Two-phonon structures of β-decay rates

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particle-hole channel

particle-particle channel

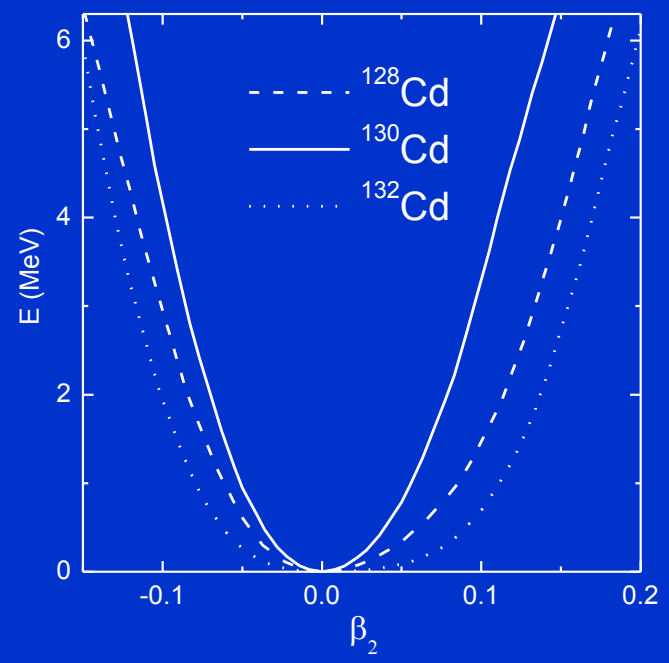
Skyrme interaction

$$V_0\left(1-\eta\frac{\rho(r_1)}{\rho_0}\right)\delta(r_1-r_2)$$

$\Downarrow$

HF-BCS calculations

To calculate binding energies of the daughter nucleus (N - 1,Z + 1) and the final nucleus (N - 1 - X, Z + 1), the blocking effect for unpaired nucleons are taken into account. Finally, the calculated *Q*-value and the neutron separation energies for the daughter nucleus are given by



$$Q_\beta = \Delta M_{n-H} + B(Z+1, N-1) \quad - B(Z, N)$$

$$S_{xn} = B(Z+1, N-1) - B(Z+1, N-1-X)$$

ΔM_{n-H}=0.782MeV is the mass difference between the neutron and the hydrogen atom

QRPA calculations $Q_\nu^+|0>$

$$Q_\nu^+ = \sum_{n,p} X_{np}^\nu A^+(n,p;JM) - (-1)^{J-M} Y_{np}^\nu A(n,p;J-M)$$

$$A^+(n,p;JM) = \sum_{m_n m_p} < j_n m_n j_p m_p | JM > \alpha_{j_n m_n}^+ \alpha_{j_p m_p}^+$$

Using the equation-of-motion approach one can get

$$\begin{pmatrix} \mathcal{A} & \mathcal{B} \\ -\mathcal{B} & -\mathcal{A} \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = \omega \begin{pmatrix} X \\ Y \end{pmatrix}$$

Making use of the finite rank separable approximation for the residual interaction enables one to perform the calculations in very large configuration spaces

Nguyen Van Giai, Ch. Stoyanov, and V. V. Voronov, Phys. Rev. C57,1204 (1998).

A.P.S., Ch. Stoyanov, V. V. Voronov, and Nguyen Van Giai, Phys. Rev. C66,034304 (2002).

A.P.S., V. V. Voronov, and Nguyen Van Giai, Prog. Theor. Phys. 128 (2012) 489.

A.P.S., and H. Sagawa, Prog. Theor. Exp. Phys. 2013 (2013)103D03.

Two-phonon structures

The coupling between one- and two-phonon terms in the wave functions of excited states are taken into account

$$|\psi_{JM\nu}\rangle = \left(\sum_i R_i(J\nu)Q^+_{JMi} + \sum_{\lambda_1 i_1 \lambda_2 i_2} P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu)\left[Q^+_{\lambda_1\mu_1 i_1}\bar{Q}^+_{\lambda_2\mu_2 i_2}\right]_{JM}\right)|0\rangle$$

$\bar{Q}^+_{\lambda_2\mu_2 i_2}|0\rangle$is the QRPA state having energy $\bar{\Omega}_{\lambda i}$. Non-charge-exchange QRPA is performed in the same way as charge-exchange QRPA. Let us now focus on the low-energy 1^+ states, since we are interested in the β-decay half-lives. We assume only the $[1^+_{i_1}\otimes 2^+_{i_2}]$ terms in order to separate the sole impact of the quadrupole excitations of parent nucleus.

The normalization condition for the wave functions is

$$\sum_i (R_i(J\nu))^2 + \sum_{\lambda_1 i_1 \lambda_2 i_2} \left(P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu)\right)^2 = 1.$$

Using the variational principle, one obtains a set of linear equations for the unknown amplitudes $R_i(J\nu)$ and $P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu)$

$$(\Omega_{\lambda i} - E_\nu)R_i(J\nu) + \sum_{\lambda_1 i_1 \lambda_2 i_2} U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji) P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu) = 0$$

$$\left(\Omega_{\lambda_1 i_1} + \bar{\Omega}_{\lambda_2 i_2} - E_\nu\right) P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu) + \sum_i U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji) R_i(J\nu) = 0$$

$U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji)$denote matrix elements coupling one- and two-phonon configurations

$$U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji) = \left\langle 0 \middle| Q_{Ji} H \left[Q^+_{\lambda_1 i_1} \bar{Q}^+_{\lambda_2 i_2} \right]_J \middle| 0 \right\rangle$$

These equations have the same form as the equations of the quasiparticle-phonon model (QPM)*[V. A. Kuzmin, V. G. Soloviev, J. Phys. G 10, 1507 (1984)],* but the single-particle spectrum and the parameters of the residual interaction are calculated with the Skyrme forces.

A.P.S., V.V.Voronov, I. N. Borzov, Nguyen Van Giai, JINR preprint E4-2012-136.

In the allowed GT approximation, the β-decay rate is expressed by summing the probabilities of the energetically allowed GT transitions ($E_m^{GT} \leq Q_\beta$) weighted with the integrated Fermi function

$$T_{1/2}^{-1} = \sum_m \lambda_{if}^m = D^{-1} \left(\frac{G_A}{G_V}\right)^2 \sum_m f_0(Z+1, A, E_m^{GT})\, B(GT)_m$$

$$E_m^{GT} = Q_\beta - E_{1_m^+}$$

where λ_{if}^m is the partial β-decay rate, $\frac{G_A}{G_V}$ =1.25, D= 6147 s. The transition energies E_m^{GT} refer to the ground state of the parent nucleus. $E_{1_m^+}$ denotes the excitation energy of the daughter nucleus.

$$E_{1_m^+} \approx E_m - E_{2qp,lowest}$$

$E_{2qp,lowest}$ corresponds the lowest two-quasiparticle energy.

E_m and $B(GT)_m$ are the solutions either of the QRPA equations, or of the equations takinginto account the two-phonon configurations.

The difference in the characteristic time scales of the β-decay and subsequent neutron emission processes justifies an assumption of their statistical independence.

The probability of the βxn emission accompanying the β decay to the one- and two-phonon excited states in the daughter nucleus can be expressed as

$$P_{xn} = T_{1/2} D^{-1} \left(\frac{G_A}{G_V}\right)^2 \sum_{m'} f_0(Z+1, A, E_{m'}^{GT})\, B(GT)_{m'}$$

$$P_{1n} \quad : \qquad Q_\beta - S_{2n} \leq E_{m'}^{GT} \leq Q_\beta - S_n$$

$$P_{2n} \quad : \qquad Q_\beta - S_{3n} \leq E_{m'}^{GT} \leq Q_\beta - S_{2n}$$

