Isotopic trends of nuclear surface properties of spherical nuclei

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Phys. Rev. C **94** (2016) 054309

Adamian G.G., Antonenko N.V., Malov L.A., Scamps G., Lacroix D.
Effects of angular dependence of surface diffuseness in deformed nuclei on Coulomb barrier.
Phys. Rev. C **90** (2014) 034322

Unified theoretical tool for nuclear structure and nuclear reactions

- HF with effective Skyrme forces or with Gogny forces, HF+BCS, HFB
- EDF (partially *ab initio* EDF, phenomenological Fayans EDF)
- *Ab initio* approaches

Average central densities, nuclear radius [internal *NN* interaction], and surface diffuseness [external part of *NN* interaction] — finite nuclear systems

Nucleus-nucleus interaction

Method I. Partially *ab initio* EDF

Nuclear binding energies, single-particle states, and ground-state densities are described by an EDF

$$\mathcal{E}(\rho,\tau,\kappa) = \mathcal{E}_{kin}(\tau) + \mathcal{E}_{int}(\rho) + \mathcal{E}_{pair}(\rho,\kappa) - \sum_{q=p,n} \lambda_q \rho_q$$

given by the kinetic-energy density $\mathcal{E}_{kin}$, the interaction energy density $\mathcal{E}_{int}$ and the pairing interaction density $\mathcal{E}_{pair}$.

$$\begin{aligned}\mathcal{E}_{int}(1,2) &= \frac{1}{2}\left(\rho^{\dagger}(1)V_{00}(1,2)\rho(2) + \rho_1^{\dagger}(1)V_{01}(1,2)\rho_1(2)\right) \\ &+ \mathcal{E}_C(1,2) + \mathcal{E}_{s.o.}(1,2) + \mathcal{E}_{res}(1,2),\end{aligned}$$

$$\rho = \rho_n + \rho_p \quad \text{and} \quad \rho_1 = \rho_n - \rho_p;$$

Variation of $\mathcal{E}$ leads to the equations for the single-particle wave functions

$$\left(-\frac{\hbar^2\nabla^2}{2m_q}+U_q+e_qU_C+U_{s.o.}\ell\cdot\sigma+\Delta_q-\varepsilon_{qj}\right)\varphi_{qjm}=0$$

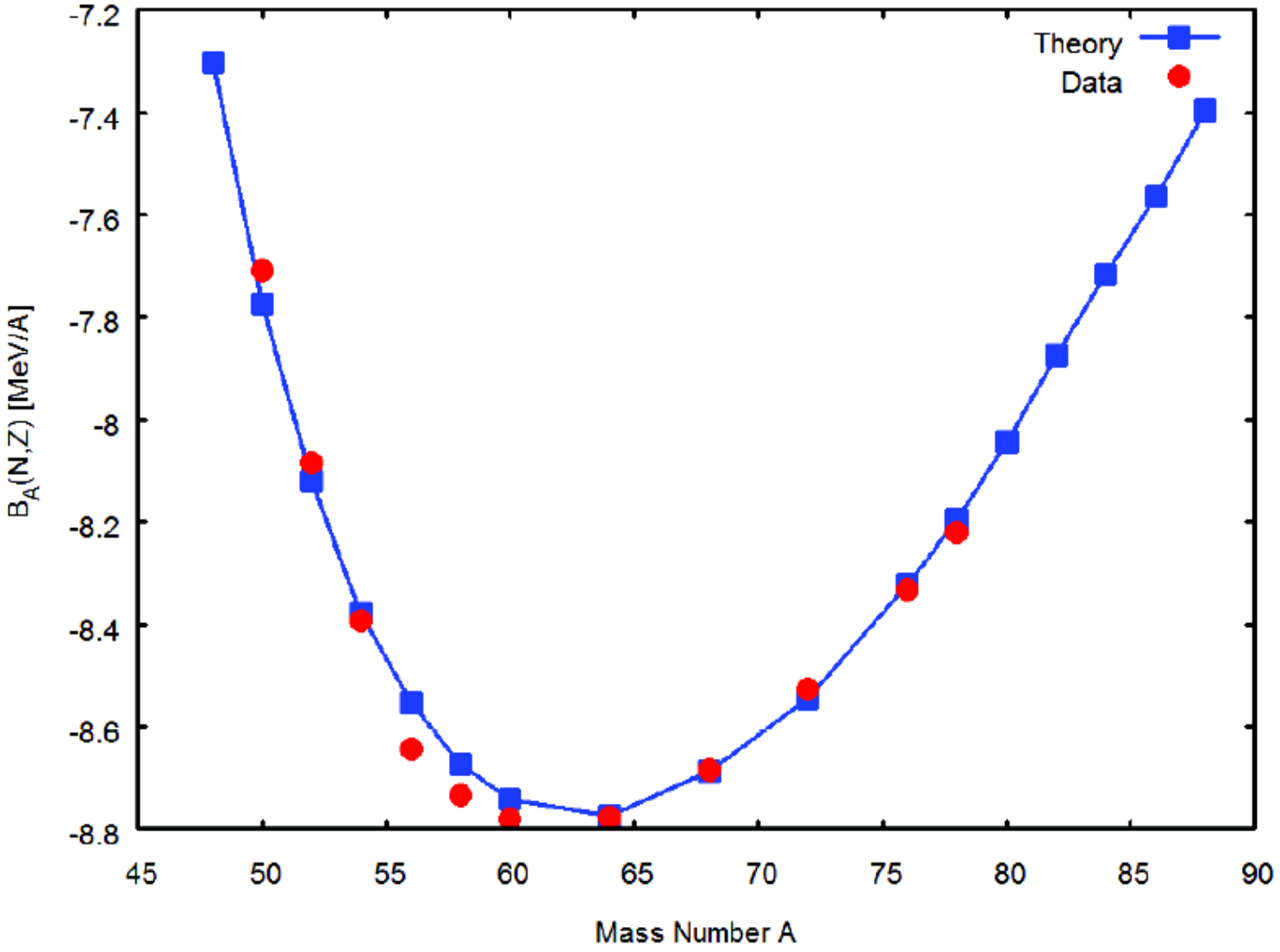
with the single-particle self-energy

$$U_q=\frac{\delta}{\delta\rho_q}\left(\mathcal{E}_{int}+\mathcal{E}_{pair}\right)$$

