FROM DINUCLEAR SYSTEMS to CLOSE BINARY STARS: APPLICATION to MASS TRANSFER

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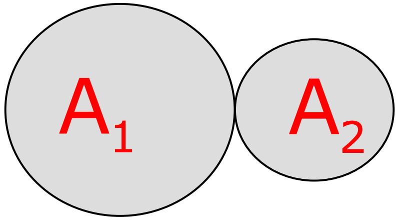
Fusion, Quasi-Fission, Multi-Nucleon Transfer Reactions

Two main collective coordinates are used for the description of these processes:

1. Relative internuclear distance R

2. Mass asymmetry coordinate $\eta=(A_1-A_2)/(A_1+A_2)$

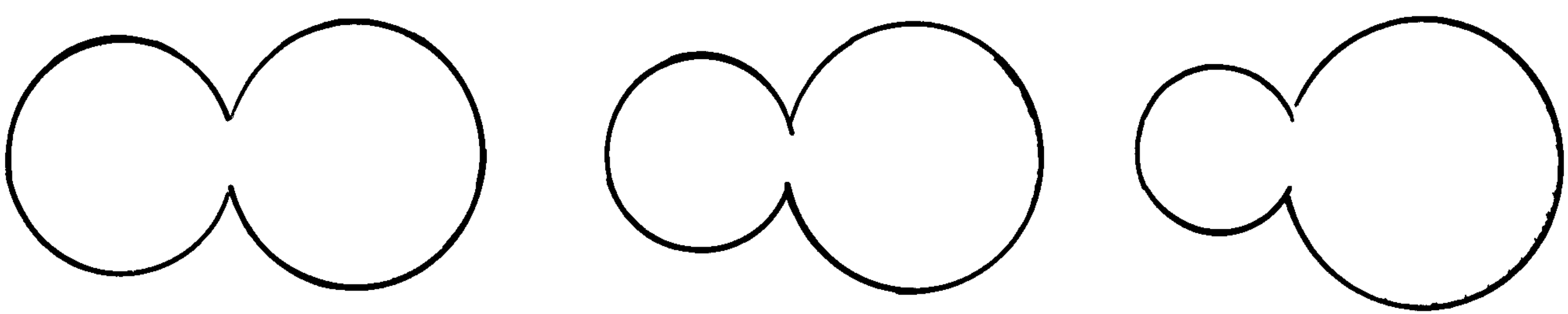


If A_1 or A_2 get small, then $|\eta|\rightarrow 1$ and system fuses.

Dinuclear system has two main degrees of freedom to describe fusion and quasifission :

1. Relative motion of nuclei, capture of target and projectile into dinuclear system, decay of the dinuclear system: quasifission

2. Transfer of nucleons between nuclei, change of mass and charge asymmetries leading to fusion and quasifission



$\eta \to 1$

Set of coordinates for the description of DNS evolution:

$$\eta_Z = (Z_1 - Z_2)/(Z_1 + Z_2)\ ,\ \ \eta = (A_1 - A_2)/(A_1 + A_2)\ ,\ \ R$$

The potential energy of DNS:

$$U(R, \eta, \eta_Z, \beta_{1,} \beta_{2,} J) = B_1 + B_2 + V(R, \eta, \eta_Z, \beta_{1,} \beta_{2,} J)$$

The nucleus-nucleus potential:

$$V(R, \eta, \eta_Z, \beta_1 \beta_{2,} J) = V_C(R, \eta_Z, \beta_{1,} \beta_2) + V_N(R, \eta, \beta_{1,} \beta_2) + V_{rot}(\eta, \beta_{1,} \beta_{2,} J)$$

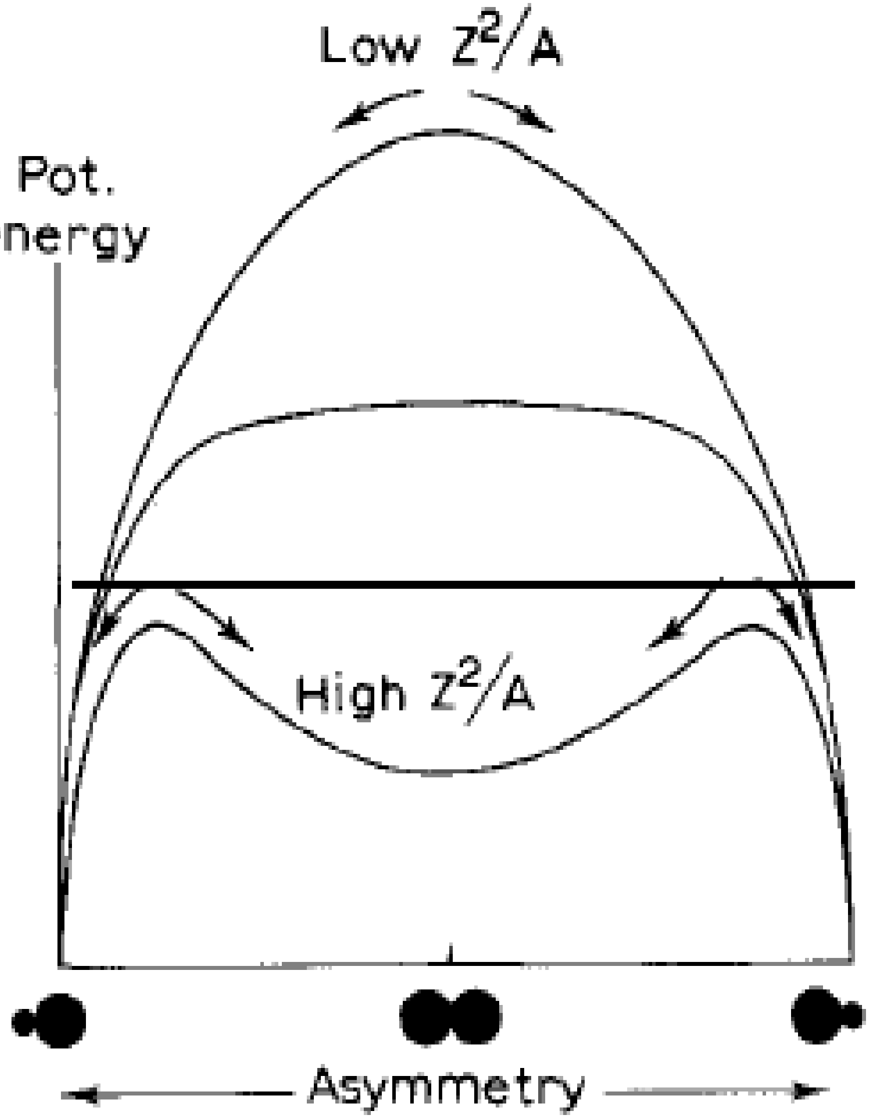
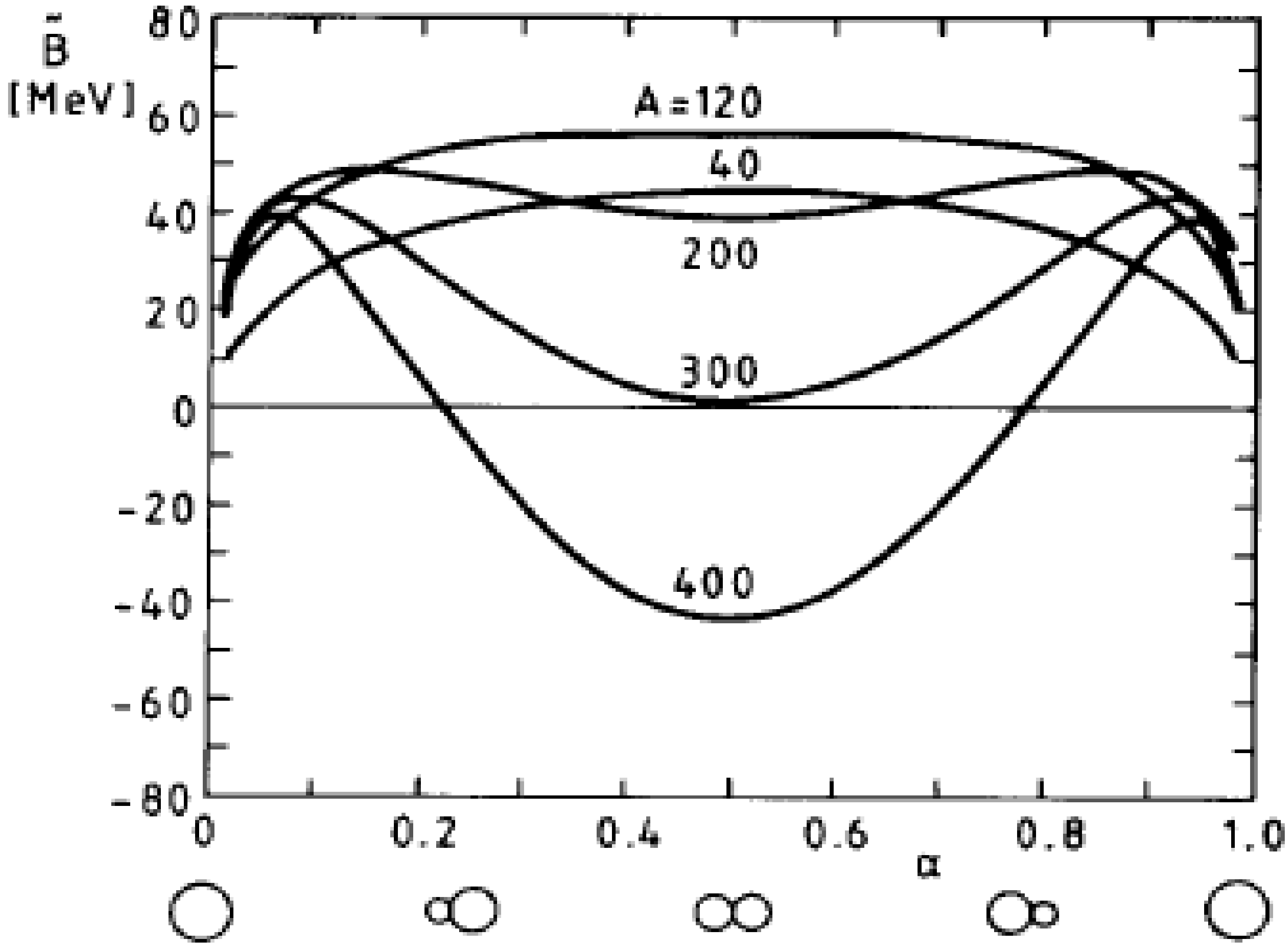


Illustration of the dependence of the potential energy of the system of two touching nuclear drops on mass asymmetry and parameter $(Z_1 + Z_2)^2/(A_1 + A_2)$.



$\bar{B}$
[MeV]
80
60
40
20
0
-20
-40
-60
-80
A = 120
40
200
300
400
0
0.2
0.4
0.6
0.8
1.0
α

From Microscopic to Macroscopic :

CLOSE BINARY STARS

Potential energy of di-star

$$\mathbf{U} = \mathbf{U_1} + \mathbf{U_2} + \mathbf{V}$$

$\mathbf{U_i}$ **- potential energy of star** "i" $(i = 1, 2)$

V - star-star interaction energy

$$\mathbf{U_i} = \mathbf{U_i^g} + \mathbf{U_i^k}$$

$\mathbf{U_i^g}$ **- gravitational energy**

$\mathbf{U_i^k}$ **- intrinsic kinetic energy**

Mechanical stellar equilibrium $\rightarrow$ Virial theorem :

$$\mathbf{U_i^k} = -\frac{\mathbf{1}}{\mathbf{2}}\mathbf{U_i^g}$$

Energy of star "i"

$$\mathrm{U_i = U_i^g + U_i^k = \omega_i \frac{GM_i^2}{R_i} + U_i^k = -\omega_i \frac{GM_i^2}{2R_i}}$$

G - gravitational constant, $\mathrm{M_i}$ - mass, $\mathrm{R_i}$ - radius

ω_i - dimensionless structural factor is determined by density profile of star

Stellar model [B.Vasiliev] well describes observables

1) temperature-radius-mass-luminosity relations

2) spectra of seismic oscillations of Sun

3) distribution of stars on their masses

4) magnetic fields of stars, planets, etc.

