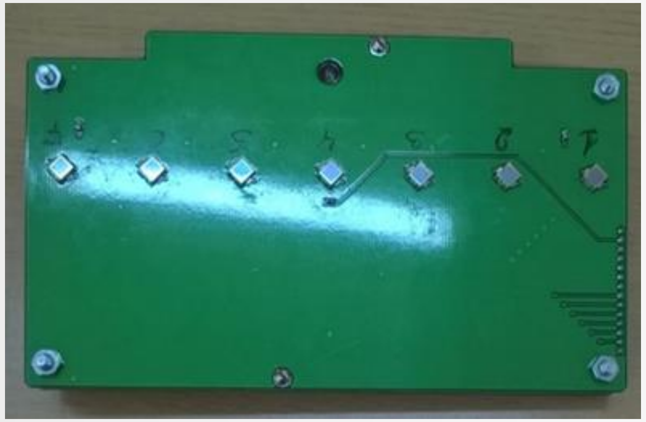


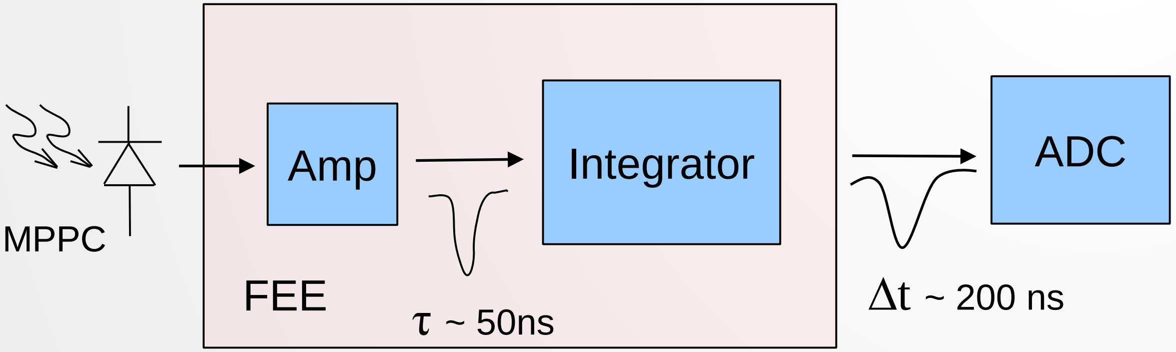


- Transverse size - 15x15cm^2 ;
- Total length - 106 cm.
- Interaction length ~4 λ_{int};
- Longitudinal segmentation – 7 sections;
- 7 photodetectors/module;
- Photodetectors – silicon photomultipliers (SiPM).

Photodiodes, FEE and readout electronics

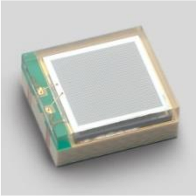
Front-End-Electronics:





7 channels: two-stage amplifiers; HV channels;
LED calibration source.

Photodetectors:



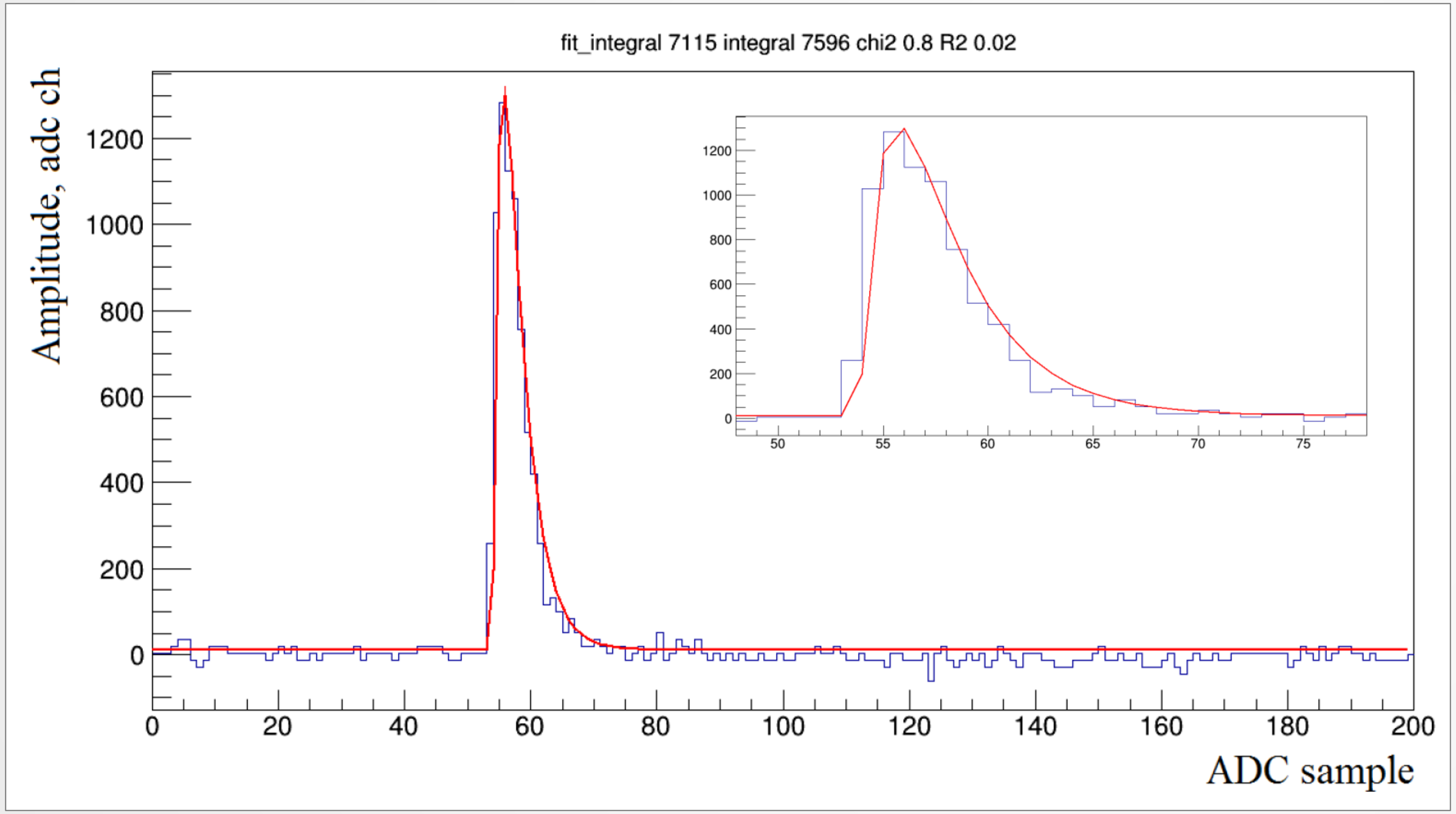
Hamamatsu MPPC:
size – 3x3 mm^2;
pixel -10x10 μm^2;
PDE~12%.