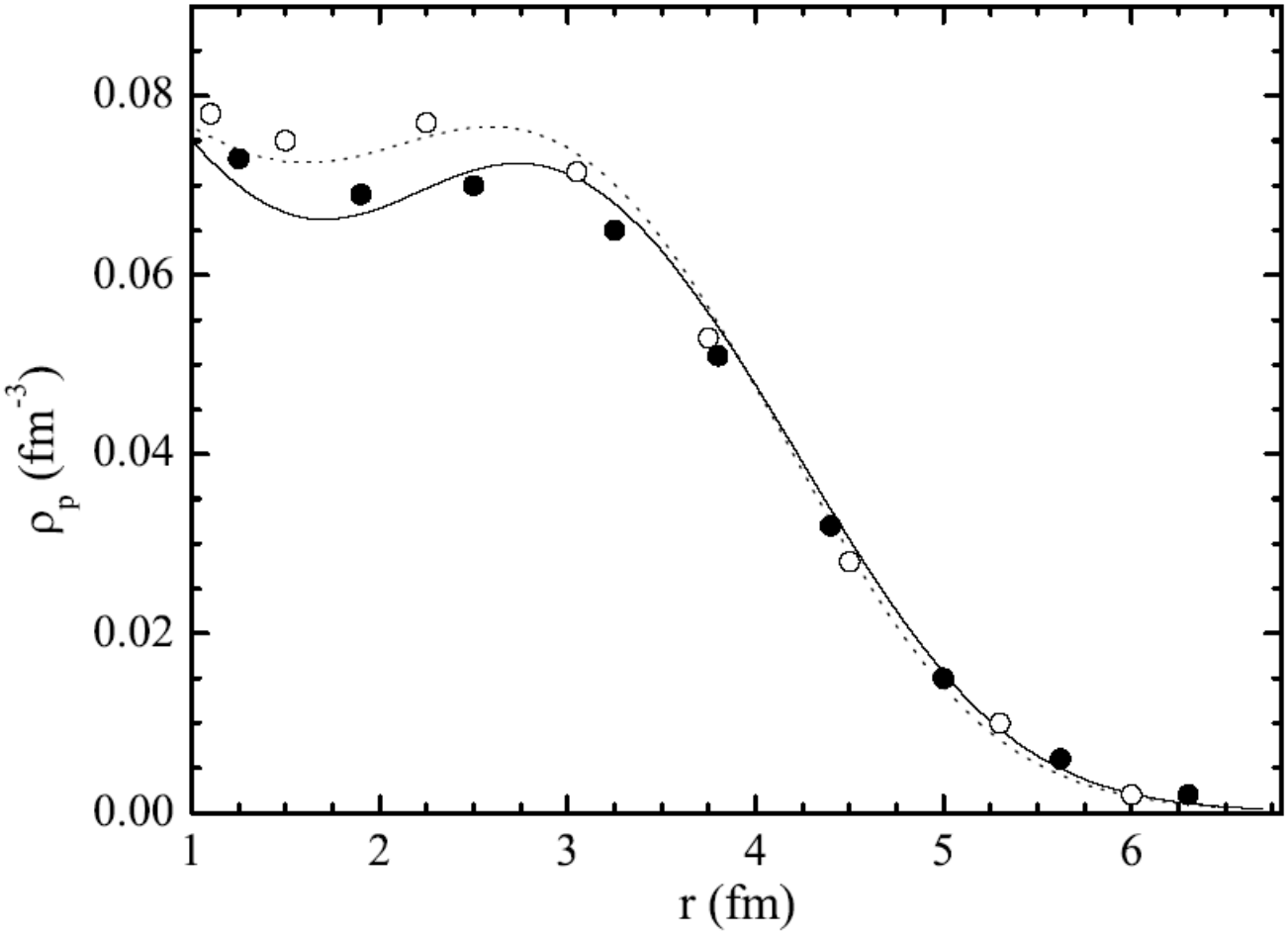
The pairing field is

$$\Delta_q=\frac{\delta}{\delta\kappa_q}\mathcal{E}_{pair}\sim V_{qq}(\rho_q)k_q$$

F.Hofmann and H.Lenske, Phys. Rev. C ***57****, 2281 (1998)*



The binding energies per nucleon for the Ni isotopic chain. The energy density functional results (solid squares connected by lines) are compared with the experimental data (solid circles).



The calculated radial distributions of the proton density for ^{58}Ni (dotted line) and ^{64}Ni (solid line) are compared with the experimental data [At. Data Nucl. Data Tables **36**, 495 (1987)] shown by open and solid symbols, respectively.

Method II. Phenomenological

Fayans EDF

$$\varepsilon = \varepsilon^{v}_{norm} + \varepsilon^{s}_{norm} + \varepsilon_{an} + \ldots$$

The in-volume central term can be schematically written as

$$\mathcal{E}^{\mathrm{v}}_{\mathrm{norm}}(x) = \frac{C_0 F_{ex} x^2}{2} \frac{1 + \alpha x^{\sigma}}{1 + \gamma x}$$

where $x = \rho/\rho_0$ is the relative density, whereas F'_{ex}, σ and γ are the parameters. $C_0 = (dn/d\varepsilon_{\mathrm{F}})^{-1} = \pi^2/mp_{\mathrm{F}}$ is the usual TFFS normalization factor, inverse density of states at the Fermi surface.

The corresponding term of the Skyrme EDF can be obtained putting γ=0.

S. A. Fayans, S. V. Tolokonnikov, E. L. Trykov, and D. Zawischa, Nucl. Phys. A **676**, 49 (2000).

Nucleon density distribution

The nucleon density distribution in the spherical nucleus is usually taken in the three-parameter symmetrized Fermi-type form

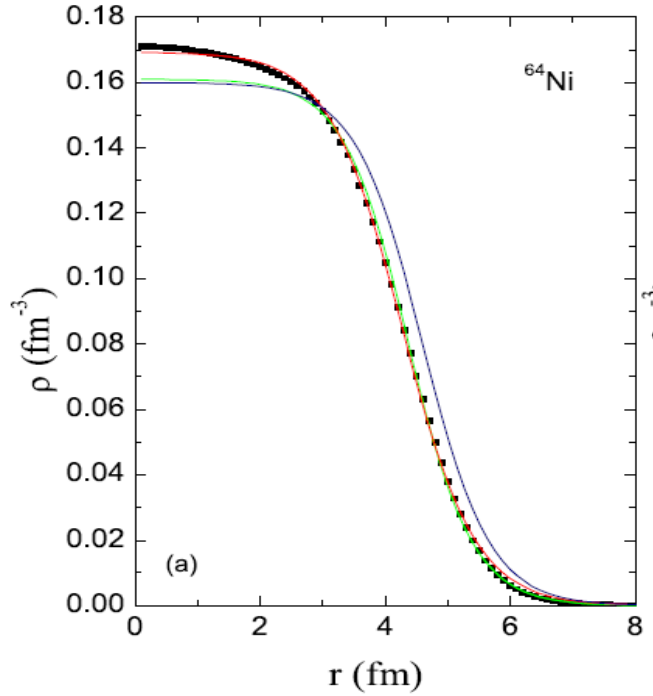
$$\rho(r) = \frac{\rho_0}{1 + \exp[(r - R)/a]},$$

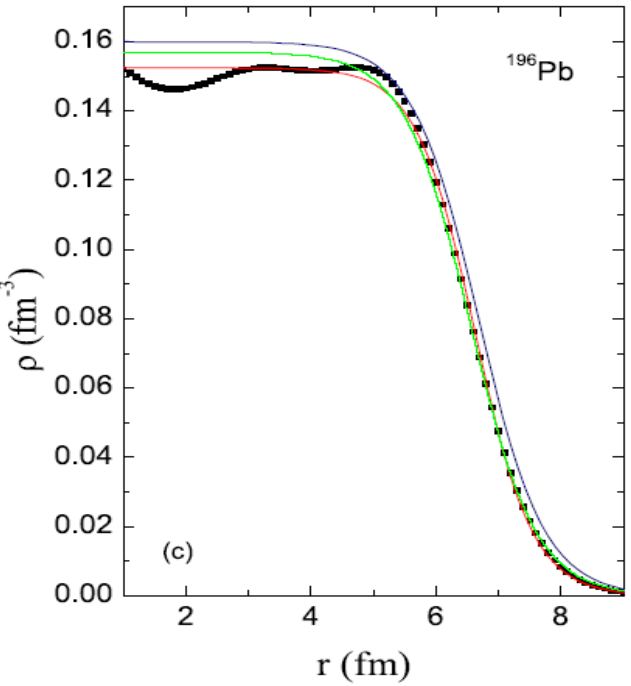
where ρ_0 is the saturated nucleon density in the center of nucleus, $R = r_0 A^{1/3}$ is the nuclear radius with the parameter r_0, and a is the nuclear diffuseness.

