

Dedicated to the memory of our dear friend and colleague Eduard E. Saperstein

# CHARGE-EXCHANGE ISOBARIC RESONANCES AND LOCAL INTERACTION PARAMETERS

Yu. S. Lutostansky

*National Research Center "Kurchatov Institute" Moscow, 123098 Russia*

---

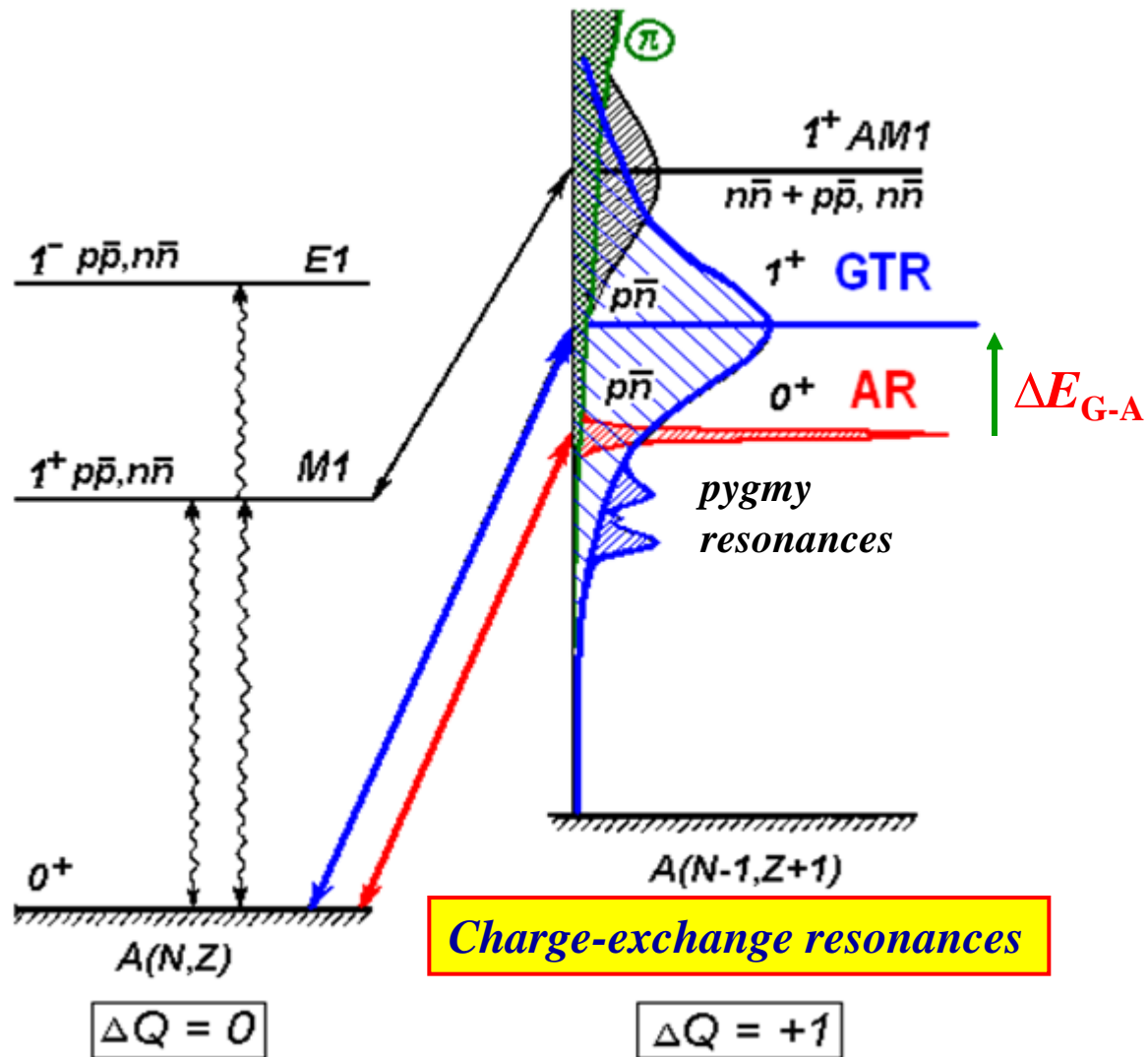
**Restoration of Wigner Supersymmetry**

**in Heavy and Superheavy Nuclei**

\*\*\*\*\*

**JINR Dubna Russia – 2019**

# NUCLEAR RESONANCES



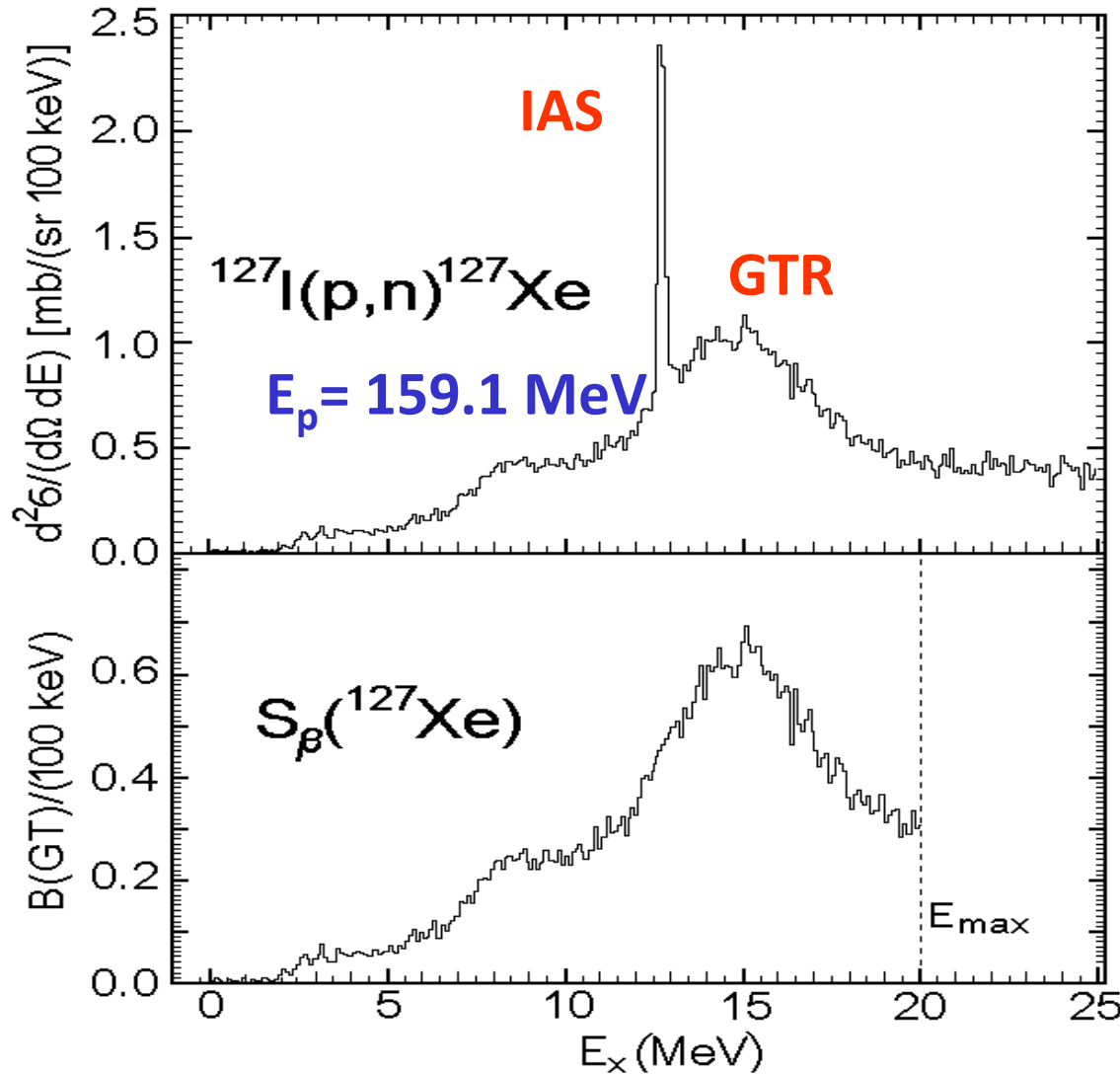
**GTR predictions**  
 Yu. V. Gaponov,  
 Yu. S. Lyutostanskii,  
*JETP Lett.* 15, 120 (1972).

**PR calculations**  
 Yu. S. Lutostansky  
*JETP Lett.* 106, 7 (2017)

$\Delta Q = 0$

$\Delta Q = +1$

# Charge-Exchange Strength Function of the Reaction $^{127}\text{I}(p,n)^{127}\text{Xe}$



First calculations:  
Yu. S. Lutostansky, N. B. Shulgina. Phys. Rev. Lett. 67, 430 (1991)  
were made long before the experiment and demonstrated good prediction accuracy.