Employing structural factor

$$\omega_i = 1.644 \left(\frac{M_\odot}{M_i}\right)^{1/4}$$

and radius

$$R_i = R_\odot \left(\frac{M_i}{M_\odot}\right)^{2/3},$$

we obtain star "*i*" energy

$$U_i = -0.822 \frac{GM_\odot^2}{R_\odot} \left(\frac{M_i}{M_\odot}\right)^{13/12}$$

$M_\odot$ - mass and $R_\odot$ - radius of Sun

Star-Star Interaction Potential :

$$\mathbf{V(R) = Q + V_{rot} = -\frac{GM_1M_2}{R} + \frac{\mu v^2}{2}}$$

Q - gravitational energy of interaction

$\mathbf{V_{rot}}$ - kinetic energy of orbital rotation

since 2 stars rotate with respect to each other around common center of mass

$\mathbf{v = GM[2/R - 1/R_m]}$ - speed

$\mathbf{R_m}$ - semimajor axis of elliptical relative orbit

$\mu = \frac{M_1M_2}{M_1+M_2}$ - reduced mass

Finally, Star-Star Interaction Potential :

$$\mathbf{V = -\frac{GM_1M_2}{2R_m} = -\omega_V G(M_1M_2)^3/2}$$

$$\omega_{\mathbf{V}} = \frac{\mathbf{1}}{\mathbf{M^2}\mu_{\mathbf{i}}^{\mathbf{2}}\mathbf{R_{m,i}}}$$

Because of Kepler's laws

$$\mathbf{R_m} = \left(\frac{\mu_{\mathbf{i}}}{\mu}\right)^{\mathbf{2}} \mathbf{R_{m,i}}$$

"i" denotes the initial (before transfer) reduced mass μ_{i} and semimajor axis $\mathbf{R_{m,i}}$

Final expression for total potential energy of di-star

$$\mathbf{U} = -\frac{\mathbf{G}}{\mathbf{2}}\left(\omega_0[\mathrm{M}_1^{13/12} + \mathrm{M}_2^{13/12}] + \omega_{\mathrm{V}}[\mathrm{M}_1\mathrm{M}_2]^3\right)$$

$$\omega_0 = 1.644\frac{\mathrm{M}_\odot^{11/12}}{\mathrm{R}_\odot}$$

$$\omega_{\mathrm{V}} = \frac{1}{\mathrm{M}^2\mu_{\mathrm{i}}^2\mathrm{R}_{\mathrm{m,i}}}$$

Using mass asymmetry coordinate η instead of masses $\mathbf{M_1 = \frac{M}{2}(1+\eta)}$ and $\mathbf{M_2 = \frac{M}{2}(1-\eta)}$:

$$\mathbf{U} = -\frac{\mathbf{G M_\odot^2}}{\mathbf{2 R_\odot}}\left(\alpha[(\mathbf{1}+\eta)^{\mathbf{13/12}} + (\mathbf{1}-\eta)^{\mathbf{13/12}}] + \beta[\mathbf{1}-\eta^{\mathbf{2}}]^{\mathbf{3}}\right)$$

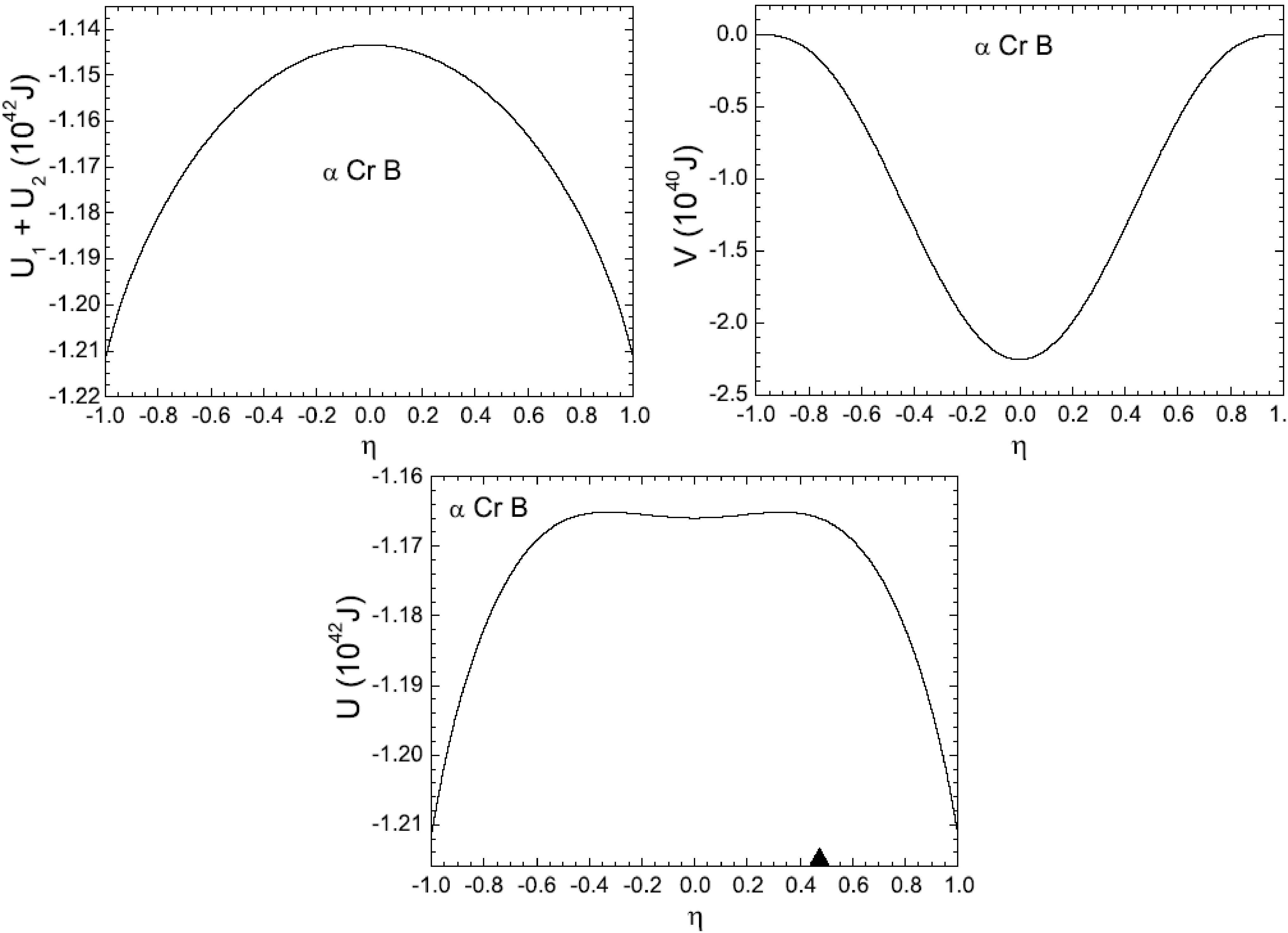
$$\alpha = \mathbf{1.644}\left(\frac{\mathbf{M}}{\mathbf{2M_\odot}}\right)^{\mathbf{13/12}}$$

$$\beta = \left(\frac{\pi^{\mathbf{2}}\mathbf{M_\odot^5 R_\odot^3}}{\mathbf{32 G}\mu_{\mathbf{i}}^{\mathbf{6}}\mathbf{P_{orb,i}^2}}\right)^{\mathbf{1/3}}\left(\frac{\mathbf{M}}{\mathbf{2M_\odot}}\right)^{\mathbf{11/3}}$$

To obtain β, we use Kepler's third law connecting semimajor axis

$$R_{m,i} = \left(\frac{GMP_{orb,i}^2}{4\pi^2} \right)^{1/3}$$

with period $P_{orb,i}$ of orb. rotation of initial di-star



U1 + U2 (10^42 J)
α Cr B
-1.14
-1.15
-1.16
-1.17
-1.18
-1.19
-1.20
-1.21
-1.22
-1.0 -0.8 -0.6 -0.4 -0.2 0.0 0.2 0.4 0.6 0.8 1.0
η
V (10^40 J)
α Cr B
0.0
-0.5
-1.0
-1.5
-2.0
-2.5
-1.0 -0.8 -0.6 -0.4 -0.2 0.0 0.2 0.4 0.6 0.8
η
U (10^42 J)
α Cr B
-1.16
-1.17
-1.18
-1.19
-1.20
-1.21
-1.0 -0.8 -0.6 -0.4 -0.2 0.0 0.2 0.4 0.6 0.8 1.0
η

Barrier in η appears as result of interplay between energy $U_1 + U_2$ of stars and star-star interaction V

Both energies have different behavior as function of η: $U_1 + U_2$ decreases, V increases with changing from $\eta = 0$ to $\eta = \pm 1$

One can study evolution of di-star in the mass asymmetry coordinate η.

Extremal points of potential energy as function of η are found by solving numerically Eq.

$$\frac{\partial \mathrm{U}}{\partial \eta} = -\frac{\mathrm{GM}_{\odot}^{2}}{2\mathrm{R}_{\odot}}\left(\frac{13}{12}\alpha[(1+\eta)^{1/12} - (1-\eta)^{1/12}] - 6\beta\eta[1-\eta^{2}]^{2}\right)$$

Eq. is solved for $\eta = \eta_{\mathrm{m}} = 0$.

At $\eta = \eta_{\rm m} = 0$ potential has extremum which is minimum if

$$\alpha < \frac{432}{13}\beta$$

or

$$L_i < [10.1 G R_\odot M_\odot^3]^{1/2} \left(\frac{M}{2M_\odot}\right)^{47/24}$$

or

$$P_{\rm orb,i} < \frac{128.5\pi}{(1-\eta_i^2)^3} \left(\frac{R_\odot^3}{GM_\odot}\right)^{1/2} \left(\frac{M}{2M_\odot}\right)^{7/8}$$

and maximum if

$$\alpha > \frac{432}{13}\beta$$

Transition point is

$$\alpha = \alpha_{\rm cr} = \frac{432}{13}\beta = \frac{27}{52}\frac{GM^5}{L_i^2}$$

If there is minimum at $\eta = 0$ ($\alpha < \alpha_{\rm cr}$), it is engulfed symmetrically by two barriers.

Expanding Eq. up to third order in η and solving it, we obtain position of these barriers at $\eta = \pm\eta_{\rm b}$,

$$\eta_{\rm b} = 2^{-1/2}\left(\frac{864^2\beta - 22464\alpha}{864^2\beta + 3289\alpha}\right)^{1/2}$$

So, at $\alpha < \alpha_{\rm cr}$ potential energy as function of η has two symmetric maxima at $\eta = \pm\eta_{\rm b}$ and minimum at $\eta = \eta_{\rm m} = 0$.

The fusion of two stars with $|\eta_{\rm i}| < \eta_{\rm b}$ can occur only by overcoming barrier at $\eta = +\eta_{\rm b}$ or $\eta = -\eta_{\rm b}$.

If $\beta \gg \frac{1}{66}\alpha$,

$$\eta_{\mathrm{b}} \to 2^{-1/2} \approx 0.71$$

Condition

$$0 < \eta_{\mathrm{b}} < 2^{-1/2}$$

means that in asymmetric system with mass ratio

$$\mathrm{M}_1/\mathrm{M}_2 > (1 + 2^{1/2})^2 \approx 6$$

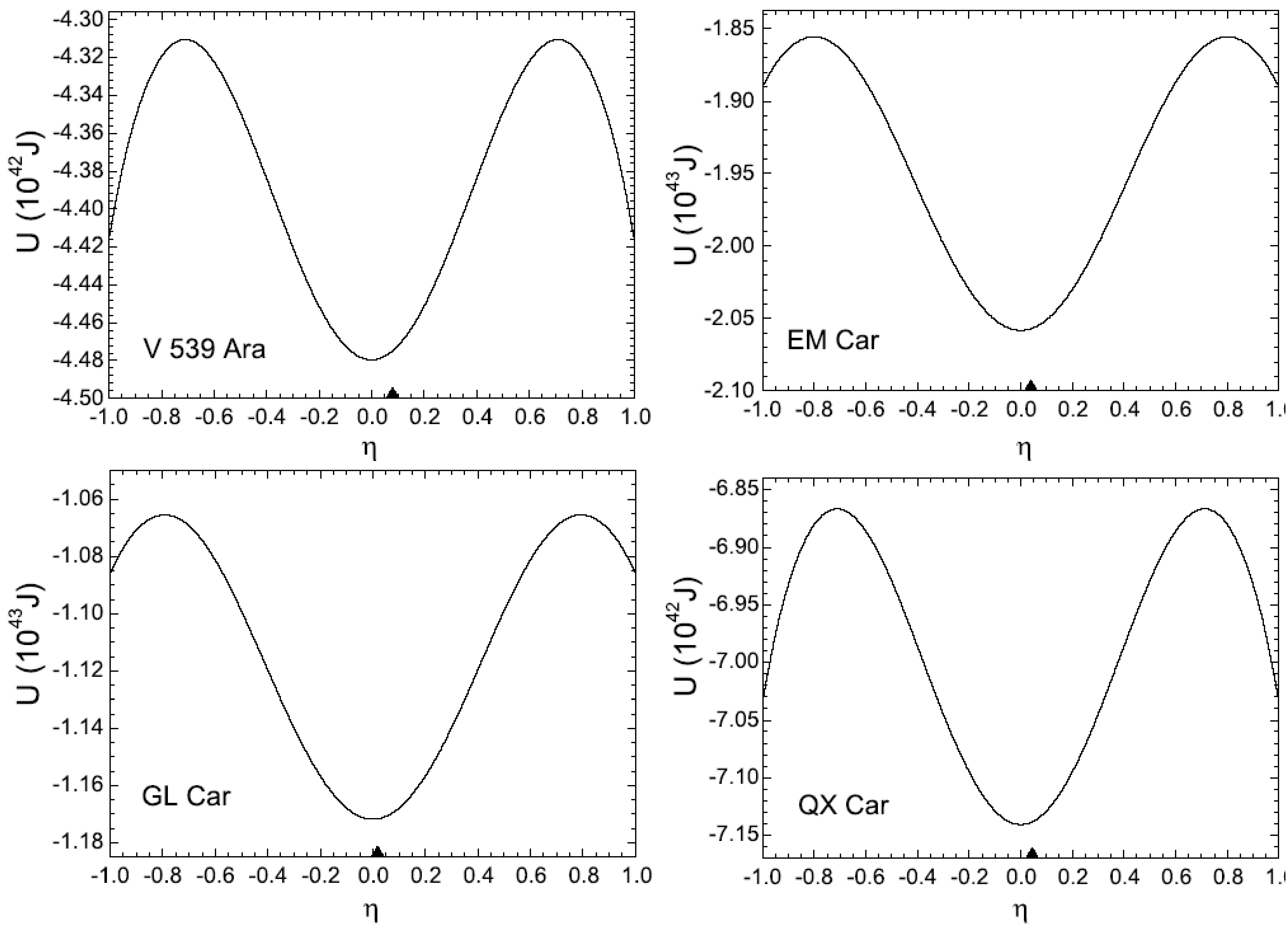
stars fuse

Thus, di-stars with $|\eta| > \eta_{\mathrm{b}}$ are unlikely to exist for sufficiently long time.