Readout electronics:

FPGA based 64 channel ADC64 board, 62.5MS/s (AFI Electronics, JINR, Dubna).

Why do we need waveform fitting

Fast signals → Few samples per signal → Large fluctuations of charge



Advantages of the fitting procedure:

- ❖ More correct determination of amplitude and charge
- ❖ Working with small signals near the noise level
- ❖ Interference and pile-up identification
- ❖ True signal recovery

Prony Least Squares method

Allows to estimate a set of complex data samples x[n] using the p-term model of exponential components:

$$\hat{x}[n] = \sum_{k=1}^{p} A_k \exp[(\alpha_k + j2\pi f_k)(n-1)T + j\theta_k] = \sum_{k=1}^{p} h_k z_k^{n-1}$$

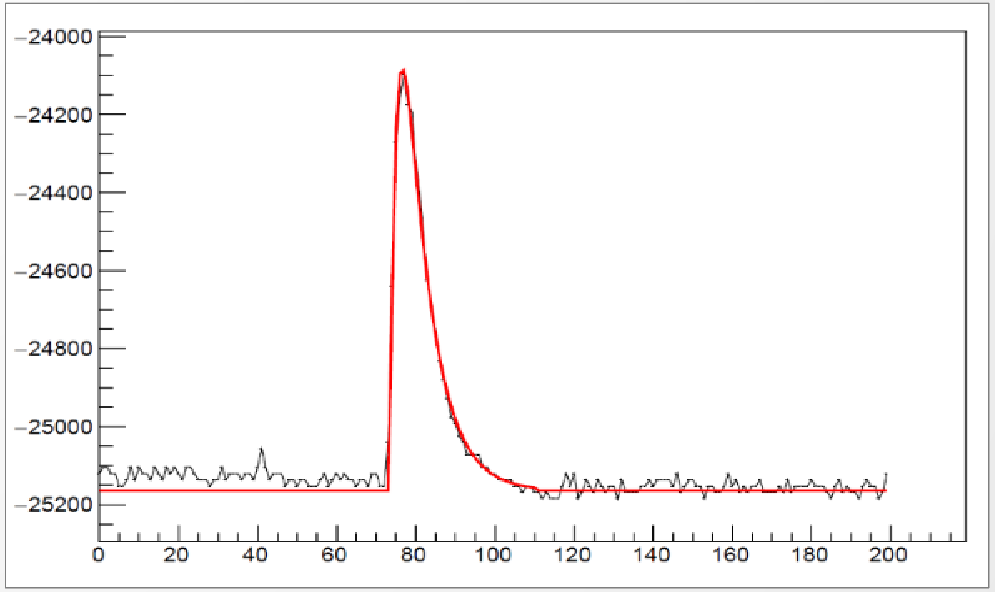
$n = 1, 2, \ldots, N$, $j^2 = -1$, T – sampling interval. $\boldsymbol{h_k} = A_k \exp(j\theta_k)$, $\boldsymbol{z_k} = \exp[(\alpha_k + j2\pi f_k)\mathrm{T}]$.

