

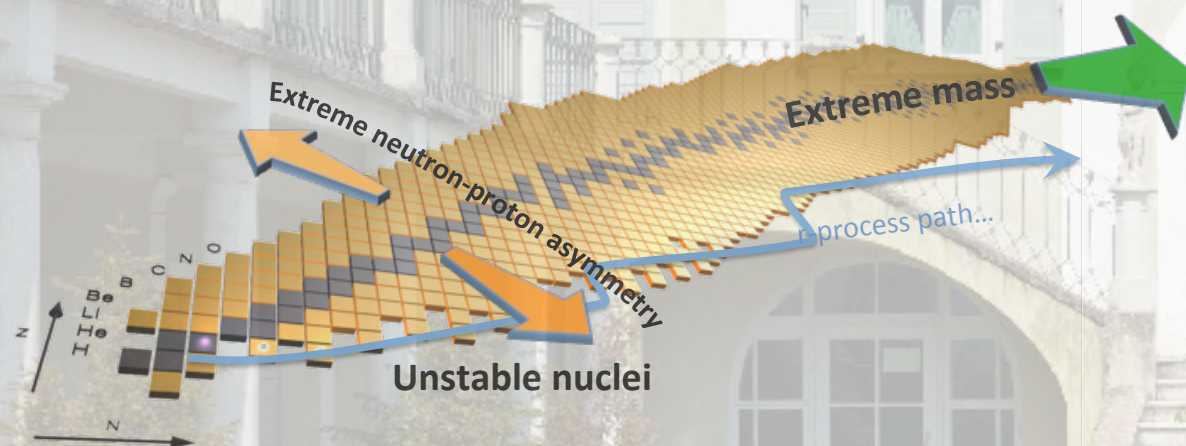


# Advances in predicting nuclear matter at finite temperature

Arianna Carbone (ECT\*-Trento, Italy)

International Workshop "Infinite and finite nuclear matter" (INFINUM)

JINR, Dubna - 20 March 2019





# Ab initio low-energy nuclear theory

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**Solve the nuclear many-body  
problem from first principles**

Employing reliable methods with predictive power

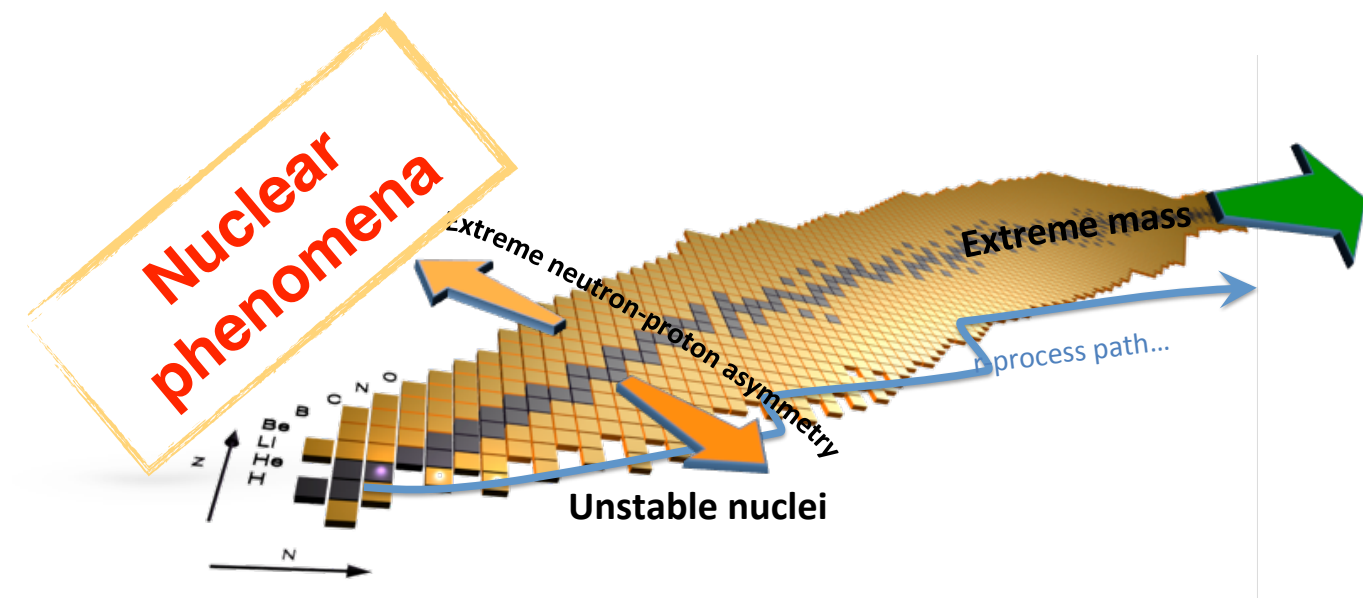


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**Solve the nuclear many-body problem from first principles**

Employing reliable methods with predictive power

- Structure and reactions of **nuclei**

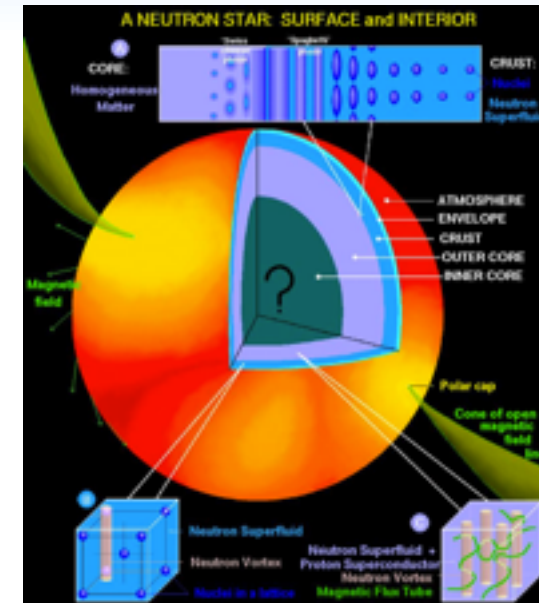


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Solve the nuclear many-body problem from first principles

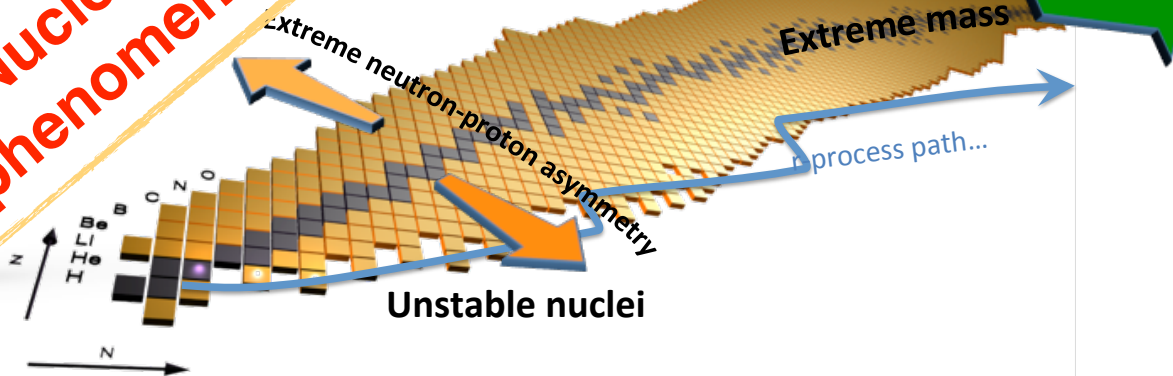
Employing reliable methods with predictive power

- Structure and reactions of **nuclei**
- Structure and dynamics of **neutron stars**



Astrophysics

Nuclear phenomena



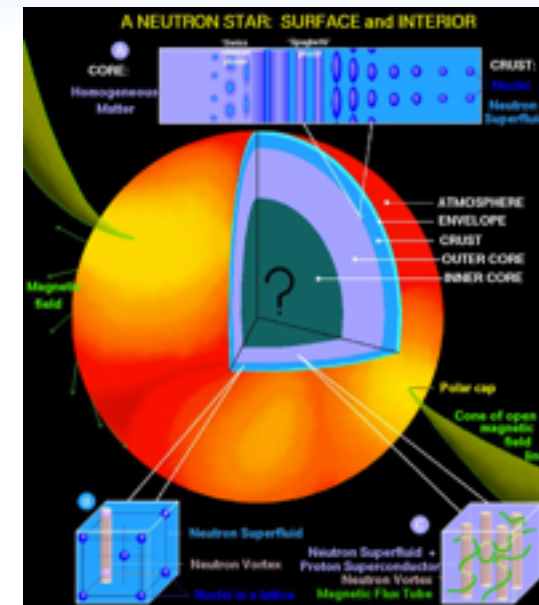


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Solve the nuclear many-body problem from first principles

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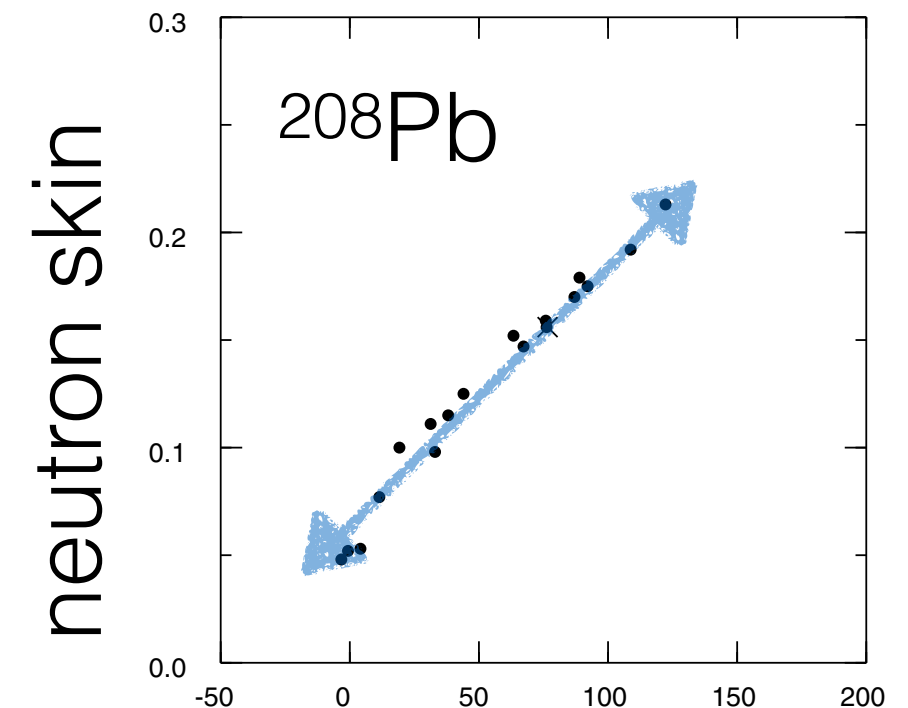
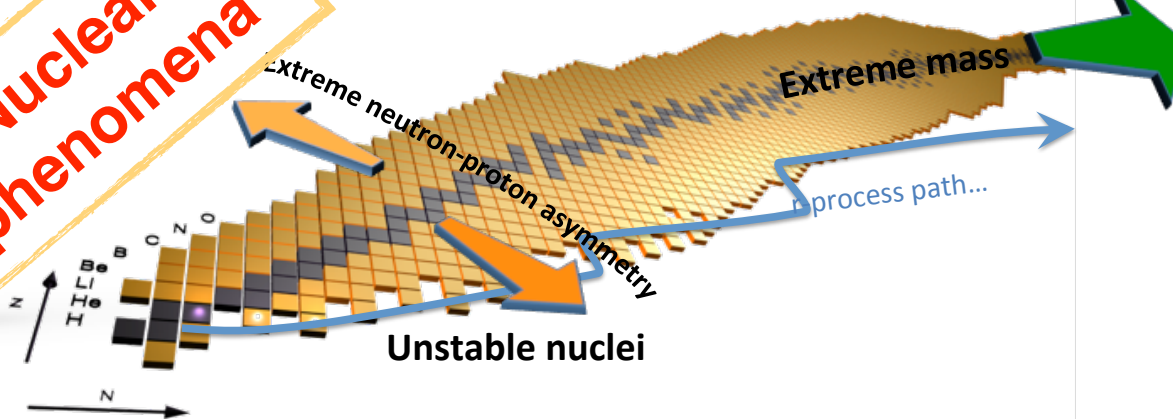
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Astrophysics

Brown (2000)

Nuclear phenomena



slope of EoS

neutron-rich nuclei and neutron matter: a strong correlation

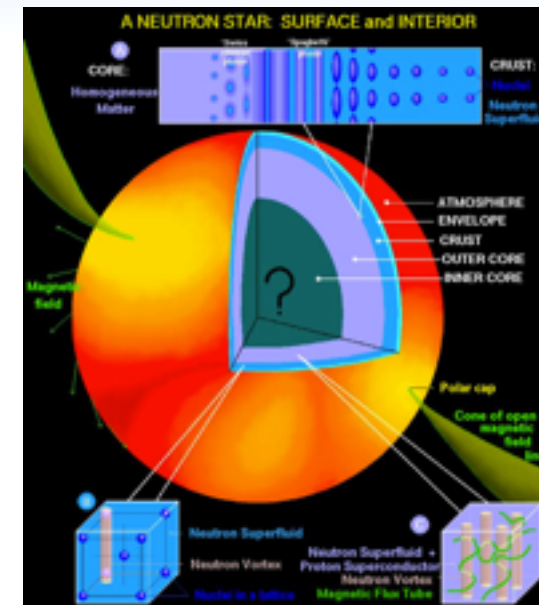


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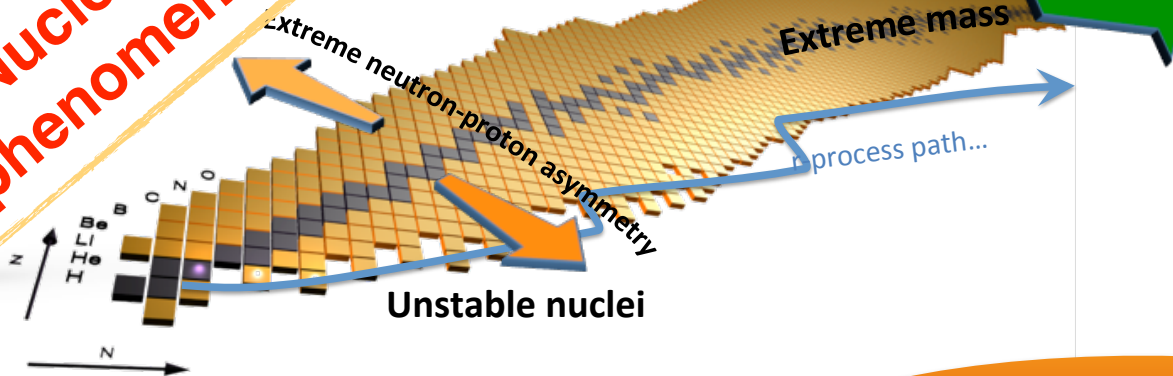
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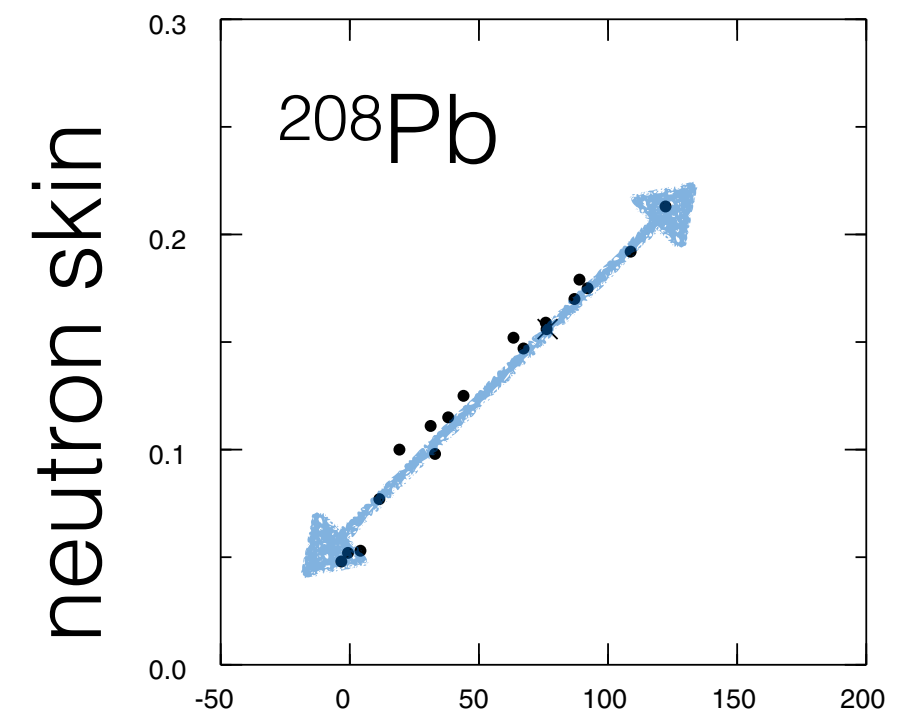
Astrophysics

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Predict infinite matter



neutron-rich nuclei and neutron matter: a strong correlation



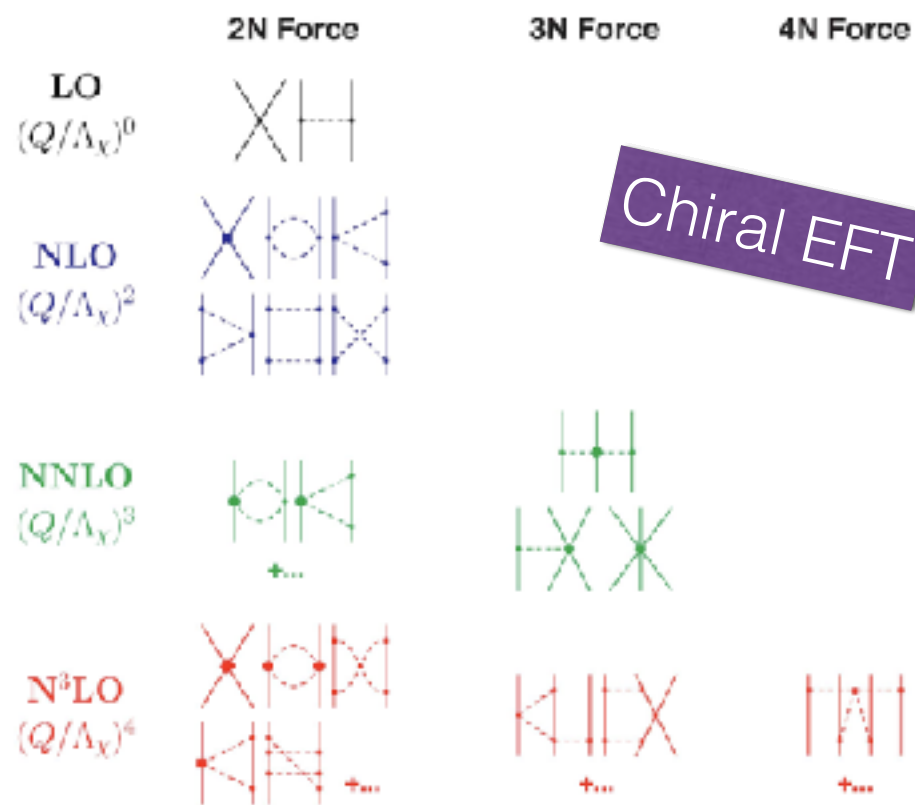


# How do we proceed in this endeavour?

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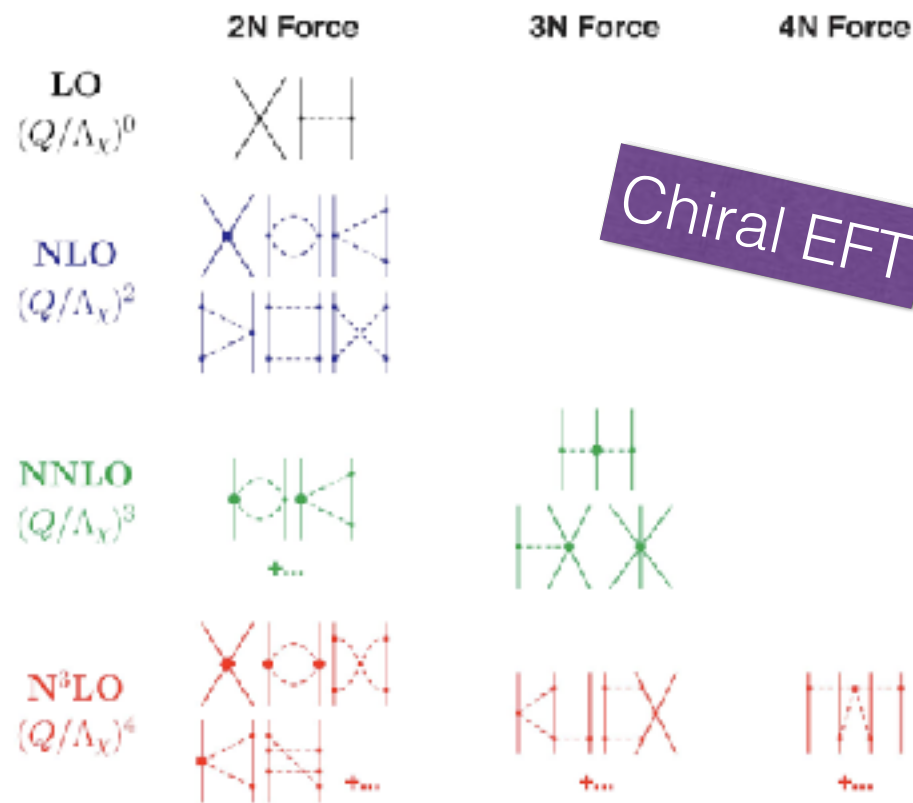
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We start from defining  
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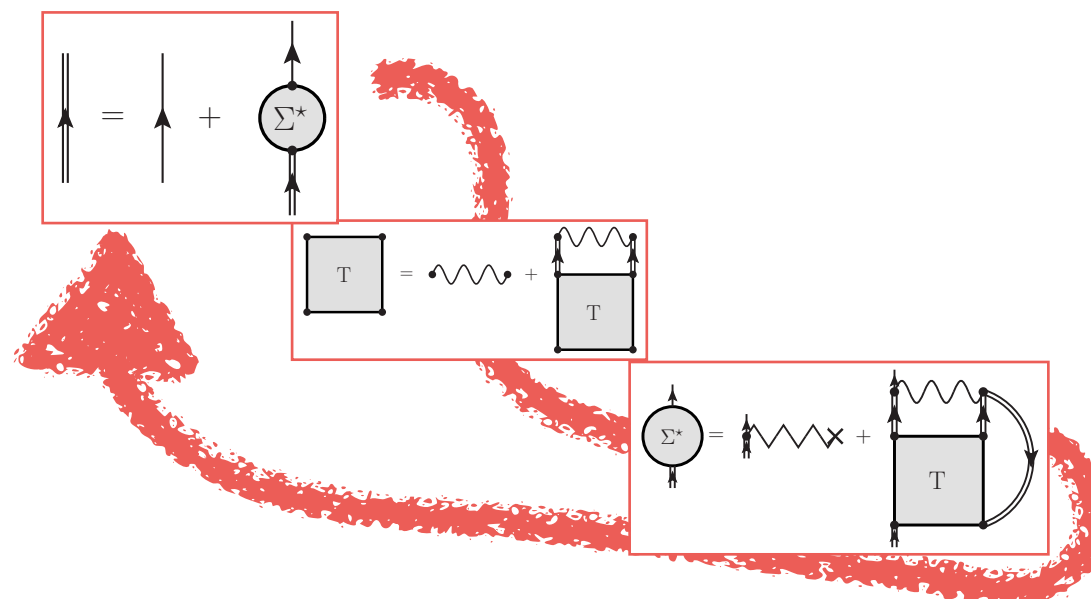
# How do we proceed in this endeavour?



Chiral EFT

We start from defining  
the Hamiltonian  
degrees of freedom and interactions

Then we solve the  
Schrödinger equation  
many-body approach



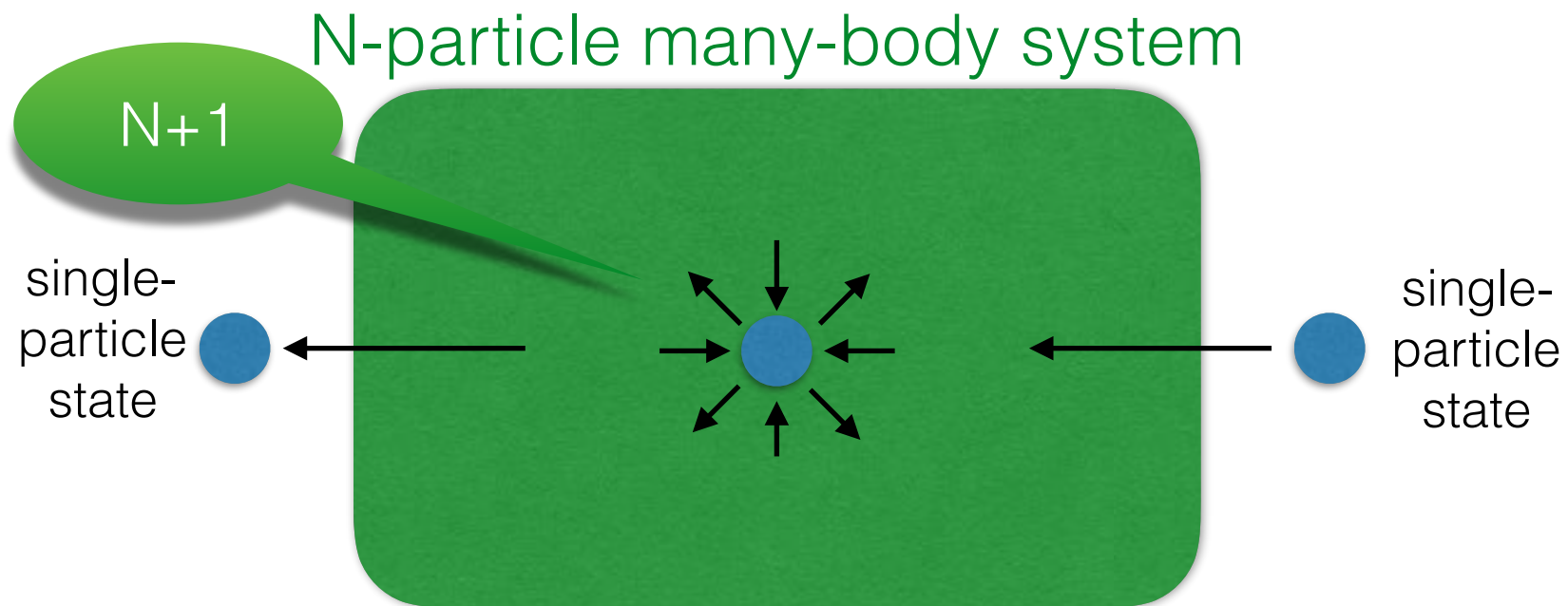
Self-consistent  
Green's function  
method

Dickhoff & Barbieri,  
PPNP **52**, 377 (2004)



# Self-consistent Green's functions

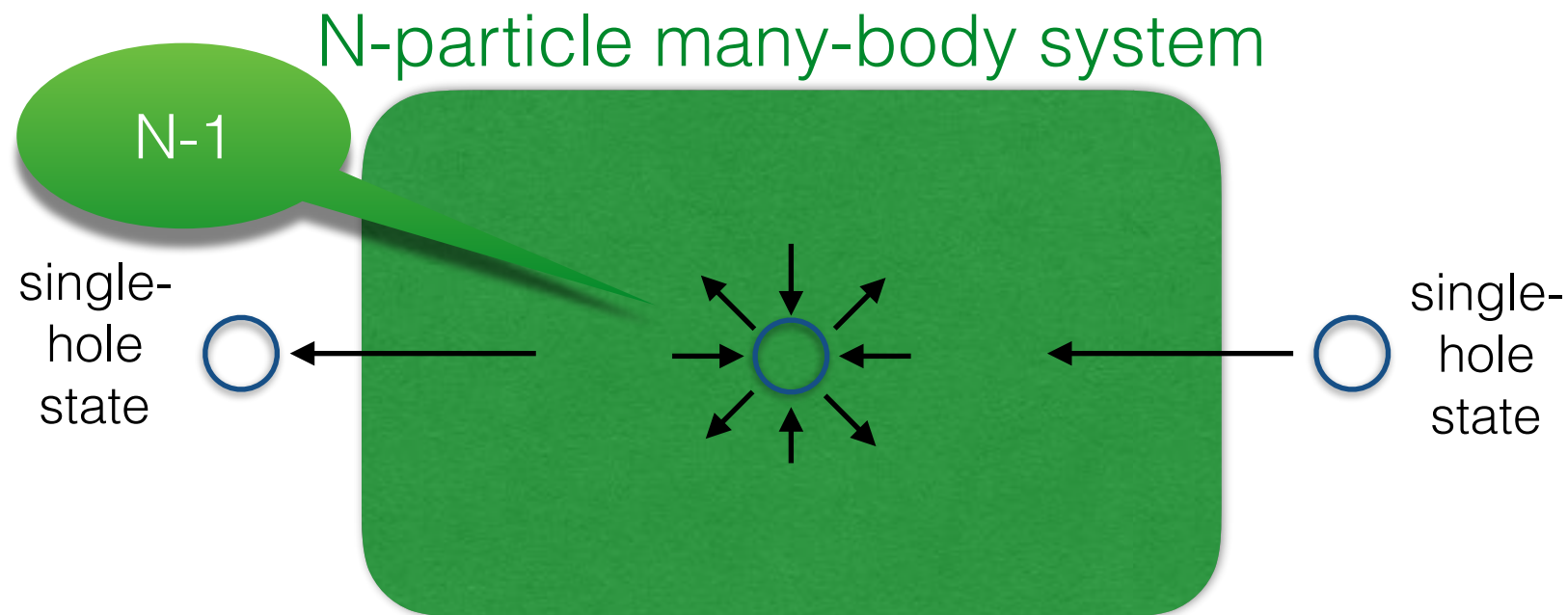
- Green's function: a tool to solve the nuclear many-body problem





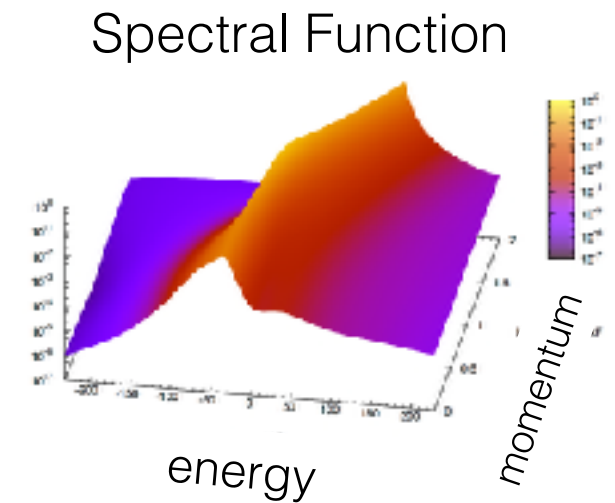
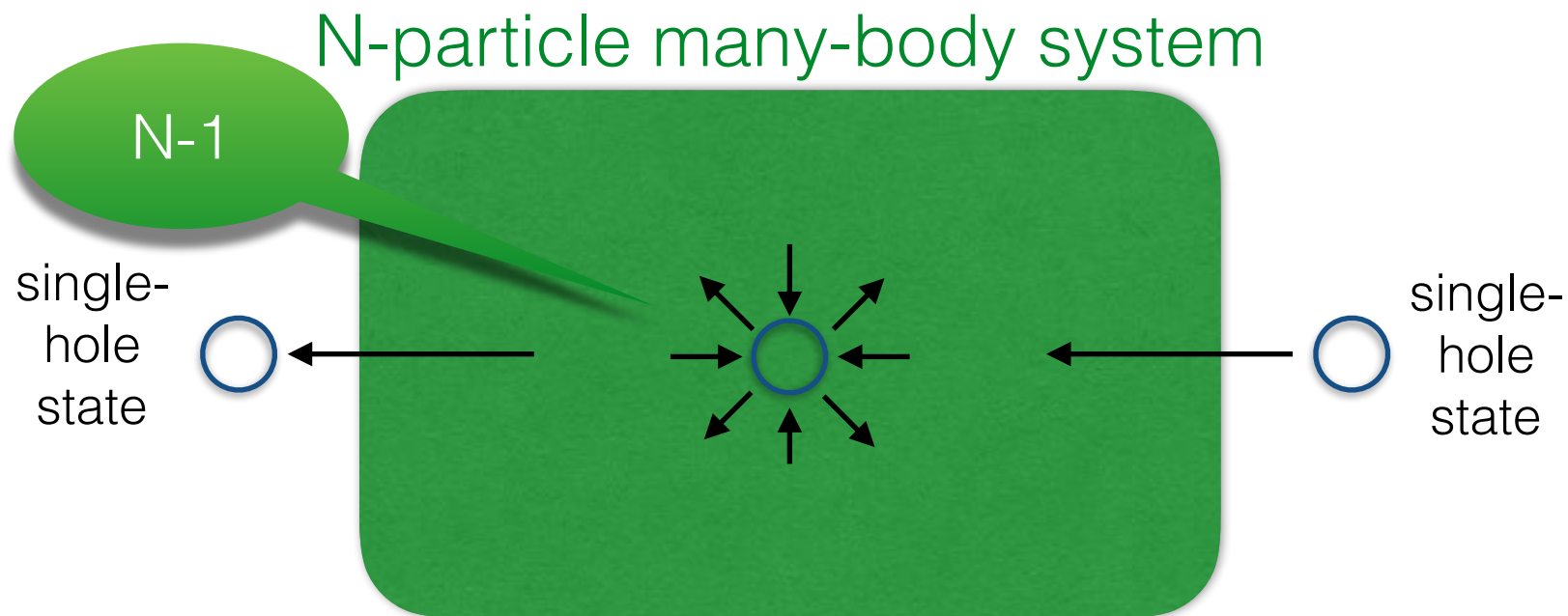
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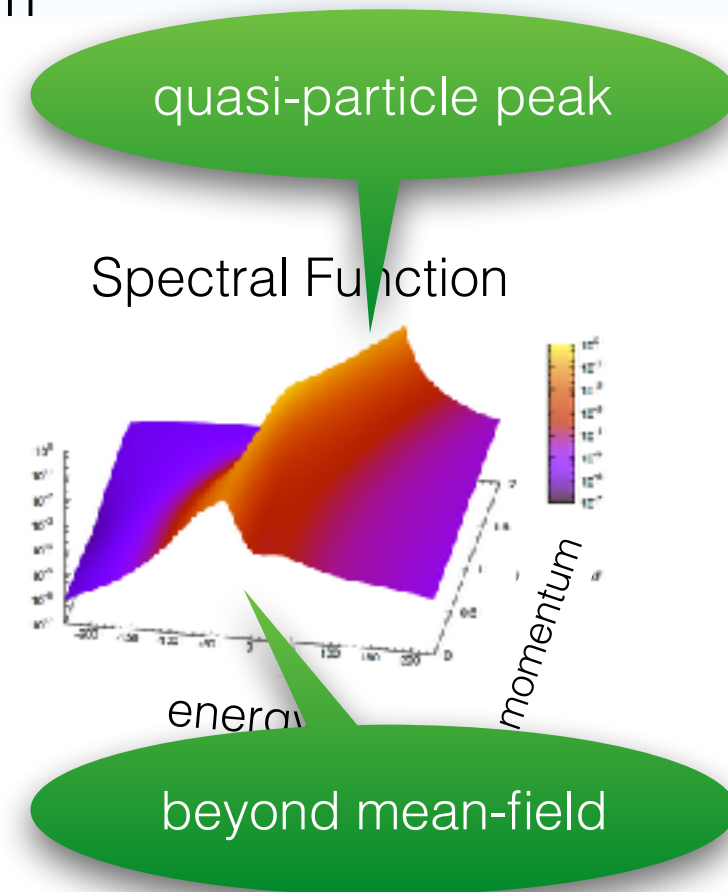
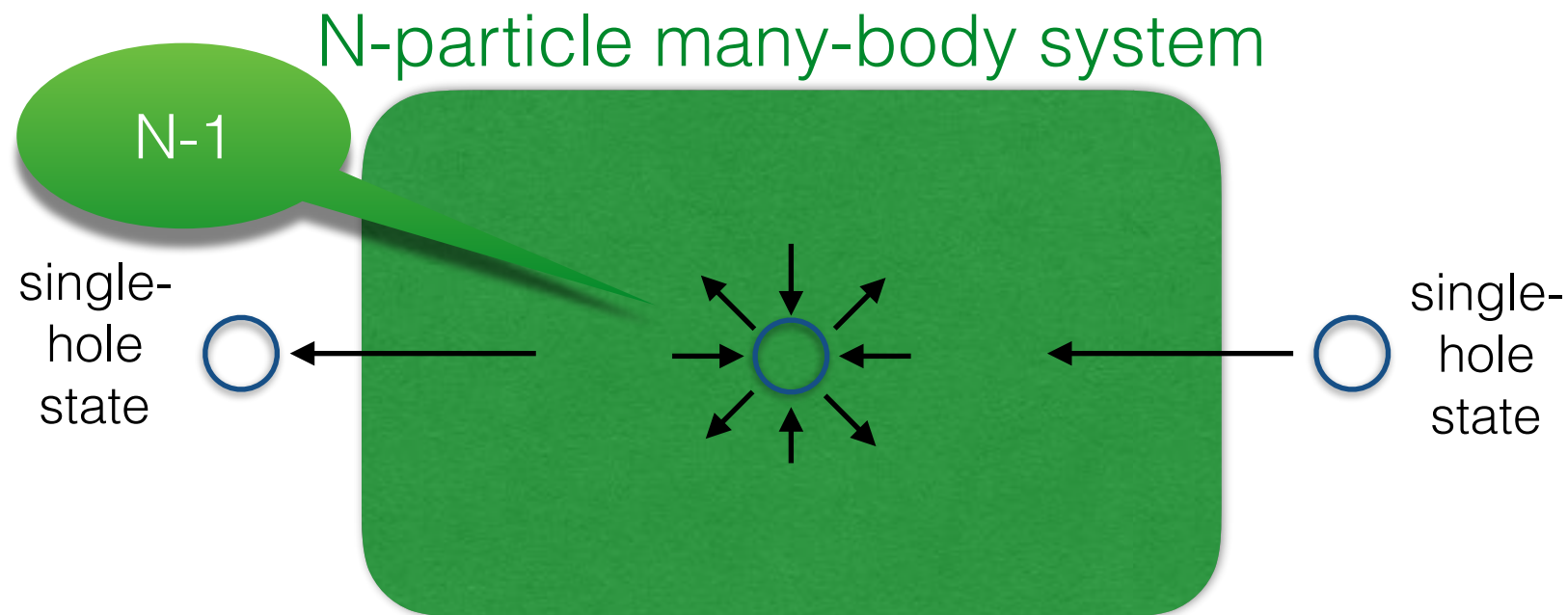
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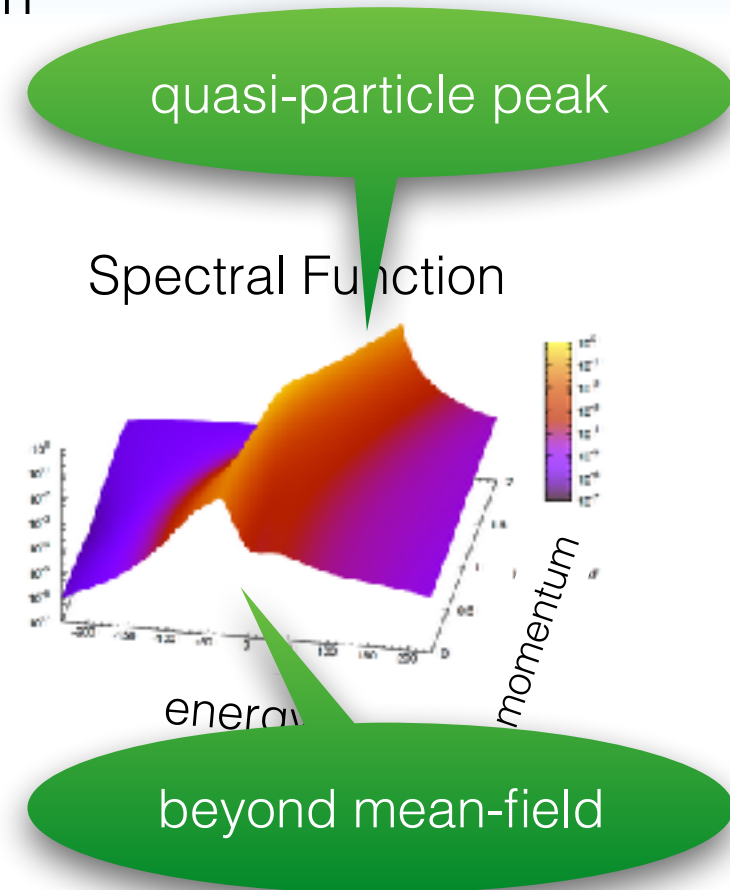
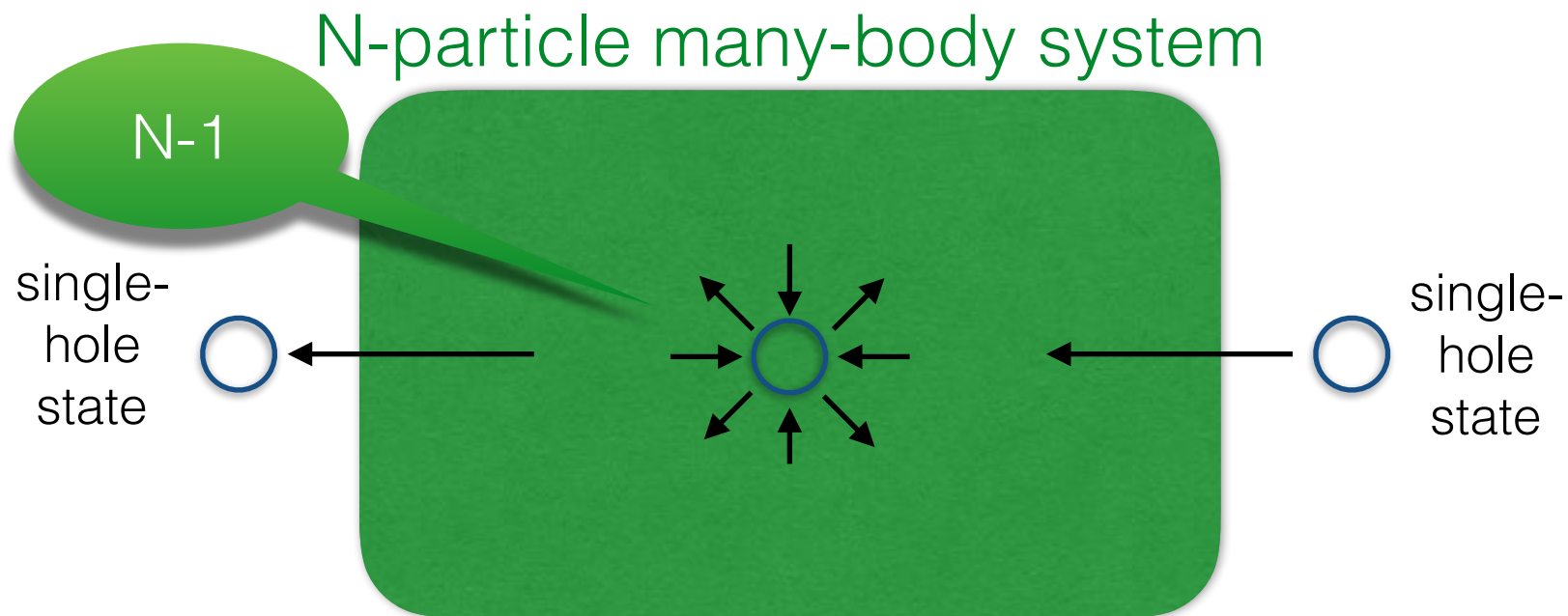
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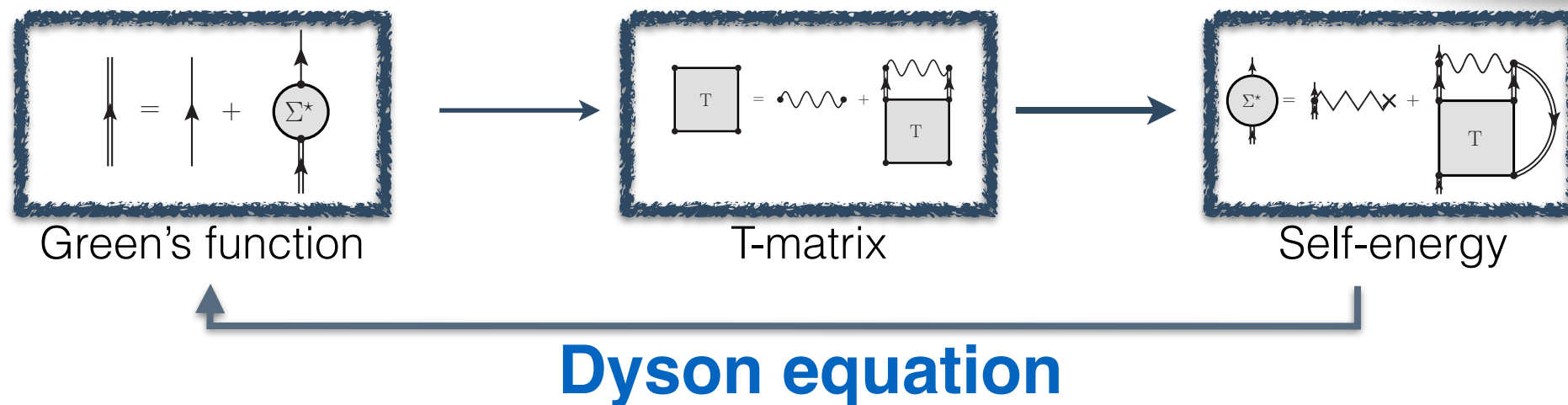


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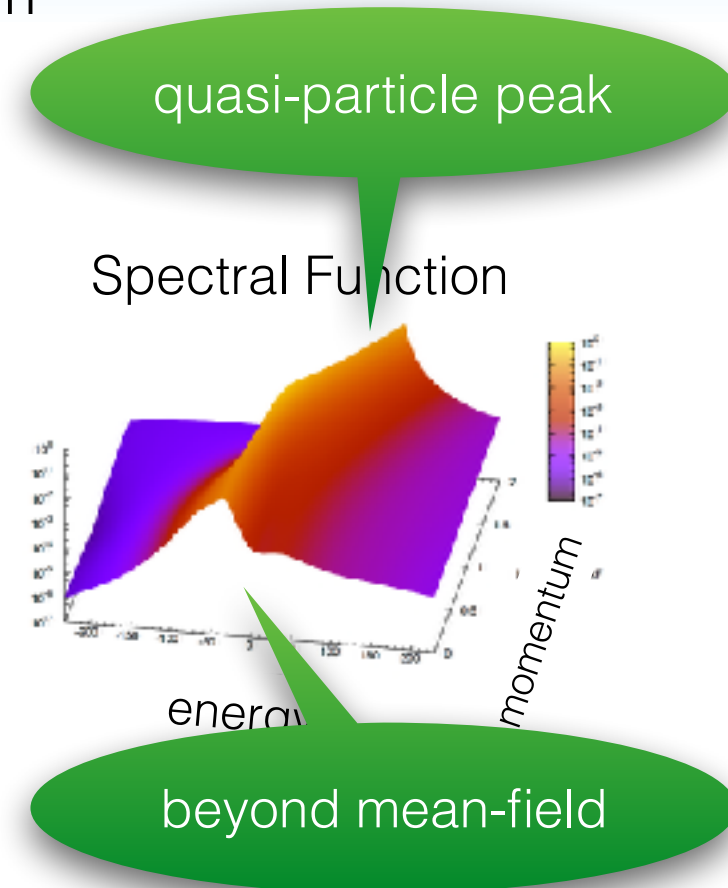
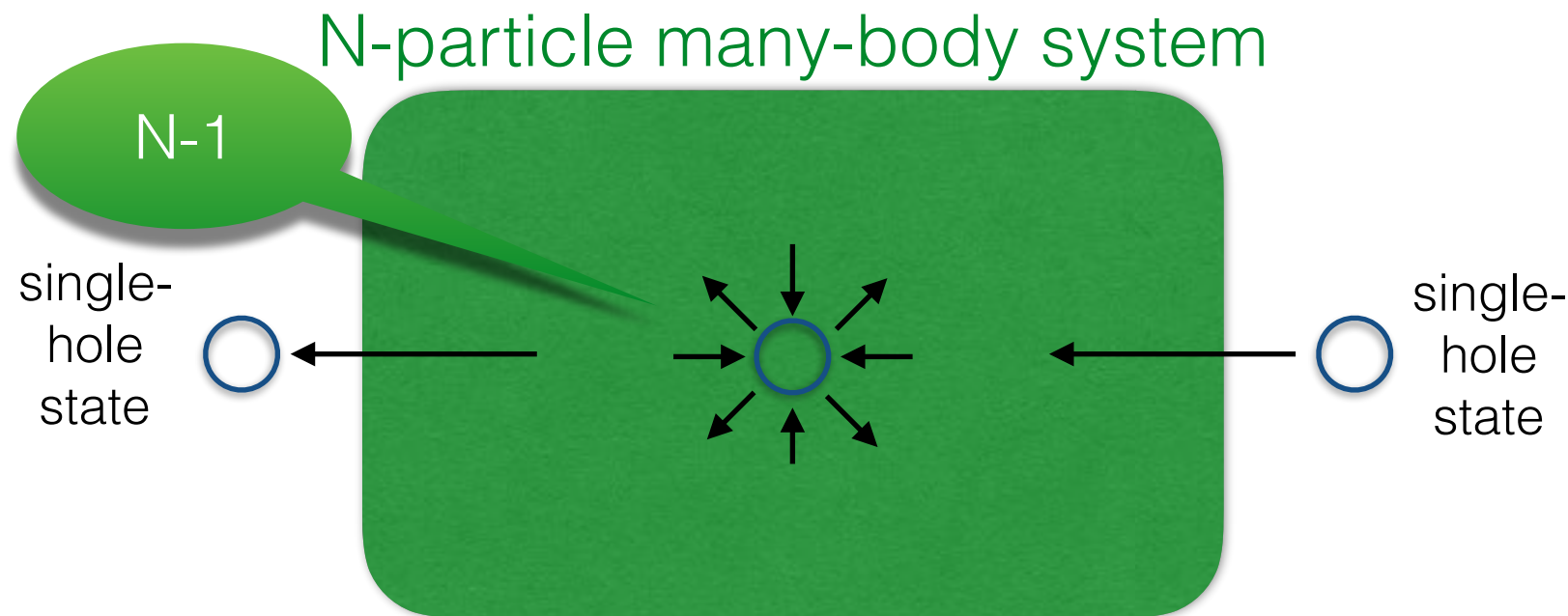


- Self-consistent nonperturbative method:

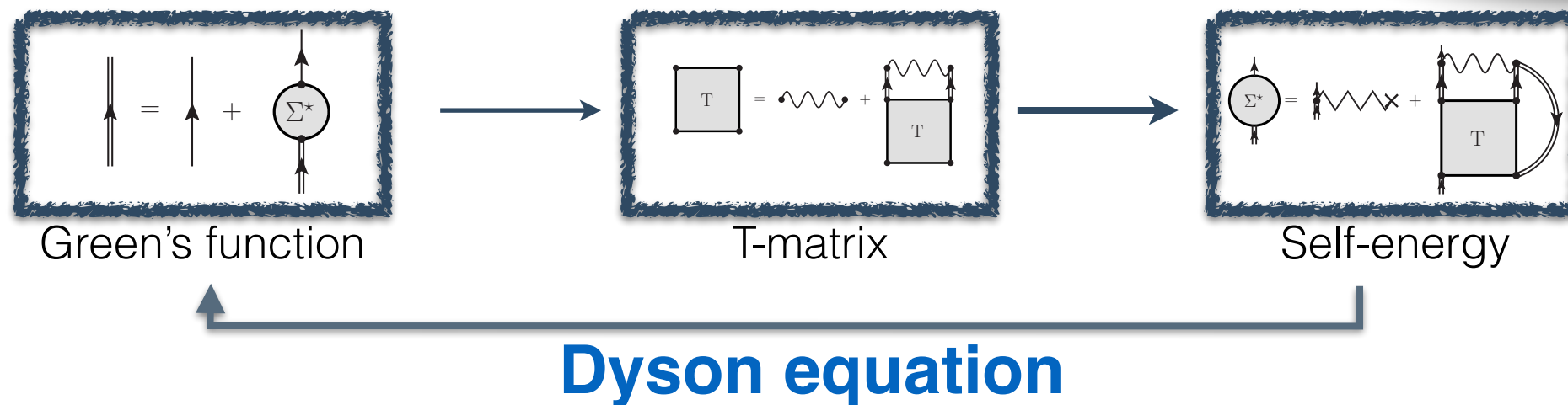


# Self-consistent Green's functions

- Green's function: a tool to solve the nuclear many-body problem



- Self-consistent nonperturbative method:



- Breakthrough: full formal extension to consistently include 3BFs

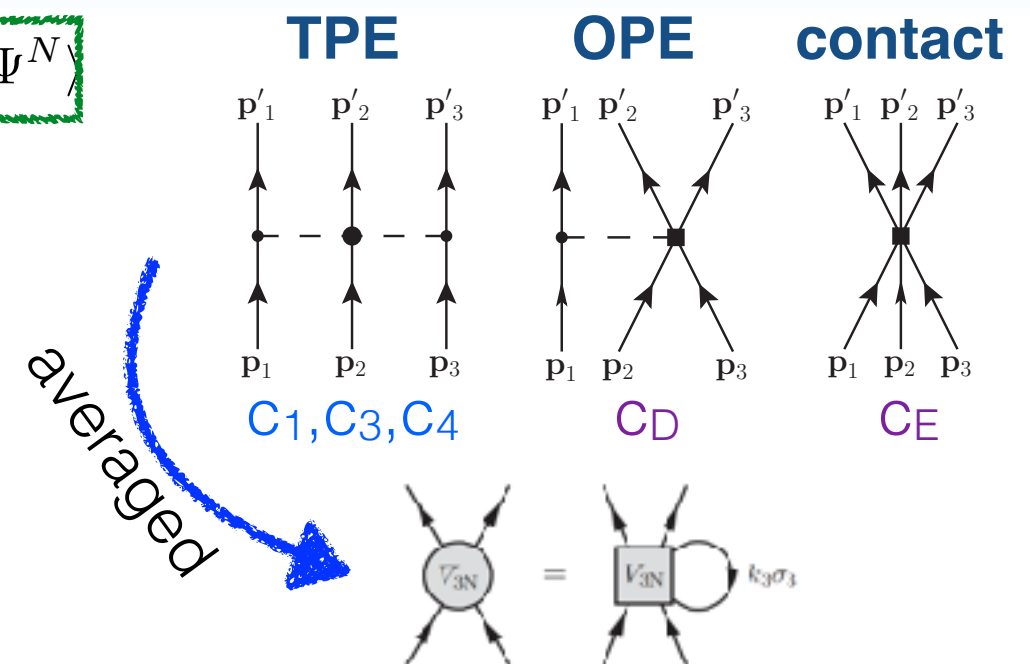
Carbone, Cipollone, Barbieri, Rios, Polls, PRC **88**, 054326 (2013)



# The need for 3-body nuclear forces

## Koltun sumrule

$$\frac{E}{A} = \frac{\nu}{\rho} \int \frac{d^3p}{(2\pi)^3} \int \frac{d\omega}{2\pi} \frac{1}{2} \left\{ \frac{p^2}{2m} + \omega \right\} \mathcal{A}(p, \omega) f(\omega) - \frac{1}{2} \langle \Psi^N | \hat{W} | \Psi^N \rangle$$

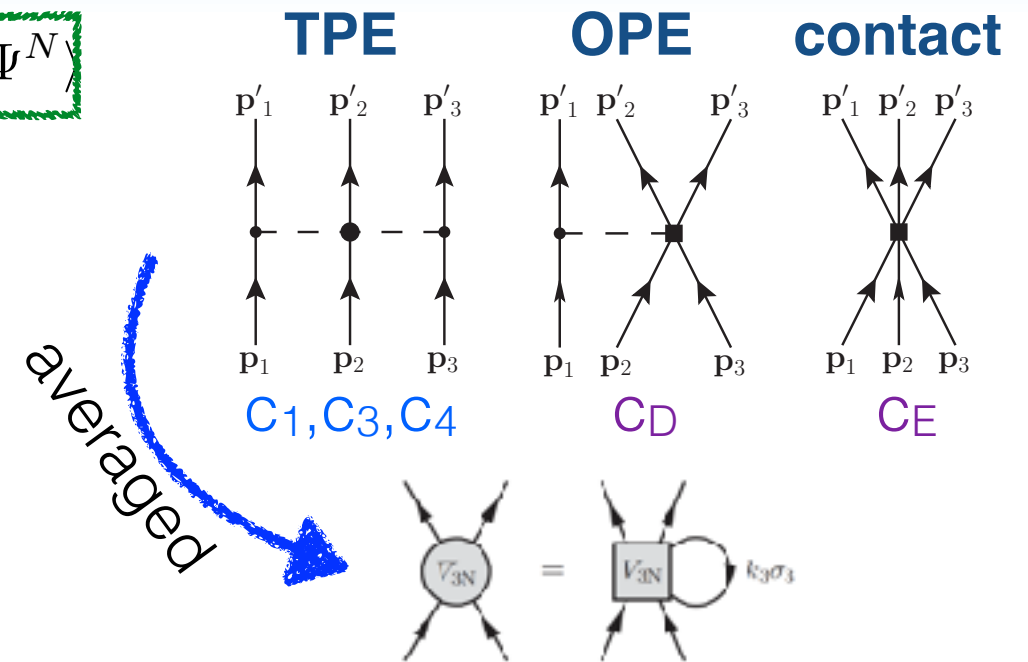
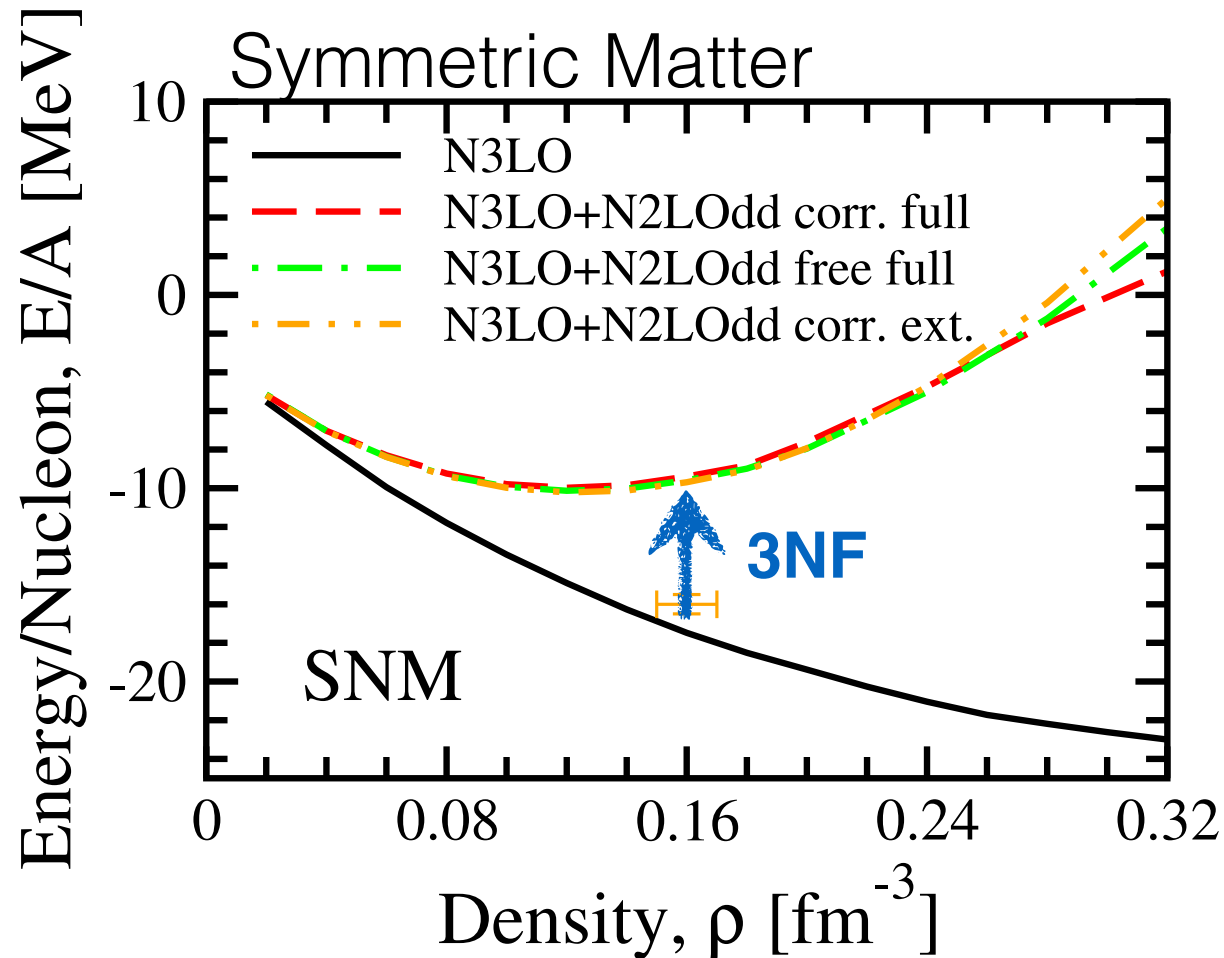




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- Improved prediction of saturation density

Carbone, Rios, Polls, PRC 88, 044302 (2013)