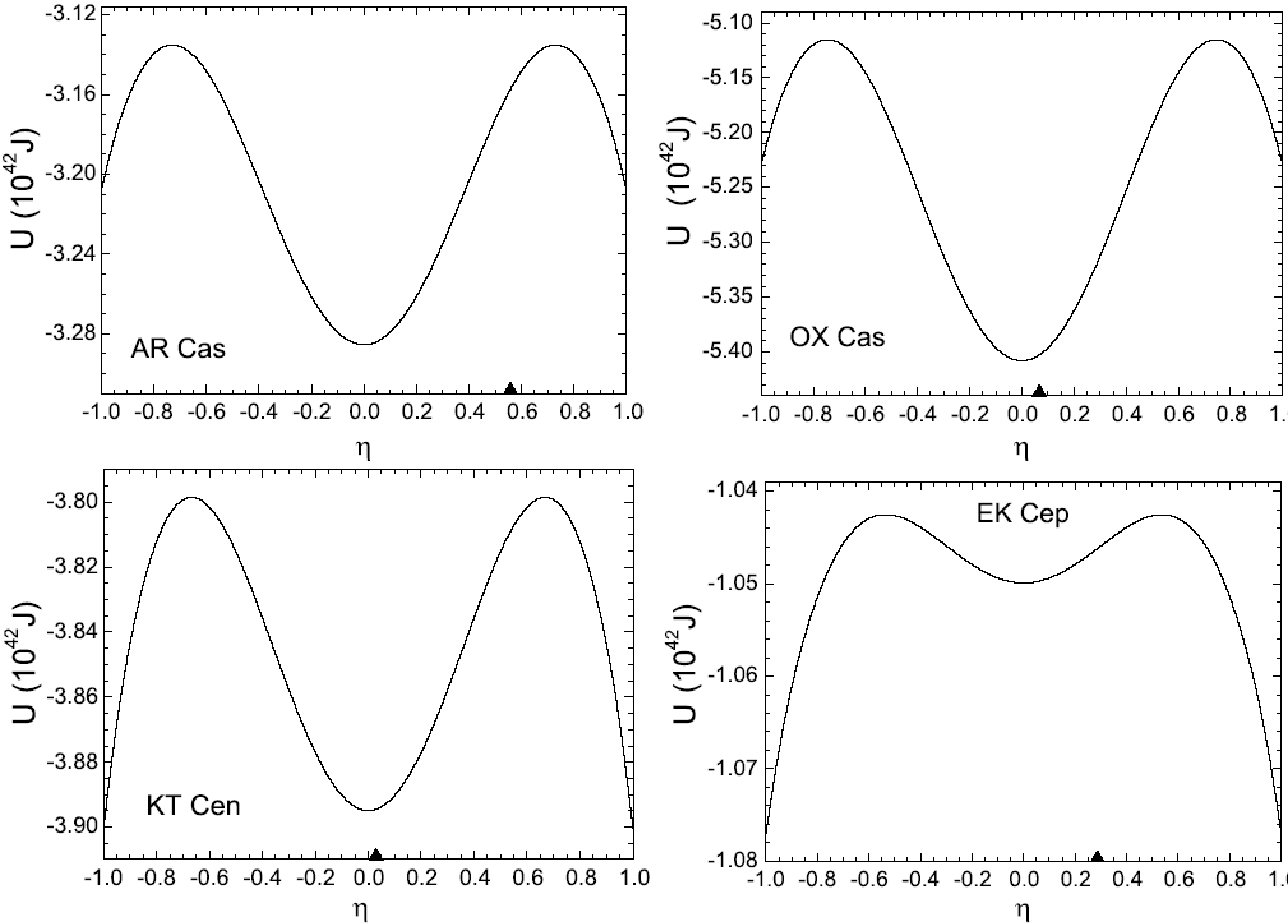
Indeed, close di-stars with large mass ratio are very rare objects in universe.

In calculations, we assume that orbital angular momentum L_i and total mass M are conserved during conservative evolution of di-star in η.

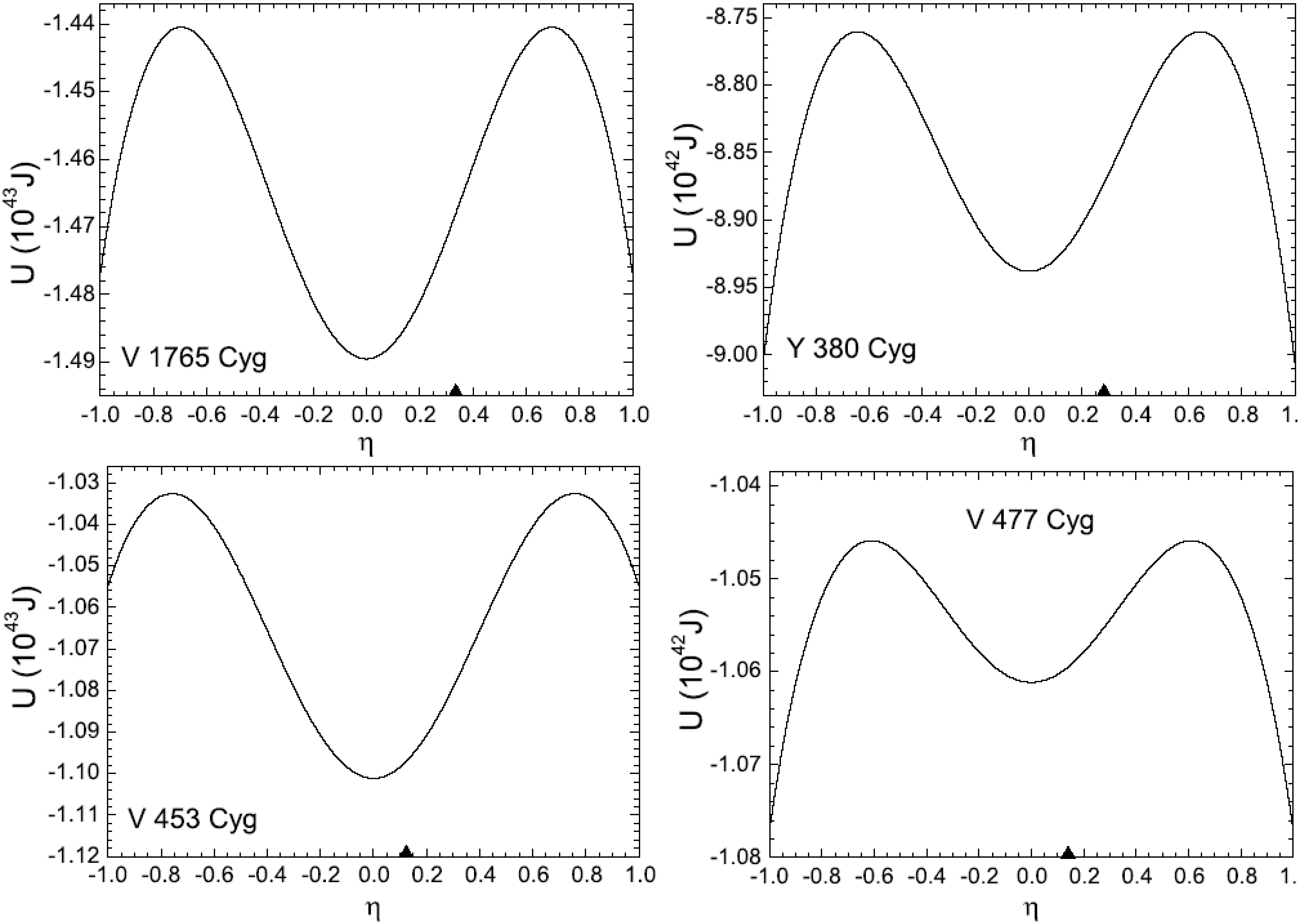
Orbital angular momentum L_i is calculated by using experimental masses M_i of stars and period $P_{orb,i}$ of their orbital rotation.



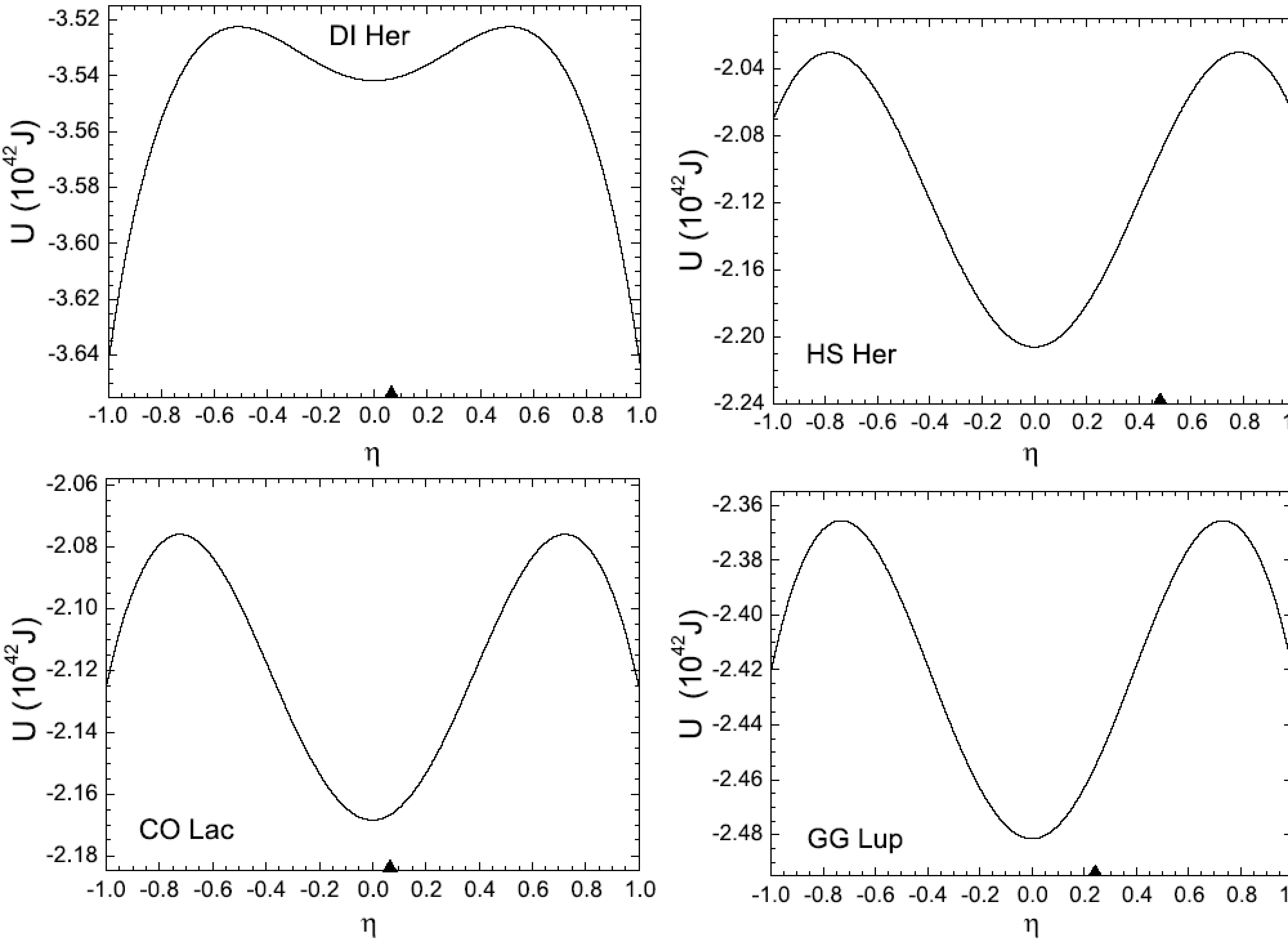
V 539 Ara
U (10^{42} J)
η
EM Car
U (10^{43} J)
η
GL Car
U (10^{43} J)
η
QX Car
U (10^{42} J)
η

