Relativistic mean-field models of neutron-star matter and nuclear liquid-gas phase transition

based on arXiv:1902.09016

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Introduction

- The equation of state (EoS) of strongly interacting hadronic matter in various regimes of density n, temperature T and isospin asymmetry $\beta = \frac{n_n - n_p}{n}$ is necessary for description of:
  - Neutron stars (NSs) : $T = 0$, $n \gg n_0$, asymmetric $\beta \sim 1$
  - Heavy-ion collisions (HICs): $T \sim m_\pi$, $n \gg n_0$, nearly symmetric $\beta \sim 0$
  - Supernova explosions and compact star mergers: $T \sim (20 - 100)$ MeV, $n \gg n_0$, asymmetric $0 < \beta \lesssim 1$
- Constraints from the NS observations can be used to select a model parametrization to be used for generalization to finite temperatures for being used in HIC/supernovae simulations
  This requires a unified hadronic EoS with many degrees of freedom included
- Any EoS is characterized by a maximum NS mass it can support from a gravitational collapse
  A viable EoS model should pass the observed maximum NS mass constraint $M > 2.01 \pm 0.04\, M_\odot$ and many other $T = 0$ constraints.

Hyperon/Δ puzzle

For realistic hyperon interaction with an increase of the density already at $n \gtrsim 2 \div 3\, n_0$ the conversion nucleons convert to more massive baryon species:

- Hyperons [N.K. Glendenning ApJ 293 (1985)], recent review [I. Vidana arXiv:1803.00504]
- Δ-isobars [A. Drago et al. Phys.Rev. C90 (2014), B.-J. Cai et al. Phys.Rev. C92 (2015)]

In standard realistic models the maximum NS mass decreases below the observed values.

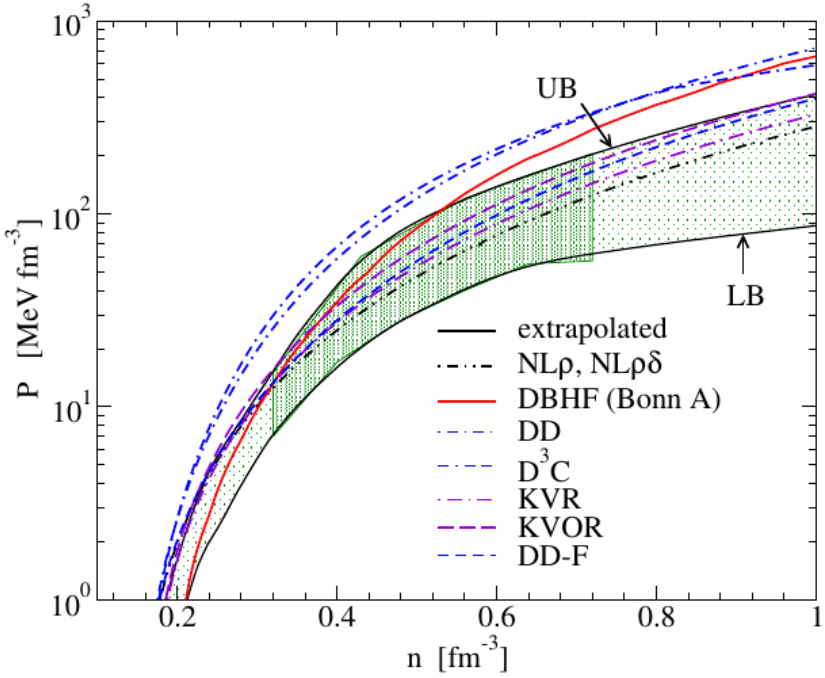
Problem can be resolved in relativistic mean-field (RMF) models by taking into account hadron mass and couplings in-medium modifications + inclusion of ϕ-meson

- Hyperons: [K. A. Maslov, E. E. Kolomeitsev and D. N. Voskresensky, Phys. Lett. B **748**, 369 (2015)]
- Δ-puzzle: [Kolomeitsev, KAM and Voskresensky, NPA 961 (2017)]

High-density EoS: contradicting constraints

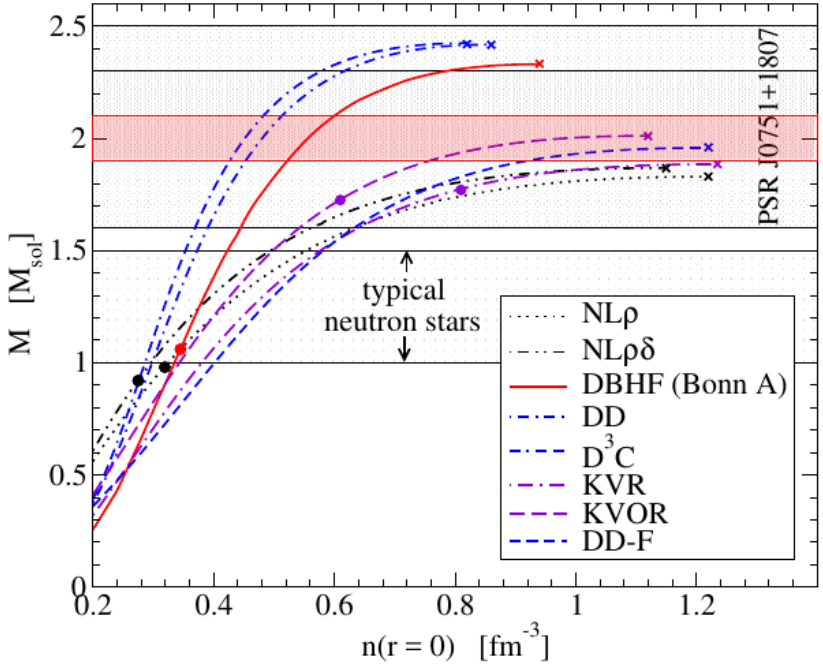
Constraint for the pressure, obtained from analyses of transverse and elliptic flows in heavy-ion collisions
Passed by rather soft EoSs

[P. Danielewicz, R. Lacey, W.G. Lynch, Science 298 (2002)]



The maximum NS mass constraint favors stiff EoS

NS cooling data ⇒ direct URCA (DU) is not operative for most stars ⇒ constraint for the proton fraction



figures from [T. Klahn et al. PRC74 (2006)]

Low-density EoS: liquid-gas phase transition

The 1st order PT from the nuclear liquid to the gas of nucleons – at low temperatures and densities below the nuclear saturation density.
In the isospin-symmetric matter the equilibrium conditions read:

$$P^I = P^{II}, \quad \mu_B^I = \mu_B^{II}$$

Not hard to describe within RMF models:

- Low densities $n \leq n_0$ – no baryons except nucleons
- Low temperatures $T \lesssim 20$ MeV – can neglect thermal excitations of mesons

Important for describing low-energy ion collisions and supernovae
Infinite to finite – effects of surface tension and Coulomb interaction?

