ROLE OF GRAVITATION
IN ELEMENTARY PARTICLE STRUCTURE

S. I. Manaenkov
NRC "Kurchatov Institute", Petersburg Nuclear Physics Institute

International Workshop INFINUM
Dubna, March 20-22, 2019

Contents

- Gravitation and Particle Physics
- Reissner-Nordström Solution
- Uniform Coordinates
- Tetrad Representation for Gravitational Field
- Action for Gravitational and Electromagnetic Fields
- Superpotential and Total Energy-Momentum Pseudo-Tensor
- Classical Electron
- Discussion of Results
- Conclusions

Gravitation and Particle Physics

- **Ratio of electrostatic and gravitational forces for two electrons**

$$\frac{\mathcal{F}_{\mathbf{el}}}{\mathcal{F}_{\mathbf{gr}}} = \frac{\mathbf{e^2}}{\mathbf{R^2}} \Big/ \frac{\mathbf{km_e^2}}{\mathbf{R^2}} = \frac{\mathbf{e^2}}{\mathbf{km_e^2}} = \mathbf{4.2 \cdot 10^{42}}; \ \frac{e}{\sqrt{k}m_e} = 2.05 \cdot 10^{21}$$

where $\mathrm{e} = 4.8 \cdot 10^{-10}$ esu, $\mathrm{m_e} = 9.11 \cdot 10^{-28}$ g, $\mathrm{k} = 6.67 \cdot 10^{-8}$ $\mathrm{cm^3\ g^{-1}\ c^{-2}}$.

- **The gravitation field does not play any role in the elementary particle structure!**
  Nevertheless this statement could be wrong.
- **Infinite electromagnetic mass of the electron**

$$m_{em} = \frac{1}{c^2}\int \frac{\vec{E}^2}{8\pi} dV = \frac{1}{c^2}\int \frac{e^2}{8\pi R^4} 4\pi R^2 dR,$$

$$m_{em}c^2 = \int_{r_{cl}/2}^{\infty} \frac{e^2 dR}{2R^2} = e^2/r_{cl} = m_e c^2,$$

where $r_{cl} = 2.8 \cdot 10^{-13}$ cm is the classical electron radius.

- **Gravitational interaction in Newtonian physics**

$$dU_{gr} = -\frac{k dm_1 dm_2}{R_{12}} = -\frac{e^4 k}{(8\pi)^2 c^4 R_{12}} \frac{dV_1}{R_1^4} \frac{dV_2}{R_2^4} < 0.$$

$U_{gr}/\mathcal{E}_{em} \sim -r_e^2/R_c^2$, where $r_e^2 = ke^2/c^4$, $r_e = 1.4 \cdot 10^{-34}$ cm.

Gravitation can play important role in classic physics at $r \sim r_e$.

- Gravitational field equations
  $R^i_k - \frac{1}{2}\delta^i_k R = \frac{8\pi k}{c^4} T^i_k, \quad T^i_k = \frac{1}{4\pi}\left\{-F^{il}F_{kl} + \frac{1}{4}\delta^i_k F_{lm}F^{lm}\right\}.$
- Reissner-Nordström (RN) solution
  Spherical coordinates: $(x^0, x^1, x^2, x^3) = (ct, r, \theta, \varphi)$
  $F^{10} = -F^{01} = F_{10} = -F_{01} = E_r = \frac{e}{r^2},$
  $T^0_{(em)0} = T^1_{(em)1} = -T^2_{(em)2} = -T^3_{(em)3} = \frac{e^2}{8\pi r^4}, \quad T^j_{(em)j} = 0.$
  $g_{00} = 1/g^{00} = \Lambda \equiv 1 - \frac{2km}{c^2 r} + \frac{ke^2}{c^4 r^2} = 1 - \frac{r_g}{r} + \frac{(r_e)^2}{r^2},$
  $r_g = \frac{2km}{c^2} = 1.35 \cdot 10^{-55}\mathrm{cm}, \ R_{Pl} = \sqrt{\frac{\hbar k}{c^3}} = 1.6 \cdot 10^{-33}\mathrm{cm},$
  $r_e^2 = \frac{ke^2}{c^4}, \quad r_e = 1.38 \cdot 10^{-34}\mathrm{cm}, \ r_e \gg r_g, \ \mathrm{R}_{Pl} \gg r_e.$
  $g_{rr} \equiv g_{11} = 1/g^{11} = -1/\Lambda = -\left[1 - \frac{2km}{c^2 r} + \frac{ke^2}{c^4 r^2}\right]^{-1},$
  $g_{\theta\theta} \equiv g_{22} = 1/g^{22} = -r^2, \quad g_{\varphi\varphi} = g_{33} = 1/g^{33} = -r^2 \sin^2\theta.$
  $ds^2 = \Lambda(dx^0)^2 - dr^2/\Lambda - r^2[d\theta^2 + \sin^2\theta d\varphi^2]$