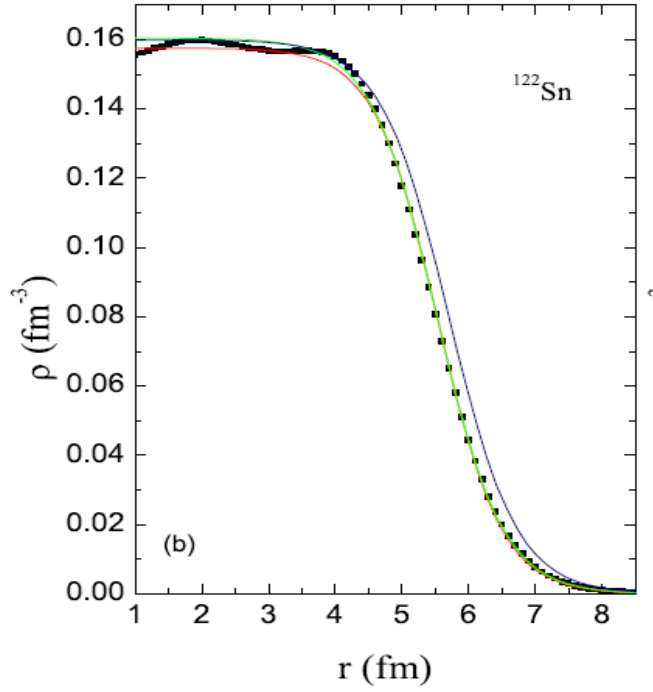
The value of

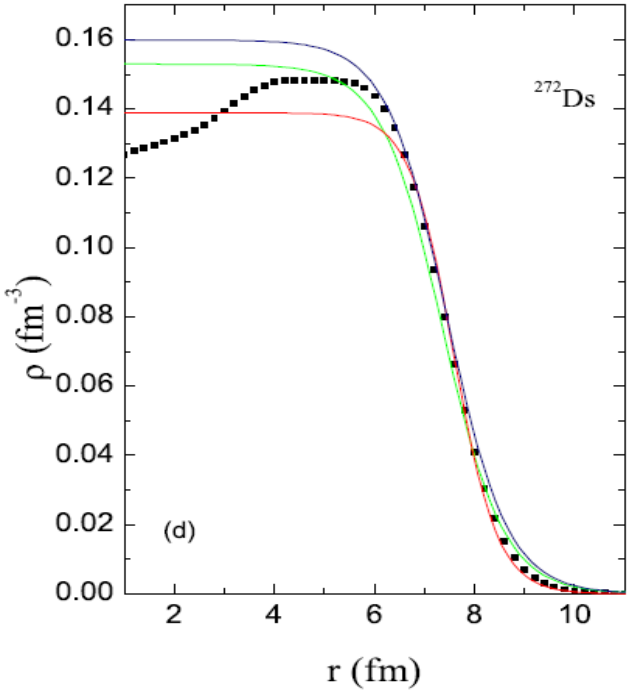
$$\rho_0 = \frac{3}{4\pi r_0^3} \frac{1}{1 + \left(\frac{\pi a}{r_0 A^{1/3}}\right)^2},$$

provides the proper normalization. Three-parameter fit of the nuclear density profile provides ρ_0=0.158, 0.162, 0.156, and 0.154 fm^{-3} for ^{64}Ni, ^{122}Sn, ^{196}Pb, and ^{276}Ds, respectively.









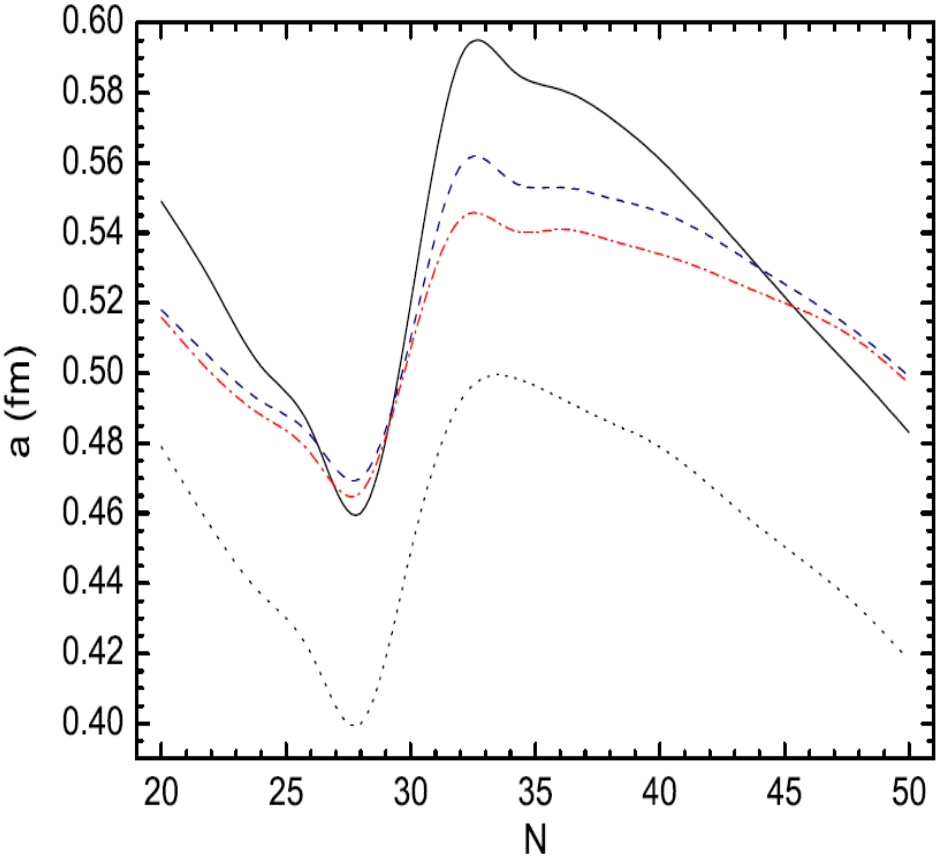
The nucleon-density distributions calculated with method I. The results of three-parameter fit are shown by red lines. The results of two-parameter fit at normalized ρ_0 are shown by green lines. The results of one-parameter fit at fixed ρ_0=0.16 fm^{-3} and r_0=1.15 fm are shown by blue lines.

If we fix the values of ρ_0=0.16 fm^{-3} and r_0=1.15 fm, the description of the density tail becomes worse, particular in the case of light nuclei. In a heavy nucleus, the density profile can be well fitted even at fixed ρ_0 and r_0.