Objects of estimation are: amplitudes of complex exponentials $\boldsymbol{A_k}$, attenuation parameters $\boldsymbol{\alpha_k}$, harmonic frequencies $\boldsymbol{f_k}$ and phases $\boldsymbol{\theta_k}$.

3 algorithm steps:

1. Composing and solving SLE p×p } $\boldsymbol{z_k}$
2. Polynomial factorization }
3. Composing and solving SLE (p+1)×(p+1) ⟶ $\boldsymbol{h_k}$

3 orders of magnitude faster than MINUIT



Fit quality assessment

Determination coefficient*

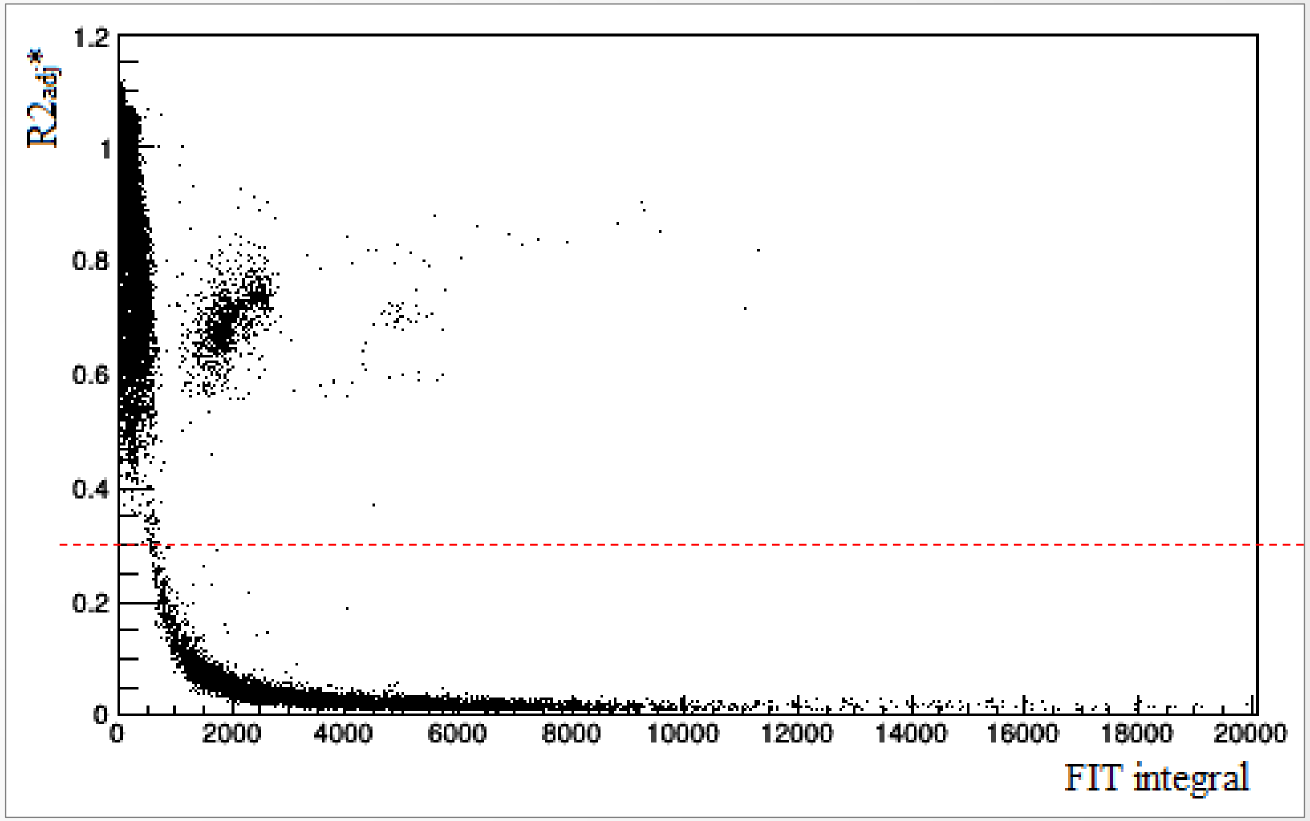
$$R^2 = \frac{\sum_{n=1}^{N}(x[n] - \hat{x}[n])^2}{\sum_{n=1}^{N}(x[n] - \overline{x})^2}$$

$x[n]$ and $\hat{x}[n]$ are the experimental and model values of the variable, respectively. $\overline{x}$ is the experimental values average.

Adjusted determination coefficient*

$$R_{adj}^2 = R^2 \frac{N-1}{N-\lambda}$$

N is the number of measurements, λ is the number of model parameters.