- Relation between radii r and ρ ($r \geq 0$, $\rho \geq \rho_{min}$)

$$r = \rho\mathcal{D}(\rho),$$

$$\mathcal{D}(\rho) = 1 + \frac{r_g}{2\rho} - \frac{r_0^2}{4\rho^2} \equiv \left[1 + \frac{r_g}{4\rho}\right]^2 - \frac{r_e^2}{4\rho^2},$$

$$\mathcal{N}(\rho) \equiv \frac{dr}{d\rho} = 1 + \frac{r_0^2}{4\rho^2} > 0, \quad r_0^2 = r_e^2 - r_g^2/4.$$

- Uniform coordinates $(\rho^0, \rho^1, \rho^2, \rho^3) = (ct, \rho_x, \rho_y, \rho_z)$

$$\rho_x = \rho \sin\theta \cos\varphi, \quad \rho_y = \rho \sin\theta \sin\varphi, \quad \rho_z = \rho \cos\theta.$$

- Spacetime interval for Reissner-Nordström (RN) solution

$$ds^2 = g_{ik} d\rho^i d\rho^k \quad g_{00} = \frac{\mathcal{N}^2}{\mathcal{D}^2}, \; g_{\mu\mu} = -\mathcal{D}^2, \mu = x, y, z.$$

$$= \frac{\mathcal{N}^2}{\mathcal{D}^2}(d\rho^0)^2 - \mathcal{D}^2(d\rho_x^2 + d\rho_y^2 + d\rho_z^2).$$

$$\mathcal{D}(\rho) = 0, \quad \rho = \rho_{min} = r_e/2 - r_g/4.$$

- Definition of tetrads

Basis of four unit mutually orthogonal four-vectors $h^i_{(a)}$ defined in every point of spacetime;

$a = 0,\ 1,\ 2,\ 3$ is the counting number of the vector $h^i_{(a)}$,

$i = 0,\ 1,\ 2,\ 3$ denotes the spacetime component of the vector $h^i_{(a)}$:
0 - ct, 1 - x, 2 - y, 3 - z.

$$h_{(a)i} = g_{ik} h^k_{(a)}, \quad h^i_{(a)} = g^{ik} h_{(a)k}, \quad h_{(a)i} h^i_{(b)} = \eta_{ab}.$$

$$\mathsf{h}^{(a)i} = \eta^{ab} h^i_{(b)}, \ h^i_{(a)} = \eta_{ab} h^{(b)i},$$

$$\eta^{ab} = \eta_{ab} = \mathrm{diag}(1, -1, -1, -1).$$

- Main properties of tetrads

$$h_{(a)i} h^{(a)}_k = g_{ik}, \quad h^i_{(a)} h^{(a)k} = g^{ik}.$$

$$g \equiv \det[\mathrm{g_{ik}}] = -|\mathrm{h}|^2, \quad |\mathrm{h}| = \det[\mathrm{h_{(a)i}}].$$

- Tetrads for Reissner-Nordström solution

  Nonzero components of tetrad four-vectors $h_{(a)\ i}$ are:

  $h_{(0)0} = \frac{\mathcal{N}}{\mathcal{D}},\ h_{(1)1} = h_{(2)2} = h_{(3)3} = \mathcal{D},$

  $h^{0}_{(0)} = \frac{\mathcal{D}}{\mathcal{N}},\ h^{1}_{(1)} = h^{2}_{(2)} = h^{3}_{(3)} = -1/\mathcal{D}.$

  $g_{ik} = h_{(a)i}h^{(a)}_{k},\ g_{00} = \frac{\mathcal{N}^2}{\mathcal{D}^2},\ g_{xx} = g_{yy} = g_{zz} = -\mathcal{D}^2,$

  $g_{ik} = 0$ if $i \neq k,\ \ |h| \equiv \mathrm{det}[\mathrm{h_{(a)i}}] = \mathcal{N}\mathcal{D}^2.$

- RN tetrads obey Euler-Lagrange equation

  $\frac{\partial \mathcal{L}_{tot}}{\partial h^{(c)}_{p}} = \frac{\partial}{\partial \rho^q}\left\{\frac{\partial \mathcal{L}_{tot}}{\partial h^{(c)}_{p,q}}\right\},$ where $h^{(c)}_{p,q} = \frac{\partial h^{(c)}_{p}}{\partial \rho^q}.$

Action for Gravitational and Electromagnetic Fields

- Lagrangian density for gravitation field

  Einstein's theory: $\mathcal{L}_g = -\frac{R\sqrt{-g}}{2\kappa}, \quad \kappa = \frac{8\pi k}{c^4}.$

  Since $R = -\kappa T^s_{(em)s} = 0$ the Hilbert-Einstein action $S_g = \frac{1}{c}\int \mathcal{L}_g d^4x = 0$ if there are gravitational and electromagnetic fields only.