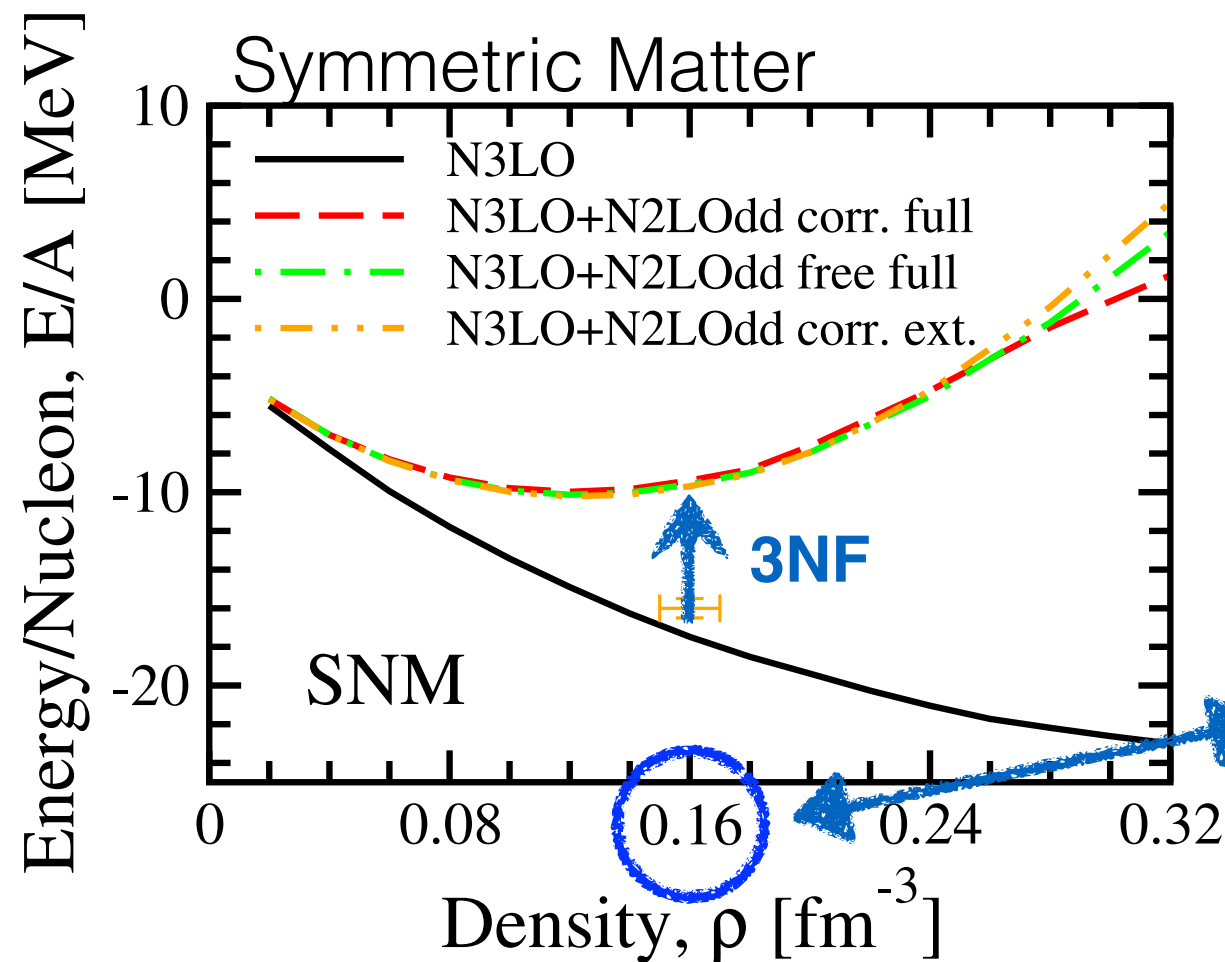
Carbone, Rios, Polls, PRC 90, 054322 (2014)



# The need for 3-body nuclear forces

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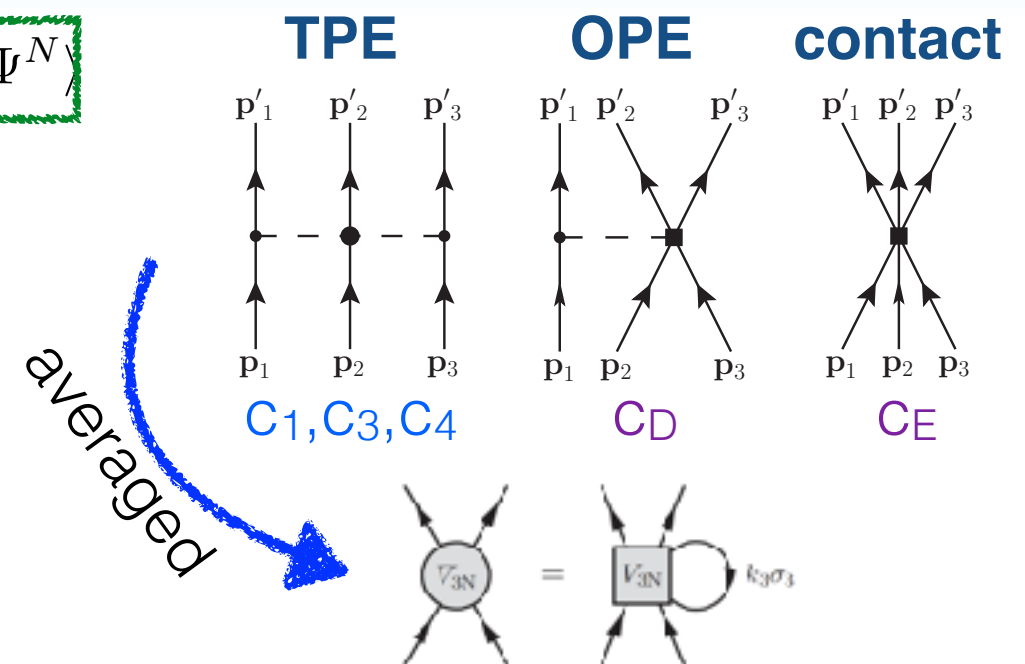
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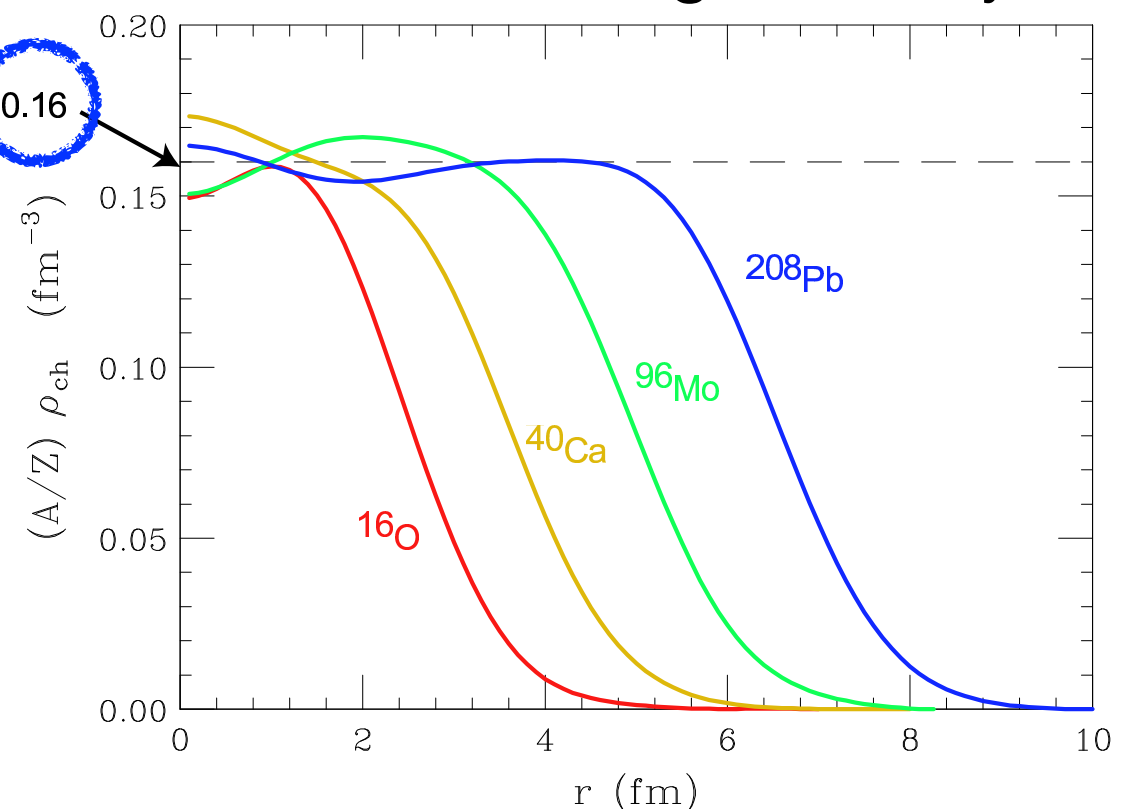
- Improved prediction of saturation density

Carbone, Rios, Polls, PRC 88, 044302 (2013)

Carbone, Rios, Polls, PRC 90, 054322 (2014)



nuclear charge density



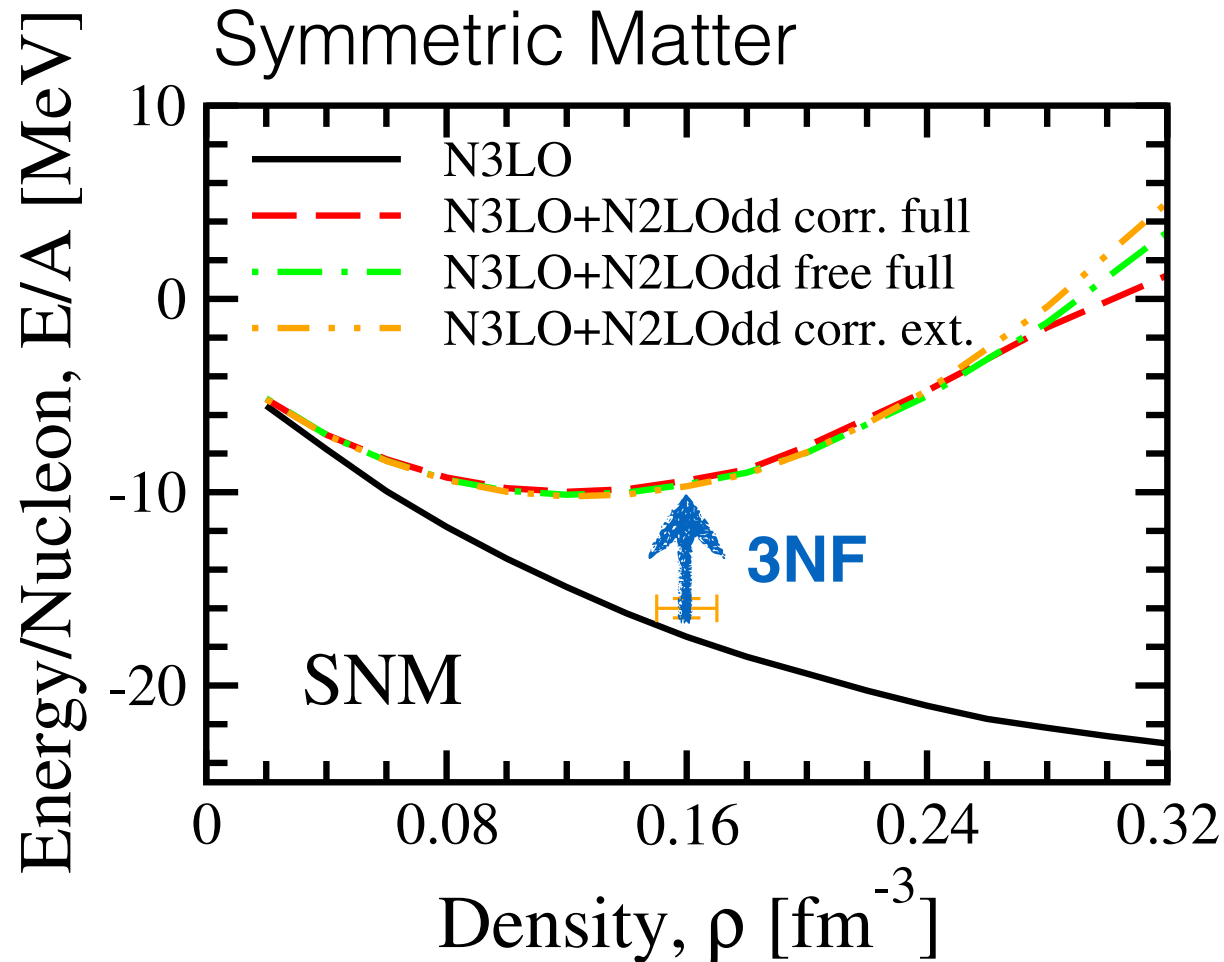
Courtesy of O. Benhar



# The need for 3-body nuclear forces

## Koltun sumrule

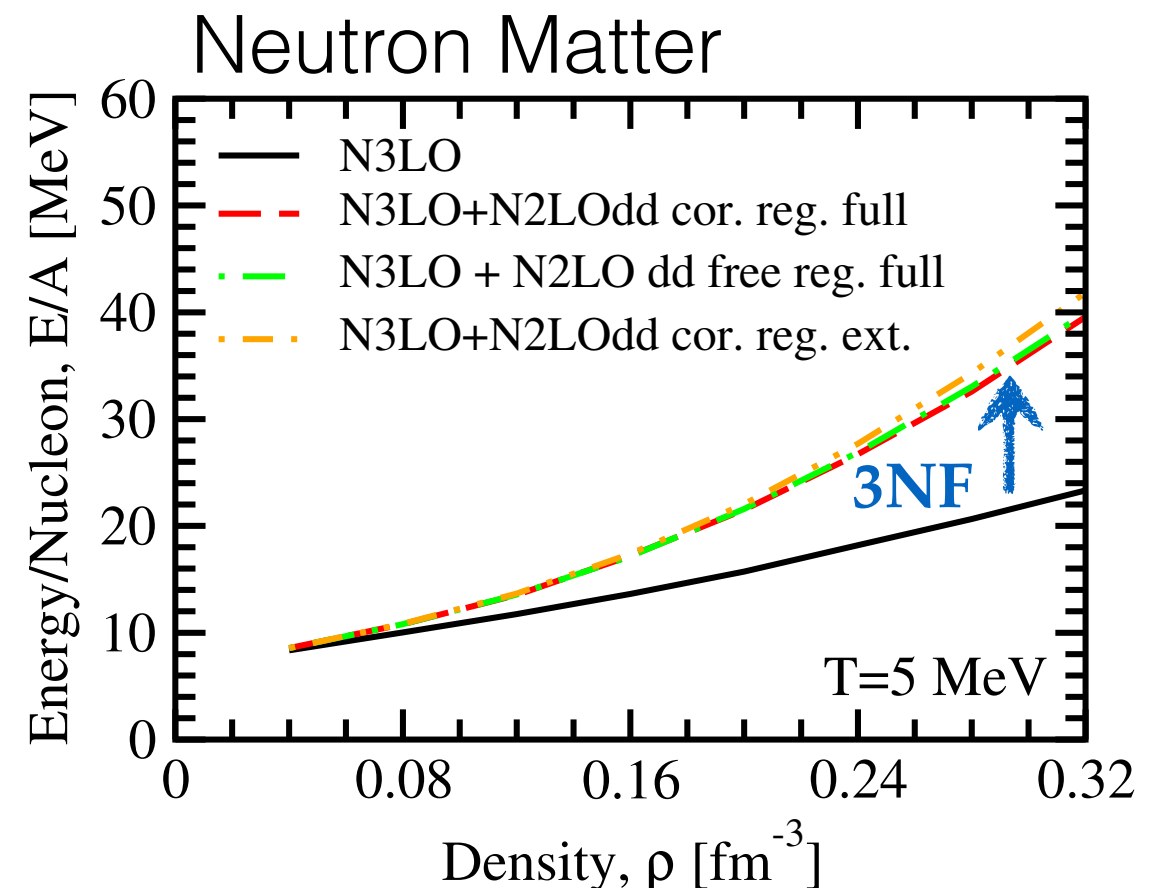
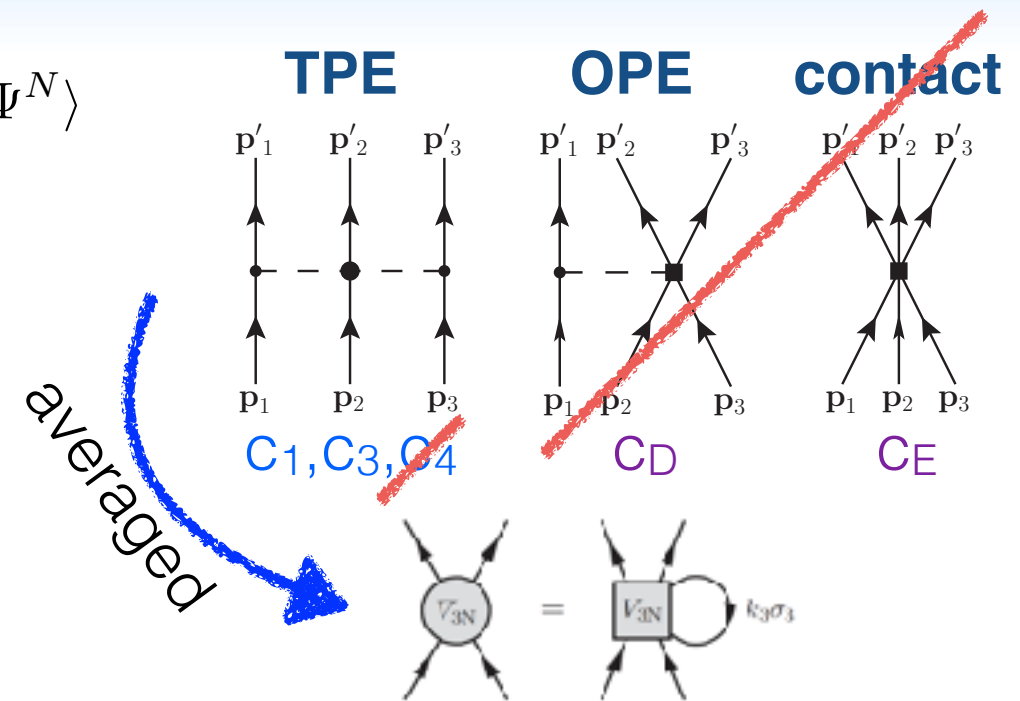
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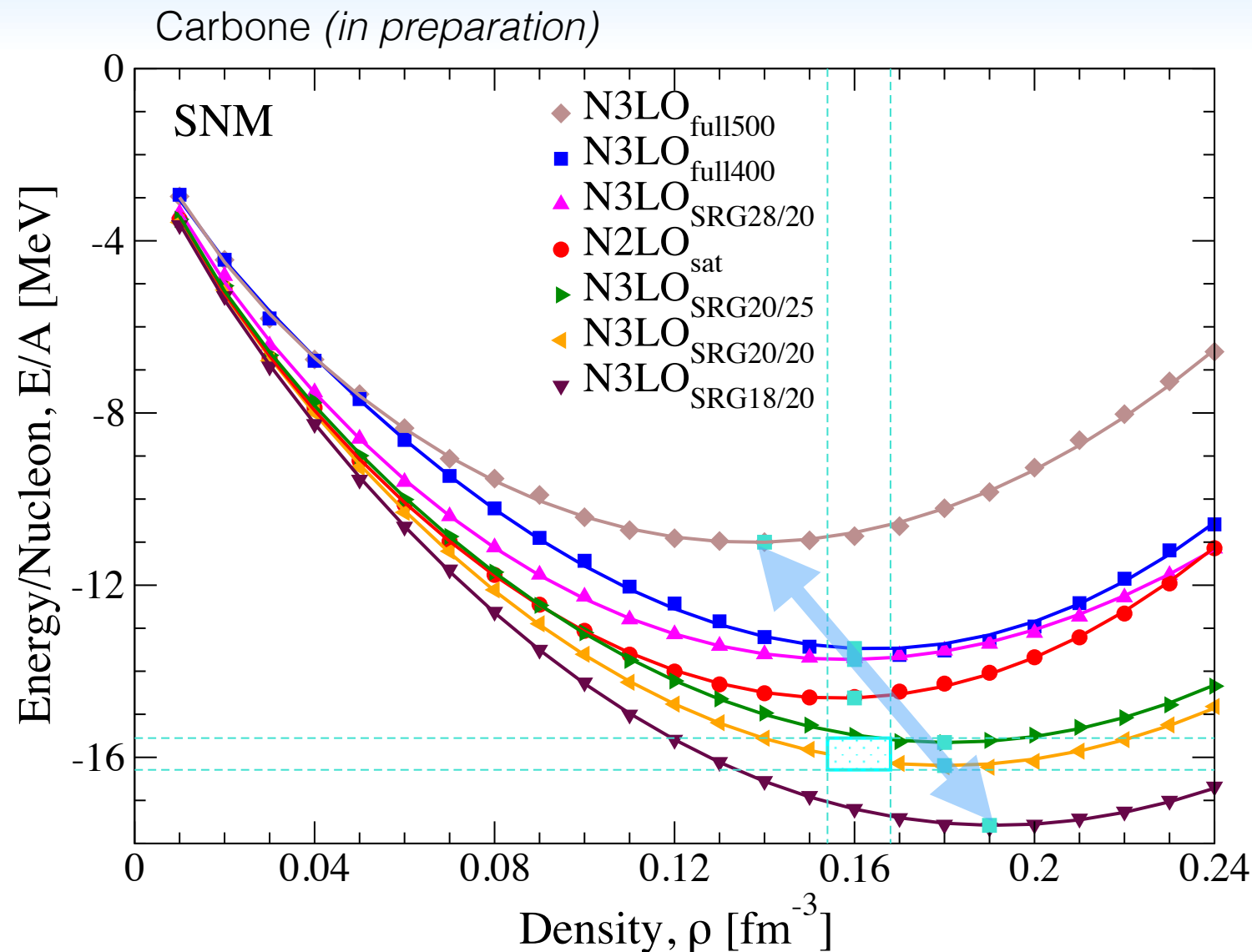
- Improved prediction of saturation density
- Neutron matter energy stiffens

Carbone, Rios, Polls, PRC 88, 044302 (2013)

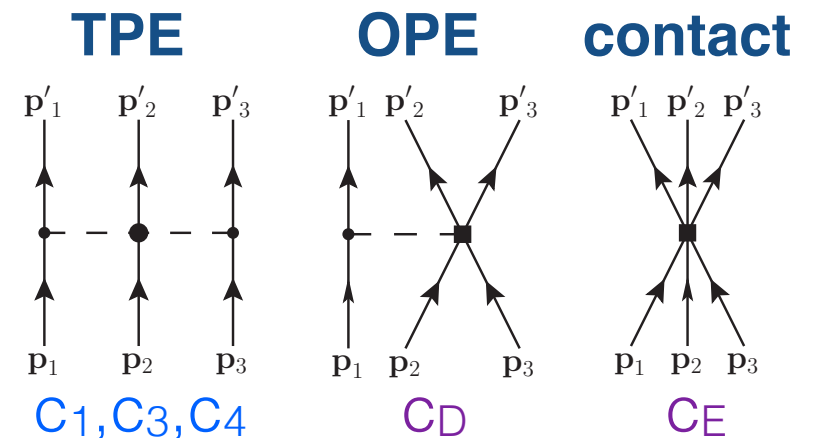
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# Saturation point according to different Hamiltonians



## Chiral hamiltonians

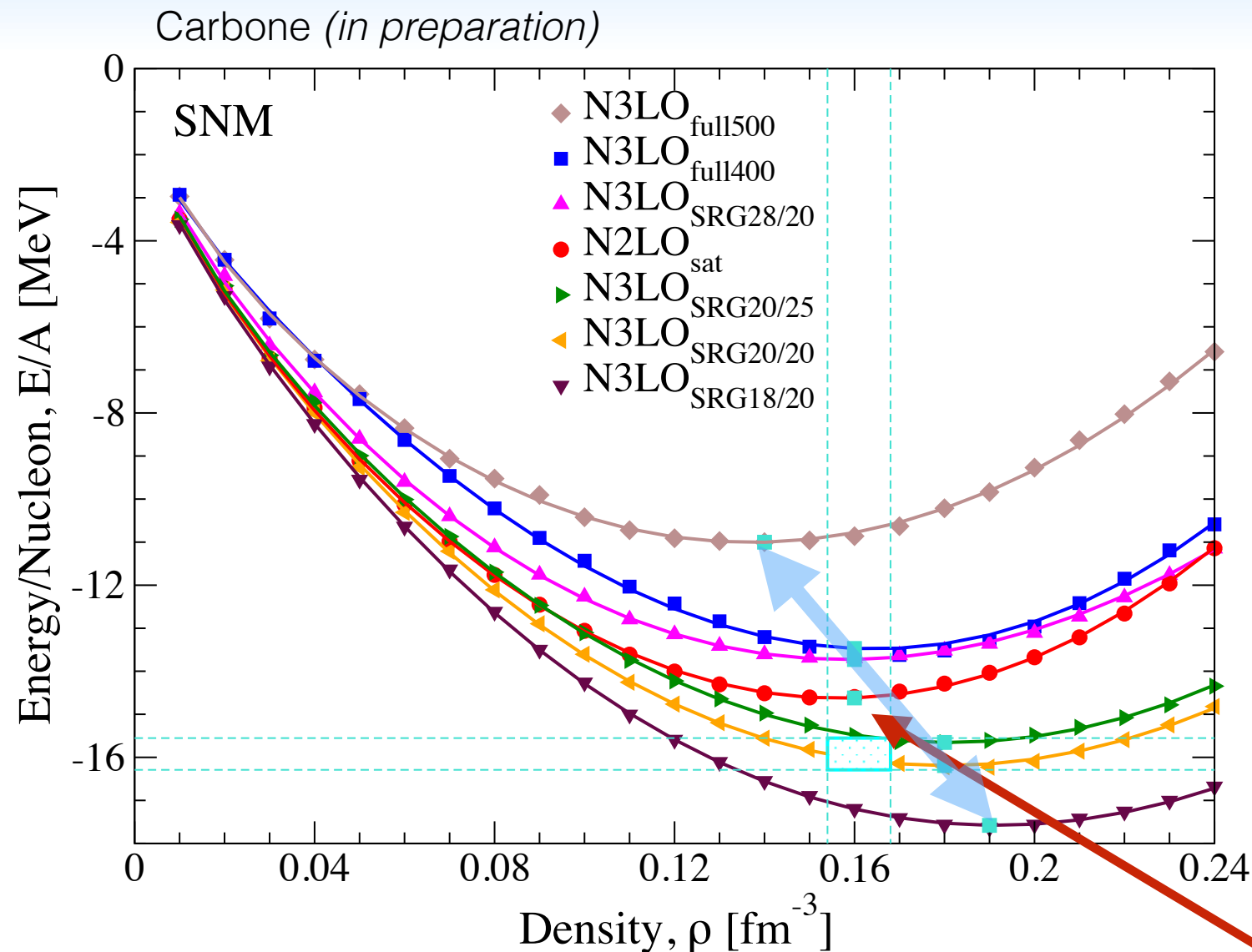


Some low-energy constants are fit to few-body properties

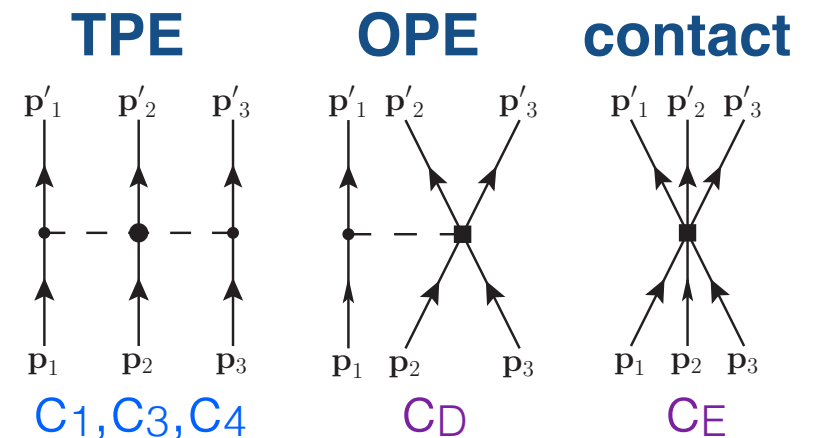
**Theoretical uncertainty band**  
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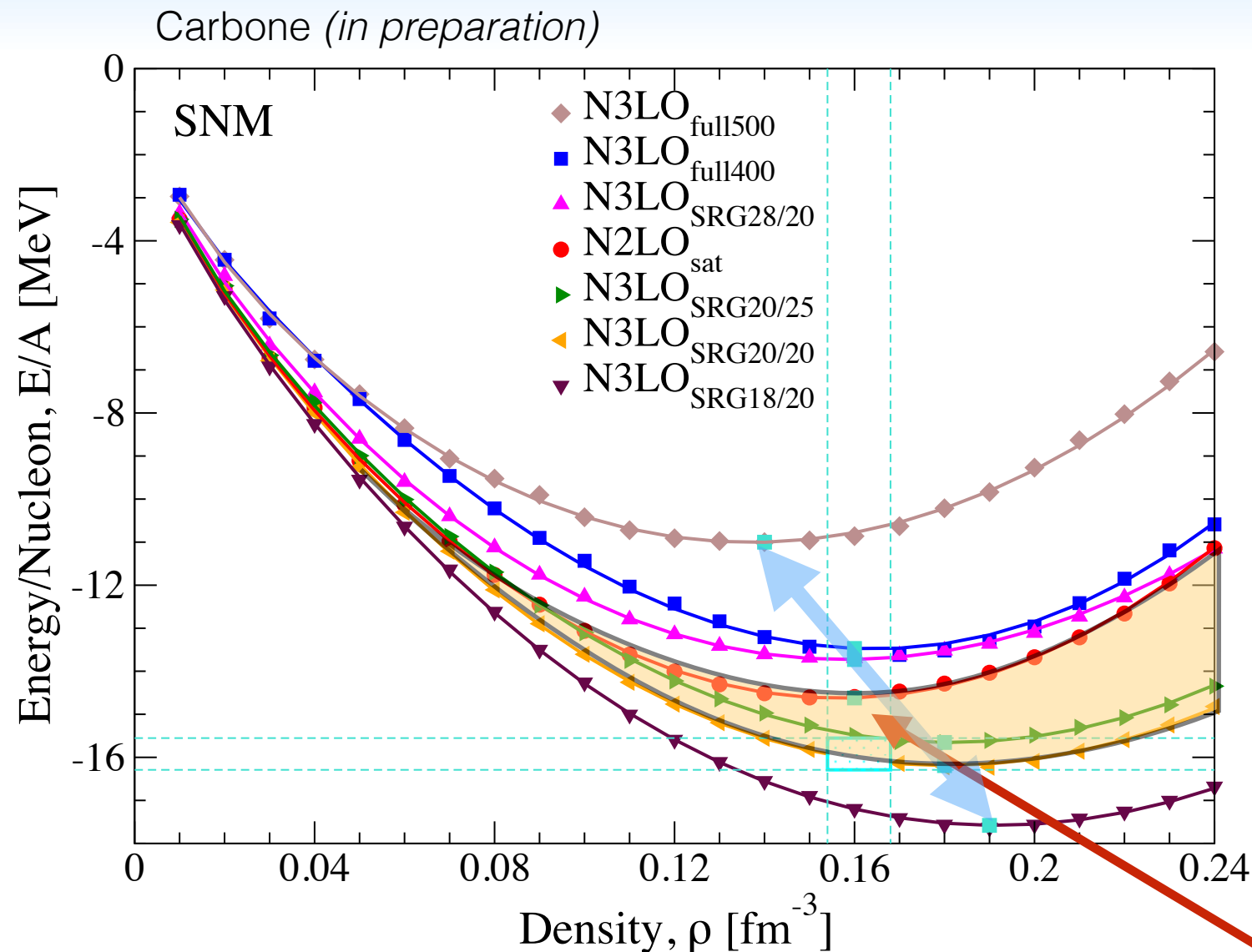


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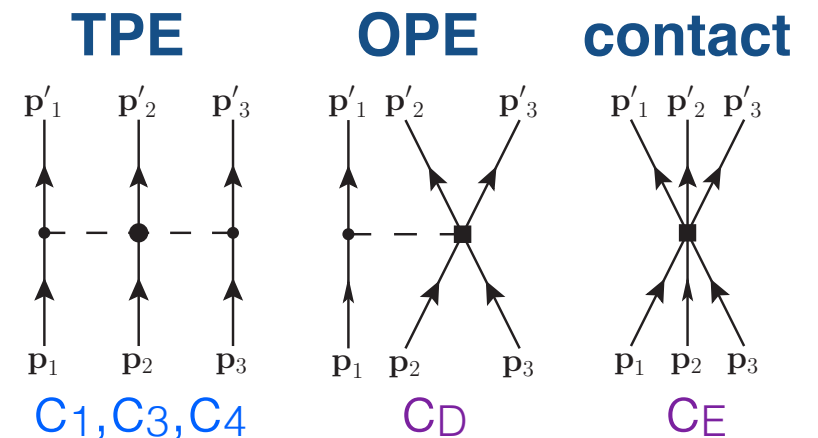
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N2LOsat (2N+3N):  
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Some low-energy constants are fit to few-body properties

**Theoretical uncertainty band**  
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- 2N N2LO<sub>sat</sub> + 3N N2LO
- 2N N3LO<sub>SRG-1</sub> + 3N N2LO
- 2N N3LO<sub>SRG-2</sub> + 3N N2LO



# Predictions for the Symmetry Energy and slope L

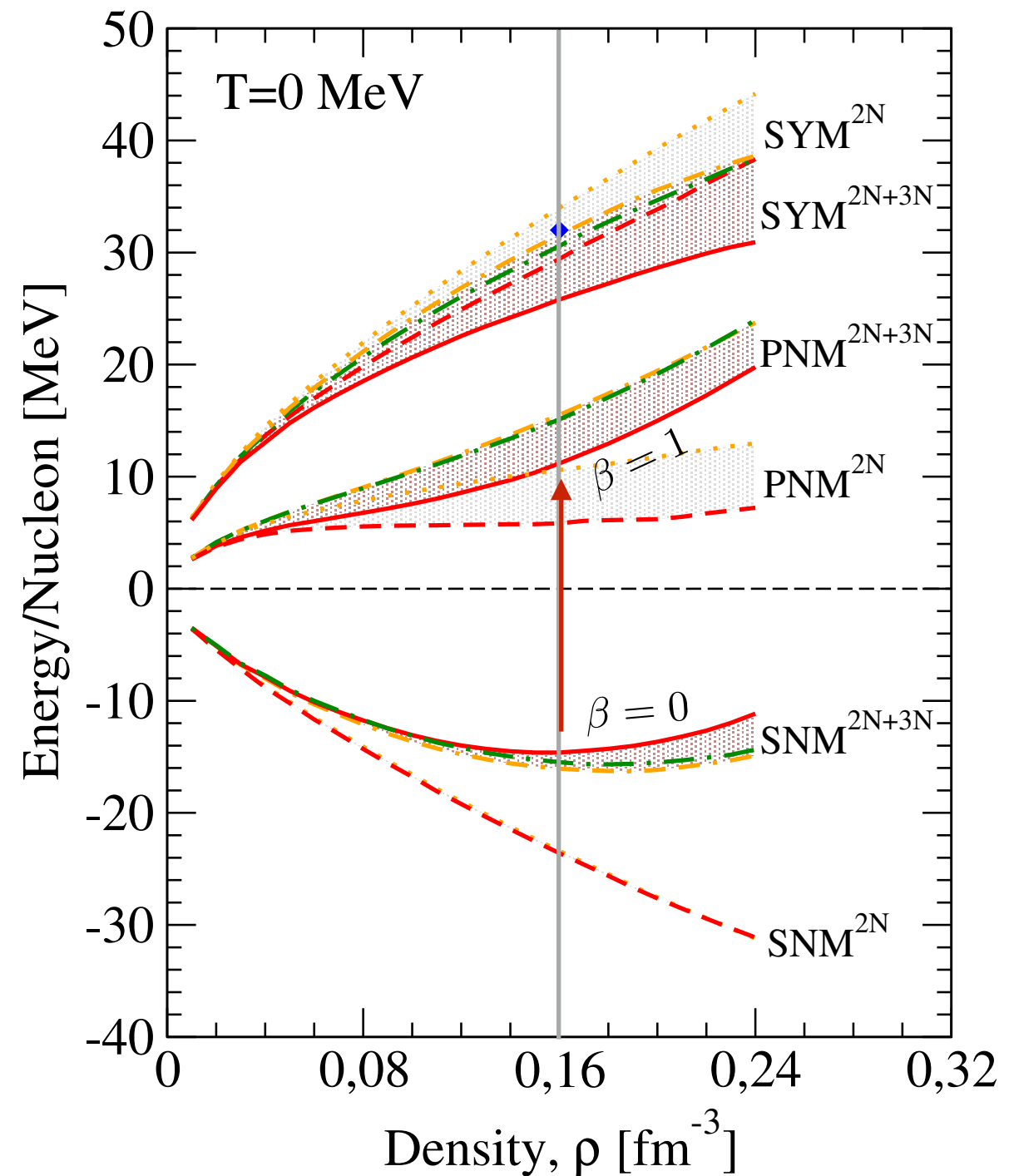
Energy of asymmetric matter

$$\frac{E}{A}(\rho, \beta) \simeq \frac{E_{\text{SNM}}}{A}(\rho) + \frac{S}{A}(\rho)\beta^2$$

$$\frac{S}{A}(\rho) \simeq \frac{E_{\text{PNM}}}{A}(\rho) - \frac{E_{\text{SNM}}}{A}(\rho)$$

Symmetry energy

	SRG1	SRG2	SAT
Sv (MeV)	31.6	30.6	25.8
L (MeV)	49.3	48.7	37.4
K (MeV)	290	270	213



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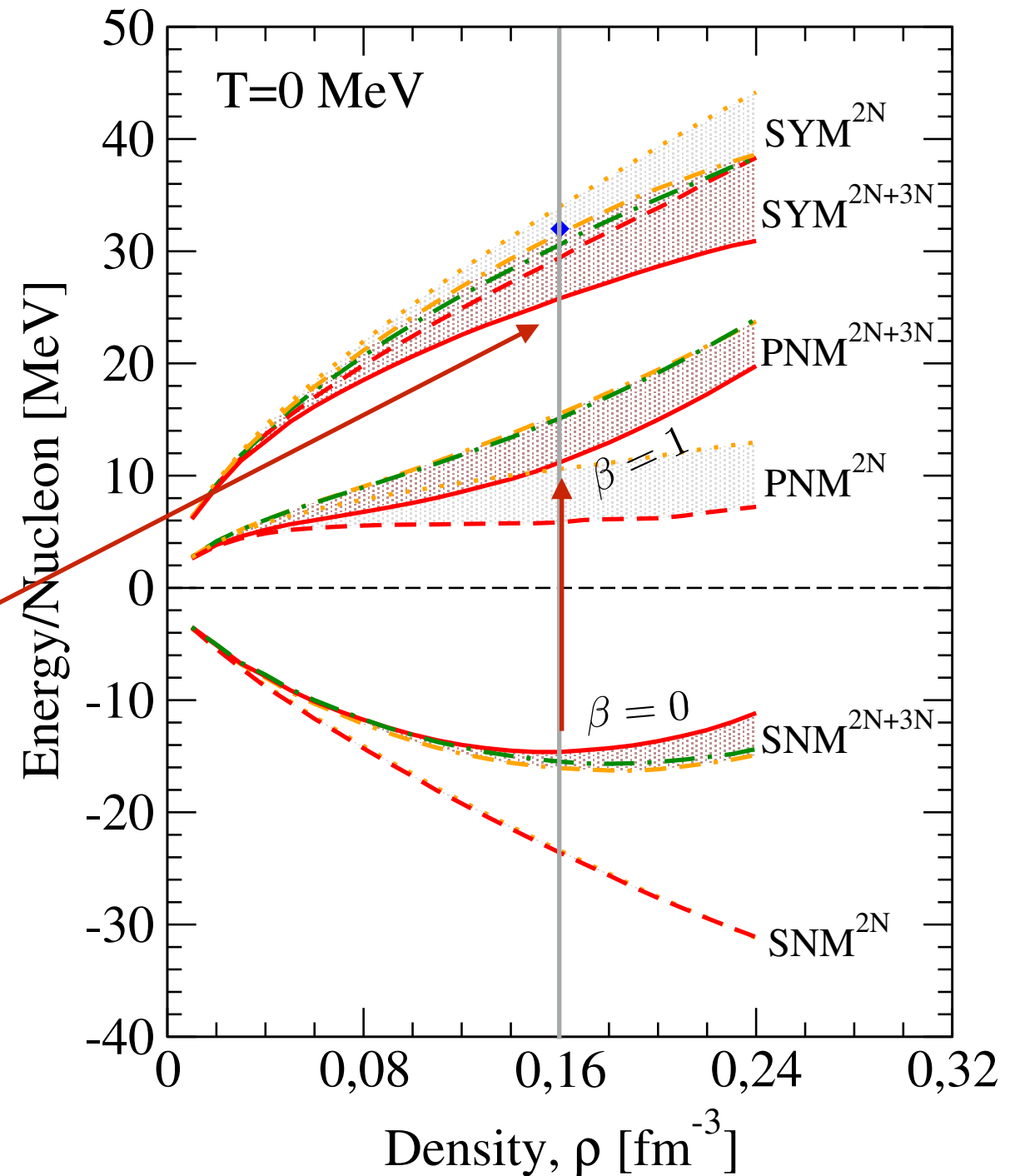
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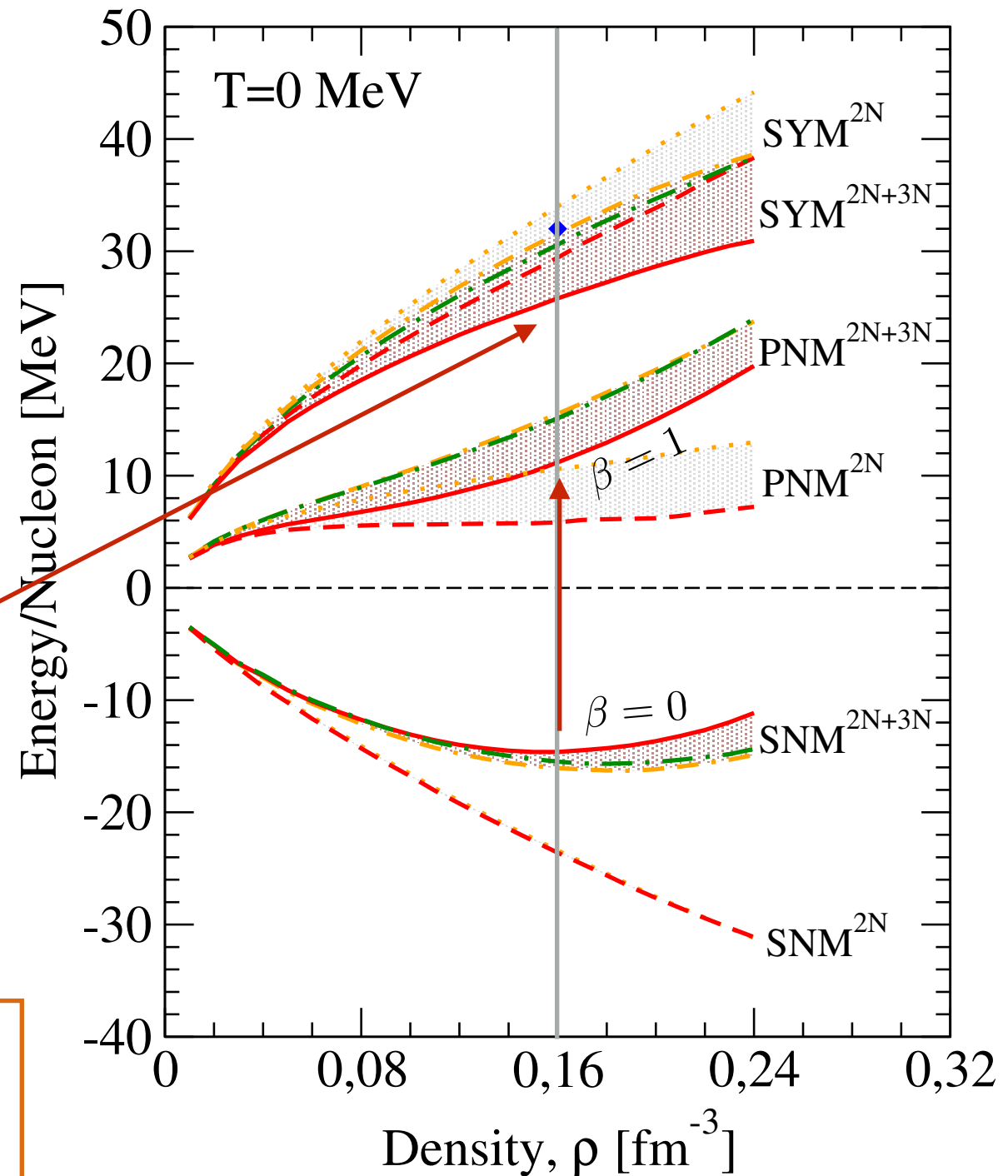
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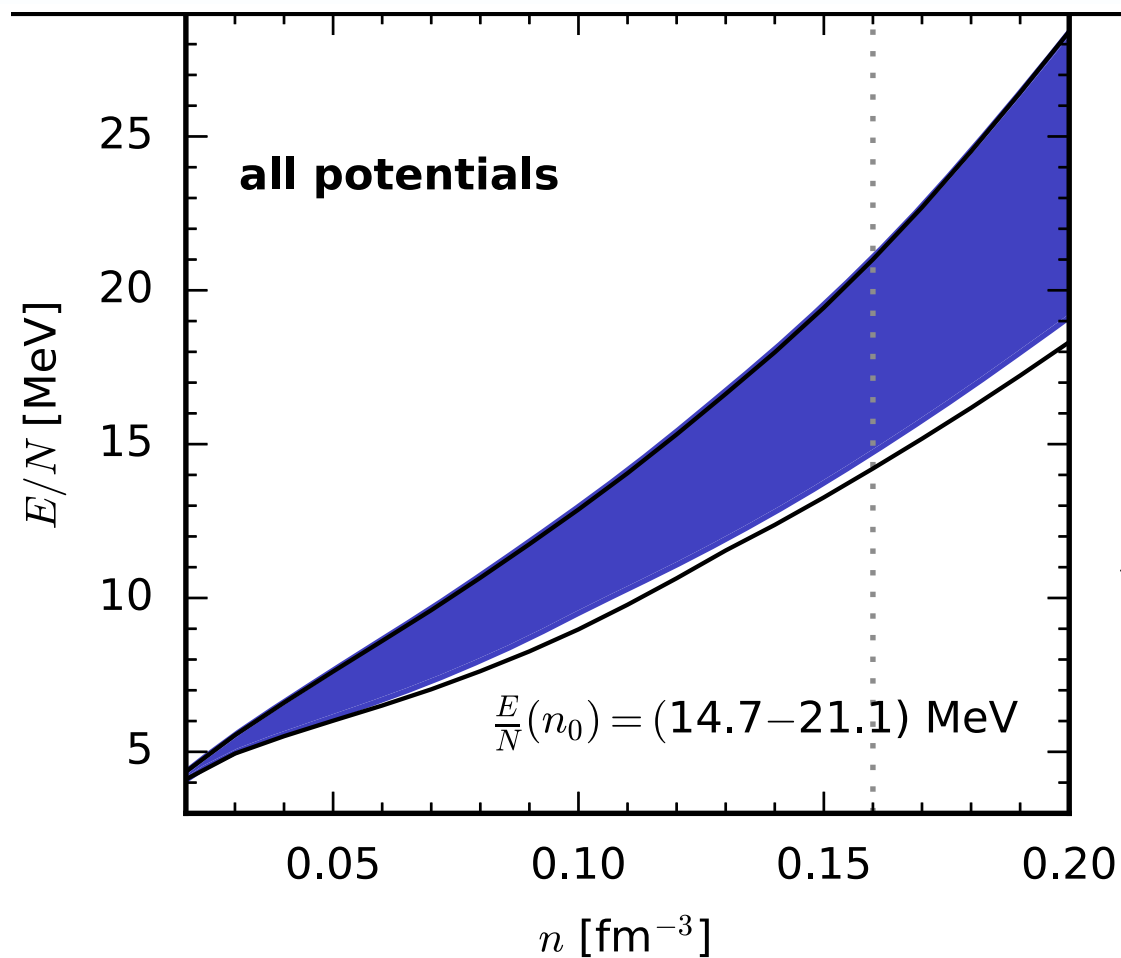
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Similar saturating points  
but different symmetry  
energy predictions



# Constraining stellar equations of state from ab initio results

Drischler, Carbone, Hebeler, Schwenk PRC94, 054307 (2016)

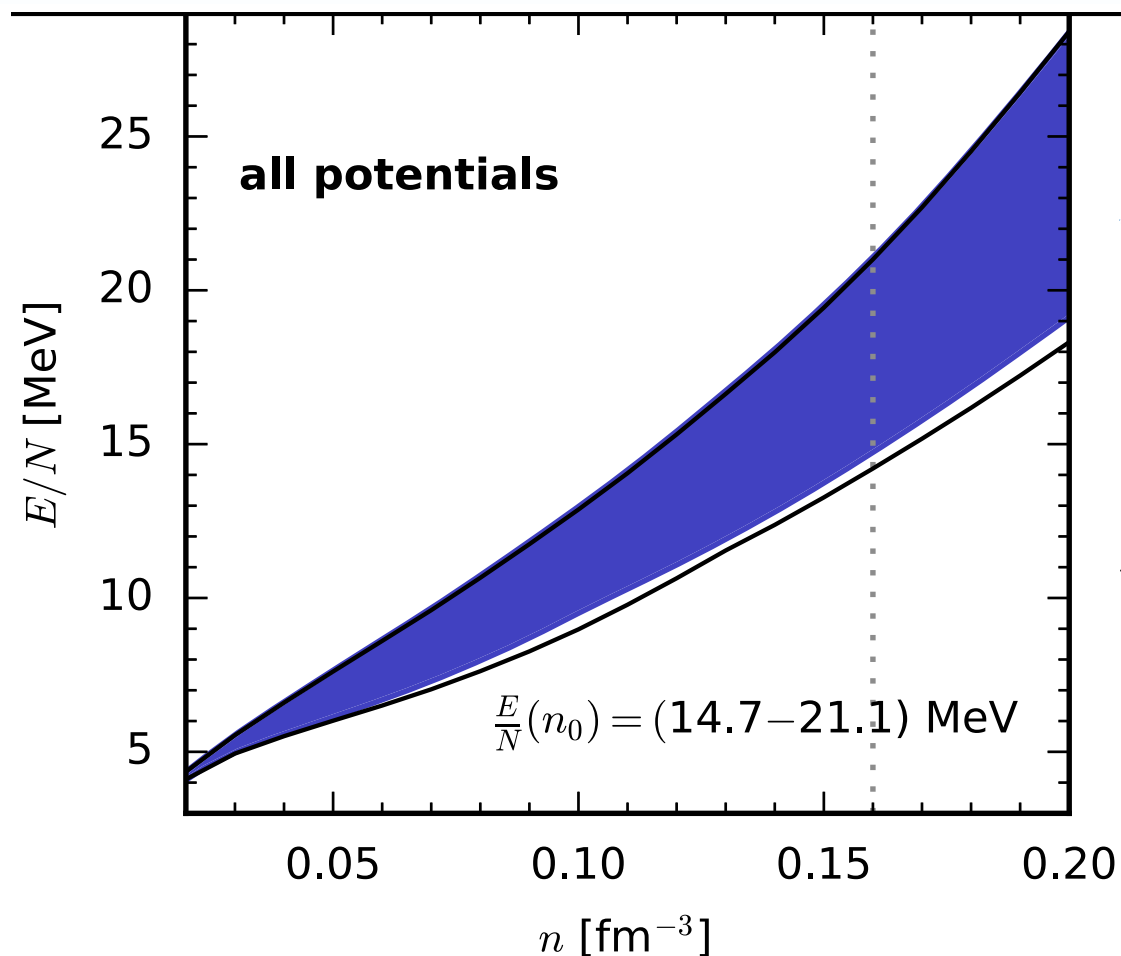


1.

Pure neutron matter  
with chiral forces

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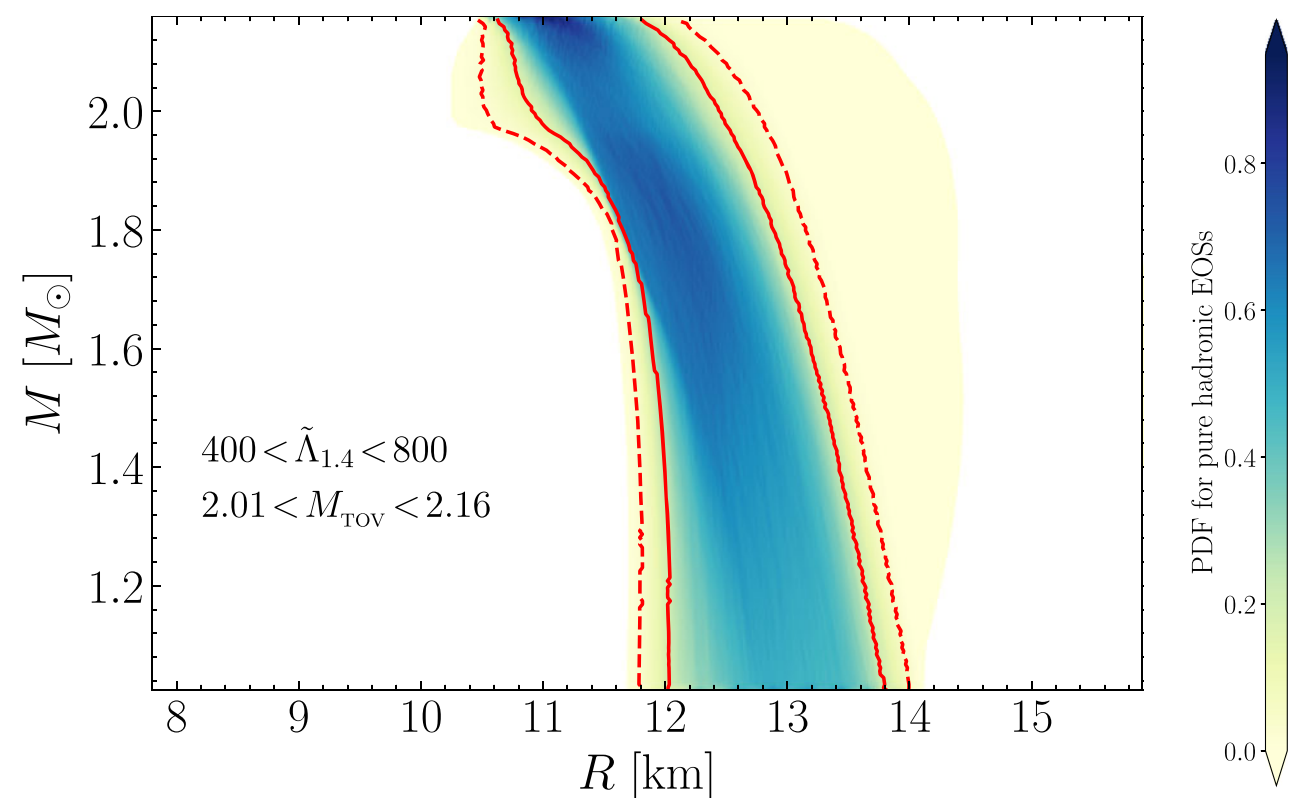
1.

Pure neutron matter  
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2.