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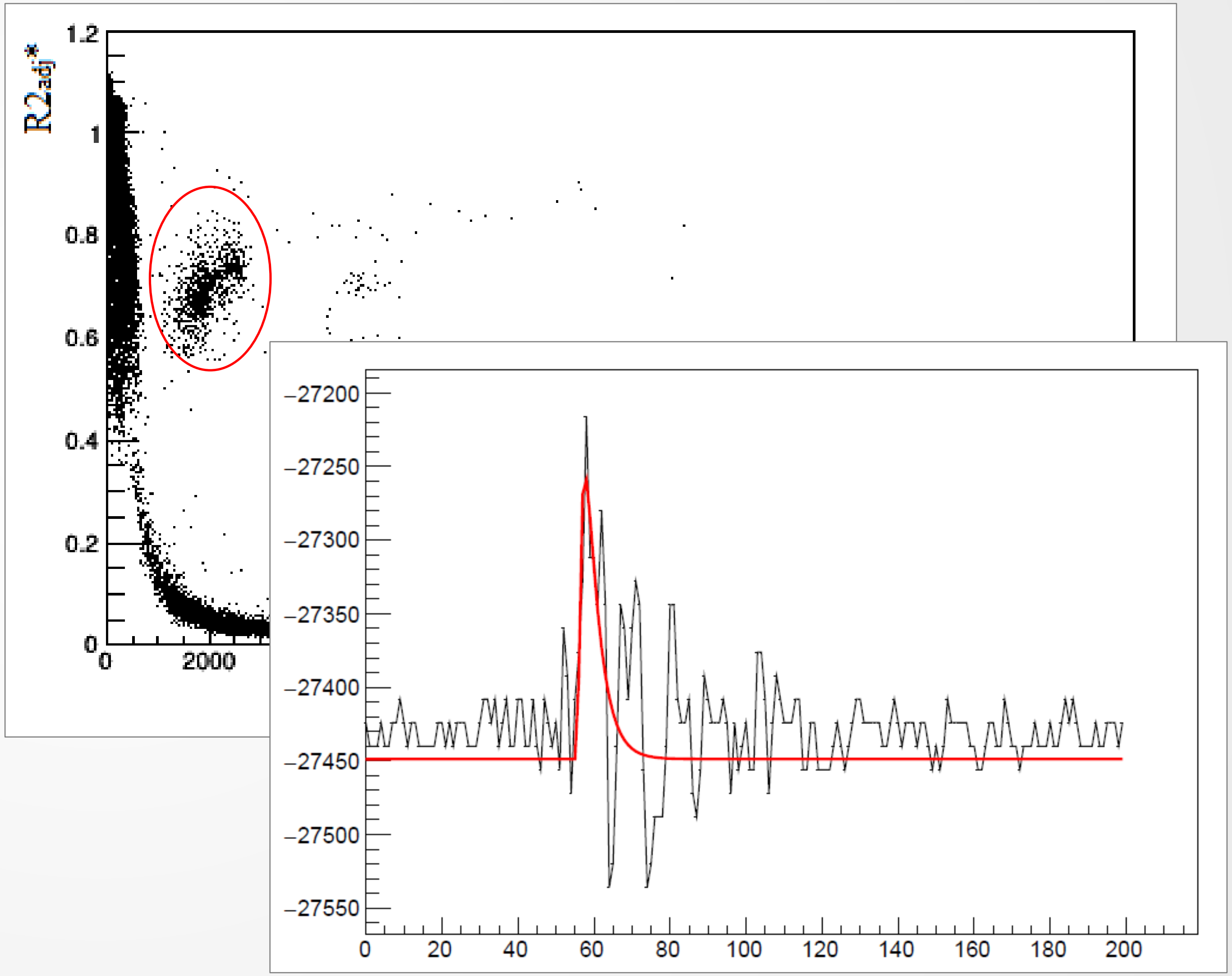
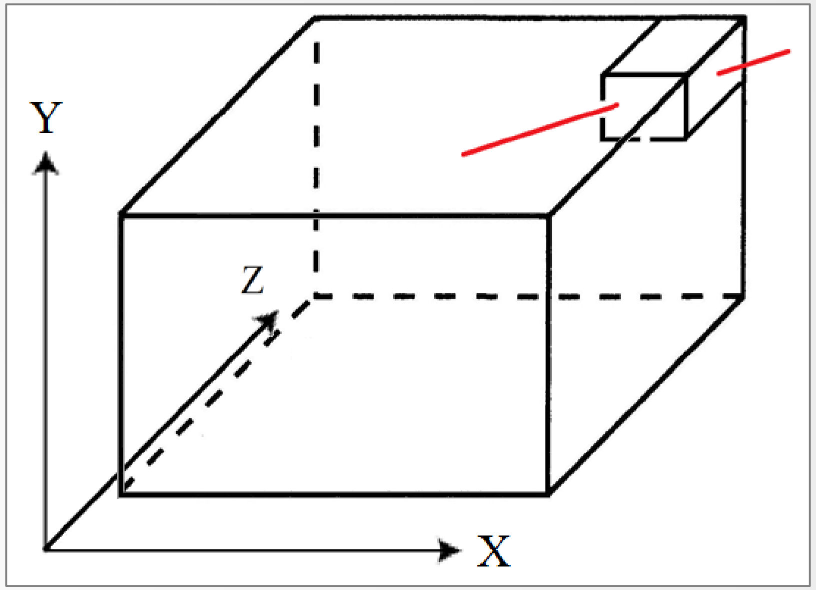