- Lagrangian density of electromagnetic field

  Electromagnetic field action

  $S_{em} = \frac{1}{c}\int \mathcal{L}_{em} d^4x = -\frac{1}{16\pi c}\int F_{ik}F^{ik}\sqrt{-g}d^4x.$

  For Reissner-Nordström solution,

  $L_{em} = \frac{1}{8\pi}\int \vec{E}^2 dV = \infty, \quad S_{em} = \int L_{em} dt = \infty.$

  The total action $S_{tot} = S_g + S_{em}$ is meaningless for RN solution in Einstein's theory.

- Lagrangian density $\mathcal{L}_g$ in tetrad representation
Møller's formula: $\mathcal{L}_g = \frac{|h|}{2\kappa}\Big(h^k_{(a);l}h^{(a)l}{}_{;k} - h^k_{(a);k}h^{(a)l}{}_{;l}\Big)$.
- Total Lagrangian density $\mathcal{L}_{tot} = \mathcal{L}_g + \mathcal{L}_{em}$ for RN solution
$\mathcal{L}_{tot} = \frac{r_0^2}{\kappa\rho^4}$, $\quad r_0^2 = r_e^2 - r_g^2/4$, $\; r_e^2 = \frac{ke^2}{c^4}$, $\; r_g = \frac{2km}{c^2}$, $\; \kappa = \frac{8\pi k}{c^4}$.
- Total Lagrangian and action for RN solution
$\mathsf{L}_{tot} = \int_{\rho\geq\rho_{min}} \mathcal{L}_{tot} d^3\rho = 4\pi\frac{r_0^2}{\kappa\rho_{min}}$, $\;\rho_{min} = r_e/2 - r_g/4$.
$L_{tot} = \frac{|e|c^2}{\sqrt{k}} + mc^2$, $\; S_{tot} = \Big(\frac{|e|c^2}{\sqrt{k}} + mc^2\Big)t$.
Total Lagrangian and action are finite for RN solution in tetrad representation in spite of singularities of electro-magnetic and gravitational fields at $\rho = \rho_{min} > 0$ $(r = 0)$.

- Total energy-momentum pseudo-tensor and superpotential

$\mathcal{T}_i^k = \frac{\partial U_i{}^{kl}}{\partial \rho^l}$,

$\mathsf{P}_i = \frac{1}{c}\int_V \mathcal{T}_i^0 d^3\rho = \frac{1}{c}\int_\Sigma U_i{}^{0\lambda} k_\lambda d\sigma.$

$U_i{}^{kl} = \frac{|h|}{\kappa}\left\{ h^k_{(a)} h^{(a)l}{}_{;i} + \left(\delta_i^k h^{(a)l} - \delta_i^l h^{(a)k}\right) h^s_{(a);s}\right\}$. (Møller)

- Superpotential for Reissner-Nordström solution

$U_0{}^{0\lambda} = -2\left(\frac{\rho^\lambda}{\kappa\rho}\right)\frac{\mathcal{N}(\rho)\mathcal{D}'(\rho)}{\mathcal{D}(\rho)}$, where $\mathcal{D}'(\rho) \equiv \frac{d\mathcal{D}}{d\rho}(\rho)$

$U_\mu{}^{\mu\lambda} = \left(\frac{\rho_\lambda}{\kappa\rho}\right)\mathcal{N}', \quad \mathcal{N}' \equiv \frac{d\mathcal{N}}{d\rho}(\rho).$

Superpotential $U_0{}^{0\lambda}$ is infinite at $\rho = \rho_{min}$ since $\mathcal{D}(\rho) = 0$, hence expression for energy becomes meaningless for $\rho_{min} > 0$.