**Ab initio uncertainty around  
saturation pins down  
probability distribution of  
neutron star radius**

$$12.00 < R_{1.4} < 13.45$$



Most, Weih, Rezzolla & Schaffner-Bielich, PRL120, 261103 (2018)



# Free energy and pressure at varying temperature

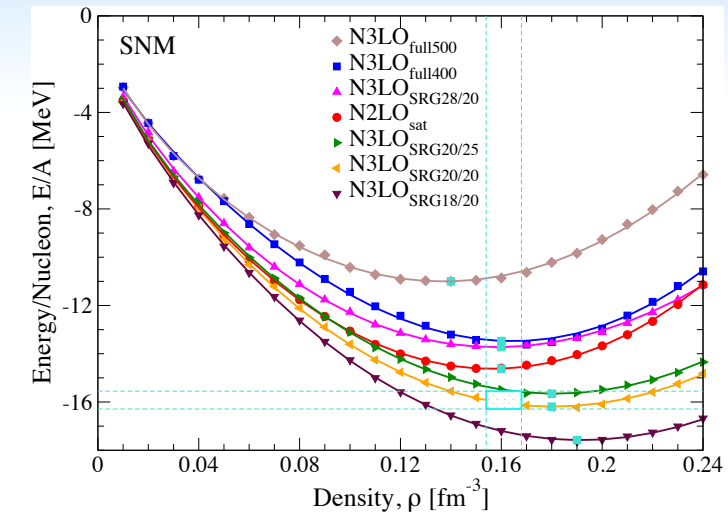
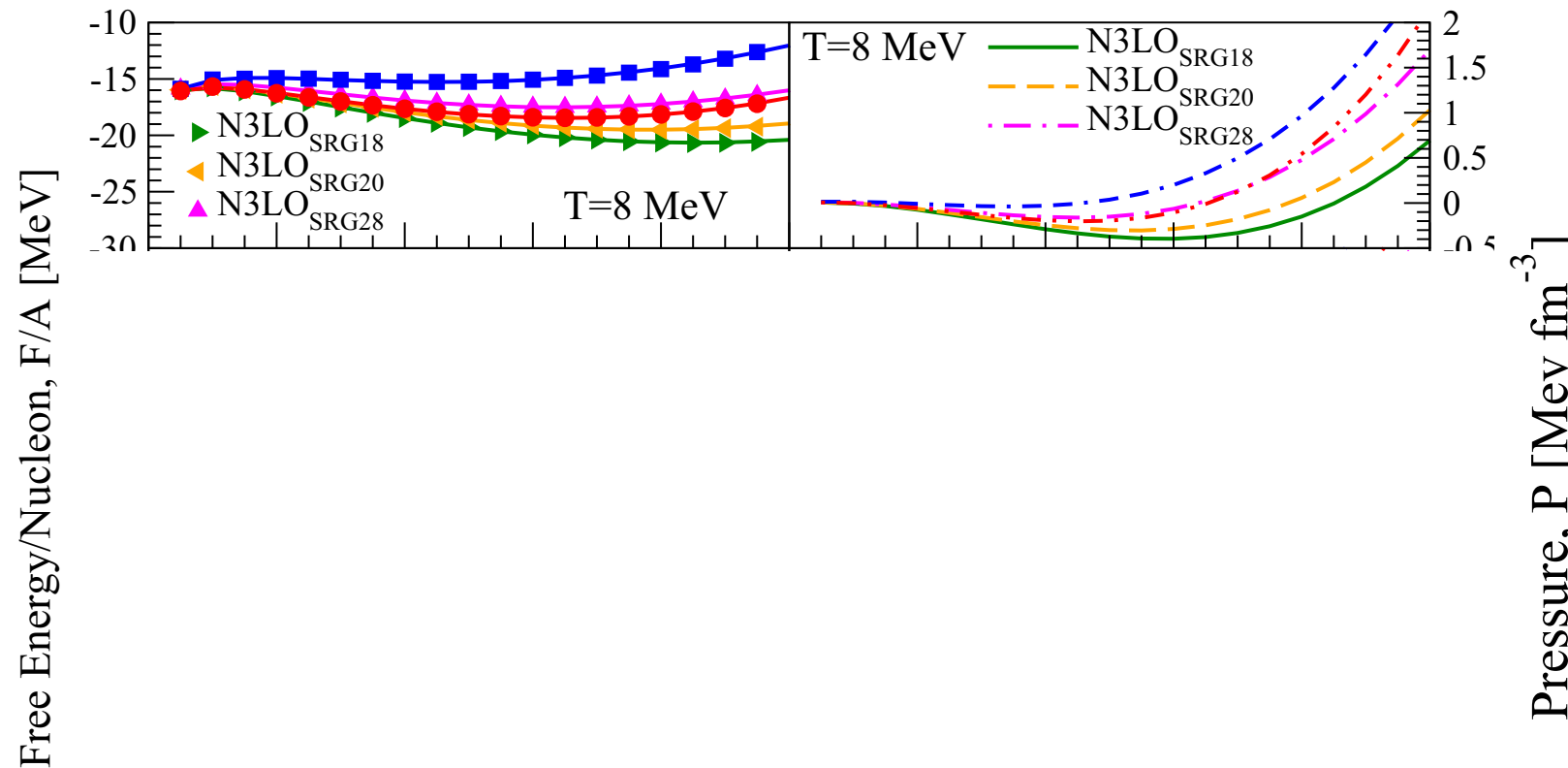
Free-energy  

$$F = E - TS$$

Pressure  

$$P = \rho(\mu - F)$$

increasing temperature



- similar behaviour to zero T energy
- Concave pressure behavior: liquid-gas phase transition

2N N3LO EM500 (SRG  $L=1.8\text{fm}^{-1}$ ) + 3N N2LO ( $L=2.0\text{fm}^{-1}$ )  
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Carbone, Polls, Rios PRC 98 025804 (2018)





# Free energy and pressure at varying temperature

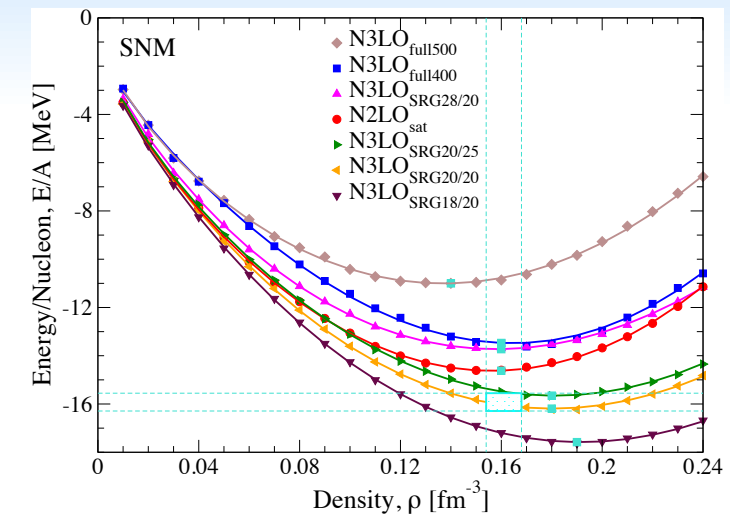
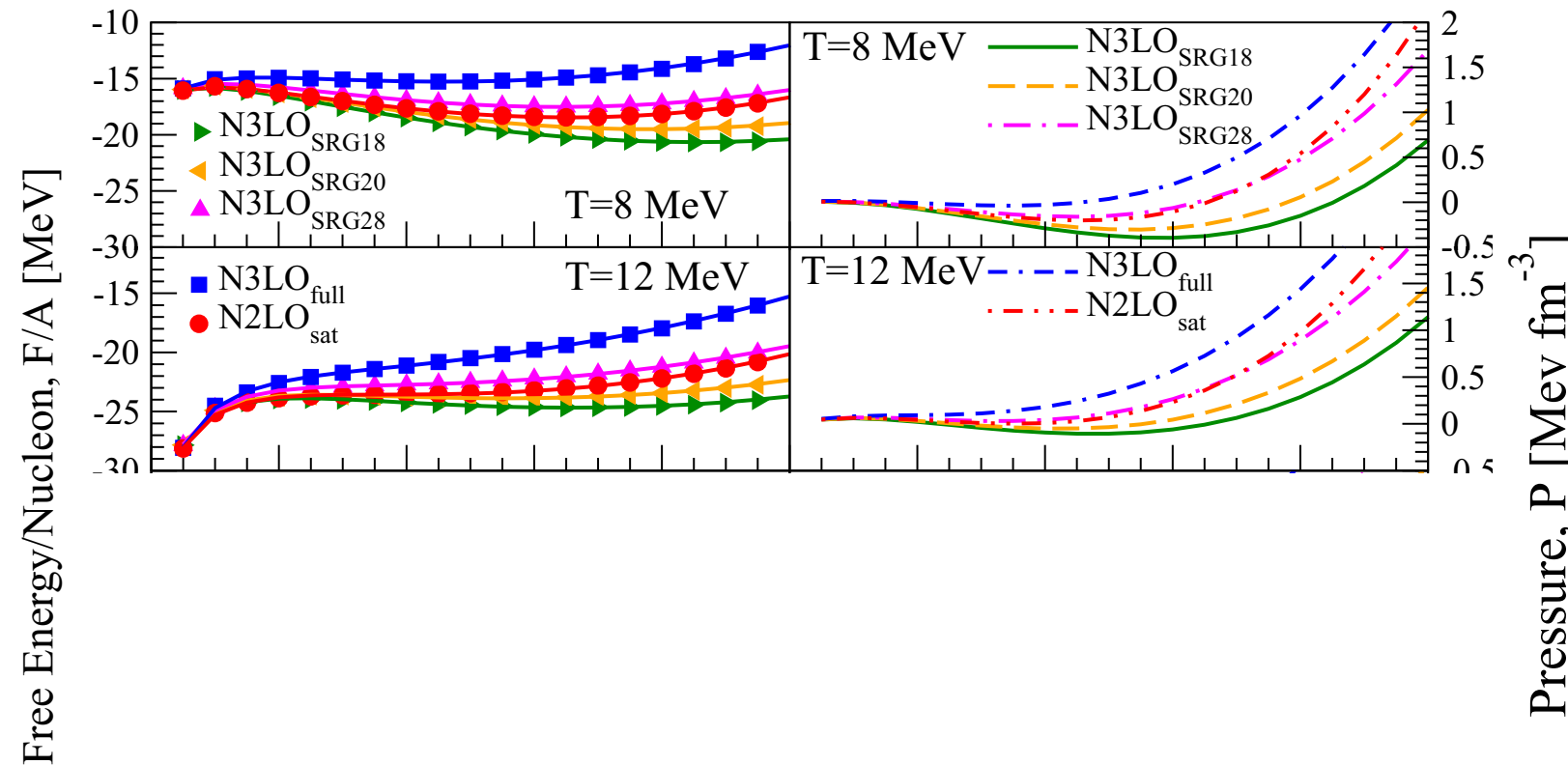
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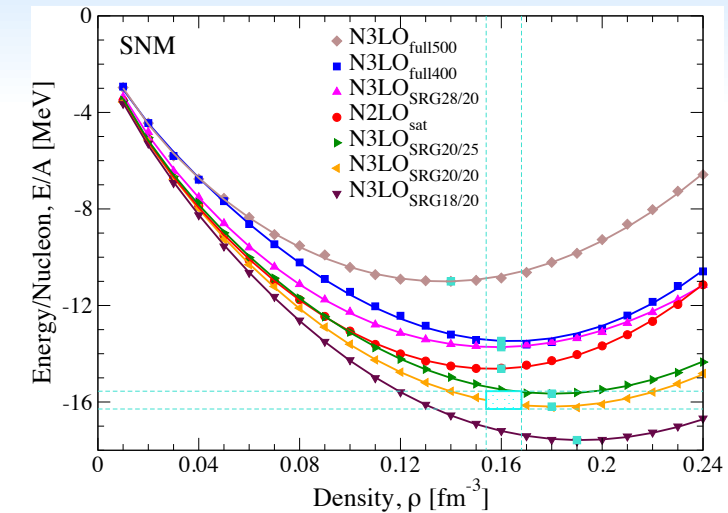
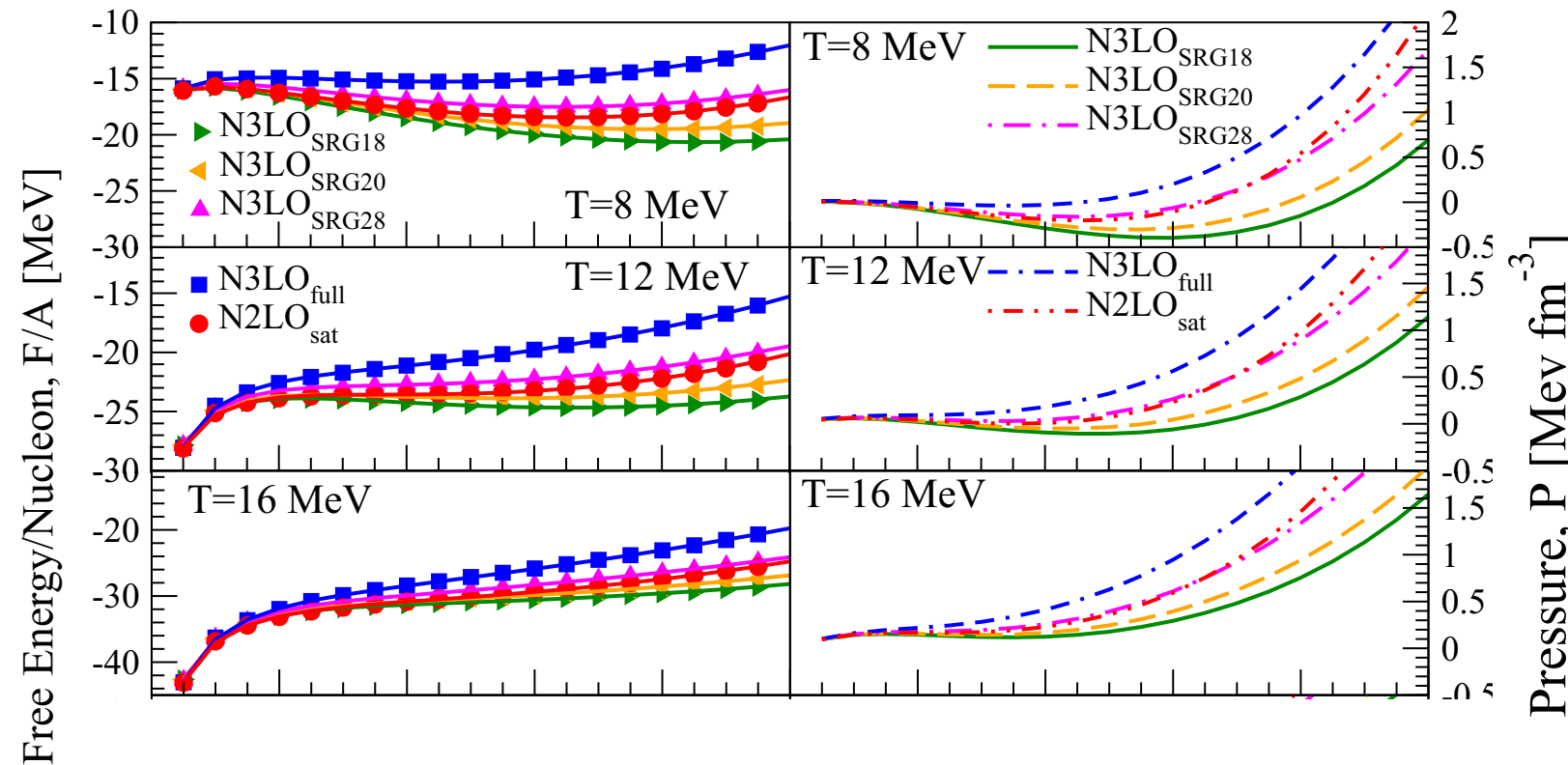


# Free energy and pressure at varying temperature

Free-energy  
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Pressure  
 $P = \rho(\mu - F)$

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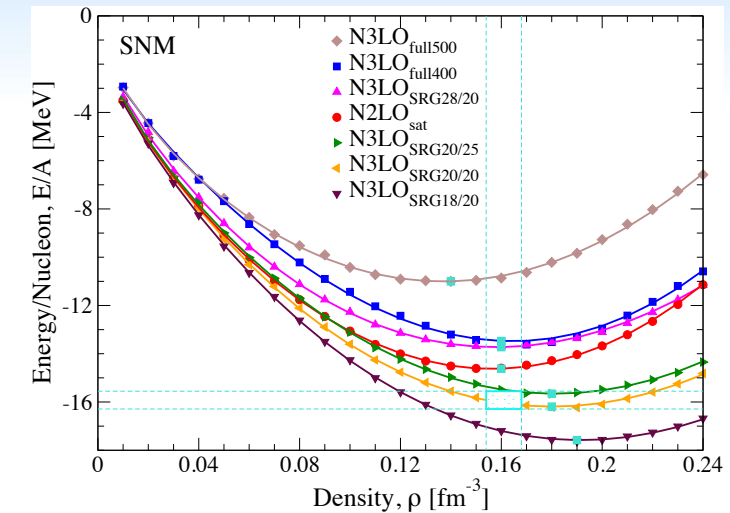
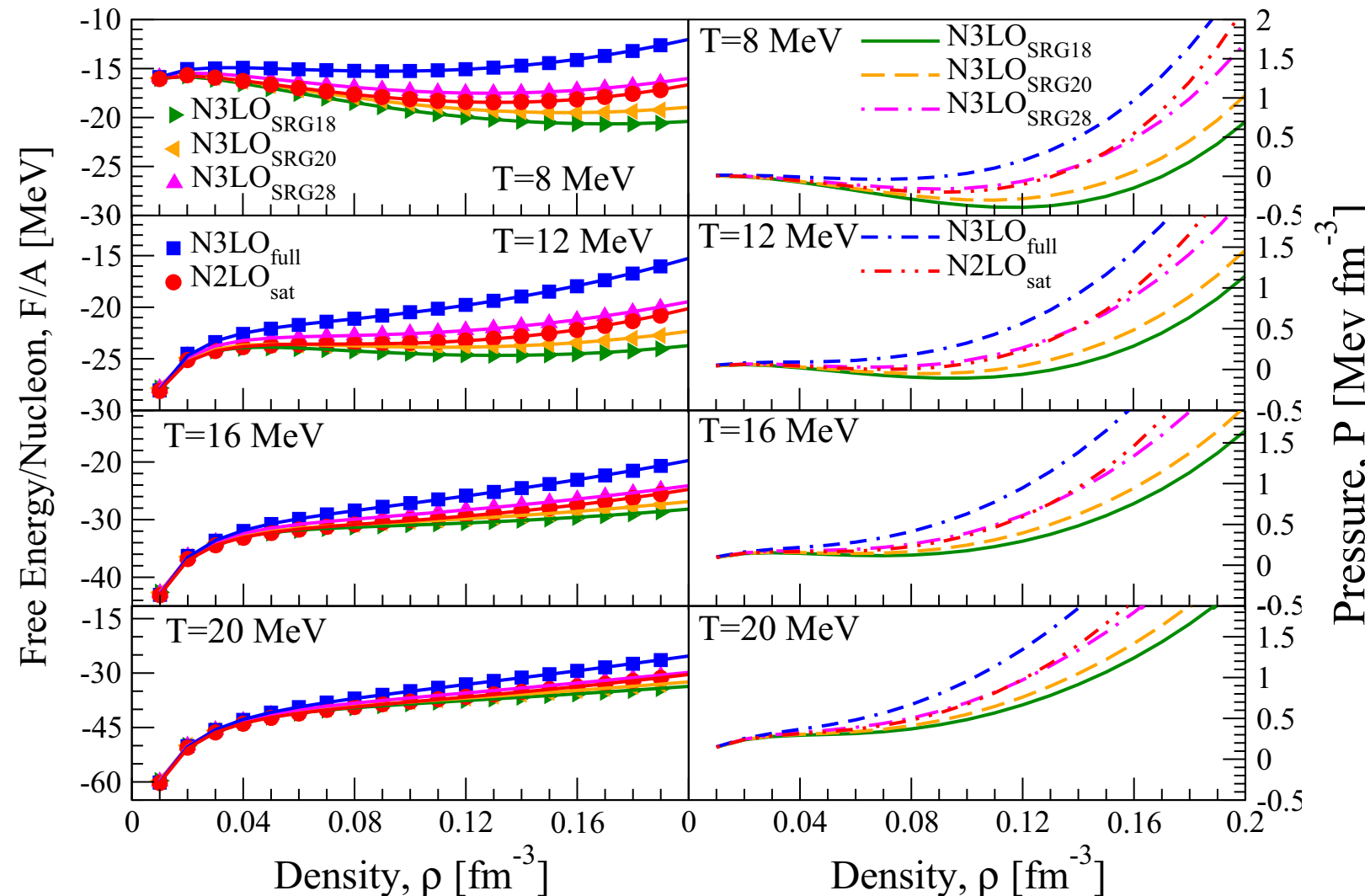
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Carbone, Polls, Rios PRC 98 025804 (2018)



# Many-body uncertainty for finite-T properties

Free-energy

$$F = E - TS$$

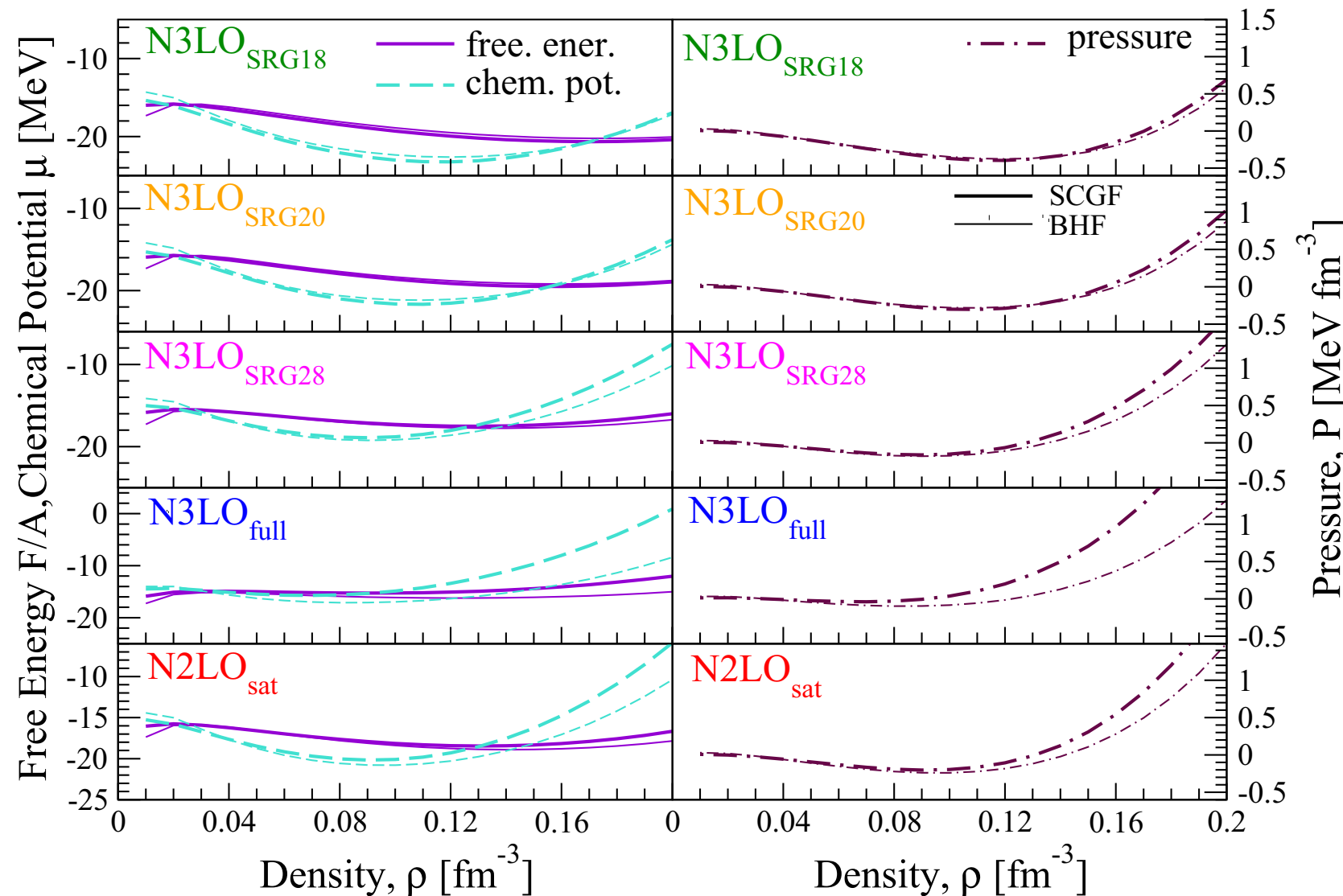
T=8 MeV

Pressure

$$P = \rho(\mu - F)$$

T=8 MeV

evolved potentials to unevolved



2N N3LO EM500 (SRG  $L=1.8\text{fm}^{-1}$ ) + 3N N2LO ( $L=2.0\text{fm}^{-1}$ )

2N N3LO EM500 (SRG  $L=2.0\text{fm}^{-1}$ ) + 3N N2LO ( $L=2.0\text{fm}^{-1}$ )

2N N3LO EM500 (SRG  $L=2.8\text{fm}^{-1}$ ) + 3N N2LO ( $L=2.0\text{fm}^{-1}$ )

N2LOsat 2N + 3N

2N N3LO EM500 + 3N N2LO

Carbone, Polls, Rios PRC 98 025804 (2018)



# Many-body uncertainty for finite-T properties

Free-energy

$$F = E - TS$$

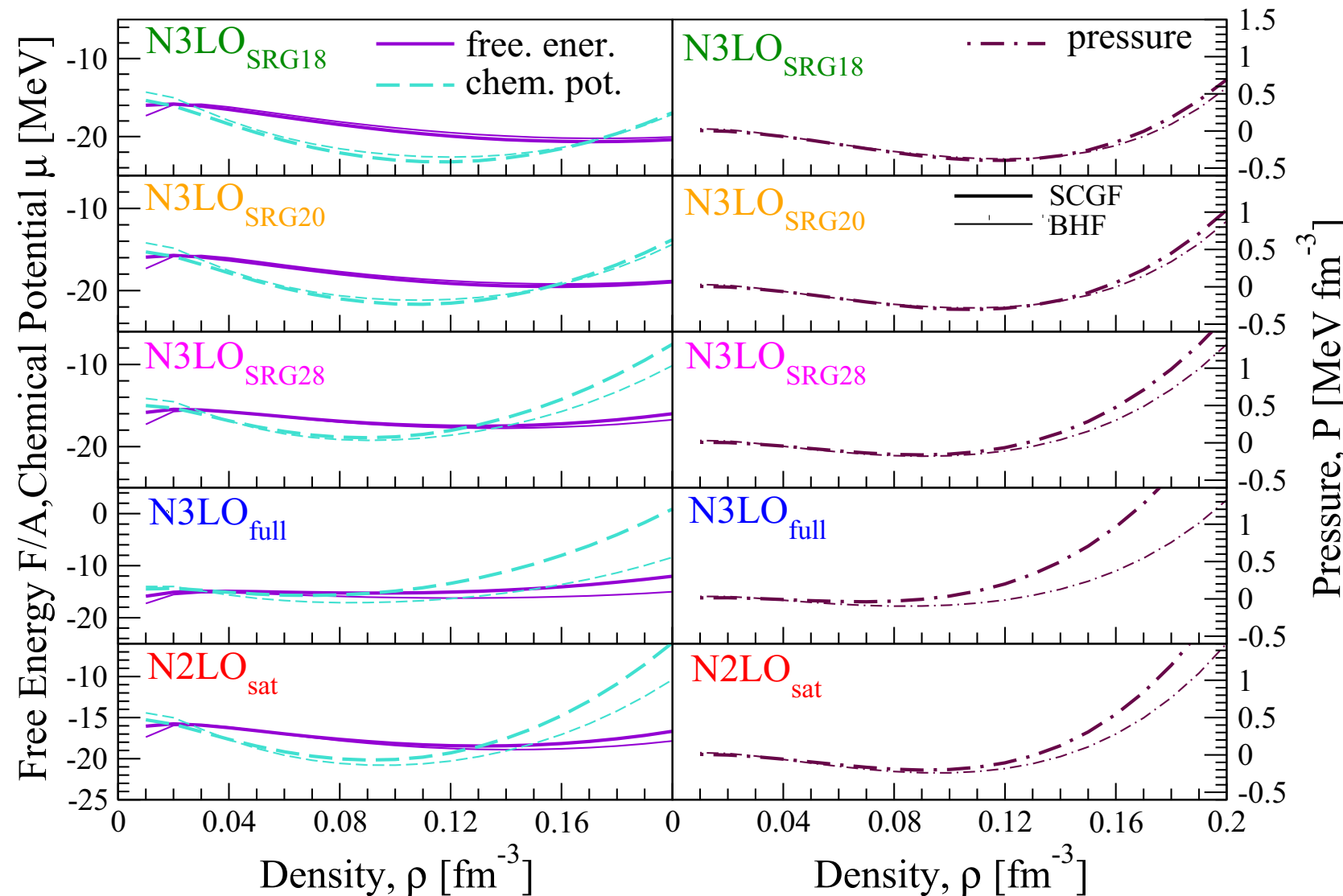
T=8 MeV

Pressure

$$P = \rho(\mu - F)$$

T=8 MeV

evolved potentials to unevolved



- Free-energy and chemical potential cross at P=0

2N N3LO EM500 (SRG L=1.8fm<sup>-1</sup>) + 3N N2LO (L=2.0fm<sup>-1</sup>)

2N N3LO EM500 (SRG L=2.0fm<sup>-1</sup>) + 3N N2LO (L=2.0m<sup>-1</sup>)

2N N3LO EM500 (SRG L=2.8fm<sup>-1</sup>) + 3N N2LO (L=2.0fm<sup>-1</sup>)

N2LOsat 2N + 3N

2N N3LO EM500 + 3N N2LO

Carbone, Polls, Rios PRC 98 025804 (2018)





# Many-body uncertainty for finite-T properties

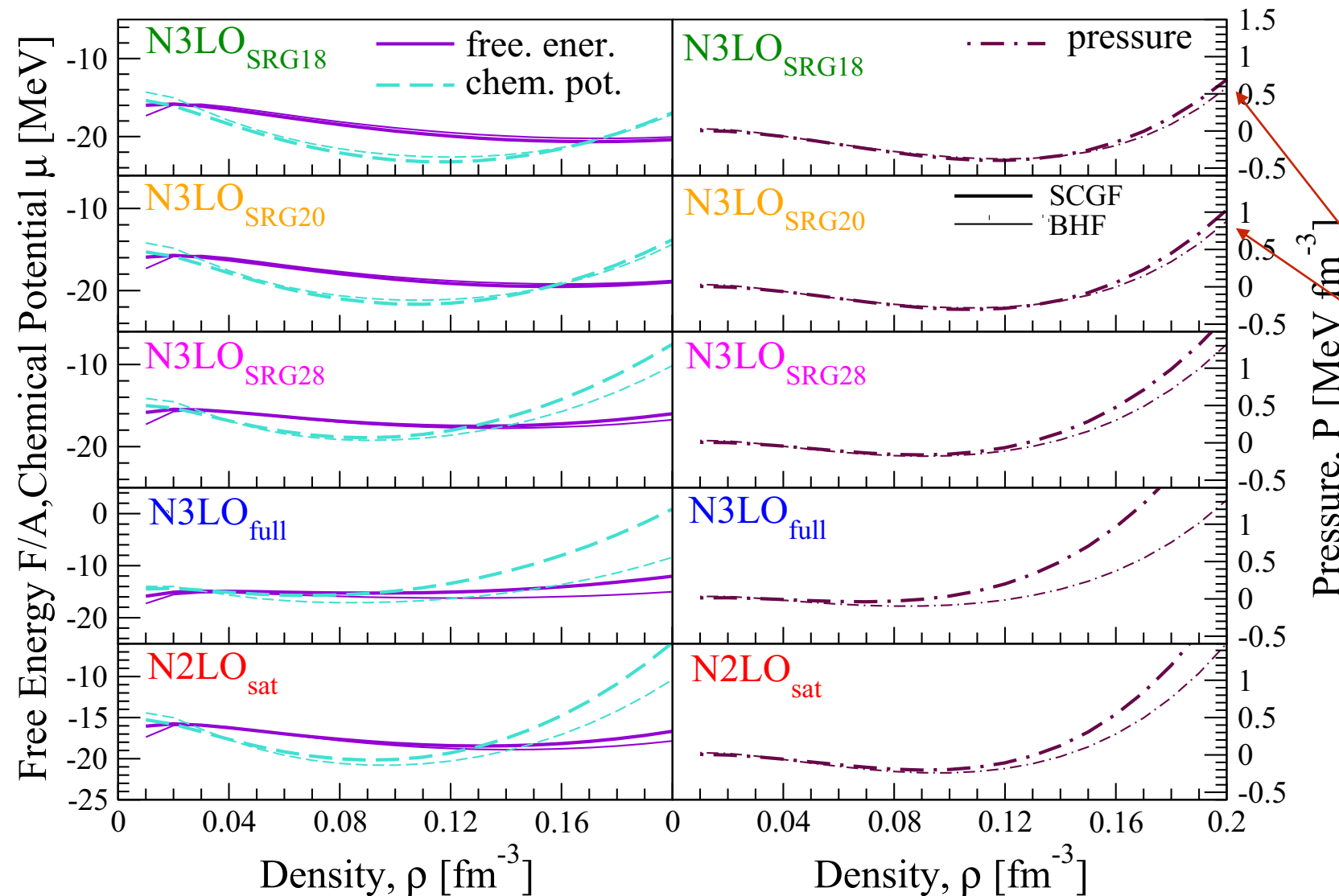
Free-energy  
 $F = E - TS$

T=8 MeV

Pressure  
 $P = \rho(\mu - F)$

T=8 MeV

evolved potentials to unevolved



- Free-energy and chemical potential cross at  $P=0$
- smaller many-body error band for evolved potentials: hole-hole states less important

2N N3LO EM500 (SRG  $L=1.8\text{fm}^{-1}$ ) + 3N N2LO ( $L=2.0\text{fm}^{-1}$ )

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N2LOsat 2N + 3N

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Carbone, Polls, Rios PRC 98 025804 (2018)



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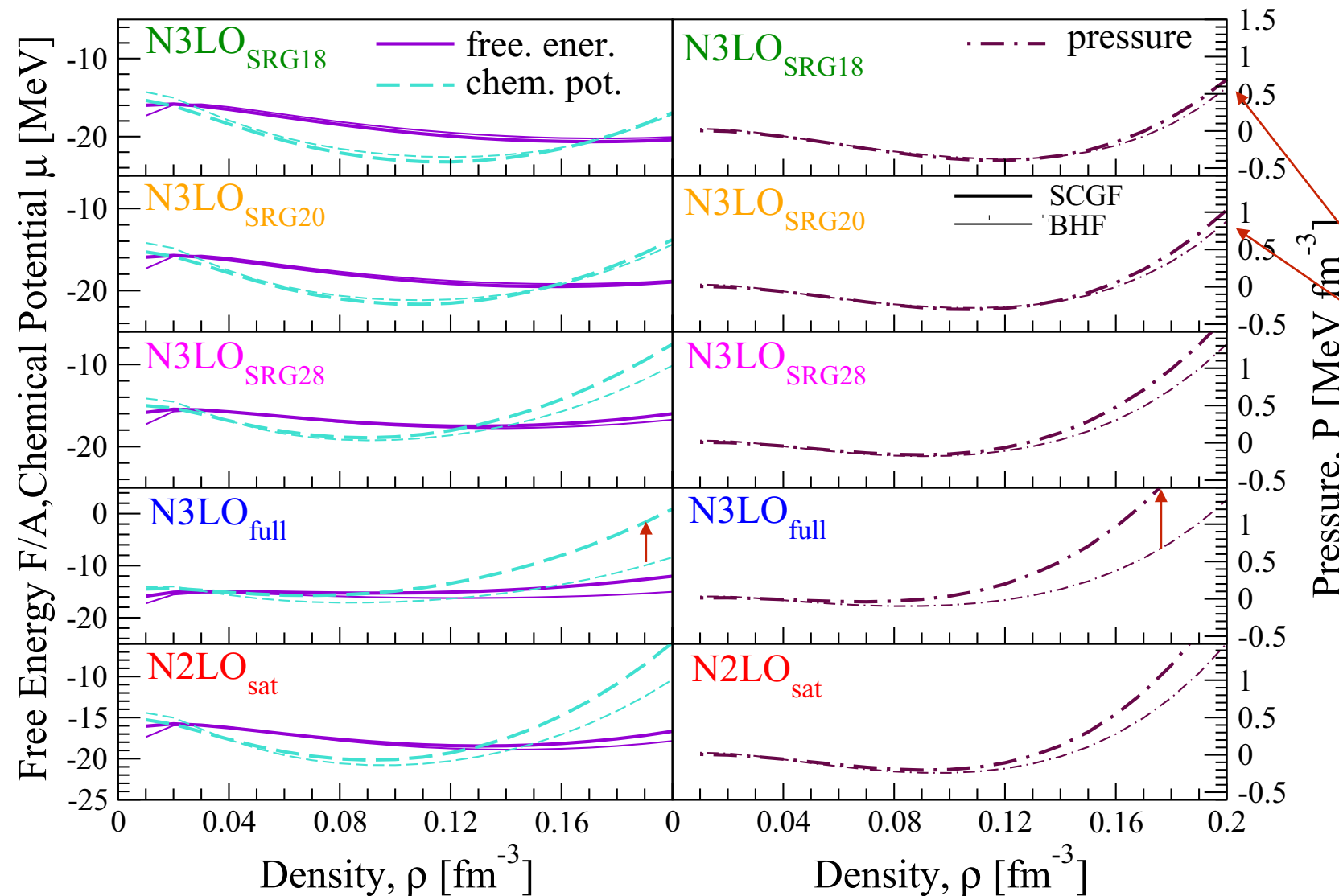
T=8 MeV

Pressure

$$P = \rho(\mu - F)$$

T=8 MeV

evolved potentials to unevolved



- Free-energy and chemical potential cross at  $P=0$
- smaller many-body error band for evolved potentials: hole-hole states less important
- uncertainty band increases with density: hole-hole states add repulsion

2N N3LO EM500 (SRG  $L=1.8\text{fm}^{-1}$ ) + 3N N2LO ( $L=2.0\text{fm}^{-1}$ )

2N N3LO EM500 (SRG  $L=2.0\text{fm}^{-1}$ ) + 3N N2LO ( $L=2.0\text{fm}^{-1}$ )

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N2LOsat 2N + 3N

2N N3LO EM500 + 3N N2LO

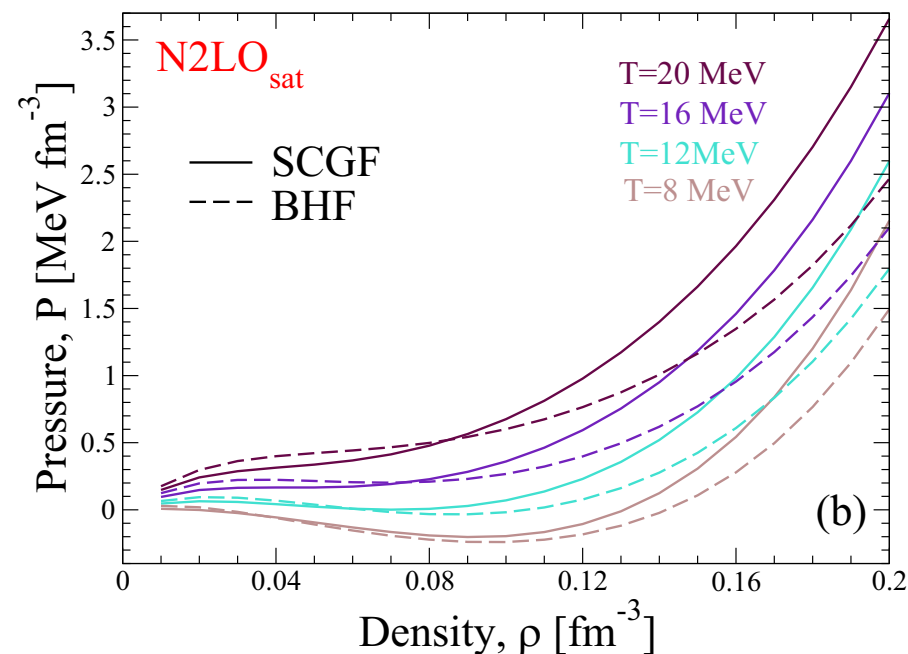
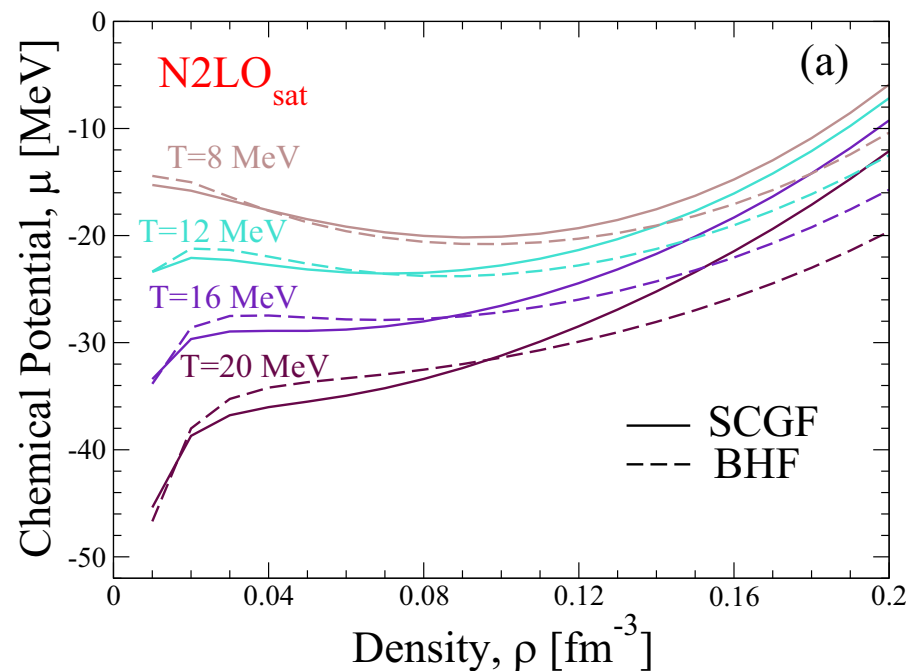
Carbone, Polls, Rios PRC 98 025804 (2018)



# The liquid-gas phase transition and critical point

Carbone, Polls, Rios PRC 98 025804 (2018)

N2LOsat (2N+3N)



Spinodal

$$\frac{\partial \mu}{\partial \rho_g} = \frac{\partial \mu}{\partial \rho_l} = 0$$

$$\frac{\partial P}{\partial \rho_g} = \frac{\partial P}{\partial \rho_l} = 0$$

Coexistence

$$\mu(\rho_g) = \mu(\rho_l)$$

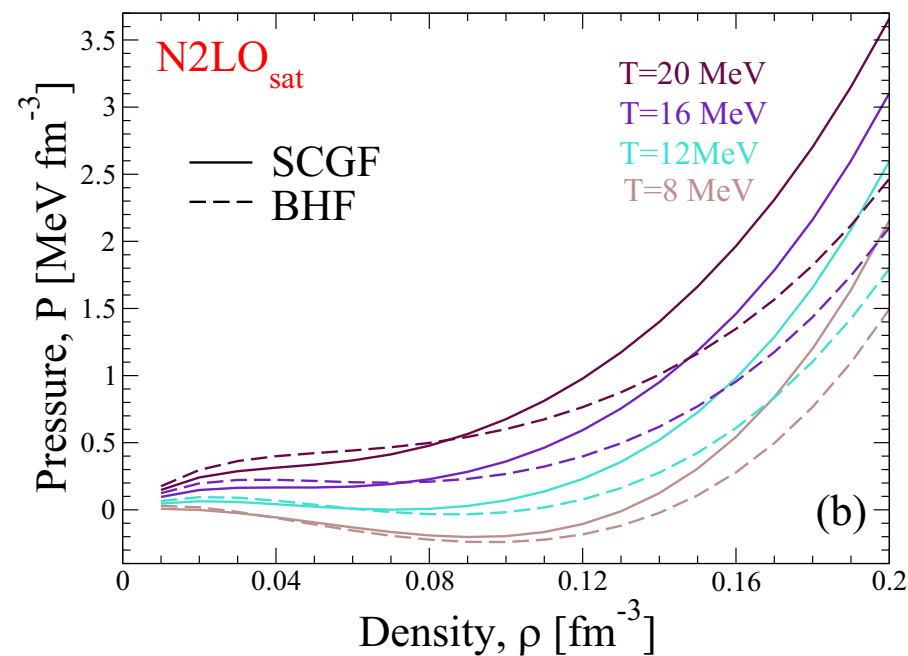
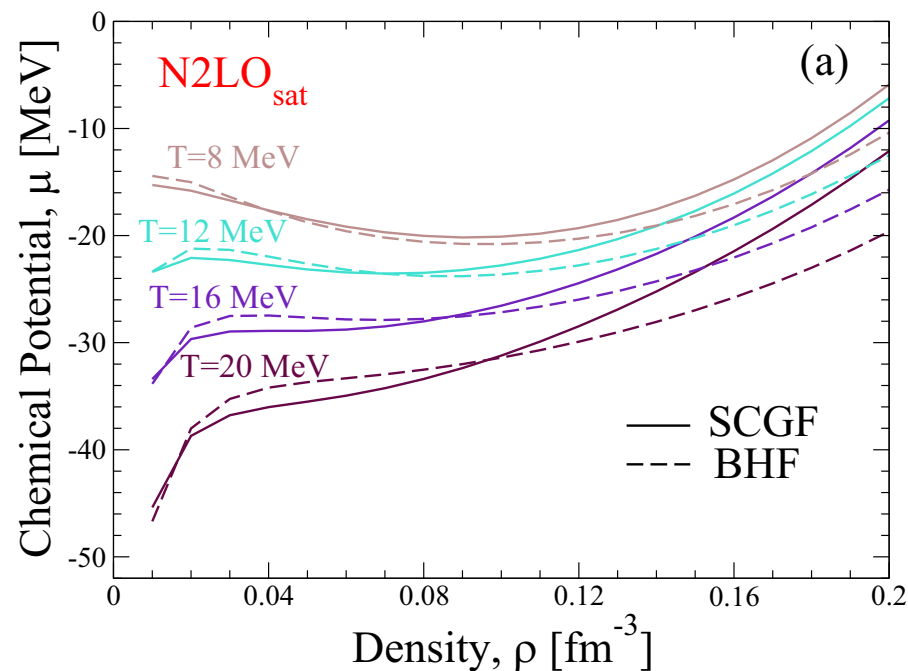
$$P(\rho_g) = P(\rho_l)$$



# The liquid-gas phase transition and critical point

Carbone, Polls, Rios PRC 98 025804 (2018)

N2LOsat (2N+3N)



Spinodal

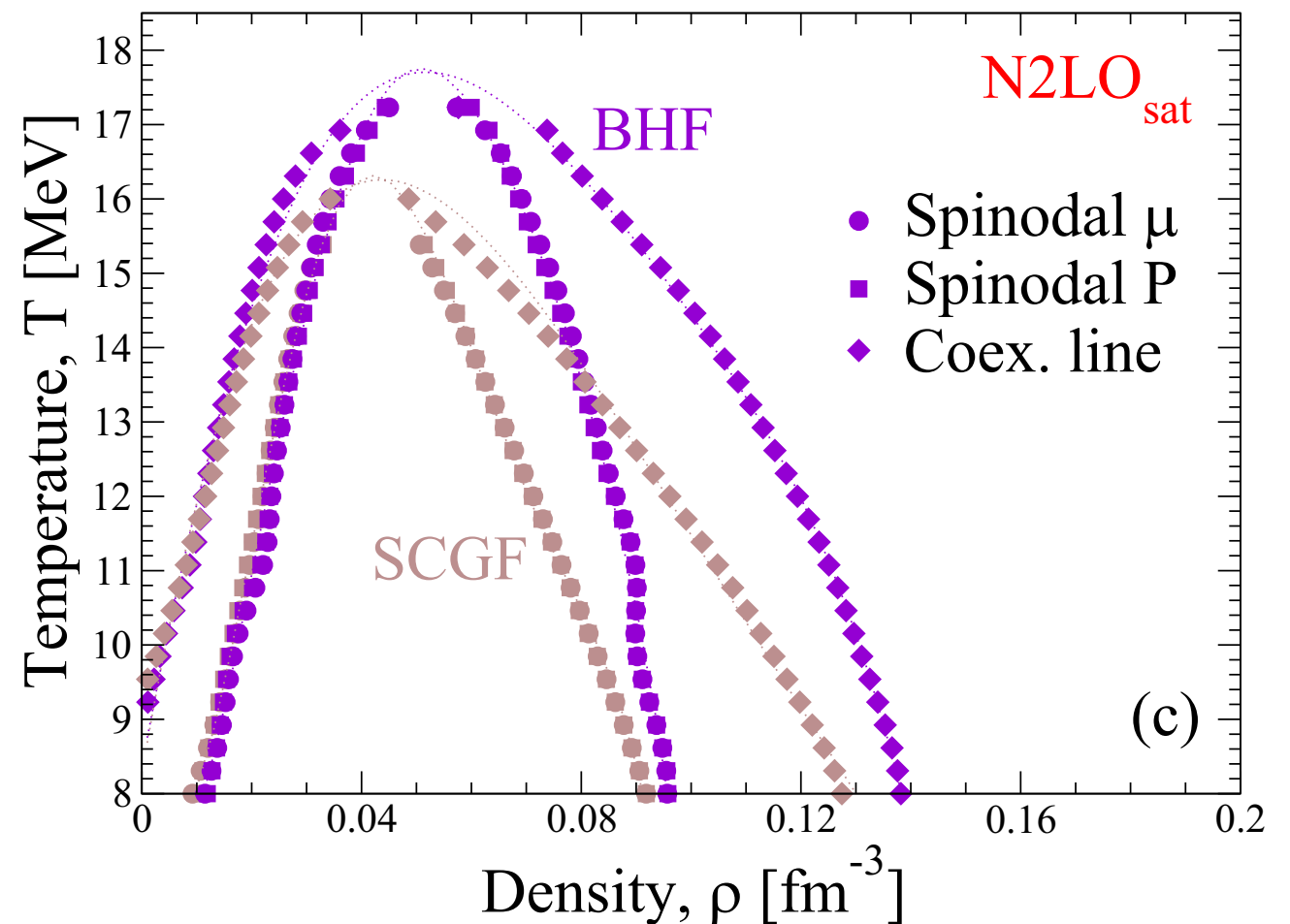
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Coexistence

$$\mu(\rho_g) = \mu(\rho_l)$$

$$P(\rho_g) = P(\rho_l)$$



- Many-body difference increase with density
- Higher critical temperature for BHF results

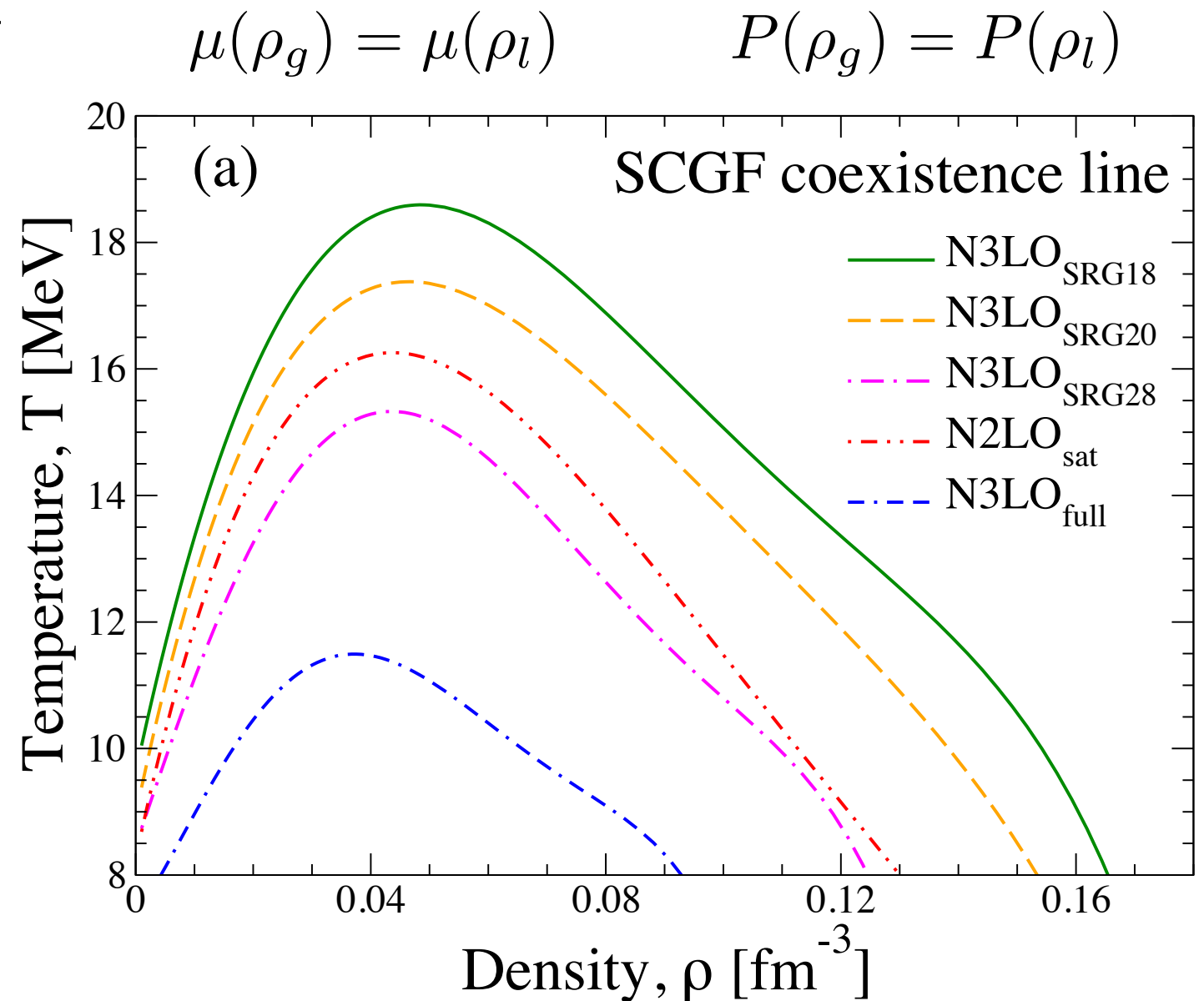


# The liquid-gas phase transition and critical point

Carbone, Polls, Rios PRC 98 025804 (2018)

SCGF	$\rho_c$ (fm <sup>-3</sup> )	$T_c$ (MeV)
N3LO <sub>SRG18</sub>	0.048	18.6
N3LO <sub>SRG20</sub>	0.047	17.4
N3LO <sub>SRG28</sub>	0.043	15.3
N2LO <sub>sat</sub>	0.043	16.3
N3LO <sub>full</sub>	0.038	11.5

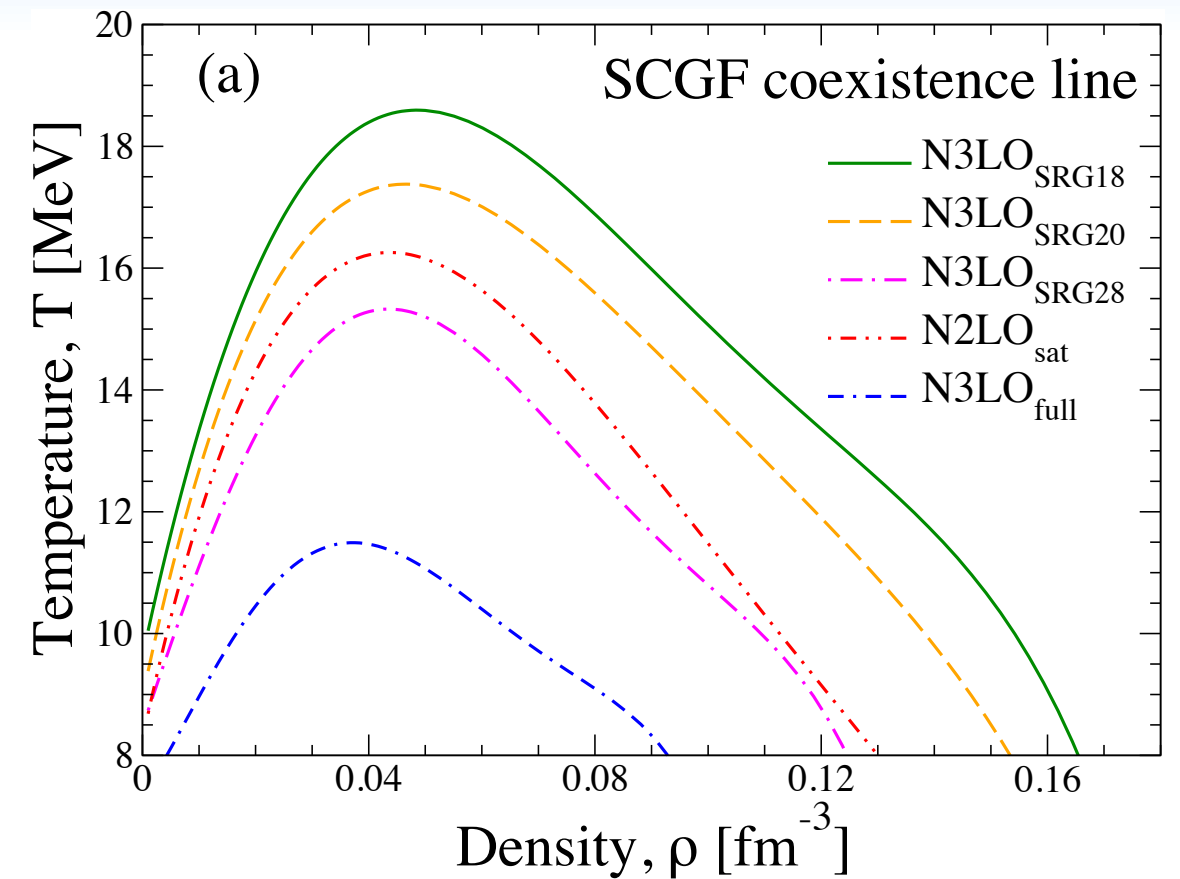
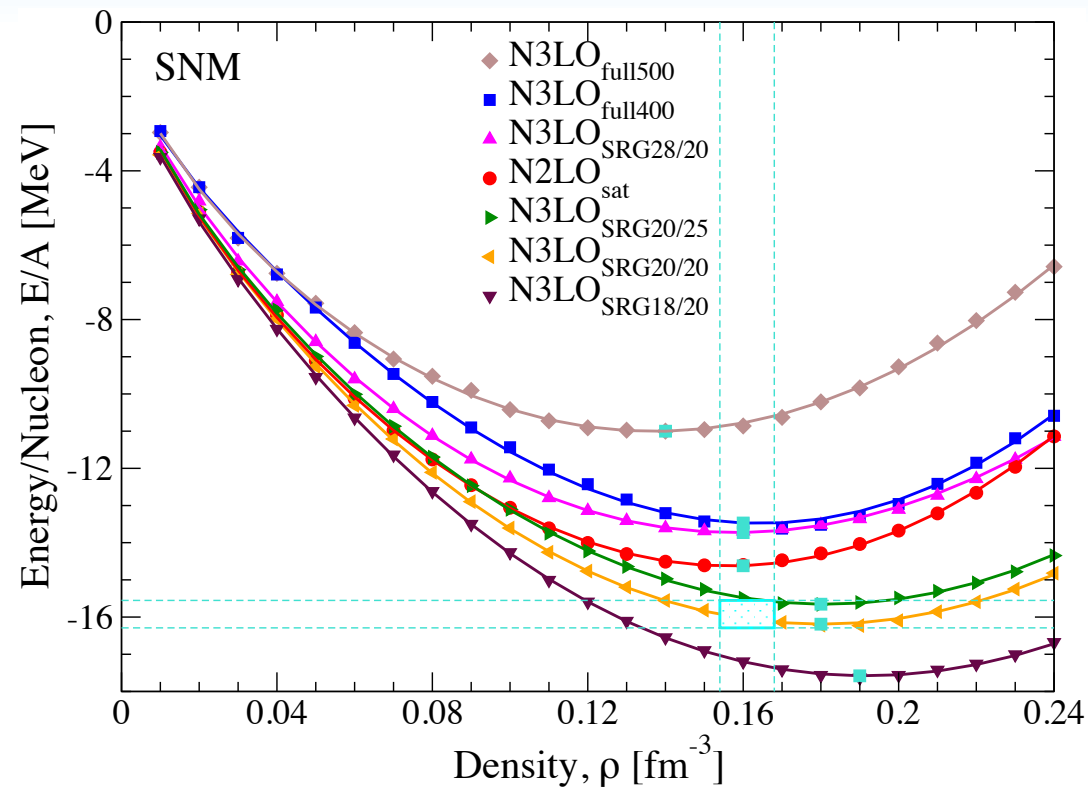
- Predicted critical temperature ~  
 $T = \sim [15-19]$  MeV
- Experimental outcomes:  
 $T = \sim [15-20]$  MeV)





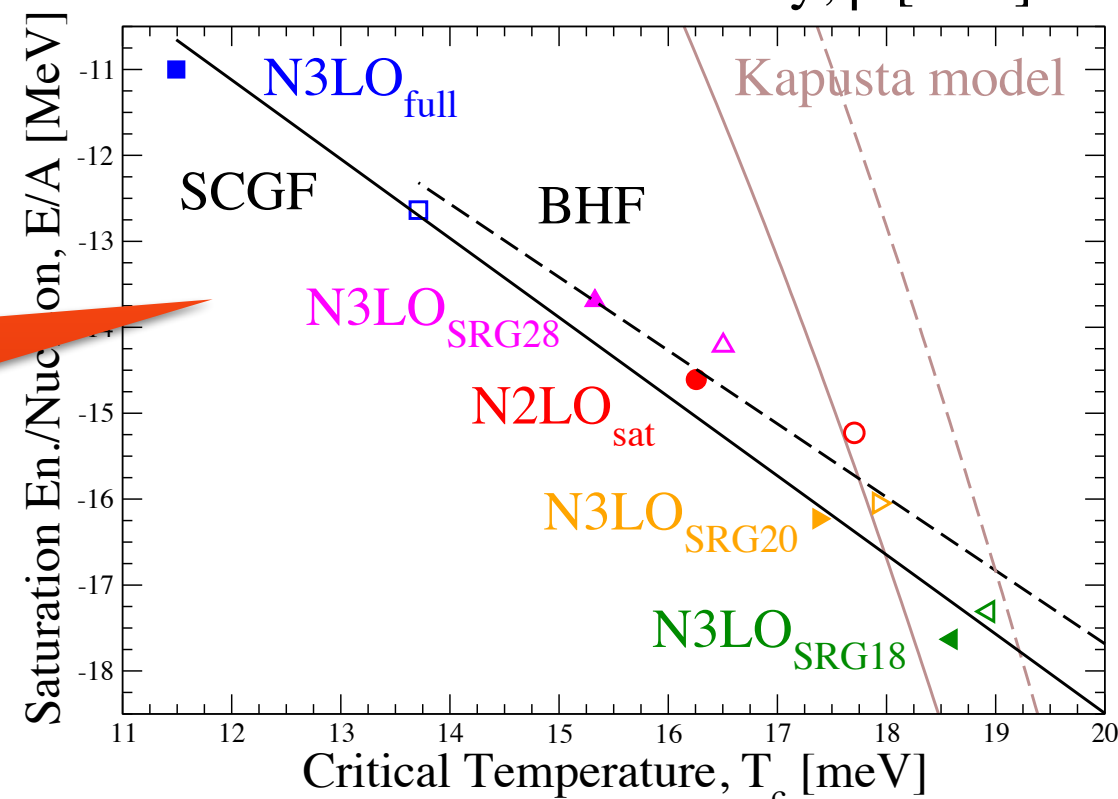
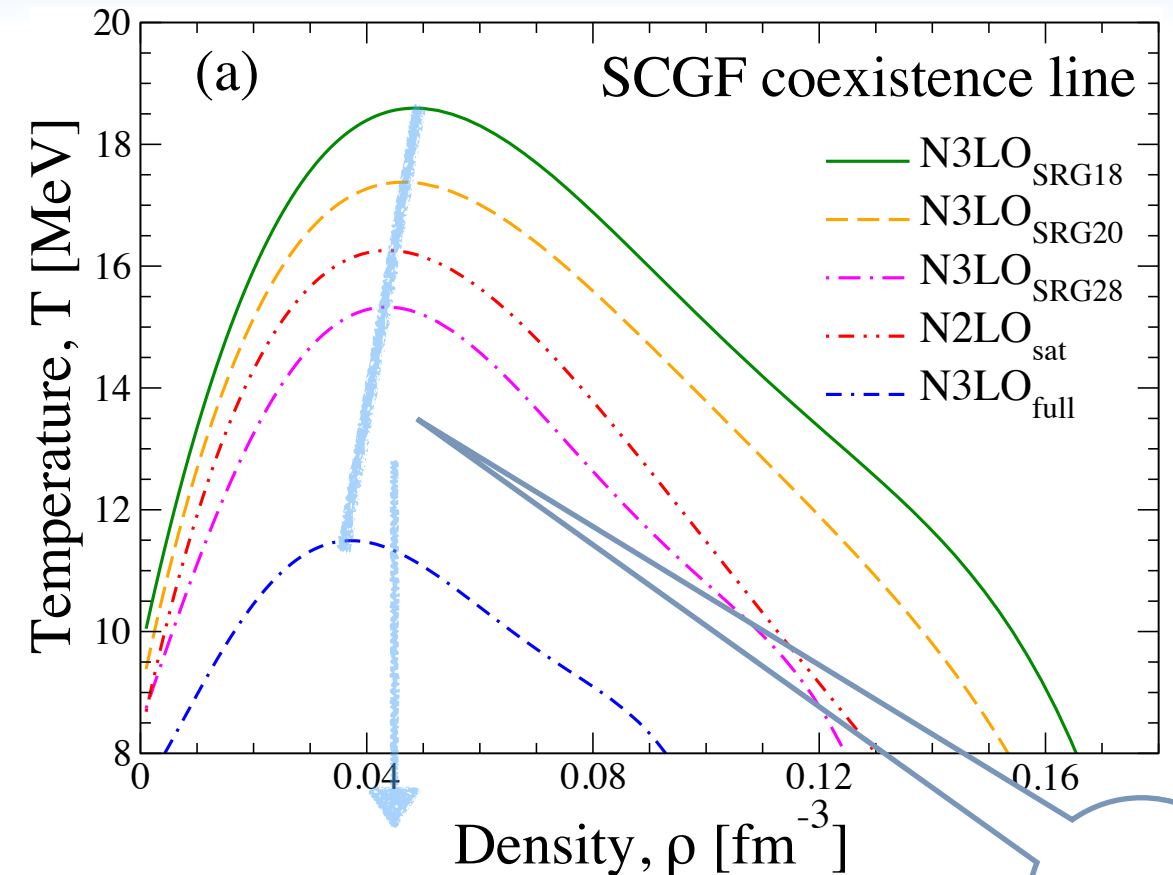
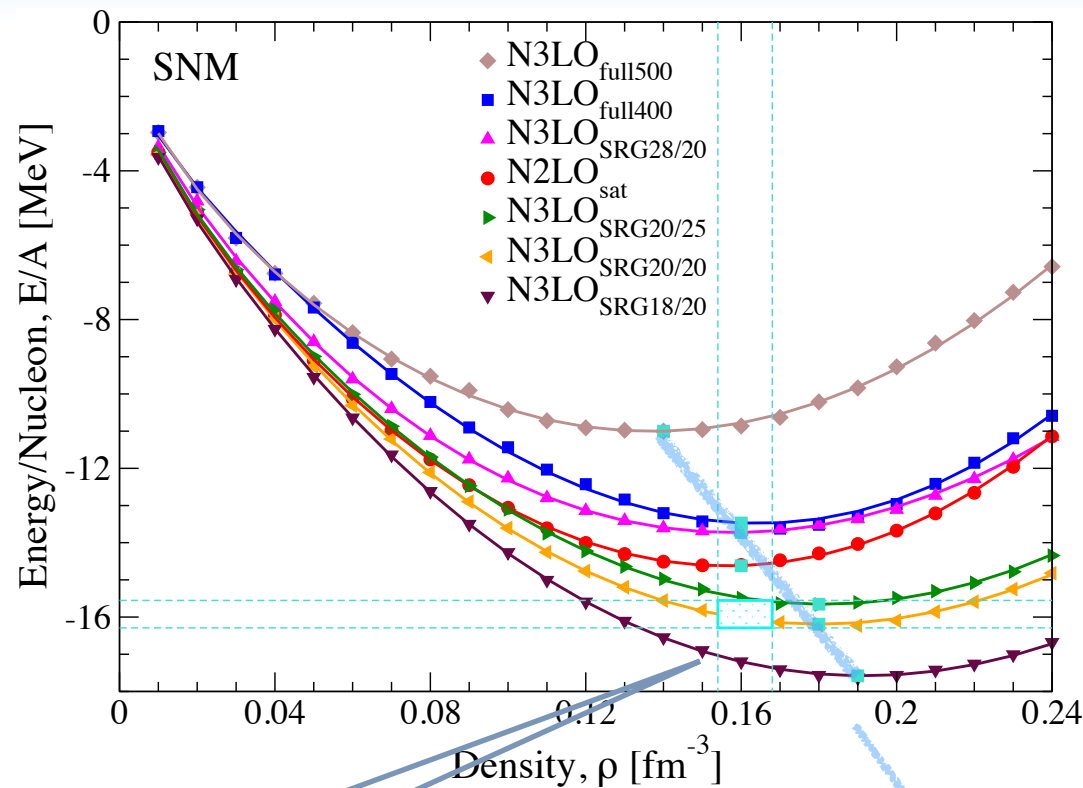
# The saturation energy vs the critical temperature

Carbone, Polls, Rios PRC 98 025804 (2018)



# The saturation energy vs the critical temperature

Carbone, Polls, Rios PRC 98 025804 (2018)



theoretical uncertainty  
bands correlate:  
helpful in pinning down the critical  
temperature



# Thermal effects in EoS for astrophysical simulations

$$P_{\text{cold}} + P_{\text{thermal}} \longrightarrow P_{\text{th}} = (\Gamma_{\text{th}} - 1)\rho E_{\text{th}}$$

Astrophysical EoS

$$\Gamma_{\text{th}} = 1 + \frac{P_{\text{th}}}{\rho E_{\text{th}}}$$

Constant value



Carbone & Schwenk (*in preparation*)

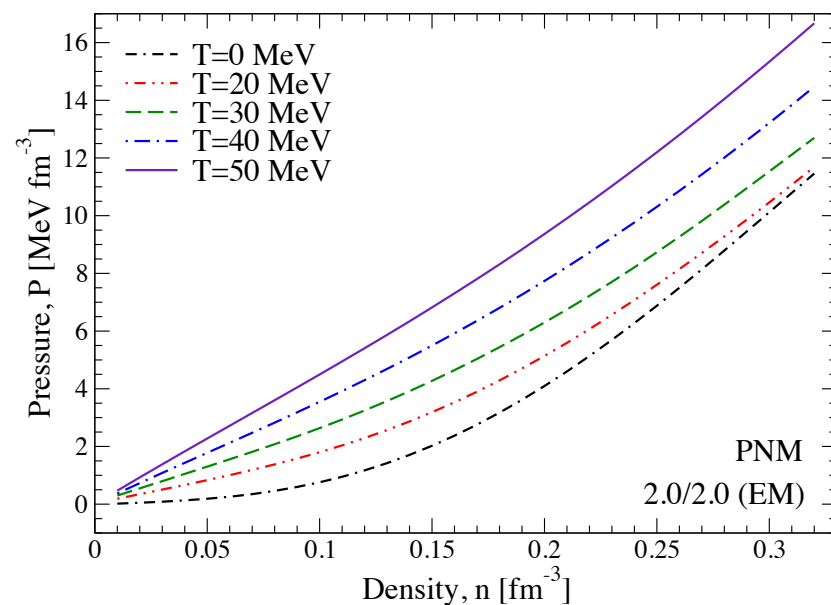
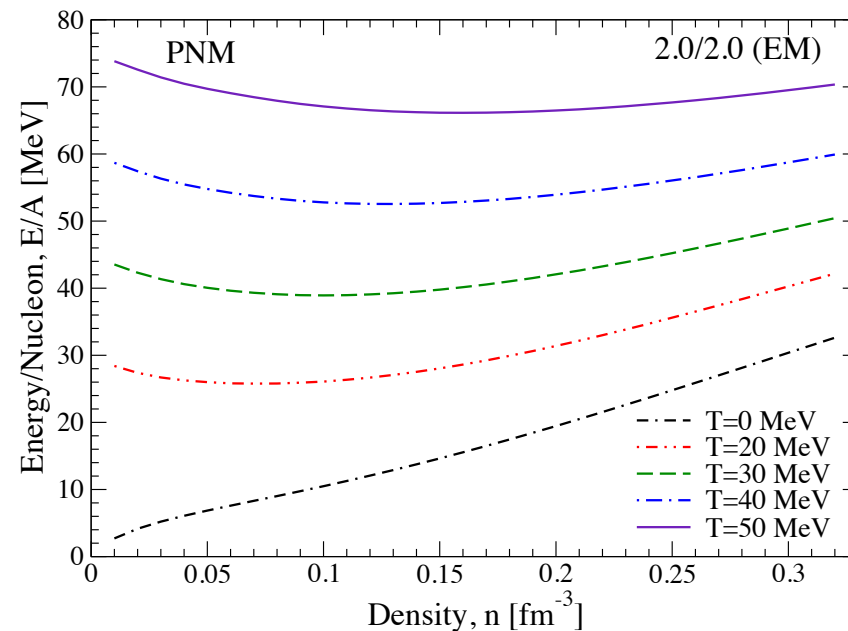
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Carbone & Schwenk (*in preparation*)