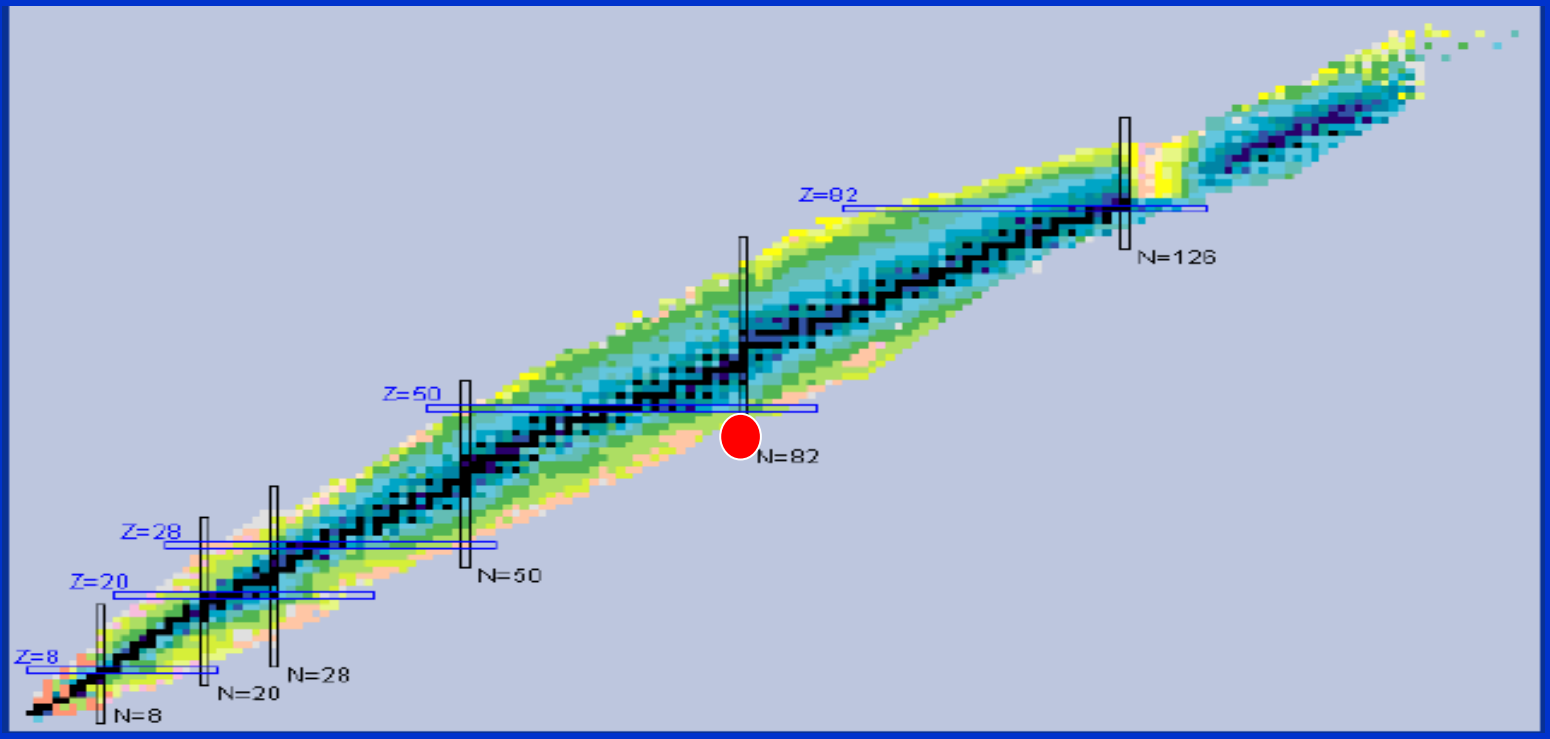
$$P_{3n} \quad : \qquad Q_\beta - S_{4n} \leq E_{m'}^{GT} \leq Q_\beta - S_{3n}$$

$$P_{4n} \quad : \qquad Q_\beta - S_{5n} \leq E_{m'}^{GT} \leq Q_\beta - S_{4n}$$

$$T_{1/2} \quad : \qquad 0 \leq E_m^{GT} \leq Q_\beta$$

A.P.S., N.N. Arsenyev, I.N. Borzov, E.O. Sushenok, Phys. Rev. C 95, 034314 (2017)

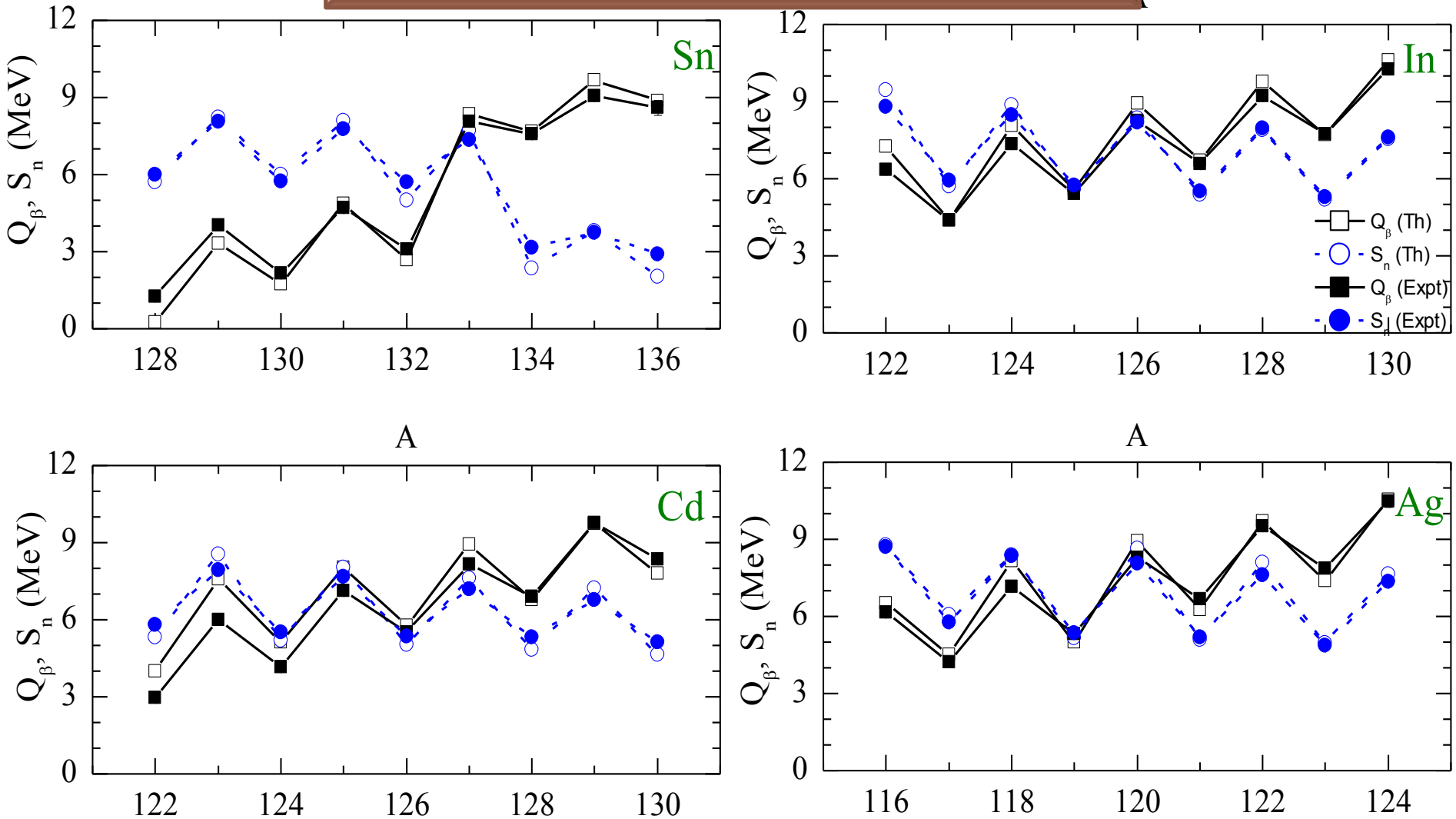
The β-decay properties of r-process "waiting-point nuclei" ^{129}Ag, ^{130}Cd, ^{131}In



We concentrate on the delayed multi-neutron emission in the region below the neutron-rich doubly magic nucleus ^{132}Sn. The β-decay rates help to reconstruct the Gamow-Teller strength distribution and it can be extracted using the 3He long-counter TETRA for β-decays studies at ISOL facility ALTO (JINR-IN2P3 collaboration). Our theoretical investigation taking into account the phonon-phonon coupling and collaboration were flanked by an appropriate experimental IPNO program for the study of low-energy spectrum populated in the β-decay of unstable nuclei near ^{78}Ni and ^{132}Sn.