# Charge-Exchange Reactions: $(p,n)$ , $(n,p)$ , $({}^3\text{He},t)$ , $(t, {}^3\text{He})$ , $({}^6\text{Li}, {}^6\text{He})$ , $(\nu, e^-)$ ...

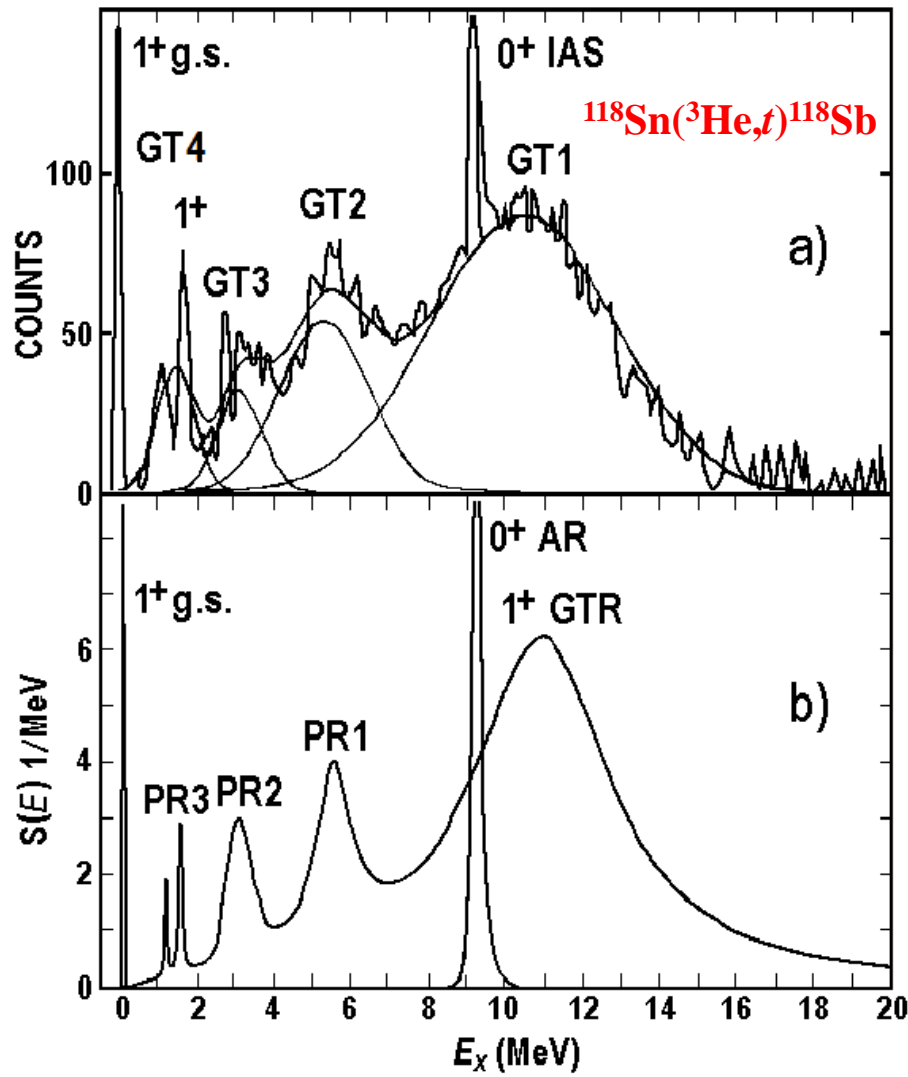


Fig. a) Experimental data [1] and fig. b) microscopic calculations for excitation spectra in the reaction  ${}^{118}\text{Sn}({}^3\text{He}, t){}^{118}\text{Sb}$ . Stand out 4 resonances: IAS = AR, GT1 = GTR and 3 pigmy resonances – GT1, GT2, GT3 in experiment (fig. a) and PR1, PR2, PR3 in calculations (fig. b).

[1]. K. Pham, J. Jänecke, D. A. Roberts, M.N. Harakeh, et al. "Fragmentation and splitting of Gamov-Teller resonances in  $\text{Sn}({}^3\text{He}, t)\text{Sb}$  charge-exchange reactions,  $A = 112-124$ ". Phys. Rev. C 51, 526-540 (1995).