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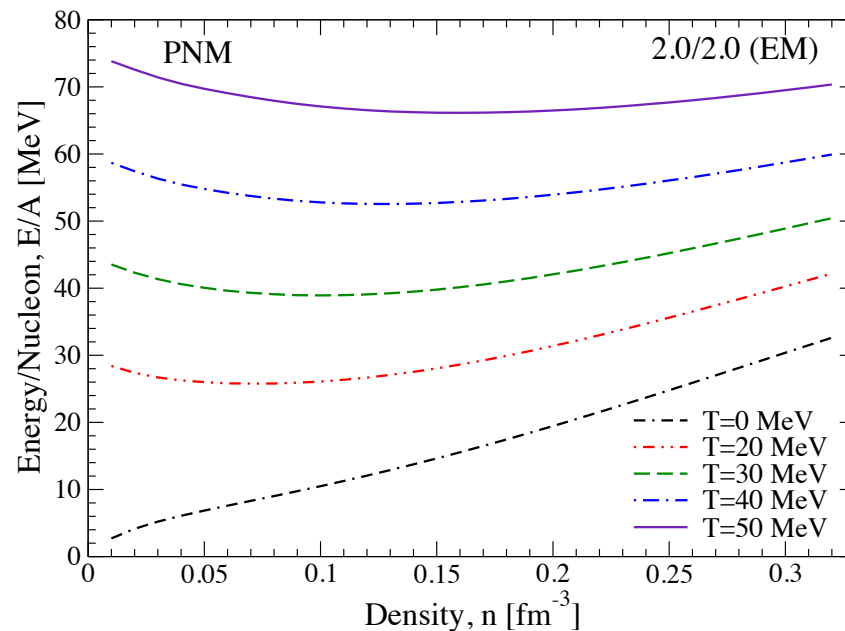
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Astrophysical EoS

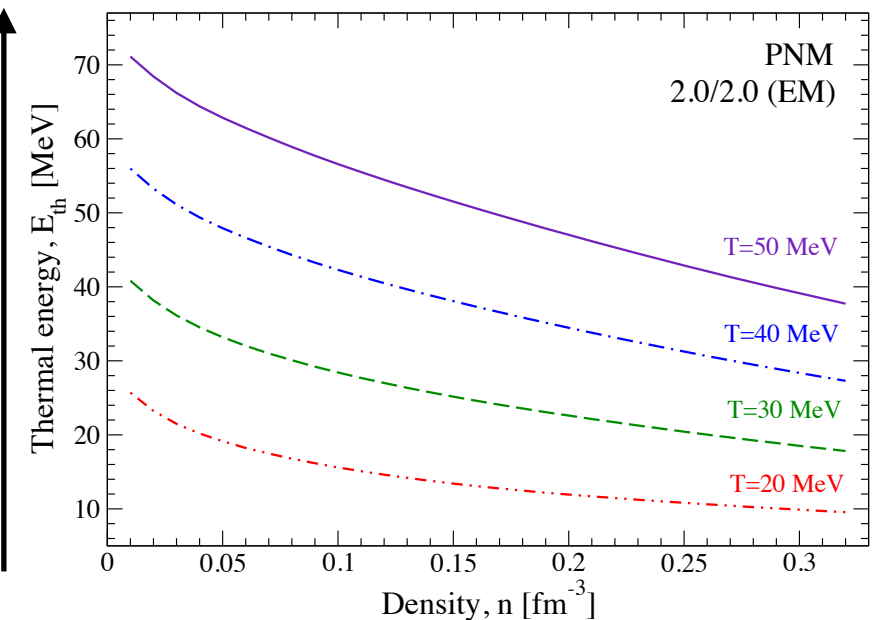
$$\Gamma_{\text{th}} = 1 + \frac{P_{\text{th}}}{\rho E_{\text{th}}}$$

Constant value

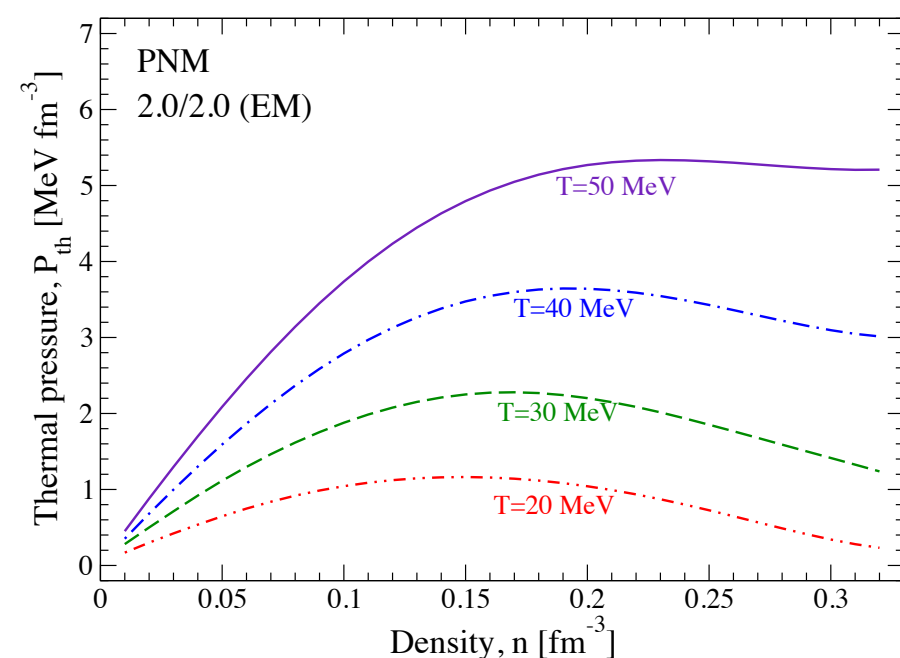
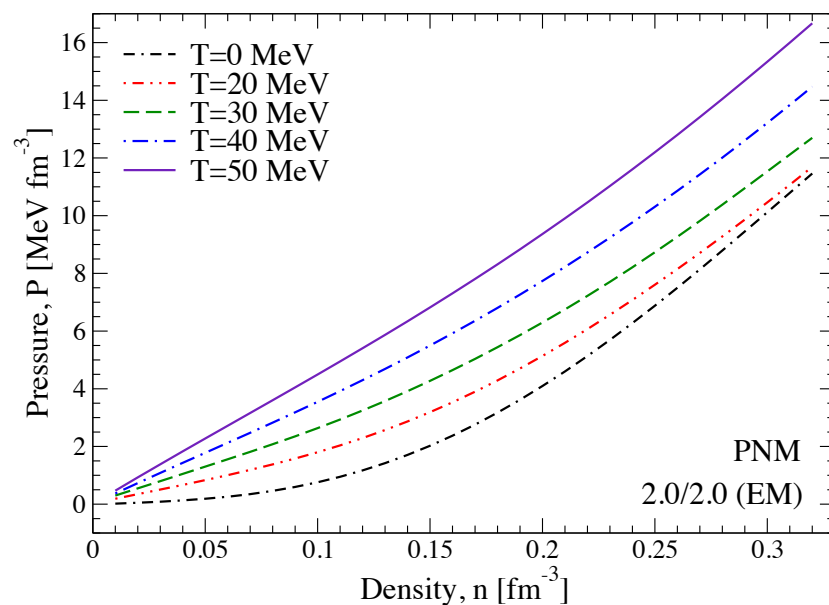
$$E_{\text{th}} = E(T) - E_0$$



increasing temperature



$$P_{\text{th}} = P(T) - P_0$$



increasing temperature

Carbone & Schwenk (*in preparation*)





# Thermal effects in EoS for astrophysical simulations

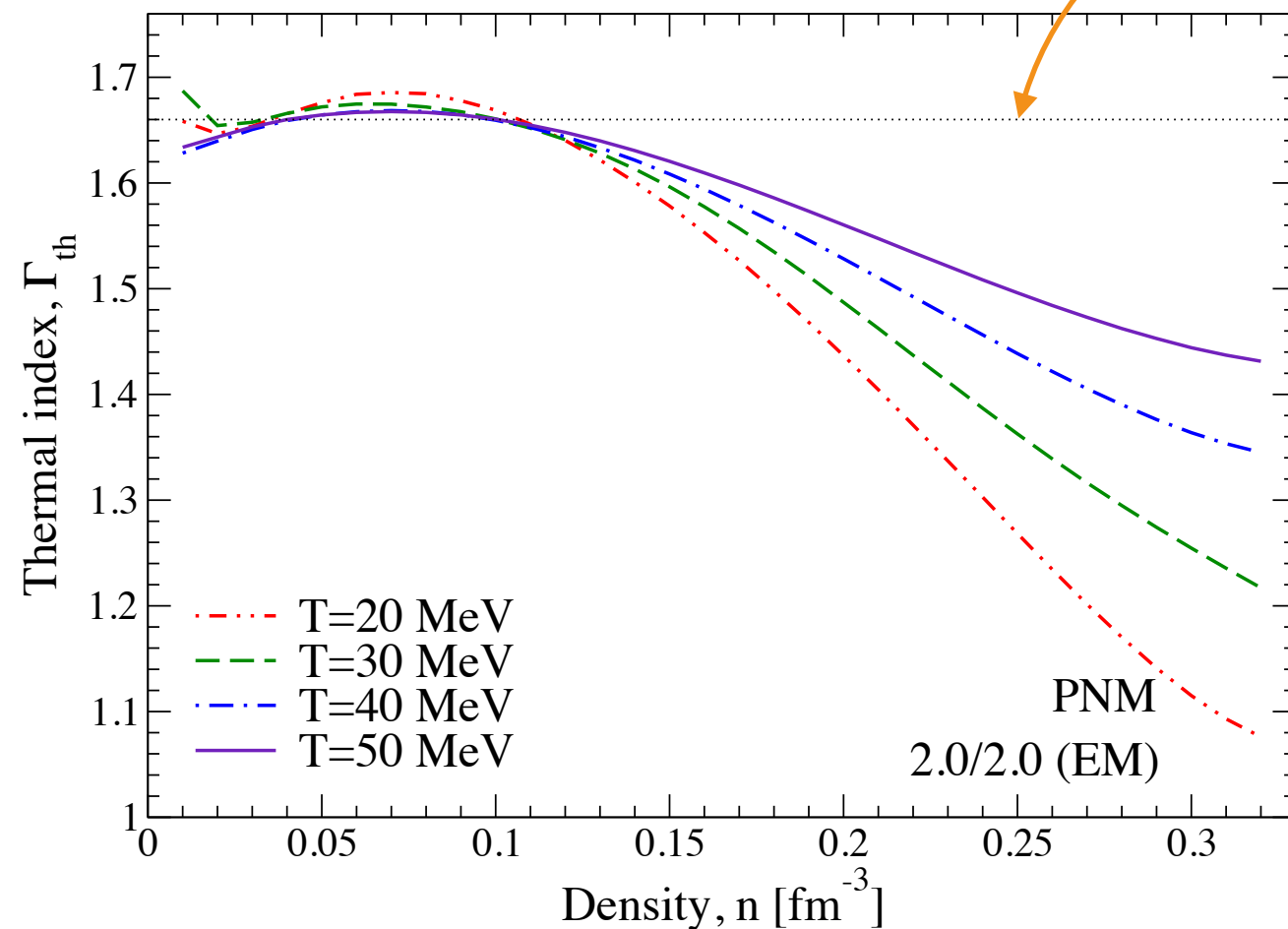
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Constant value

ideal gas



Carbone & Schwenk (*in preparation*)



# Thermal effects in EoS for astrophysical simulations

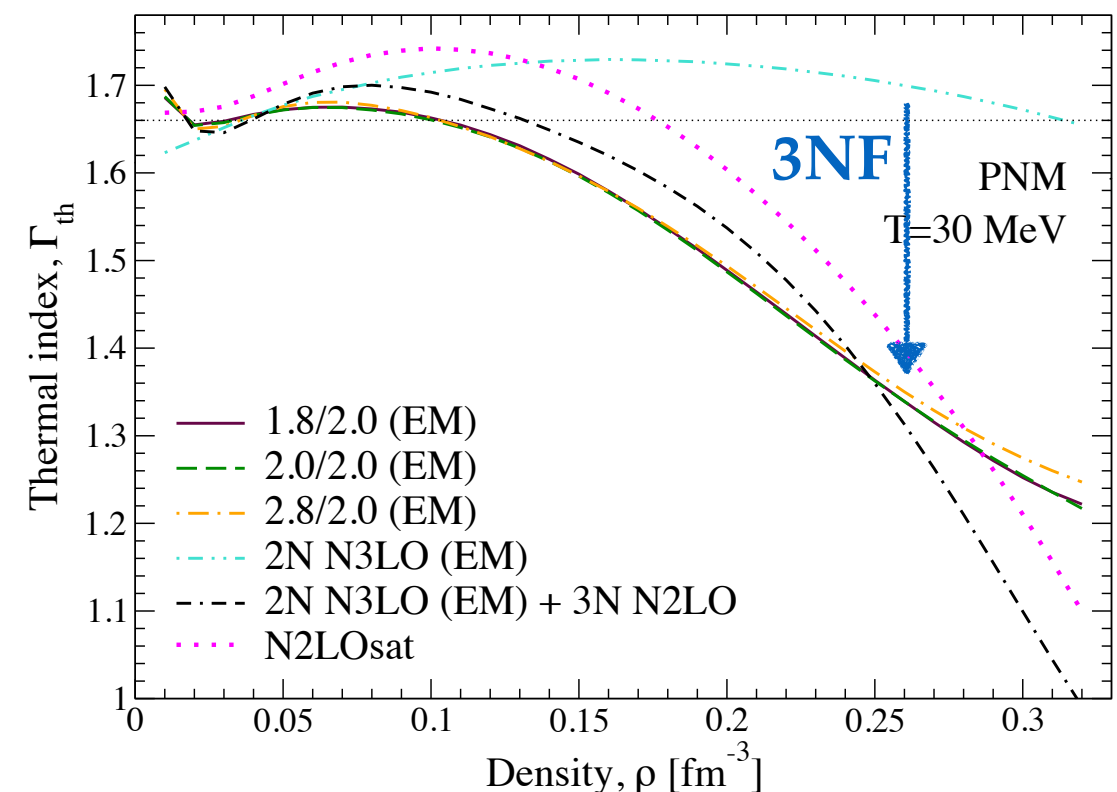
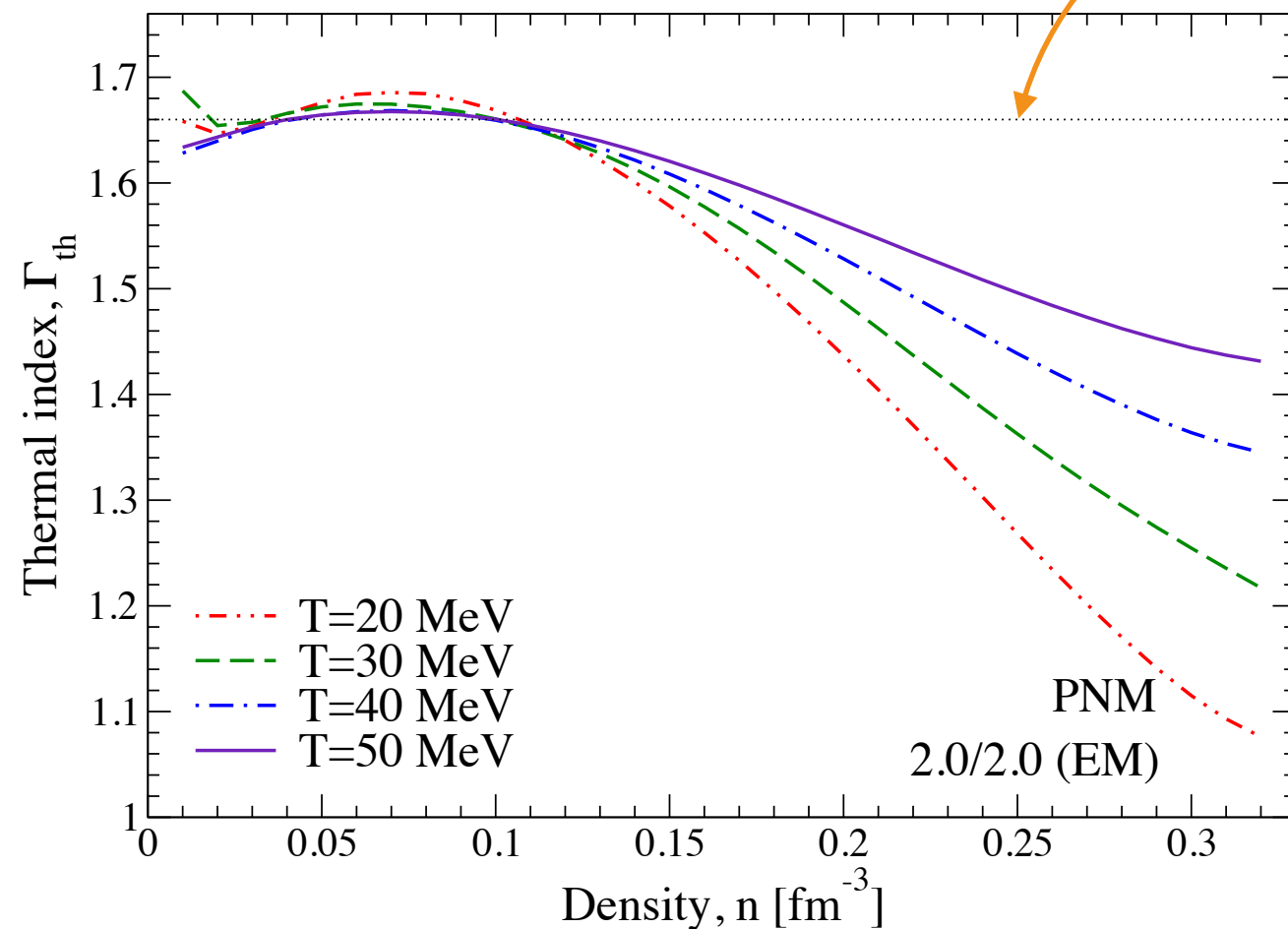
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Astrophysical EoS

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Constant value

ideal gas



suppression due to 3-body forces

Carbone & Schwenk (*in preparation*)



# Thermal effects in EoS for astrophysical simulations

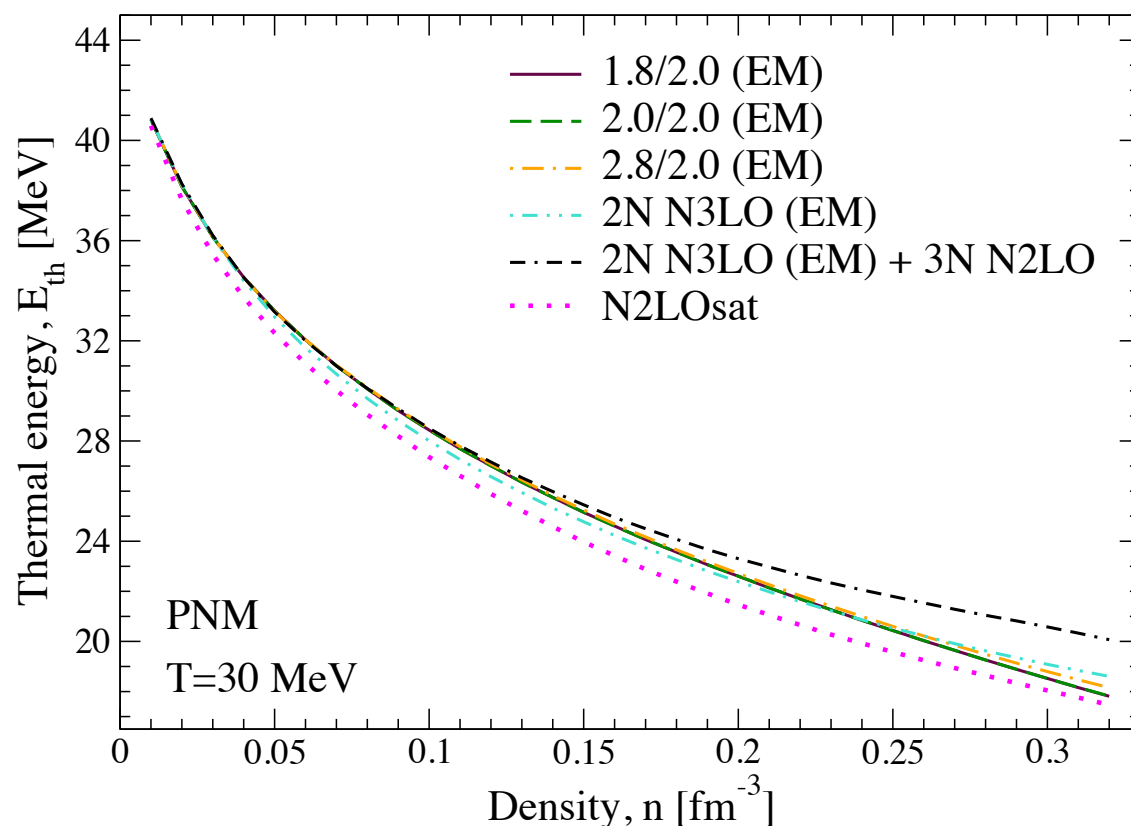
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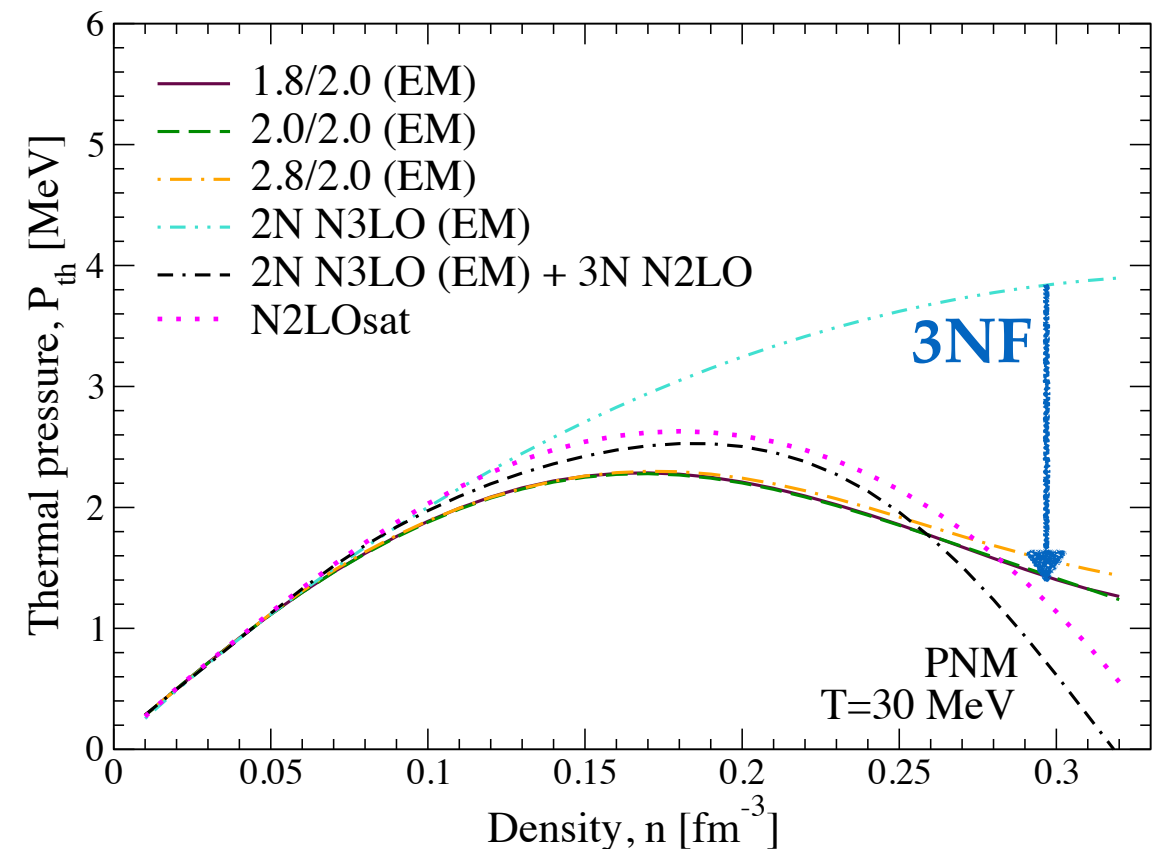
Astrophysical EoS

Constant value

$$E_{\text{th}} = E(T) - E_0$$



$$P_{\text{th}} = P(T) - P_0$$



suppression due to 3-body forces

Carbone & Schwenk (*in preparation*)



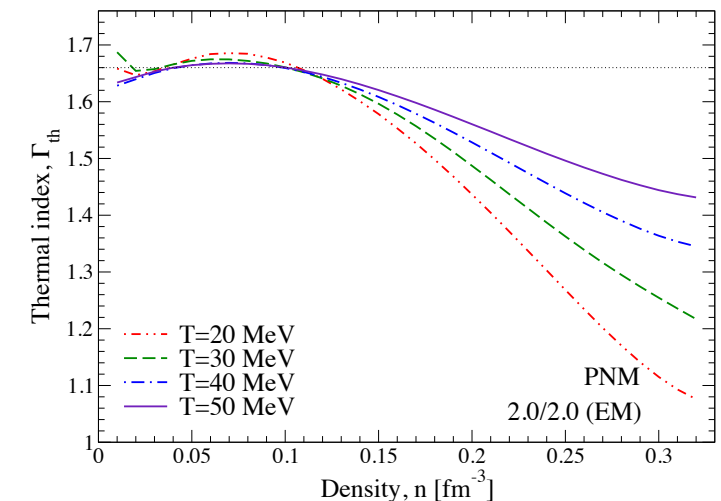
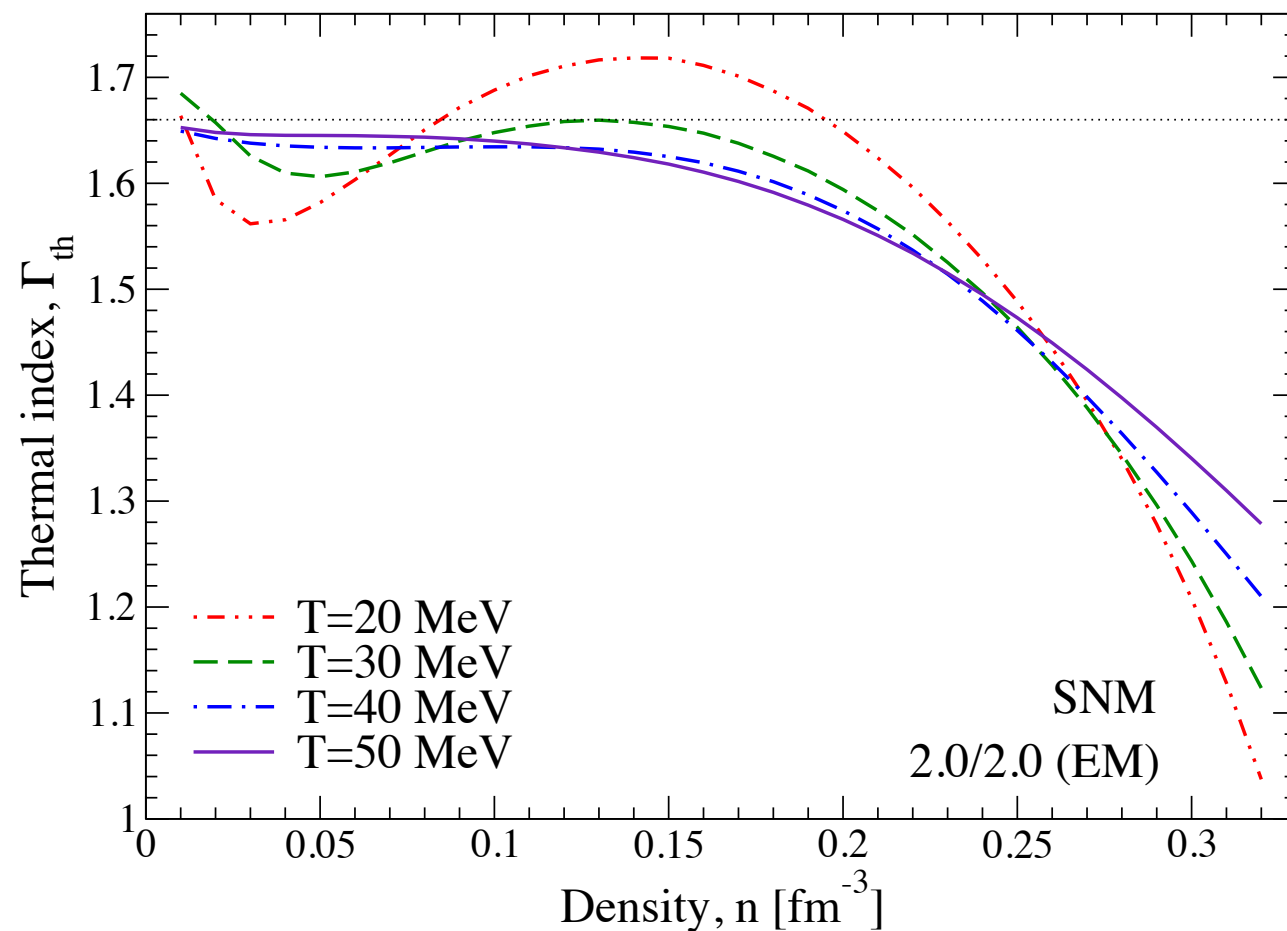
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Constant value



- strength of thermal effects
- stiffness of pressure at  $T=0$

Carbone & Schwenk (*in preparation*)



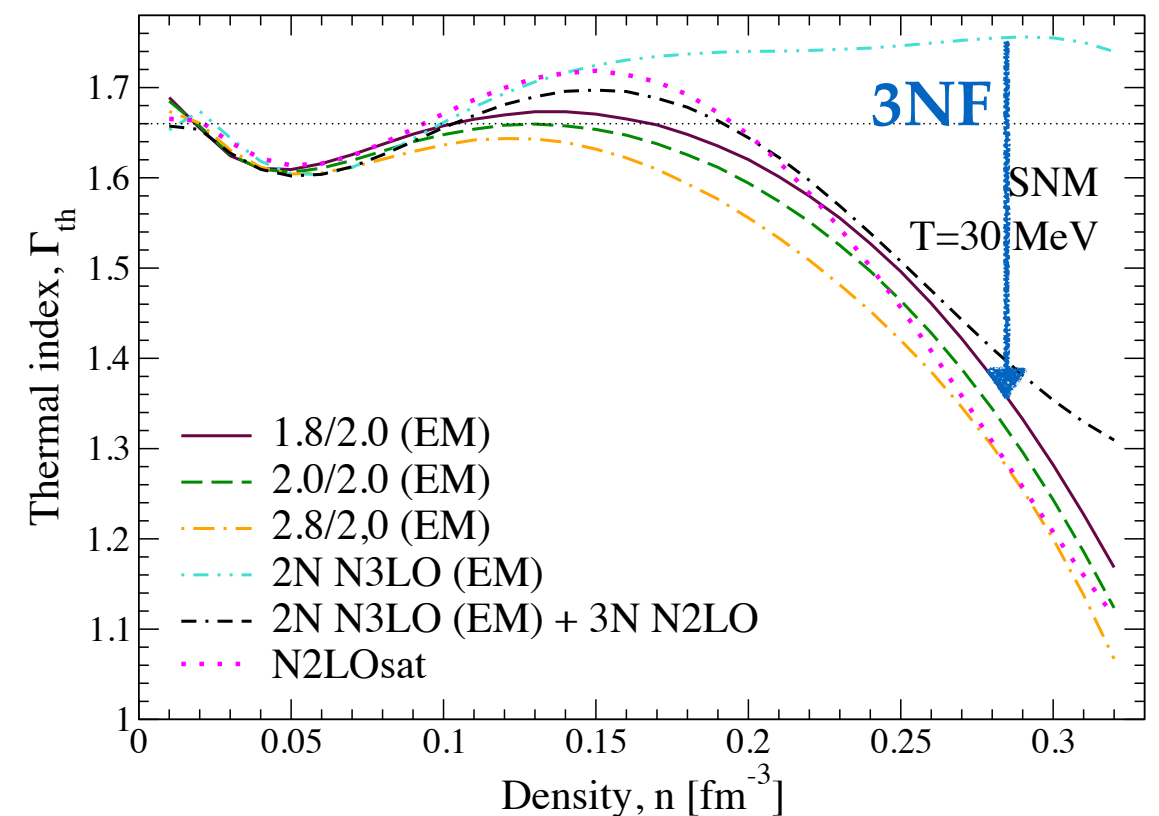
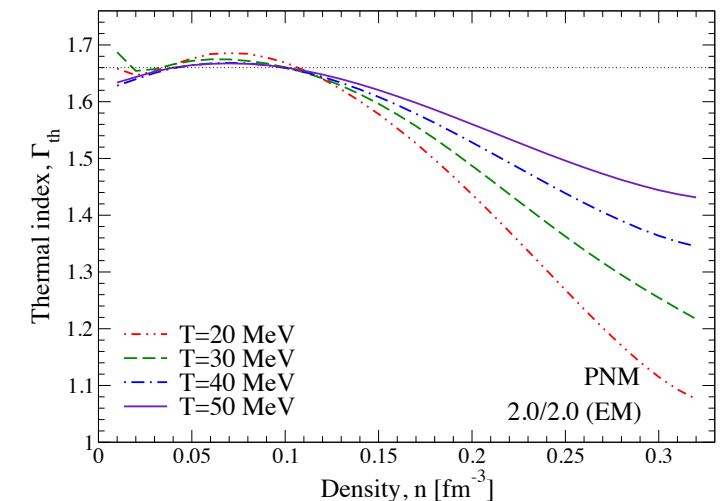
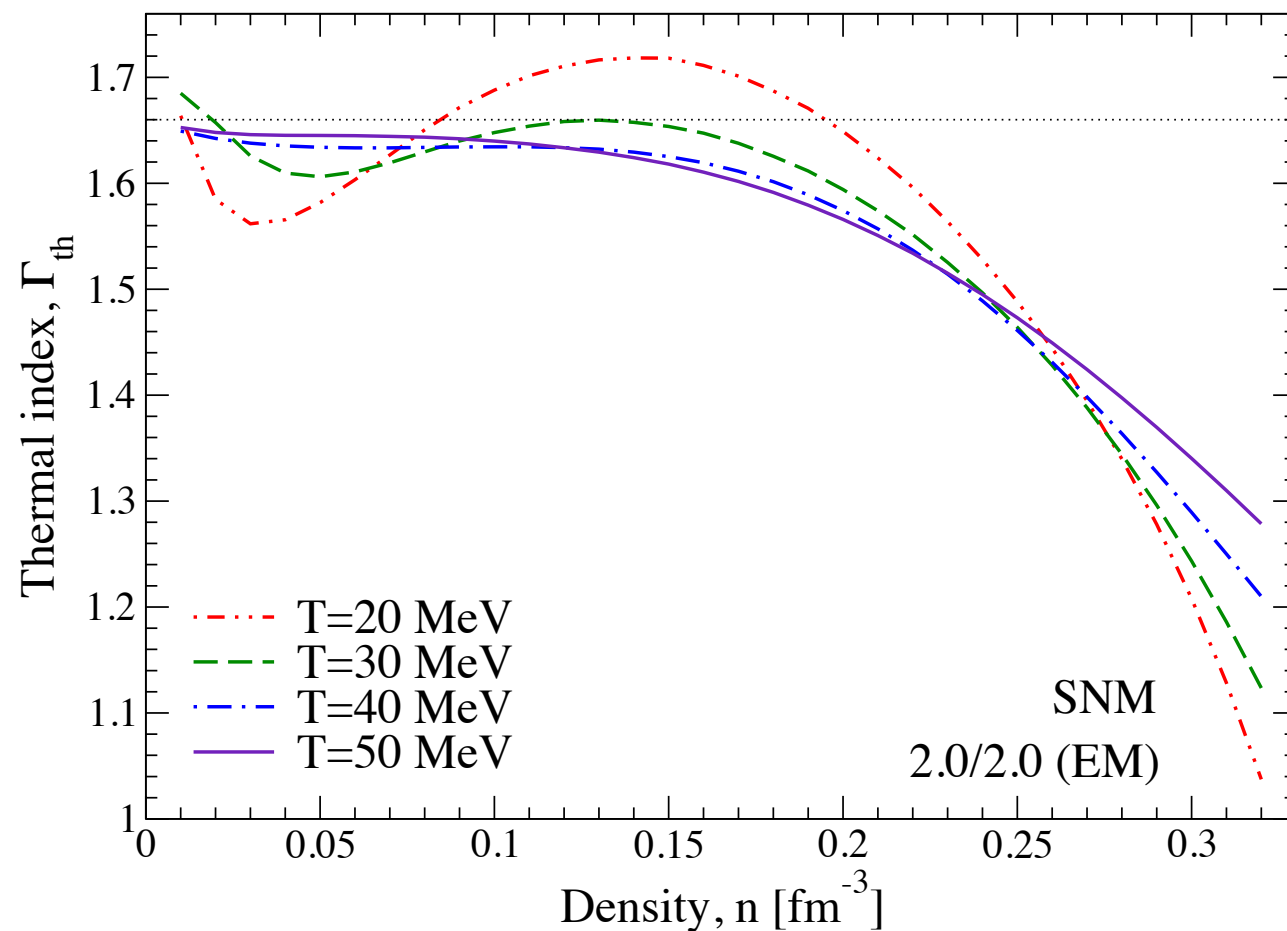
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Carbone & Schwenk (*in preparation*)

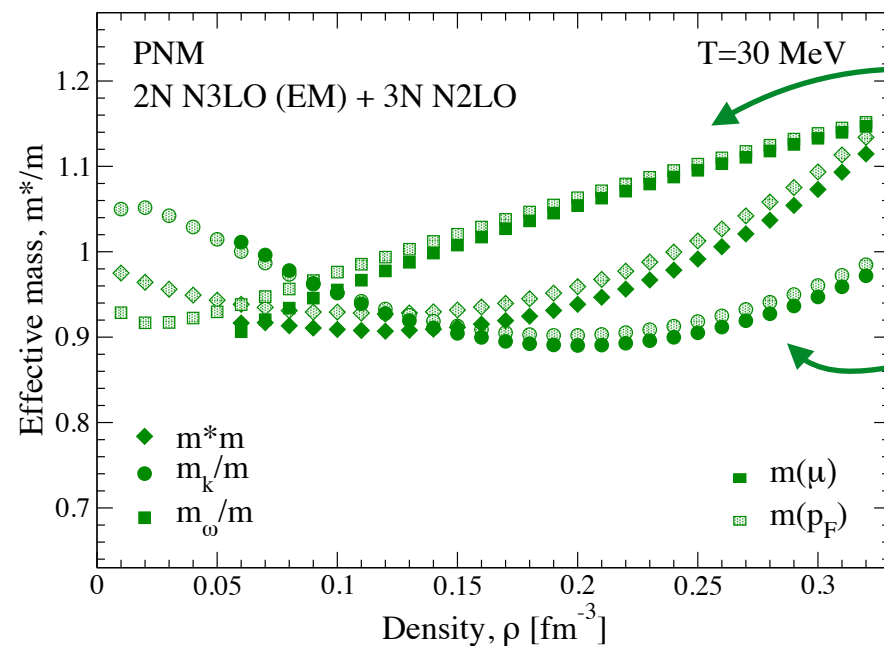
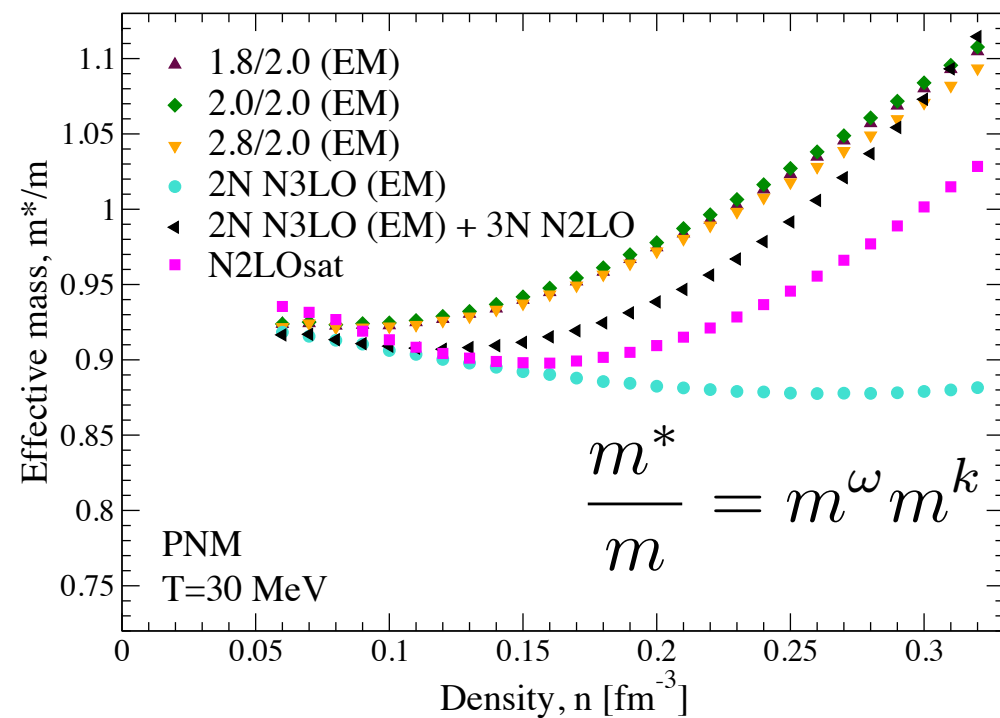




# Thermal index parametrized via effective mass

$$\Gamma_{\text{th}} = 1 + \frac{P_{\text{th}}}{\rho E_{\text{th}}}$$

$$\Gamma_{\text{th}} = \frac{5}{3} - \frac{\rho}{m^*} \frac{dm^*}{d\rho}$$



$$m^\omega = 1 - \frac{\partial \text{Re}\Sigma(p, \omega)}{\partial \omega}$$

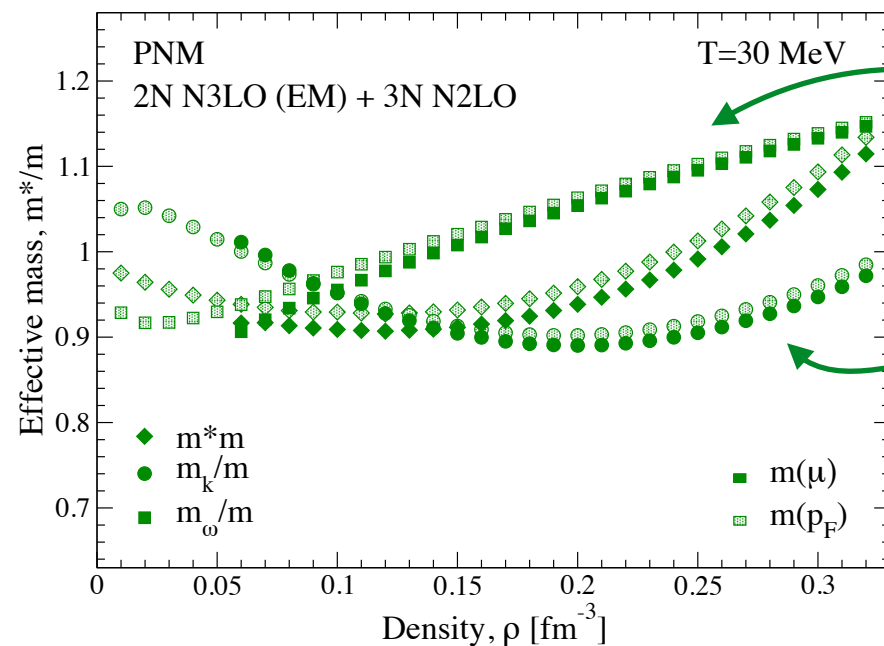
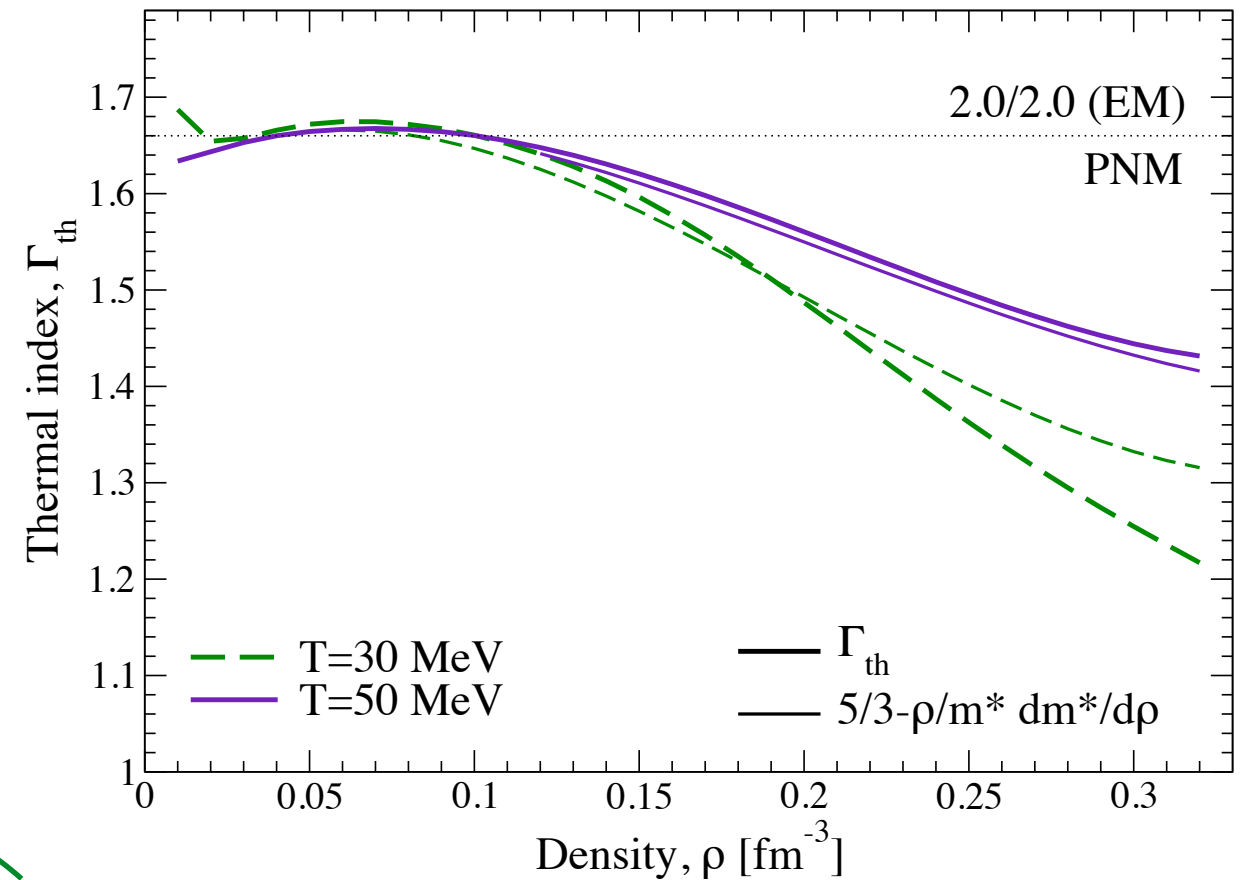
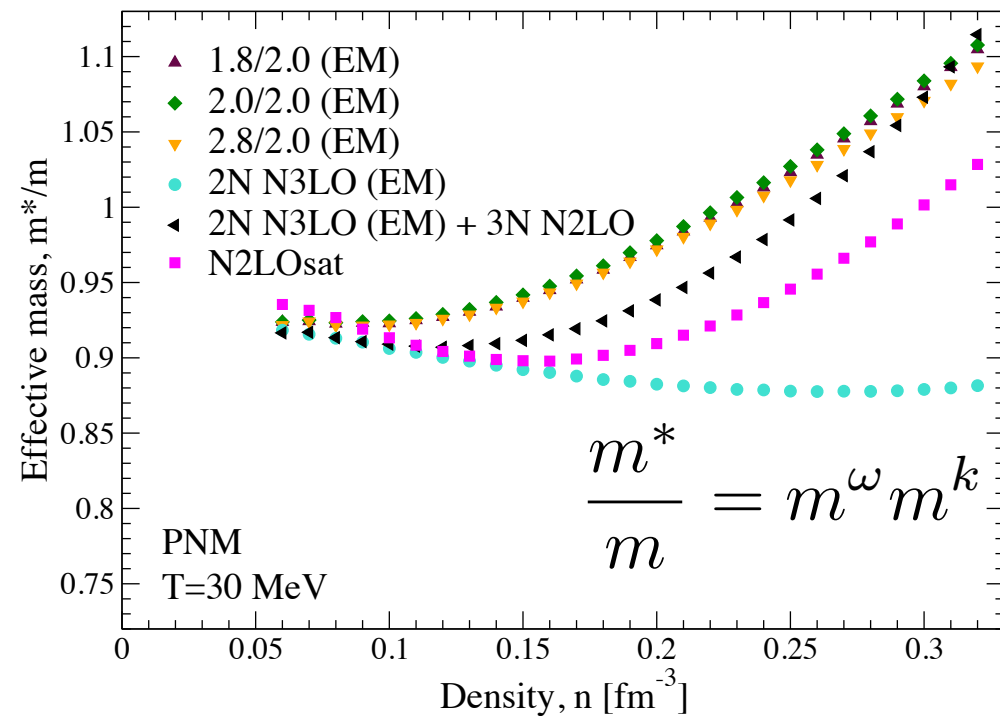
$$m^k = \left( 1 + \frac{m}{p} \frac{\partial \text{Re}\Sigma(p, \omega)}{\partial p} \right)^{-1}$$

Carbone & Schwenk (*in preparation*)



# Thermal index parametrized via effective mass

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Carbone & Schwenk (*in preparation*)



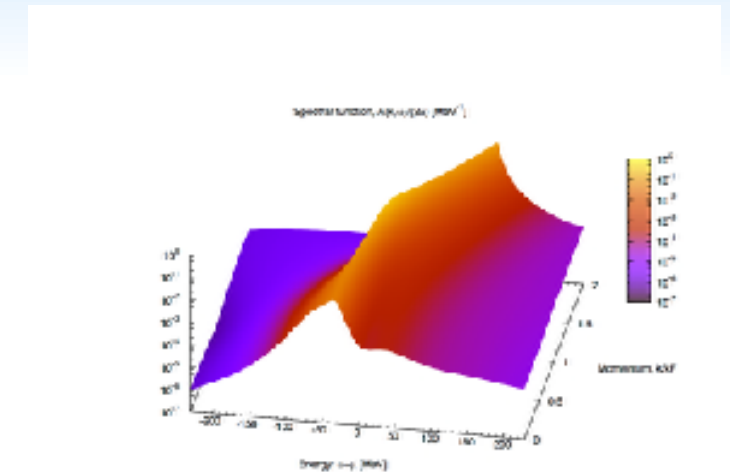
# Take home messages

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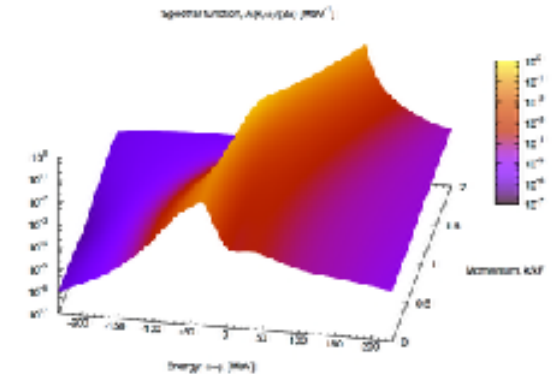
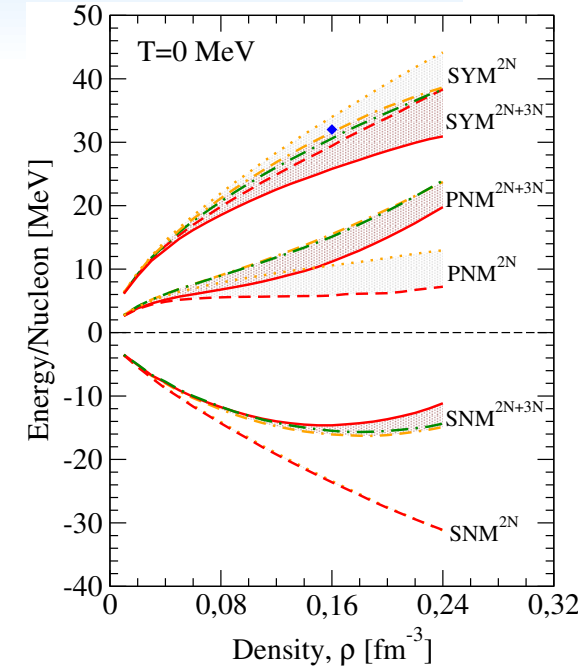
# Take home messages

- ★ Convenient approach to tackle down properties of nuclear systems



# Take home messages

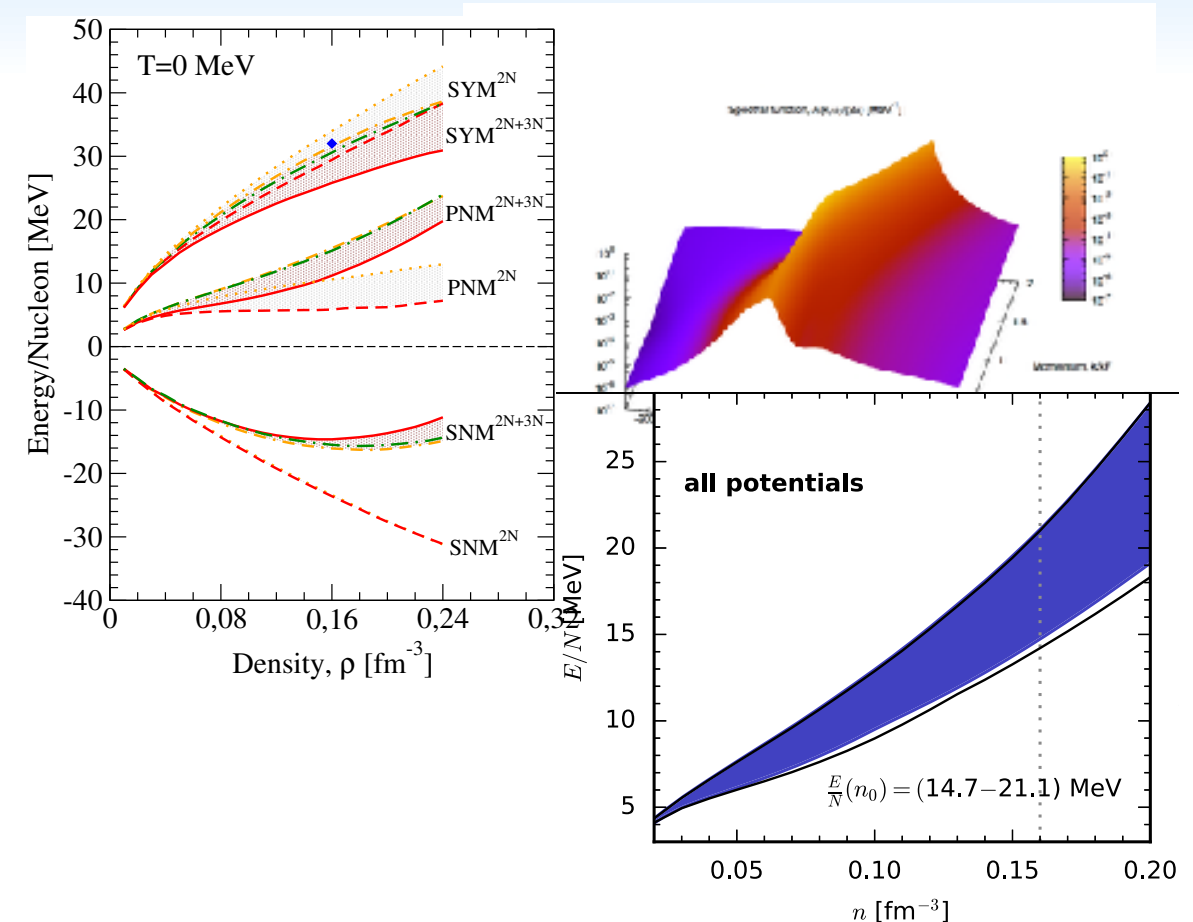
- ★ Convenient approach to tackle down properties of nuclear systems
- ★ Pinning saturation point is not enough for reasonable predictions of symmetry energy





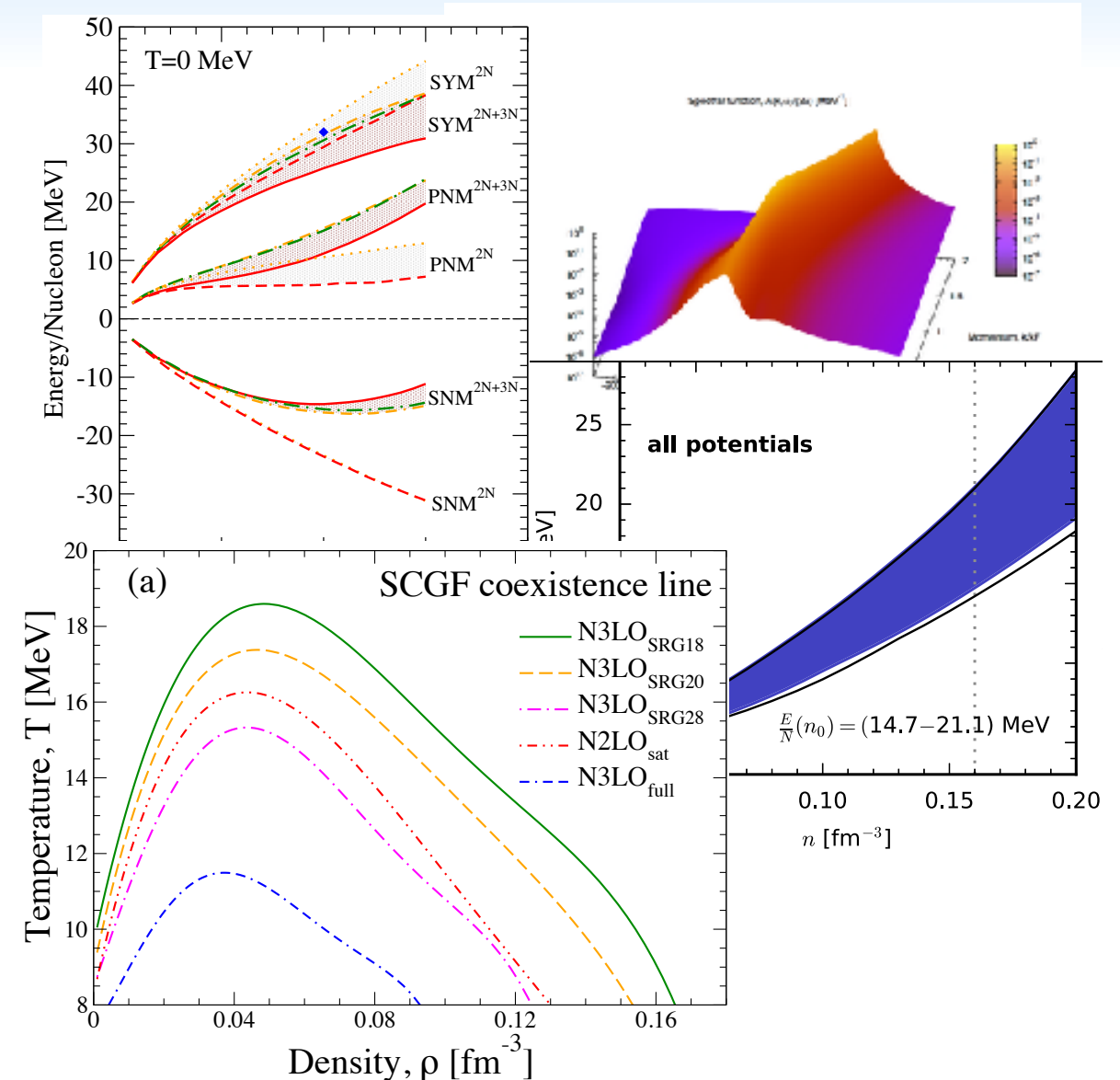
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- ★ Convenient approach to tackle down properties of nuclear systems
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- ★ Ab initio PNM can constrain NS radius