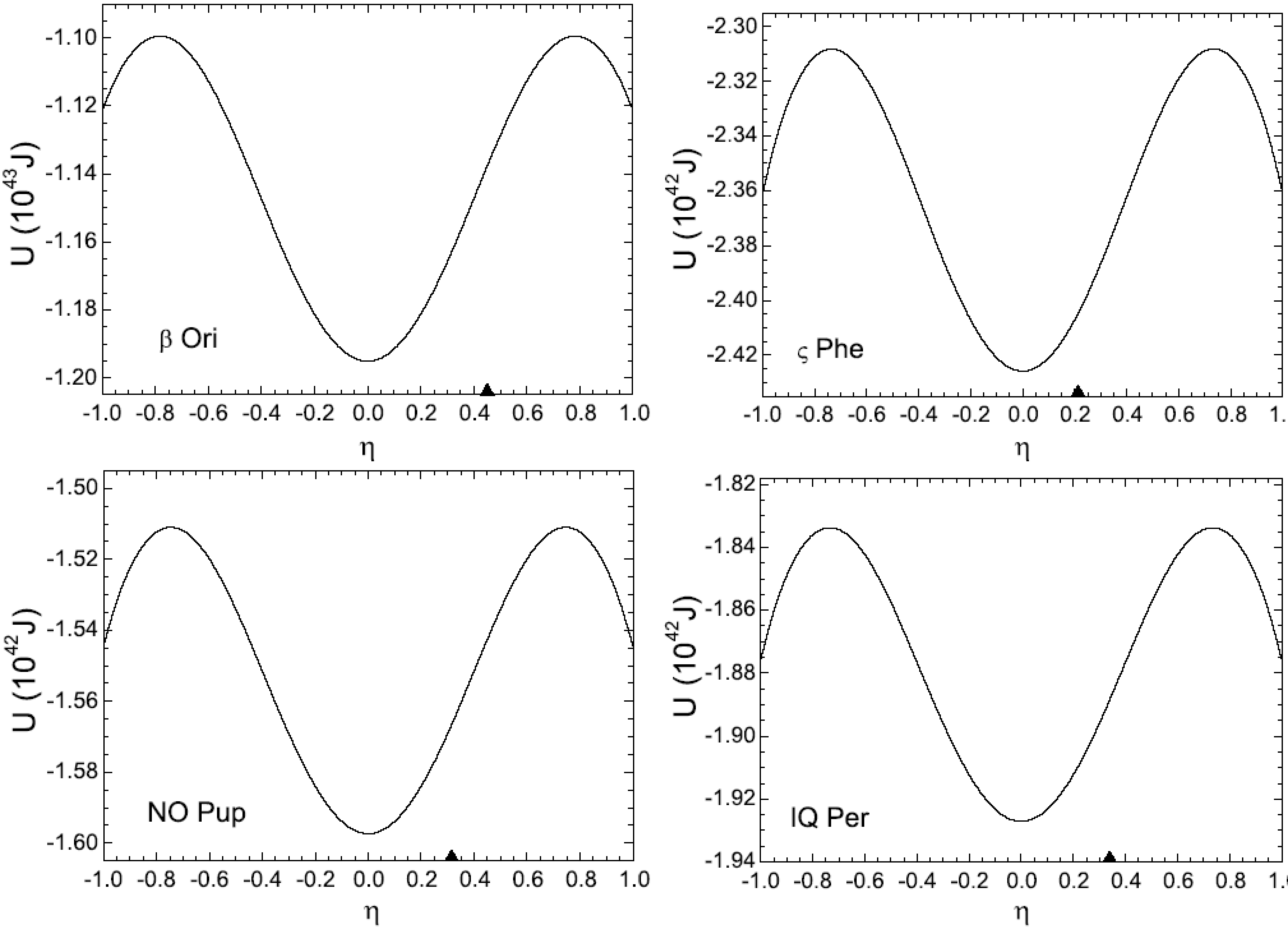


AR Cas
U (10^42 J)
η
-3.12
-3.16
-3.20
-3.24
-3.28
-1.0 -0.8 -0.6 -0.4 -0.2 0.0 0.2 0.4 0.6 0.8 1.0
OX Cas
-5.10
-5.15
-5.20
-5.25
-5.30
-5.35
-5.40
KT Cen
-3.80
-3.82
-3.84
-3.86
-3.88
-3.90
EK Cep
-1.04
-1.05
-1.06
-1.07
-1.08

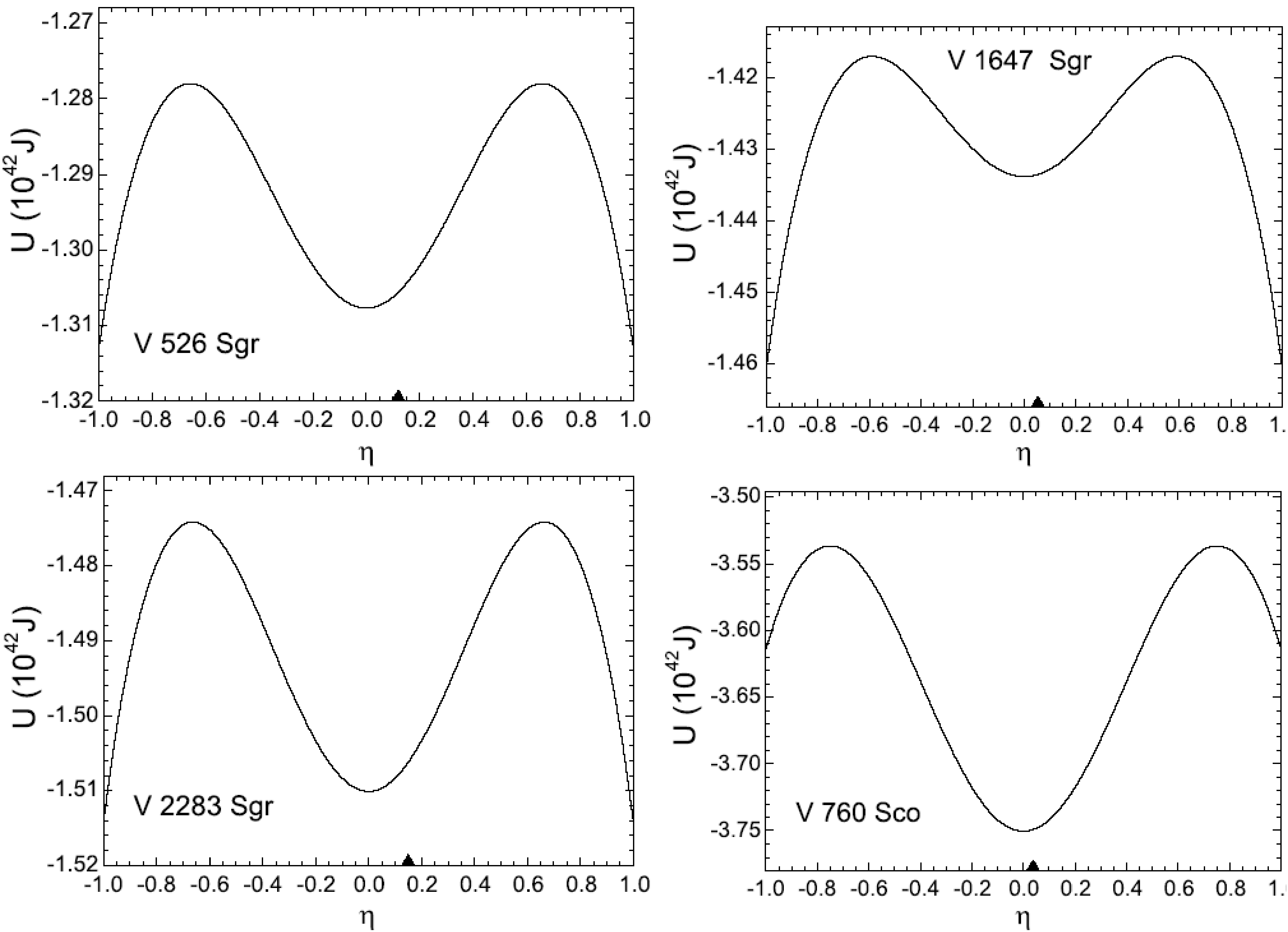


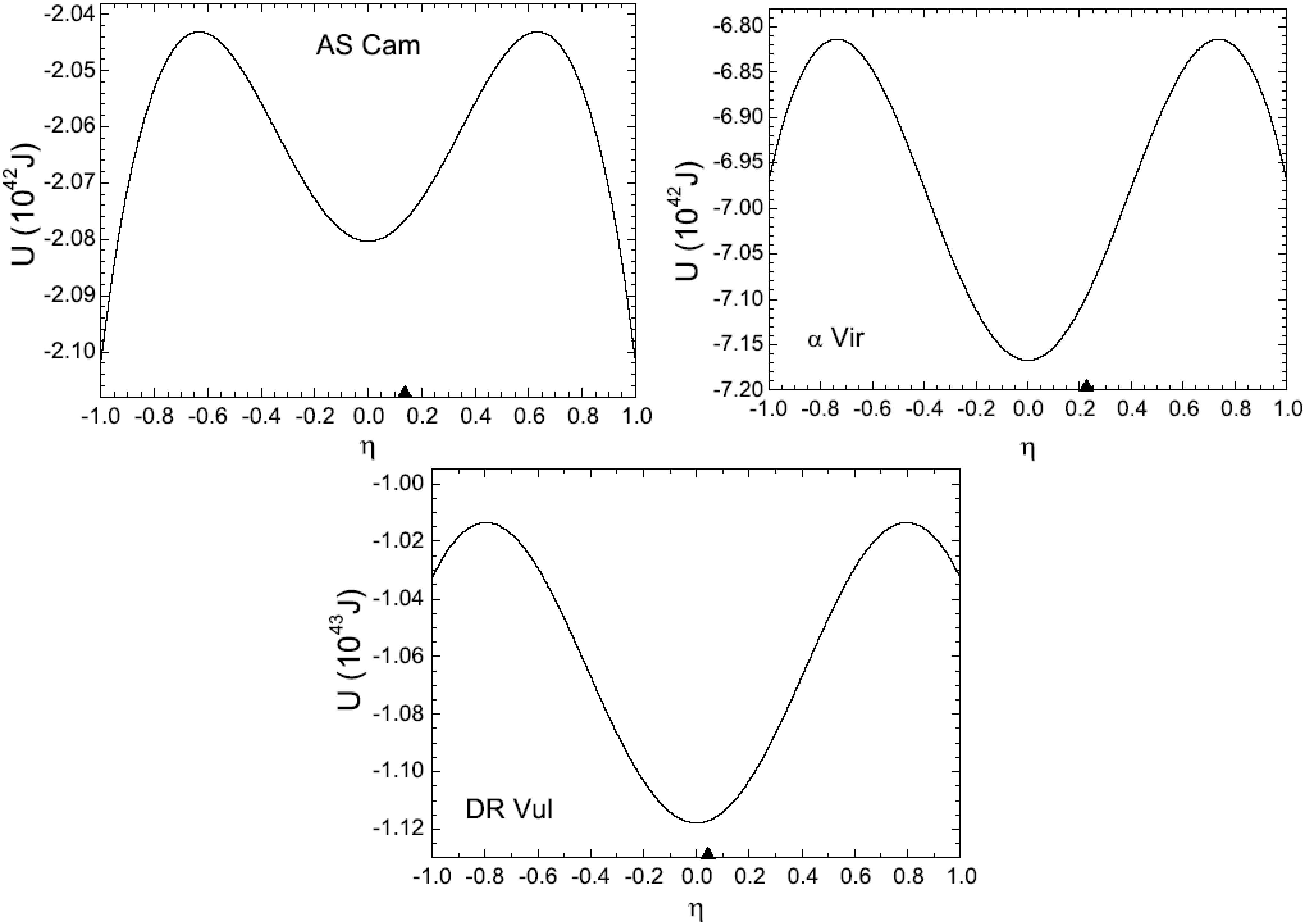
V 1765 Cyg
U (10^{43}J)
η
-1.44
-1.45
-1.46
-1.47
-1.48
-1.49
-1.0 -0.8 -0.6 -0.4 -0.2 0.0 0.2 0.4 0.6 0.8 1.0
Y 380 Cyg
U (10^{42}J)
η
-8.75
-8.80
-8.85
-8.90
-8.95
-9.00
-1.0 -0.8 -0.6 -0.4 -0.2 0.0 0.2 0.4 0.6 0.8 1.
V 453 Cyg
U (10^{43}J)
η
-1.03
-1.04
-1.05
-1.06
-1.07
-1.08
-1.09
-1.10
-1.11
-1.12
-1.0 -0.8 -0.6 -0.4 -0.2 0.0 0.2 0.4 0.6 0.8 1.0
V 477 Cyg
U (10^{42}J)
η
-1.04
-1.05
-1.06
-1.07
-1.08
-1.0 -0.8 -0.6 -0.4 -0.2 0.0 0.2 0.4 0.6 0.8 1.