EoS frameworks

Microscopic

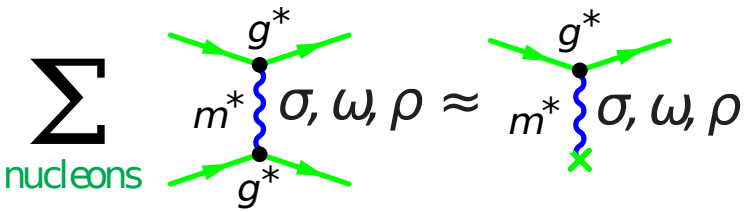
- Based on baryon-baryon potential + a many-body method
- Robust at low densities, large uncertainties at large densities
- Non-relativistic – acausal at large densities

Phenomenological

- Relatively simple models with parameters fitted to describe the experimental data / robust theoretical results
- Causal for all densities - important for NSs and HICs

Relativistic mean-field models

Meson-exchange picture of the interaction with classical meson fields



Additional flexibility needed to describe all the data

- Density-dependent couplings
- Various meson fields and self-interactions
- Field-dependent couplings and meson masses

RMF model with scaled hadron masses and couplings

E. E. Kolomeitsev and D. N. Voskresensky NPA 759 (2005) 373

- Walecka-type model with in-medium change of masses and coupling constants of all hadrons in terms of the scalar field σ:

$$m_i^* = m_i \Phi_i(\sigma),\ g_{mB}^* = g_{mB}\chi_m(\sigma),$$
$$m = \{\text{mesons}\},\ B = \{\text{baryons}\},\ i = B \cup m$$

- Common decrease of hadron masses [Brown, Rho PRL 66 (1991), Phys. Rept. 363 (2002)]:

$$\frac{m_N^*}{m_N} \simeq \frac{m_\sigma^*}{m_\sigma} \simeq \frac{m_\omega^*}{m_\omega} \simeq \frac{m_\rho^*}{m_\rho}$$

- In the infinite matter only $\eta_m(\sigma) = \frac{\Phi_m^2(\sigma)}{\chi_m^2(\sigma)}$ enter the EoS - we define them phenomenologically to pass the constraints

Below we use the dimensionless scalar field $f(n) \equiv \dfrac{g_{\sigma N}\chi_\sigma(\sigma)\sigma}{m_N}$

Generalized relativistic mean-field model

E.E.K., D.N.V. NPA 759 (2005)

E. E. K., K. A. M. and D. N. V., NPA 961 (2017)

$$\mathcal{L} = \mathcal{L}_{\rm bar} + \mathcal{L}_{\rm mes} + \mathcal{L}_l,$$

$$\mathcal{L}_{\rm bar} = \sum_{i=b\cup r} (\bar{\Psi}_i \left(i D_\mu^{(i)} \gamma^\mu - m_i \Phi_i(\sigma) \right) \Psi_i,$$

$$D_\mu^{(i)} = \partial_\mu + i g_{\omega i} \chi_{\omega i}(\sigma) \omega_\mu + i g_{\rho i} \chi_{\rho i}(\sigma) \vec{t} \vec{\rho}_\mu + i g_{\phi i} \chi_{\phi i}(\sigma) \phi_\mu,$$

$$\{b\} = (N, \Lambda, \Sigma^{\pm,0}, \Xi^{-,0}, \Delta^-, \Delta^0, \Delta^+, \Delta^{++})$$

$$\mathcal{L}_{\rm mes} = \frac{\partial_\mu \sigma \partial^\mu \sigma}{2} - \frac{m_\sigma^2 \Phi_\sigma^2(\sigma) \sigma^2}{2} - U(\sigma) +$$

$$+ \frac{m_\omega^2 \Phi_\omega^2(\sigma) \omega_\mu \omega^\mu}{2} - \frac{\omega_{\mu\nu} \omega^{\mu\nu}}{4} + \frac{m_\rho^2 \Phi_\rho^2(\sigma) \vec{\rho}_\mu \vec{\rho}^\mu}{2} - \frac{\rho_{\mu\nu} \rho^{\mu\nu}}{4} +$$

$$+ \frac{m_\phi^2 \Phi_\phi^2(\sigma) \phi_\mu \phi^\mu}{2} - \frac{\phi_{\mu\nu} \phi^{\mu\nu}}{4},$$

$$\omega_{\mu\nu} = \partial_\nu \omega_\mu - \partial_\mu \omega_\nu, \quad \vec{\rho}_{\mu\nu} = \partial_\nu \vec{\rho}_\mu - \partial_\mu \vec{\rho}_\nu + g_\rho \chi'_\rho [\vec{\rho}_\mu \times \vec{\rho}_\nu],$$

$$\phi_{\mu\nu} = \partial_\nu \phi_\mu - \partial_\mu \phi_\nu,$$

$$\mathcal{L}_l = \sum_l \bar{\psi}_l (i \partial_\mu \gamma^\mu - m_l) \psi_l, \quad \{l\} = (e, \mu).$$

Finite T: Pressure

$$P[\mu_B, \mu_Q, f, T] = T \sum_b (2S_b + 1) \int_0^\infty \frac{dp\, p^2}{2\pi^2} \ln[1 + e^{-\beta(\epsilon_b^*(p) - \mu_b^*)}] - \frac{m_N^4 f^2}{2C_\sigma^2} \eta_\sigma(f)$$

$$+ \frac{C_\omega^2}{2m_N^2 \eta_\omega(f)} n_V^2 + \frac{C_\rho^2}{2m_N^2 \eta_\rho(f)} n_I^2, \quad \epsilon_b^*(p) = \sqrt{p^2 + m_b^{*2}}, \quad \beta = 1/T$$

$$\mu_b^* = \mu_B - Q_b \mu_Q - x_{\omega b} \frac{C_\omega^2 n_V}{m_N^2 \eta_\omega(f)} - t_{3b} x_{\rho b} \frac{C_\rho^2 n_I}{m_N^2 \eta_\rho(f)}$$

$$n_V = \sum_b x_{\omega b} n_b, \quad n_I = \sum_b x_{\rho b} t_{3b} n_b, \quad n_b = (2S_b + 1) \int_0^\infty \frac{dp\, p^2}{2\pi^2} f_b(p; \mu_b^*, T),$$

$$f_b(p; \mu, T) = \frac{1}{1 + e^{\beta(\epsilon_b^*(p) - \mu)}}, \quad Q_b, S_b - \text{charge and spin of a baryon } b$$

Scaling functions

In the homogeneous medium $\eta_M = \Phi_M^2(f)/\chi_{Mb}^2(f)$,

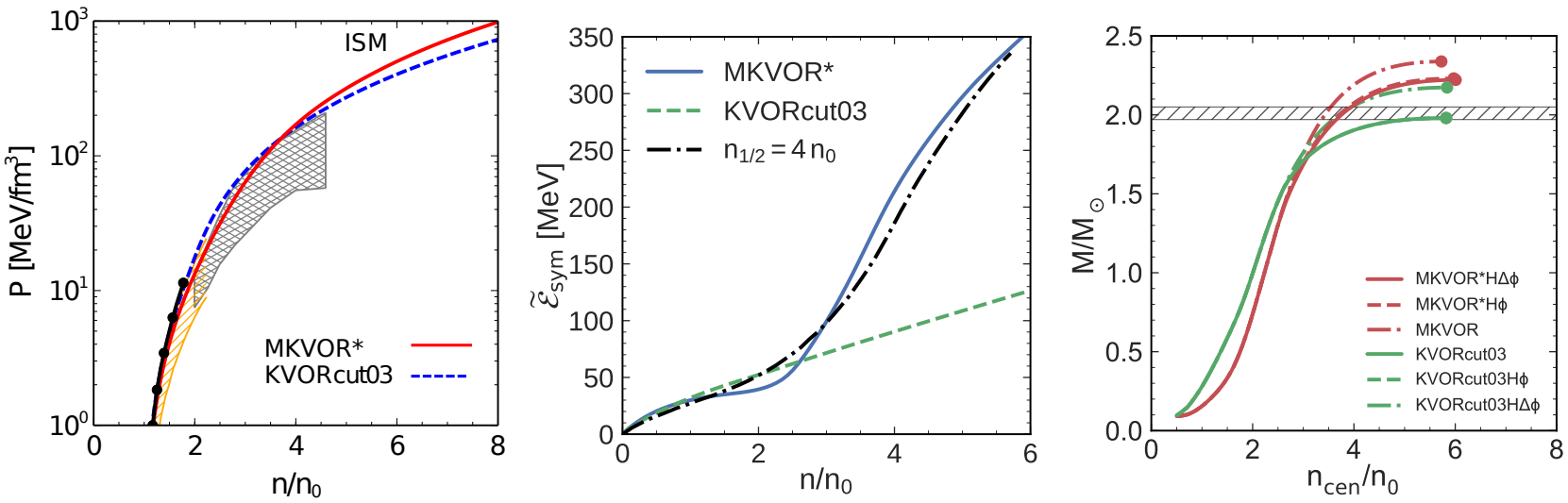
$$\Phi_N(f) = \Phi_m(f) = 1 - f, \text{universal scaling of hadron masses}$$

$$\frac{\partial P}{\partial f} = 0 - \text{e.o.m. for the scalar field}$$

Working models

Initial model: KVOR [E.E.K., D.N.V. NPA 759 (2005)] described many constraints, but only without hyperons ⇒ need for enhancement [K.A.M, E.E.K., D.N.V. PRC 92 (2015), NPA 950 (2016)].