βn –decay window

T43



D.Testov, A. Severyukhin, D. Verney, F. Ibrahim, Yu. Penionzhkevich, N. Arsenyev, M. Lebois, I. Matea, V. Smirnov, E. Sokol, B. Roussiere, I. Stefan, D. Susuki, Jh. Wilson, in preparartion

Excitation 2_1^+ energies of the parent nuclei

T43

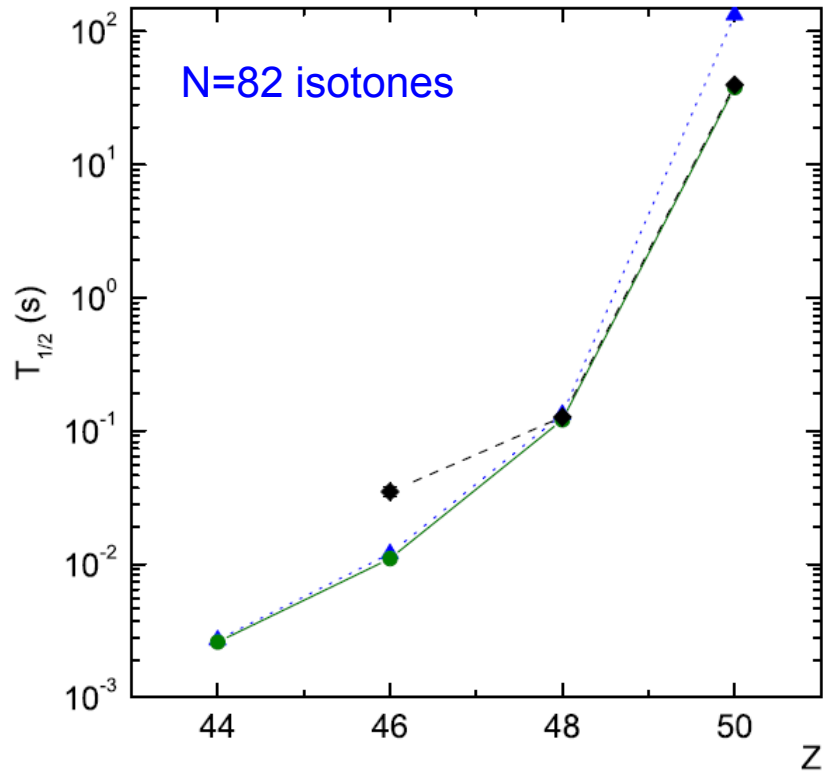
The $2^+_{1_{\{QRPA\}}}$ state is the lowest collective excitation which leads to the maximal matrix elements coupling one- and two-phonon configurations. The calculated energies and the B(E2) values for up-transitions to the $2^+_{1_{\{QRPA\}}}$ states reproduce the experimental data satisfactory.

	E_x (MeV)		B(E2) $(e^2 fm^4)$	
	EXPT	QRPA	EXPT	QRPA
^{126}Cd	0.652	0.8	2700±600	3400
^{128}Cd	0.646	1.1		2100
^{130}Cd	1.325	1.3		1100
^{132}Cd		1.1		1500
^{134}Cd		1.1		1700

A.P.S., N.N. Arsenyev, I.N. Borzov, E.O. Sushenok, Phys. Rev. C 95, 034314 (2017)

The evolution of the β-decay half-lives

Nucleus	Half-life (ms) QRPA	PPC	Expt.
^{126}Cd	334	263	513±6
^{128}Cd	212	180	245±5
^{130}Cd	133	121	127±2
^{132}Cd	42	38	82±4
^{134}Cd	36	32	65±15



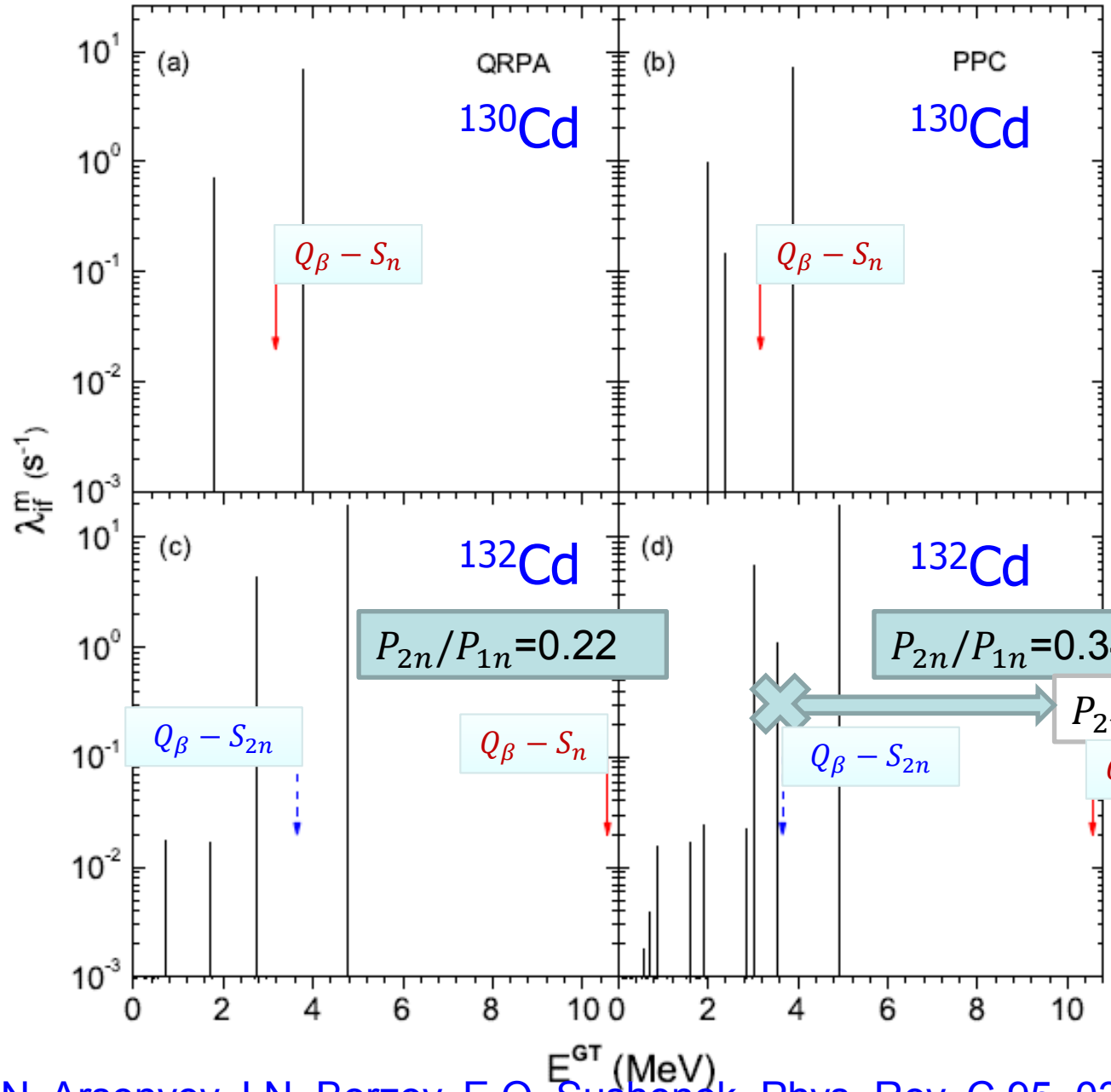
$$T_{1/2}^{-1} = \sum_m \lambda_{if}^m = D^{-1}\left(\frac{G_A}{G_V}\right)^2 \times \sum_m f_0(Z+1, A, E_m^{GT})\, B(GT)_m$$

The largest contribution in the calculated β-decay half-life comes from the $[1_1^+]_{QRPA}$ configuration, but the two-phonon contributions are appreciable. The inclusion of the two-phonon terms results in an increase of the transition energy.