DI Her
U (10^42 J)
-3.52
-3.54
-3.56
-3.58
-3.60
-3.62
-3.64
-1.0 -0.8 -0.6 -0.4 -0.2 0.0 0.2 0.4 0.6 0.8 1.0
η
HS Her
U (10^42 J)
-2.04
-2.08
-2.12
-2.16
-2.20
-2.24
-1.0 -0.8 -0.6 -0.4 -0.2 0.0 0.2 0.4 0.6 0.8
η
CO Lac
U (10^42 J)
-2.06
-2.08
-2.10
-2.12
-2.14
-2.16
-2.18
-1.0 -0.8 -0.6 -0.4 -0.2 0.0 0.2 0.4 0.6 0.8 1.0
η
GG Lup
U (10^42 J)
-2.36
-2.38
-2.40
-2.42
-2.44
-2.46
-2.48
-1.0 -0.8 -0.6 -0.4 -0.2 0.0 0.2 0.4 0.6 0.8
η



β Ori
U (10^{43} J)
ς Phe
U (10^{42} J)
NO Pup
U (10^{42} J)
IQ Per
U (10^{42} J)
η


V 526 Sgr
V 1647 Sgr
V 2283 Sgr
V 760 Sco
U (10^42 J)
η
-1.0 -0.8 -0.6 -0.4 -0.2 0.0 0.2 0.4 0.6 0.8 1.0
-1.27 -1.28 -1.29 -1.30 -1.31 -1.32
-1.42 -1.43 -1.44 -1.45 -1.46
-1.47 -1.48 -1.49 -1.50 -1.51 -1.52
-3.50 -3.55 -3.60 -3.65 -3.70 -3.75


AS Cam
U (10^42 J)
η
α Vir
U (10^42 J)
η
DR Vul
U (10^43 J)
η

Potentials U(η) for di-stars looks like potentials for microscopic dinuclear systems !

Di-stars undergo mass symmetrization.

Symmetrization of asymmetric binary star leads to decrease of potential energy U or transformation of potential energy into internal energy of stars

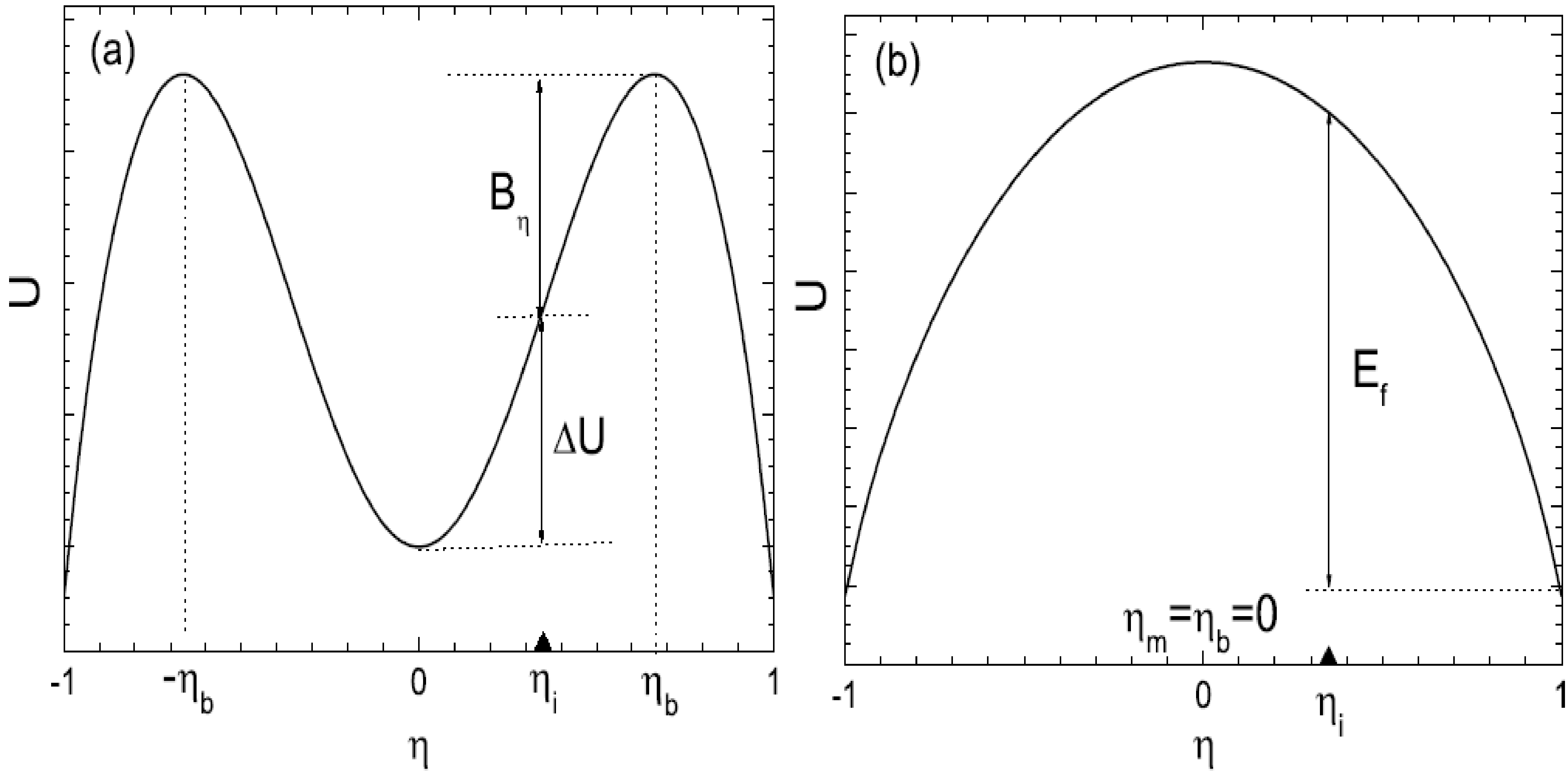


FIG. 1: The schematical drawings of the driving potential energy of the star-star system at $\alpha < \alpha_{cr}$ (a), and $\alpha > \alpha_{cr}$ (b). The arrows on x-axis show the corresponding initial binary stars. The notations used in the text are indicated.

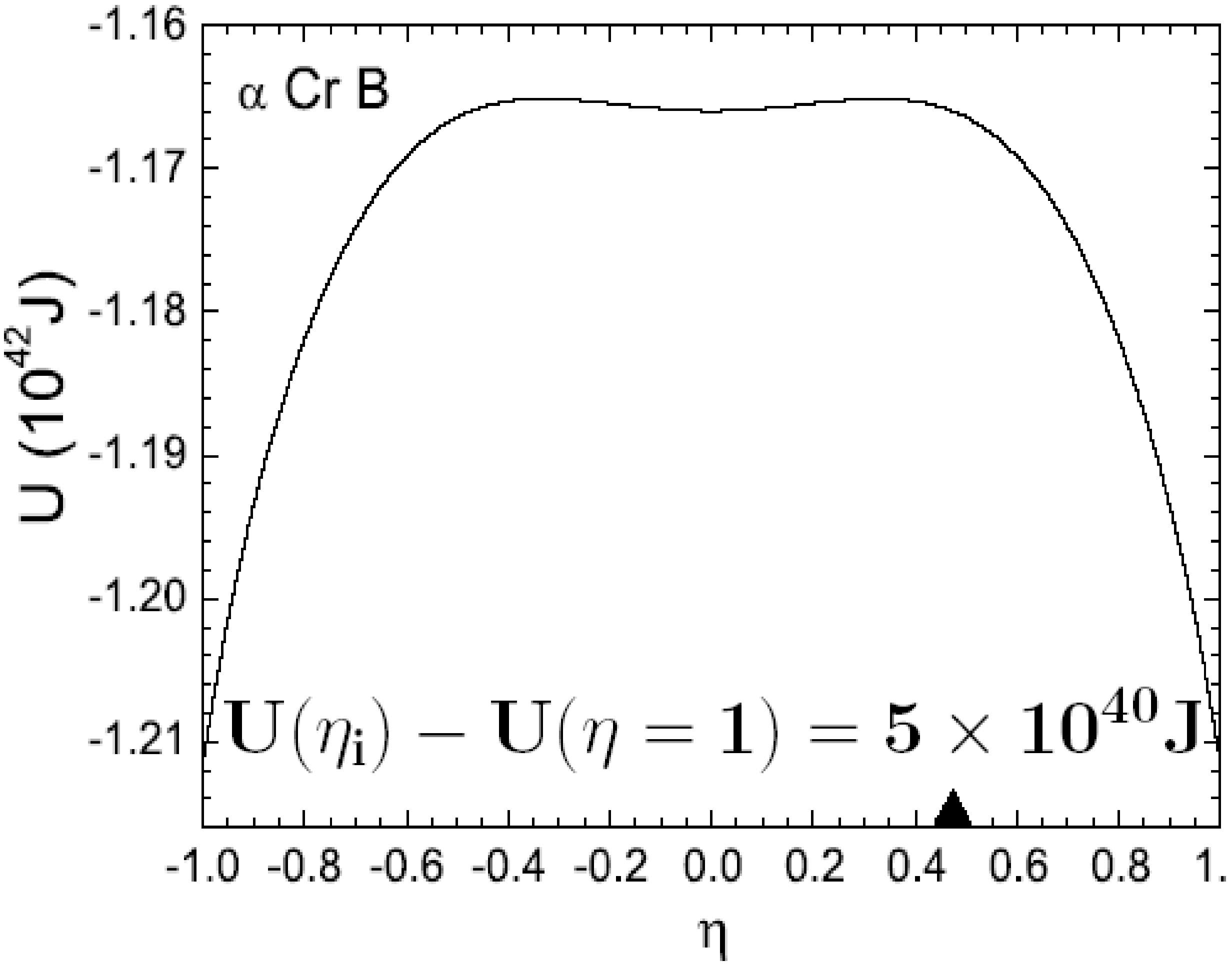
For close di-stars β Ori (η_i=0.451), V 1765 Cyg (η_i=0.335), HS Her (η_i=0.481), AR Cas (η_i=0.558), internal energies of stars increase during symmetrization by amount

$$\Delta U = U(\eta_i) - U(\eta = 0) \approx 10^{41-42} J.$$

Because most of close di-stars are asymmetric ones, symmetrization process leads to release of large amount of energy in these systems and can be important source of energy in Universe.

If $|\eta_i| > \eta_b$ or $\eta_b = 0$, di-star is unstable, evolves to mono-star system, enforcing fusion of stars.

We found only one close di-star α Cr B ($M_1 = 2.58 M_\odot$, $M_2 = 0.92 M_\odot$), for which $|\eta_i| = 0.47 > \eta_b = 0.33$.