New muon calibration approach

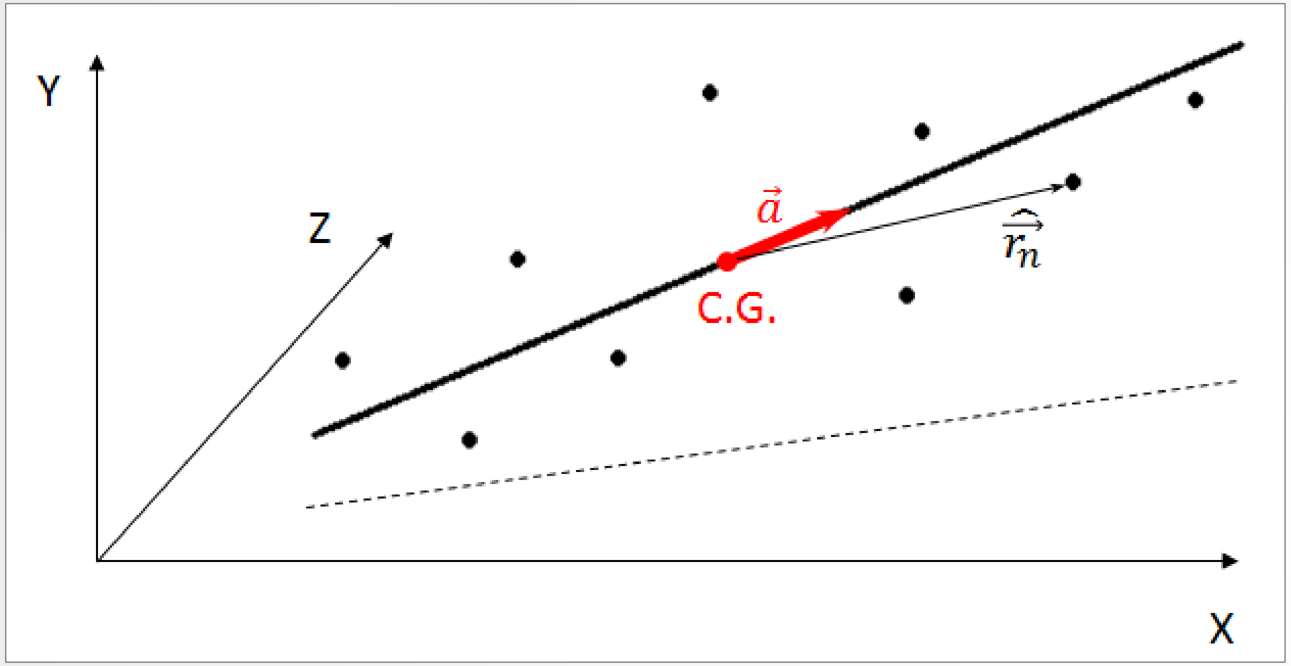
Cosmic muons deposit different amounts of energy in the calorimeter sections depending on the position and direction of the particle track. This should be taken into account when conducting a muon calibration.



Calibration approach:

- ❖ Reconstruct muon tracks using signals selected with fit QA
- ❖ Determine the thickness of the scintillator passed by track in each cell
- ❖ Make corrections when calculating energy deposition

Muon track reconstruction



$$\sum_{n=1}^{N}\left(\frac{(\hat{\vec{r}}[n],\vec{a})}{|\vec{a}|}\right)^2 \to \max \qquad \varphi = \sum_{n=1}^{N} \hat{r}_i a_i \hat{r}_j a_j \to max$$

Maximizing the quadratic form φ on the unit vector $\vec{a}$. The quadratic form is maximal on the eigenvector corresponding to the maximal eigenvalue.

- ❖ Selection of triggered sections by fit QA
- ❖ Shift reference system to the center of gravity