KVORcut03	**MKVOR***
Stiff in NS matter and nuclear matter	Stiff in NS matter, soft in nuclear matter



- Modification of ω–meson (KVORcut03) and ρ–meson (MKVOR*) properties
- Pass the flow constraint and many more together with the maximum NS mass constraint with both **hyperons** and Δ-**isobars** included
- MKVOR describes the cooling data with hyperons (KVORcut03 not checked) [H. Grigorian, D. Voskresensky, KM NPA980 (2018)]
- "$n_{1/2}$" – see [Y.-L. Ma and M. Rho Phys.Rev. D99 (2019)]

Low-density: bulk nuclear matter properties

Energy per particle expansion:

$$\mathcal{E} = \mathcal{E}_0 + \frac{K}{18}\epsilon^2 - \frac{K^{'}}{162}\epsilon^3 + ... + \beta^2 \left(\mathcal{E}_{\rm sym} + \frac{L}{3}\epsilon + \frac{K_{\rm sym}}{18}\epsilon^2 ... \right),$$
$$\epsilon = (n - n_0)/n_0, \quad \beta = [(n_n - n_p)/n_0]_{n_0}$$

Coefficients are accessible experimentally and are to be used to determine $C_\sigma, C_\omega, C_\rho$ and parameters of the scaling function $\eta_\sigma(f)$.
We adopt the values consistent with available data within uncertainties

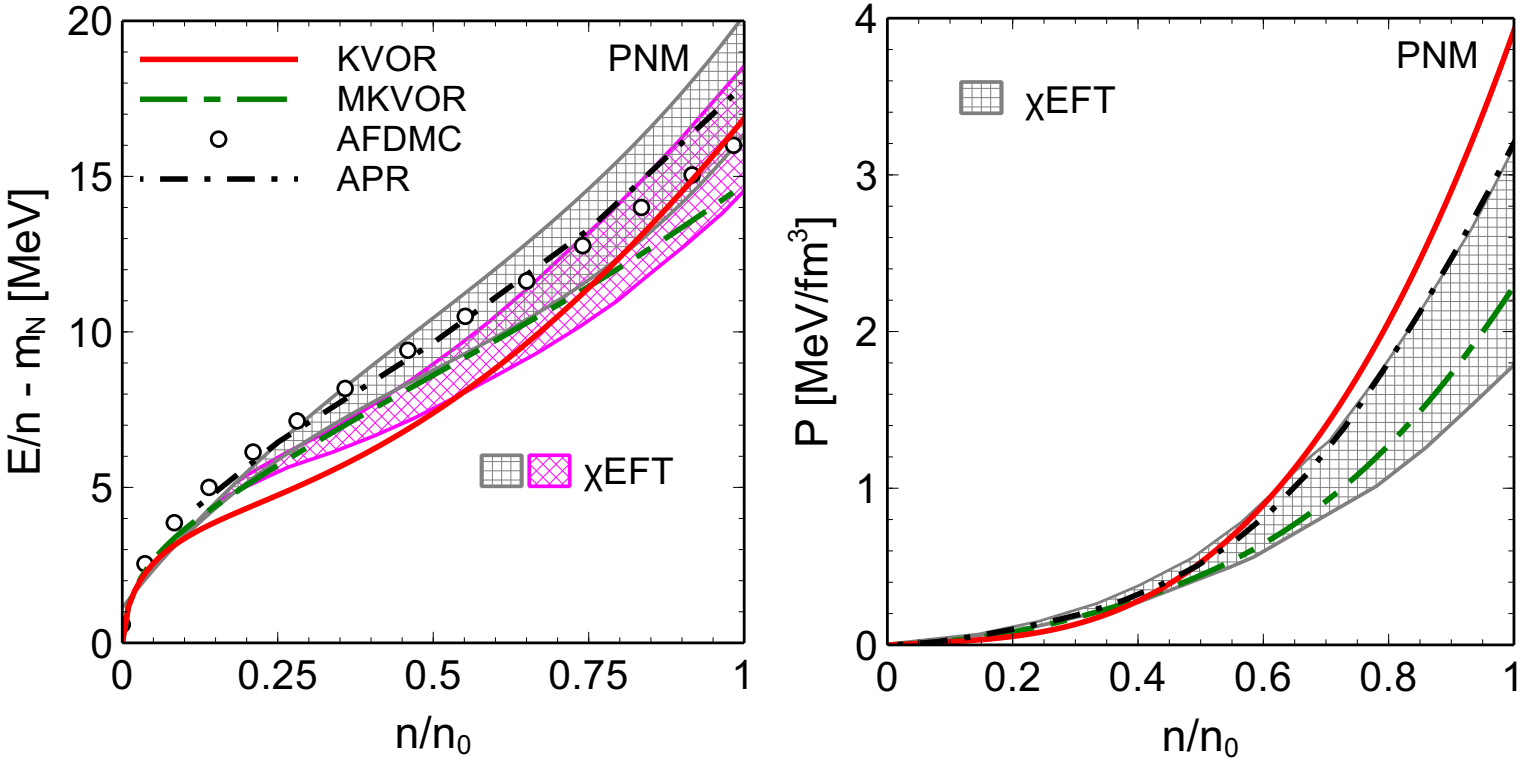
$$n_0 = 0.16 \text{ fm}^{-3}, \quad \mathcal{E}_0 = -16 \text{ MeV}, \quad K = 250 \text{ MeV},$$

KVORcut03: $\mathcal{E}_{\rm sym} = 32$ MeV, $\quad m_N^*(n_0)/m_N = 0.805$
MKVOR : $\mathcal{E}_{\rm sym} = 30$ MeV, $\quad m_N^*(n_0)/m_N = 0.73$

Low-density behavior of EoSs

Comparison with the results of:

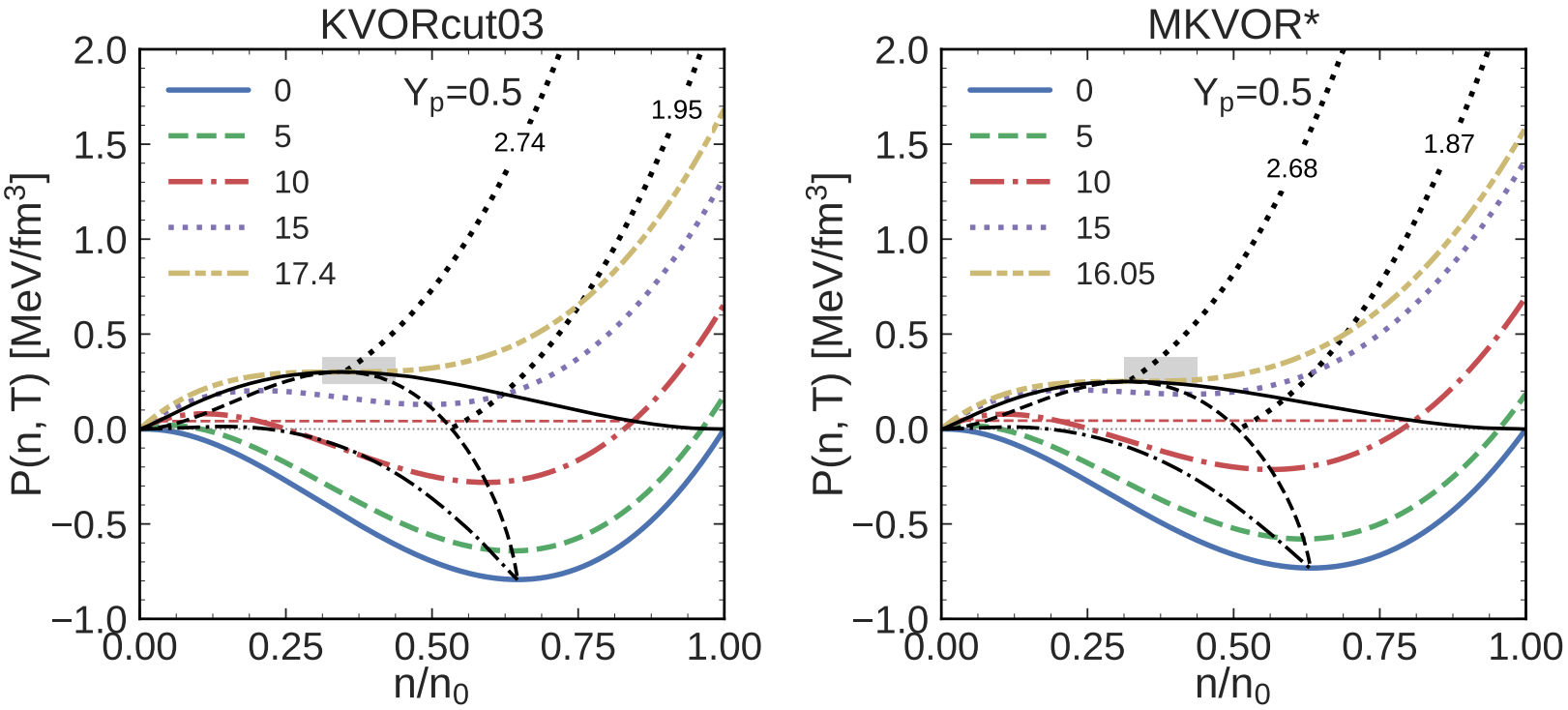
- Chiral effective field theory (χEFT), [K, Hebeler et al. EPJ A50 (2014)]
- Auxiliary field diffusion Monte-Carlo [S. Gandolfi et al. MNRAS 404 (2010)]
- APR EoS