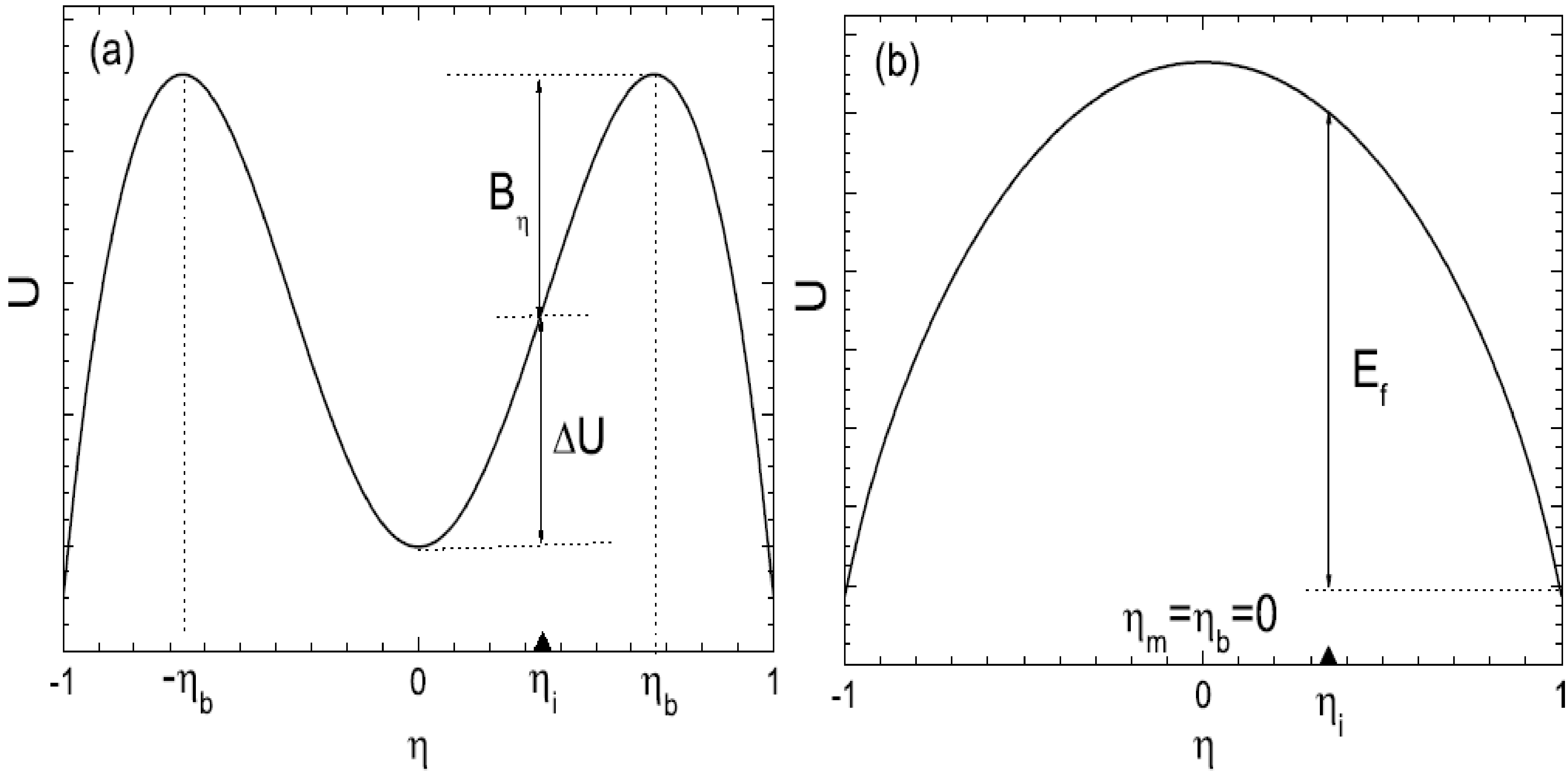
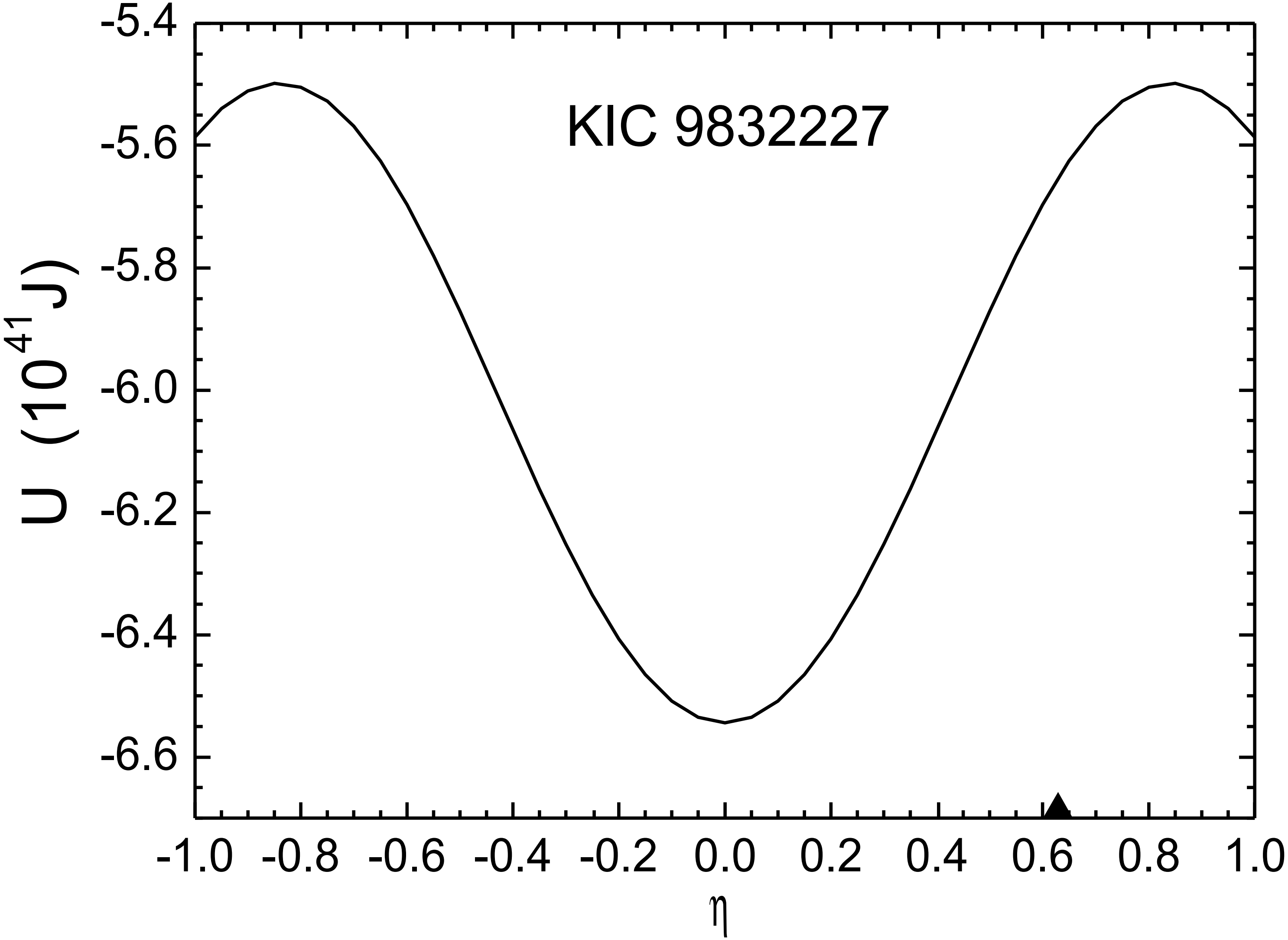


FIG. 1: The schematical drawings of the driving potential energy of the star-star system at $\alpha < \alpha_{cr}$ (a), and $\alpha > \alpha_{cr}$ (b). The arrows on x-axis show the corresponding initial binary stars. The notations used in the text are indicated.

Di-star	$\frac{M_1}{M_\odot}$	$\frac{M_2}{M_\odot}$	ω_1	ω_2	β/α	ΔU (J)	B_η (J)
V 346 Cen	11.8	8.4	0.89	0.97	0.137	3×10^{40}	2×10^{41}
Y 380 Cyg	14.3	8.0	0.85	1.27	0.103	7×10^{40}	10^{41}
V 453 Cyg	14.5	11.3	0.84	0.9	0.213	4×10^{40}	7×10^{41}
GG Lup	4.12	2.51	1.15	1.3	0.173	3×10^{40}	10^{41}
NO Pup	2.88	1.5	1.26	1.49	0.192	3×10^{40}	6×10^{40}
IQ Per	3.51	1.73	1.2	1.43	0.177	4×10^{40}	6×10^{40}
α Vir	10.8	6.8	0.91	0.99	0.180	7×10^{40}	3×10^{41}
AR Cas	6.7	1.9	1.02	1.40	0.170	10^{41}	2×10^{40}
HS Her	4.25	1.49	0.89	1.49	0.258	10^{41}	10^{41}
V 1765 Cyg	23.5	11.7	0.75	0.89	0.138	2×10^{41}	3×10^{41}
β Ori	19.8	7.5	0.87	0.99	0.259	6×10^{41}	4×10^{41}

Because fusion barriers B_η are quite large for di-stars with $|\eta_{\rm i}| < \eta_{\rm b}$, formation of mono-star from di-star by thermal diffusion in η is suppressed.

Minimum in $U(\eta)$ disappears, stars fuses as result of release of matter from one of stars or increase of orbital momentum due to strong external perturbation by third object or spin-orbital coupling in di-star.


KIC 9832227
U (10^41 J)
η
-5.4
-5.6
-5.8
-6.0
-6.2
-6.4
-6.6
-1.0
-0.8
-0.6
-0.4
-0.2
0.0
0.2
0.4
0.6
0.8
1.0

Summary

Mass asymmetry plays comparable important role in macroscopic as well as in microscopic object

Mass asymmetry can govern fusion, symmetrization of di-stars

Di-star evolution depends on total mass, initial orbital momentum, mass ratio

In di-stars, except αCr B, symmetrization occurs

Symmetrization will lead to $M_1/M_2 \to 1$, $T_1/T_2 \to 1$, $L_1/L_2 \to 1$, $R_1/R_2 \to 1$ which are observables

Symmetrization leads to release of huge energy ($\sim 10^{41-42}$ J).

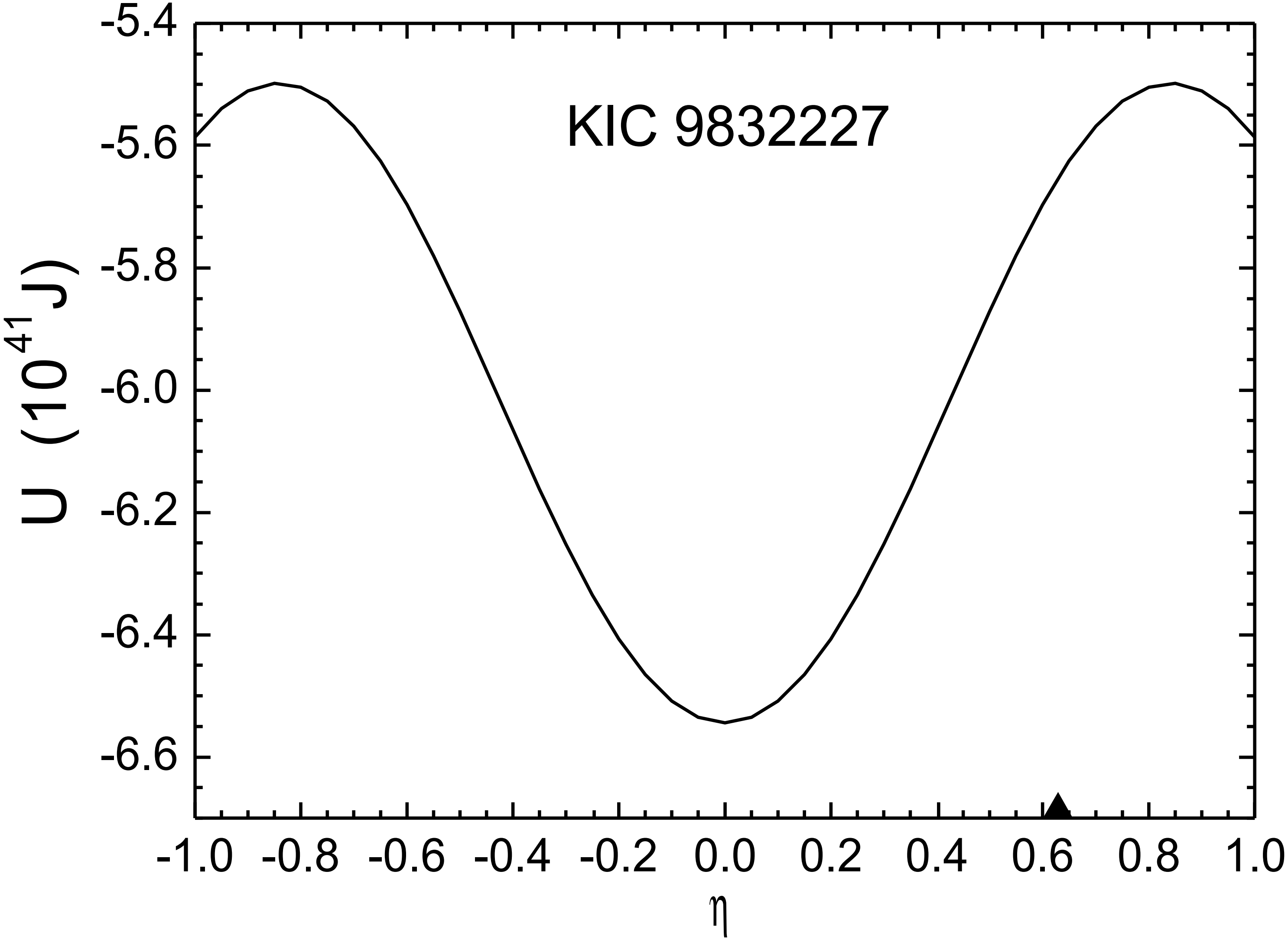
Di-stars are one of sources of energy in Universe

Merger are rare events

1) Release of matter from one of stars or

2) Increase of orbital momentum due to strong external perturbation by third object or due to spin-orbital coupling can lead to disappearance of minimum in $U(\eta)$ and fusion of stars

THANK YOU !