$$\vec{R}_{C.G.} = \frac{1}{N}\sum_{n=1}^{N} E[n]\,\vec{r}[n].$$

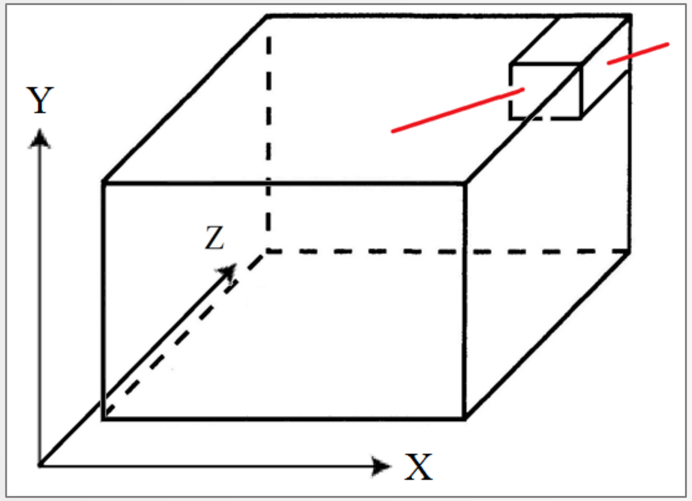
- ❖ Extremum search

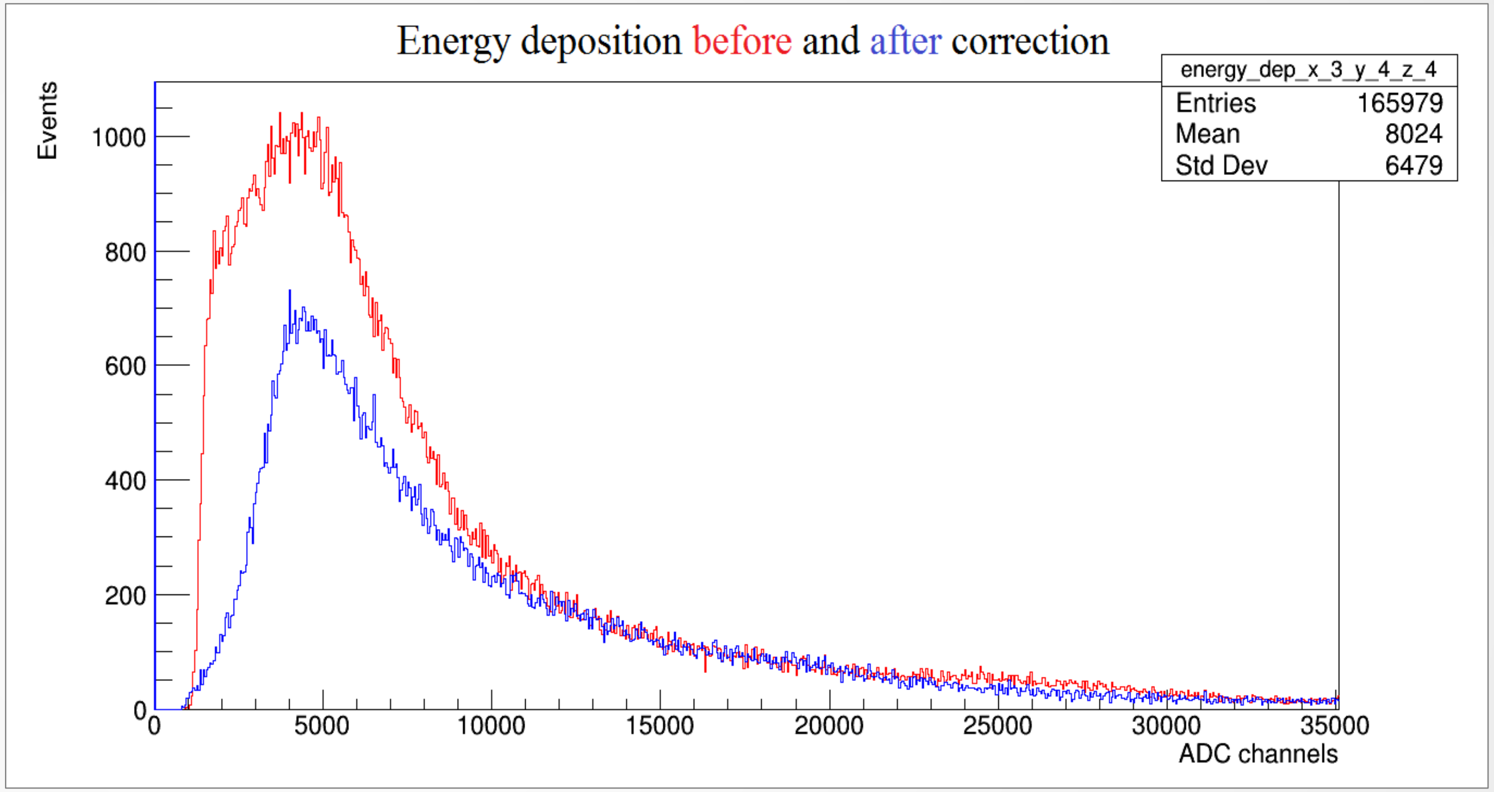
$$\sum_{n=1}^{N}\left(\hat{\vec{r}}^{\,2}[n] - \left(\frac{(\hat{\vec{r}}[n],\vec{a})}{|\vec{a}|}\right)^2\right) \to min$$

$$M = \begin{pmatrix} \sum_{n=1}^{N} r_n^x\, r_n^x & \sum_{n=1}^{N} r_n^x\, r_n^y & \sum_{n=1}^{N} r_n^x\, r_n^z \\ \sum_{n=1}^{N} r_n^y\, r_n^x & \sum_{n=1}^{N} r_n^y\, r_n^y & \sum_{n=1}^{N} r_n^y\, r_n^z \\ \sum_{n=1}^{N} r_n^x\, r_n^z & \sum_{n=1}^{N} r_n^y\, r_n^z & \sum_{n=1}^{N} r_n^z\, r_n^z \end{pmatrix}$$

Adjusted charge calculation

Calculation of the thickness of scintillator material traversed by the particle track by enumerating 6 faces of each triggered section.