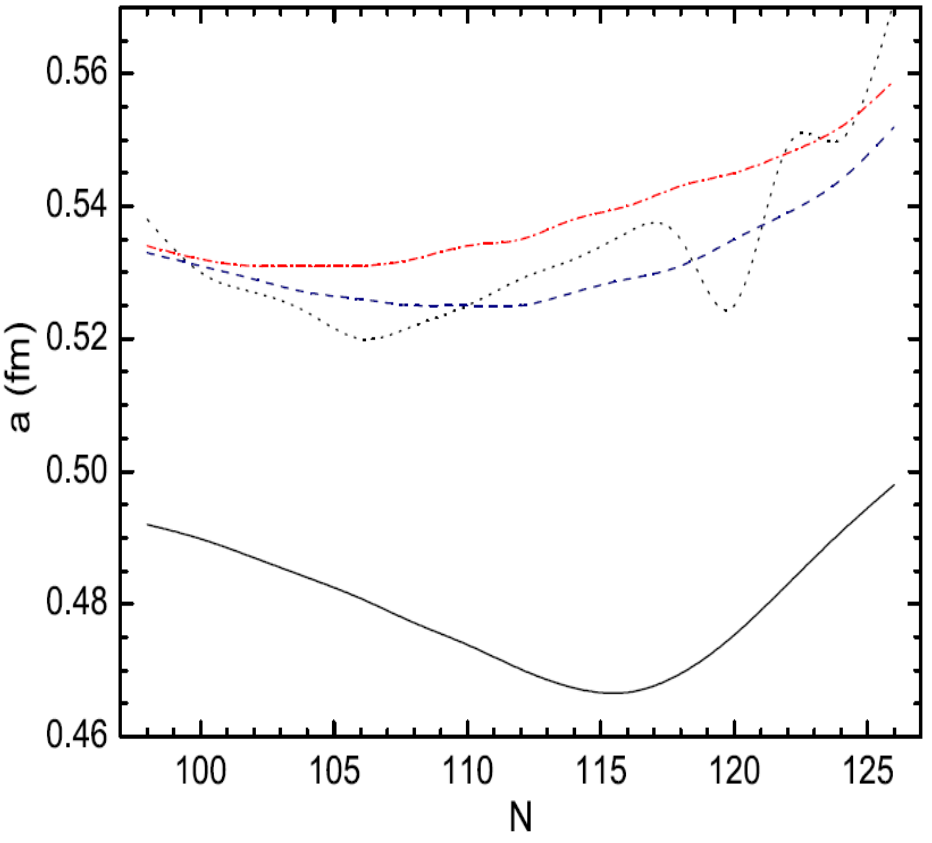
We found that r_0 is weakly dependent on the neutron number of nucleus.

$$r_0 = Z^{1/37}\ (\mathrm{fm})$$

r_0 varies from 1.07 fm for Mg to 1.135 fm for Ds



The comparison of isotopic dependencies of diffuseness of Ni obtained with method I (solid line), method II (dotted line), and approach of PRC **88**, 064327 (2013) (dashed and dash-dotted lines are resulted from the SLy4 and SKMS interactions, respectively).

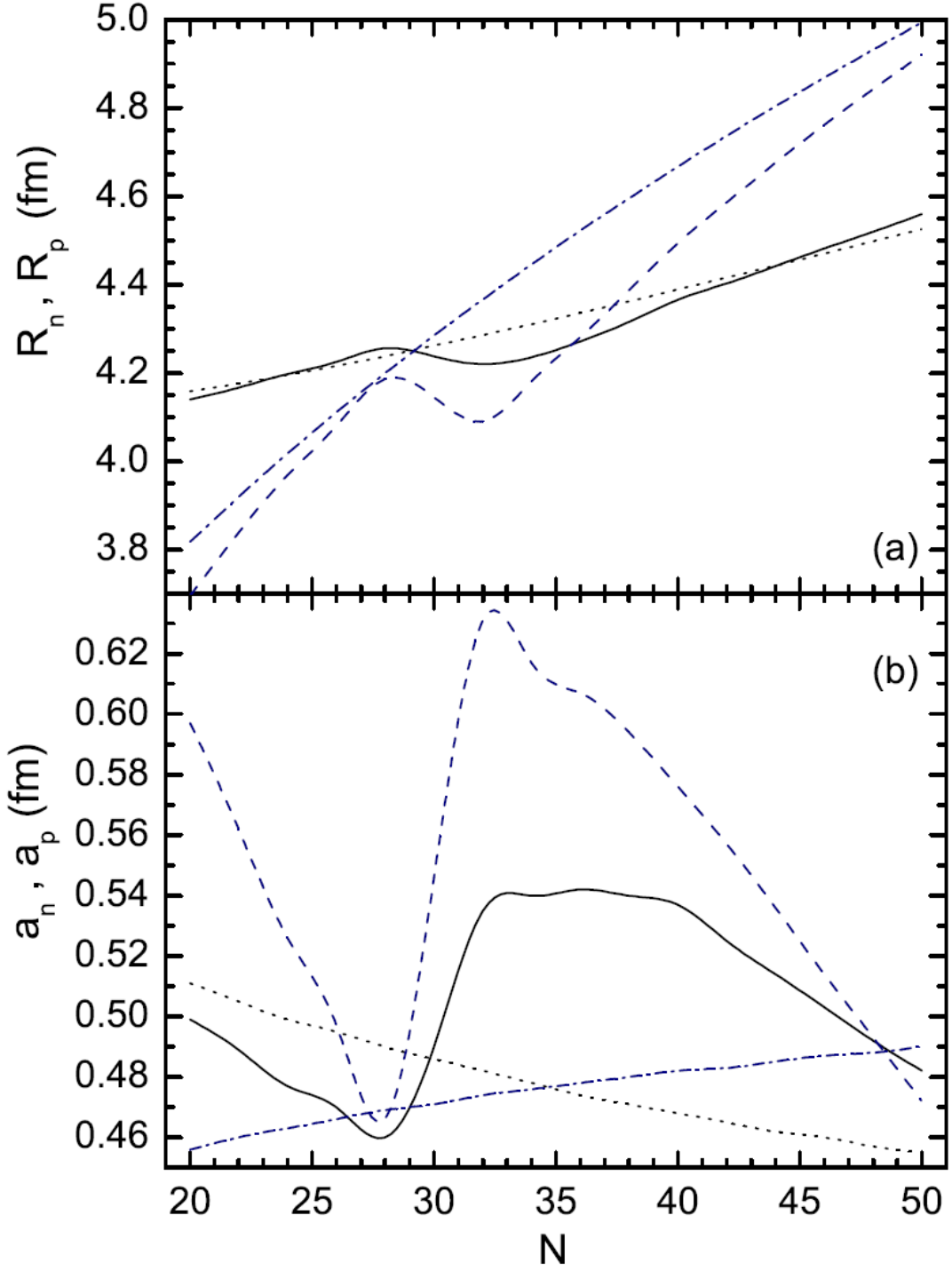


The same, but for Pb.

