Pion condensation in dense baryonic/quark matter with isospin and chiral imbalance

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Broad Group

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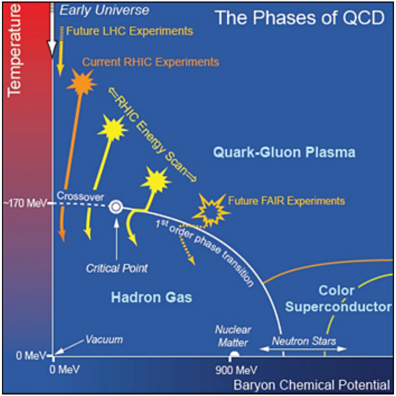
H. Abuki, M. Ruggieri, J. O. Andersen, L. Kyllingstad et al details can be found in

Eur.Phys.J. C79 (2019) no.2, 151, arXiv:1812.00772 [hep-ph],
Phys.Rev. D98 (2018) no.5, 054030 arXiv:1804.01014 [hep-ph],
Phys.Rev. D97 (2018) no.5, 054036 arXiv:1710.09706 [hep-ph],
Phys.Rev. D95 (2017) no.10, 105010 arXiv:1704.01477 [hep-ph],
Phys. Rev. D **86** (2012) 085011 [arXiv:1206.2519 [hep-ph]],
Int. J. Mod. Phys. A **27** (2012) 1250162 [arXiv:1106.2928[hep-ph]],
Phys. Rev. D **78** (2008) 014002 [arXiv:0801.4254 [hep-ph]],
J. Phys. G **37** (2009) 015003 [hep-ph/0701033],
J. Phys. G **32** (2006) 599 [hep-ph/0507007],
Eur. Phys. J. C **46** (2006) 771 [hep-ph/0510222].

QCD at finite temperature and nonzero chemical potential

QCD at nonzero temperature and baryon chemical potential plays a fundamental role in many different physical systems. (QCD at extreme conditions)

- neutron stars
- heavy ion collision experiments
- Early Universe



Lattice QCD

First principle calculation – lattice Monte Carlo simulations, LQCD

lattice QCD at non-zero baryon chemical potential μ_B

Lattice QCD

non-zero baryon chemical potential μ_B

sign problem — complex determinant

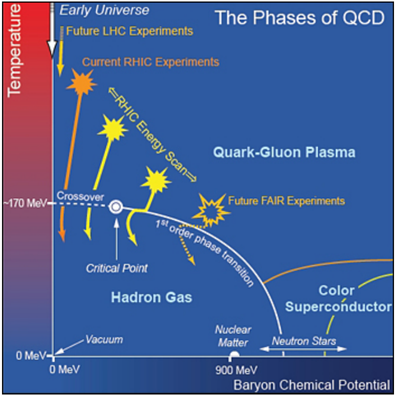
$$(Det(D(\mu)))^\dagger = Det(D(-\mu^\dagger))$$

QCD at non-zero baryon density

QCD at nonzero baryon chemical potential

in effective models

Nambu–Jona-Lasinio model



NJL model

NJL model can be considered as **effective field theory for QCD**.

the model is **nonrenormalizable**
Valid up to $E < \Lambda \approx 1$ GeV

Parameters G, Λ, m_0

dof– **quarks**
no gluons only **four-fermion interaction**
attractive feature — dynamical CSB

Relative simplicity allow to consider hot and dense QCD in the framework of NJL model and explore the QCD phase structure (diagram).