A.P.S., N.N. Arsenyev, I.N. Borzov, E.O. Sushenok, Phys. Rev. C 95, 034314 (2017)

The phonon-phonon coupling effect on the β-transition rates

T43



$P_{n\,tot}$=13.7%

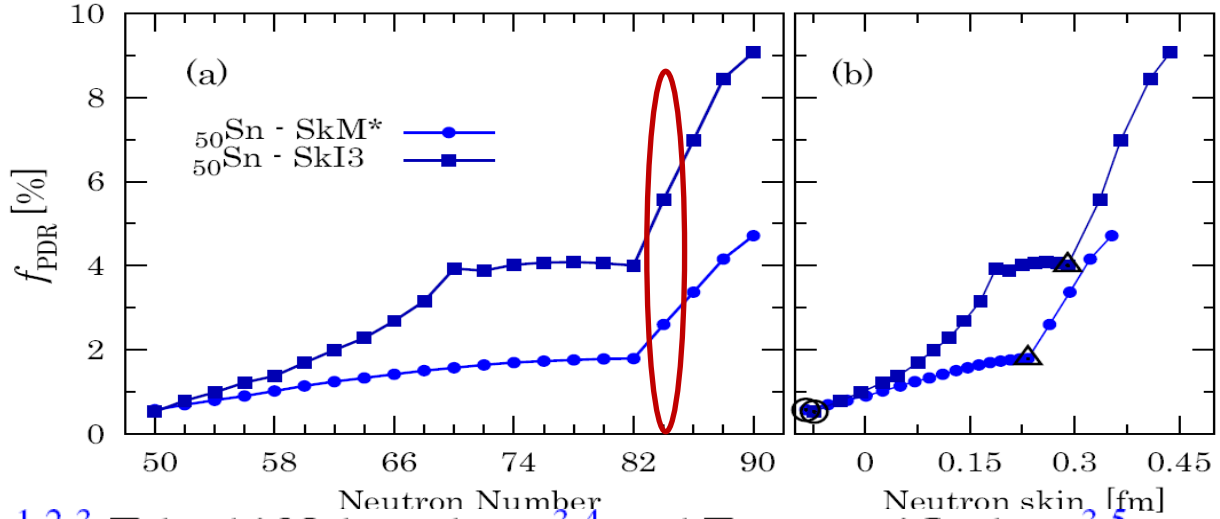
EXPT 3.5±1%

$P_{n\,tot}$=100%

EXPT 60±15%

A.P.S., N.N. Arsenyev, I.N. Borzov, E.O. Sushenok, Phys. Rev. C 95, 034314 (2017)

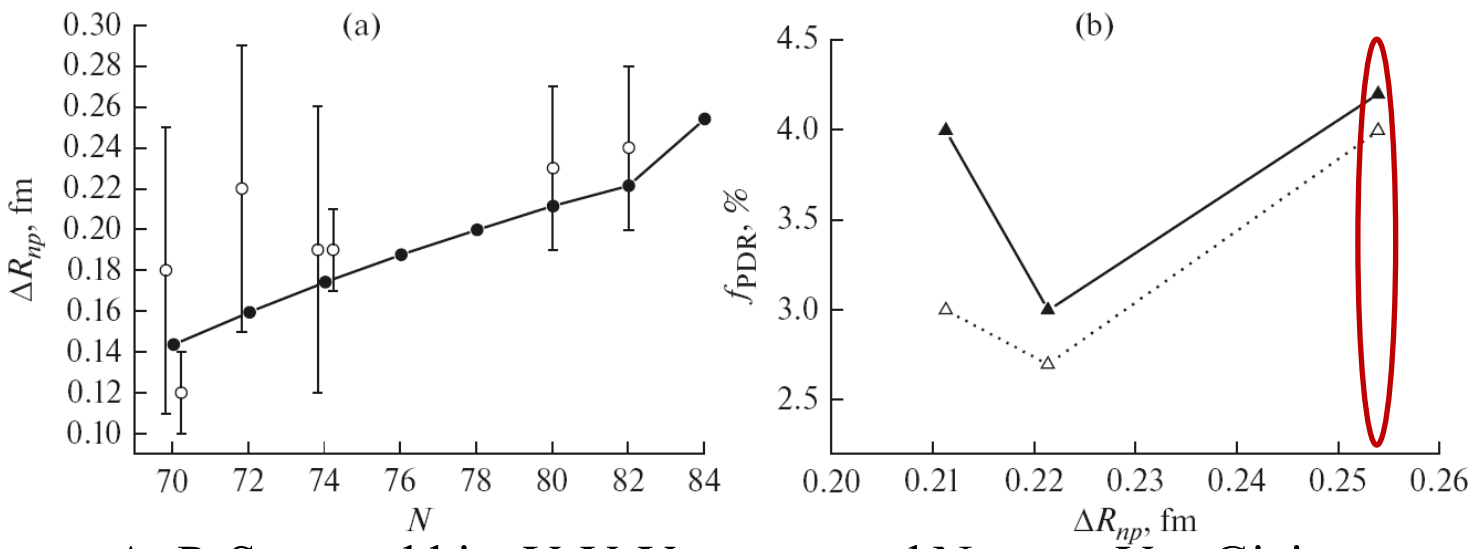
The neutron skin thickness and the pygmy E1 resonance strength



^{134}Sn

Shuichiro Ebata,[1,2,3] Takashi Nakatsukasa,[3,4] and Tsunenori Inakura[3,5]

PHYSICAL REVIEW C **90**, 024303 (2014)



N. N. Arsenyev, A. P. Severyukhin, V. V. Voronov, and Nguyen Van Giai

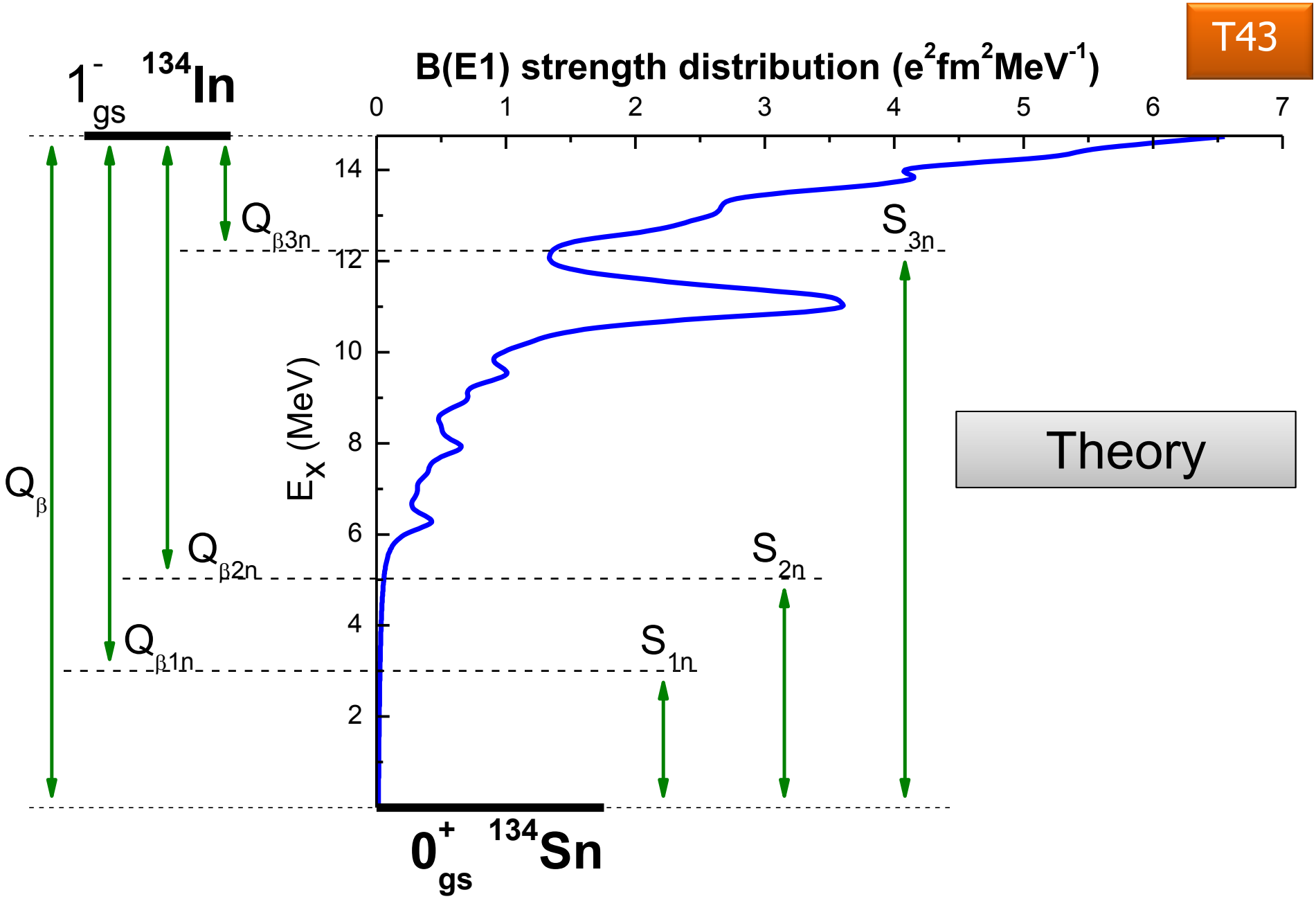
Physics of Particles and Nuclei, 2017, Vol. 48, No. 6, pp. 876–878.