MKVOR is consistent with χEFT at low densities despite the parameterization was chosen basing only on the high-density properties

Liquid-gas PT - results

Pressure for various temperatures T [MeV]



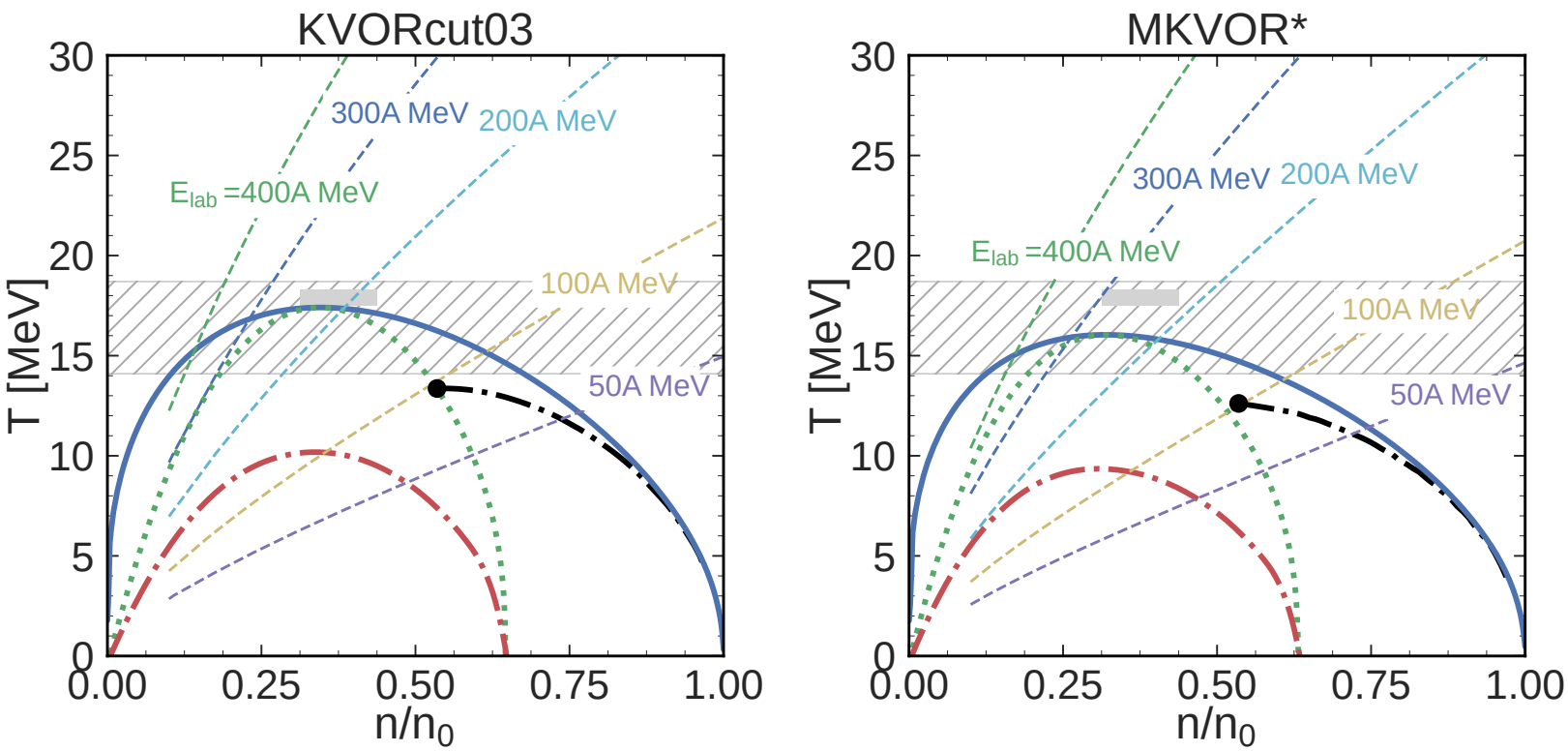
Dashed lines – isothermal spinodal region with $v_s^2 = \frac{dP}{dE} < 0 \Rightarrow$ mechanically unstable

Dash-dotted – adiabatic spinodal region

KVORcut03 well within the box, MKVOR passes marginally

$T - n$ plane

Dashed region – [A. Carbone et al. Phys.Rev. C98 (2018)]



Dashed lines – isothermal spinodal region with $v_s^2 = \dfrac{dP}{dE} < 0 \Rightarrow$ mechanically unstable