Based on the calculations of proton and neutron densities, the following expressions

$$
\begin{aligned}
R_p &= 1.249A^{1/3} - 0.5401 - 0.9582\frac{N-Z}{A}\ (\mathrm{fm}),\\
R_n &= 1.2131A^{1/3} - 0.4415 + 0.8931\frac{N-Z}{A}\ (\mathrm{fm}),\\
a_p &= 0.4899 - 0.1236\frac{N-Z}{A}\ (\mathrm{fm}),\\
a_n &= 0.4686 + 0.0741\frac{N-Z}{A}\ (\mathrm{fm})
\end{aligned}
$$

can be suggested to estimate the proton and neutron radii as well as the diffuseness for protons and neutrons. These expressions are suitable for estimating the smooth part of isotopic dependence of nuclear radius and diffuseness.

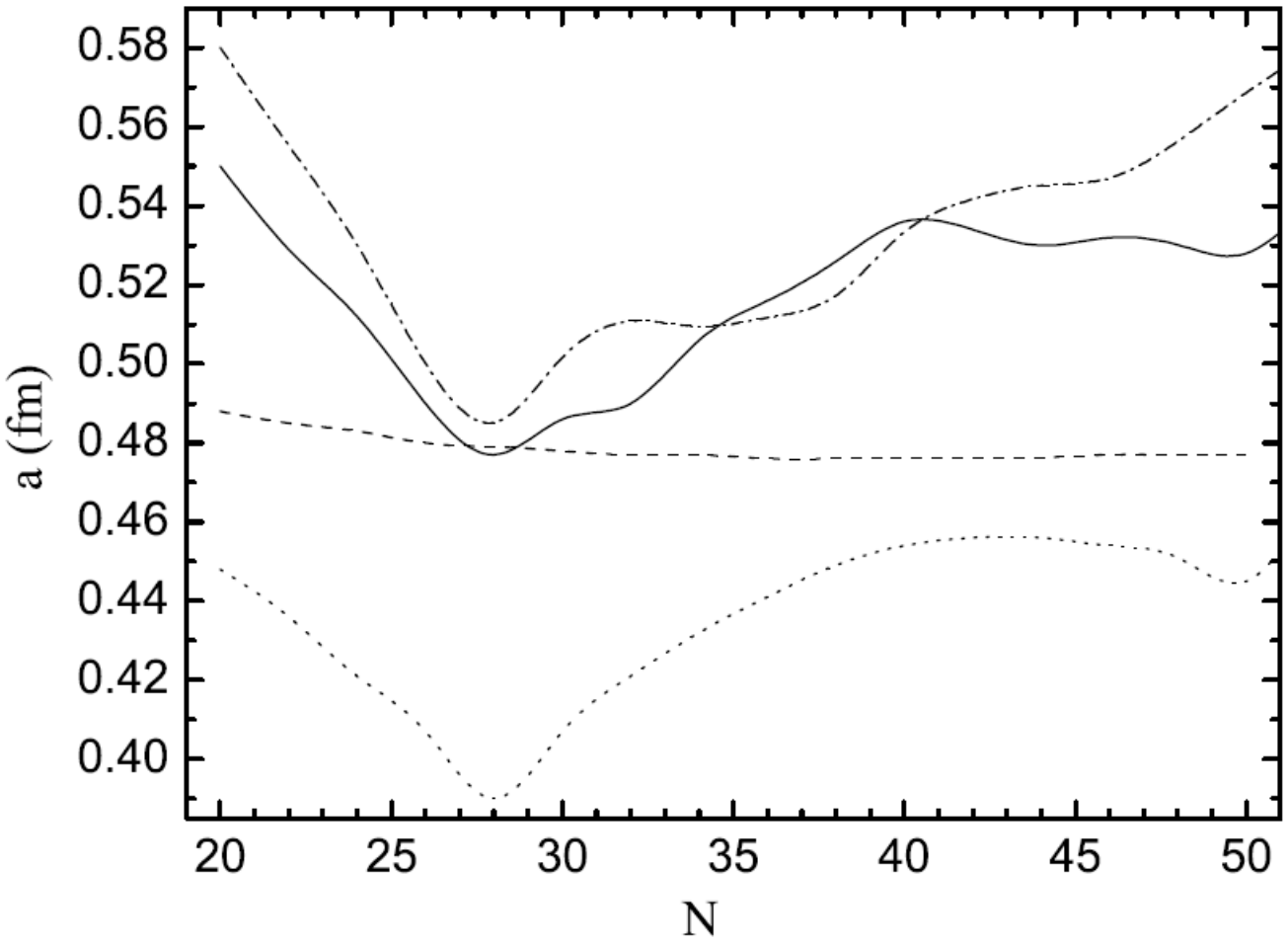


Comparison of isotopic dependencies of proton (solid line) and neutron (dashed line) radii (a), and proton (solid line) and neutron (dashed line) diffuseness (b) obtained with method I for Ni isotopes with the smooth parts of those.

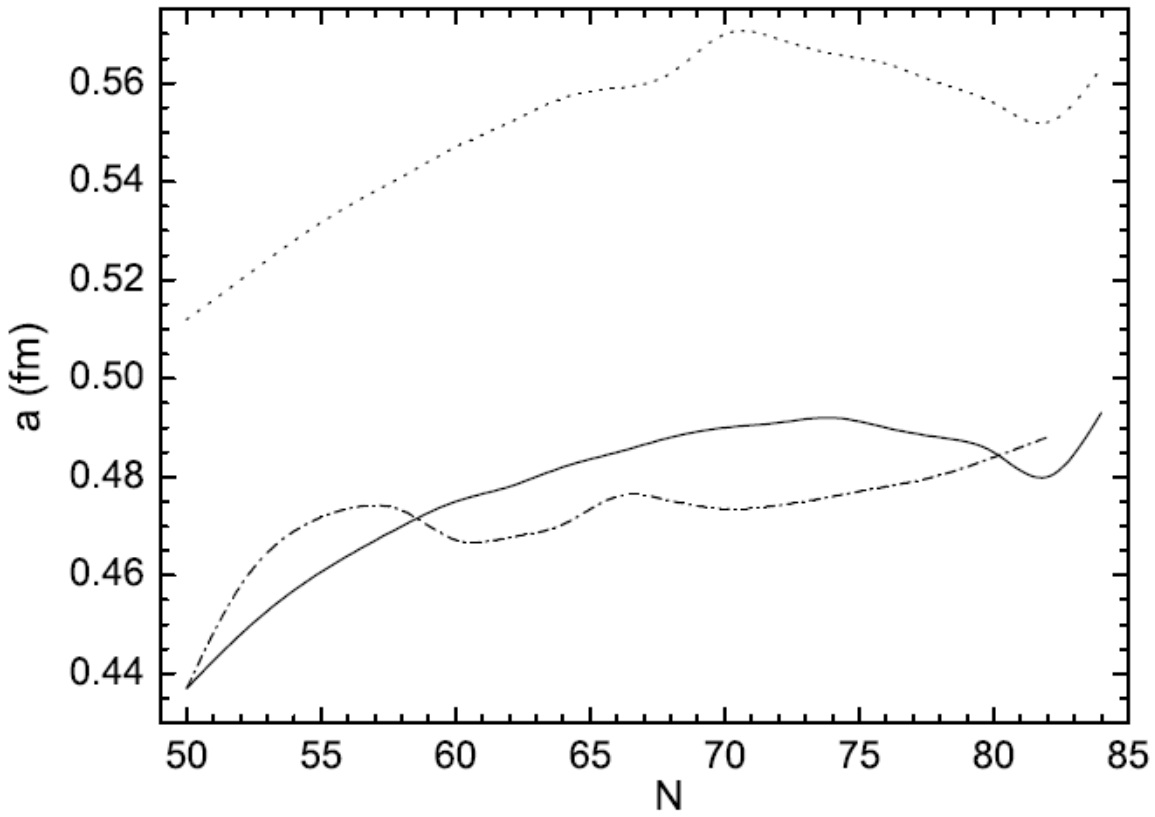
The dependencies of a on N are rather complicated to be fitted with a simple formula. One can consider the dependence of a on the neutron separation energy S_n. Indeed, the value of a is proportional to $1/\sqrt{S_n}$. In this case one can suggest the simple parametrization