KIC 9832227
U (10^41 J)
-5.4
-5.6
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-6.0
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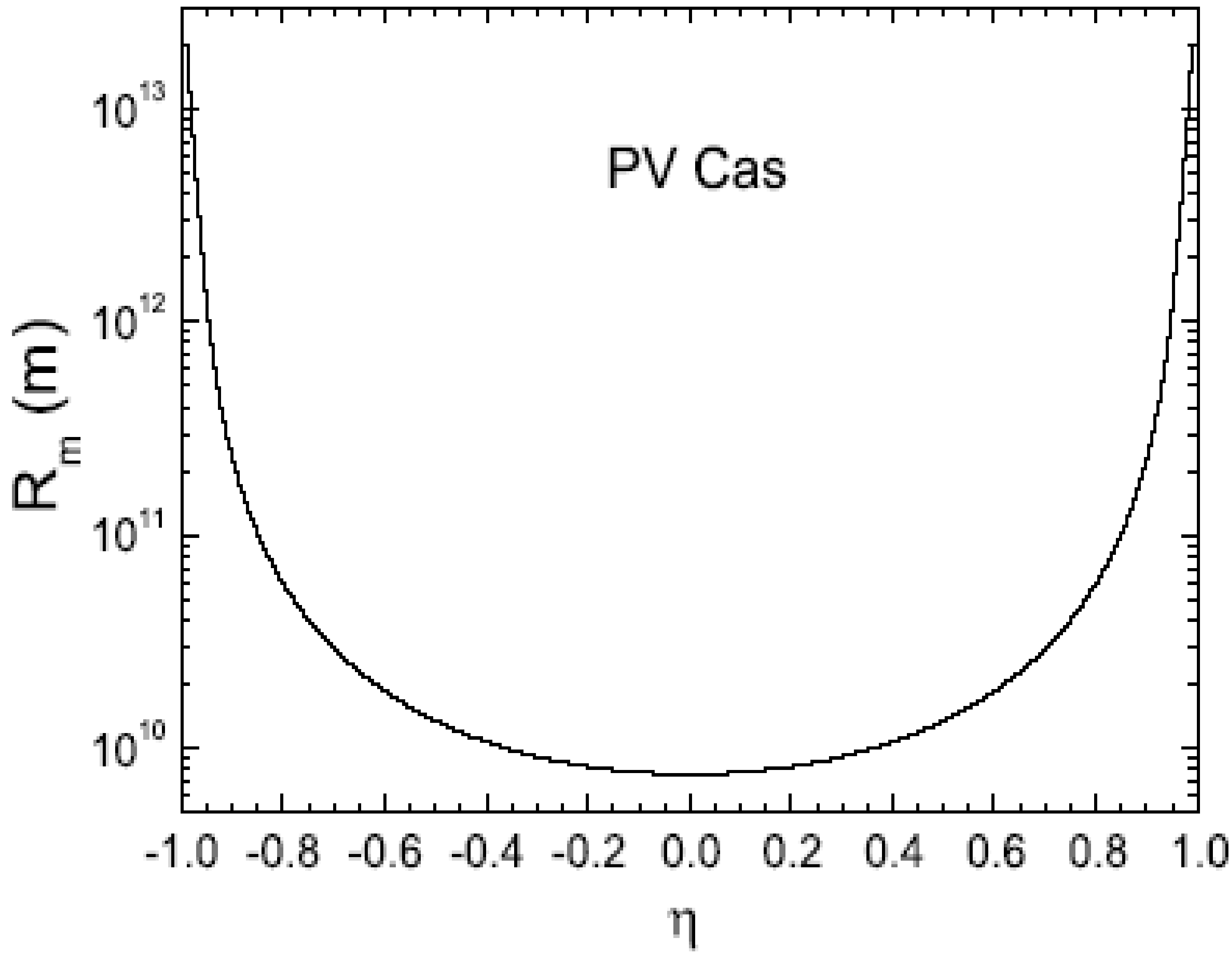
-6.4
-6.6
-1.0 -0.8 -0.6 -0.4 -0.2 0.0 0.2 0.4 0.6 0.8 1.0
η



PV Cas
R_m (m)
10^{13}
10^{12}
10^{11}
10^{10}
-1.0 -0.8 -0.6 -0.4 -0.2 0.0 0.2 0.4 0.6 0.8 1.0
η

Mass transfer between stars in di-star is closely related to their radii, which will take place when star exceeds its Roche lobe.

As seen from our calculations, ratio between radii of star and corresponding Roche lobe weakly depends on η.

Evolution of di-star system depends on initial mass asymmetry $\eta = \eta_{\rm i}$ at its formation.

If original di-star is asymmetric, but $|\eta_{\rm i}| < \eta_{\rm b}$, then it is energetically favorable to evolve in η to configuration in global minimum at $\eta = 0$, to form symmetric di-star.

Symmetrization of asymmetric binary star leads to decrease of potential energy U or transformation of potential energy into internal energy of stars.

If $\beta \gg \frac{1}{66}\alpha$,

$$\eta_{\rm b} \to 2^{-1/2} \approx 0.71$$

Condition

$$0 < \eta_{\rm b} < 2^{-1/2}$$

means that in asymmetric system with mass ratio

$$M_1/M_2 > (1 + 2^{1/2})^2 \approx 6$$

stars fuse

Thus, di-stars with $|\eta| > \eta_{\rm b}$ are unlikely to exist for sufficiently long time.

Indeed, close di-stars with large mass ratio are very rare objects in universe.

If there is minimum at $\eta = 0$ ($\alpha < \alpha_{\mathrm{cr}}$), it is engulfed symmetrically by two barriers.

Expanding Eq. up to third order in η and solving it, we obtain position of these barriers at $\eta = \pm\eta_{\mathrm{b}}$,

$$\eta_{\mathrm{b}} = 2^{-1/2}\left(\frac{864^2\beta - 22464\alpha}{864^2\beta + 3289\alpha}\right)^{1/2}$$

So, at $\alpha < \alpha_{\mathrm{cr}}$ potential energy as function of η has two symmetric maxima at $\eta = \pm\eta_{\mathrm{b}}$ and minimum at $\eta = \eta_{\mathrm{m}} = 0$.

The fusion of two stars with $|\eta_{\mathrm{i}}| < \eta_{\mathrm{b}}$ can occur only by overcoming barrier at $\eta = +\eta_{\mathrm{b}}$ or $\eta = -\eta_{\mathrm{b}}$.

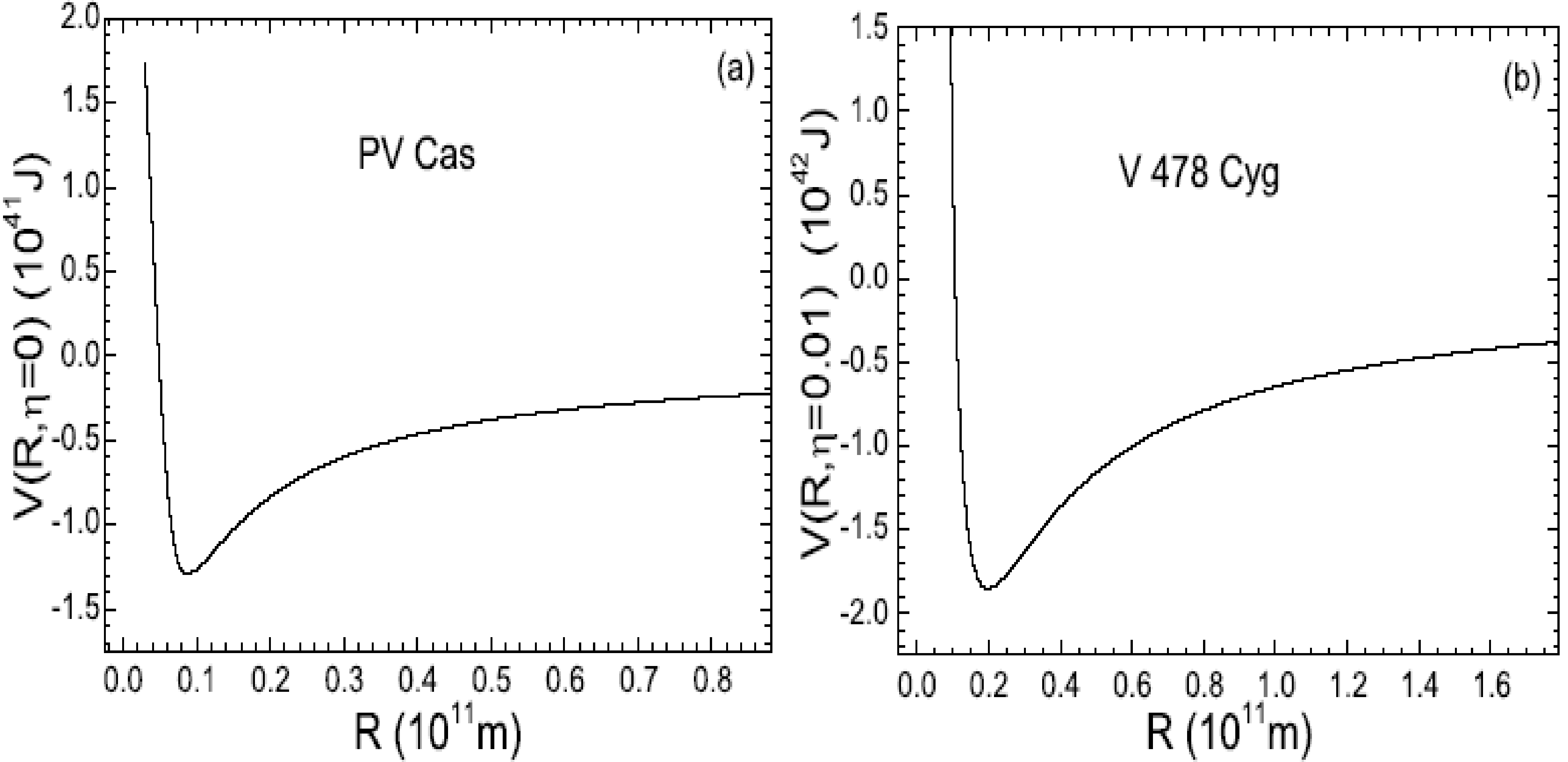
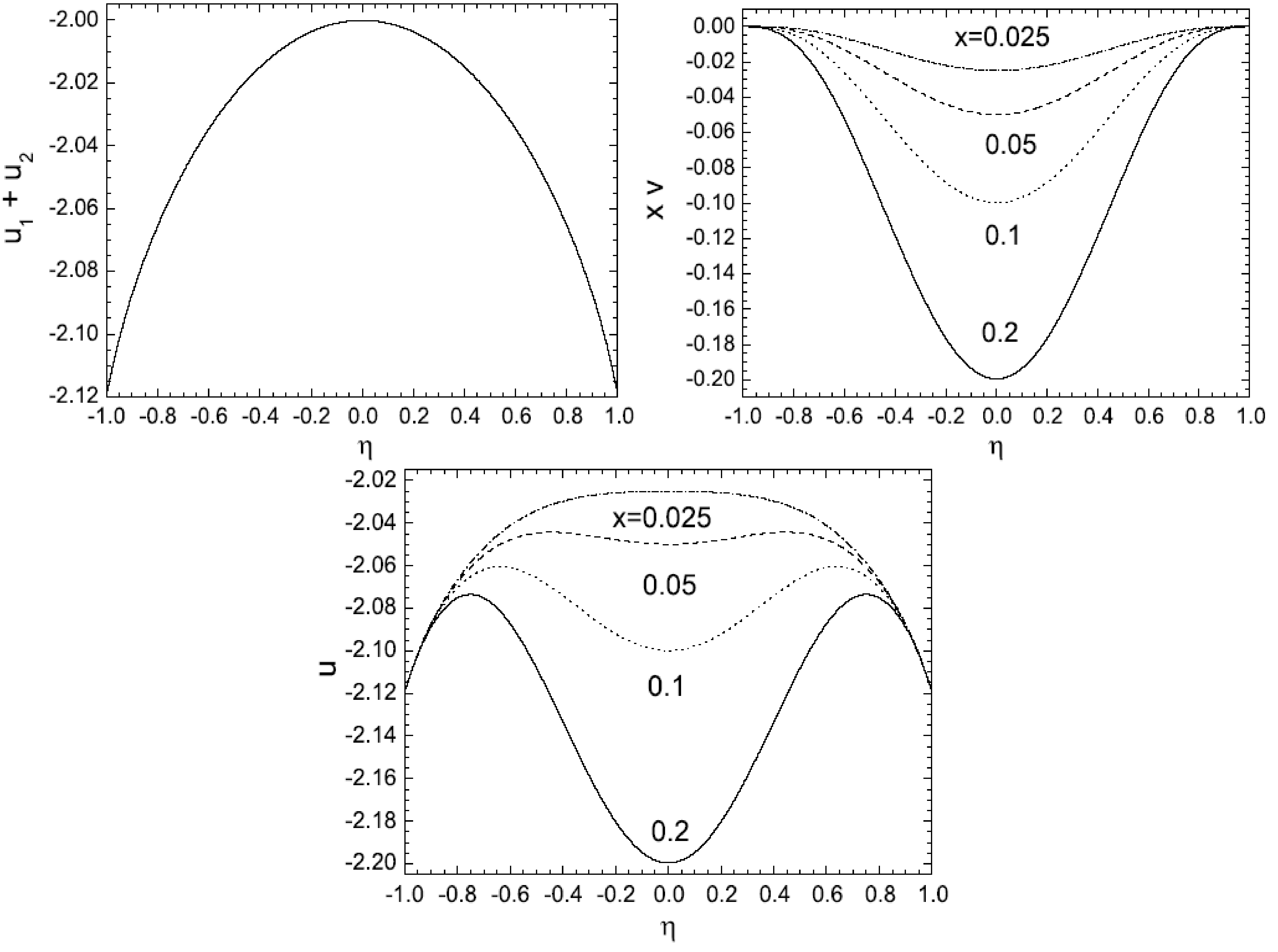


Fig. 2. The star-star interaction potentials for the di-stars PV Cas ($M_1 = M_2 = 2.79M_\odot$, $R_1 = R_2 = 2.264R_\odot$, $T_1 = T_2 = 11200$ K) and V 478 Cyg ($M_1 = 16.30M_\odot$, $M_2 = 16.60M_\odot$, $R_1 = R_2 = 7.422R_\odot$, $T_1 = T_2 = 29800$ K) vs R.