How to identify E1 strength distribution of ^{134}Sn ?

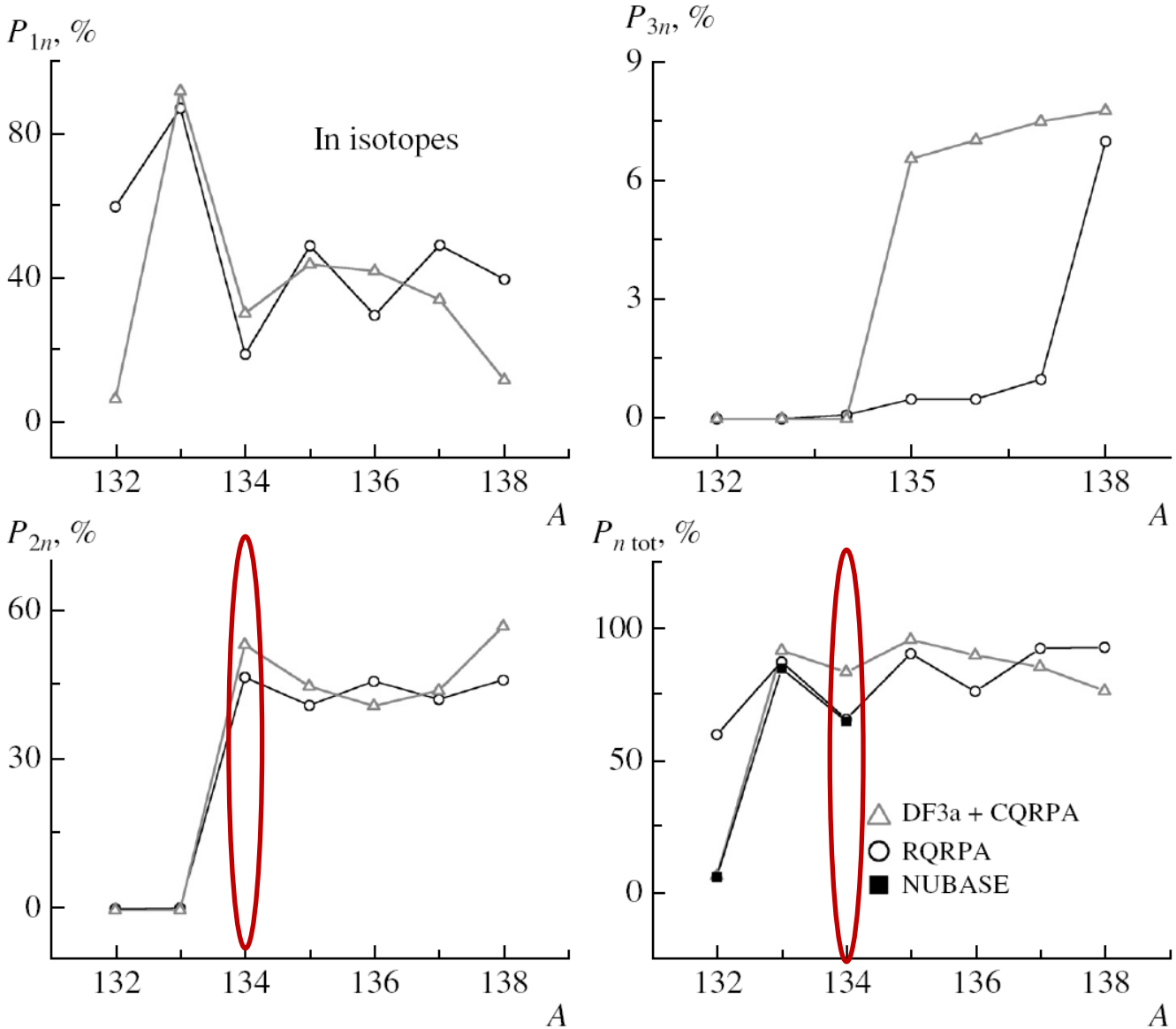
Expt	$T_{1/2}, s$	P_n, %
^{134}In	0.140(4)	65(10)
^{134}Sn	1.050 (11)	17

The E1 strength distribution of ^{130,132}Sn have been studied extensively, but little is known about low-energy 1^- states of ^{134}Sn. For a more firm interpretation of the experimental results, the model is improved to β-decay of odd-odd nuclei. We have studied the β-decay rates of ^{134}In. The inclusion of the two-phonon configurations plays the key role in our calculation. The probabilities of β-delayed neutron emission help to identify E1 strength distribution of ^{134}Sn.

Q_β, MeV (^{134}In)		S_n, MeV (^{134}Sn)		S_{2n}, MeV (^{134}Sn)	
EXPT	T43	EXPT	T43	EXPT	T43
14.8(3)	14.8	3.629	3.0	6.031	5.0


T43
B(E1) strength distribution (e²fm²MeV⁻¹)
0
1
2
3
4
5
6
7
1⁻gs 134In
0+gs 134Sn
Qβ
Qβ3n
Qβ2n
Qβ1n
S3n
S2n
S1n
Ex (MeV)
14
12
10
8
6
4
2
Theory

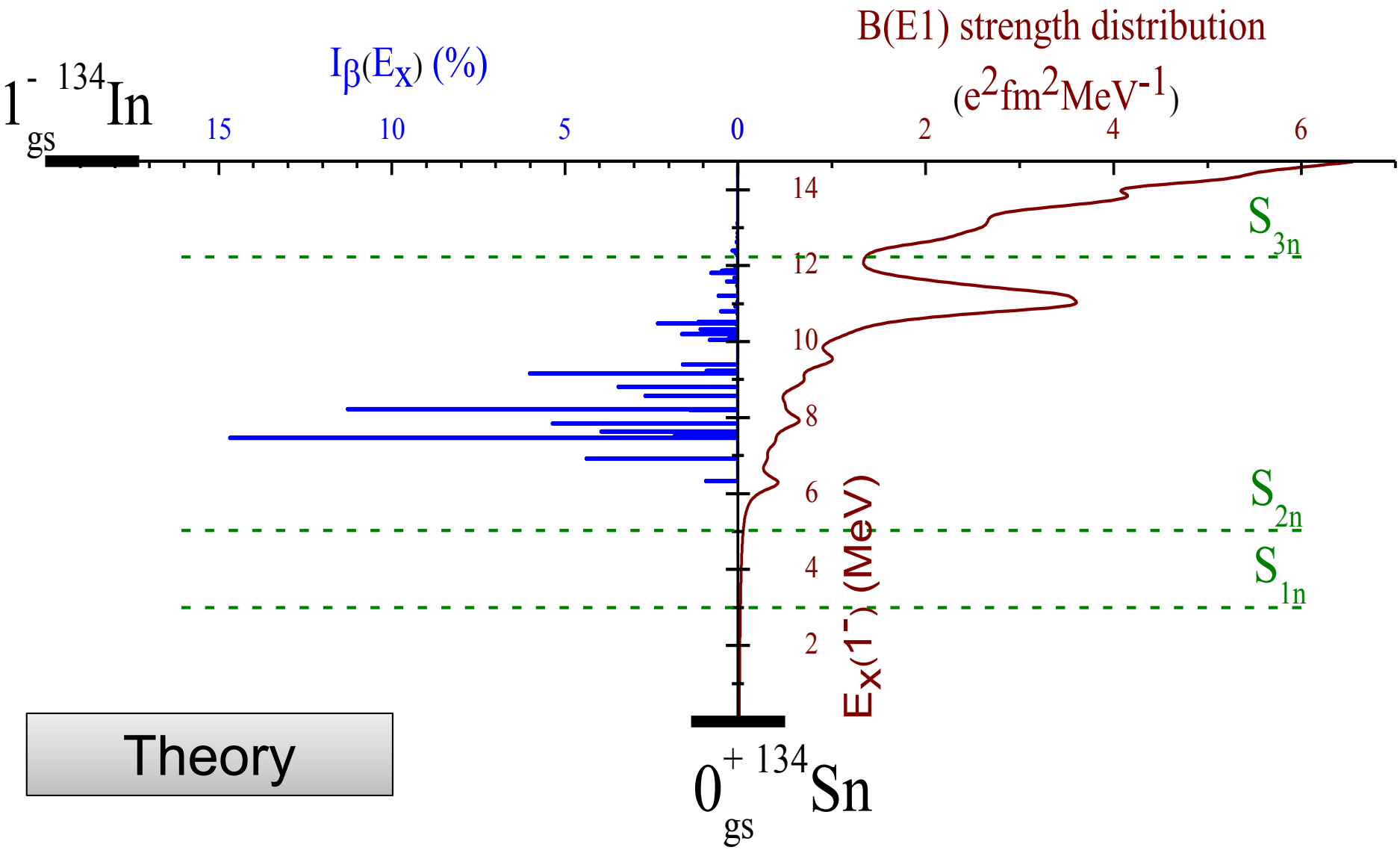
Self-consistent (global) models of beta-decay strength function