- Special case $r_0 = 0$
  $\mathcal{D}(\rho) = 1 + \frac{r_g}{2\rho}$, $\mathcal{N} \equiv 1$, $\mathcal{N}' \equiv \frac{d\mathcal{N}}{d\rho}(\rho) \equiv 0$.
  $r_e = r_g/2, \;\; \rho_{min} = 0, \;\; m = |e|/\sqrt{k} = m_{cl}$.
  For $m = m_{cl}$, nonzero components of superpotential are
  ${U_0}^{0\lambda} = \frac{m_{cl}c^2}{4\pi}\frac{n_\lambda}{\rho^2(1+r_e/\rho)}$, where $n_\lambda = \rho_\lambda/\rho$. ${U_\mu}^{\mu\lambda} = \frac{n_\lambda}{\kappa}\mathcal{N}' \equiv 0$.
- Consequence for the energy-momentum pseudo-tensor
  $\mathcal{T}_0^0 = \frac{\partial {U_0}^{0\lambda}}{\partial \rho^\lambda} = \frac{k m_{cl}^2 c^2}{4\pi}\frac{1}{\rho^4(1+r_e/\rho)^2} = \frac{e^2}{4\pi}\frac{1}{\rho^2(\rho+r_e)^2}$.
  $\mathcal{E} = \int_0^\infty \mathcal{T}_0^0 4\pi\rho^2 d\rho = \int_0^\infty \frac{e^2 dr}{(\rho+r_e)^2} = \frac{e^2}{r_e} = \frac{e^2}{\sqrt{ke^2/c^2}} = m_{cl}c^2$.
  The integral is convergent. $\mathcal{E} \equiv m_{in}c^2 = \sqrt{\frac{e^2}{k}}c^2$.
  The stress-tensor $\mathcal{T}_\lambda^\mu$ is zero identically.
  There is no pressure of any part of electron on others.

- Asymptotic behaviour at $\rho \to \infty$ $\quad g_{00} \approx 1 - \frac{2km_{gr}}{c^2\rho}$.

  For RN solution $g_{00} = \frac{\mathcal{N}^2}{\mathcal{D}^2} \approx 1 - \frac{2km}{c^2\rho}$, therefore $m = m_{gr}$.
- Equivalence principle

  For $m = m_{cl}$ the mass m is the total inertial mass m_{in} of the electromegnetic and gravitation fields.

  Since $m_{in} = m = m_{gr}$, then due to equivalence principle, there is no need in the contribution of a point-like particle with a bare mass $m_b > 0$ to the total inertial mass.

  The classical electron is the system of the electromagnetic and gravitational fields having a singularity.

  The total mass is $m_{cl} = |e|/\sqrt{k} = 1.86 \cdot 10^{-6}$ g, if the electric charge is chosen equal to $e = -4.8 \cdot 10^{-10}$ esu.

- Lagrangian and lagrangian density
  $L_{tot} = 2m_{cl}c^2, \mathcal{L}_{tot} = 0$. Paradox: $L_{tot} = \int \mathcal{L}_{tot} d^3\rho$.
- Internal geometry
  The distance between points with $\rho = \rho_1$ and $\rho = \rho_2$ is
  $l_{12} = \int_{\rho_1}^{\rho_2} \mathcal{D}(\rho) d\rho = \int_{\rho_1}^{\rho_2} d\rho (1 + r_e/\rho) = \rho_2 - \rho_1 + r_e \ln\left[\frac{\rho_2}{\rho_1}\right]$.
  For $\rho_1 \to 0$ $l_{12} \to \infty$.
  The distance between any point with $\rho > 0$ and the singular point ($\rho = 0$) is infinite.
- Surface charge density Since $r = \rho + r_e$, the sphere of radius r_e corresponds to the point with $\rho = 0$.
  Surface area is $S = 4\pi r_e^2$.
  Surface charge density $\sigma = \frac{e}{4\pi r_e^2} = \frac{c^4}{4\pi e k}$.

- Contribution of gravitation to inertial mass

  Switch off gravitation: $k = 0$, $r_e = \frac{\sqrt{k e^2}}{c^2} = 0$, $\mathcal{T}_0^0 = \frac{e^2}{4\pi}\frac{1}{\rho^4}$.

  The total inertial mass is infinite

  Switch on gravitation: $k > 0$, $r_e > 0$.

  $\mathcal{T}_0^0 = \frac{e^2}{4\pi}\frac{1}{\rho^2(\rho+r_e)^2}$, $\mathcal{E} = \int \mathcal{T}_0^0 d^3\rho = m_{cl}c^2 < \infty$.

  The total inertial mass is finite and equal to the gravitational mass of the system of the electromagnetic and gravitational fields only. No contribution of point-charge.

- Classical electron is a black hole

  $r_g = 2r_e$, $r = \rho + r_e$, $g_{00} = \Lambda = (1 - \frac{r_e}{r})^2$.

  Sphere with $r = r_e$ is the event horizon ($g_{00} = 0$).

  The external observer cannot get any information from the region with $r < r_e$.