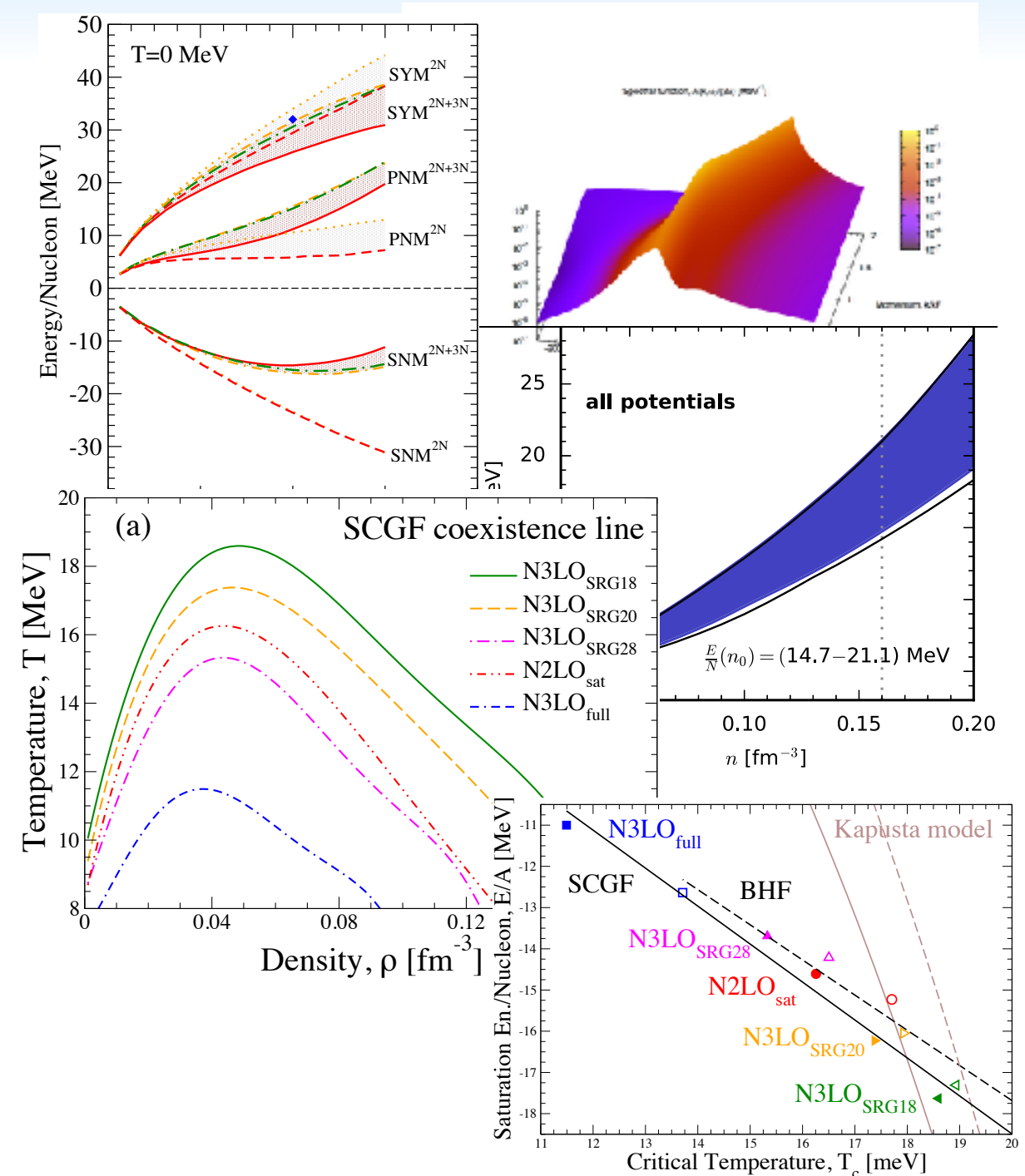
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- ★ Convenient approach to tackle down properties of nuclear systems
- ★ Pinning saturation point is not enough for reasonable predictions of symmetry energy
- ★ Ab initio PNM can constrain NS radius
- ★ Acceptable results for the liquid-gas critical temperature



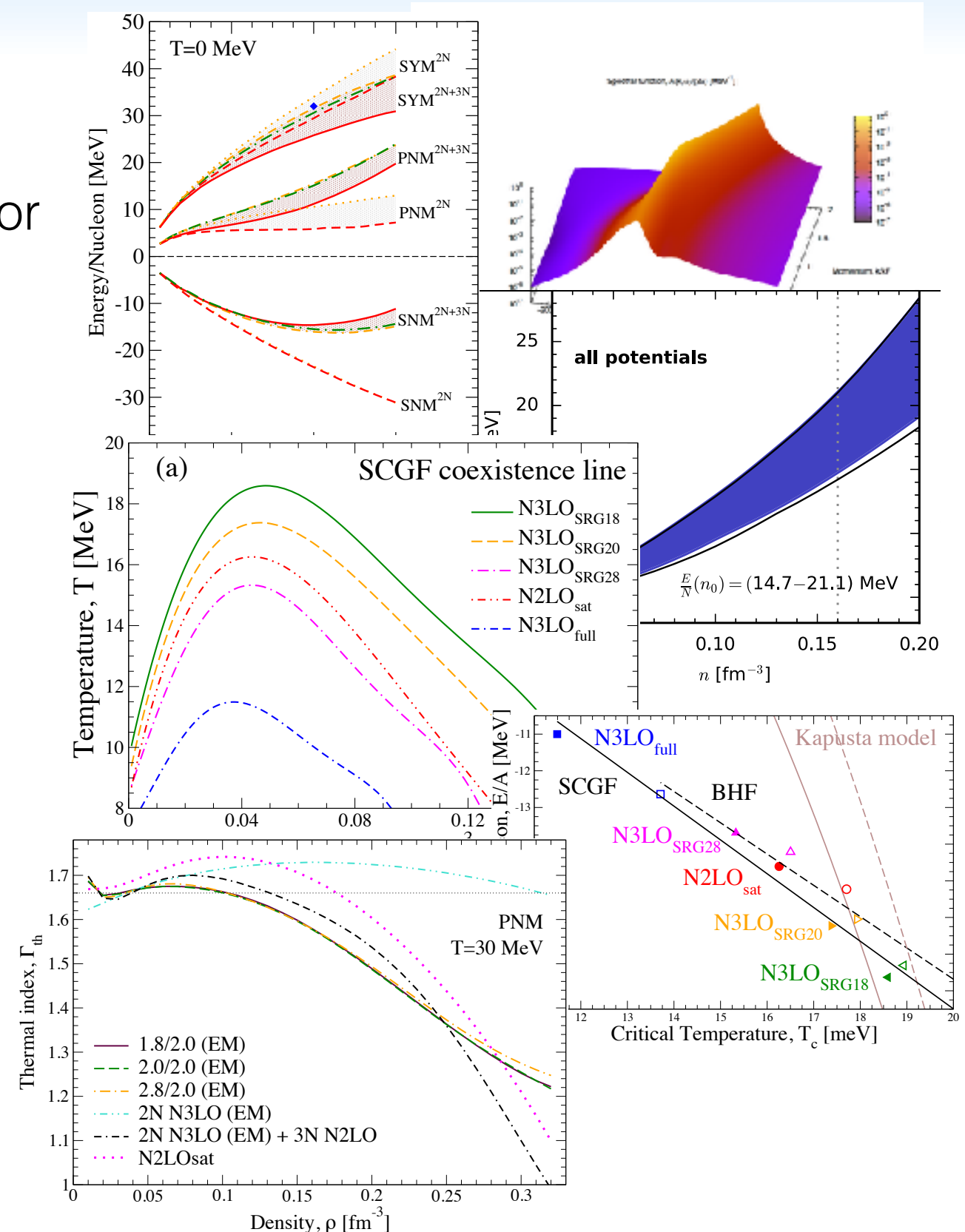
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- ★ Correlations between  $E_{\text{sat}}$  and  $T_c$



# Take home messages

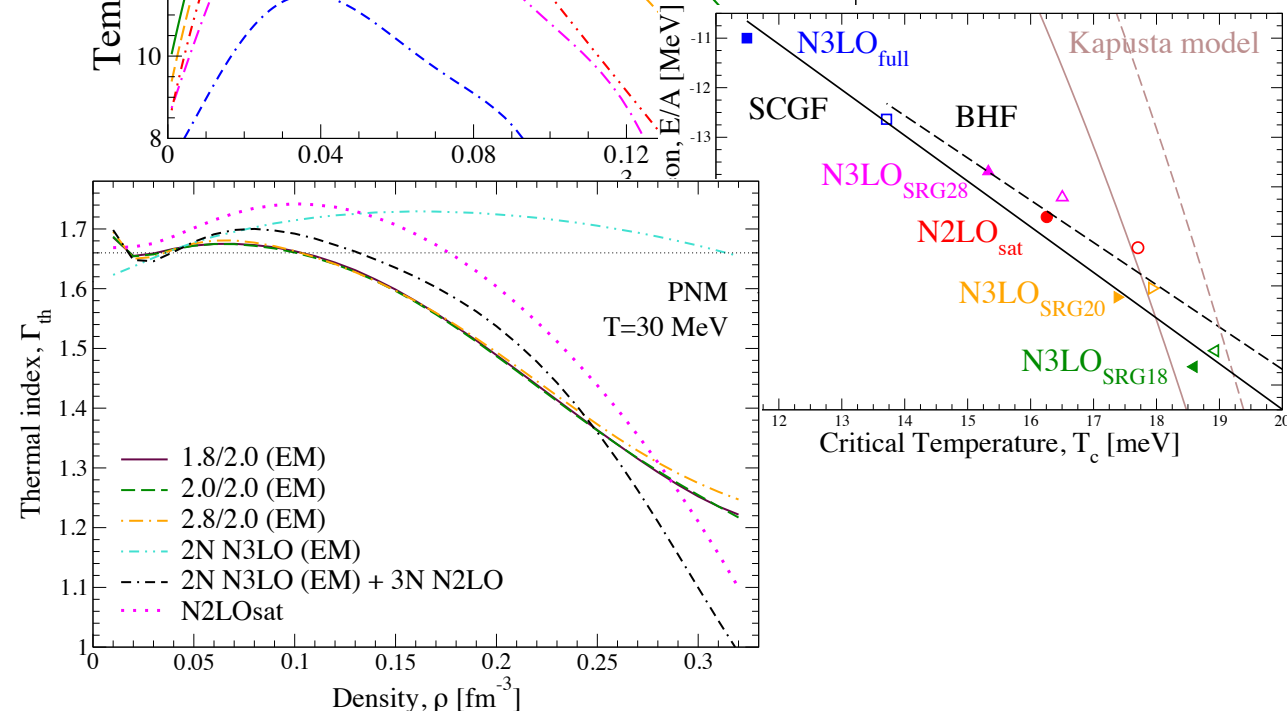
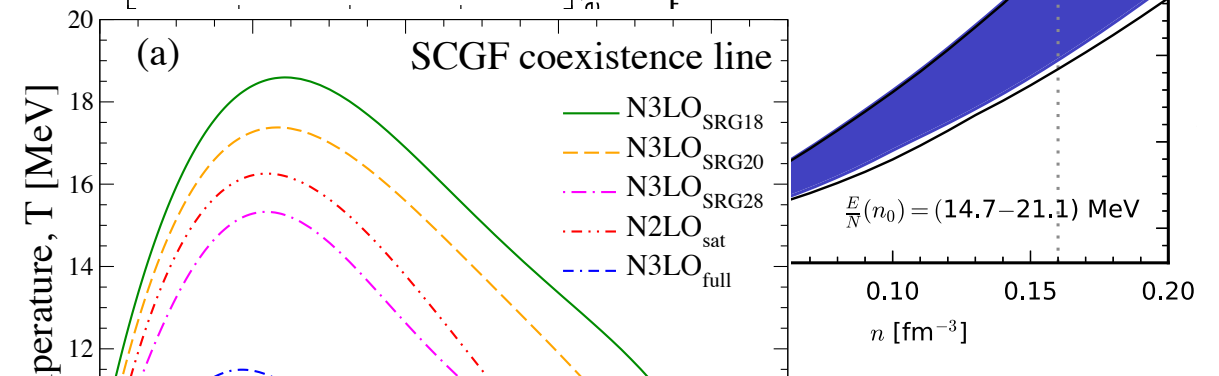
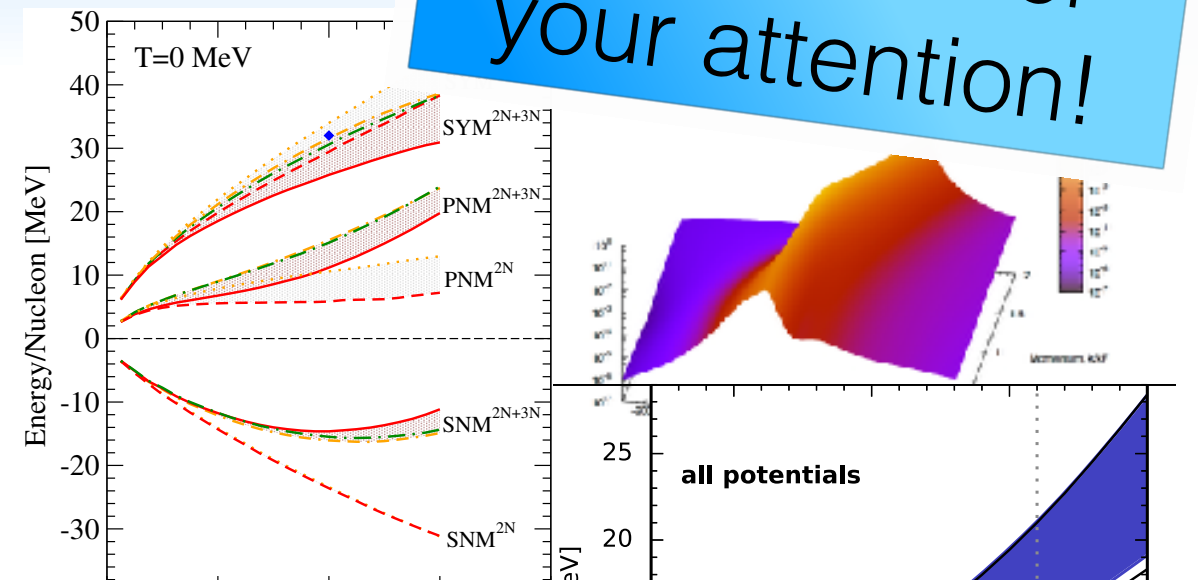
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- ★ Correlations between  $E_{\text{sat}}$  and  $T_c$
- ★ Thermal effects are important for astrophysical applications



# Take home messages

- ★ Convenient approach to tackle down properties of nuclear systems
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- ★ Ab initio PNM can constrain NS radius
- ★ Acceptable results for the liquid-gas critical temperature
- ★ Correlations between  $E_{\text{sat}}$  and  $T_c$
- ★ Ther...

Thank you for  
your attention!



Universitat de Barcelona

UNIVERSITY OF SURREY

TECHNISCHE UNIVERSITÄT DARMSTADT

A. Polls

A. Rios

C. Barbieri

C. Drischler, P. Klos, K. Hebeler, A. Schwenk





Backup

# Extended SCGF approach

Carbone, Cipollone, Barbieri, Rios, Polls, PRC **88**, 054326 (2013)

**2B**

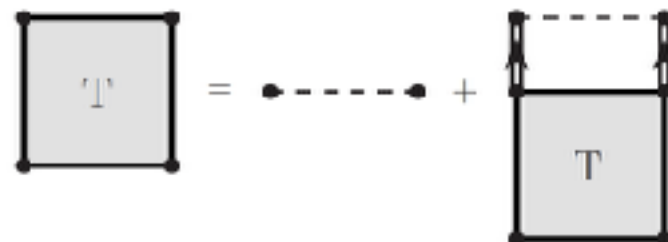
1. define effective interactions:

Interaction



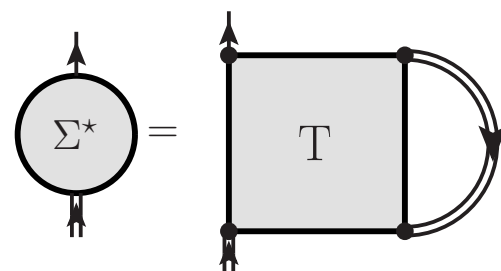
2. modify ladder approximation:

T-matrix

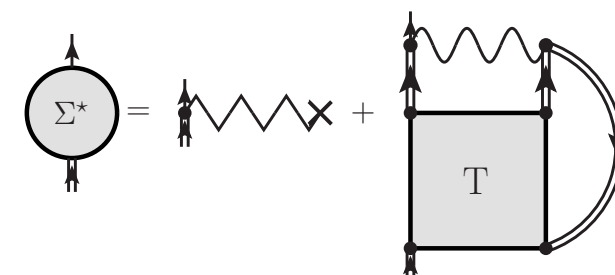
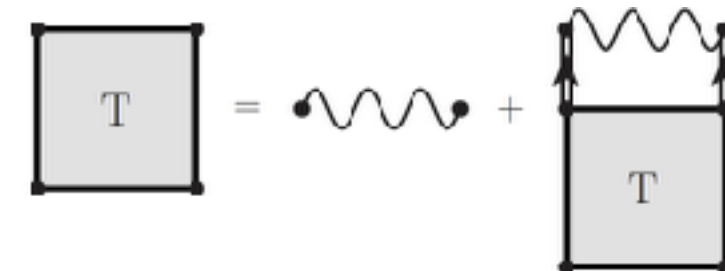
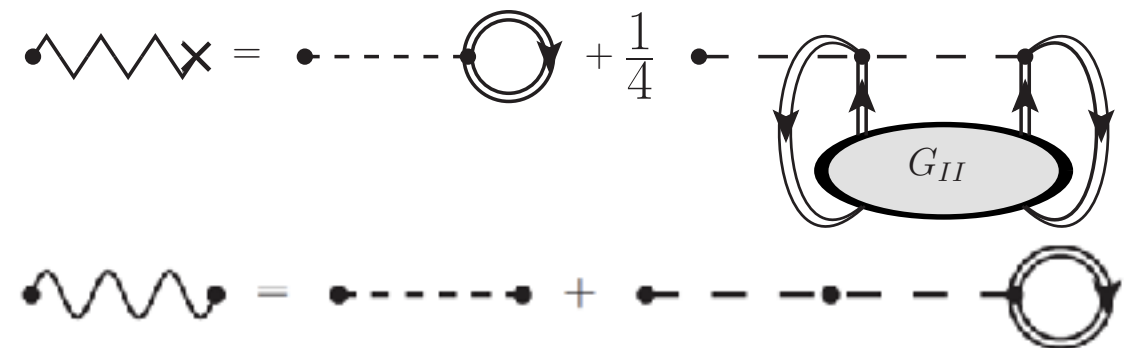


3. correct diagrams counting:

Self-energy



**2B + 3B**



# Extended SCGF approach

Carbone, Cipollone, Barbieri, Rios, Polls, PRC **88**, 054326 (2013)

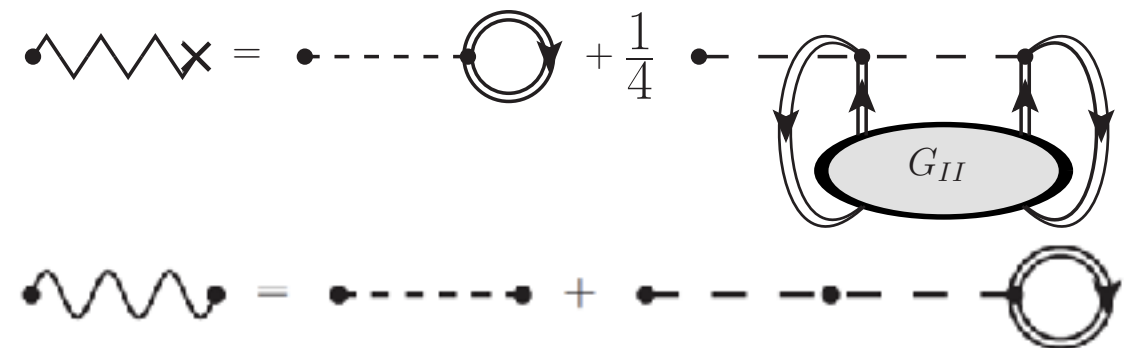
**2B**



**2B + 3B**

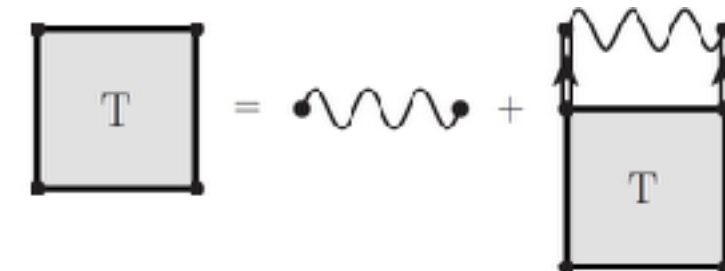
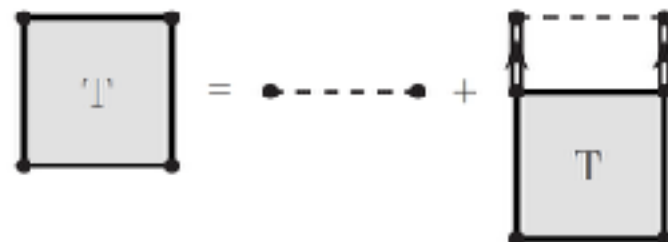
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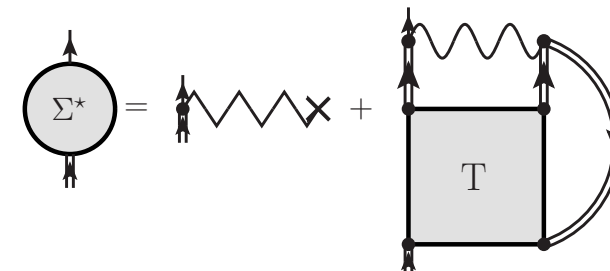
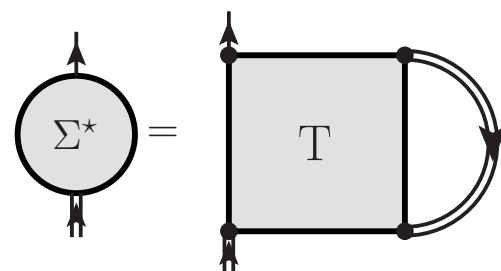
2. modify ladder approximation:

T-matrix



3. correct diagrams counting:

Self-energy



# Extended SCGF approach

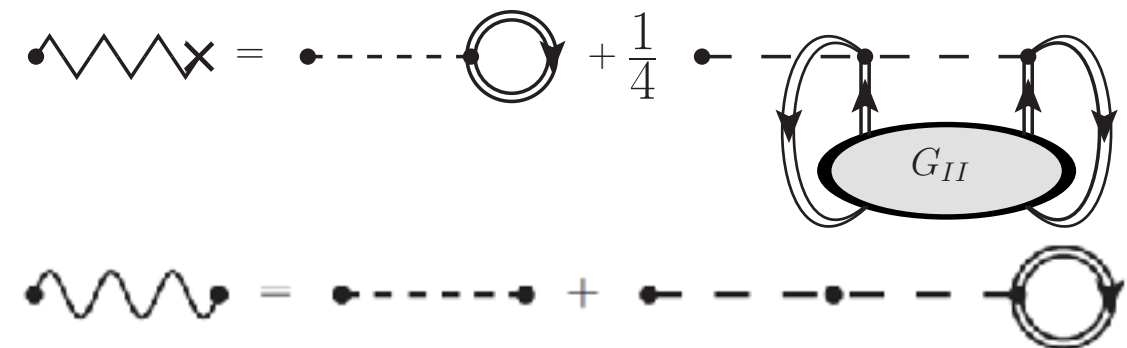
Carbone, Cipollone, Barbieri, Rios, Polls, PRC **88**, 054326 (2013)

**2B**

**2B + 3B**

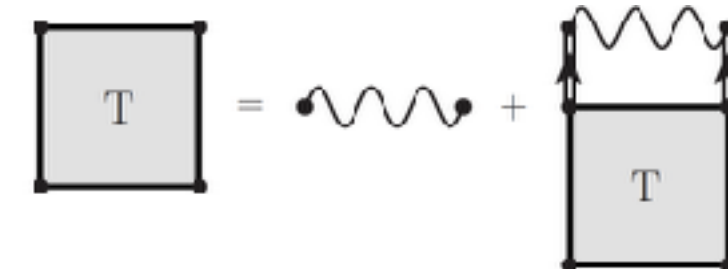
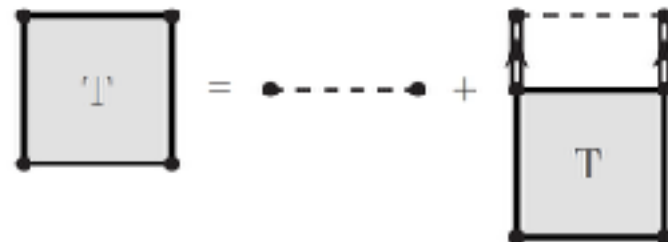
1. define effective interactions:

Interaction



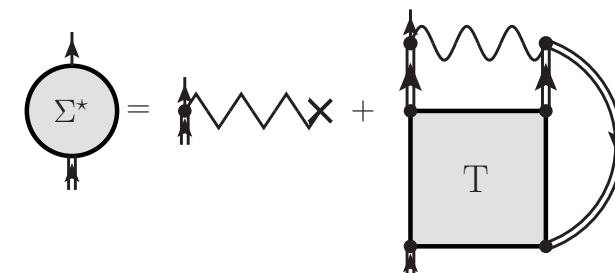
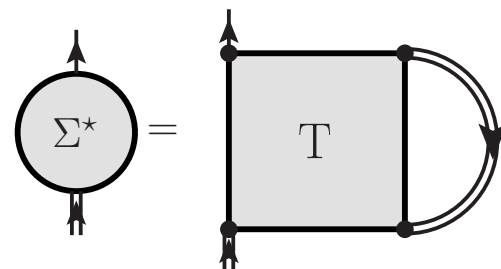
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# Extended SCGF approach

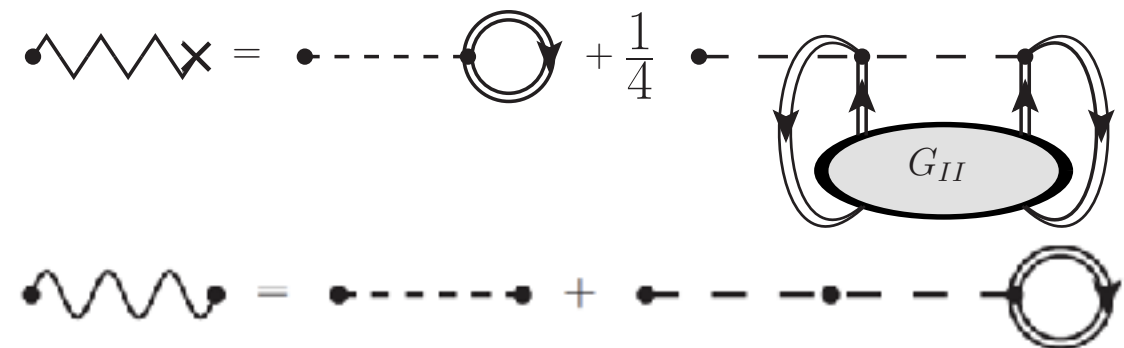
Carbone, Cipollone, Barbieri, Rios, Polls, PRC **88**, 054326 (2013)

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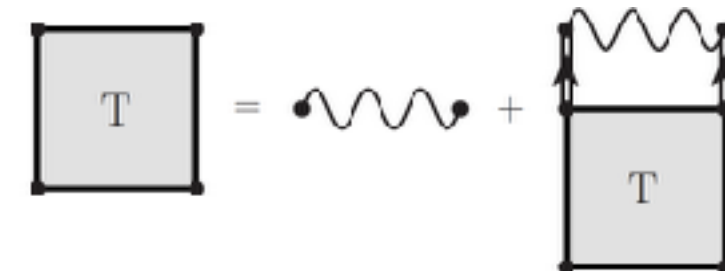
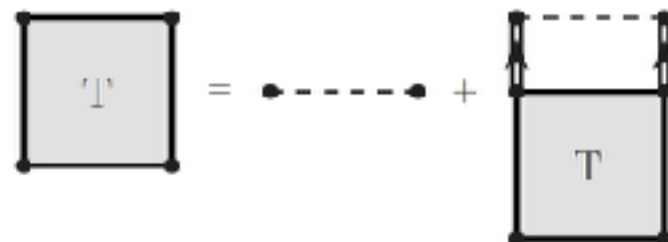
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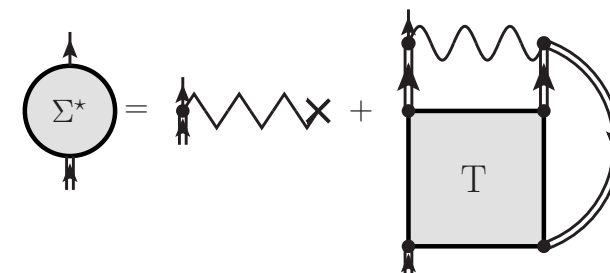
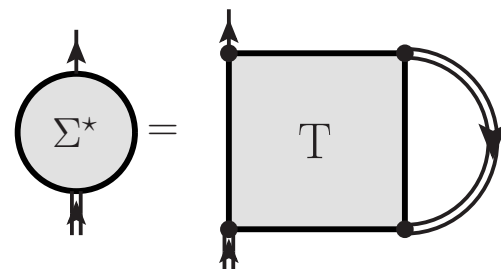
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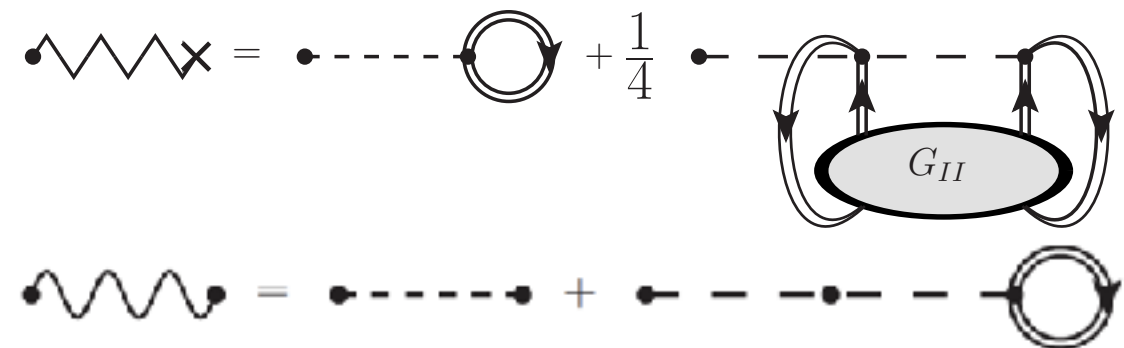
Carbone, Cipollone, Barbieri, Rios, Polls, PRC **88**, 054326 (2013)

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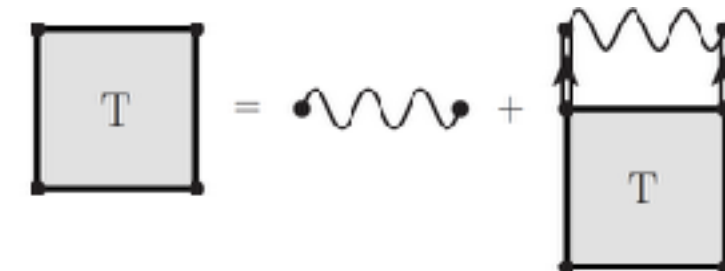
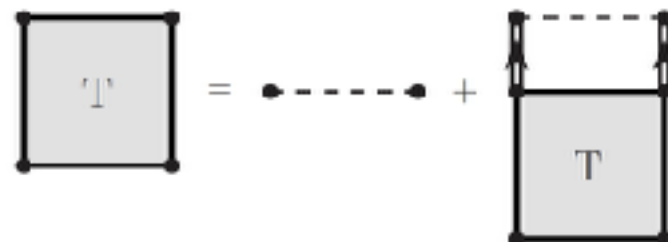
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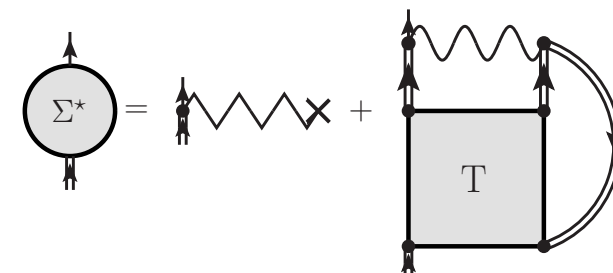
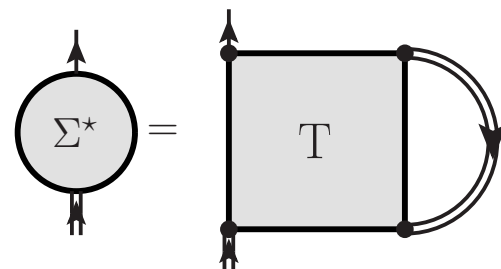
2. modify ladder approximation:

T-matrix



3. correct diagrams counting:

Self-energy



Diagrammatic equation for the dressed propagator:

$$\text{Double line with arrow} = \text{Single line with arrow} + \text{Single line with arrow} \circ \Sigma^*$$

# 2B

# Interaction

# T-matrix

# Self-energy

**2B + 3B**

Diagrammatic equations for the renormalization of the two-point function:

Top equation: A self-energy diagram (a wavy line with a cross) is equal to a tadpole diagram (a dashed line with a bubble) plus  $\frac{1}{4}$  times a diagram with a bubble labeled  $G_{II}$ .

Bottom equation: A tadpole diagram (a wavy line with a cross) is equal to a tadpole diagram (a dashed line with a bubble) plus  $\frac{1}{4}$  times a diagram with a bubble labeled  $G_{II}$ . The right-hand side of this equation is crossed out with a blue diagonal line.

$$T = \text{wavy line with dots} + T \text{ with wavy line on top}$$

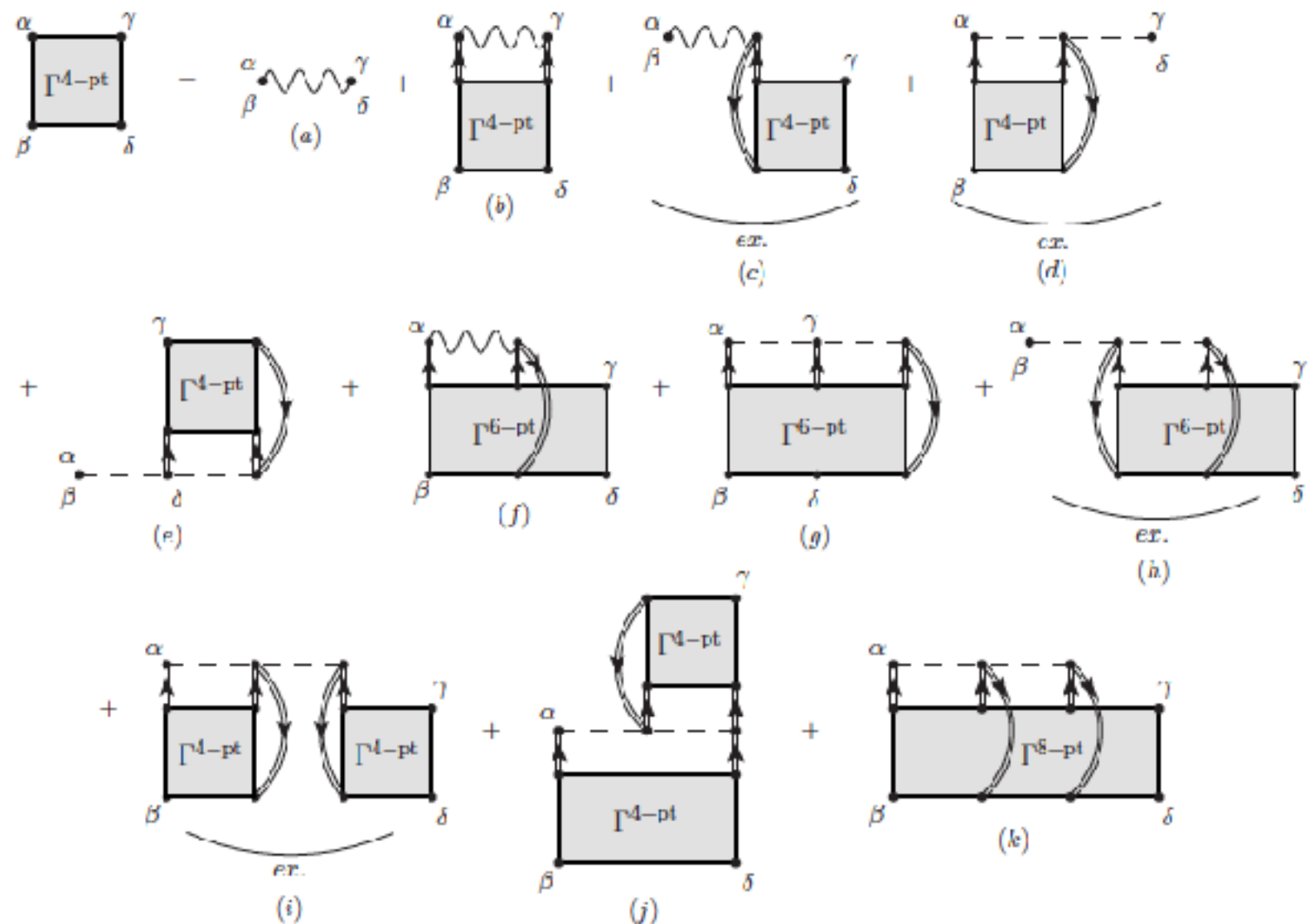
The diagram shows a self-energy loop  $\Sigma^*$  (a circle with two external legs) equal to the sum of two terms. The first term is a zigzag line with two external legs. The second term is a square loop labeled  $T$  with two external legs, and a wavy line connecting the top two vertices of the square.

# The 4-pt vertex function

Carbone, Cipollone, Barbieri, Rios, Polls, PRC **88**, 054326 (2013)

Obtain the interacting vertex function including 3body forces:

The 4-pt vertex function:



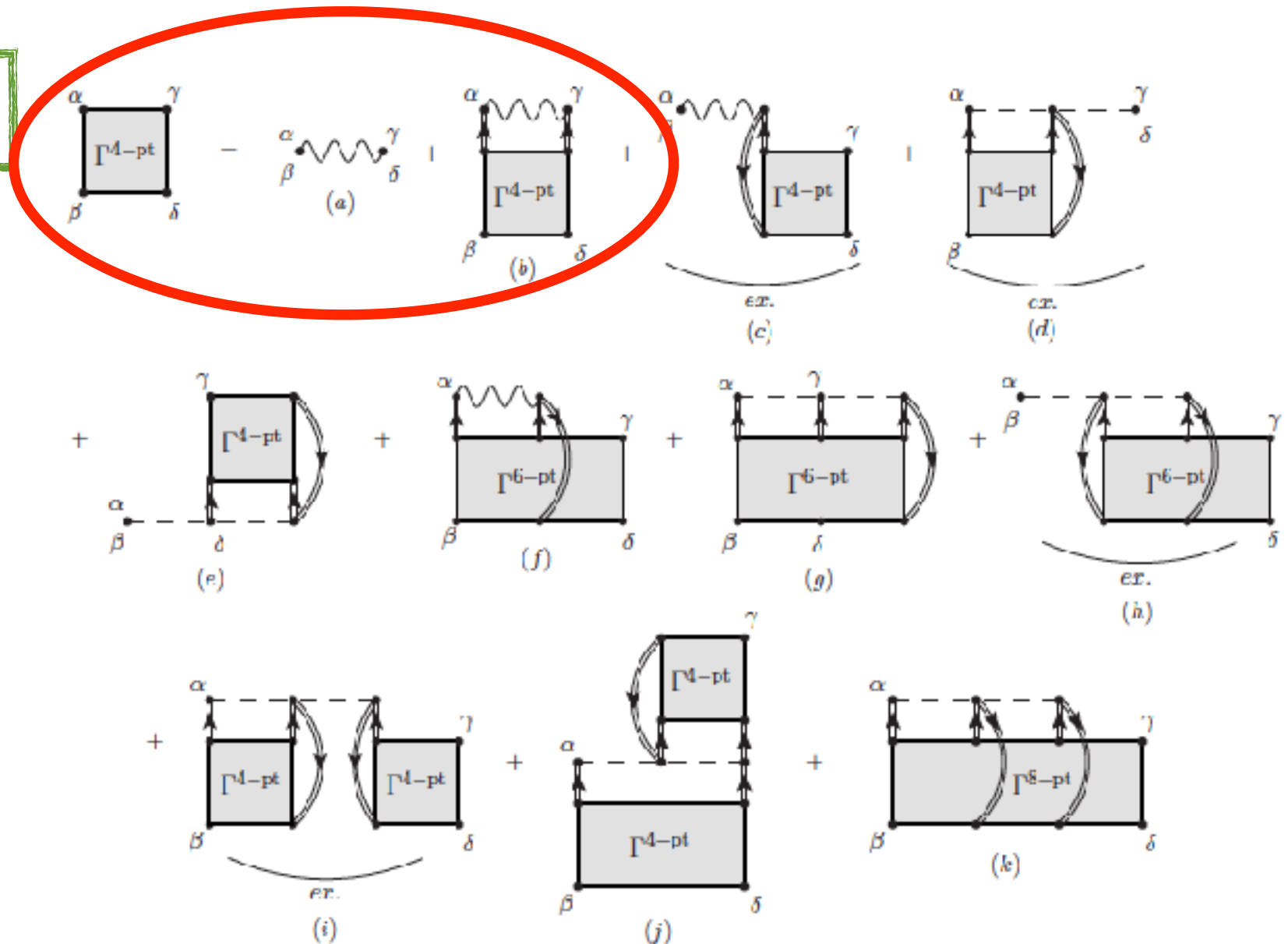
It's an equation including the 4-pt, 6-pt and 8-pt interacting vertex functions!

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Carbone, Cipollone, Barbieri, Rios, Polls, PRC **88**, 054326 (2013)

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# The single-particle self-energy

Carbone, Cipollone, Barbieri, Rios, Polls, PRC **88**, 054326 (2013)

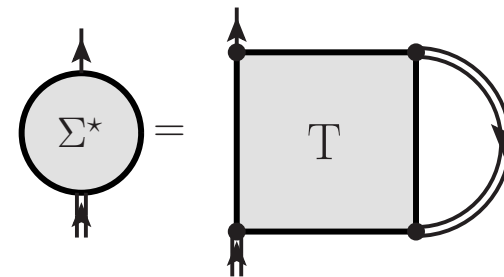
Calculate self-energy paying attention to the effective terms

Self-energy:

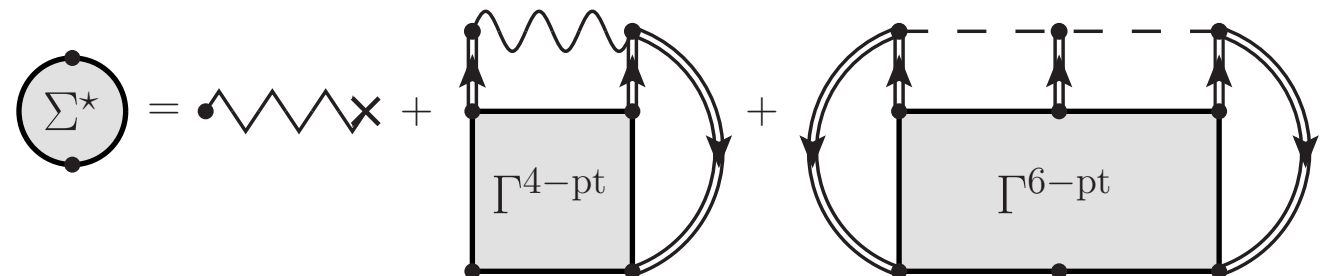
**2B**



**2B + 3B**



Residual 3NF





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Carbone, Cipollone, Barbieri, Rios, Polls, PRC **88**, 054326 (2013)

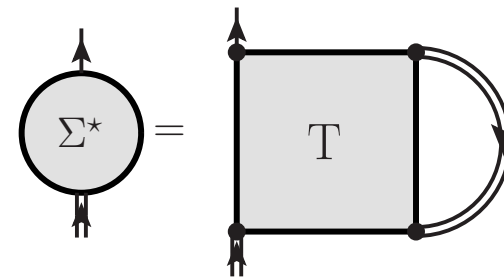
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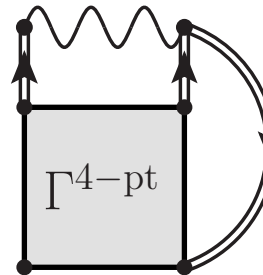
**2B**



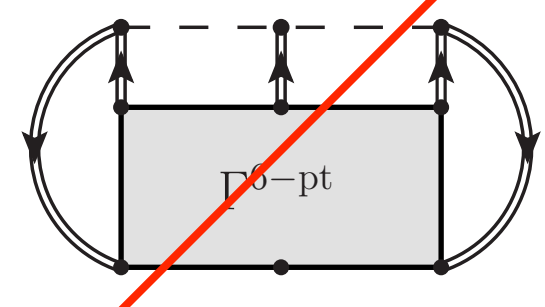
**2B + 3B**



=



+



Residual 3NF

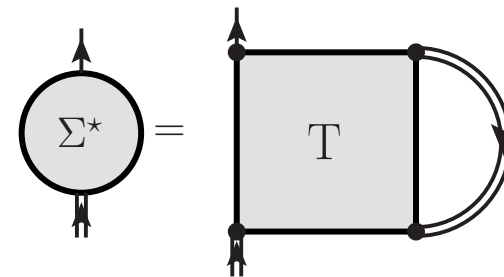
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Carbone, Cipollone, Barbieri, Rios, Polls, PRC **88**, 054326 (2013)

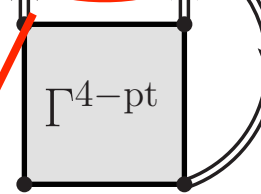
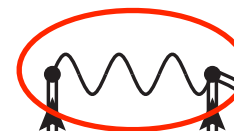
Calculate self-energy paying attention to the effective terms

Self-energy:

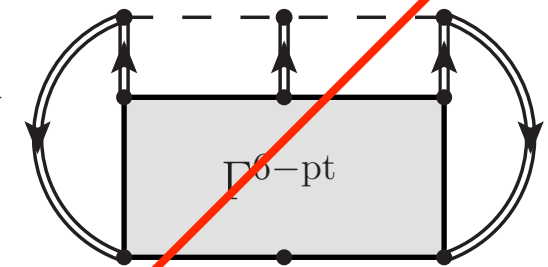
**2B**



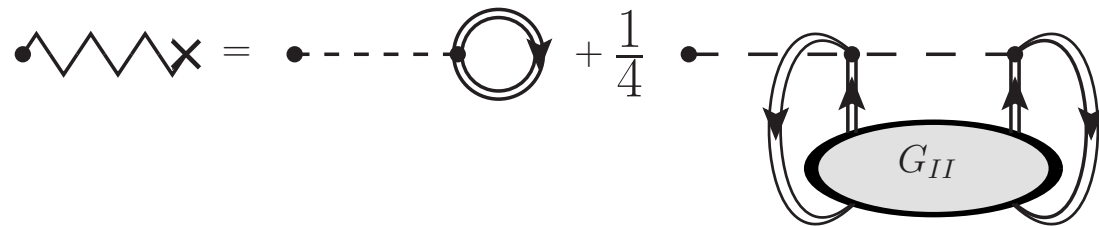
**2B + 3B**



+



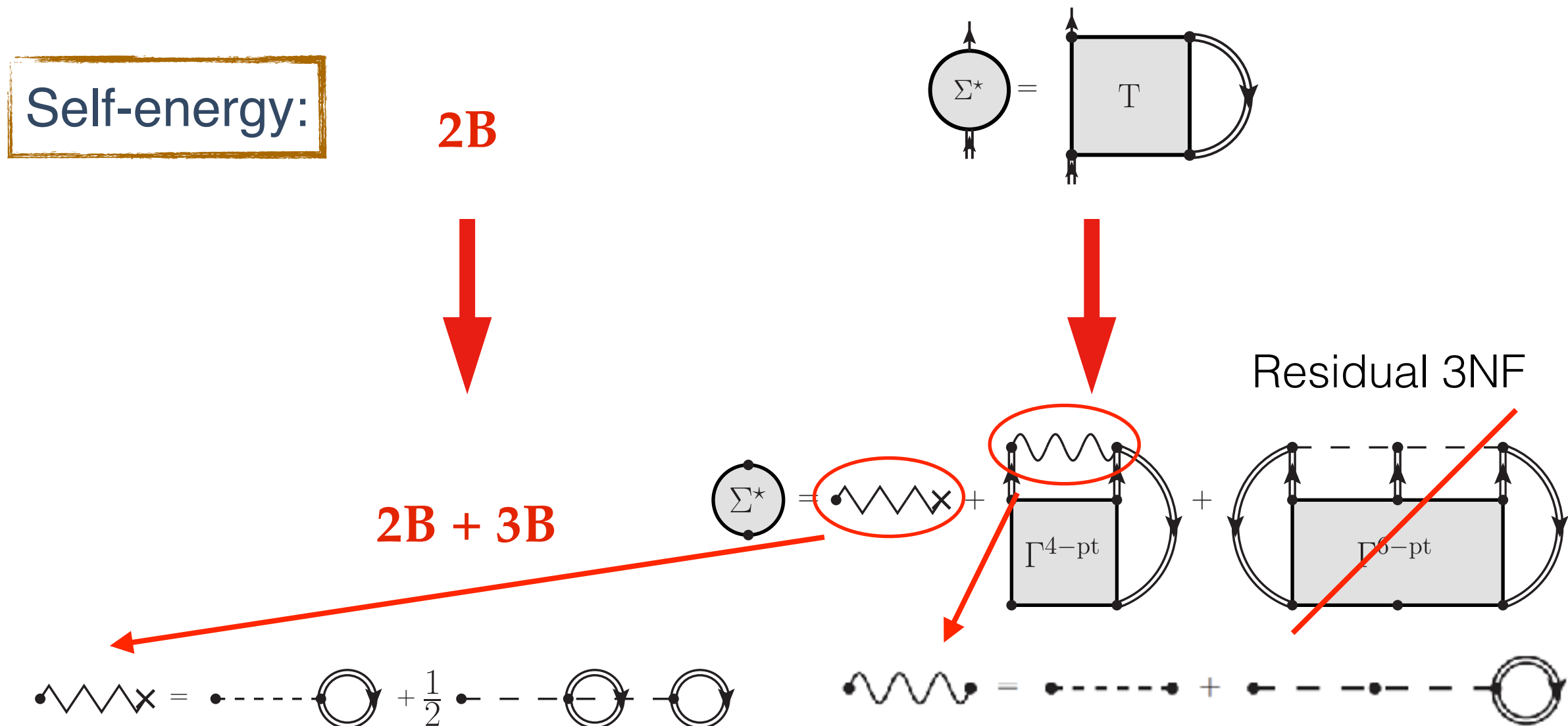
Residual 3NF



# The single-particle self-energy

Carbone, Cipollone, Barbieri, Rios, Polls, PRC **88**, 054326 (2013)

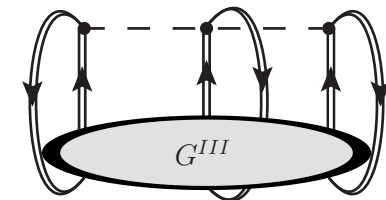
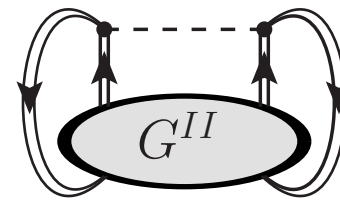
Calculate self-energy paying attention to the effective terms



# Define a new sum rule

Carbone, Cipollone, Barbieri, Rios, Polls, PRC **88**, 054326 (2013)

$$E^N = \langle \Psi^N | \hat{H} | \Psi^N \rangle = \langle \Psi^N | \hat{T} | \Psi^N \rangle + \langle \Psi^N | \hat{V} | \Psi^N \rangle + \langle \Psi^N | \hat{W} | \Psi^N \rangle$$



- Galitskii-Migdal-Koltun sumrule modified:

$$\sum_{\alpha} \int_{-\infty}^{E^N - E^{N-1}} d\omega \omega \frac{1}{\pi} \text{Im} G_{\alpha\alpha}(\omega) = \langle \Psi^N | \hat{T} | \Psi^N \rangle + 2 \langle \Psi^N | \hat{V} | \Psi^N \rangle + 3 \langle \Psi^N | \hat{W} | \Psi^N \rangle$$

$$\frac{E}{A} = \frac{\nu}{\rho} \int \frac{d^3 p}{(2\pi)^3} \int \frac{d\omega}{2\pi} \frac{1}{2} \left\{ \frac{p^2}{2m} + \omega \right\} \mathcal{A}(p, \omega) f(\omega) - \frac{1}{2} \langle \Psi^N | \hat{W} | \Psi^N \rangle$$

# Define a new sum rule

Carbone, Cipollone, Barbieri, Rios, Polls, PRC **88**, 054326 (2013)

$$E^N = \langle \Psi^N | \hat{H} | \Psi^N \rangle = \underbrace{\langle \Psi^N | \hat{T} | \Psi^N \rangle}_{\text{Diagram 1}} + \underbrace{\langle \Psi^N | \hat{V} | \Psi^N \rangle}_{\text{Diagram 2}} + \underbrace{\langle \Psi^N | \hat{W} | \Psi^N \rangle}_{\text{Diagram 3}}$$

- Galitskii-Migdal-Koltun sumrule modified:

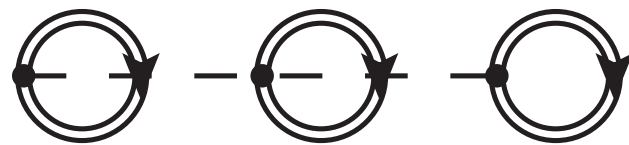
$$\sum_{\alpha} \int_{-\infty}^{E^N - E^{N-1}} d\omega \omega \frac{1}{\pi} \text{Im} G_{\alpha\alpha}(\omega) = \underbrace{\langle \Psi^N | \hat{T} | \Psi^N \rangle}_{\text{Green oval}} + \underbrace{2}_{\text{Red circle}} \underbrace{\langle \Psi^N | \hat{V} | \Psi^N \rangle}_{\text{Red oval}} + \underbrace{3}_{\text{Blue circle}} \underbrace{\langle \Psi^N | \hat{W} | \Psi^N \rangle}_{\text{Blue oval}}$$

$$\frac{E}{A} = \frac{\nu}{\rho} \int \frac{d^3 p}{(2\pi)^3} \int \frac{d\omega}{2\pi} \underbrace{\frac{1}{2}}_{\text{Red oval}} \left\{ \frac{p^2}{2m} + \omega \right\} \mathcal{A}(p, \omega) f(\omega) - \underbrace{\frac{1}{2}}_{\text{Blue oval}} \langle \Psi^N | \hat{W} | \Psi^N \rangle$$

# Modified Koltun sum rule

Carbone, Cipollone, Barbieri, Rios, Polls, PRC **88**, 054326 (2013)

$$\frac{E}{A} = \frac{\nu}{\rho} \int \frac{d^3p}{(2\pi)^3} \int \frac{d\omega}{2\pi} \frac{1}{2} \left\{ \frac{p^2}{2m} + \omega \right\} \mathcal{A}(p, \omega) f(\omega) - \frac{1}{2} \langle \Psi^N | \hat{W} | \Psi^N \rangle$$



1st order fully dressed

$$\frac{E}{A} = \frac{\nu}{\rho} \int \frac{d^3p}{(2\pi)^3} \int \frac{d\omega}{2\pi} \frac{1}{2} \left\{ \frac{p^2}{2m} + \omega - \frac{1}{3} \Sigma_{HF}^{3NF}(p) \right\} \mathcal{A}(p, \omega) f(\omega)$$



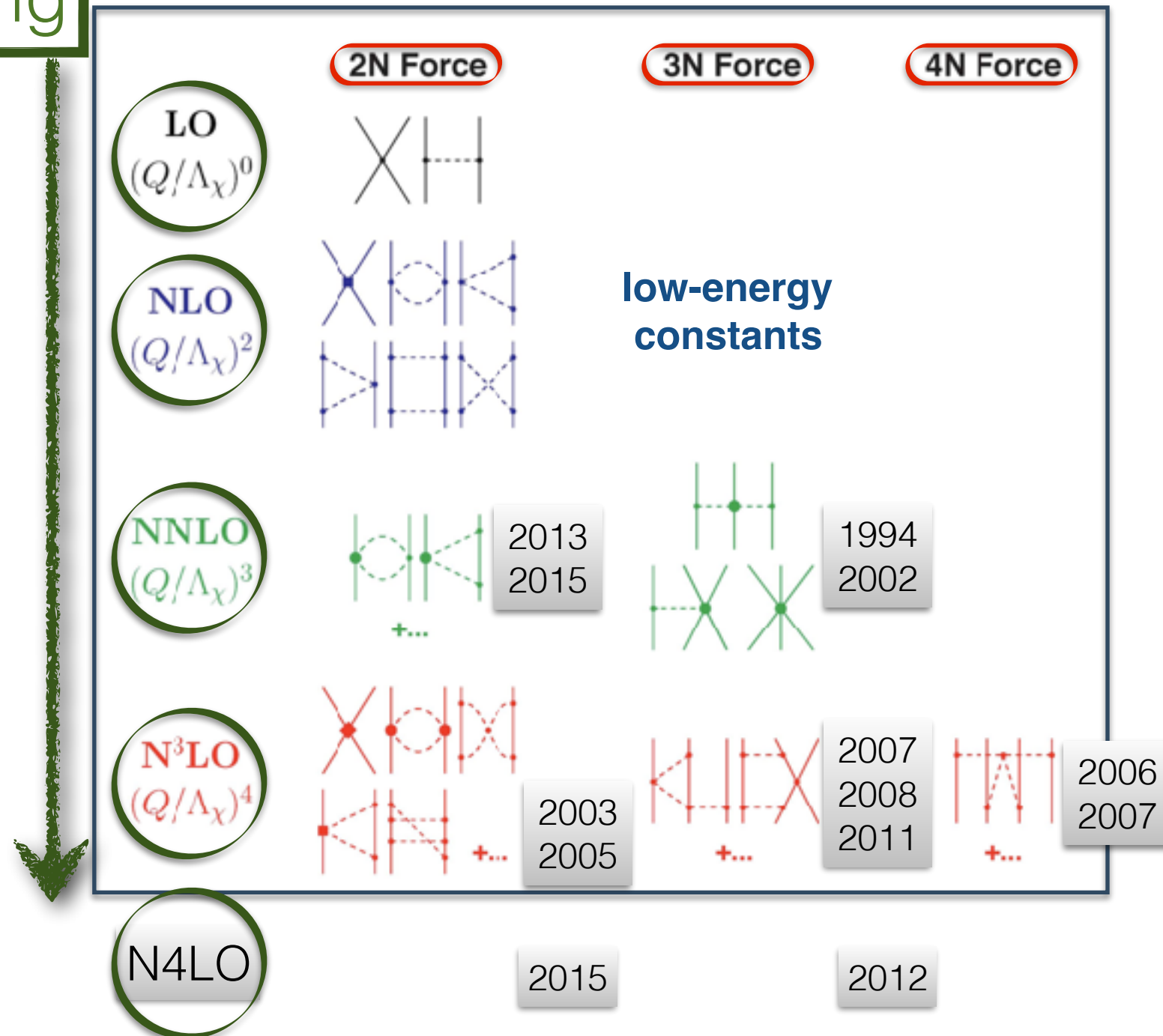
# Why nuclear matter from chiral EFT?