Nambu–Jona-Lasinio model

Nambu–Jona-Lasinio model

$$\mathcal{L} = \bar{q}\gamma^{\nu}\mathrm{i}\partial_{\nu}q + \frac{G}{N_c}\Big[(\bar{q}q)^2 + (\bar{q}\mathrm{i}\gamma^5 q)^2\Big]$$

$$q \to e^{i\gamma_5\alpha}q$$

continuous symmetry

$$\widetilde{\mathcal{L}} = \bar{q}\Big[\gamma^{\rho}\mathrm{i}\partial_{\rho} - \sigma - \mathrm{i}\gamma^5\pi\Big]q - \frac{N_c}{4G}\Big[\sigma^2 + \pi^2\Big].$$

Chiral symmetry breaking

$1/N_c$ expansion, leading order

$$\langle\bar{q}q\rangle \neq 0$$

$$\langle\sigma\rangle \neq 0 \quad \longrightarrow \quad \widetilde{\mathcal{L}} = \bar{q}\Big[\gamma^{\rho}\mathrm{i}\partial_{\rho} - \langle\sigma\rangle\Big]q$$

Dense matter with isotopic imbalance:

Different types of chemical potentials

Different types of chemical potentials: dense matter with isotopic imbalance

Baryon chemical potential μ_B

Allow to consider systems with non-zero baryon densities.

$$\frac{\mu_B}{3}\bar{q}\gamma^0 q = \mu\bar{q}\gamma^0 q,$$

Different types of chemical potentials: dense matter with isotopic imbalance

Baryon chemical potential μ_B

Allow to consider systems with non-zero baryon densities.

$$\frac{\mu_B}{3}\bar{q}\gamma^0 q = \mu\bar{q}\gamma^0 q,$$

Isotopic chemical potential μ_I

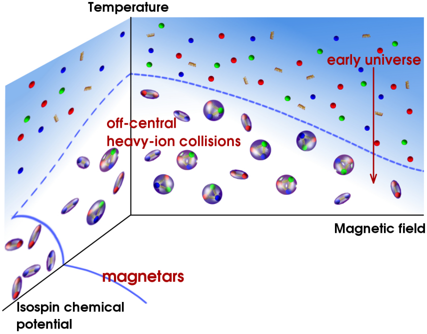
Allow to consider systems with isotopic imbalance.

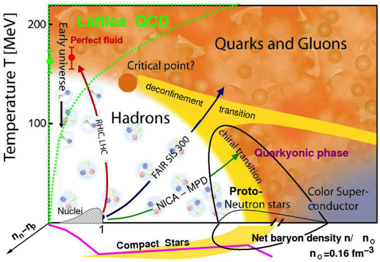
$$n_I = n_u - n_d \quad \longleftrightarrow \quad \mu_I = \mu_u - \mu_d$$

The corresponding term in the Lagrangian is $\frac{\mu_I}{2}\bar{q}\gamma^0\tau_3 q$

QCD phase diagram with isotopic imbalance

neutron stars, heavy ion collisions have isotopic imbalance





Model and its Lagrangian

We consider a NJL model, which describes dense quark matter with two massless quark flavors (u and d quarks).

$$\mathcal{L} = \bar{q}\Big[\gamma^\nu \mathrm{i}\partial_\nu + \frac{\mu_B}{3}\gamma^0 + \frac{\mu_I}{2}\tau_3\gamma^0\Big]q +$$

$$\frac{G}{N_c}\Big[(\bar{q}q)^2 + (\bar{q}\mathrm{i}\gamma^5\vec{\tau}q)^2\Big]$$

q is the flavor doublet, $q = (q_u, q_d)^T$, where q_u and q_d are four-component Dirac spinors as well as color N_c-plets; τ_k $(k = 1, 2, 3)$ are Pauli matrices.

Equivalent Lagrangian

To find the thermodynamic potential we use a semi-bosonized version of the Lagrangian

$$\widetilde{L} = \bar{q}\Big[\gamma^\rho \mathrm{i}\partial_\rho + \mu\gamma^0 + \nu\tau_3\gamma^0 - \sigma - \mathrm{i}\gamma^5\pi_a\tau_a\Big]q - \frac{N_c}{4G}\Big[\sigma\sigma + \pi_a\pi_a\Big].$$

$$\sigma(x) = -2\frac{G}{N_c}(\bar{q}q); \quad \pi_a(x) = -2\frac{G}{N_c}(\bar{q}\mathrm{i}\gamma^5\tau_a q).$$