$$a(N) = a_0 \frac{N}{2N - N_0} \sqrt{S_0/S_n} \ (\mathrm{fm}),$$

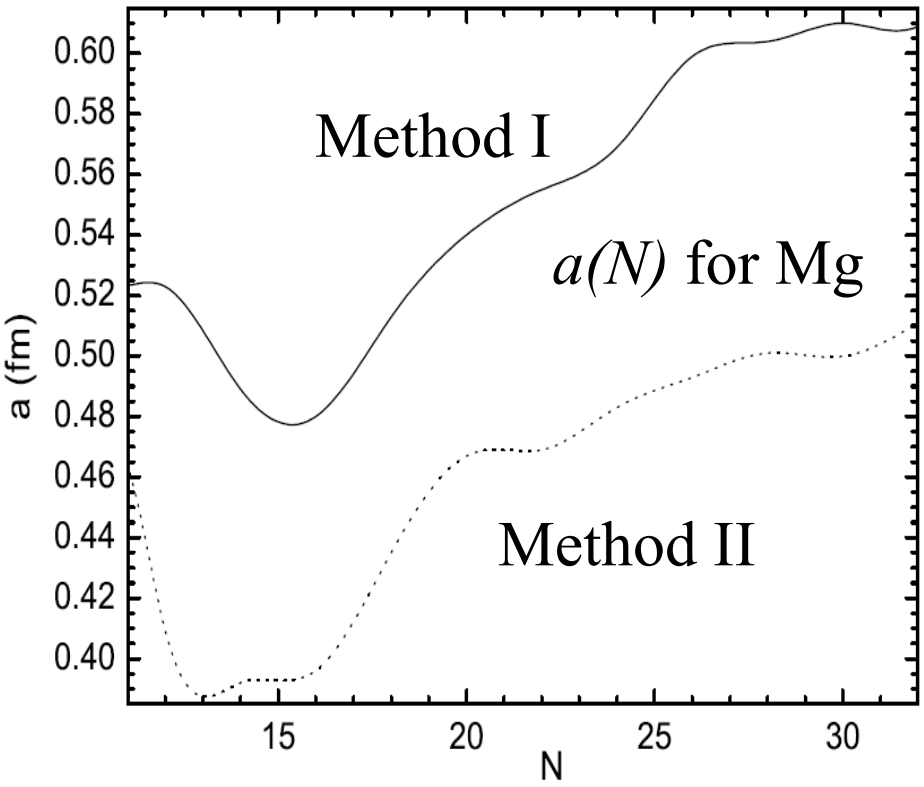
where a_0 and S_0 are the diffuseness and neutron separation energy, respectively, in the isotope with neutron number N_0.

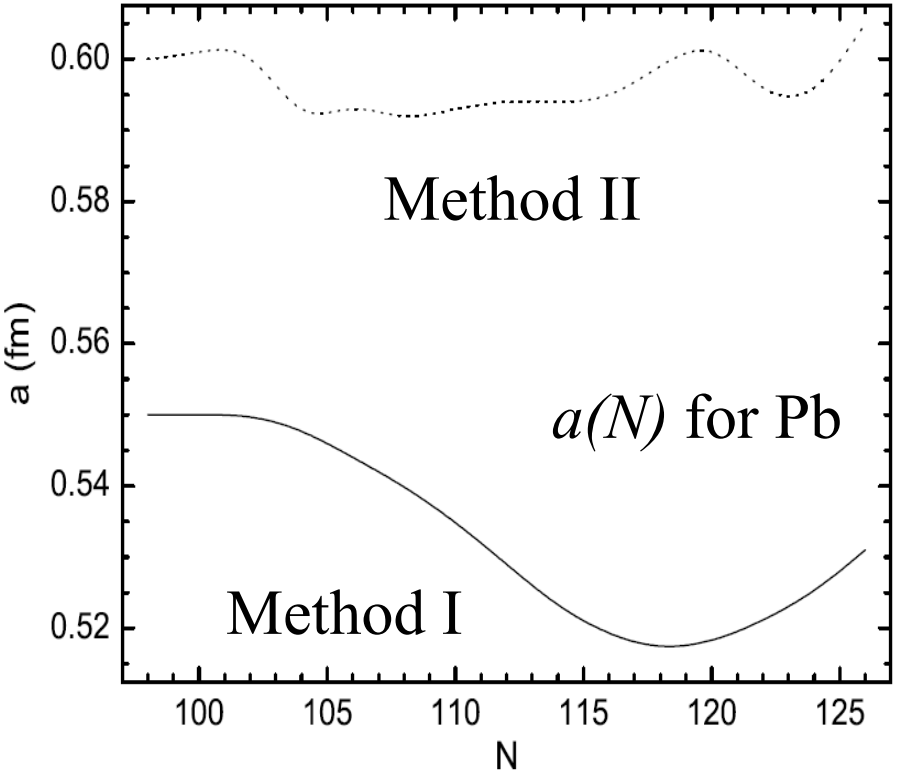


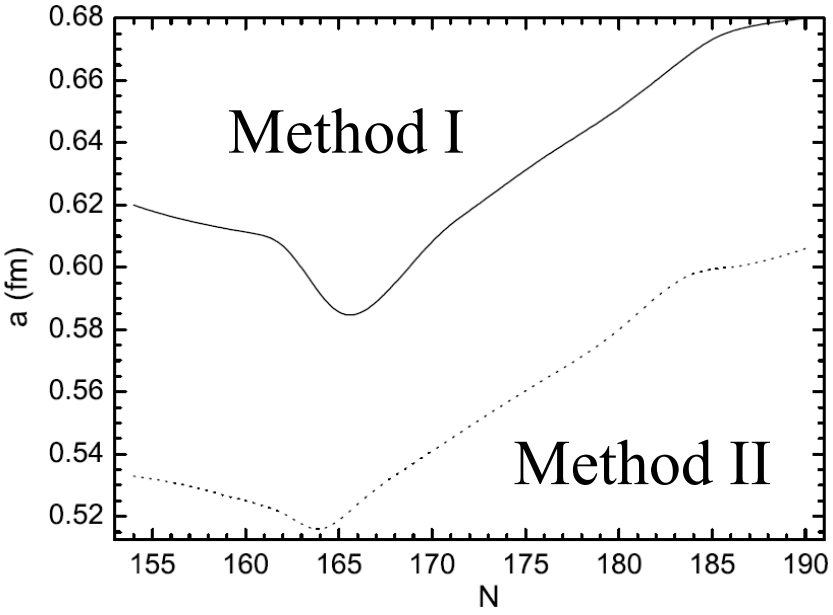
a(N) for spherical Ni. The fit of the solid line at N_0=28 is presented by dash-dotted line. The smooth isotopic dependence of *a* is presented by dashed line.



a(N) for spherical Sn. The fit of the solid line at N_0=50 is presented by dash-dotted line.