## Power counting

- Effective theory of QCD
- Nucleons & pions as d.o.f.
- Power counting expansion
- Hierarchy of many-body forces
- Theoretical uncertainties

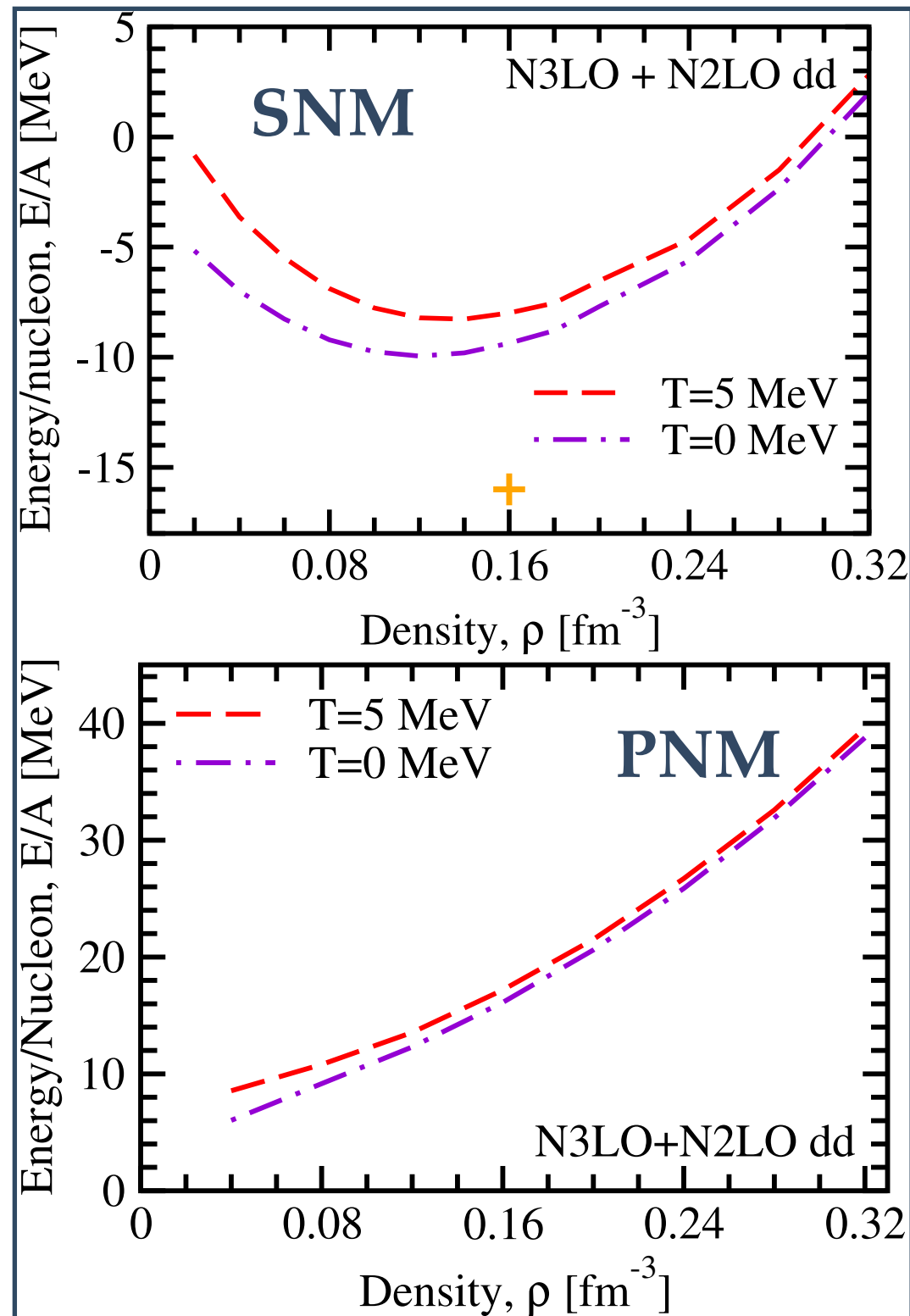
Over 20 years of ongoing improvement

Epelbaum *et al.*, Rev. Mod. Phys. 81, 1773(2009)  
Machleidt *et al.*, Phys. Rep. 503, 1 (2011)



# Temperature dependence

Macroscopic  
properties



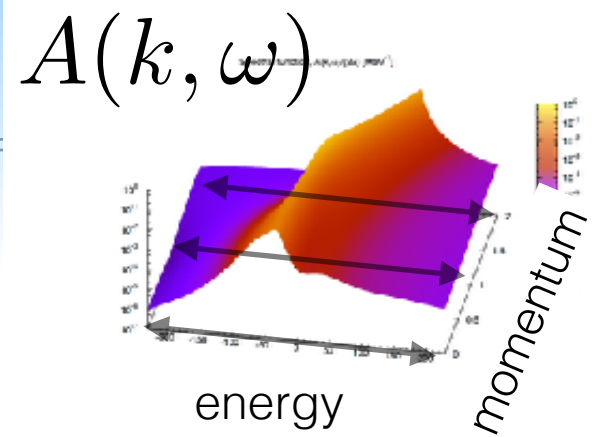
- ★ overcome pairing instability at zero- $T$
- ★ exploit the Sommerfeld expansion for low temperatures
  - ★ energy:  $e \sim e_0 + a T^2$
  - ★ free-energy:  $f \sim f_0 - a T^2$
- ★ semi-sum is an estimate of zero- $T$  results

$$\Delta E_{\text{SNM}}(\rho_0) \sim 1.4 \text{ MeV}, \Delta E_{\text{PNM}}(\rho_0) \sim 1 \text{ MeV}$$

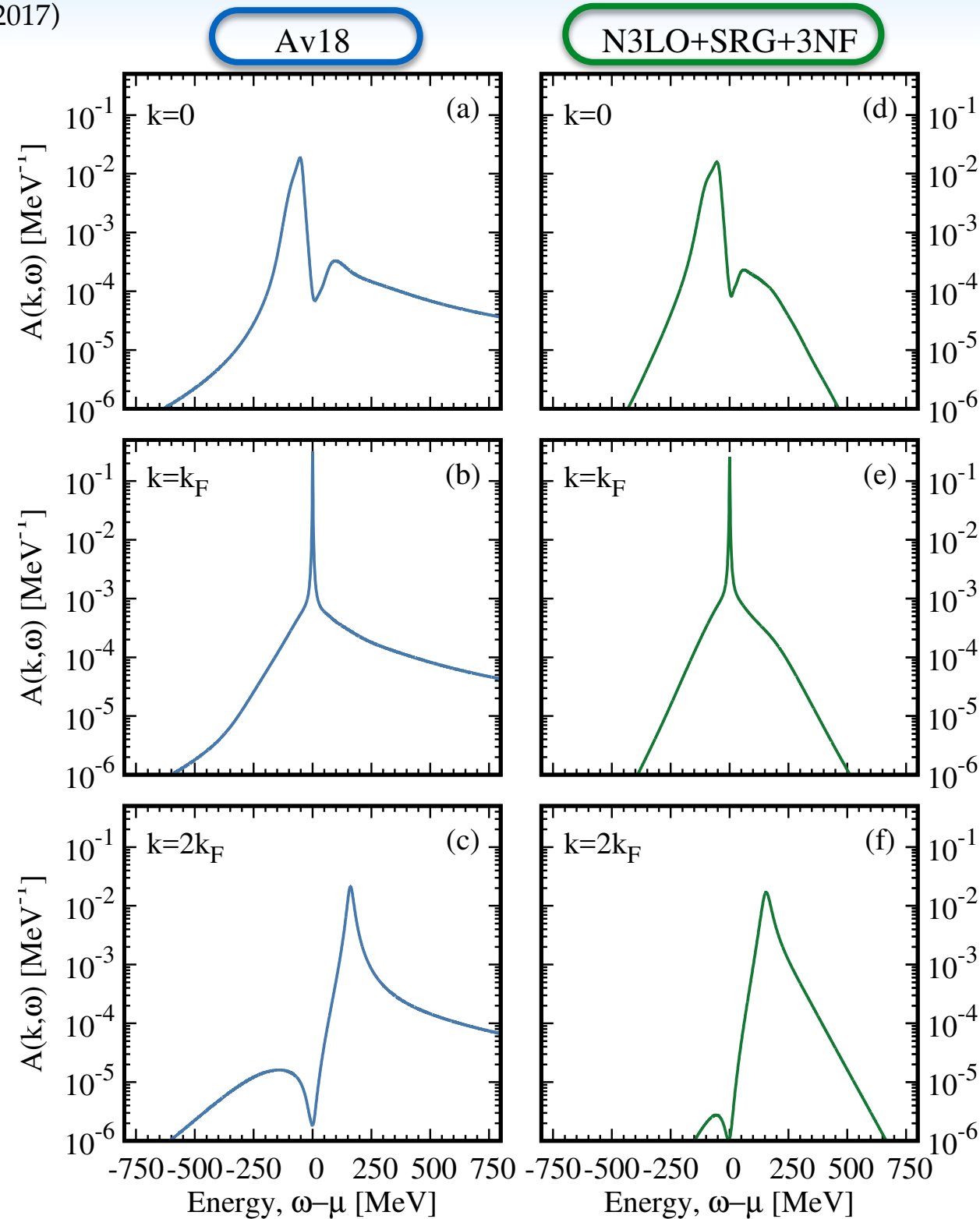
# Spectral vision of the nuclear interaction

Rios, Carbone, Polls, PRC 96, 014003 (2017)

Hard-core



Soft-core

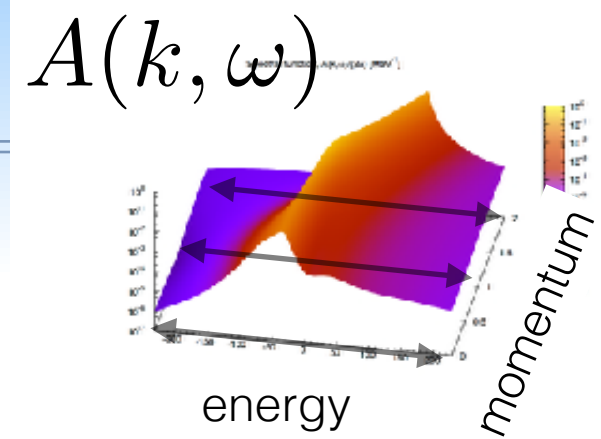


$\rho = 0.20 \text{ fm}^{-3}$   
 $k_F = 283 \text{ MeV}$   
 SNM



# Spectral vision of the nuclear interaction

Rios, Carbone, Polls, PRC 96, 014003 (2017)



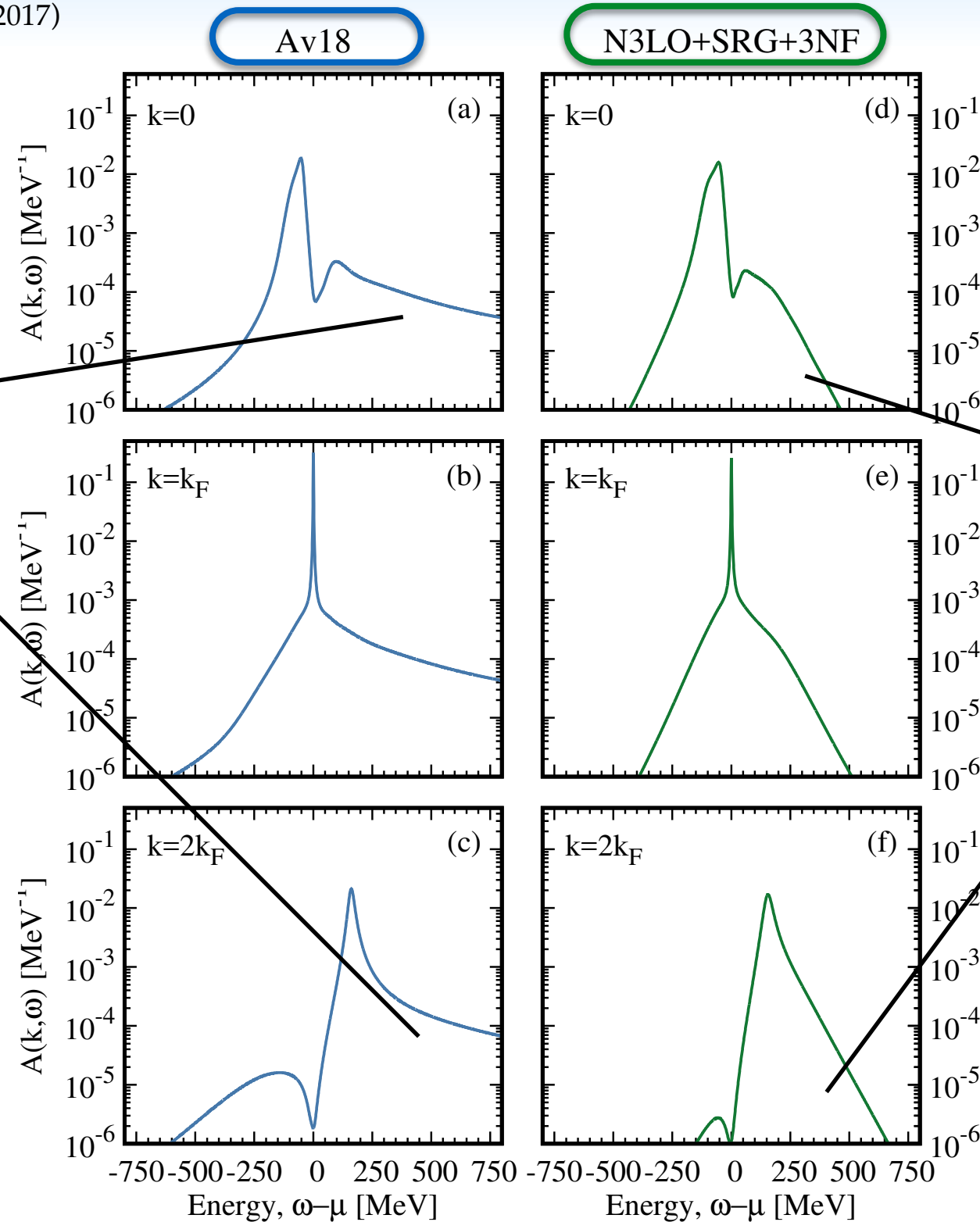
Hard-core

Soft-core

high-energy tails populated

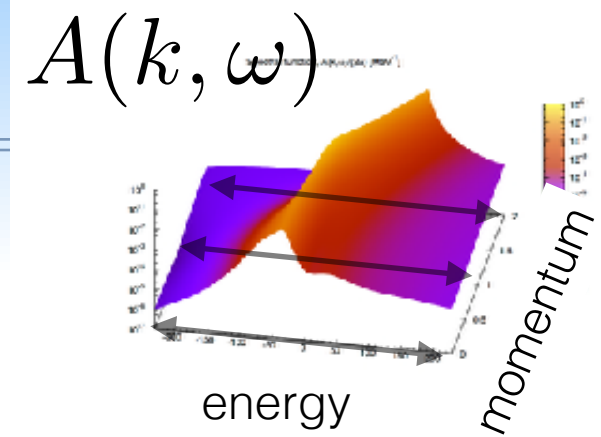
high-energy tails suppressed

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Rios, Carbone, Polls, PRC 96, 014003 (2017)



Hard-core

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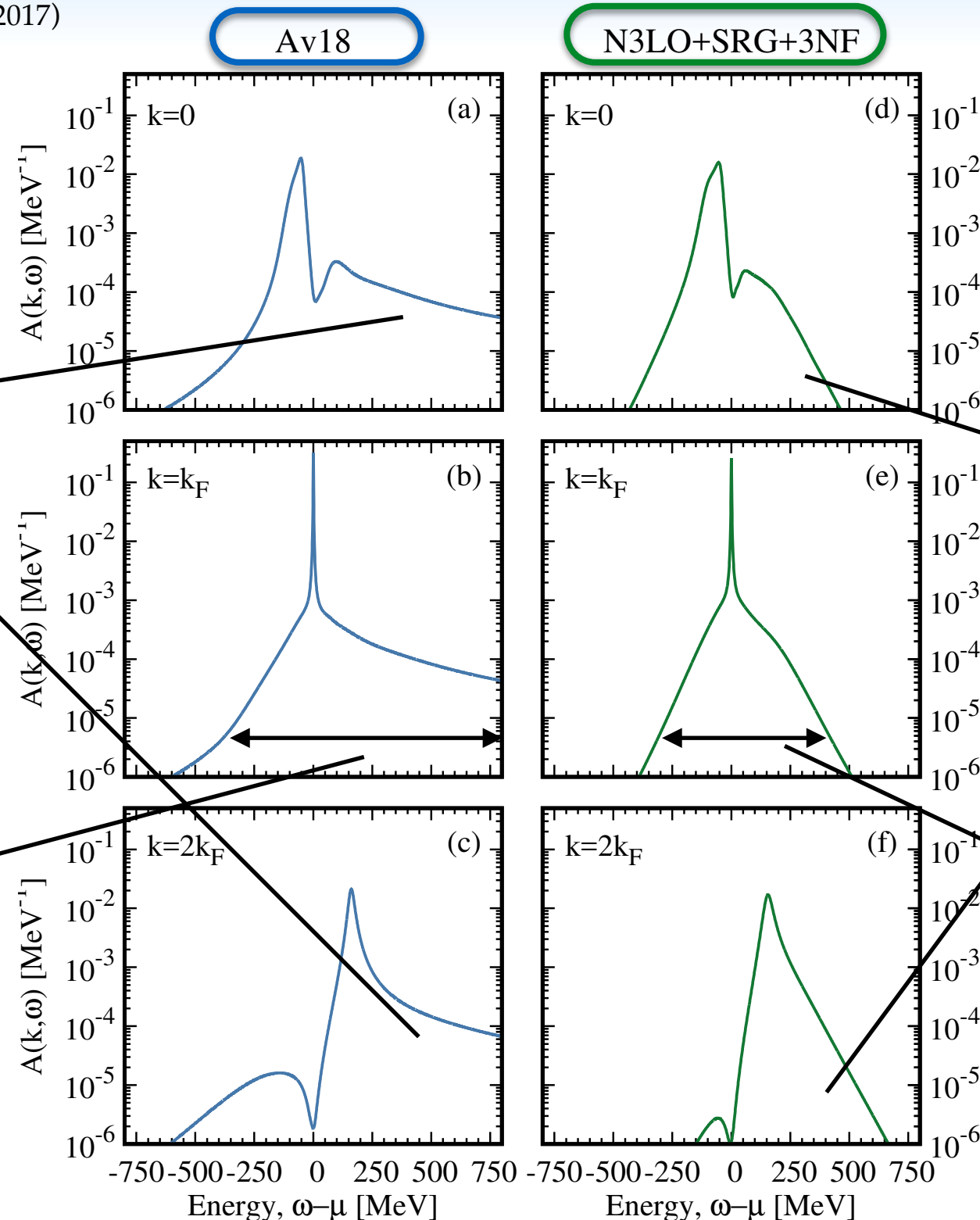
bigger fragmentation

smaller fragmentation

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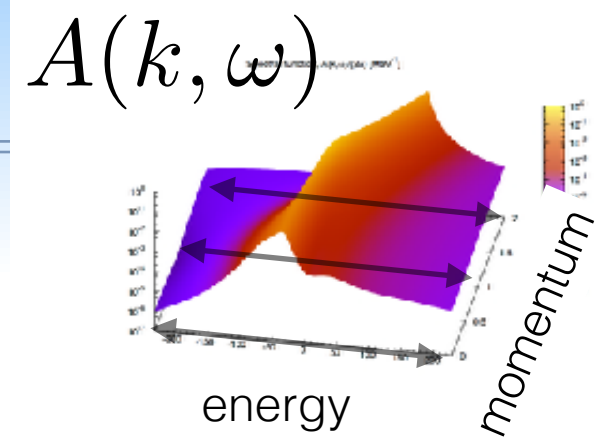
SNM





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Rios, Carbone, Polls, PRC 96, 014003 (2017)



Hard-core

Soft-core

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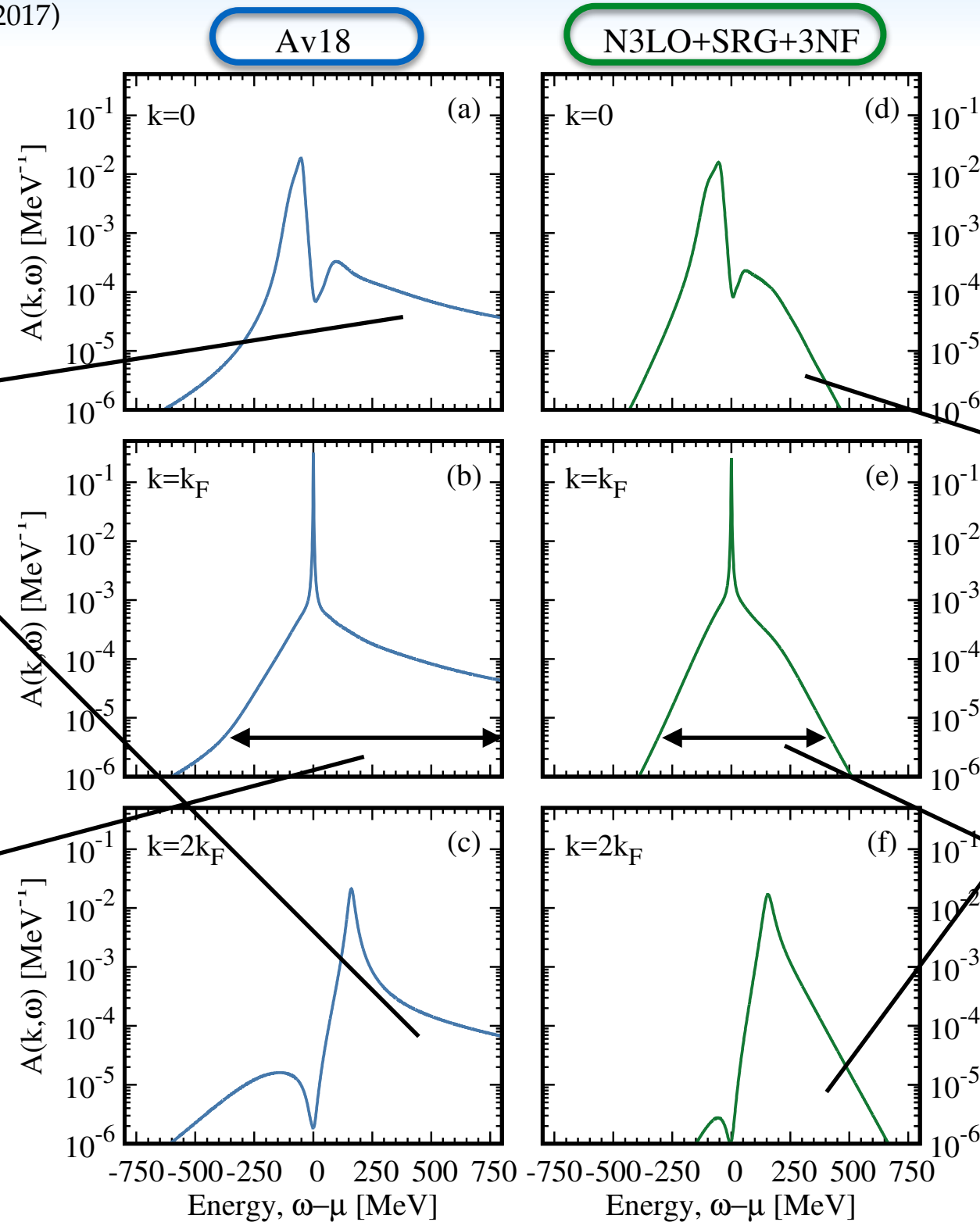
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SNM



$$m_k^{(n)} = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \omega^n \mathcal{A}_k(\omega)$$

Understand fragmentation of nuclear states from sum rules





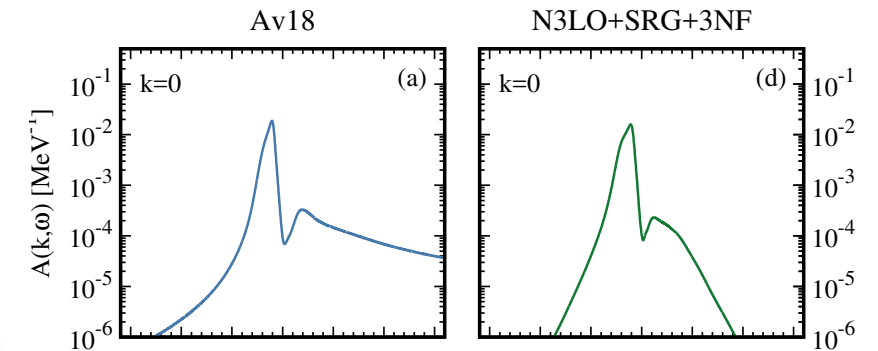
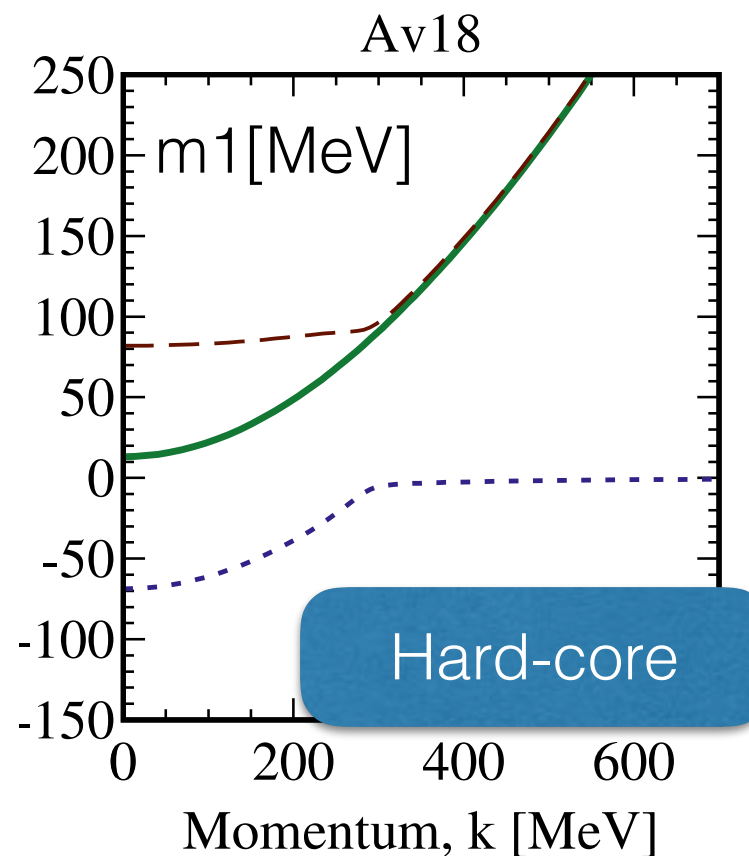
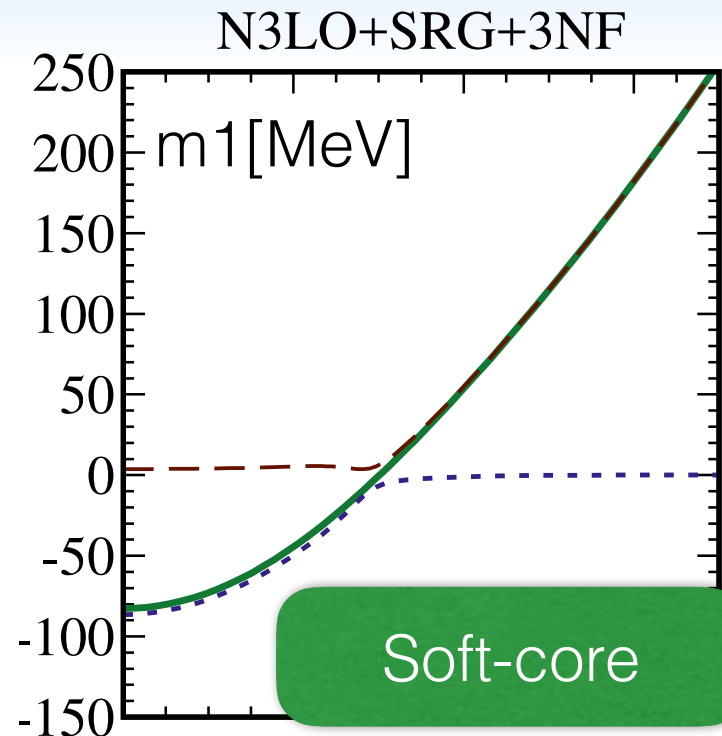
# Understanding nuclear states fragmentation

$$m_k^{(1)} = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \omega \mathcal{A}_k(\omega)$$

mean distribution

$$m_k^{(1)} = \frac{\hbar^2 k^2}{2m} + \sum_k^{\infty}$$

Hartree-Fock like energy:  
increasing with k



$$\rho = 0.20 \text{ fm}^{-3}$$

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SNM

# Understanding nuclear states fragmentation

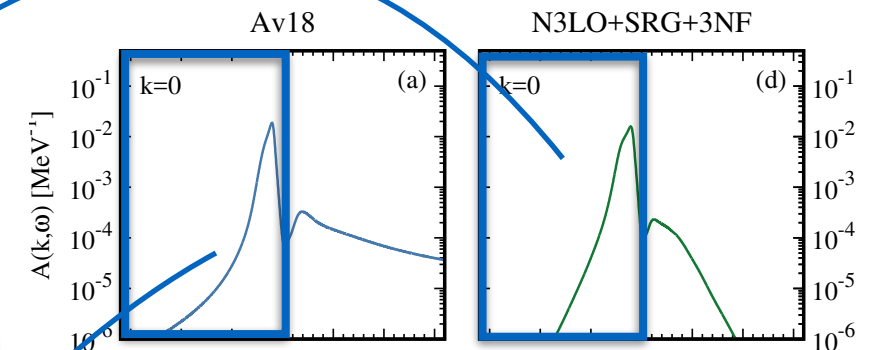
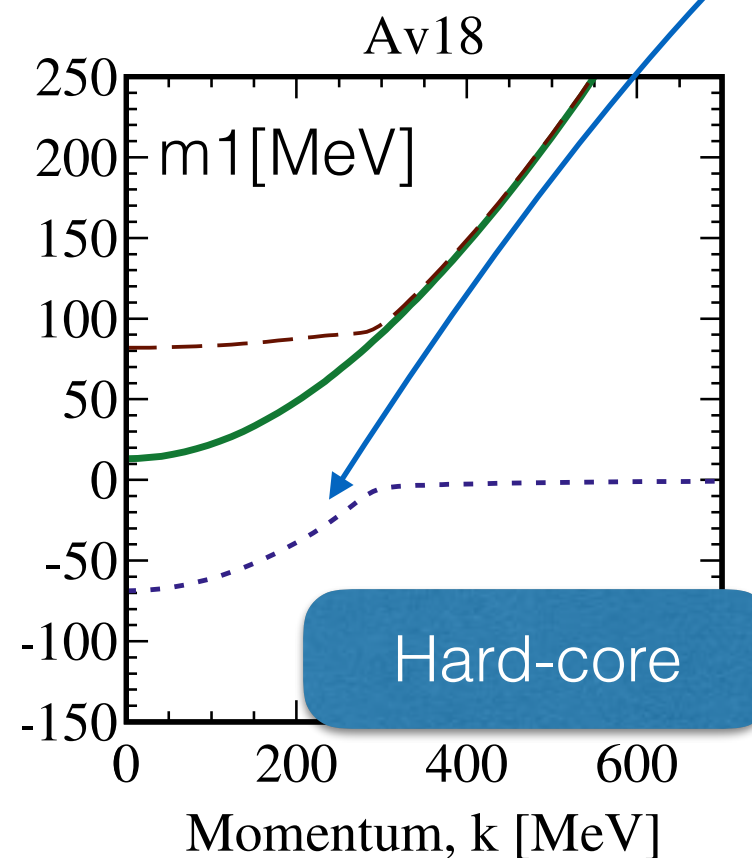
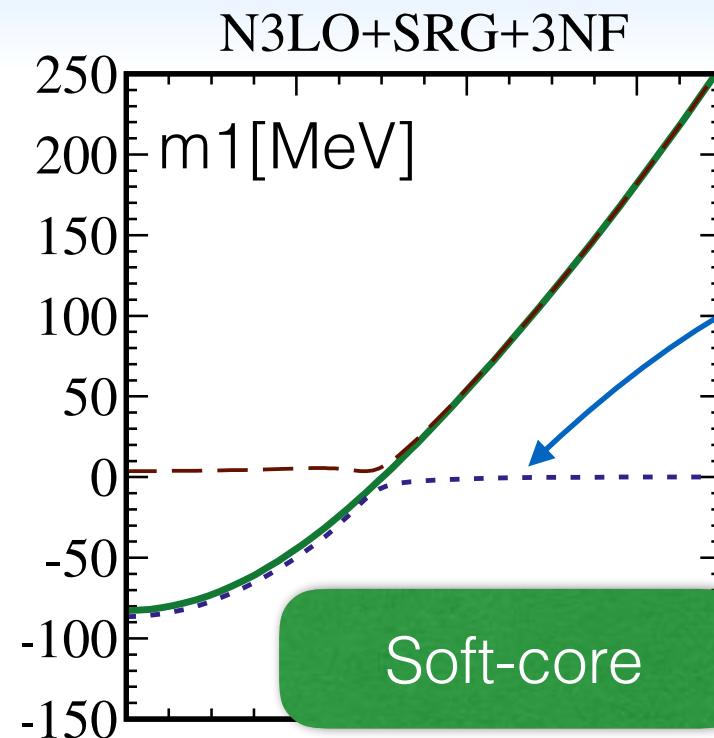
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Hartree-Fock like energy:  
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1. • strength of integrated peak below  $k_F$   
• strongly suppressed above  $k_F$



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SNM

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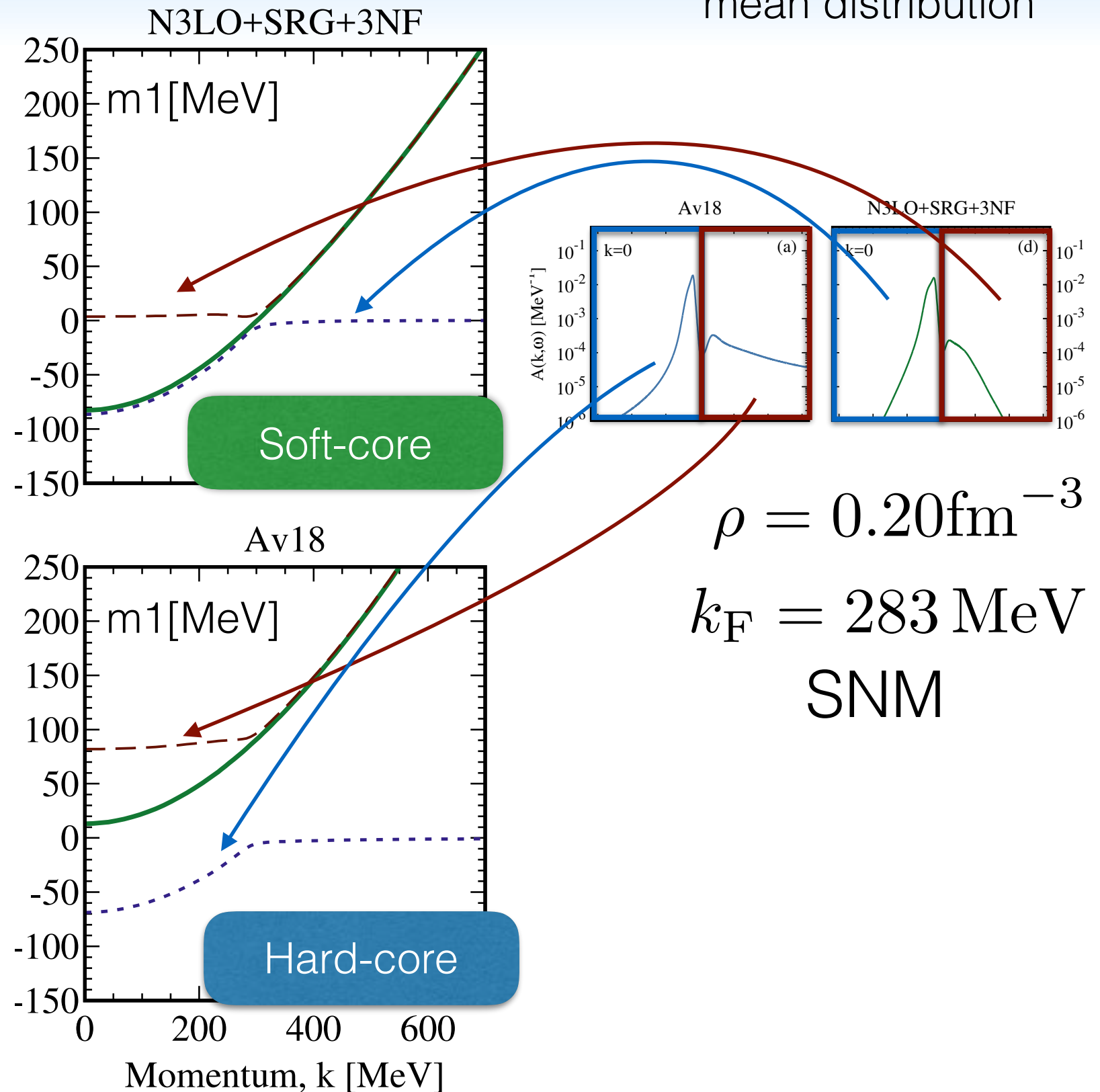
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• steep increase above  $k_F$ , peak integrated

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SNM

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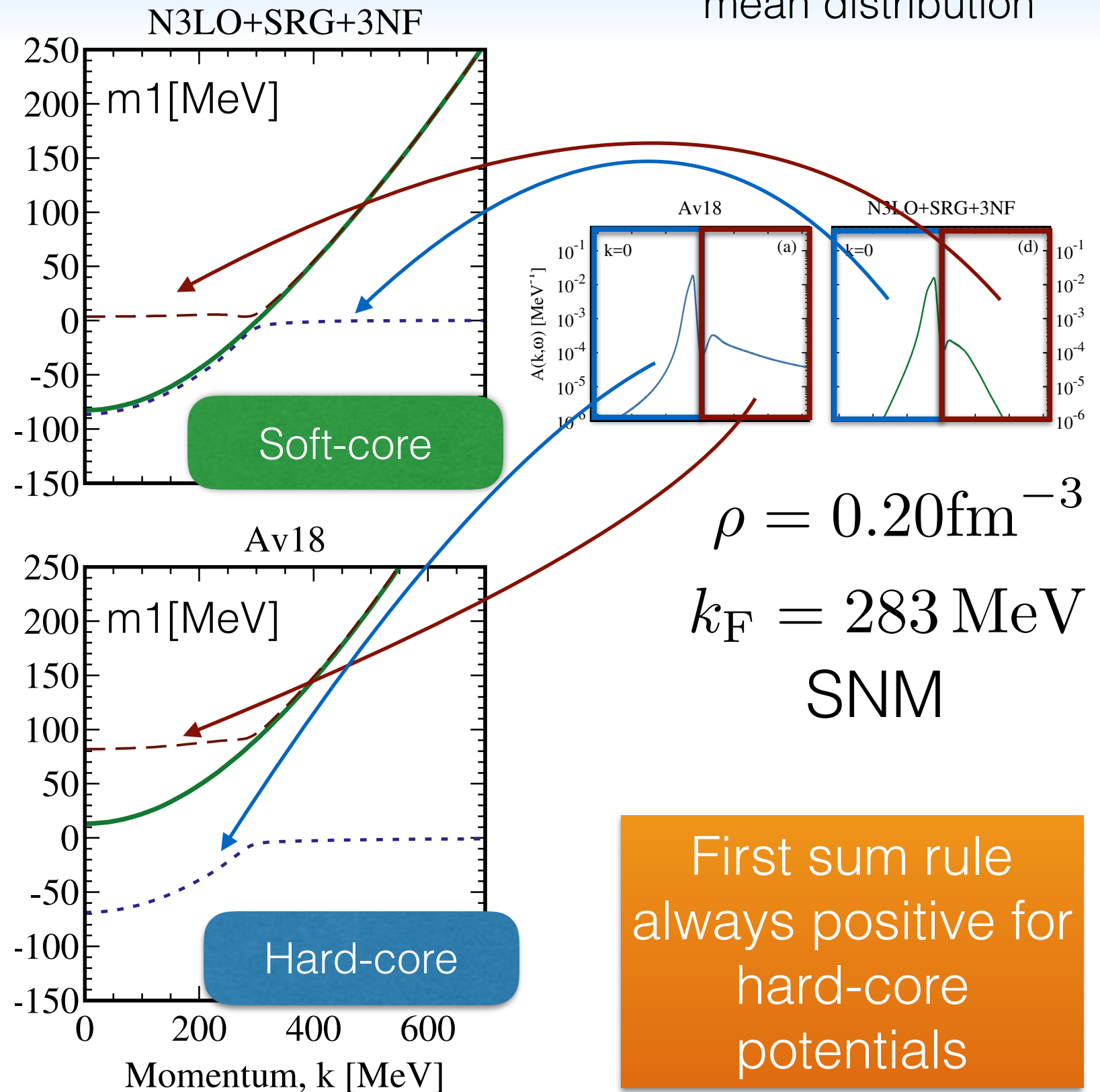
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Rios, Carbone, Polls, PRC 96, 014003 (2017)

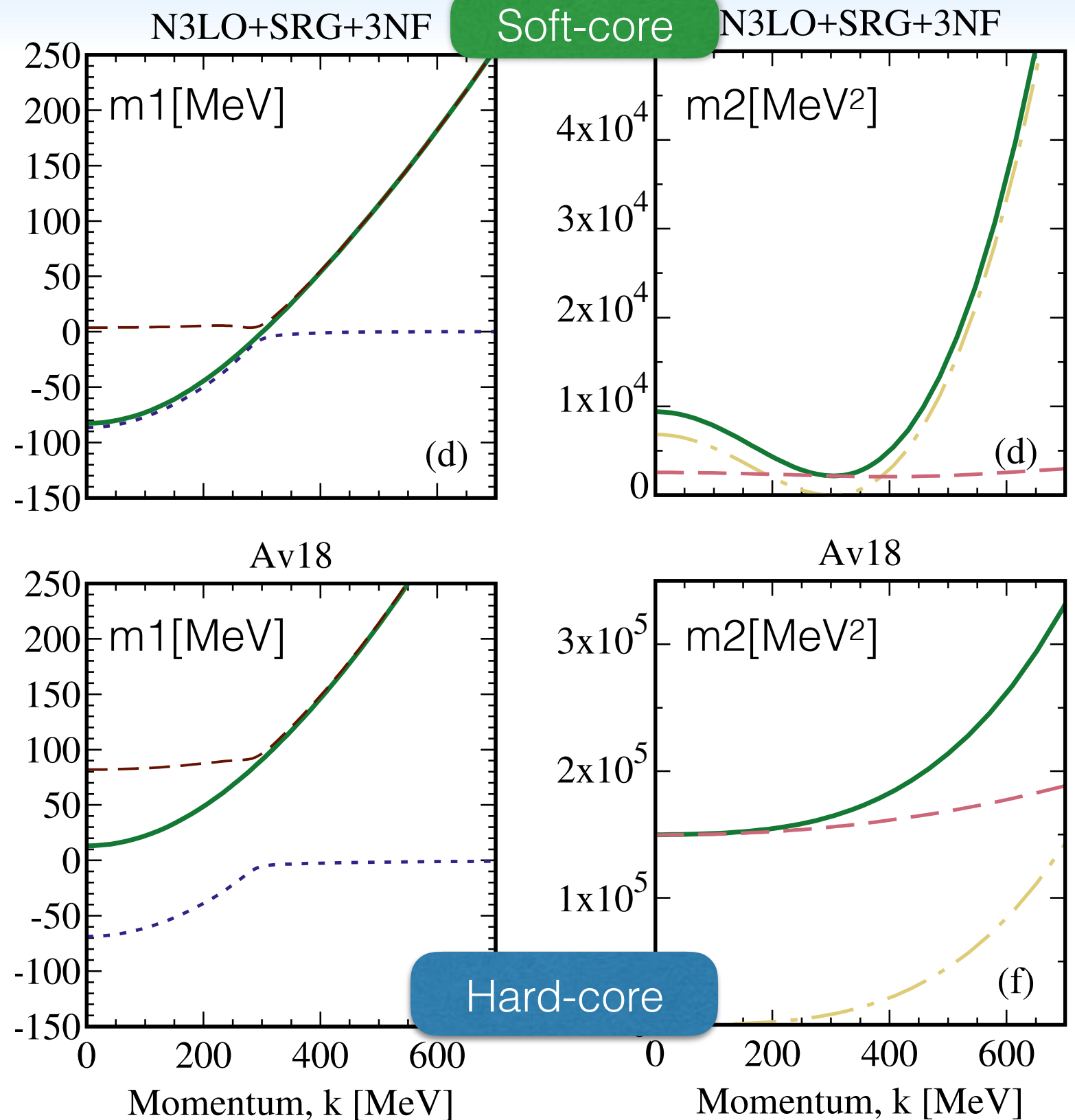


# Understanding nuclear forces from sum rules

$$m_k^{(2)} = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \omega^2 \mathcal{A}_k(\omega)$$

Rios, Carbone, Polls, PRC 96, 014003 (2017)

$$m_k^{(2)} = [m_k^{(1)}]^2 + \sigma_k^2$$



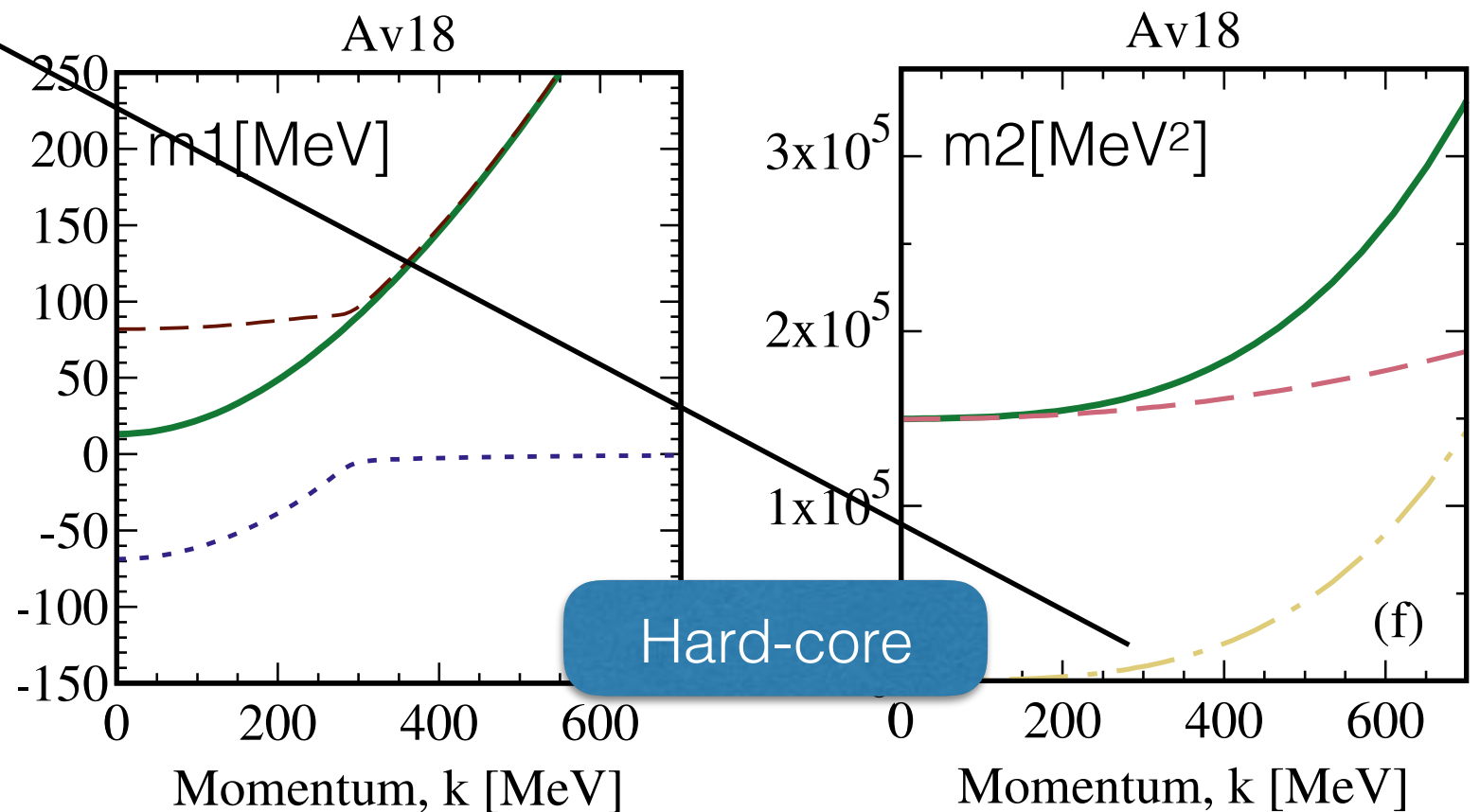
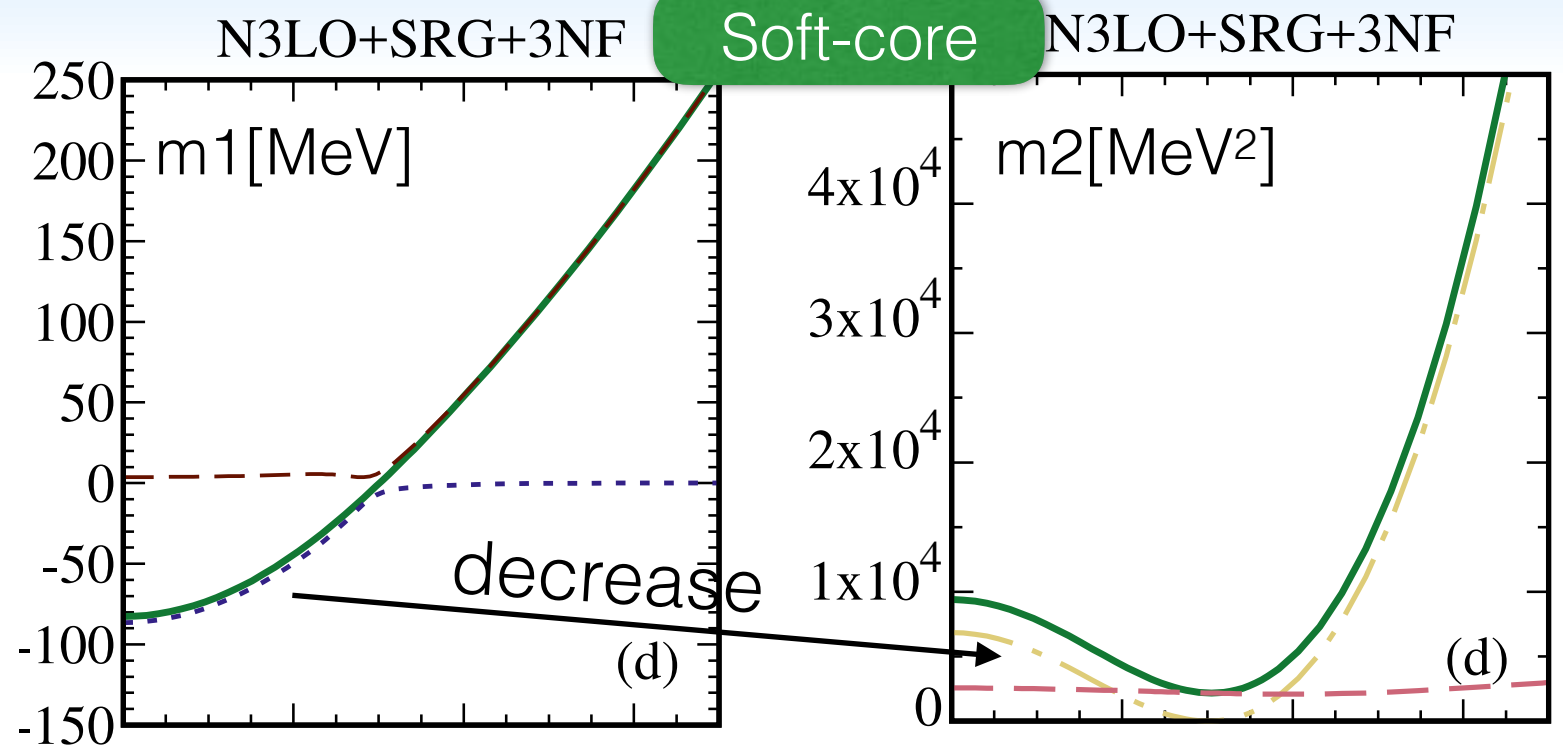
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Rios, Carbone, Polls, PRC 96, 014003 (2017)

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Quadratic first sum rule always increasing for hard-core potentials





# Understanding nuclear forces from sum rules

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Rios, Carbone, Polls, PRC 96, 014003 (2017)

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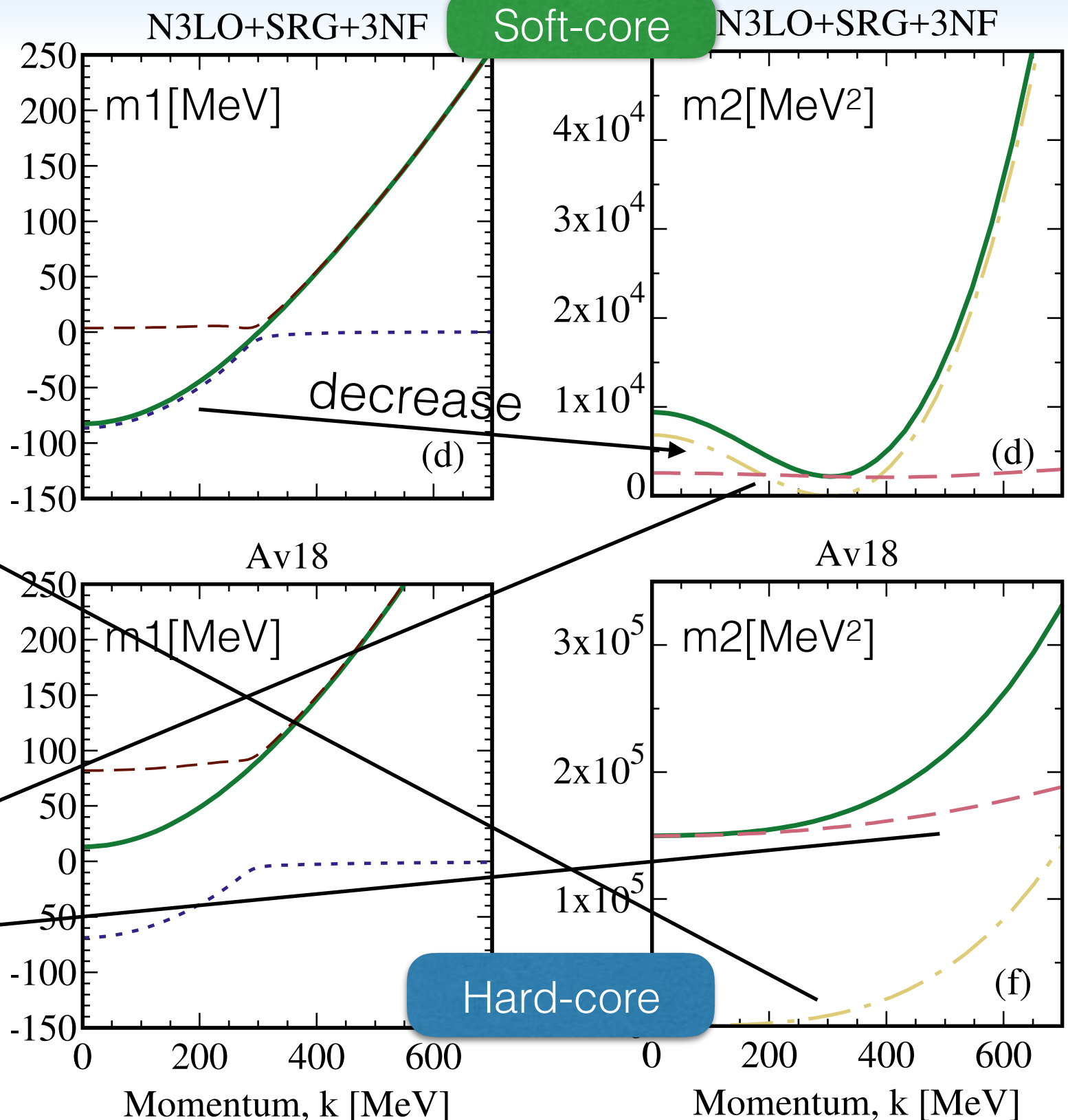
Quadratic first sum rule always increasing for hard-core potentials

variance of the spectral function

negligible for soft forces increasing for hard ones

$$\sigma_k^2 = - \int_{-\infty}^{+\infty} \frac{d\omega}{\pi} \text{Im} \Sigma_k(\omega)$$

beyond mean-field effects



# The variance as a proxy of softness

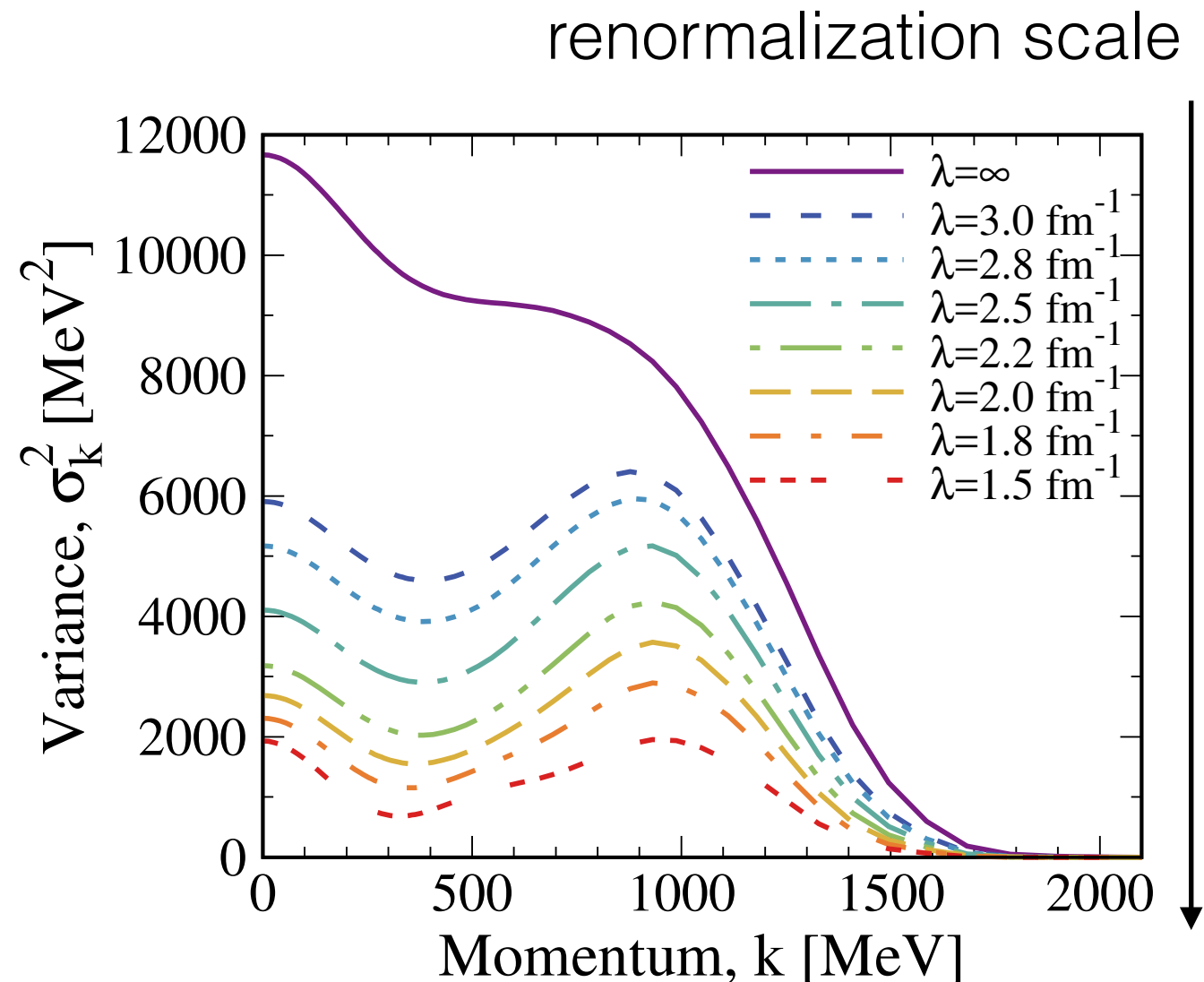
$$\rho = 0.20 \text{ fm}^{-3}$$
$$k_F = 283 \text{ MeV}$$

Rios, Carbone, Polls, PRC 96, 014003 (2017)

SNM

$$\sigma_k^2 = - \int_{-\infty}^{+\infty} \frac{d\omega}{\pi} \text{Im} \Sigma_k(\omega)$$

- beyond-mean field effects
- strongly suppressed in soft forces
- less fragmentation of states
- narrower quasi-particle peaks
- variance varies with RG scale



# The variance as a proxy of softness

$$\rho = 0.20 \text{ fm}^{-3}$$

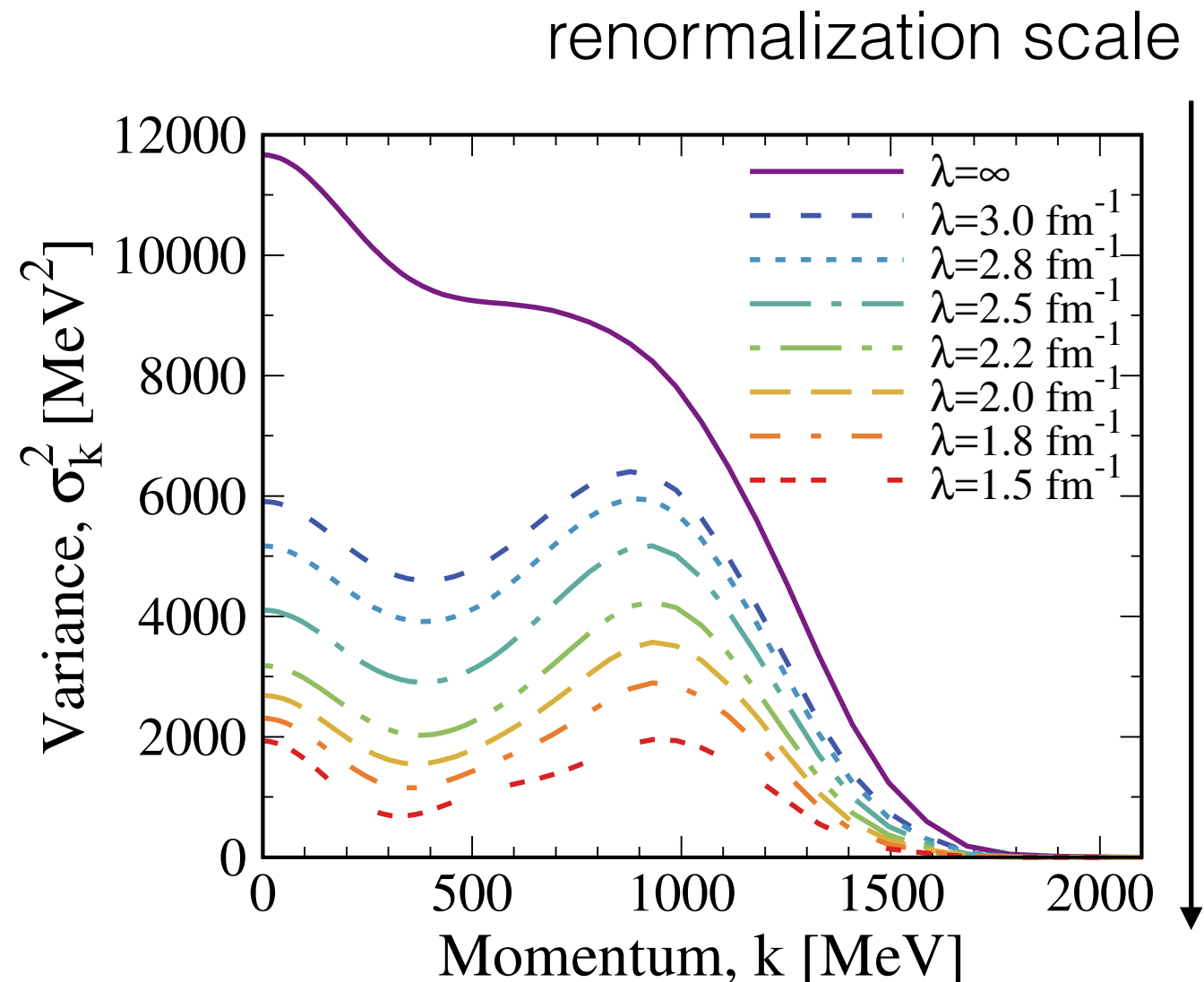
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SNM

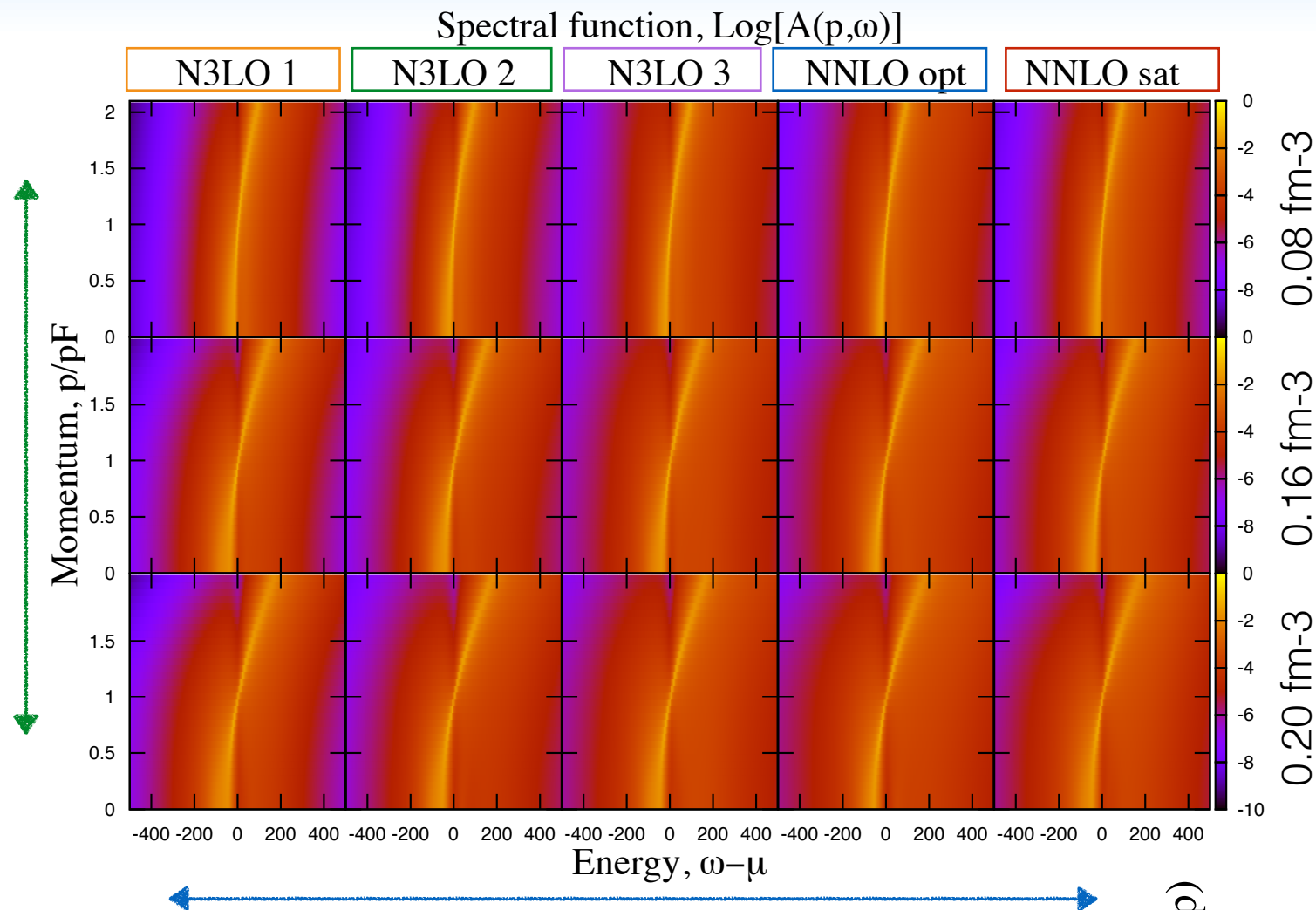
$$\sigma_k^2 = - \int_{-\infty}^{+\infty} \frac{d\omega}{\pi} \text{Im} \Sigma_k(\omega)$$