Condansates ansatz $\langle\sigma(x)\rangle$ and $\langle\pi_a(x)\rangle$ do not depend on spacetime coordinates x,

$$\langle\sigma(x)\rangle = M, \quad \langle\pi_1(x)\rangle = \Delta, \quad \langle\pi_2(x)\rangle = 0, \quad \langle\pi_3(x)\rangle = 0. \quad (1)$$

where M and Δ are already constant quantities.

thermodynamic potential

the thermodynamic potential can be obtained in the large N_c limit

$$\Omega(M, \Delta)$$

No mixed phase $(M \neq 0, \Delta \neq 0)$

Pion condensation history

In the early 1970s Migdal suggested the possibility of pion condensation in a nuclear medium
A.B. Migdal, E. E. Saperstein, D. N. Voskresensky et al

A.B. Migdal, Zh. Eksp. Teor. Fiz. 61, 2210 (1971) [Sov. Phys. JETP 36, 1052 (1973)]; A. B. Migdal, E. E. Saperstein, M. A. Troitsky and D. N. Voskresensky, Phys. Rept. 192, 179 (1990).
R.F. Sawyer, Phys. Rev. Lett. 29, 382 (1972);

In medium pion mass properties and RMF.
pion condensation is highly unlikely to be realized in nature in matter of neutron star, A. Ohnishi D. Jido T. Sekihara, and K. Tsubakihara, Phys. Rev. C80, 038202 (2009) . .

Pion condensation in NJL model, chiral limit

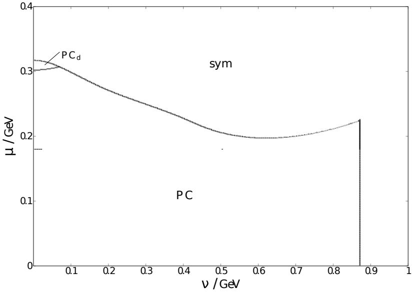
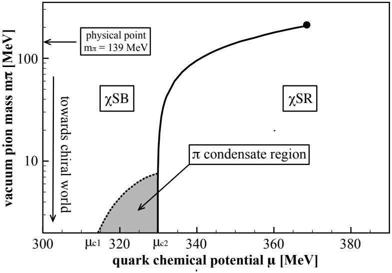


Figure: (ν, μ) phase diagram in NJL model in the chiral limit.

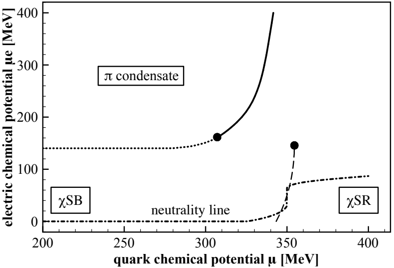
K. G. Klimenko, D. Ebert
J.Phys. G32 (2006) 599-608;
Eur.Phys.J.C46:771-776,(2006)

PC phenomenon maybe could be realized in **dense baryonic matter**
even in charge neutral case

Pion condensation in NJL model: physical point and the case of electric neutrality



(μ, m_0) phase portrait.



(μ, μ_e) phase portrait.

No PC condensation in the neutral case at the physical point

(H. Abuki, R. Anglani, M. Ruggieri etc.
Phys. Rev. D **79** (2009) 034032.

Pion condensation in NJL model, physical point

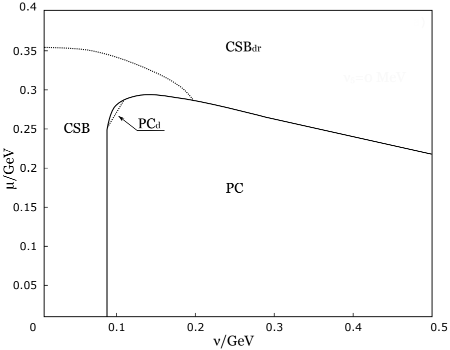


Figure: (ν, μ) phase diagram in NJL model at physical point.