I. N. Borzov *Physics of Atomic Nuclei, 2016, Vol. 79, No. 6, pp. 910–923.*

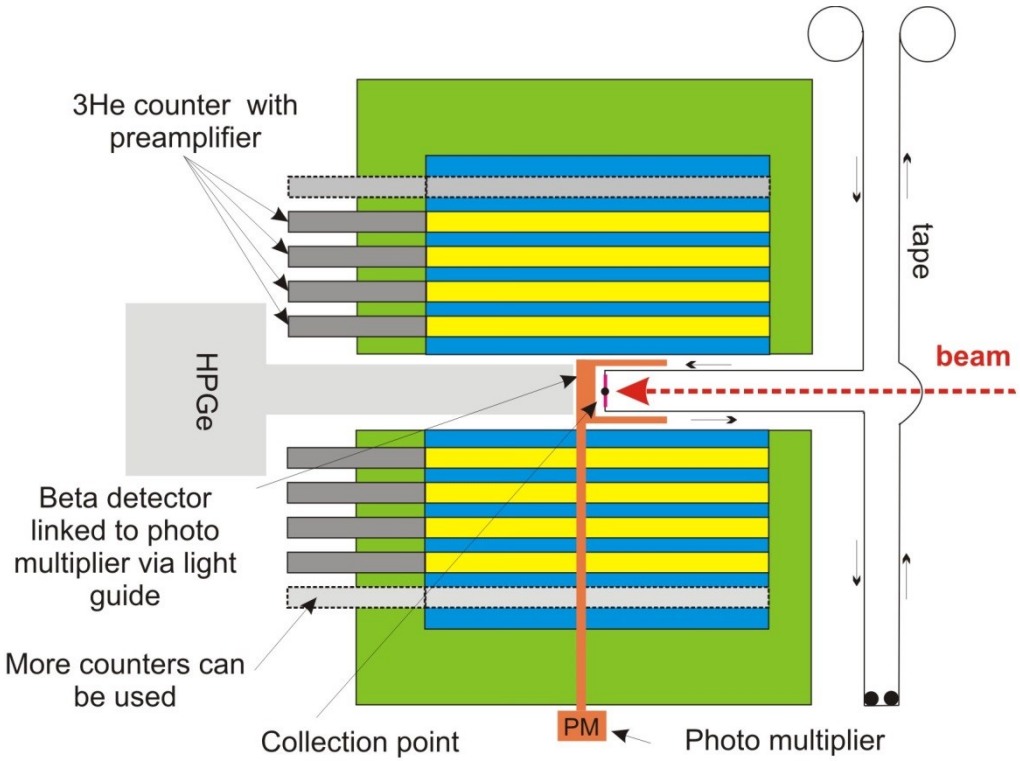
E1 strength distribution of ^{134}Sn and β2n-decay of ^{134}In

T43



Theory

Neutron long counter TETRA for beta-decays studies at ISOL facility ALTO (JINR-IN2P3 collaboration)



TETRA allows Investigation of nuclei with large neutron excess:

- Absolute measurements of P1n, P2n values
- Gamma-spectroscopy

ε(1n) ~50% (measured with ^{252}Cf)
ε(gamma)~1% (1MeV)
ε(beta) ~ 50%

D. Testov et. al, JINST *JINST* **10** P09011
D. Testov et. al. Nucl.Inst.&Meth.Sec.A 815 96 (2016)

Courtesy of D. Testov

Towards the random matrix limit of the Gaussian orthogonal ensembles (GOE)

The model is extended by enlarging the configurational space. A natural question arises: what the complexity of the configurational space should be enough in order to understand the half-life and the β-delayed neutron emission at extreme N/Z ratio? This restriction can be justified by the rough estimate from the random matrix theory. We generalize the approach based on the ideas of the random matrix distribution of the coupling between one-phonon and two-phonon states generated in the QRPA. This approach has been invented to describe a gross structure of the spreading widths of monopole, dipole, and quadrupole giant resonances.

A.P. S., S. Aberg, N.N. Arsenyev, R.G. Nazmitdinov, Phys. Rev. C 95, 061305(R) (2017)
A.P. S., S. Aberg, N.N. Arsenyev, R.G. Nazmitdinov, Phys. Rev. C 97, 059802 (2018)
A.P. S., S. Aberg, N.N. Arsenyev, R.G. Nazmitdinov, Phys. Rev. C 98, 044319 (2018)

Editors' Suggestion

Discovery of ^{60}Ca and Implications For the Stability of ^{70}Ca

O. B. Tarasov,[1,2,3] D. S. Ahn,[2] D. Bazin,[1] N. Fukuda,[2] A. Gade,[1,4] M. Hausmann,[5] N. Inabe,[2] S. Ishikawa,[6] N. Iwasa,[6] K. Kawata,[7] T. Komatsubara,[2] T. Kubo,[5] K. Kusaka,[2] D. J. Morrissey,[1,8] M. Ohtake,[2] H. Otsu,[2] M. Portillo,[5] T. Sakakibara,[6] H. Sakurai,[2] H. Sato,[2] B. M. Sherrill,[1,4] Y. Shimizu,[2] A. Stolz,[1] T. Sumikama,[2] H. Suzuki,[2] H. Takeda,[2] M. Thoennessen,[1,4] H. Ueno,[2] Y. Yanagisawa,[2] and K. Yoshida[2]