- beyond-mean field effects
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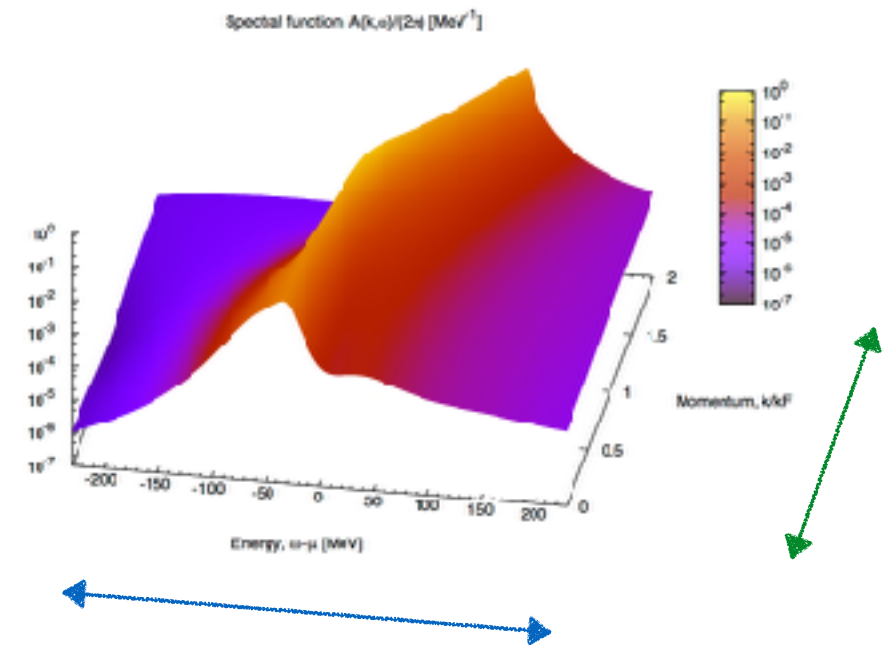


Sum rules provide a measurement of hardness (or softness) of nuclear potential  
Relation to effective single-particle energies in finite nuclei

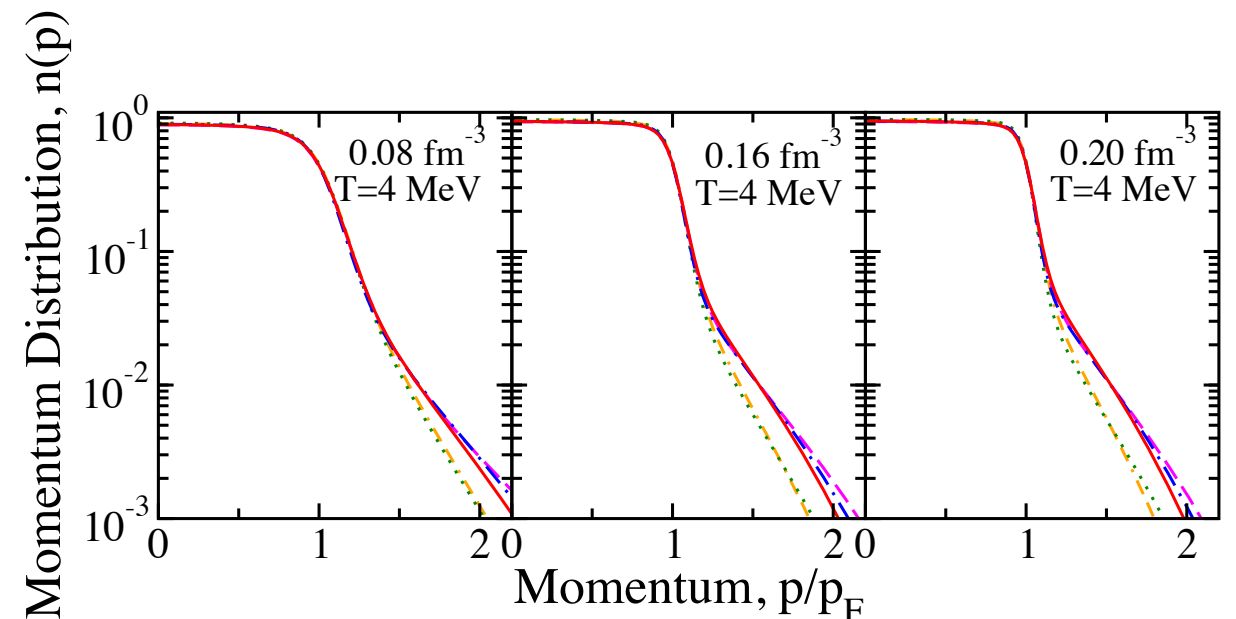
# Momentum distribution according to different Hamiltonians



$$n(p) = \int \frac{d\omega}{2\pi} \mathcal{A}(p, \omega) f(\omega)$$



- energy tails affected by the cutoff on the NN force
- high-momentum region also affected by cutoff and density dependence
- effects clearly visible in momentum distribution



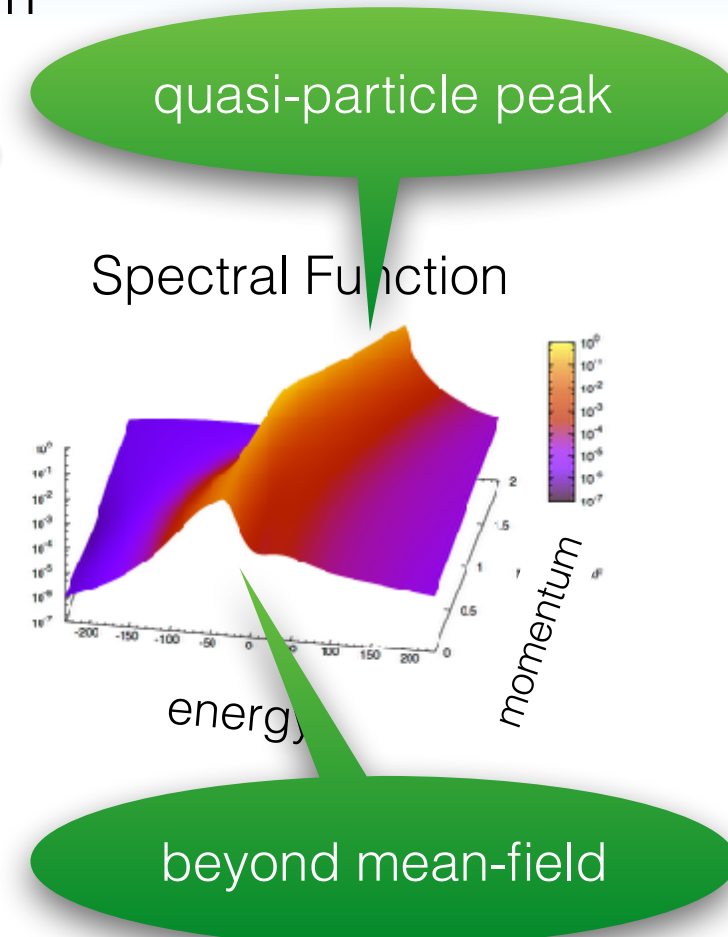
# Self-consistent Green's functions

- Green's function: a tool to solve the nuclear many-body problem

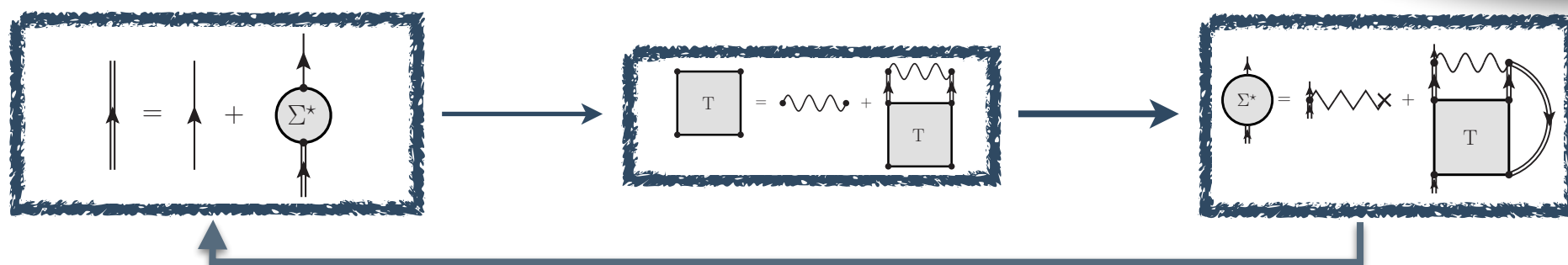
$$G_{\alpha\beta}(\omega) = \sum_m \frac{\langle \Psi_0^N | a_\alpha | \Psi_m^{N+1} \rangle \langle \Psi_m^{N+1} | a_\beta^\dagger | \Psi_0^N \rangle}{\omega - (E_m^{N+1} - E_0^N) + i\eta} + \sum_n \frac{\langle \Psi_0^N | a_\beta^\dagger | \Psi_n^{N-1} \rangle \langle \Psi_n^{N-1} | a_\alpha | \Psi_0^N \rangle}{\omega - (E_0^N - E_n^{N-1}) + i\eta}$$

*particle exits* *particle enters* *hole exits* *hole enters*

*energy with an added particle* *energy with a removed particle*



- Self-consistent nonperturbative method:



**Dyson equation**

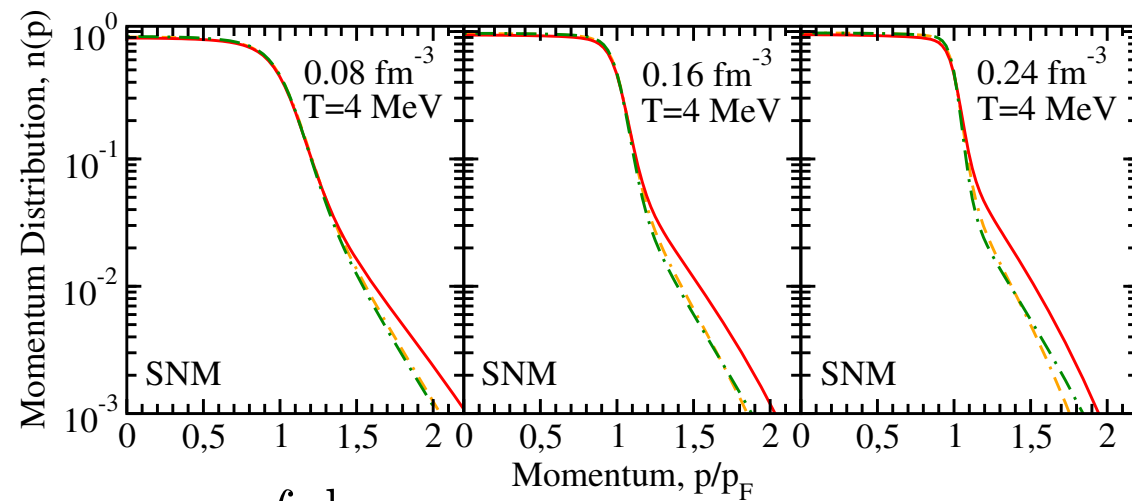
- Breakthrough: full formal extension to consistently include 3BFs

Carbone, Cipollone, Barbieri, Rios, Polls, PRC **88**, 054326 (2013)

# From microscopic... to macroscopic

**Symmetric nuclear matter**

The microscopic picture: momentum distribution



$$n(p) = \int \frac{d\omega}{2\pi} \mathcal{A}(p, \omega) f(\omega)$$

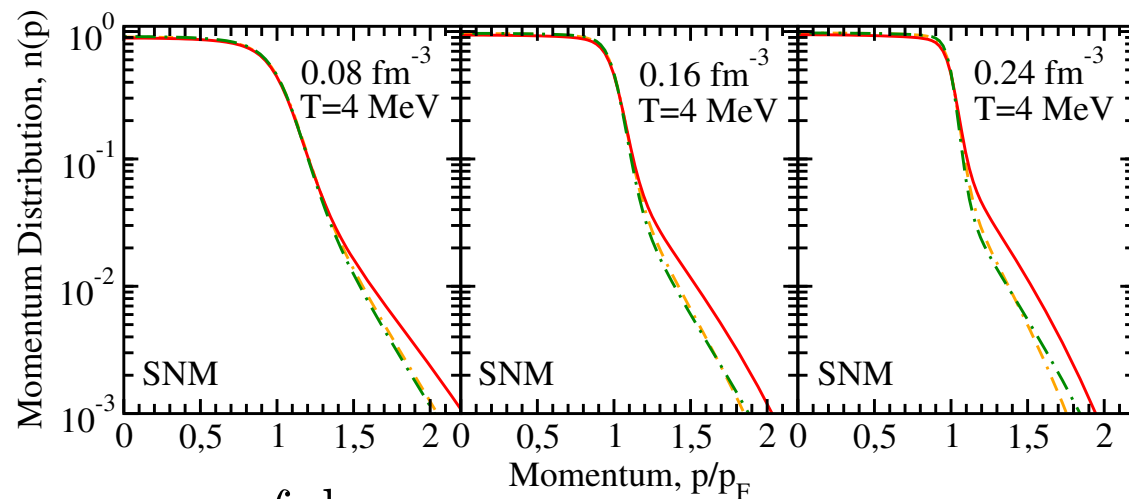
- N2LOsat high-momentum states



# From microscopic... to macroscopic

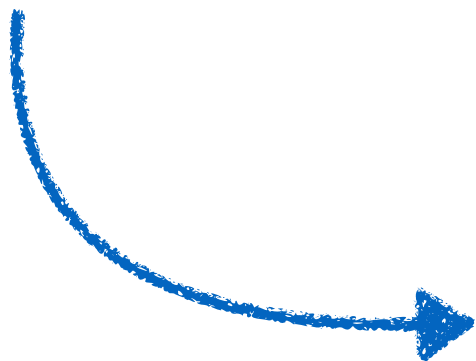
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The microscopic picture: momentum distribution



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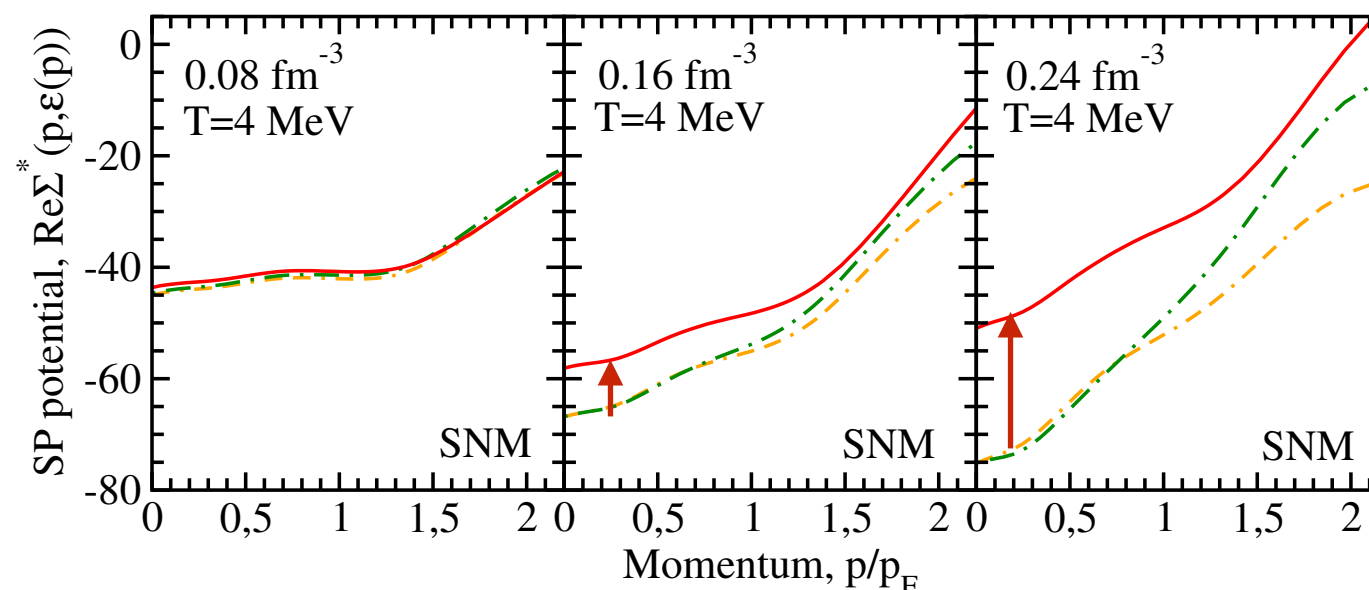
- N2LOsat high-momentum states



- 3NF effects as density increases
- N2LOsat more repulsive

...start seeing the big picture: the self-energy

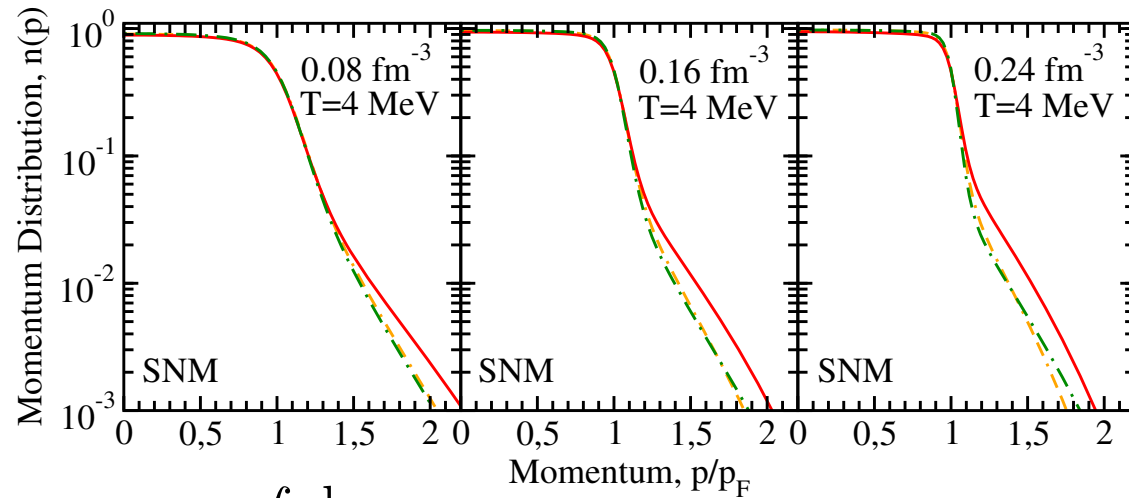
$$\varepsilon_{qp}(p) = \frac{p^2}{2m} + \text{Re}\Sigma^*(p, \varepsilon_{qp}(p))$$



# From microscopic... to macroscopic

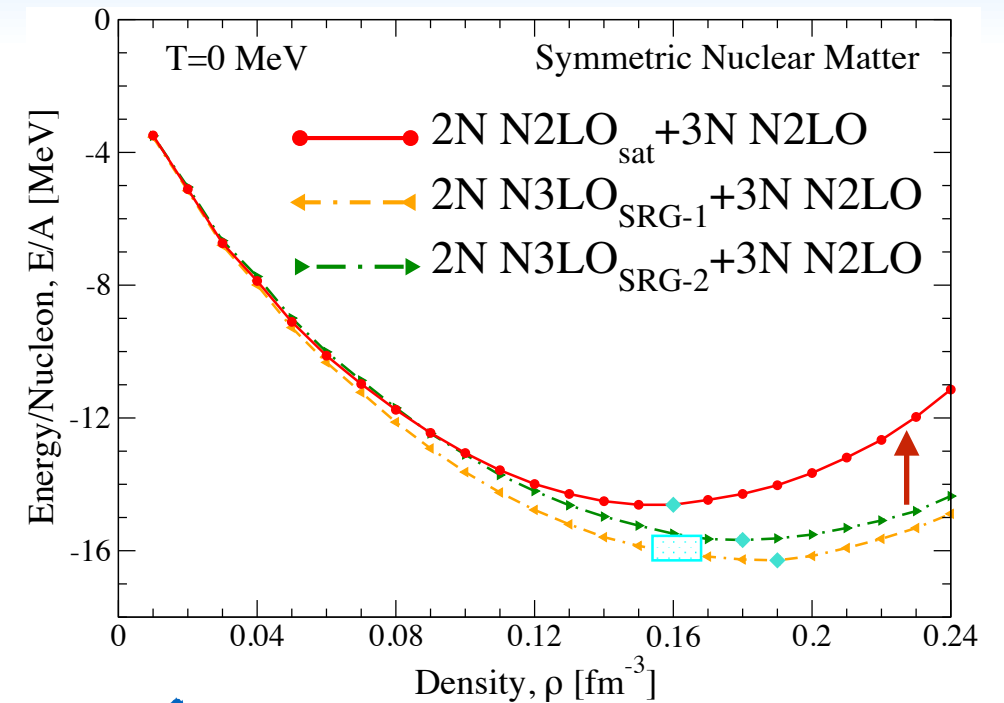
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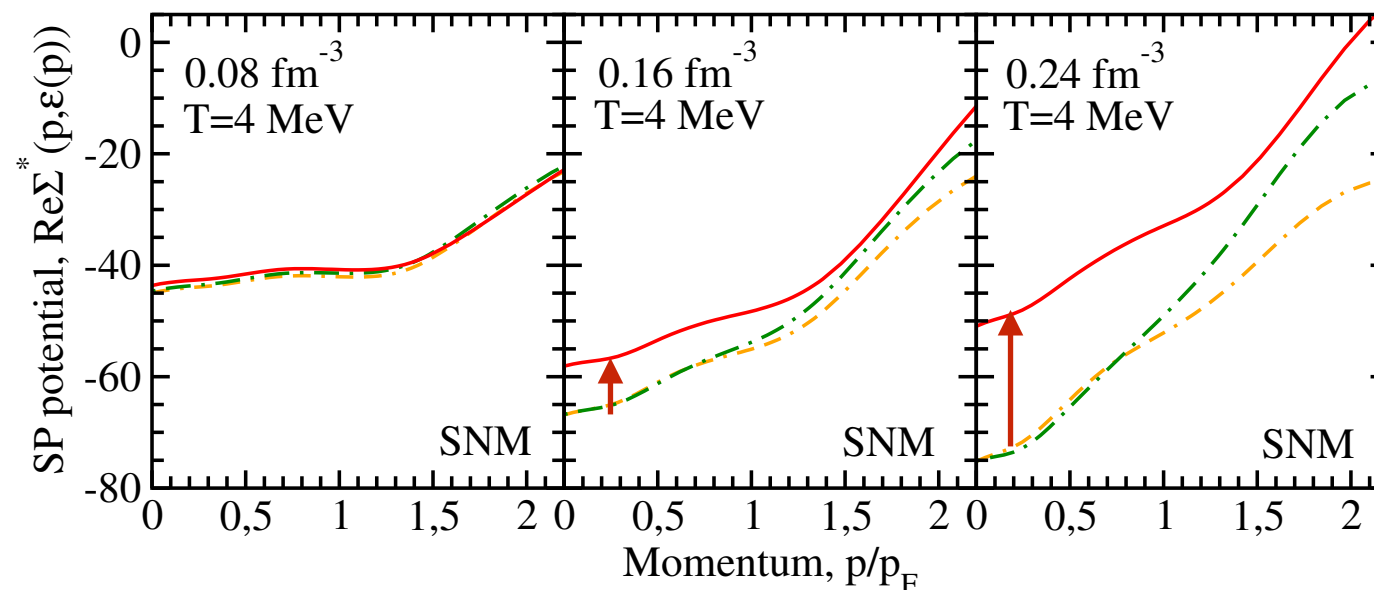
- N2LOsat high-momentum states



..the macroscopic picture:  
total energy more repulsive

...start seeing the big  
picture: the self-energy

$$\varepsilon_{qp}(p) = \frac{p^2}{2m} + \text{Re}\Sigma^*(p, \varepsilon_{qp}(p))$$

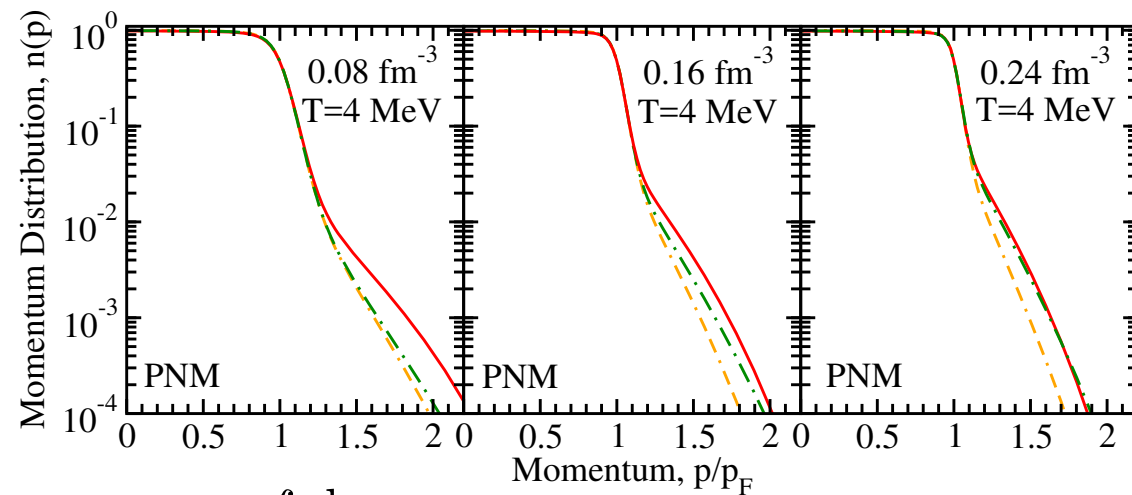


- 3NF effects as density increases
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# From microscopic... to macroscopic

*Pure neutron matter*

The microscopic picture: momentum distribution



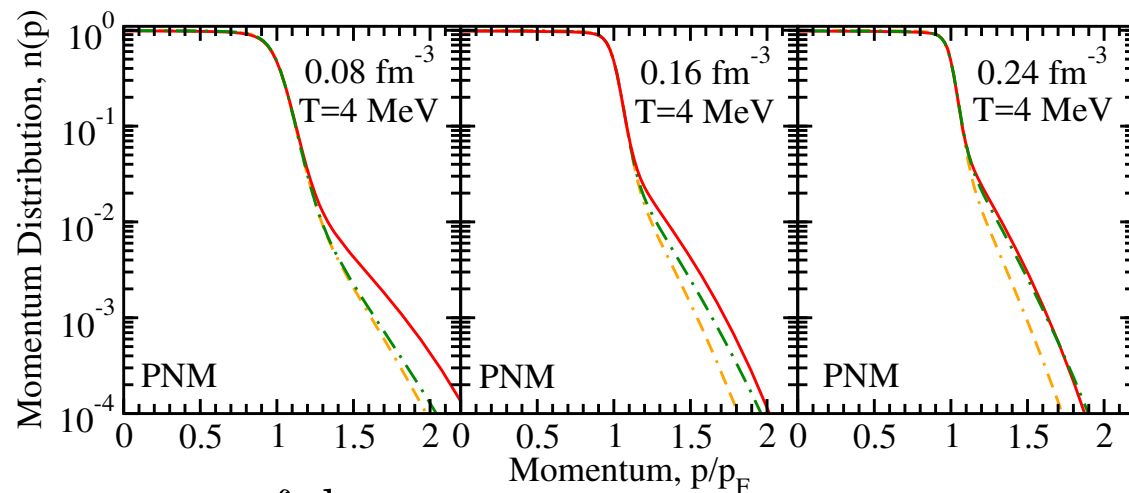
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- N2LOsat high-momentum states

# From microscopic... to macroscopic

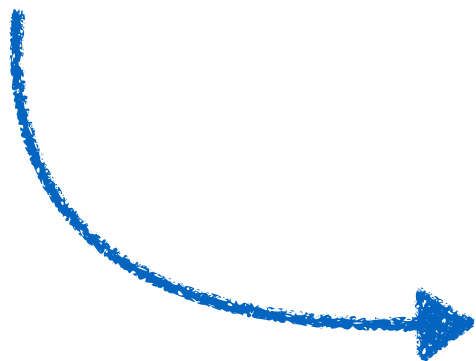
Pure neutron matter

The microscopic picture: momentum distribution



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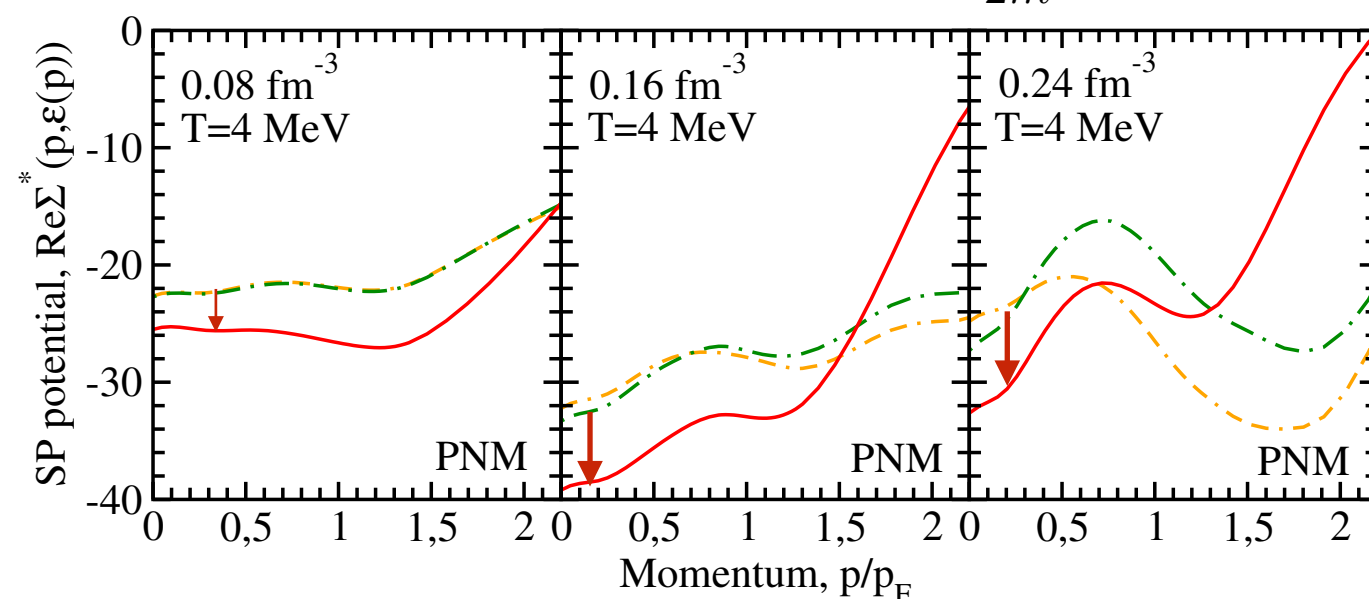
- N2LOsat high-momentum states



- 3NF effects are reversed
- N2LOsat more attractive

...start seeing the big picture: the self-energy

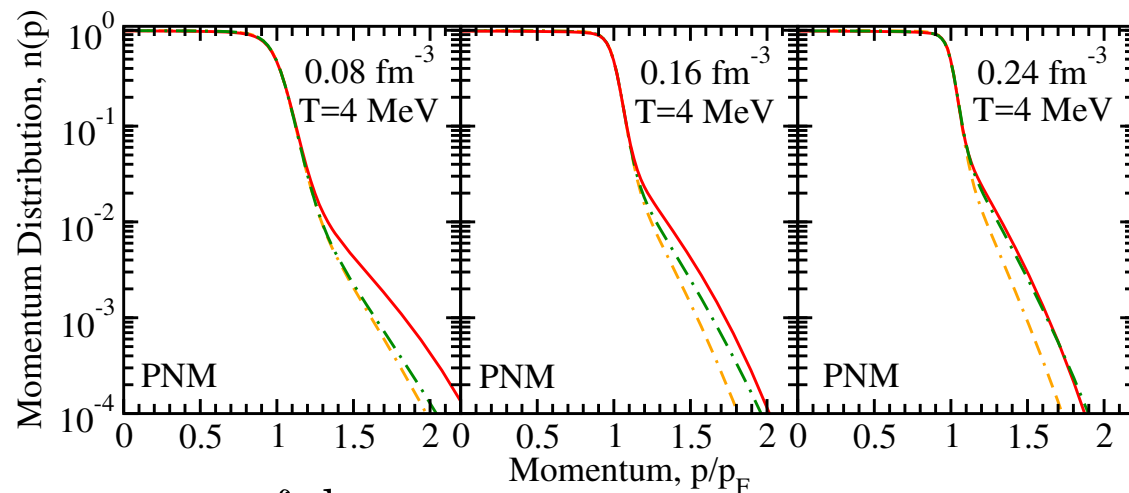
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# From microscopic... to macroscopic

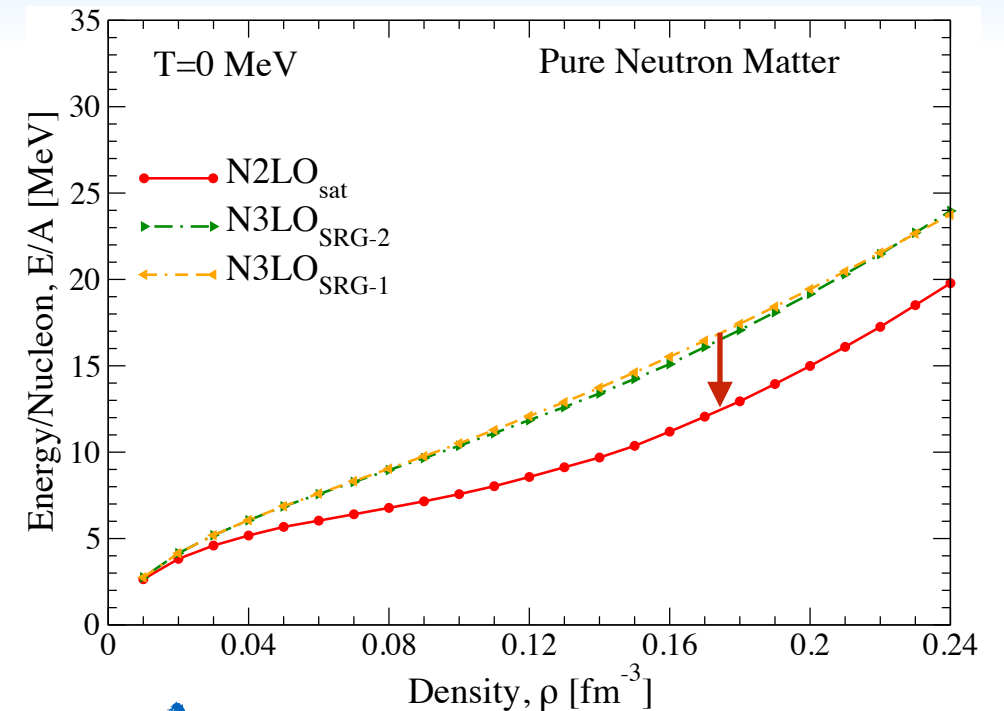
Pure neutron matter

The microscopic picture: momentum distribution



$$n(p) = \int \frac{d\omega}{2\pi} \mathcal{A}(p, \omega) f(\omega)$$

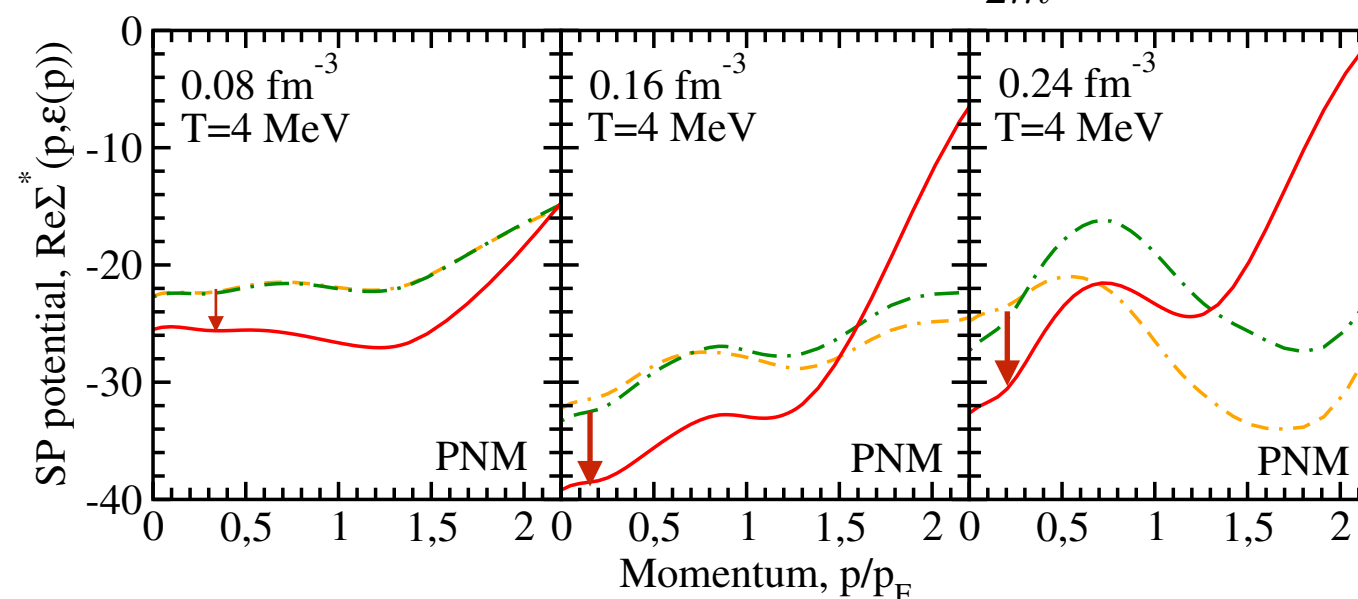
- N2LOsat high-momentum states



..the macroscopic picture:  
total energy more attractive

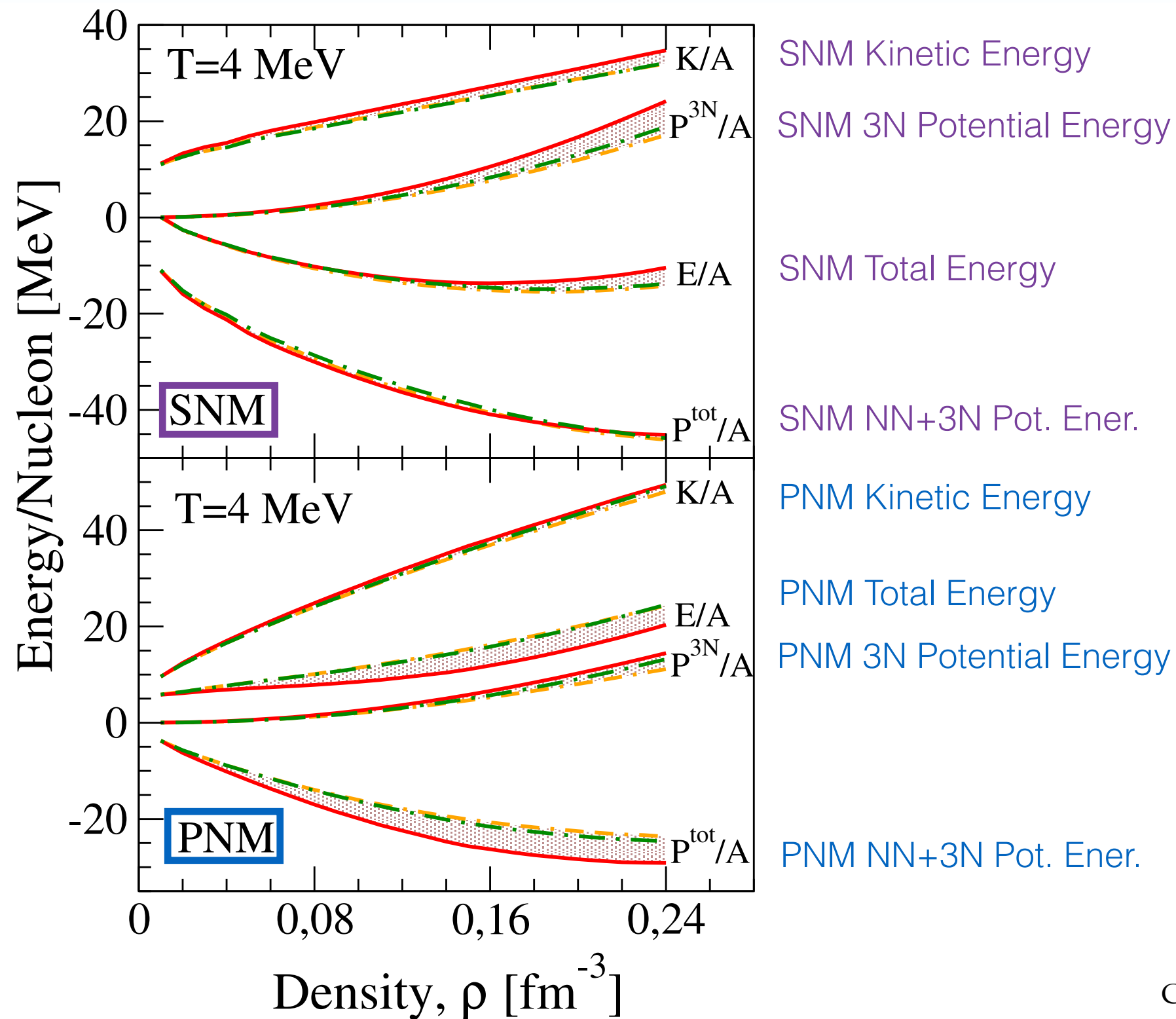
...start seeing the big  
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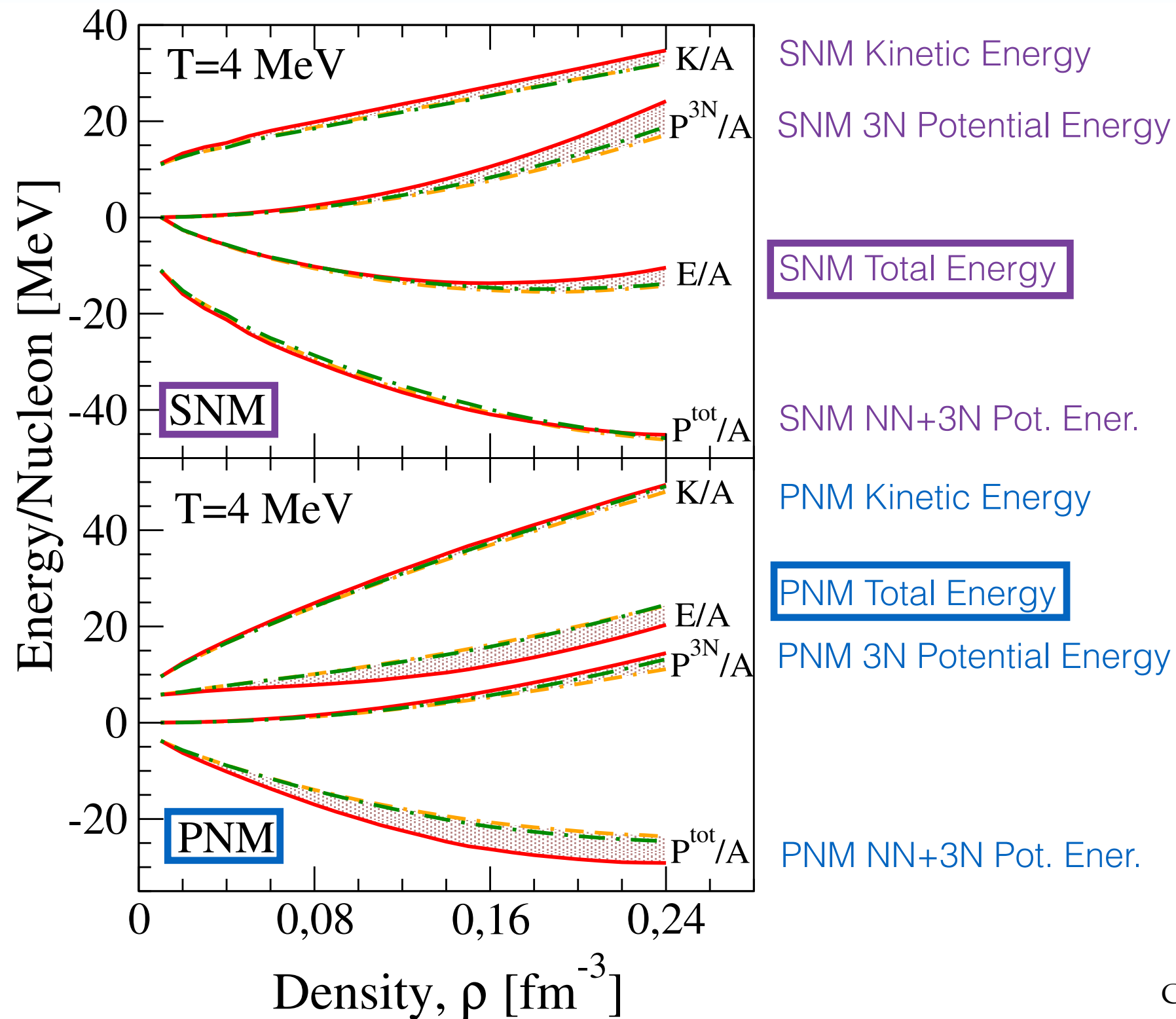
# Energy contributions: SNM vs PNM



Carbone (*in preparation*)



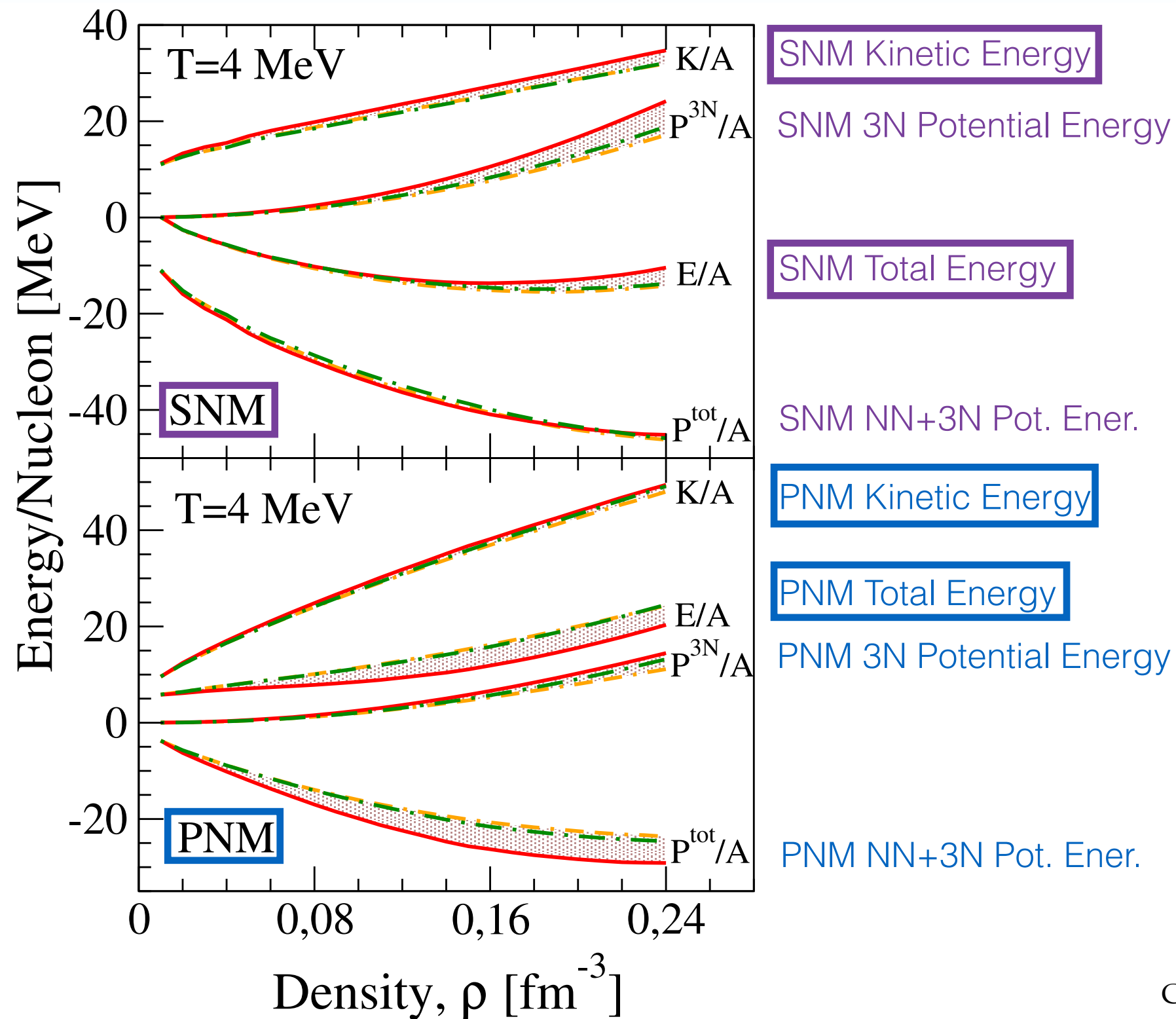
# Energy contributions: SNM vs PNM



**N2LOsat vs N3LO (SRG)**

Carbone (*in preparation*)

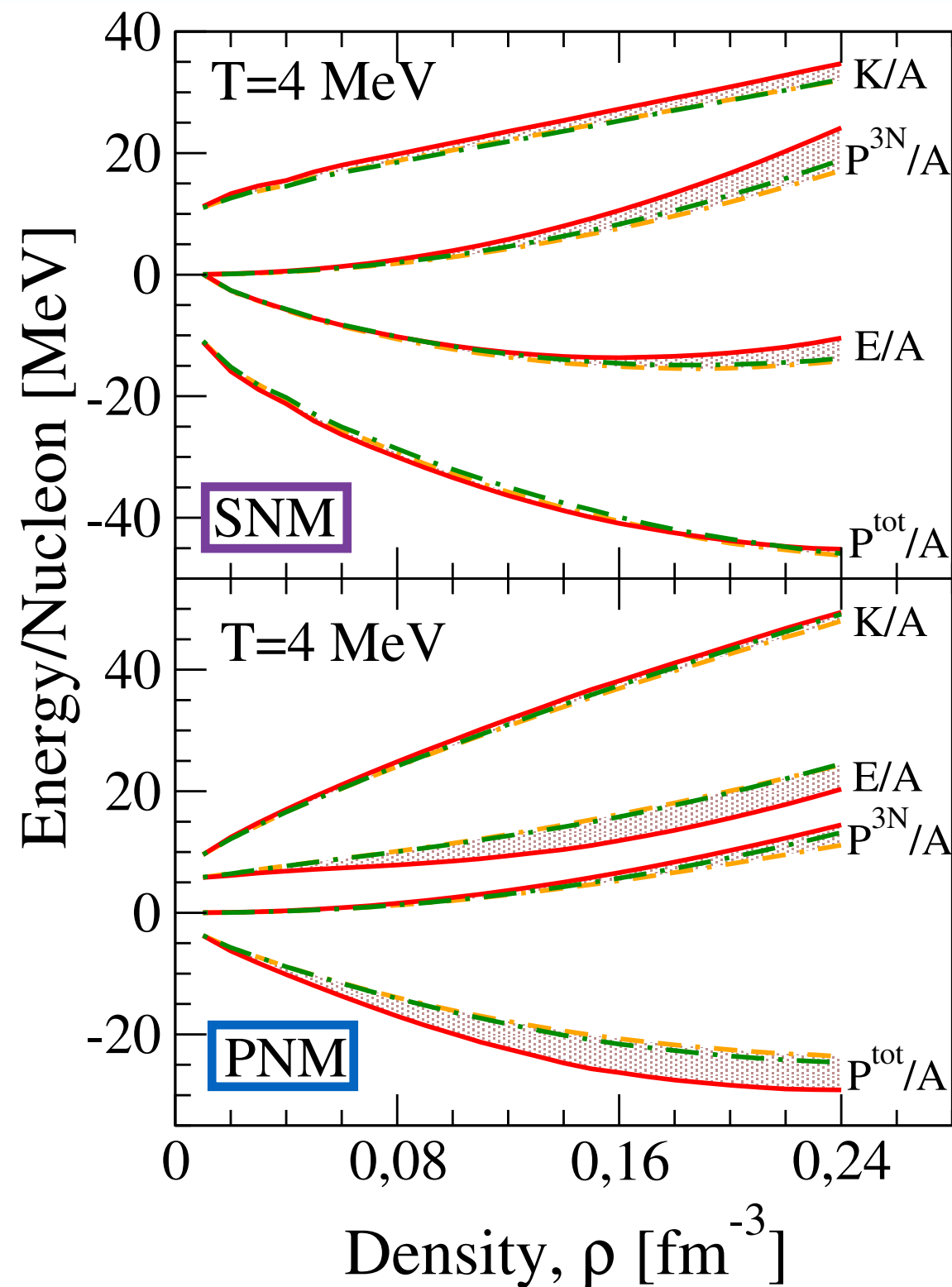
# Energy contributions: SNM vs PNM



**N2LOsat vs N3LO (SRG)**  
 ● higher Kinetic Energy

Carbone (*in preparation*)

# Energy contributions: SNM vs PNM

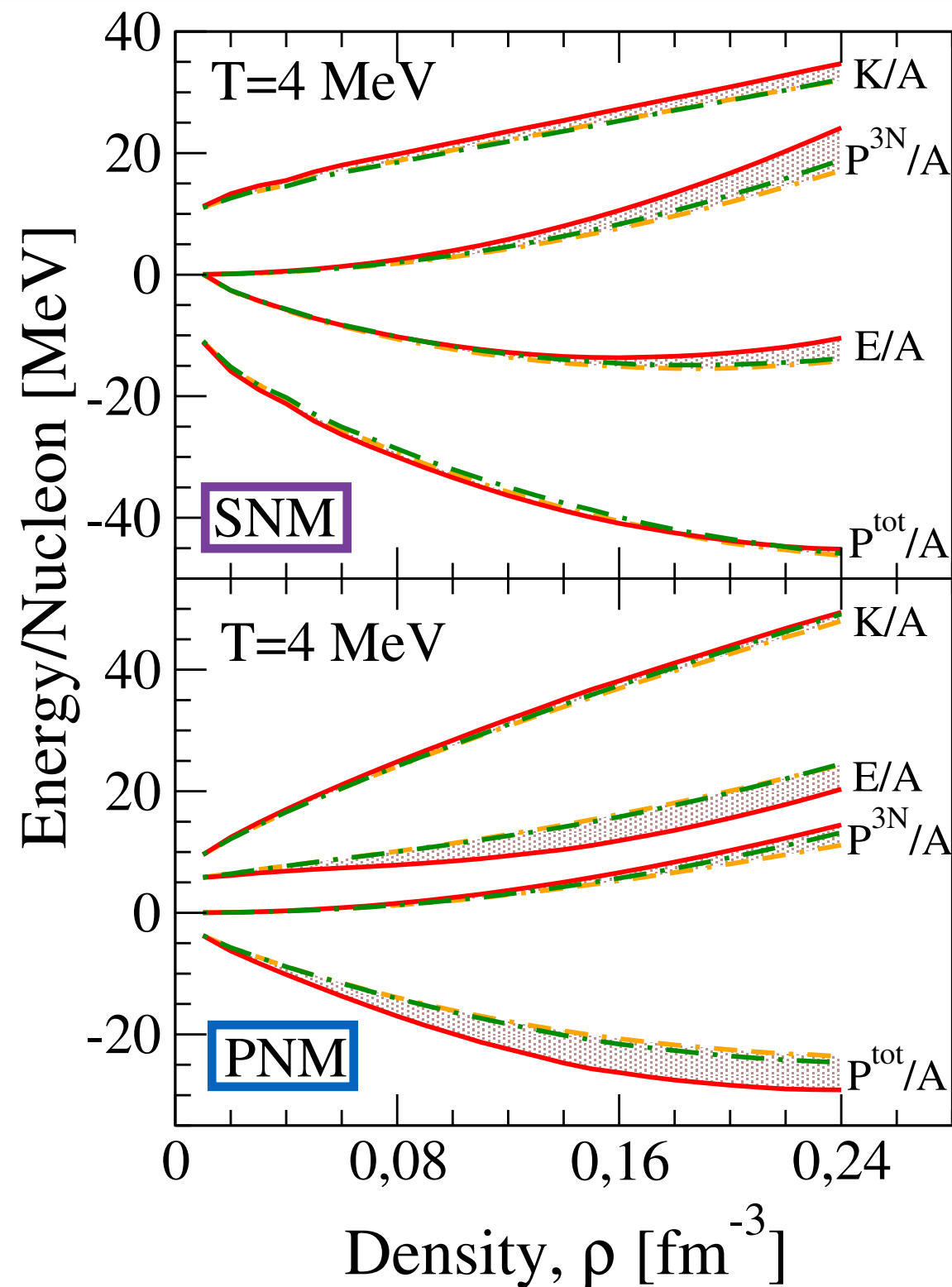


**N2LOsat vs N3LO (SRG)**

- higher Kinetic Energy
- higher 3N Potential Energy

Carbone (*in preparation*)

# Energy contributions: SNM vs PNM



SNM Kinetic Energy

SNM 3N Potential Energy

SNM Total Energy

SNM NN+3N Pot. Ener.

PNM Kinetic Energy

PNM Total Energy

PNM 3N Potential Energy

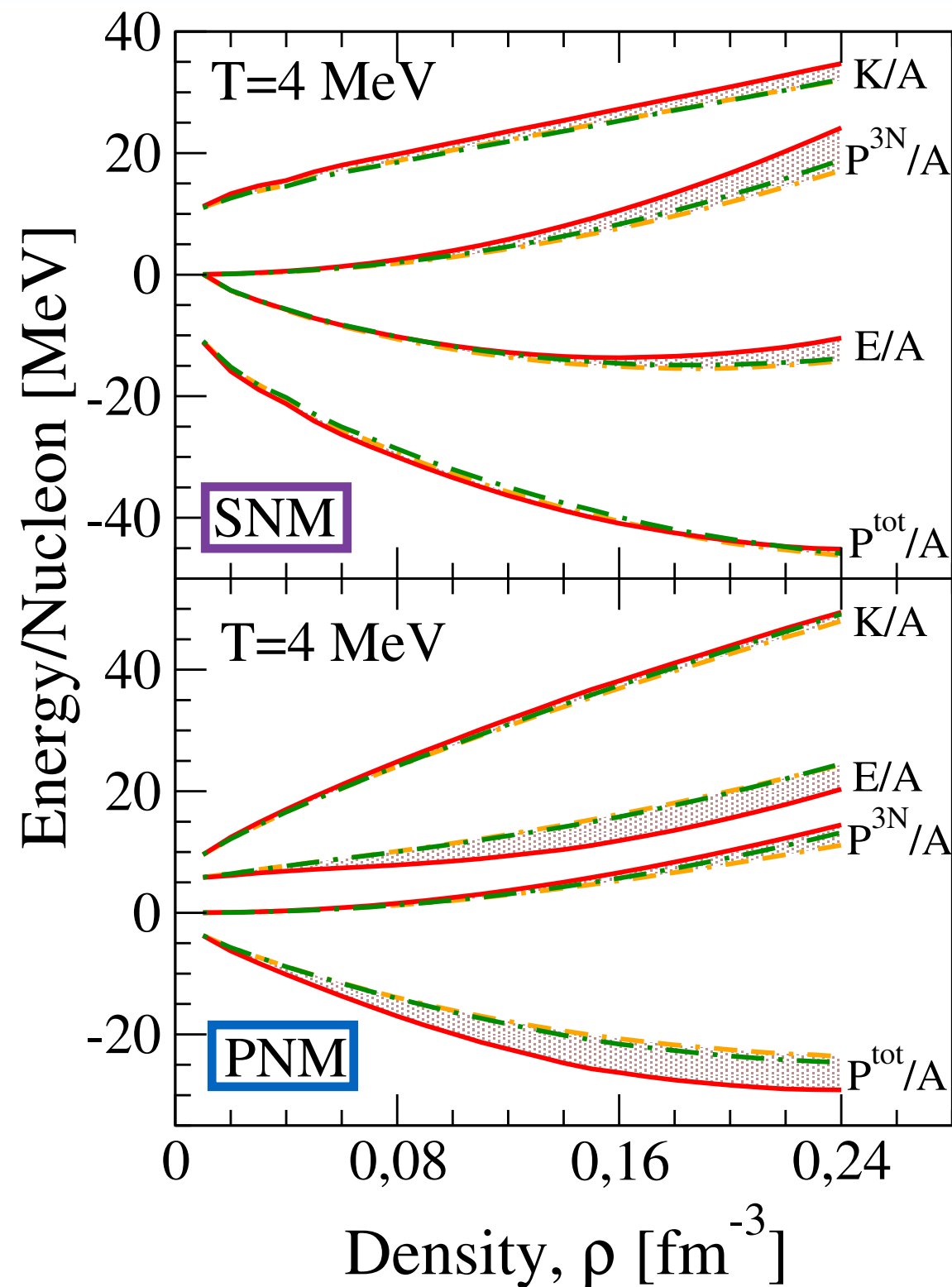
PNM NN+3N Pot. Ener.

**N2LOsat vs N3LO (SRG)**

- higher Kinetic Energy
- higher 3N Potential Energy
- PNM NN+3N Potential Energy smaller

Carbone (*in preparation*)

# Energy contributions: SNM vs PNM



SNM Kinetic Energy

SNM 3N Potential Energy

SNM Total Energy

SNM NN+3N Pot. Ener.

PNM Kinetic Energy

PNM Total Energy

PNM 3N Potential Energy

PNM NN+3N Pot. Ener.