But the analysis has been performed in the **chiral limit (zero current quark mass)**

At the physical point (physical values of quark masses) **PC phenomenon** in **dense baryonic matter** is almost extinct from the phase diagram.
even without charge neutral condition

(1+1)-dimensional Gross-Neveu (GN) or NJL_2 model consideration

Conditions promoting PC in dense baryonic matter

(1+1)-dimensional Gross-Neveu (GN) or NJL_2 model consideration

The NJL_2 model Lagrangian has the form

$$L = \bar{q}\gamma^{\nu} \mathrm{i}\partial_{\nu} q + \frac{G}{N_c}\left[(\bar{q}q)^2 + (\bar{q}\mathrm{i}\gamma^5\vec{\tau}q)^2\right],$$

where the quark field $q(x) \equiv q_{i\alpha}(x)$ is a flavor doublet ($i = u, d$), it is a two-component Dirac spinor

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \qquad \gamma^1 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}; \qquad \gamma^5 = \gamma^0\gamma^1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

(1+1)- dimensional GN, NJL_2 model

(1+1)-dimensional Gross-Neveu (GN) possesses a lot of common features with QCD

- renormalizability
- asymptotic freedom
- spontaneous chiral symmetry breaking in vacuum
- dimensional transmutation
- have the similar $\mu_B - T$ phase diagrams

NJL_2 model
laboratory for the qualitative simulation of specific properties of QCD **at arbitrary energies**

Finite size effects

Finite size effects

To simulate the finite size effect one puts our (1+1)-dim system into a restricted space region $0 \leq x \leq L$

and consider the model in spacetime $R^1 \times S^1$

and with quantum fields satisfying

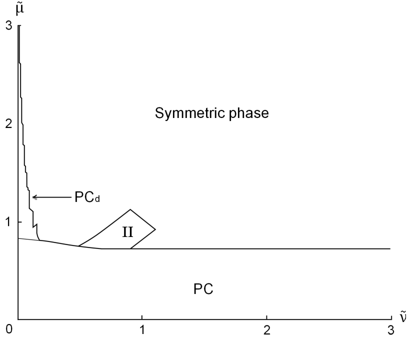
$$q(t, x + L) = e^{i\pi\alpha} q(t, x),$$

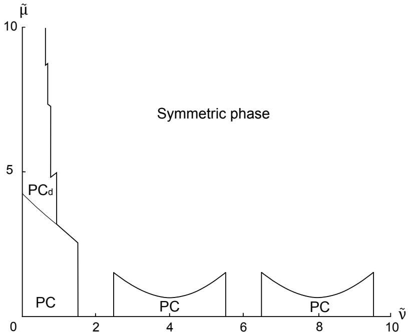
where $0 \leq \alpha \leq 2$ is the parameter fixing the boundary conditions,

$\alpha = 0$ – periodic boundary condition

$\alpha = 1$ – antiperiodic boundary condition

Pion condensation and finite size effects





If the system is **confined (finite size effects) PC condensation in dense quark matter** can appear

(D. Ebert, T. G. Khunjua, K. G. Klimenko and V. Ch. Zhukovsky, Int. J. Mod. Phys. A **27** (2012) 1250162)

Inhomogeneous pion condensation

Inhomogeneous pion condensation

when $\mu \neq 0$, $\mu_I \neq 0$

$$\langle\sigma(x)\rangle = M, \ \langle\pi_3(x)\rangle = 0, \ \langle\pi_1(x)\rangle = \Delta\cos(2bx), \ \langle\pi_2(x)\rangle = \Delta\sin(2bx)$$

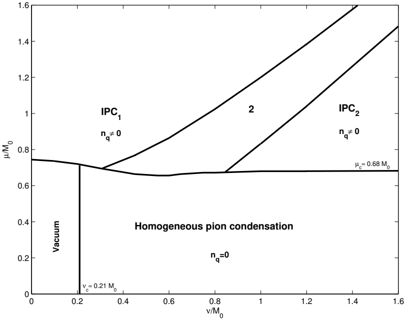


Figure: (ν, μ) phase diagram.