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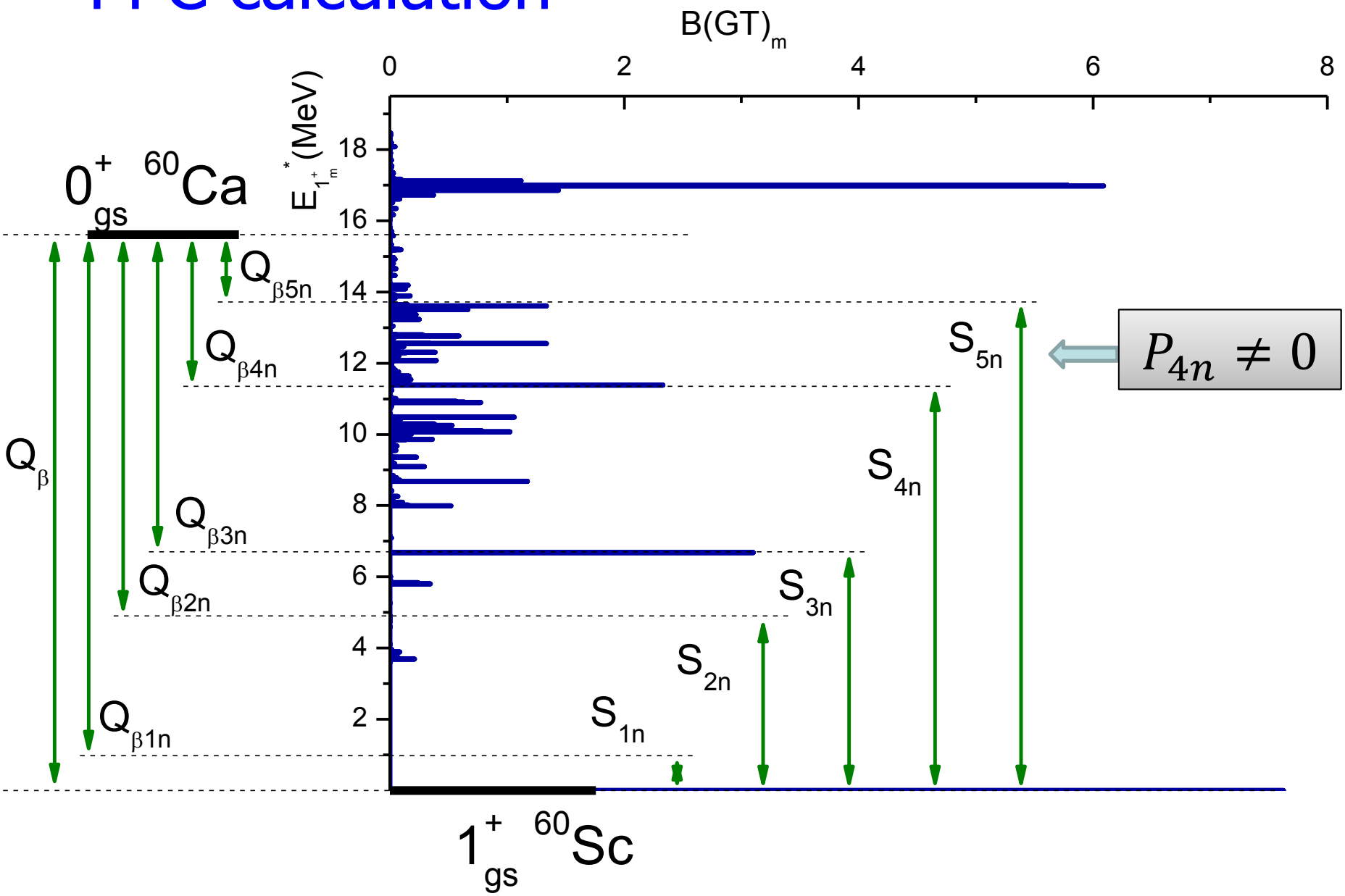
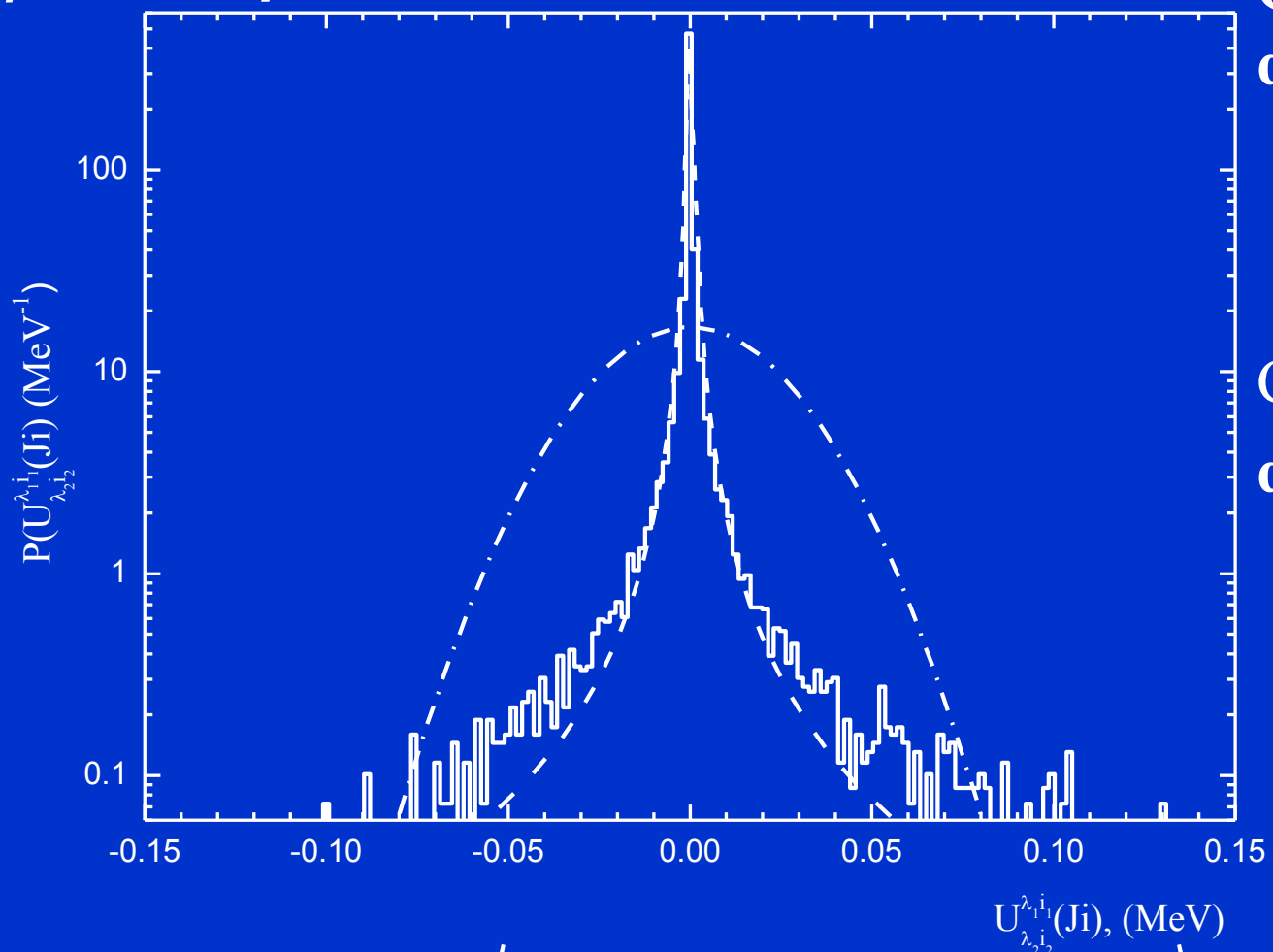
The discovery of the important neutron-rich nucleus $^{60}_{20}\mathrm{Ca}_{40}$ and seven others near the limits of nuclear stability is reported from the fragmentation of a 345 MeV/u ^{70}Zn projectile beam on ^{9}Be targets at the radioactive ion-beam factory of the RIKEN Nishina Center. The produced fragments were analyzed and unambiguously identified using the BigRIPS two-stage in-flight separator. The eight new neutron-rich nuclei discovered, ^{47}P, ^{49}S, ^{52}Cl, ^{54}Ar, ^{57}K, ^{59,60}Ca, and ^{62}Sc, are the most neutron-rich isotopes of the respective elements. In addition, one event consistent with ^{59}K was registered. The results are compared with the drip lines predicted by a variety of mass models and it is found that the models in best agreement with the observed limits of existence in the explored region tend to predict the even-mass Ca isotopes to be bound out to at least ^{70}Ca.