One can express potential energy in units of

$$\mathbf{u} = \mathbf{u}_1 + \mathbf{u}_2 + \mathbf{x}\mathbf{v} = (1+\eta)^{13/12} + (1-\eta)^{13/12} + \mathbf{x}(1-\eta^2)^3$$

$$\mathbf{x} = \beta/\alpha = \frac{\mathrm{G}\mathrm{M}_\odot^3\mathrm{R}_\odot}{3.288\mathrm{L}_\mathrm{i}^2}\left(\frac{\mathrm{M}}{2\mathrm{M}_\odot}\right)^{47/12}$$



u1 + u2
η
x v
x=0.025
0.05
0.1
0.2
η
u
x=0.025
0.05
0.1
0.2
η

Stability of di-star depends on

1) Angular momentum L_i or period $P_{orb,i}$ of orb. rotation and reduced mass μ_i

2) Total mass M

Increase of M or decrease of L_i leads to more stable di-star system.

Average star density

$$\frac{\rho_i}{\rho_\odot} = \frac{M_\odot}{M_i} = \frac{2M_\odot}{M}(1 \pm \eta)^{-1},$$

charge

$$\frac{Z_i}{Z_\odot} = \left(\frac{M_\odot}{M_i}\right)^{5/12} = \left(\frac{2M_\odot}{M}\right)^{5/12} (1 \pm \eta)^{-5/12},$$

mass

$$\frac{A_i}{A_\odot} = \left(\frac{M_\odot}{M_i}\right)^{11/12} = \left(\frac{2M_\odot}{M}\right)^{11/12} (1 \pm \eta)^{-11/12}$$

numbers of nuclei forming star increase with decreasing mass M_i of di-star

Structural factor depends on mass of star:

$$\frac{\omega_{\mathrm{i}}}{\omega_\odot} = \left(\frac{\mathrm{M}_\odot}{\mathrm{M}_{\mathrm{i}}}\right)^{1/4} = \left(\frac{2\mathrm{M}_\odot}{\mathrm{M}}\right)^{1/4} (1 \pm \eta)^{-1/4}$$

structural factor $\omega_\odot$ of Sun,

the signs plus and minus correspond to ω_1 and ω_2

The change of η from 0 to 1 leads to the change of ω_1 by about of 16%.

Energy of star "i"

$$\mathrm{U_i} = -\omega_\mathrm{i}\frac{\mathrm{GM_i^2}}{\mathrm{2R_i}}$$

G - gravitational constant, $\mathrm{M_i}$ - mass, $\mathrm{R_i}$ - radius of s

Dimensionless structural factor

$$\omega_\mathrm{i} = \int_0^1 \mathrm{dq_{x_i} q_{x_i}/x_i}$$

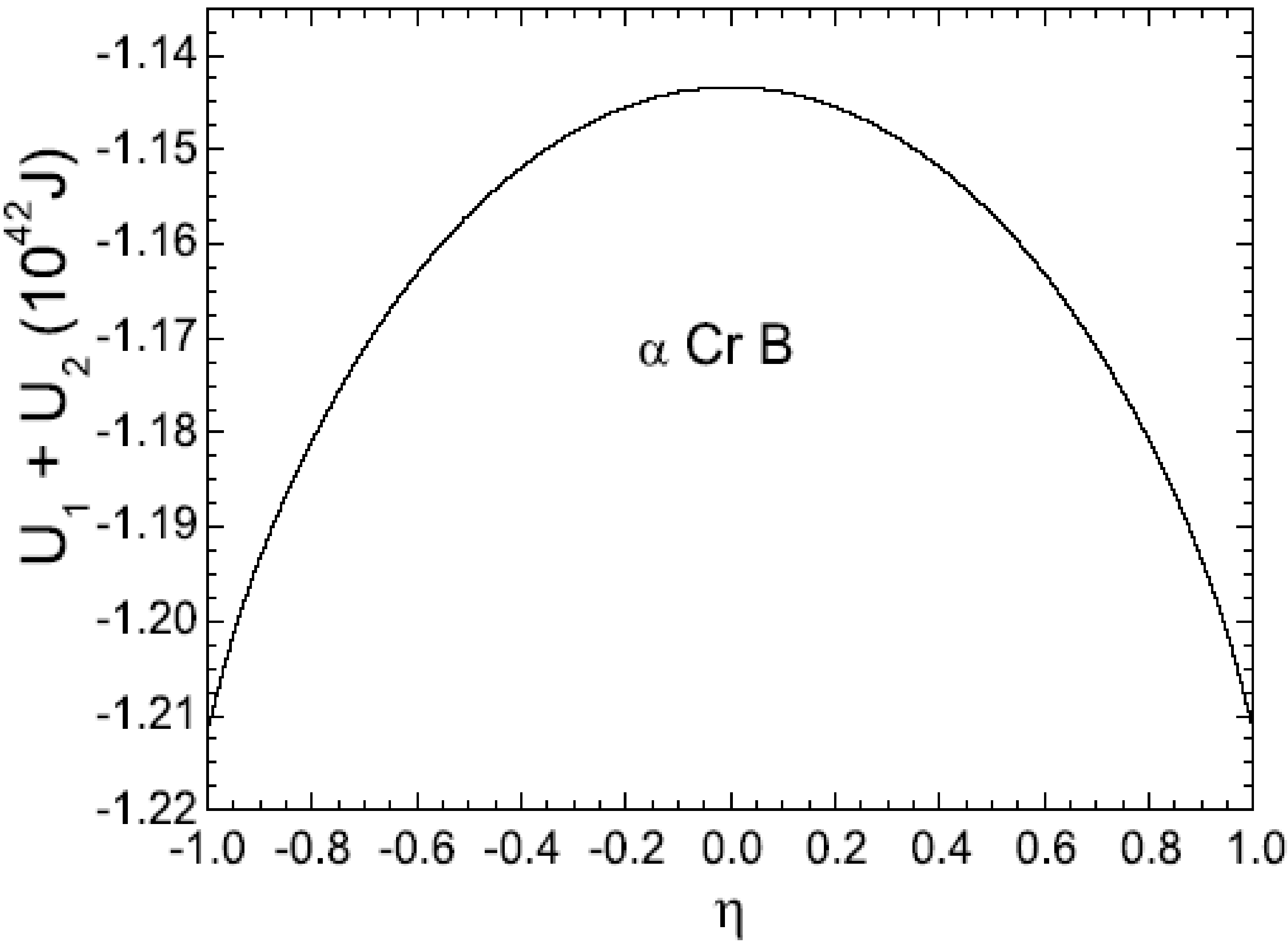
is determined by density profile $\rho_\mathrm{i}(\mathrm{r})$ of star,

radius fraction $\mathrm{x_i = r/R_i}$ and mass fraction

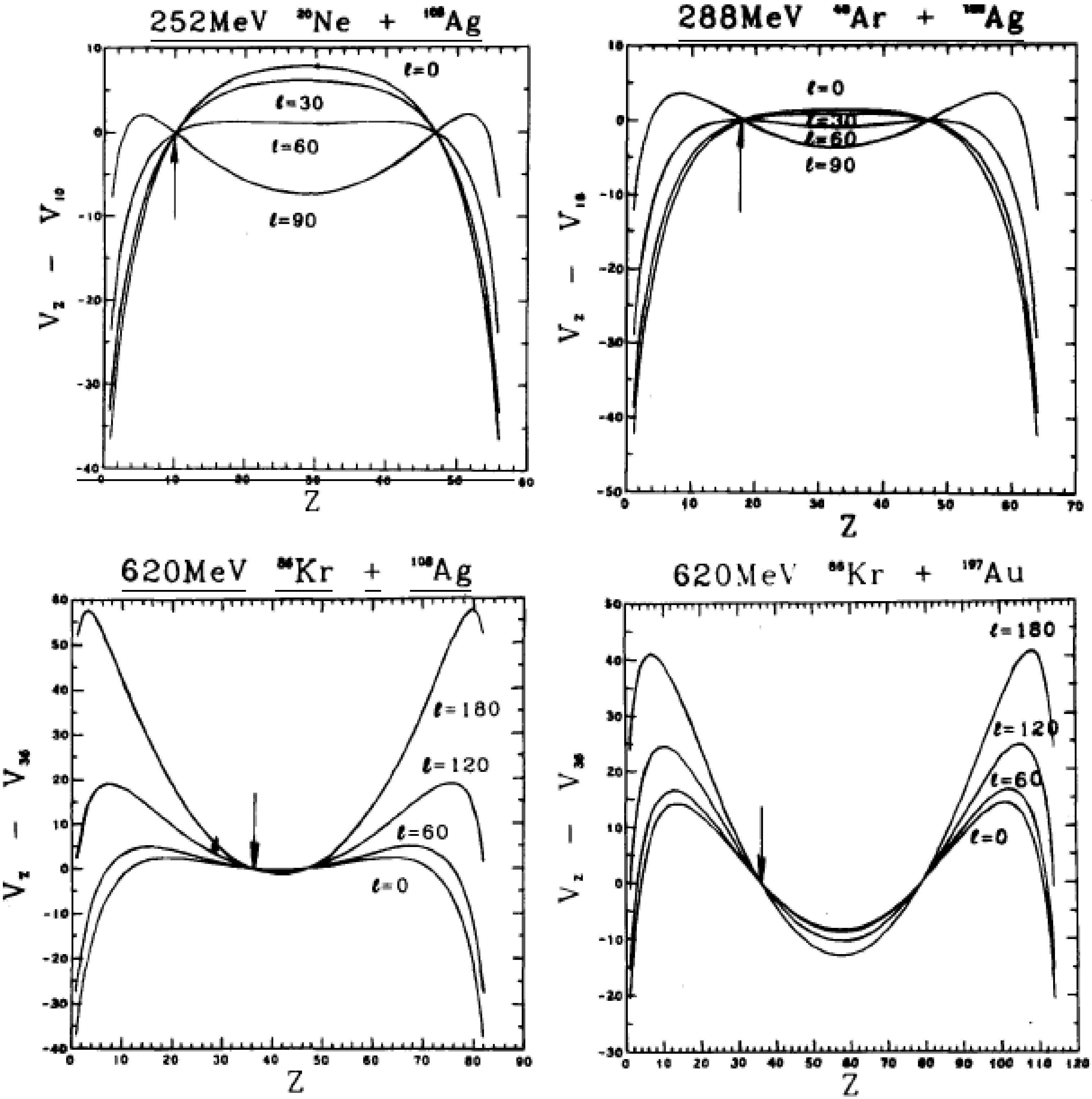
$$\mathrm{q_{x_i} = M_{x_i}/M_i} = \int_0^\mathrm{r} \mathrm{dr'r'^2}\rho_\mathrm{i}(\mathrm{r'}) / \int_0^{\mathrm{R_i}} \mathrm{dr'r'^2}\rho_\mathrm{i}(\mathrm{r'})$$

of star at distance r from center of star

Because $\mathrm{x_i} \leq 1$, $\omega_\mathrm{i} \geq 1/2$.



-1.14
-1.15
-1.16
-1.17
-1.18
-1.19
-1.20
-1.21
-1.22
$U_1 + U_2$ (10^{42}J)
α Cr B
-1.0 -0.8 -0.6 -0.4 -0.2 0.0 0.2 0.4 0.6 0.8 1.0
η



252MeV ²⁰Ne + ¹⁰⁸Ag
ℓ=0
ℓ=30
ℓ=60
ℓ=90
V_z − V_10
Z
288MeV ⁴⁰Ar + ¹⁰⁸Ag
ℓ=0
ℓ=30
ℓ=60
ℓ=90
V_z − V_18
Z
620MeV ⁸⁶Kr + ¹⁰⁸Ag
ℓ=180
ℓ=120
ℓ=60
ℓ=0
V_z − V_36
Z
620MeV ⁸⁶Kr + ¹⁹⁷Au
ℓ=180
ℓ=120
ℓ=60
ℓ=0
V_z − V_36
Z