Inhomogeneous PC phase in dense baryonic matter can be generated

N. V. Gubina, K. G. Klimenko, S. G. Kurbanov, V. Ch. Zhukovsky, 10.1103/PhysRevD.86.085011

Chiral imbalance.

Different types of chemical potentials: chiral imbalance

chiral (axial) chemical potential

Allow to consider systems with chiral imbalance (difference between between densities of left-handed and right-handed quarks).

$$n_5 = n_R - n_L \quad \longleftrightarrow \quad \mu_5 = \mu_R - \mu_L$$

The corresponding term in the Lagrangian is

$$\mu_5 \bar{q} \gamma^0 \gamma^5 q$$

chiral (axial) isotopic chemical potential

Allow to consider systems with chiral isospin imbalance

$$\mu_{I5} = \mu_{u5} - \mu_{d5}$$

so the corresponding density is

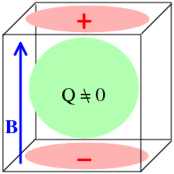
$$n_{I5} = n_{u5} - n_{d5}$$

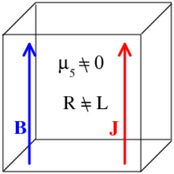
$$n_{I5} \quad \longleftrightarrow \quad \mu_{I5}$$

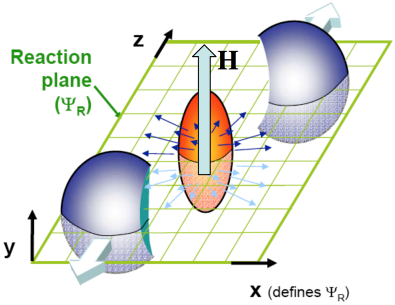
Term in the Lagrangian — $\frac{\mu_{I5}}{2}\bar{q}\tau_3\gamma^0\gamma^5 q$

If one has all four chemical potential, one can consider different densities n_{uL}, n_{dL}, n_{uR} and n_{dR}

Chiral magnetic effect



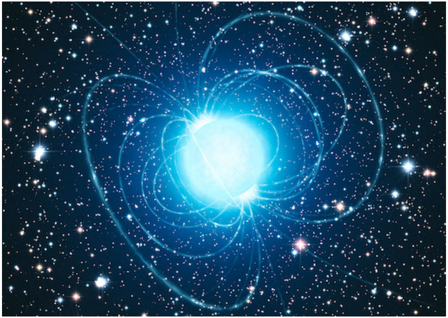




$$\vec{J} = c\mu_5 \vec{B}, \qquad c = \frac{e^2}{2\pi^2}$$

A. Vilenkin, PhysRevD.22.3080,
K. Fukushima, D. E. Kharzeev and H. J. Warringa, Phys. Rev. D **78** (2008) 074033 [arXiv:0808.3382 [hep-ph]].

Generation of chiral imbalance in compact stars



Due to high baryon densities, magnetic fields and vorticity

- Chiral separation effect CSE
- Chiral Vortical effect CVE

Chiral separation effect

Chiral magnetic (CME) effect has the form

$$\vec{J} = c\mu_5 \vec{H}$$

There is a dual effect so-called chiral sepration effect (CSE) (Son and Zhitnitsky 2004, Metlitski and Zhitnitsky 2005)

$$\vec{J_5} = c\mu \vec{H}, \quad J_5^\mu = \langle \bar{\psi}\gamma^\mu\gamma^5\psi \rangle$$

Then the phenomena looks very similar and dual.