PPC calculation



Towards the random matrix limit of the GOE

β decay of ^{60}Ca



Gaussian distribution

$$\frac{1}{\sigma\sqrt{2\pi}}\exp\left(\frac{-x^2}{2\sigma^2}\right)$$

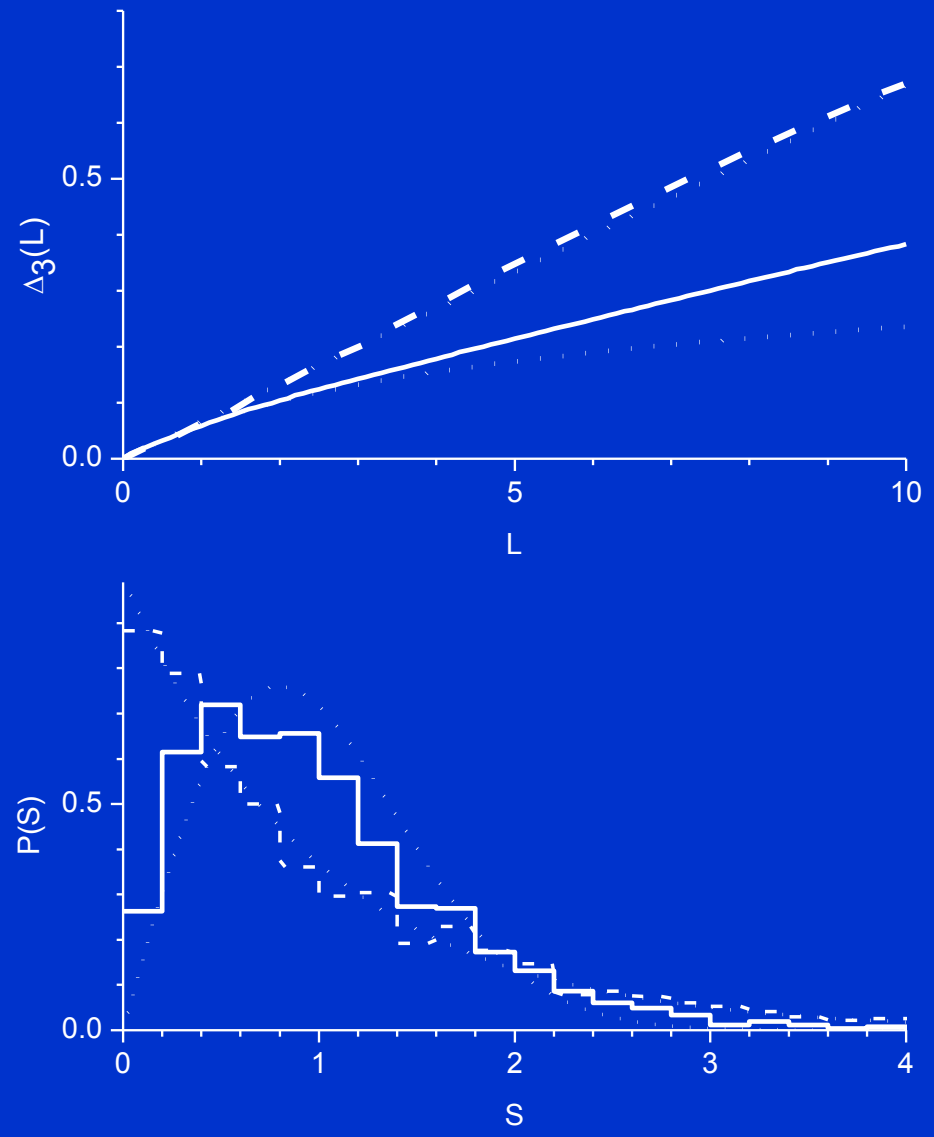
Cauchy **distribution**

$$\frac{1}{\pi b\left[1+\left(\frac{x}{b}\right)^2\right]}$$

$$U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji) = \left\langle 0 \middle| Q_{Ji} H \left[Q^+_{\lambda_1 i_1} \bar{Q}^+_{\lambda_2 i_2}\right]_J \middle| 0 \right\rangle$$

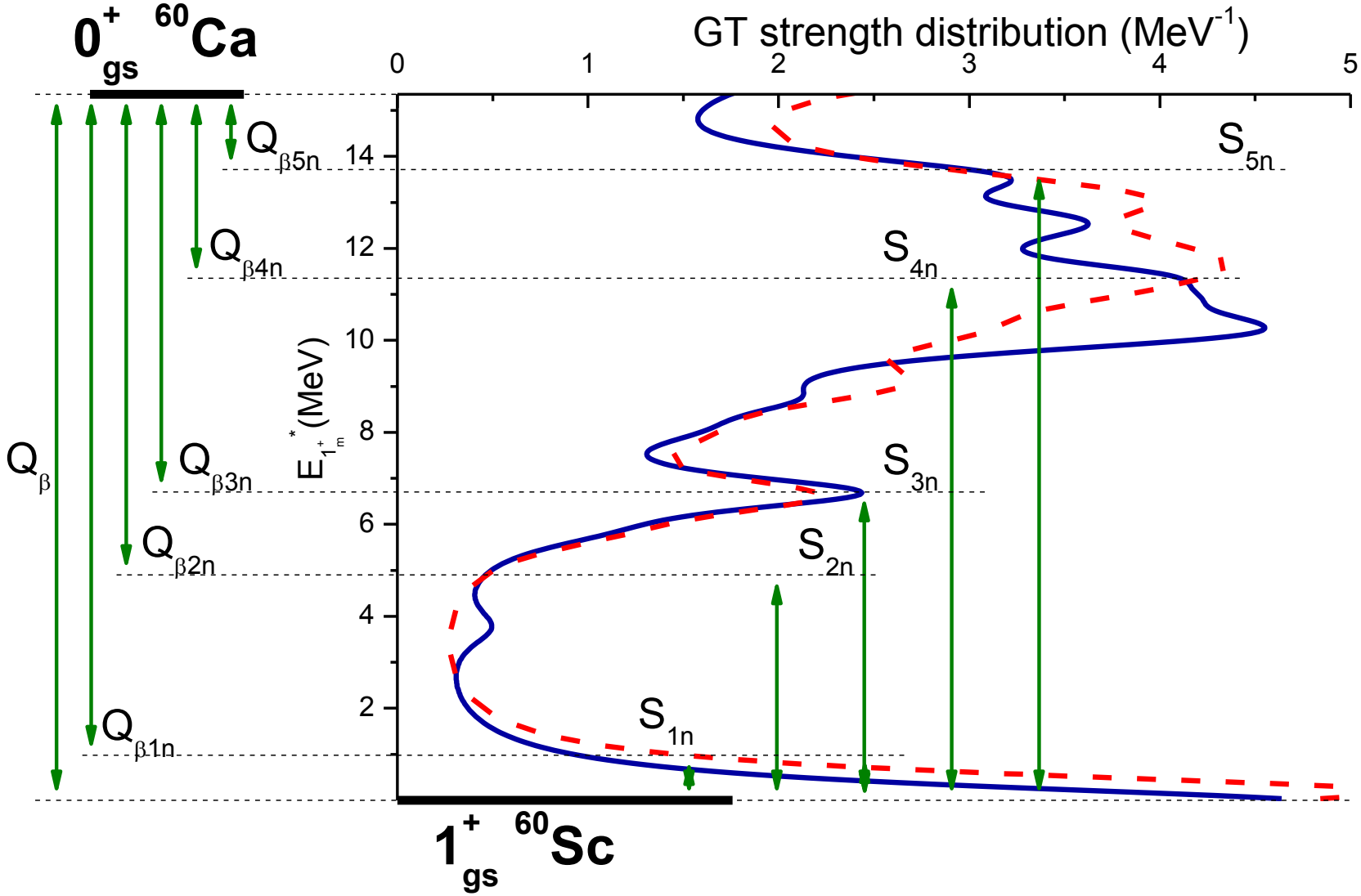
A.P. S., N.N. Arsenyev, I.N. Borzov, R.G. Nazmitdinov, S. Aberg, arXiv:1810.04932

Towards the random matrix limit of the GOE



We use two typical measures to analyze the unfolded spectrum: the nearest-neighbor spacing distribution and the spectral rigidity of Dyson and Metha.

A.P. S., N.N. Arsenyev, I.N. Borzov, R.G. Nazmitdinov, S. Aberg, arXiv:1810.04932



$T_{1/2}$ ms			$P_{n\ tot}$		
QRPA	PPC (microscopic)	PPC (random)	QRPA	PPC (microscopic)	PPC (random)
0.30	0.28	0.29	4.8%	6.1%	5.3%

Summary

- Starting from the Skyrme mean-field calculations, we have studied the effects of the phonon-phonon coupling on the β-delayed multi-neutron emission. Our results represent the first successful comparison between experimental probabilities of the neutron emission and those calculated with the Skyrme interaction.
- We predict a non-zero probability of the neutron emission in the case of ^{126}Cd . Among our initial motivation was the search for multi-neutron emission in the case ^{132}Cd in comparison to the N=82 isotone ^{130}Cd. The inclusion of the phonon-phonon coupling results in the 55% increase of the ratio P_{2n}/P_{1n} .
- The model is improved to β-decay of odd-odd nuclei. As an illustration of the method we have studied the β-decay rates of ^{134}In. The inclusion of the two-phonon configurations plays the key role in our calculation. The probabilities of β-delayed neutron emission help to identify E1 strength distribution of ^{134}Sn and it can be extracted using 3He long counter TETRA for β-decays studies at ISOL facility ALTO (JINR-IN2P3 collaboration).
- The model is extended by enlarging the variational space. The restriction of the two-phonon configurations can be justified by the rough estimate from the random matrix theory, which demonstrate the expected unimportance of other two-phonon composition on the half-life and the neutron-emission probability. To illustrate this evaluation the statistical properties of the 1^+ spectrum populated in the β-decay of ^{60}Ca are discussed.

Thanks for collaboration:

N.N. Arsenyev – BLTP, JINR

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E.O. Sushenok– BLTP, JINR

V.V. Voronov– BLTP, JINR

Y.E. Penionzhkevich – FLNR, JINR

D. Testov– FLNR, JINR

F. Ibrahim – IPN, Orsay

Nguyen Van Giai – IPN, Orsay

D. Verney – IPN, Orsay

S. Aberg– Lund University