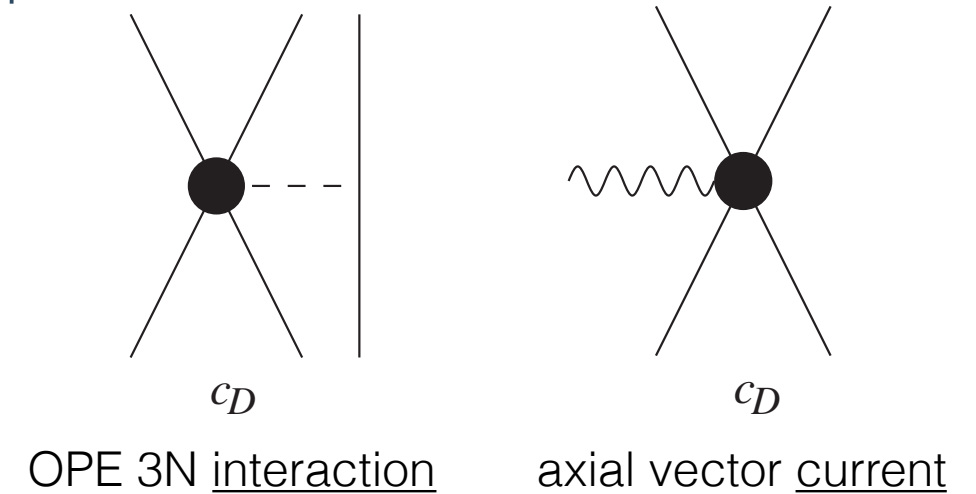
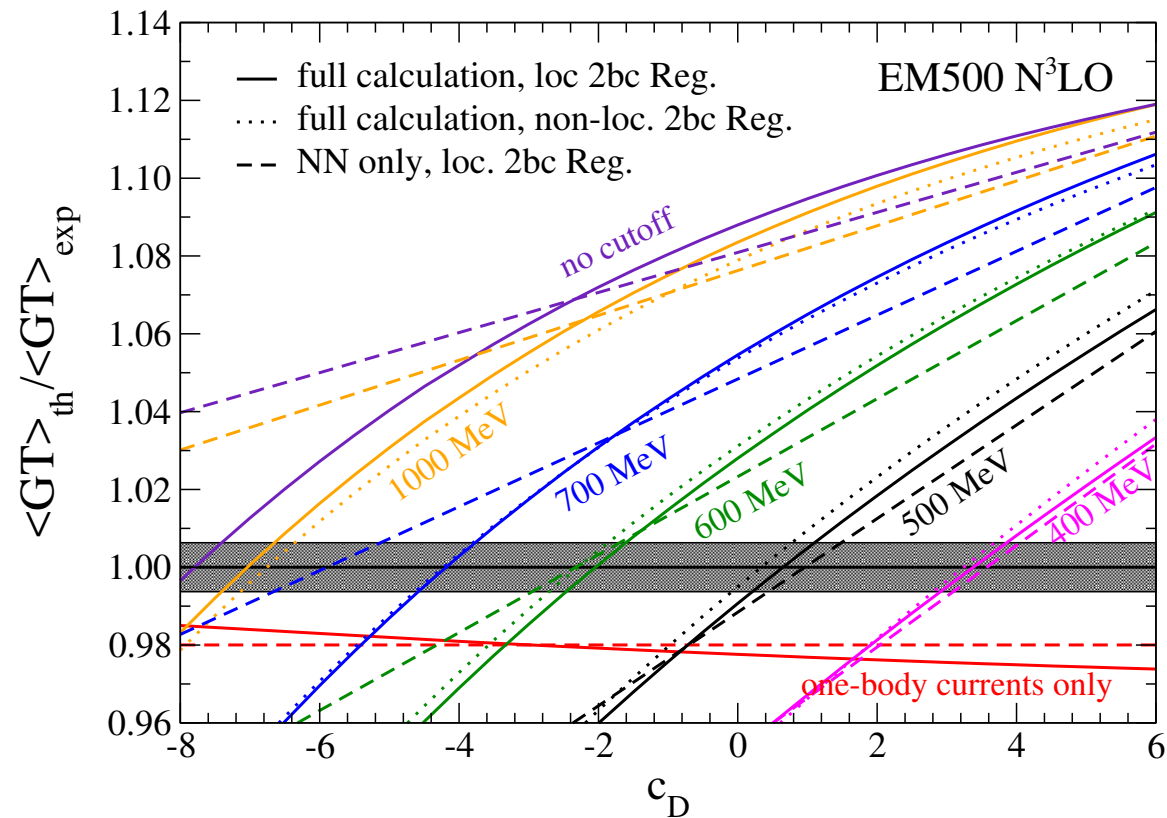
## N2LOsat vs N3LO (SRG)

- higher Kinetic Energy
- higher 3N Potential Energy
- PNM NN+3N Potential Energy smaller
- PNM Total Energy more attractive

Carbone (*in preparation*)

# Uncertainties due to fitting procedures

- Triton beta-decay is experimentally precisely known
- Constraints on the  $c_D$  coupling

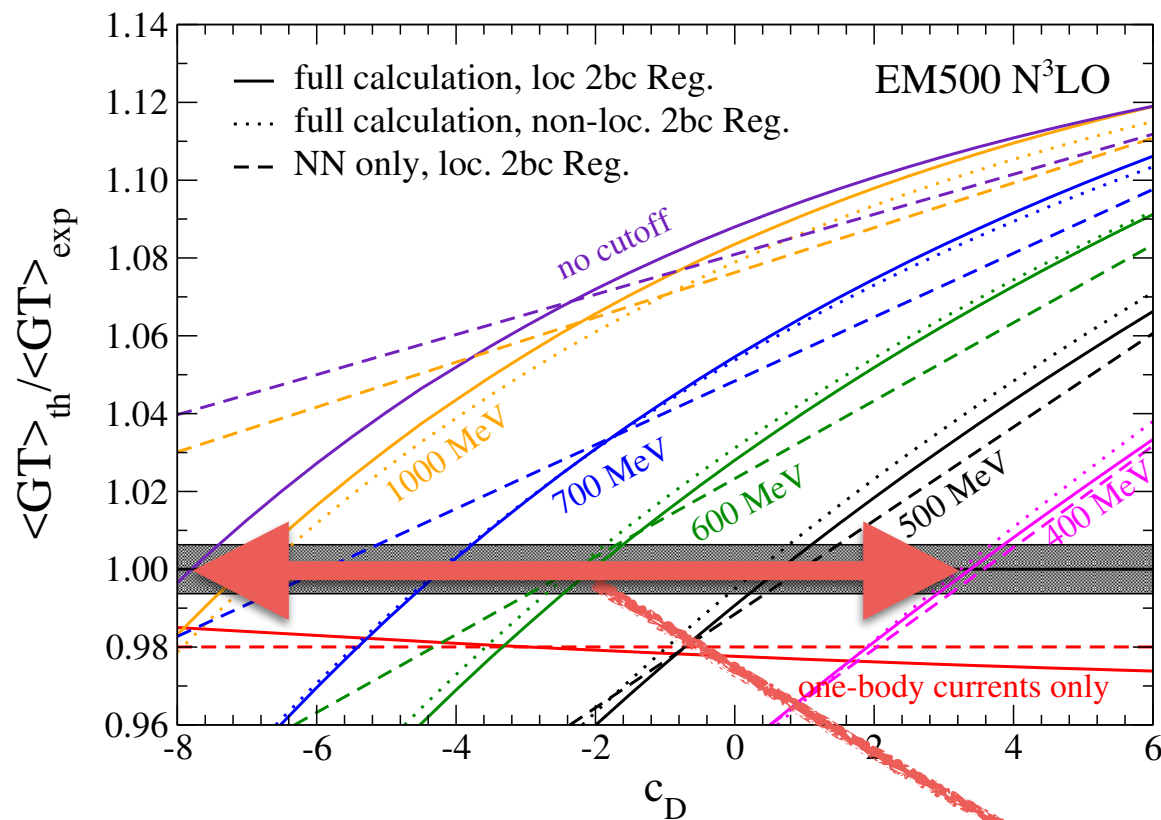


Klos, Carbone, Hebeler, Menéndez, Schwenk, EPJA 53, 168 (2017)

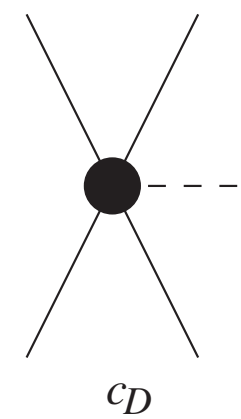


# Uncertainties due to fitting procedures

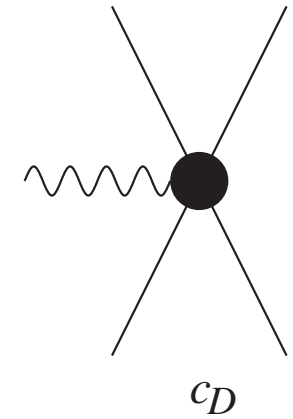
- Triton beta-decay is experimentally precisely known
- Constraints on the  $c_D$  coupling



- **Visible effect on the prediction of the saturation point**
- Energy and density range:  
 $E \sim [-11; -14] \text{ MeV}$ ;  $\rho \sim [0.13-0.16] \text{ fm}^{-3}$

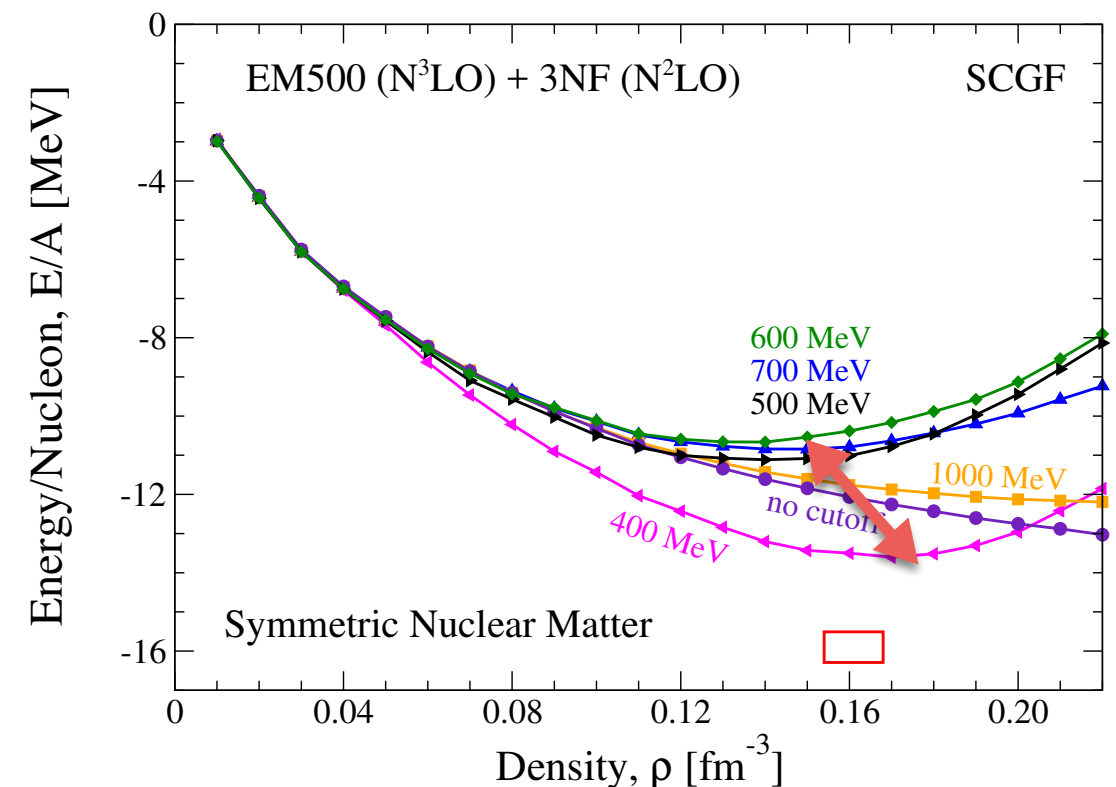


OPE 3N interaction



axial vector current

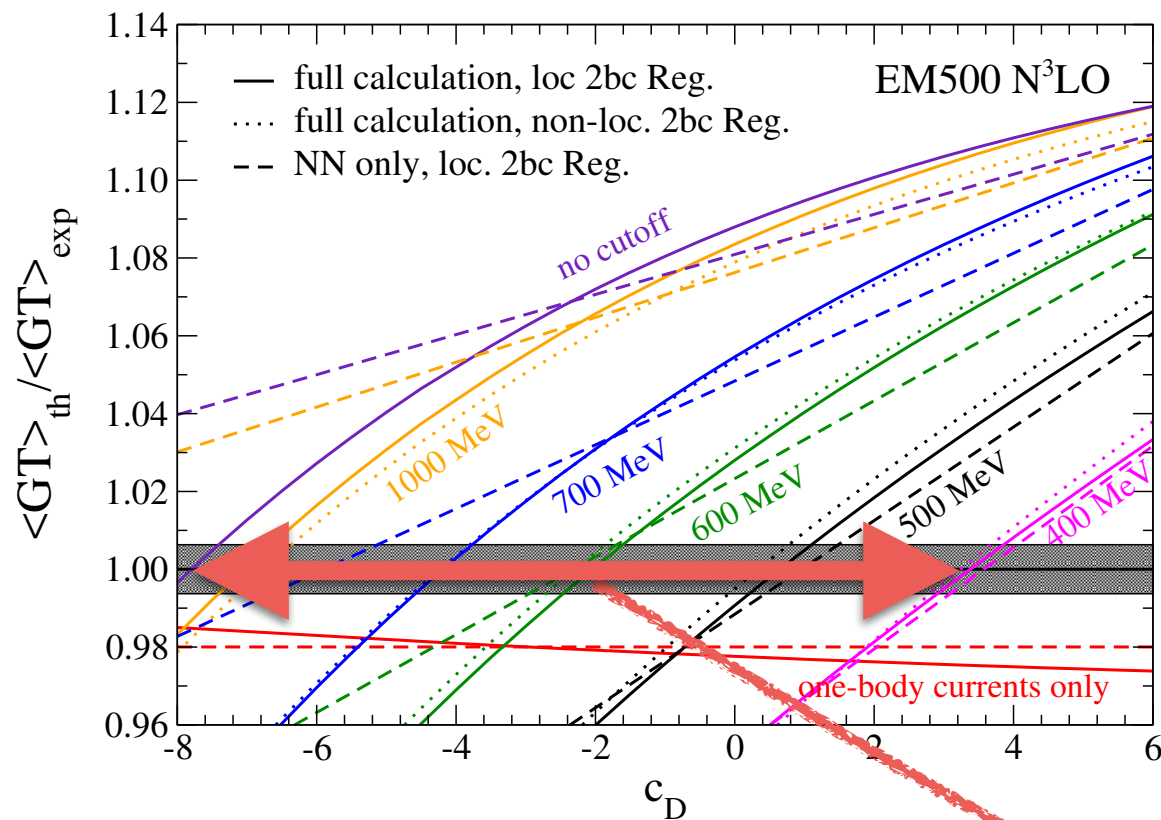
## Cutoff dependence on the current



Klos, Carbone, Hebeler, Menéndez, Schwenk, EPJA 53, 168 (2017)

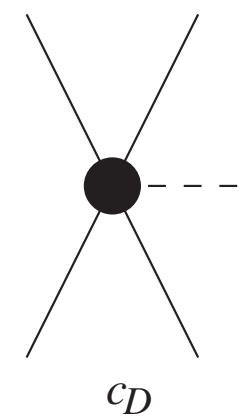
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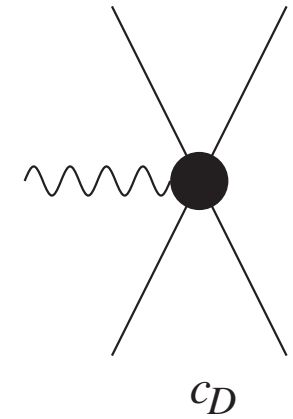


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- Energy and density range:  
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Understand new ways to fit the LECs

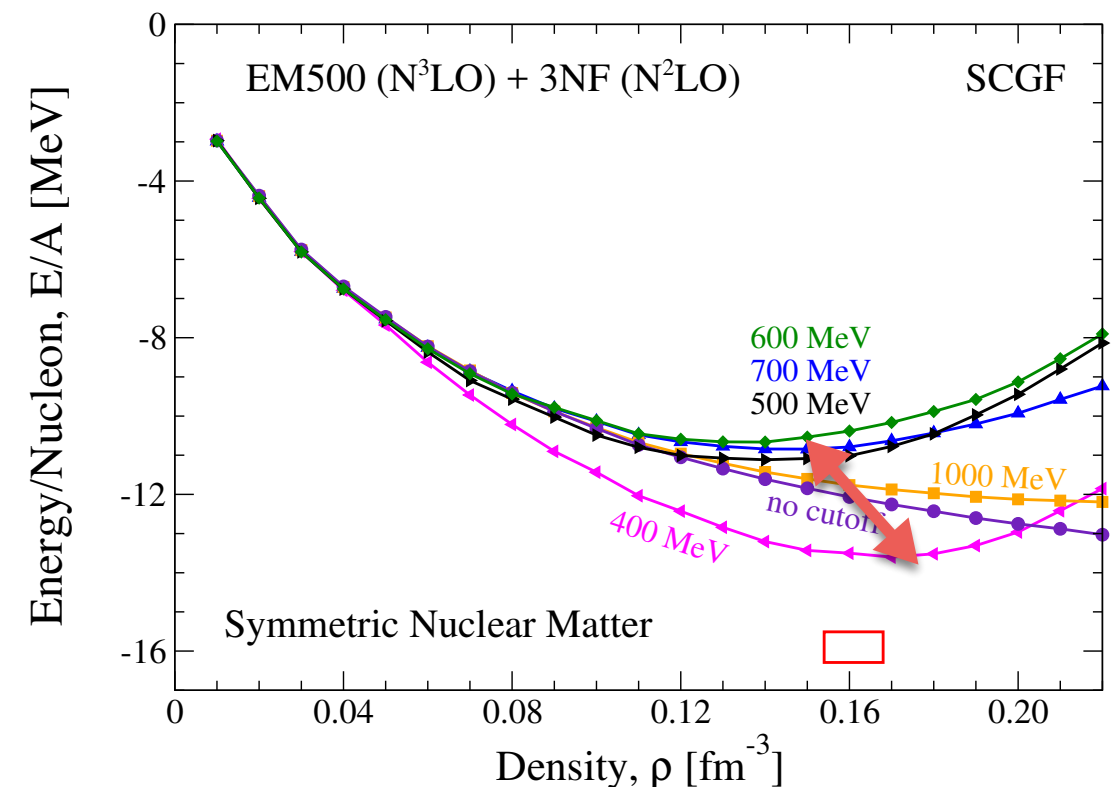


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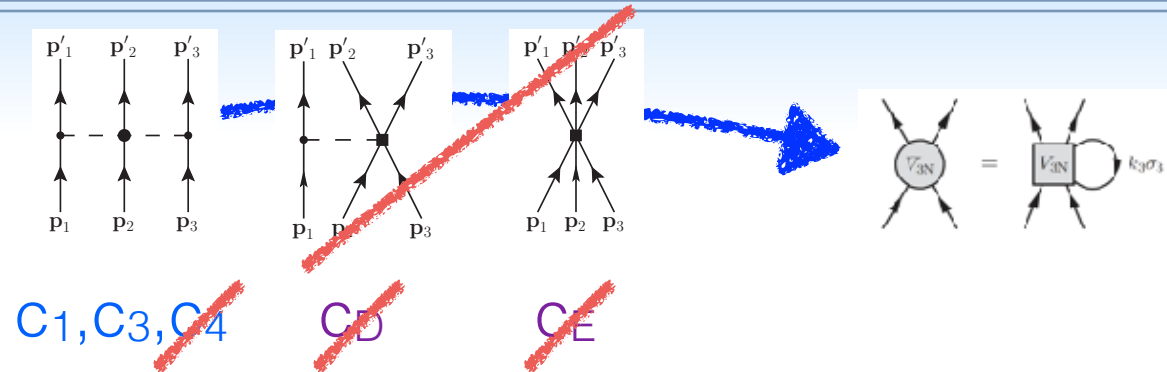
**Cutoff dependence on the current**



Klos, Carbone, Hebeler, Menéndez, Schwenk, EPJA 53, 168 (2017)

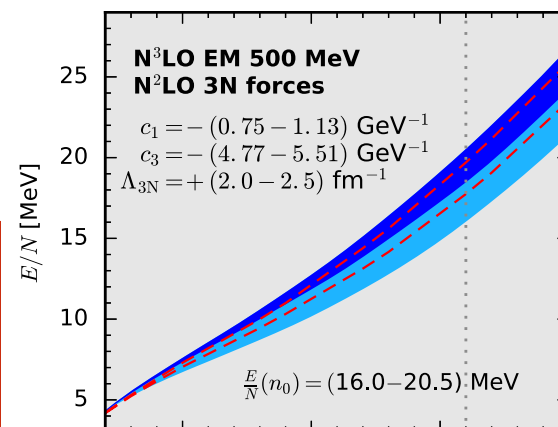


# Pure neutron matter with $2N + 3N$ at N3LO



Improved 3NF matrix elements Hebeler et al. 2015  
Partial-wave based 3NF average Drischler 2014-2015

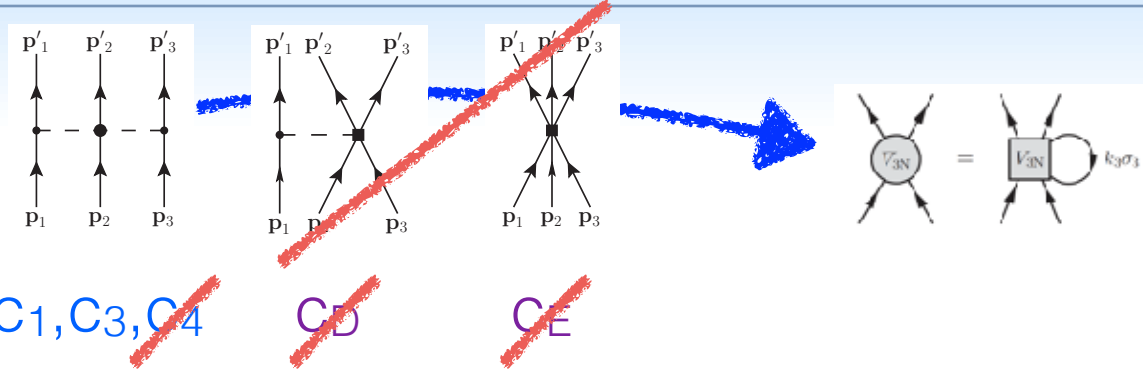
## EM N3LO500



	2N Force	3N Force	4N Force
<b>LO</b> ( $Q/\Lambda_\chi$ ) <sup>0</sup>			
<b>NLO</b> ( $Q/\Lambda_\chi$ ) <sup>2</sup>			
<b>NNLO</b> ( $Q/\Lambda_\chi$ ) <sup>3</sup>			
<b>N<sup>3</sup>LO</b> ( $Q/\Lambda_\chi$ ) <sup>4</sup>			

Drischler, Carbone, Hebeler, Schwenk PRC94, 054307 (2016)

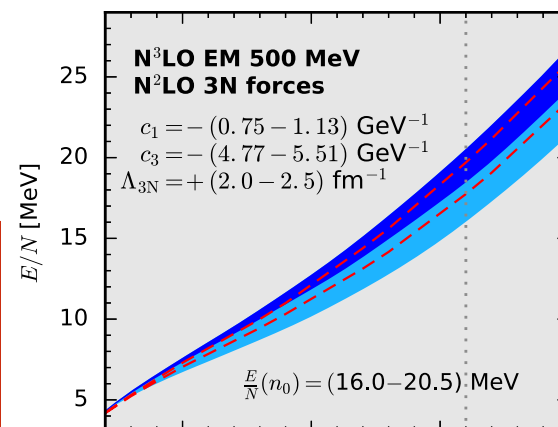
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1. many-body approximation uncertainty : **MBPT** vs **SCGF**

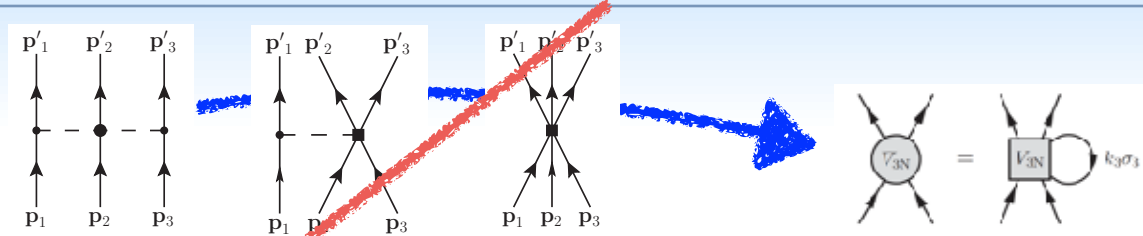
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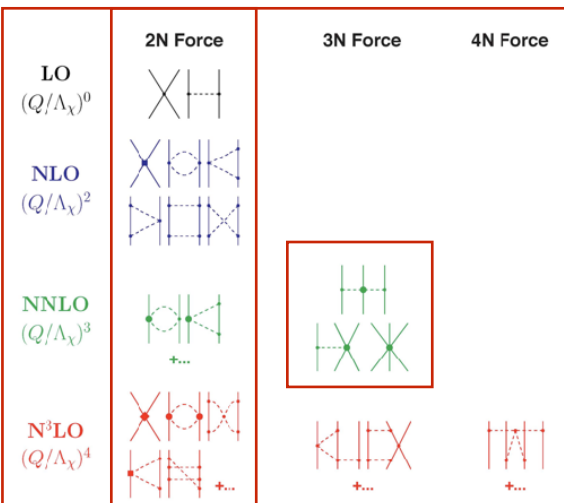
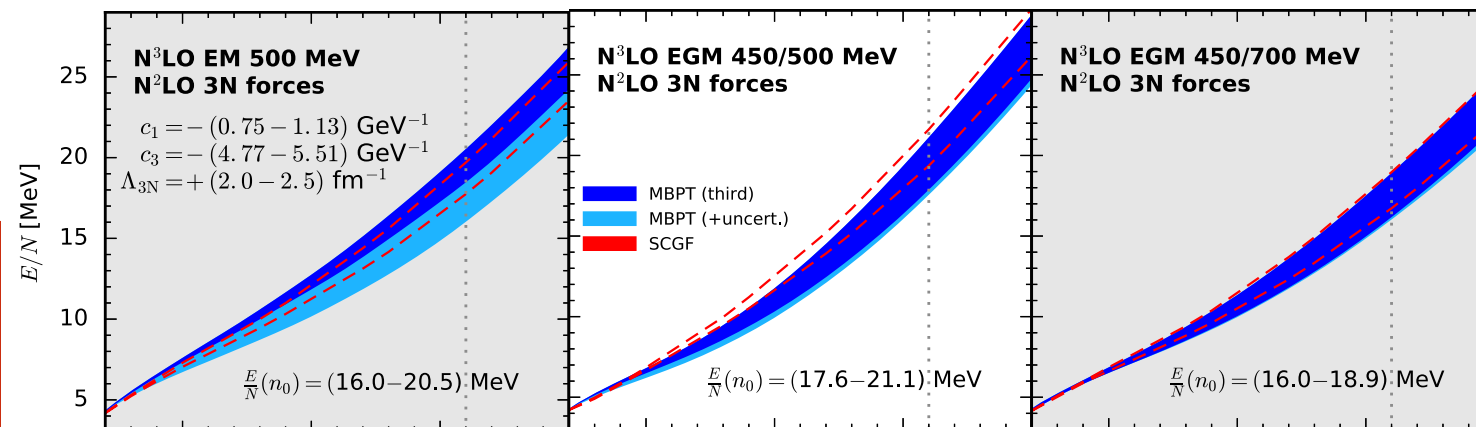
1. many-body approximation uncertainty : **MBPT** vs **SCGF**

How perturbative is the potential:  
**MBPT** vs **SCGF**  
band shrinks

EM N3LO500

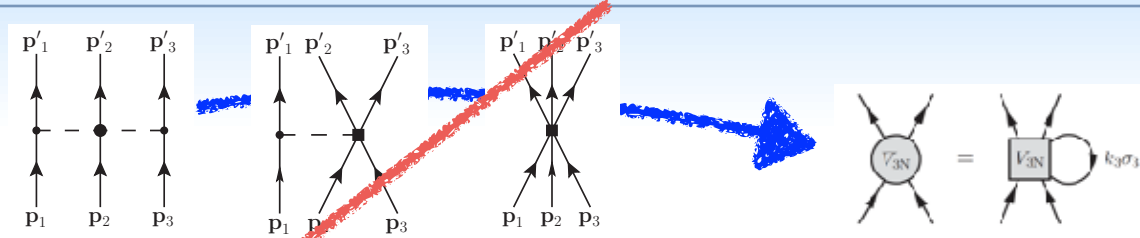
EGM 450/500

EGM 450/700



Drischler, Carbone, Hebeler, Schwenk PRC94, 054307 (2016)

# Pure neutron matter with $2N + 3N$ at N3LO



Improved 3NF matrix elements Hebeler et al. 2015  
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$C_1, C_3, C_4$

$C_2$

$C_5$

1. many-body approximation uncertainty : **MBPT** vs **SCGF**

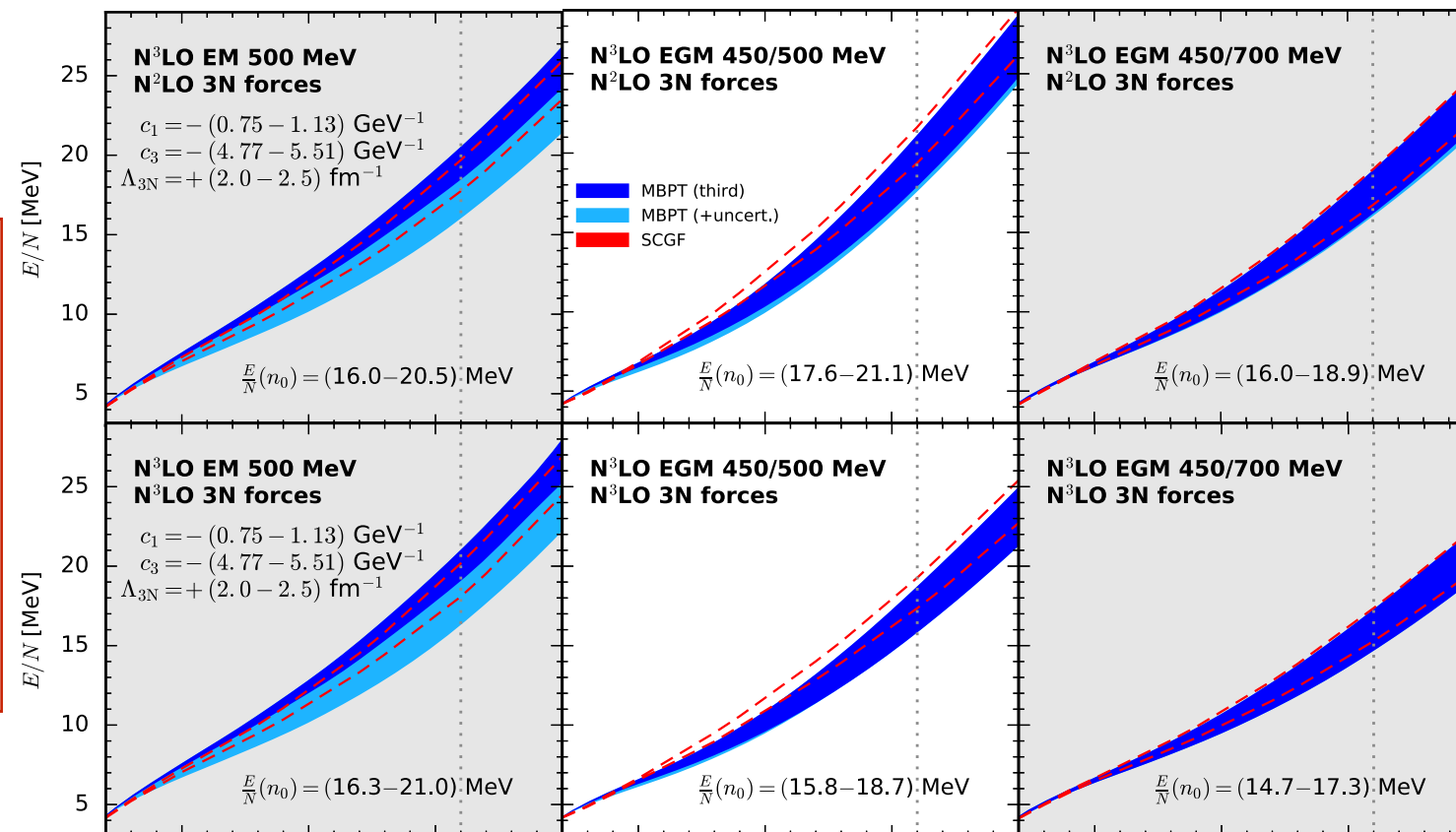
How perturbative is the potential:

**MBPT** vs **SCGF**  
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EM N3LO500

EGM 450/500

EGM 450/700



2.

chiral 3NFs  
uncertainty:  
N2LO vs N3LO

N3LO 3NF shift in  
energy bands

3N N2LO

3N N3LO

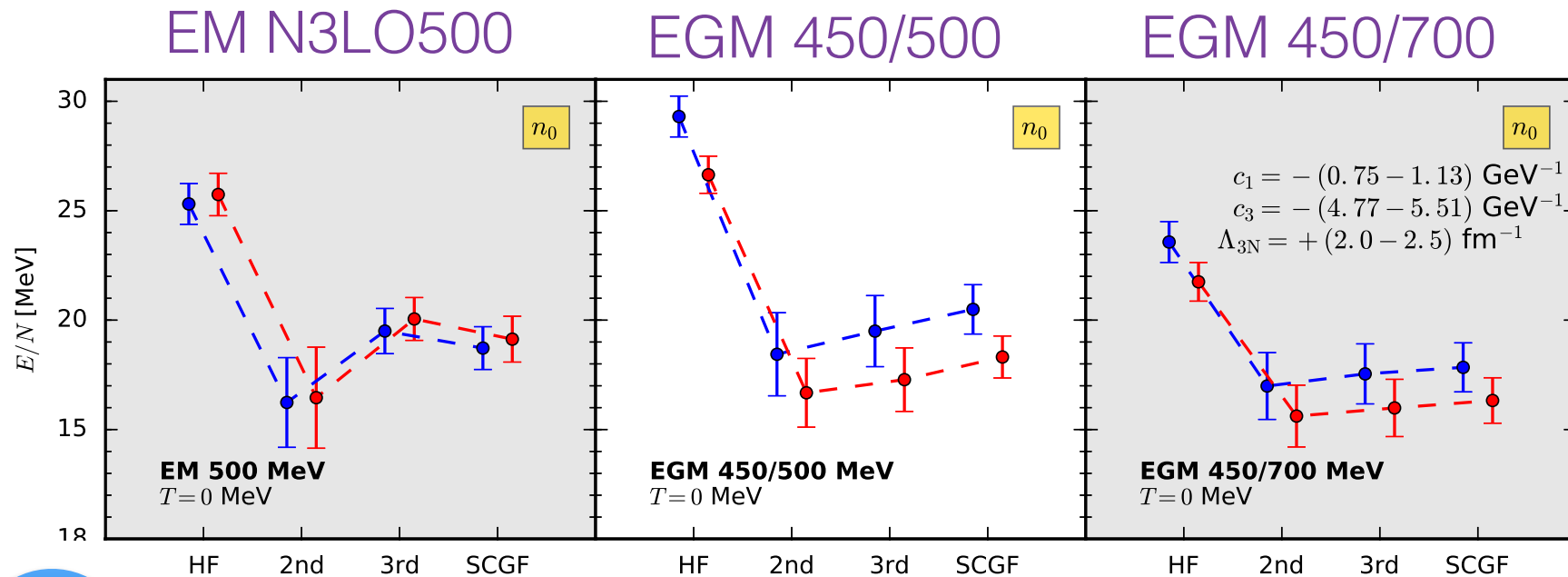
Drischler, Carbone, Hebeler, Schwenk PRC94, 054307 (2016)



# Many-body convergence at full 2N+3N N3LO in PNM

Improved 3NF matrix elements Hebeler et al. 2015  
Partial-wave based 3NF average Drischler 2014-2015

Energy/Nucleon at saturation density:  $0.16 \text{ fm}^{-3}$



1.

many-body truncation  
attractive 2nd order  
repulsive 3rd order

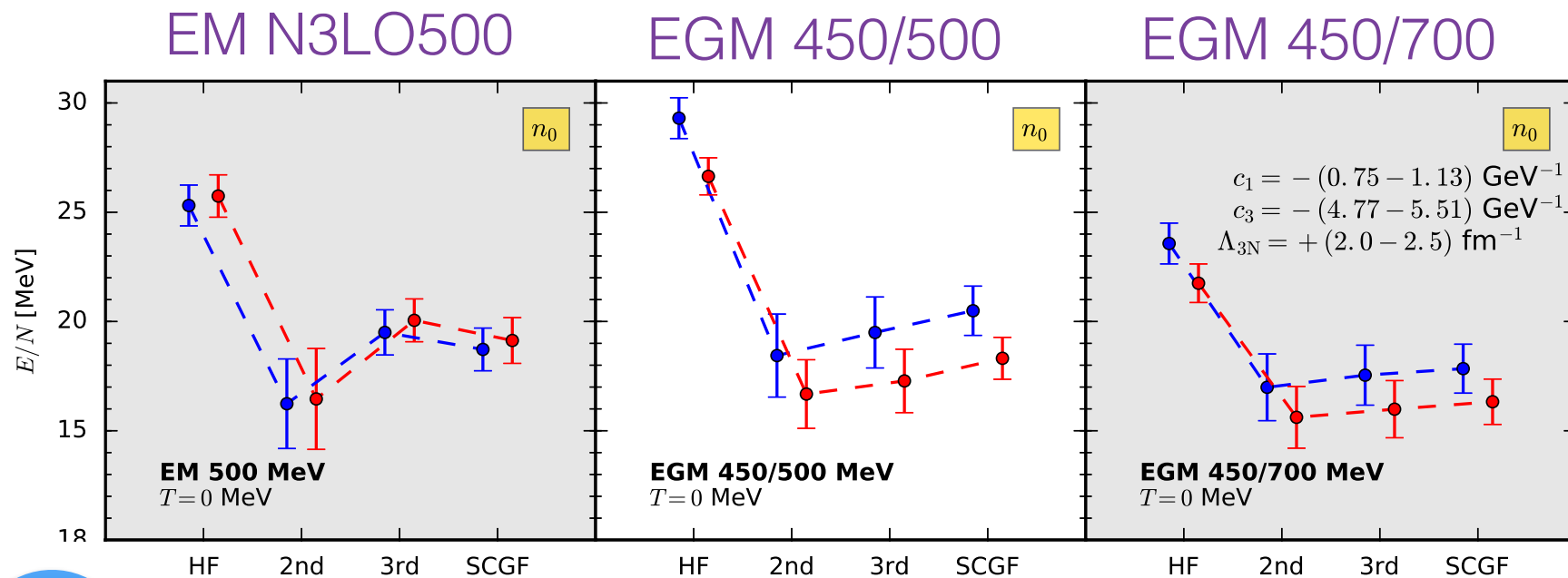
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2. EM N3LO500 EGM 450/500 EGM 450/700  
How perturbative is the potential: smaller beyond of 3rd order

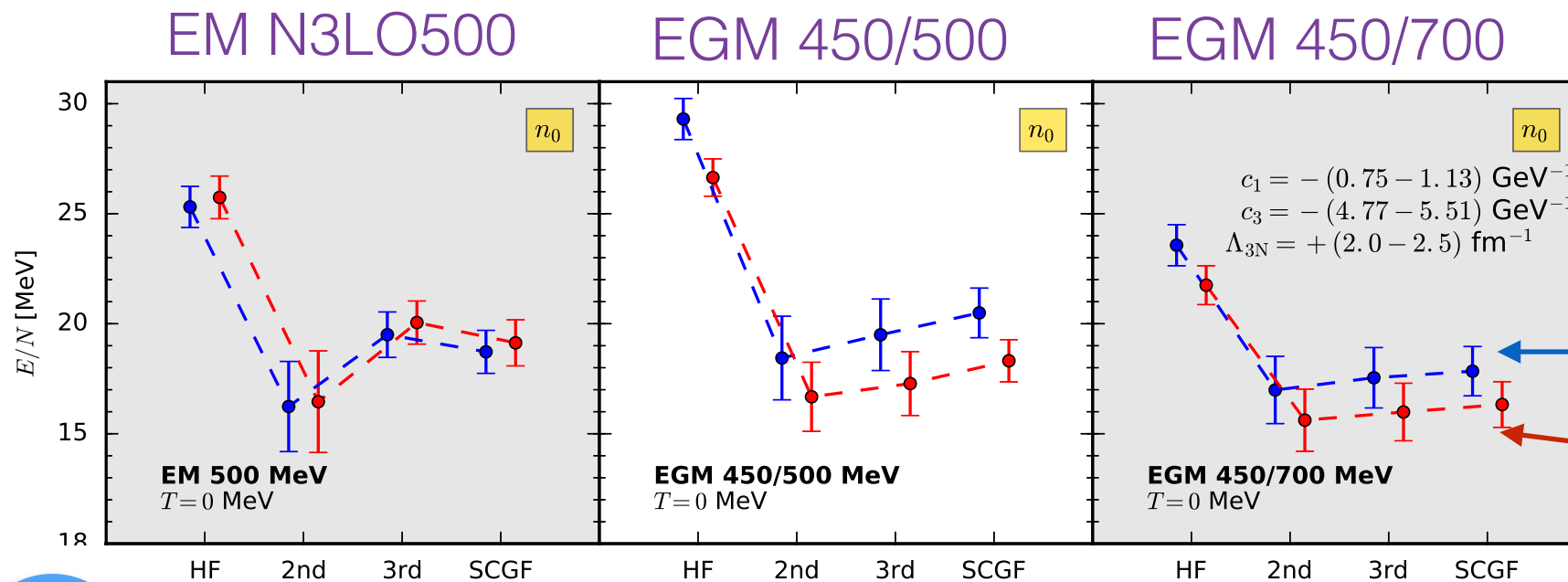
Drischler, Carbone, Hebeler, Schwenk PRC94, 054307 (2016)



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Energy/Nucleon at saturation density:  $0.16 \text{ fm}^{-3}$



3.

3N N2LO  
3N N3LO

1.

many-body truncation  
attractive 2nd order  
repulsive 3rd order

2.

EM N3LO500      EGM 450/500      EGM 450/700

How perturbative is the potential: smaller beyond of 3rd order

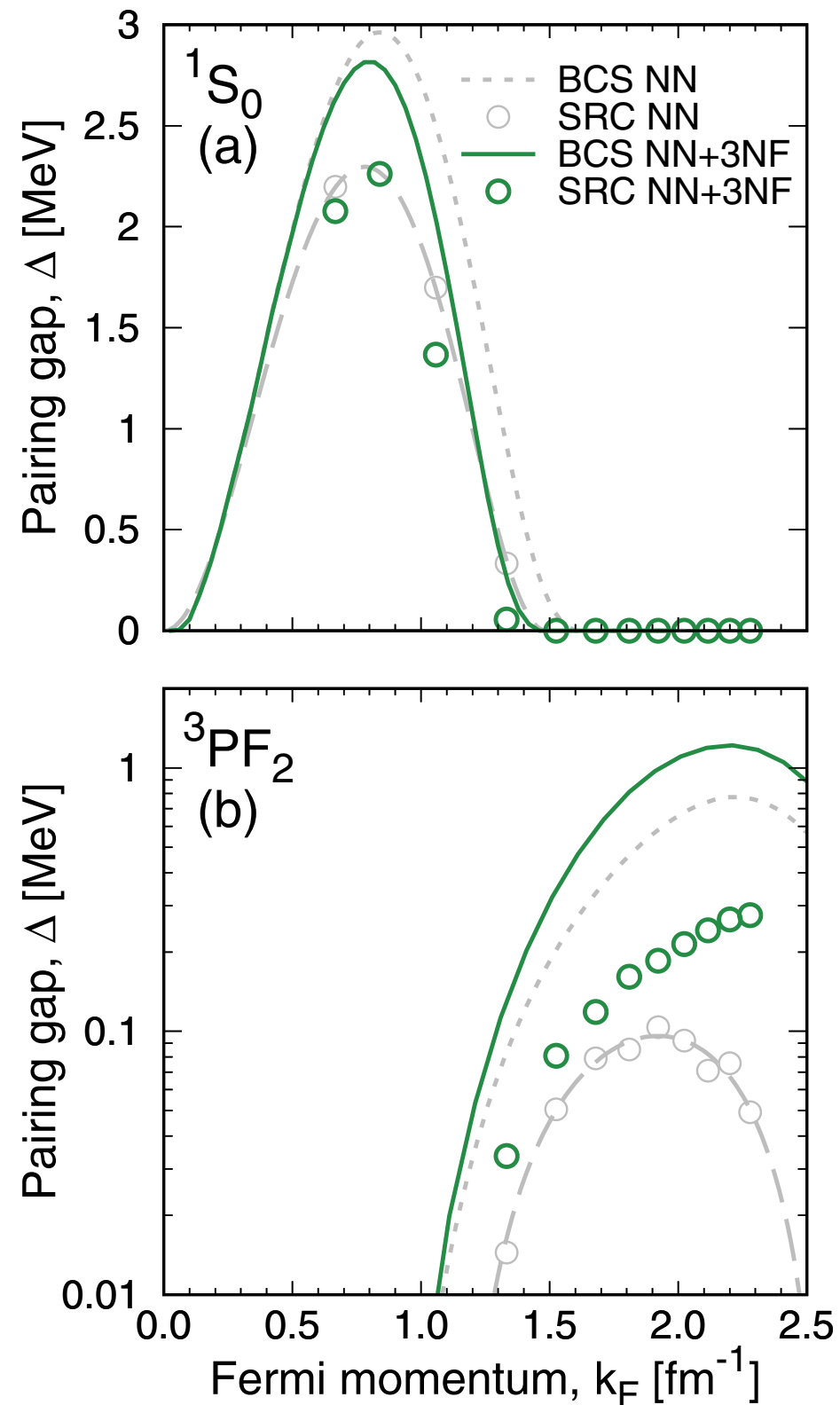
Drischler, Carbone, Hebeler, Schwenk PRC94, 054307 (2016)

	2N Force	3N Force	4N Force
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N <sup>3</sup> LO ( $Q/\Lambda_\chi$ ) <sup>4</sup>			



# The pairing gap in neutron matter

$$\Delta_L^{JST}(k) = - \sum_{L'} \int_0^\infty \frac{dk' k'^2}{\pi} \frac{\langle k | V_{LL'}^{JST} | k' \rangle}{\xi(k')} \Delta_{L'}^{JST}(k')$$

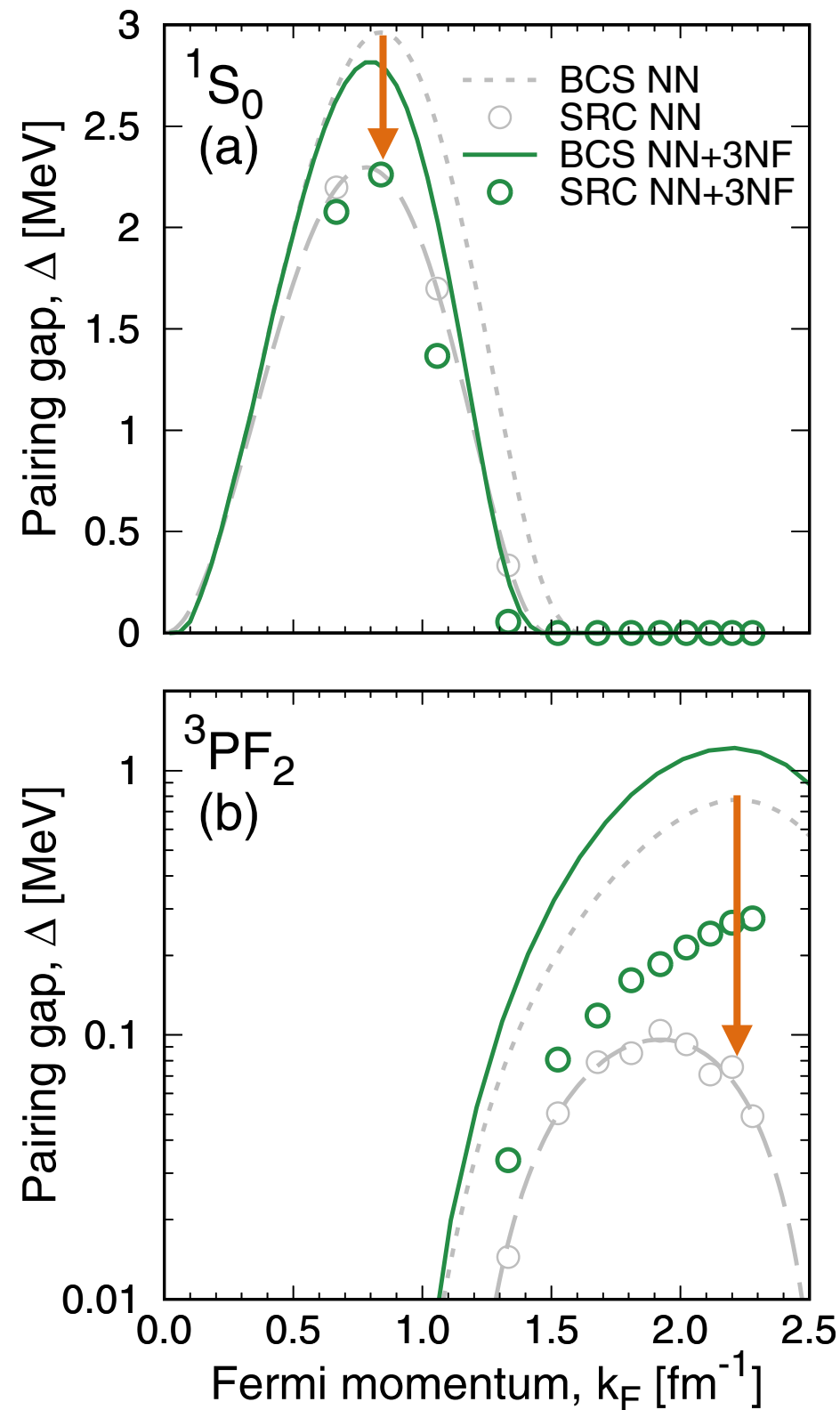


Ding et al., PRC94, 025802 (2016)

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$$\Delta_L^{JST}(k) = - \sum_{L'} \int_0^\infty \frac{dk' k'^2}{\pi} \frac{\langle k | V_{LL'}^{JST} | k' \rangle}{\xi(k')} \Delta_{L'}^{JST}(k')$$

Beyond BCS:  
correlations  
strongly reduce  
gap



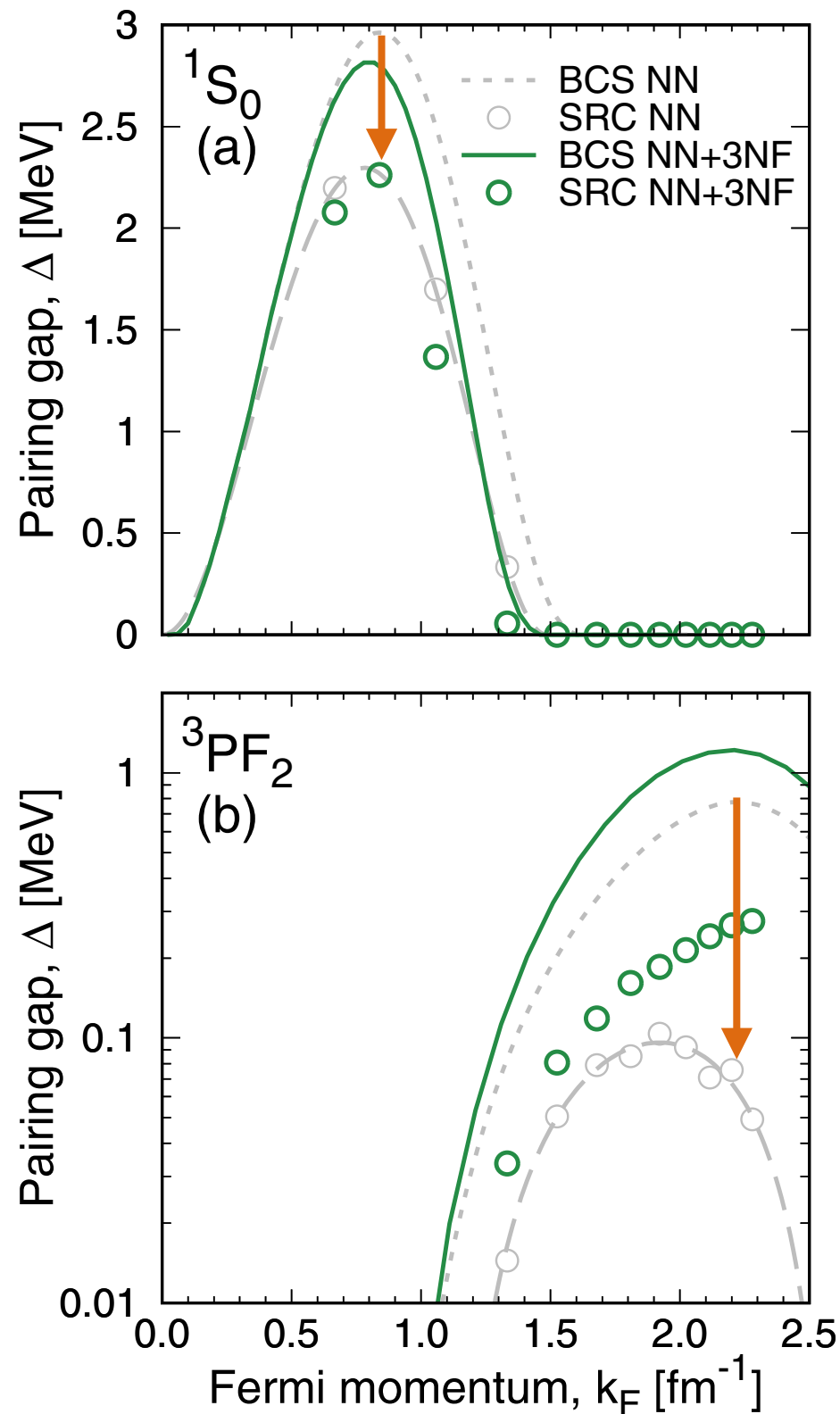
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Beyond BCS:  
correlations  
strongly reduce  
gap

- effect of 3NFs:
  - $^1S_0$ : weaker, repulsive, lower densities
  - $^3P_2$ : stronger, attractive, higher densities



Ding et al., PRC94, 025802 (2016)

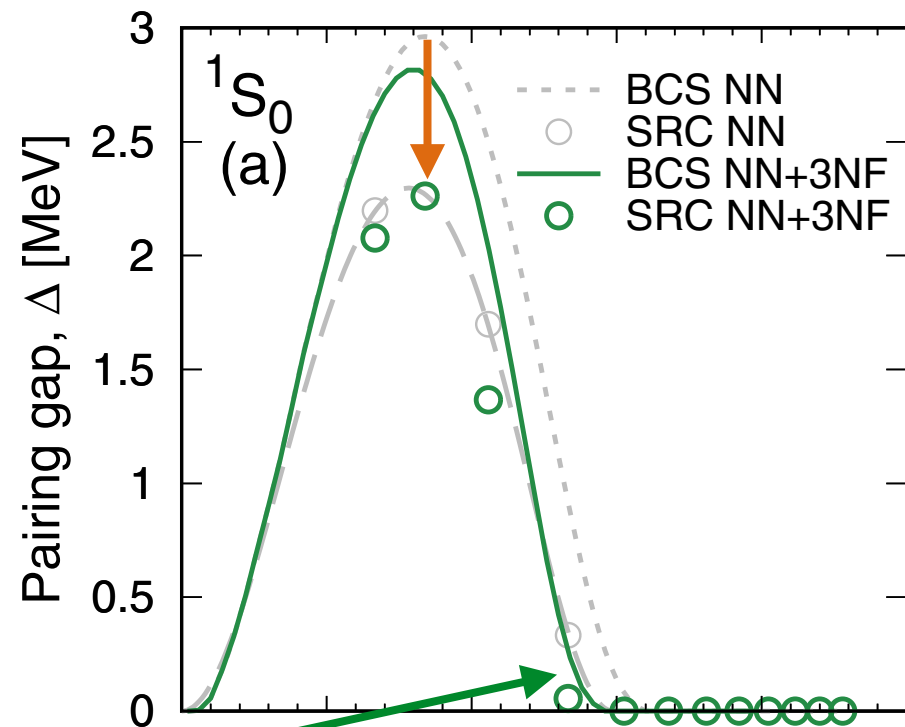


# The pairing gap in neutron matter

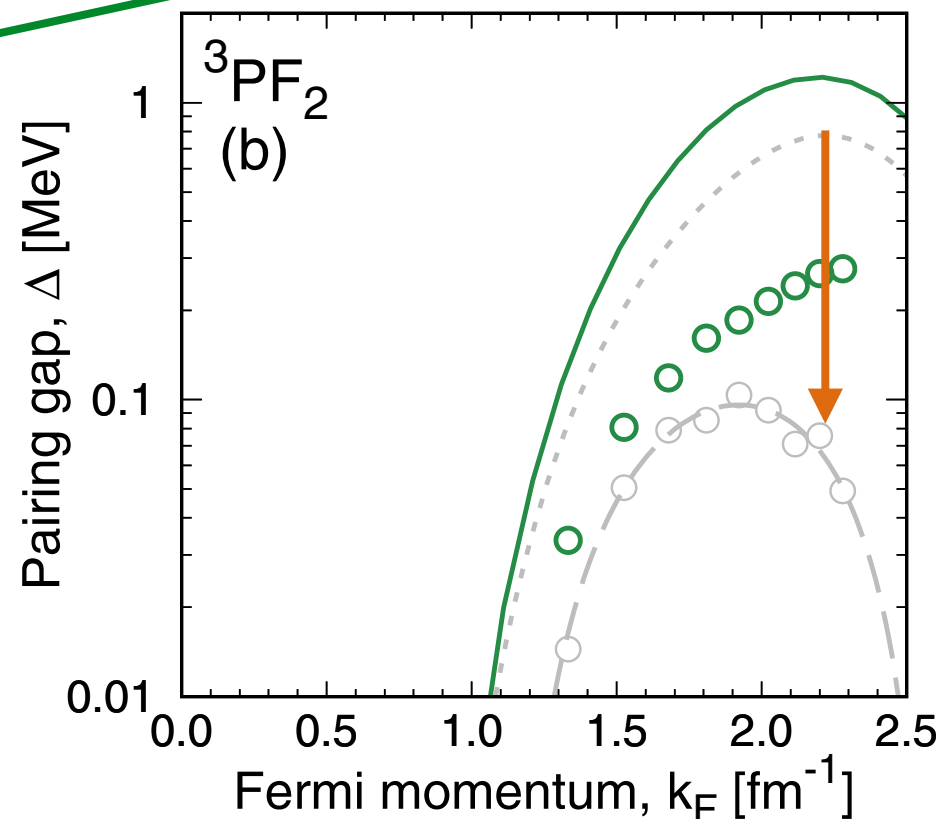
$$\Delta_L^{JST}(k) = - \sum_{L'} \int_0^\infty \frac{dk' k'^2}{\pi} \frac{\langle k | V_{LL'}^{JST} | k' \rangle}{\xi(k')} \Delta_{L'}^{JST}(k')$$

Beyond BCS:  
correlations  
strongly reduce  
gap

- effect of 3NFs:
  - $^1S_0$ : weaker, repulsive, lower densities
  - $^3P_2$ : stronger, attractive, higher densities



- $^1S_0$  max 2.3 MeV, closes at  $\sim 1.5 \text{ fm}^{-1}$
- $^3P_2$  repulsive, pairing smaller



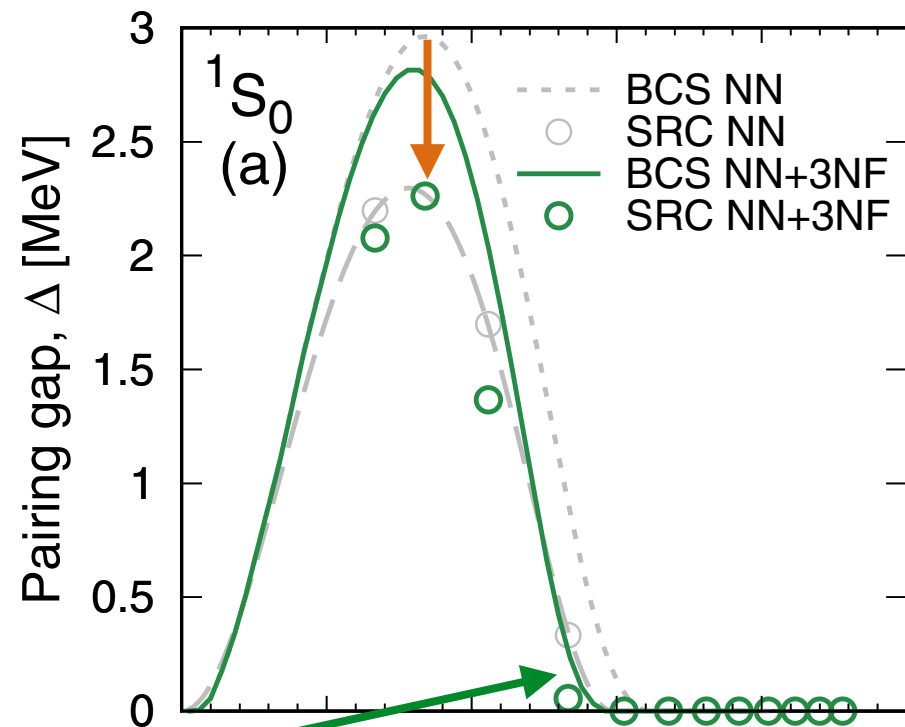
Ding et al., PRC94, 025802 (2016)

# The pairing gap in neutron matter

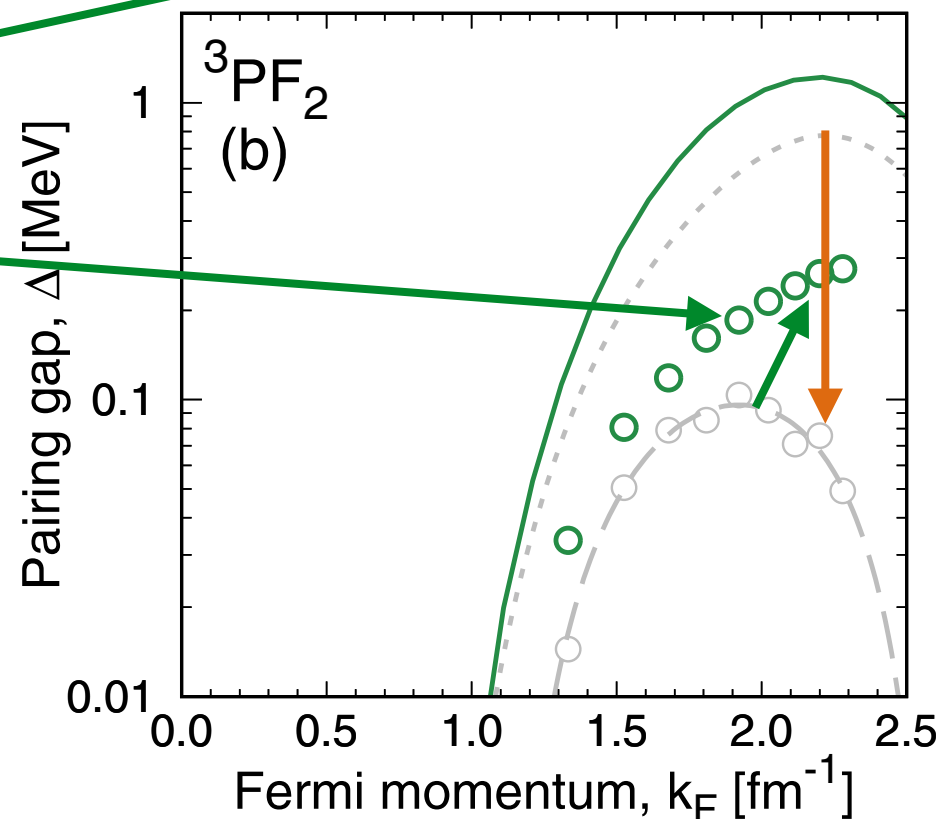
$$\Delta_L^{JST}(k) = - \sum_{L'} \int_0^\infty \frac{dk' k'^2}{\pi} \frac{\langle k | V_{LL'}^{JST} | k' \rangle}{\xi(k')} \Delta_{L'}^{JST}(k')$$

Beyond BCS:  
correlations  
strongly reduce  
gap

- effect of 3NFs:
  - 1S0: weaker, repulsive, lower densities
  - 3PF2: stronger, attractive, higher densities



- 1S0 max 2.3 MeV, closes at ~1.5 fm<sup>-1</sup>
- 3NFs repulsive, pairing smaller



- No closure for 3PF2 gap with 3N
- limits of applicability of chiral forces

Ding et al., PRC94, 025802 (2016)