Discussion of Results

- Quantum effects: they start from Compton wavelength $\lambda_C = \hbar/(m_e c) \approx 3.9 \cdot 10^{-13}$ cm $\gg r_e = 1.4 \cdot 10^{-34}$ cm. Divergence of self-energy graph in QED is $\int dr/r$, while the integral for energy in classical electrodynamics (CE) is divergent as $\int dr/r^2$ at small distances.
  The fractional contribution of gravitation to the inertial mass of the electron in QED can be much less than in CE.
- Spin of electron: system of an electric charge and magnetic dipole can have the angular momentum $s = \hbar/2$.
- Leptons and quarks: It is not excluded that leptons are localized systems of the electromagnetic, weak-boson, and gravitational fields, while quarks are systems of the gluon, electromagnetic, weak-boson, and gravitational fields.

Conclusions

- Classical electron is a system of electromagnetic and gravitational fields localized in a space region of a range $r_e \sim 10^{-34}$ cm. It is described with the Reissner-Nordström solution in the tetrad representation with parameters e and m related with equation $m = \sqrt{e^2/k}$ where e is the experimental electrical charge of the electron.
- The total Lagrangian density of this system and action are finite for the tetrad representation.
- The total inertial mass of the electromagnetic and gravitational fields is equal to $\sqrt{e^2/k}$. It is equal to the gravitational mass of the classical electron.
- There is no need in an additional point-like particle having the charge e and any bare mass since the equivalence principle for the classical electron is fulfilled.
- There is no pressure of any part of electron on others. There is no need in non-electromagnetic and non-gravitational forces preventing the classical electron disintegration.

- Scalar curvature in tetrad representation

$$h_{(a)k;l;m} - h_{(a)k;m;l} = h_{(a)s}R^{s}{}_{klm},$$
$$R^{i}{}_{klm} = h^{(a)i}(h_{(a)k;l;m} - h_{(a)k;m;l}),$$
$$R_{km} = R^{i}{}_{kim} = h^{(a)i}(h_{(a)k;i;m} - h_{(a)k;m;i}),$$
$$R = R^{m}_{m} = h^{(a)i}(h^{m}_{(a);i;m} - h^{m}_{(a);m;i}) =$$
$$(h^{(a)i}h^{m}_{(a);i})_{;m} - (h^{(a)i}h^{m}_{(a);m})_{;i} - h^{(a)i}_{;m}h^{m}_{(a);i} + h^{(a)i}_{;i}h^{m}_{(a);m}.$$

- Lagrangian density of gravitation field

$$\mathcal{L}_g = -\frac{R\sqrt{-g}}{2\kappa} = \frac{\sqrt{-g}}{2\kappa}(h^{(a)i}_{;m}h^{m}_{(a);i} - h^{(a)i}_{;i}h^{m}_{(a);m}) + \frac{\partial W^i}{\partial x^i}$$

Since for any four-vector B^i $\sqrt{-g}B^{i}_{;i} = \frac{\partial(\sqrt{-g}B^i)}{\partial x^i}$,

then $\sqrt{-g}(h^{(a)i}h^{m}_{(a);i})_{;m} = \dfrac{\partial(\sqrt{-g}(h^{(a)m}h^{i}_{(a);m}))}{\partial x^i}$,

$$\sqrt{-g}(h^{(a)i}h^{m}_{(a);m})_{;i} = \frac{\partial(\sqrt{-g}(h^{(a)i}h^{m}_{(a);m})}{\partial x^i}.$$

- Lagrangian density for gravitation field

Therefore $W^i = \frac{1}{2\kappa}[\sqrt{-g}(h^{(a)i}h^m_{(a);m} - h^{(a)m}h^i_{(a);m})]$
$= \frac{\sqrt{-g}}{\kappa}h^{(a)i}h^m_{(a);m}$.

- Arbitrariness of tetrad choice

$g_{ik} = \eta_{ab}h^{(a)}_i h^{(b)}_k$, $\tilde{g}_{ik} = \eta_{ab}\tilde{h}^{(a)}_i \tilde{h}^{(b)}_k$,
$\tilde{h}^{(a)}_i = M^a_c h^{(c)}_i$, $\tilde{g}_{ik} = \eta_{ab}M^a_c h^{(c)}_i M^b_g h^{(g)}_k$.
If $\eta_{ab}M^a_c M^b_g = \eta_{cg}$, then $\tilde{g}_{ik} = g_{ik}$.
M^a_c is matrix of Lorentz transformations (six parameters).
If tetrad $h^{(a)}_i$ obeys Euler-Lagrange equations it is unique.