Method I
a(N) for Mg
Method II
a (fm)
N
0.60
0.58
0.56
0.54
0.52
0.50
0.48
0.46
0.44
0.42
0.40
15
20
25
30


Method II
a(N) for Pb
Method I
a (fm)
N
0.60
0.58
0.56
0.54
0.52
100
105
110
115
120
125


Method I
Method II
a (fm)
N
0.68
0.66
0.64
0.62
0.60
0.58
0.56
0.54
0.52
155
160
165
170
175
180
185
190

a(N) for Ds

For Ni and Sn, the functions *a(N)* have the minima at *N*=28, 50, and 82. There is also the minimum of *a* at *N*=162-164 in Ds. In the single-particle schemes of spherical nuclei, these neutron shells and sub-shells are closed by the levels with large orbital angular momenta ($l \geq 7$). Thus, the centrifugal barrier restricts the diffuseness of some closed shell nuclei.

For Mg and Pb, the magic neutron numbers *N*=20 and 126 are not reflected in the *a(N)* dependence because these shells are closed by the levels with small orbital angular momenta.

One can conclude that the last occupied single-particle level is responsible for the peculiarities in the isotopic dependence of the nuclear diffuseness.

Nucleus-nucleus interaction potential

The *NN* interaction is

$$F = \frac{\delta^2 \mathcal{E}^{\rm v}_{\rm norm}(\rho)}{\delta \rho^2} = C_0 \frac{F_{ex}}{2(1+\gamma x)^3}[2 - 2\alpha x^\sigma - \alpha\sigma x^\sigma (1+\gamma x)(3+\sigma+\gamma x(1+\sigma))].$$

At $x \to 0$ we get external NN interaction and $F \to C_0 F_{ex}$. The NN interaction at saturation density is found for $x \to 1$.

at $\gamma = 0$ and $\sigma = 1$,

$$F = C_0[F_{ex} + (F_{in} - F_{ex})x]$$

has the Migdal form with $F_{in} = F_{ex}(1 - 3\alpha)$.

The nucleus-nucleus interaction potential V is represented as the sum

$$V(R) = V_C(R) + V_N(R) + V_R(R)$$

of the Coulomb, nuclear, and centrifugal potentials.

The double-folding procedure

$$V_N(R) = \int d\mathbf{r_1} d\mathbf{r_2}\, \rho_1(\mathbf{r_1})\rho_2(\mathbf{R} - \mathbf{r_2})F(\mathbf{r_2} - \mathbf{r_1}).$$

The effective *NN* forces

$$F(\mathbf{r_2} - \mathbf{r_1}) = C_0 \left(F_{in} x(\mathbf{r_1}) + F_{ex} \left(1 - x(\mathbf{r_1})\right)\right) \delta(\mathbf{r_2} - \mathbf{r_1}),$$

$$\rho(\mathbf{r_1}) = \rho_1(\mathbf{r_1}) + \rho_2(\mathbf{R} - \mathbf{r_2})$$

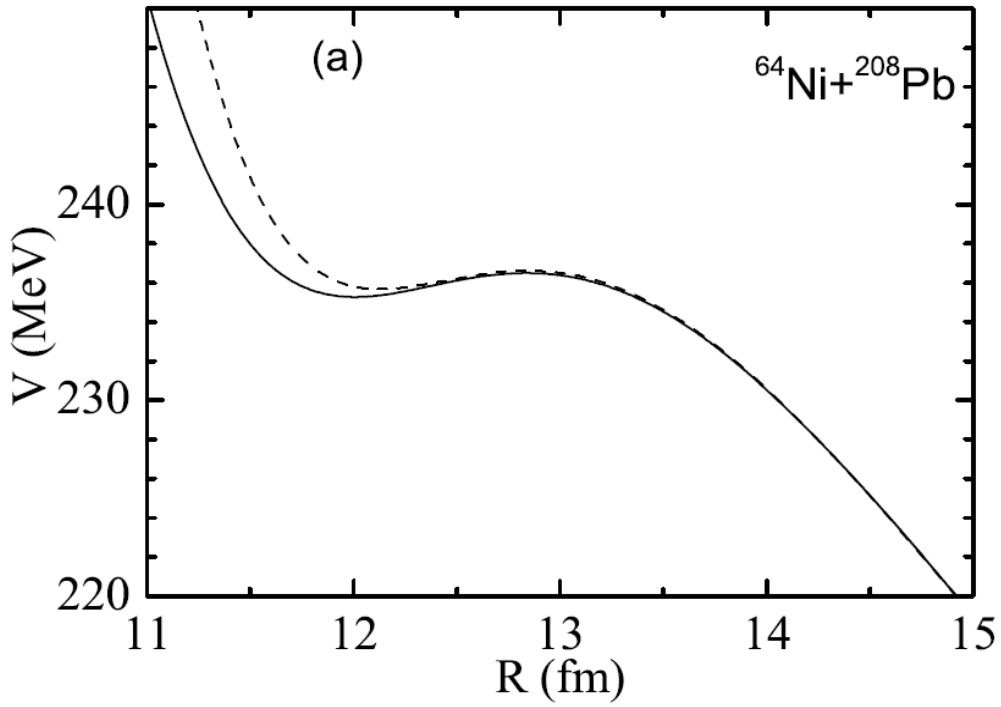
The Landau-Migdal parameters F_{in} = 0.09 and F_{ex}=-2.59, C_0 = 300 MeV fm^3 (ρ_0=0.17 fm^{-3}) were determined from a fit to experimentally measured properties of nuclei