Dash-dotted – adiabatic spinodal region

$$T_c = (17.9 \pm 0.4) \text{ MeV}$$

KVORcut03: $T_c = 17.4$ MeV – almost passes,

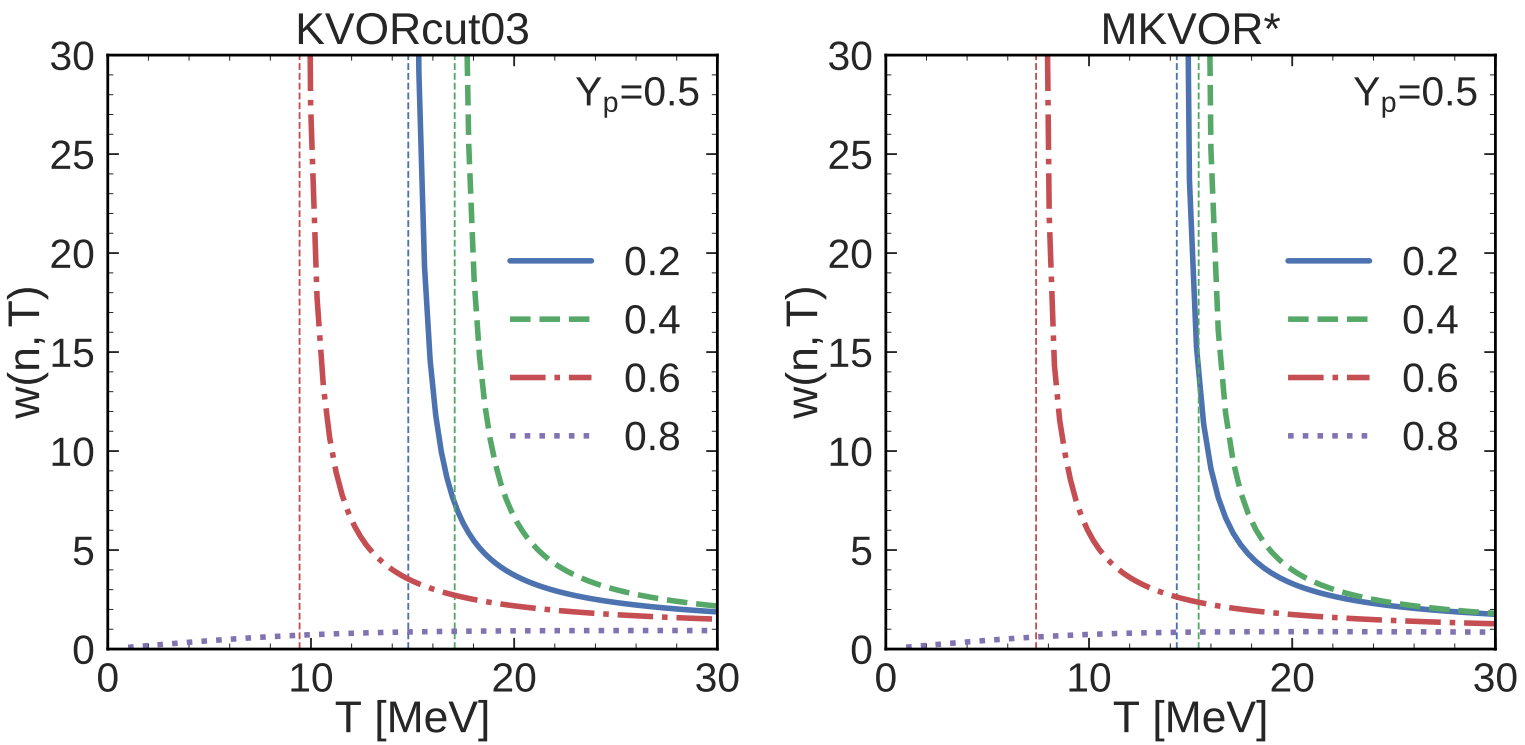
MKVOR: $T_c = 16.05$ MeV – slightly lower

See [arXiv:1902.09016] for details on how we obtain trajectories $\mathcal{E}_{\rm lab}$

Isospin symmetric case - scaled variance

Quantity characterizing the particle number fluctuations in an event-by-event analysis, $\langle \dots \rangle$ – event-by-event averaging

$$w[n,T] = \frac{\langle (N - \langle N \rangle)^2 \rangle}{\langle N \rangle} = \frac{T}{n}\left(\frac{\partial n}{\partial \mu}\right)_T$$



The variance diverges at the spinodal border

Liquid-gas at finite isospin density

Heavy nuclei are not symmetric (e.g. $Y_p = Z/A \simeq 0.4$ for Au + Au); supernova simulations require EoS of warm asymmetric matter

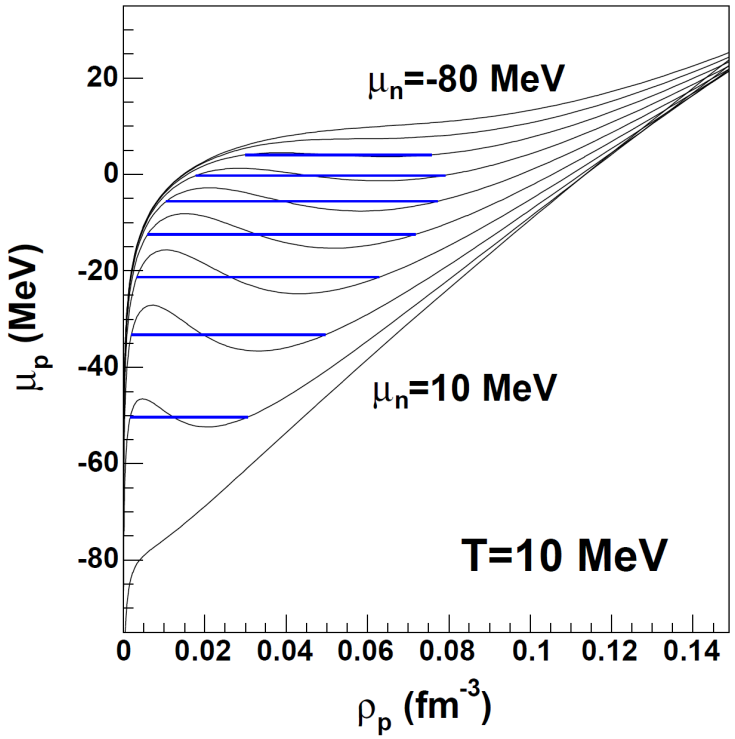
Construction of the PT:

- Continuity of two chemical potentials:

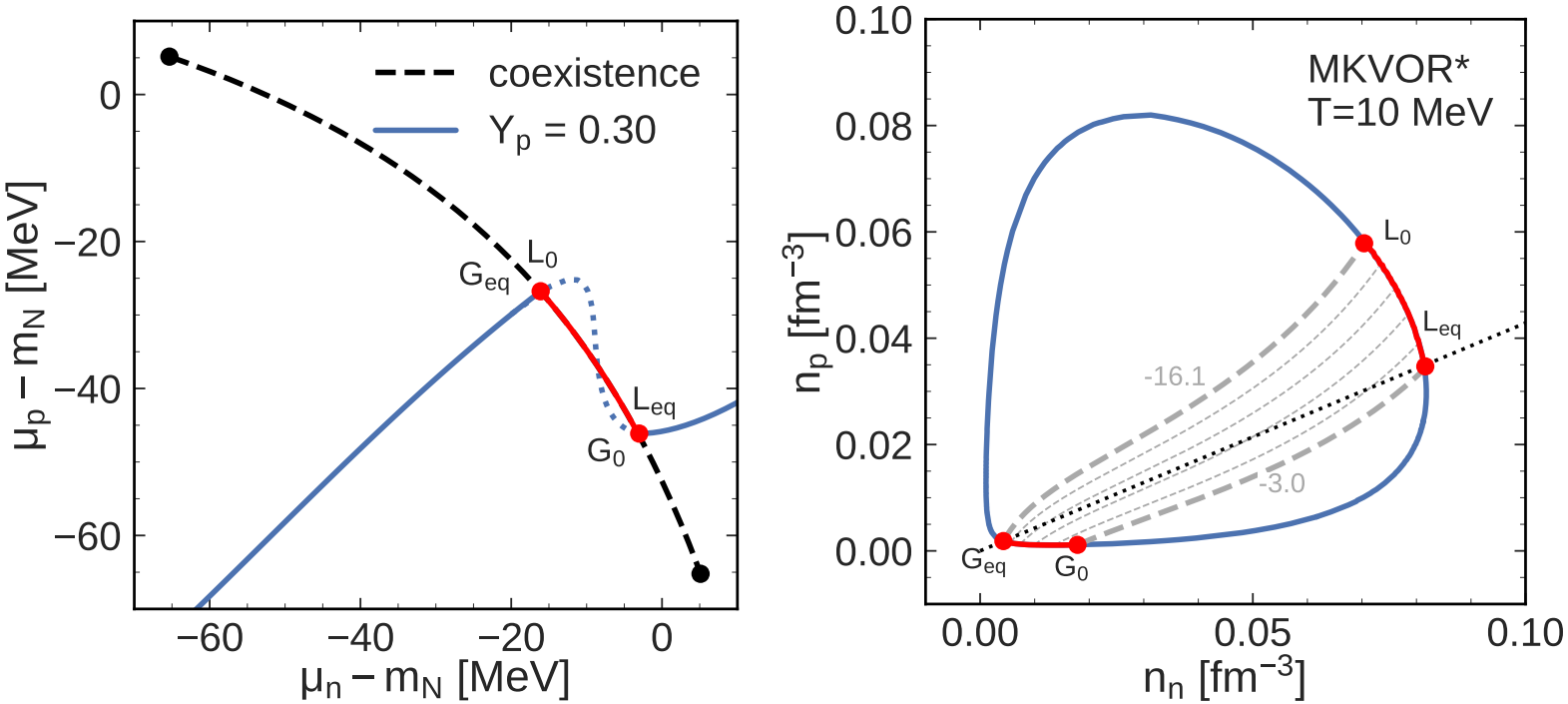
$$\mu_B^I = \mu_B^{II}, \quad \mu_Q^I = \mu_Q^{II}.$$