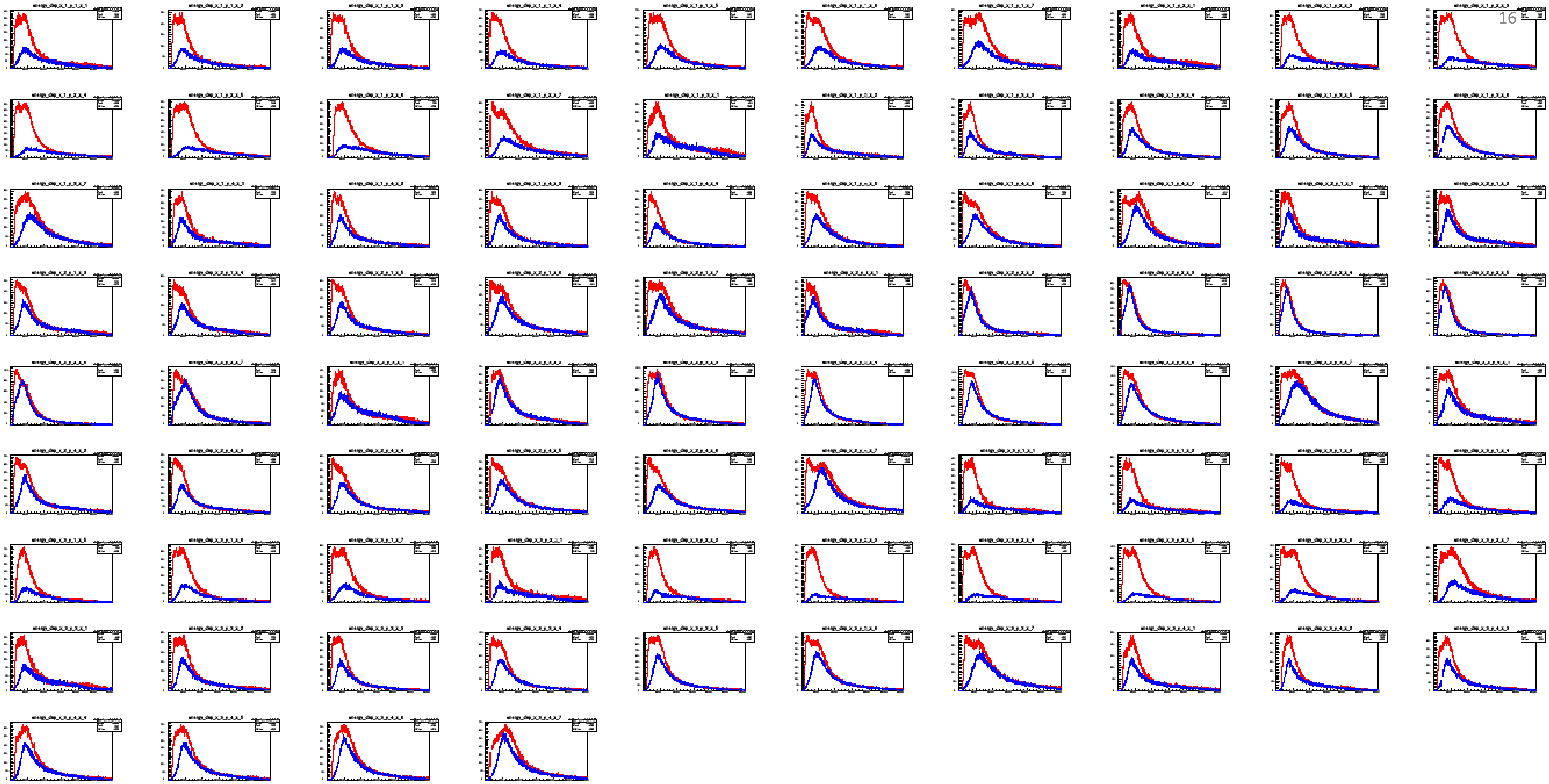
The adjusted charge is considered as if the particle has passed straight through the section, traversing 6×4 mm of the scintillator. In the case when the track did not pass through the section, it is impossible to correct the charge, the adjusted energy deposition is considered to be zero.