$$\vec{J_V} = c\mu_A \vec{H}, \quad \vec{J_A} = c\mu_V \vec{H}$$

Chiral separation effect in a two flavoured system

Let us consider the system with u and d quark flavours

$$\vec{J}_{5u} = \frac{N_c q_u}{2\pi^2} \mu_u \vec{H}$$

and for d quark sector the axial current is

$$\vec{J}_{5d} = \frac{N_c q_d}{2\pi^2} \mu_d \vec{H}$$

Now let us calculate the chiral current

$$\vec{J}_5 = \vec{J}_u^5 + \vec{J}_d^5 = \frac{N_c}{2\pi^2}(q_u \mu_u + q_d \mu_d)\vec{H}$$

Now let us express it in terms of μ and ν, taking into account that $\mu_u = \mu + \nu$ and $\mu_d = \mu - \nu$ one has

$$\vec{J}_5 = \frac{N_c}{2\pi^2}[(q_u + q_d)\mu + (q_u - q_d)\nu)]\vec{H}$$

Chiral separation effect in a two flavoured system

Chiral isospin current and charge

$$\vec{J}_{I5} = \vec{J}_{5u} - \vec{J}_{5d} = \frac{N_c}{2\pi^2}(q_u\mu_u - q_d\mu_d)\vec{H}$$

Expressing it in terms of μ and ν

$$\vec{J}^{I5} = \frac{N_c}{2\pi^2}[(q_u - q_d)\mu + (q_u + q_d)\nu)]\vec{H}$$

Chiral separation effect in a two flavoured system

The chiral charge:

$$Q_5 = \int d^3x \langle \bar{\psi} \gamma^0 \gamma^5 \psi \rangle \iff \mu_5$$

The chiral isospin charge

$$Q_{I5} = \int d^3x \langle \bar{\psi} \gamma^0 \gamma^5 \tau_3 \psi \rangle \iff \mu_{I5}$$

Thanks to Igor Shovkovy

Chiral Vortical Effect (CVE)

Vorticity

$$\vec{\omega} = \frac{1}{2}\vec{\nabla} \times \vec{v}$$

Chiral Vortical Effect (CVE) quantifies the generation of a vector current J along the vorticity direction:

$$\vec{J} = \frac{1}{\pi^2}\mu\mu_5\vec{\omega}$$

Axial current can be generated by the rotation as well

$$\vec{J_5} = \left[\frac{1}{6}T^2 + \frac{1}{2\pi^2}(\mu^2 + \mu_5^2)\right]\vec{\omega}$$

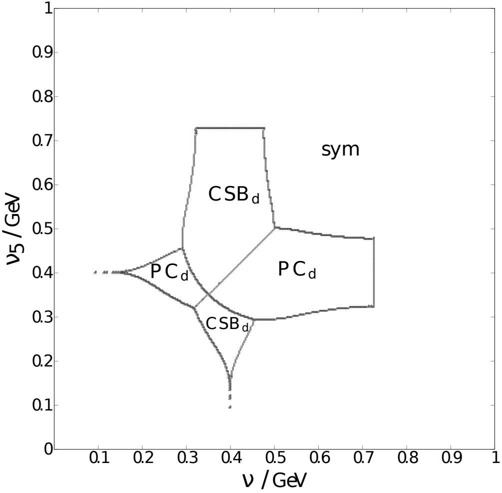
Chiral imbalance generation due to CVE

$$\vec{J}_5 = \vec{J}_5^u + \vec{J}_5^d = \left[\frac{1}{3}T^2 + \frac{1}{2\pi^2}(\mu^2 + \nu^2)\right]\vec{\omega}$$

$$\vec{J}_{I5} = \vec{J}_5^u - \vec{J}_5^d = \left[\frac{2}{\pi^2}\mu\nu\right]\vec{\omega}$$

Chiral isospin imbalance

Chiral isospin imbalance generate PC phenomenon in dense qurark matter



Chiral imbalance in the form of μ_5 chemical potential. (ν, μ_5) phase diagram

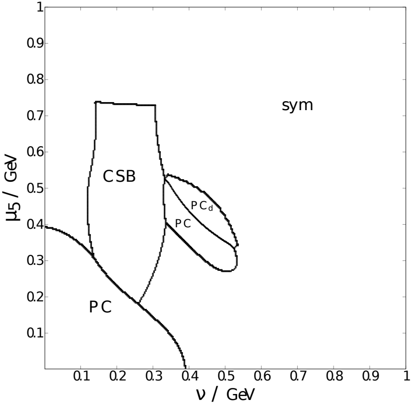


Figure: (ν, μ_5) phase diagram at $\mu = 0.23$ GeV.