- Easy way to solve: use the mixed thermodynamic potential $\Omega'[n_p, \mu_B]$ and perform a Maxwell construction in terms of n_p for a given μ_B

  [Ducoin Chomaz Gulminelli NPA 771 (2006)]



Phase transition with isospin asymmetry



dotted line – $Y_p = 0.3$

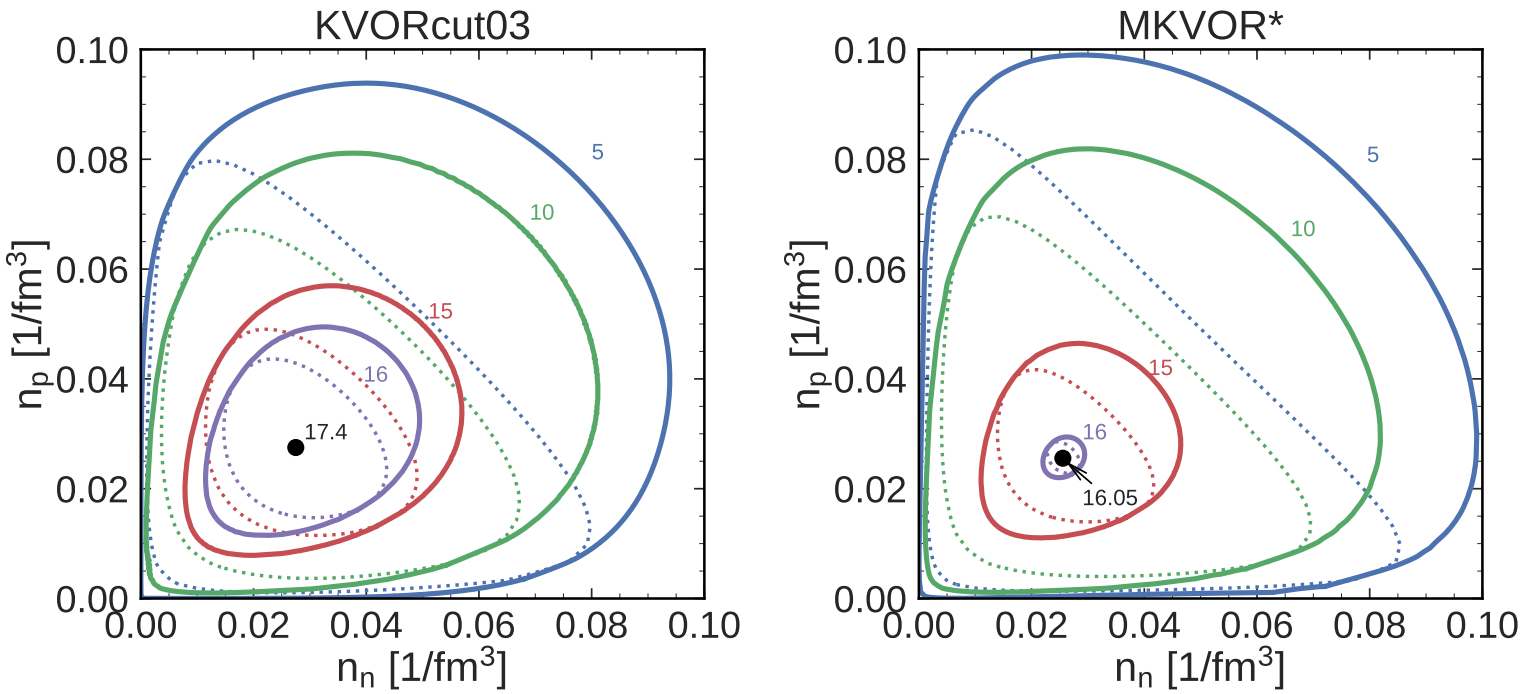
- $G_{\rm eq}$, L_0 – infinitesimal liquid droplet in the gas phase
- G_0, $L_{\rm eq}$ – infinitesimal gas bubble in the liquid phase

Liquid fraction at L_0 is closer to the symmetric matter

Critical areas in $n_n - n_p$ plane

Phase coexistence borders for various temperatures

Dotted lines – spinodal region $\det\left[\frac{\partial \mu_i}{\partial \rho_j}\right] = 0, \;\; i,j = n,p$



Shape of the coexistence borders depends on the L parameter ($L \simeq 40$ MeV for MKVOR and $\simeq 70$ MeV for KVORcut03), not contradicting to findings of [N. Alam et al. PRC 95(5) (2017)] where the variation of L was studied.

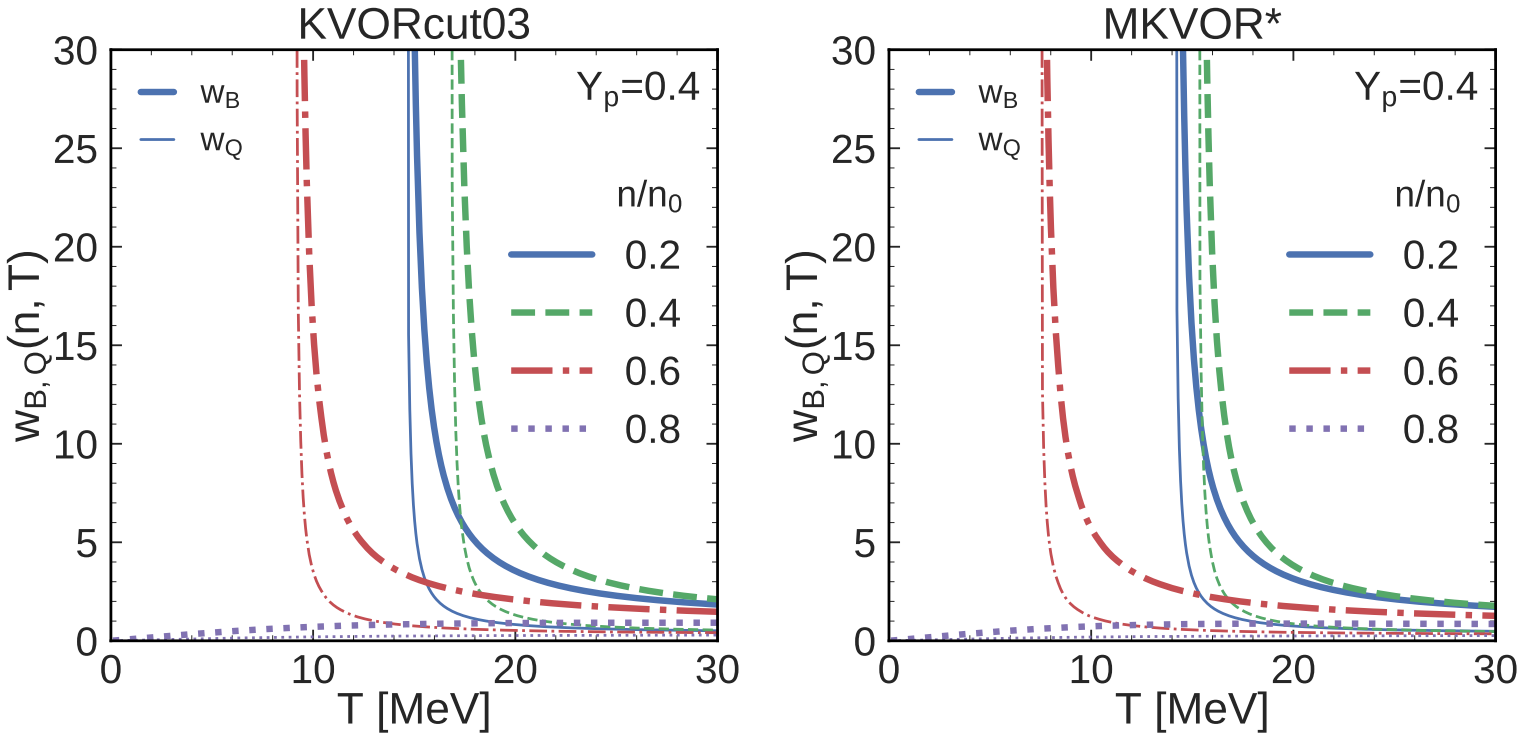
Isospin symmetric case - scaled variance