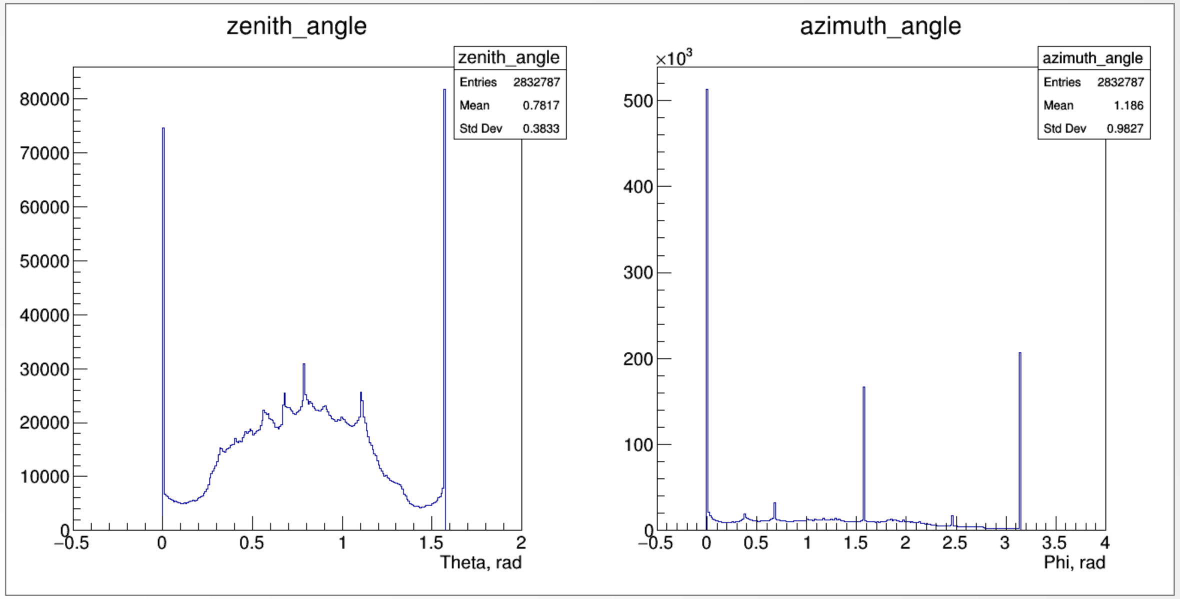
Summary

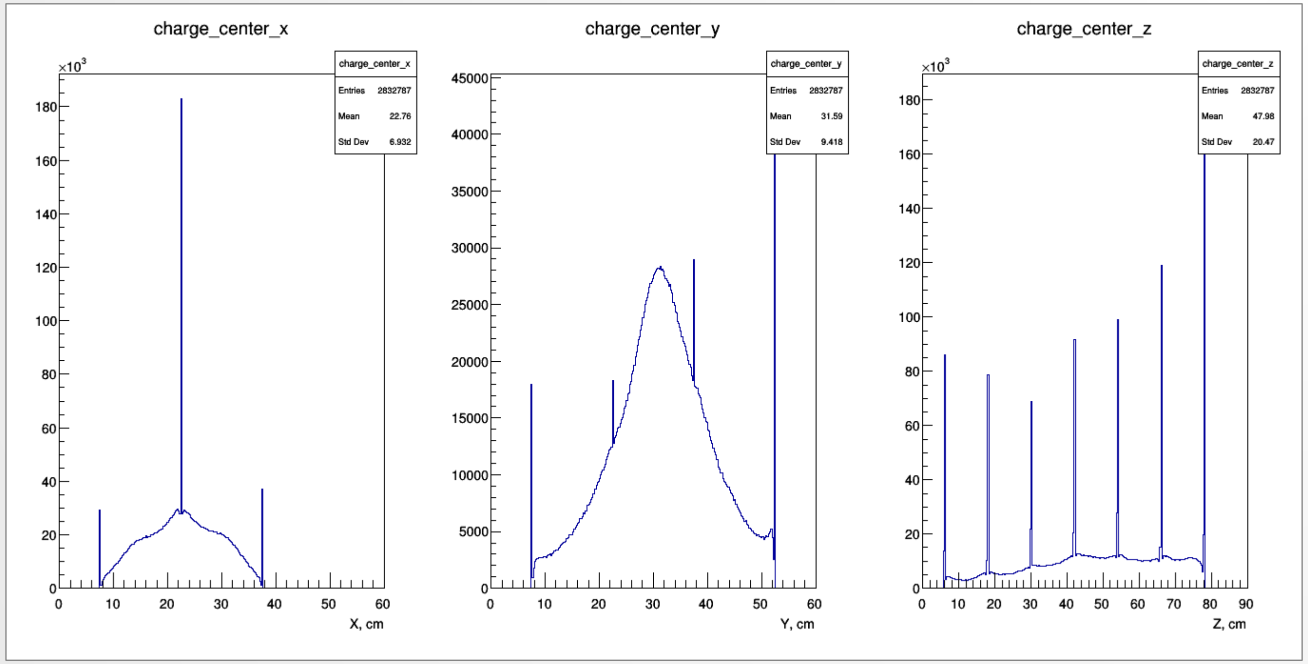
- A new method for fitting signals is developed
- The application of the fit QA is shown
- New approach to the muon calibration is implemented

Thank you for your attention

Backup




zenith_angle
zenith_angle
Entries 2832787
Mean 0.7817
Std Dev 0.3833
Theta, rad
azimuth_angle
azimuth_angle
Entries 2832787
Mean 1.186
Std Dev 0.9827
Phi, rad


charge_center_x
charge_center_x
Entries 2832787
Mean 22.76
Std Dev 6.932
X, cm
charge_center_y
charge_center_y
Entries 2832787
Mean 31.59
Std Dev 9.418
Y, cm
charge_center_z
charge_center_z
Entries 2832787
Mean 47.98
Std Dev 20.47
Z, cm