$\mu_5 \rightarrow PC_d$

No that widespread and only at rather low baryon densities

Consideration of the general case μ, μ_I, μ_{I5} and μ_5

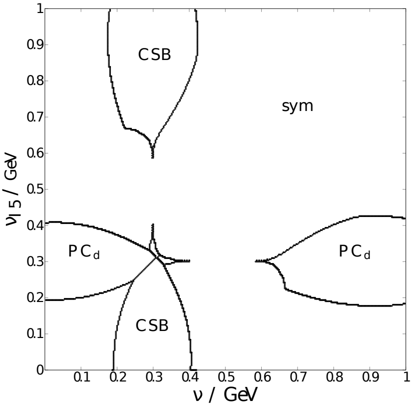


Figure: (ν, ν_5) phase diagram at $\mu_5 = 0.5$ GeV and $\mu = 0.3$ GeV.

generation of PC$_d$ phase is even **more widespread**

possible even **for zero isospin asymmetry**

Charge neutrality condition

the general case (μ, μ_I, μ_{I5}, μ_5)

consider charge neutrality case $\quad \rightarrow \quad \nu = \mu_I/2 = \nu(\mu, \nu_5, \mu_5)$

Charge neutrality condition

-physical quark mass and electric neutrality - no pion condensation in dense medium
H. Abuki, R. Anglani, R. Gatto, M. Pellicoro, M. Ruggieri
Phys.Rev.D79:034032,2009 arXiv:0809.2658 [hep-ph]

-Chiral isospin chemical potential μ_{I5} generates PC$_d$

-can this generation happen in the case of neutrality condition

Charge neutrality condition

It can be shown that the PC_d phase can be generated by chiral imbalance in the case of charge neutrality condition

non-zero $\mu_5 \to \mathrm{PC}_d$ phase in neutral quark matter

Conclusions

In dense $\mu_B \neq 0$ isotopically asymmetric $\mu_I \neq 0$ quark matter

PC_d is not realized

But there could be conditions promoting this phenomenon

- finite size effect (in NJL_2 model)
- inhomogeneous PC phase (in NJL_2 model)
- chiral imbalance (in NJL_2 and NJL models)

$\mu_B \neq 0$ - dense quark matter

$\mu_I \neq 0$ isotopically asymmetric

$\mu_5 \neq 0$ and $\mu_{I5} \neq 0$ chirally asymmetric

Dualities

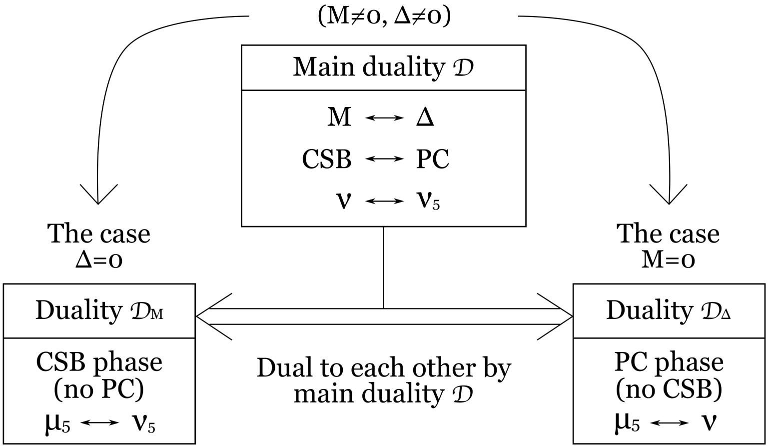


Figure: Dualities

Thanks for the attention