Variance matrix:

$$w_{ij} \equiv \frac{T}{n}\left[\frac{\partial \mu_i}{\partial n_j}\right]^{-1}$$

Fluctuations of the conserved charges:

Bold lines : $w_B = w_{nn} + w_{pp} + 2w_{np}$, thin lines: $w_Q = w_{pp}$

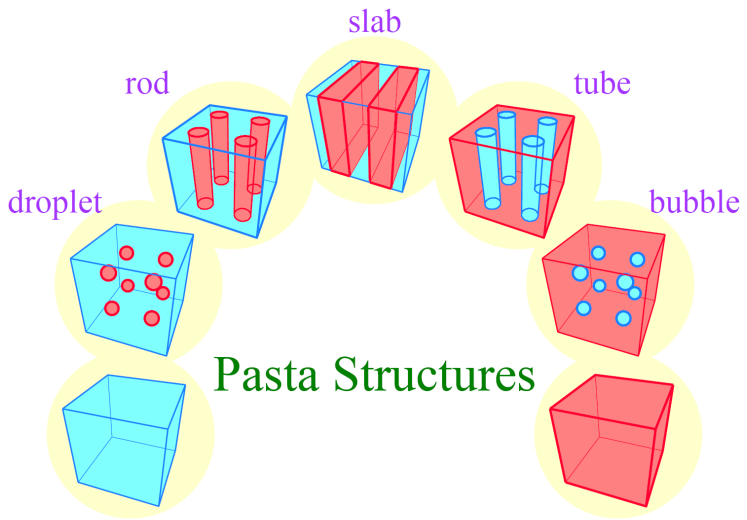


The variance diverges at the spinodal border

Pasta phases and the LG phase transition

Gibbs conditions $\Leftrightarrow$ global charge neutrality in the asymmetric matter:
non-uniform charge distribution is possible
Electric field potential: $\mu_Q \to \mu_Q - V(\vec{r})$

Finite size effects:
surface tension + Coulomb force =



Can be found in

- NS inner crust

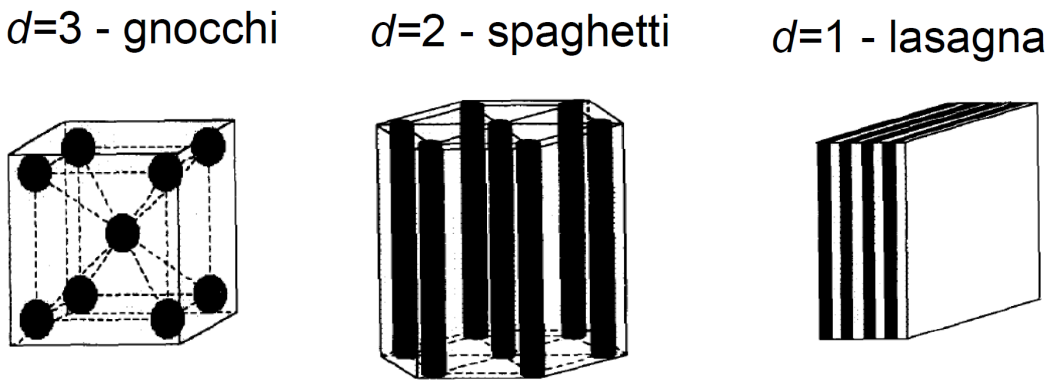
[N. Chamel, P. Haensel Living Rev.Rel. 11 (2008) 10]

- NS core: any possible 1st order PT, e.g.

quark–hadron PT [D.N. Voskresensky, M. Yasuhira, T. Tatsumi NPA723 (2003)]
K^- condensation [T. Maruyama, T. Tatsumi, D.N. Voskresensky, T. Tanigawa, T. Endo, S. Chiba (JAERI, Kyoto)PRC73 (2006)

Treatment of the electric field

Wigner-Seitz approximation with the cell radius R_W (consider d=3)



Poisson equation ($p = H, Q$)

$$\Delta V^{(p)}(r) = 4\pi e^2 n_{\rm ch}^{(p)}[\mu_B, \mu_e - V^{(p)}(r)]$$

$\Rightarrow$ nonuniform electron density distribution and charge screening

Linearized around some $V_{\rm ref}$: Debye screening lengths

$$\Delta\delta V^{(p)}(r) = 4\pi e^2 n_{\rm ch}^{(p)}[\mu_B, \mu_e - V_{\rm ref}] + (\lambda_D^{(p)})^{-2}\delta V(r),$$

$$\delta V^{(p)}(r) = V(r) - V_{\rm ref}^{(p)}, \quad (\lambda_D^{(p)})^{-2} \equiv -4\pi e^2 \left(\frac{\partial n_{\rm ch}^{(p)}}{\partial \mu_e}\right)_{\mu_B}$$

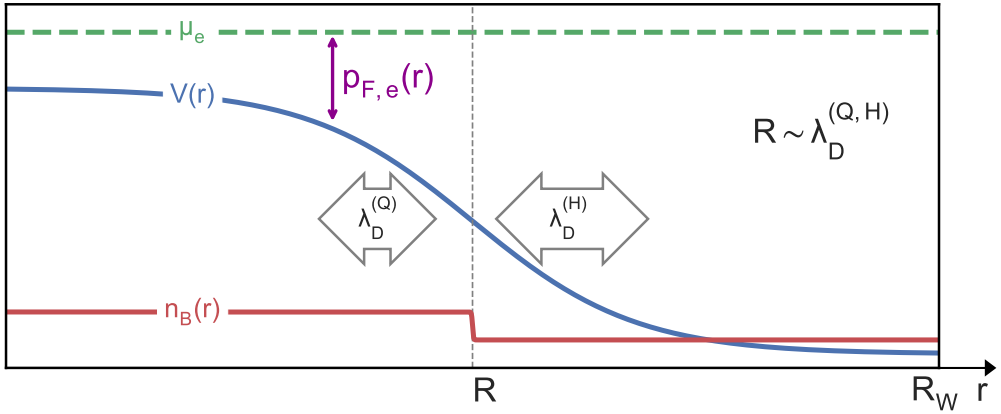
Linearized equation can be solved analytically

[D.N. Voskresensky, M. Yasuhira, T.Tatsumi PLB 541 (2002), NPA 723 (2003)]

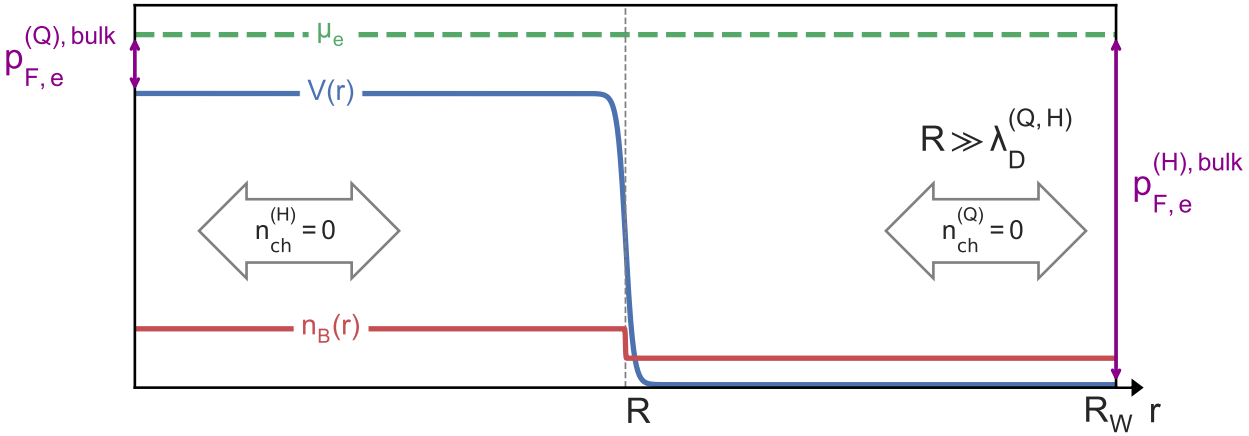
NS matter: e^- and screening effects

1. Case $R \sim \lambda_D$: smooth non-uniform electron density distribution

$\lambda_D \sim 1/e^2 \gg$ diffuseness layer thickness $l \sim 1$ fm $\Rightarrow$ neglected in $n_B(r)$ profile