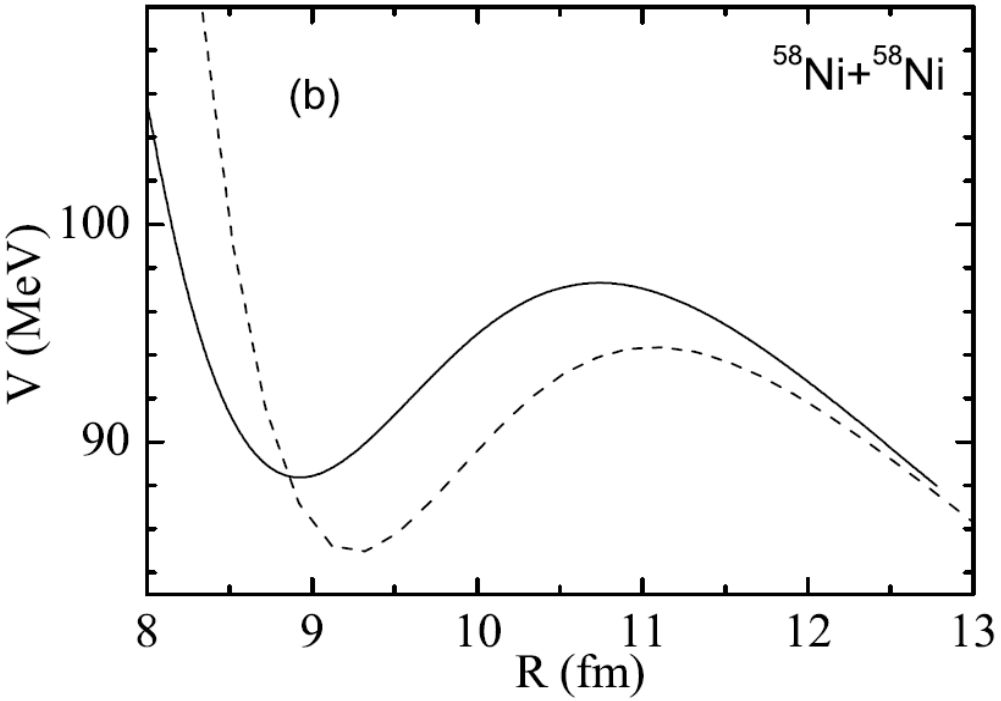
$r_0 = 1.15 - 1.16$ fm $\qquad a = 0.53 - 0.56$ fm

As seen, the values of a and r_0 found in the present work are smaller than those used in our previous calculations of the nucleus-nucleus potential. If they were used with the Landau-Migdal interaction, the Coulomb barrier barriers would be higher than those resulted from the experimental data. Using the values of a and r_0 found, we calculate the nucleus-nucleus potential with F defined in the general form. We set σ=1/3 and look for the values of F_{ex}, α, and γ which provide a good description of the Coulomb barrier heights.

$$F_{ex} = -10.8,\ \alpha = 0.534,\ \gamma = 0.4$$

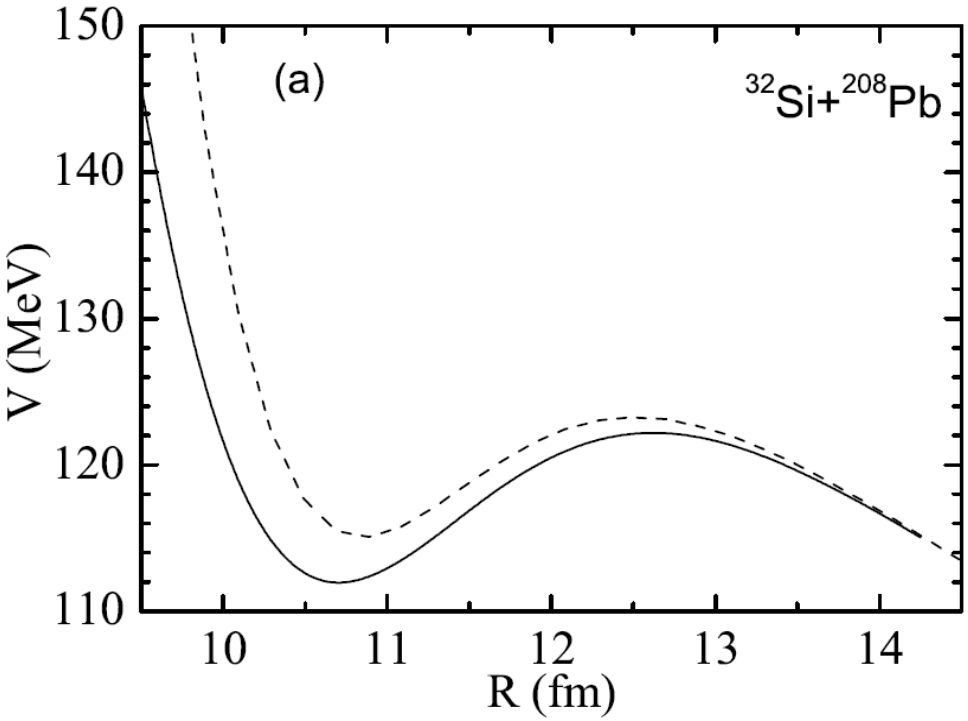
$$F_{in} = 0.062$$

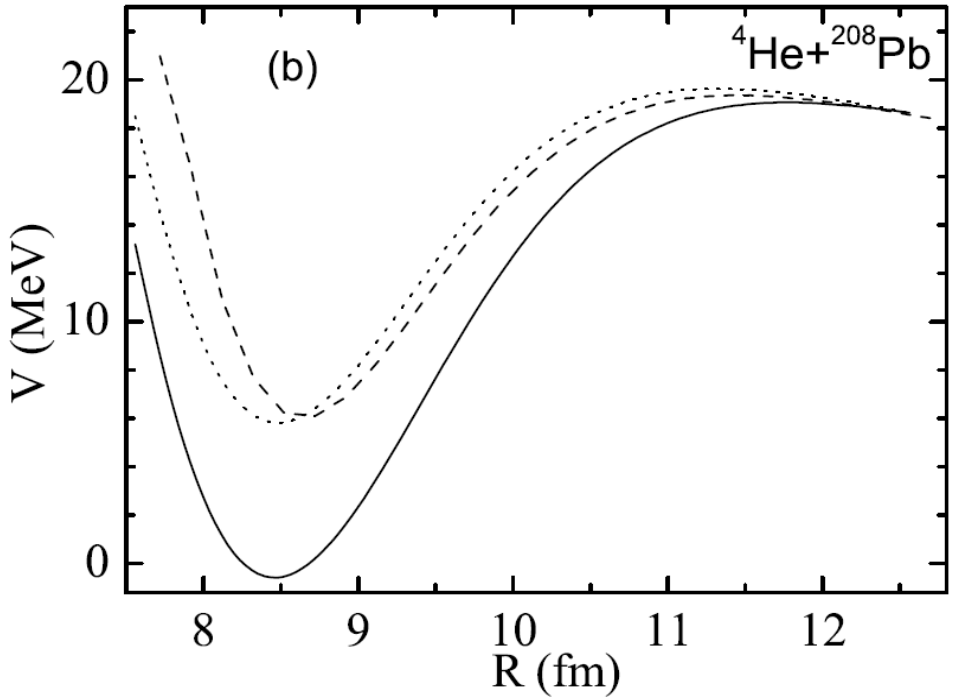




with nucleon densities of method I (solid lines)

with the Landau-Migdal parameters and adopted ρ (dashed lines)





with nucleon densities of method I (solid lines)

with the Landau-Migdal parameters and adopted ρ (dashed lines)

Summary

*** The nuclear diffuseness quite strongly depends on the neutron number.**

*** The simple parametrization of *a(N)* was suggested.**

*** The value of the diffuseness is related to the attraction strength (the external constant F_{ex}) of the *NN* interaction. The external constant of the density dependent *NN* forces mainly predetermines the height of the Coulomb barrier, that is a measurable value.**

*** If the parameters of the EDF or of the corresponding effective *NN* interaction are found form the fit of nuclear ground-state characteristics, one should check whether this interaction is able to reproduce the height of the Coulomb barrier between two interacting nuclei.**

Thank you.