2. Case of large droplets $R \gg \lambda_D^{(Q,H)}$: electric fields contributes only in the thin border layer $\Rightarrow$ contribution to the effective surface tension



Far from the border – bulk solution with $n_{\rm ch}^{(Q,H)} = 0$ – Maxwell case

There is a critical surface tension, such that for $\sigma > \sigma_c$ the solution is Maxwell

Depends on the model combination [K.M., N. Yasutake et al. arXiv:1812.11889]

Heavy-ion collisions

When the droplet appears the charge densities shift from $n_Q = Y_p n$

$$\delta n_Q^{\mathrm{I}} = n_Q^{\mathrm{I}} - n_Q, \quad \delta n_Q^{\mathrm{II}} = n_Q^{\mathrm{II}} - n_Q,$$
$$V^{\mathrm{I,II}} = V_0 + \delta V^{\mathrm{I,II}}$$

In an idealistic case $R \ll \lambda_D^{\mathrm{I,II}}$ screening plays no role

$$\Delta V_0 = 4\pi e^2 n_Q\,, \quad \Delta\delta V^{\mathrm{I,II}} = 4\pi e^2 \delta n_Q^{\mathrm{I,II}}\,, \tag{1}$$

The finite-size contribution to the free energy per cell is
Heiselberg, Pethick, Staubo Phys. Rev. Lett. 70 (1993)

$$\delta F_{\mathrm{finite}} = 2\pi e^2 (\delta n_Q^{\mathrm{I}} - \delta n_Q^{\mathrm{II}})^2 R^2 \Phi_d(\chi) + \frac{\chi \sigma d}{R},$$

Φ_d is a dimensionless function for a given geometry d and
$\chi = V_{\mathrm{I}}/V_{\mathrm{II}} = (R/R_{WS})^d$
The optimal droplet radius is

$$R_m^d = \Big[\frac{\sigma d (1-\chi)^2}{4\pi e^2 (\delta n_Q^{\mathrm{I}})^2 \Phi_d(\chi)}\Big]^{1/3} \simeq (3-5)\,\text{fm for}\,\chi \to 0.$$

The optimal geometry is determined by minimization of δF_{finite} over d
For the liquid immersed in the vapor:

$$\begin{array}{ll} \text{droplets for} & 0 < \chi < 0.22, \\ \text{rods for} & 0.22 < \chi < 0.35, \\ \text{slabs for} & 0.35 < \chi < 0.5. \end{array}$$

... and for vice versa $\chi \to (1 - \chi)$.

Summary

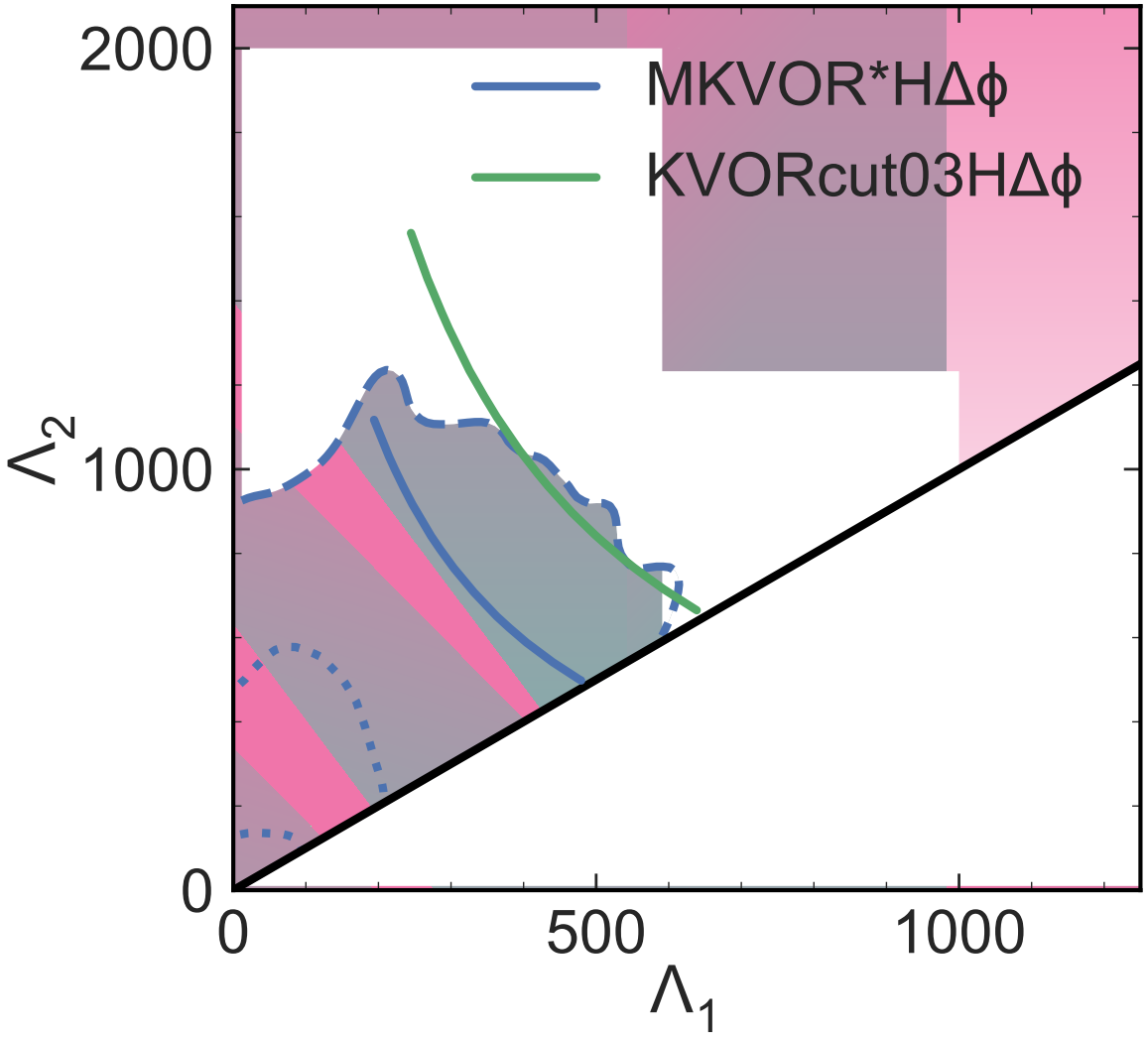
- RMF models with scaled hadron masses and couplings constructed to describe neutron stars give reasonable properties of the nuclear liquid-gas phase transition
- Critical temperature T_c is lower in the MKVOR model due to the lower effective nucleons mass – tension with large $M_{\max}$?
- Weak model dependence, no anomalies in the MKVOR model

Possible formation of pasta-like droplets in HICs

After taking into account the surface tension and Coulomb force in low-energy HICs:

- Structures of various shapes can form without electric neutrality
- Estimate for the size is compatible with a few WS cells within an expanded fireball
- Do they have time to form?

Backup


MKVOR*HΔϕ
KVORcut03HΔϕ
2000
1000
0
0
500
1000
Λ_2
Λ_1