# ISOBARIC STATES MICROSCOPIC DESCRIPTION - 1

The Gamow–Teller resonance and other charge-exchange excitations of nuclei are described in Migdal TFFS-theory by the system of equations for the effective field:

$$V_{pn} = e_q V_{pn}^\omega + \sum_{p'n'} \Gamma_{np,n'p'}^\omega \rho_{p'n'} \quad V_{pn}^h = \sum_{p'n'} \Gamma_{np,n'p'}^\omega \rho_{p'n'}^h$$

$$d_{pn}^1 = \sum_{p'n'} \Gamma_{np,n'p'}^\xi \varphi_{p'n'}^1 \quad d_{pn}^2 = \sum_{p'n'} \Gamma_{np,n'p'}^\xi \varphi_{p'n'}^2$$

where  $V_{pn}$  and  $V_{pn}^h$  are the effective fields of quasi-particles and holes, respectively;

$V_{pn}^\omega$  is an external charge-exchange field;  $d_{pn}^1$  and  $d_{pn}^2$  are effective vertex functions that describe change of the pairing gap  $\Delta$  in an external field;

$\Gamma^\omega$  and  $\Gamma^\xi$  are the amplitudes of the effective nucleon–nucleon interaction in, the particle–hole and the particle–particle channel;

$\rho$ ,  $\rho^h$ ,  $\varphi^1$  and  $\varphi^2$  are the corresponding transition densities.

-----  
 Effects associated with change of the pairing gap in external field are negligible small, so we set  $d_{pn}^1 = d_{pn}^2 = \mathbf{0}$ , what is valid in our case for external fields having zero diagonal elements  
 -----

Width:  $\Gamma = -2 \operatorname{Im} [\sum (\varepsilon + iI)] = \Gamma = \alpha \cdot \varepsilon |\varepsilon| + \beta \varepsilon^3 + \gamma \varepsilon^2 / |\varepsilon| + O(\varepsilon^4) \dots$ , where  $\alpha \approx \varepsilon_F^{-1}$

$$\Gamma_i(\omega_i) = 0,018 \omega_i^2 \text{ MeV}$$

# ISOBARIC STATES MICROSCOPIC DESCRIPTION - 2

For the GT effective nuclear field, system of equations in the energetic  $\lambda$ -representation has the form [FFST Migdal A. B.]:

$$\left. \begin{aligned} V_{\lambda\lambda'} &= V_{\lambda\lambda'}^\omega + \sum_{\lambda_1\lambda_2} \Gamma_{\lambda\lambda'\lambda_1\lambda_2}^\omega A_{\lambda_1\lambda_2} V_{\lambda_2\lambda_1} + \sum_{v_1v_2} \Gamma_{\lambda\lambda'v_1v_2}^\omega A_{v_1v_2} V_{v_2v_1}; \\ V_{v_1v_2} &= \sum_{\lambda_1\lambda_2} \Gamma_{v_1v_2\lambda_1\lambda_2}^\omega A_{\lambda_1\lambda_2} V_{\lambda_2\lambda_1} + \sum_{v_1v_2} \Gamma_{v_1v_2v_1v_2}^\omega A_{v_1v_2} V_{v_2v_1}; \\ V^\omega &= e_q \sigma \tau^+; \quad A_{\lambda\lambda'}^{(p\bar{n})} = \frac{n_\lambda^n (1 - n_{\lambda'}^p)}{\varepsilon_\lambda^n - \varepsilon_{\lambda'}^p + \omega}; \quad A_{\lambda\lambda'}^{(n\bar{p})} = \frac{n_\lambda^p (1 - n_{\lambda'}^n)}{\varepsilon_\lambda^p - \varepsilon_{\lambda'}^n - \omega}. \end{aligned} \right\} \begin{array}{l} \text{G -T selection rules:} \\ \Delta j = 0; \pm 1 \\ \Delta j = +1: j = l+1/2 \rightarrow j = l-1/2 \\ \Delta j = 0: j = l \pm 1/2 \rightarrow j = l \pm 1/2 \\ \Delta j = -1: j = l-1/2 \rightarrow j = l+1/2 \\ j = l-1/2 \rightarrow j = l-1/2 \end{array}$$

where  $n_\lambda$  and  $\varepsilon_\lambda$  are, respectively, the occupation numbers and energies of states  $\lambda$ .

## Local nucleon-nucleon $\delta$ -interaction $\Gamma^\omega$ in the Landau-Migdal form:

$$\Gamma^\omega = C_0 (f' + g' \sigma_1 \sigma_2) \tau_1 \tau_2 \delta(r_1 - r_2)$$

where coupling constants of:  $f'=1.35$  – isospin-isospin and  $g'=1.22$  – spin-isospin quasi-particle interaction with  $L = 0$ .

*Constants  $f'$  and  $g'$  are the phenomenological parameters.*

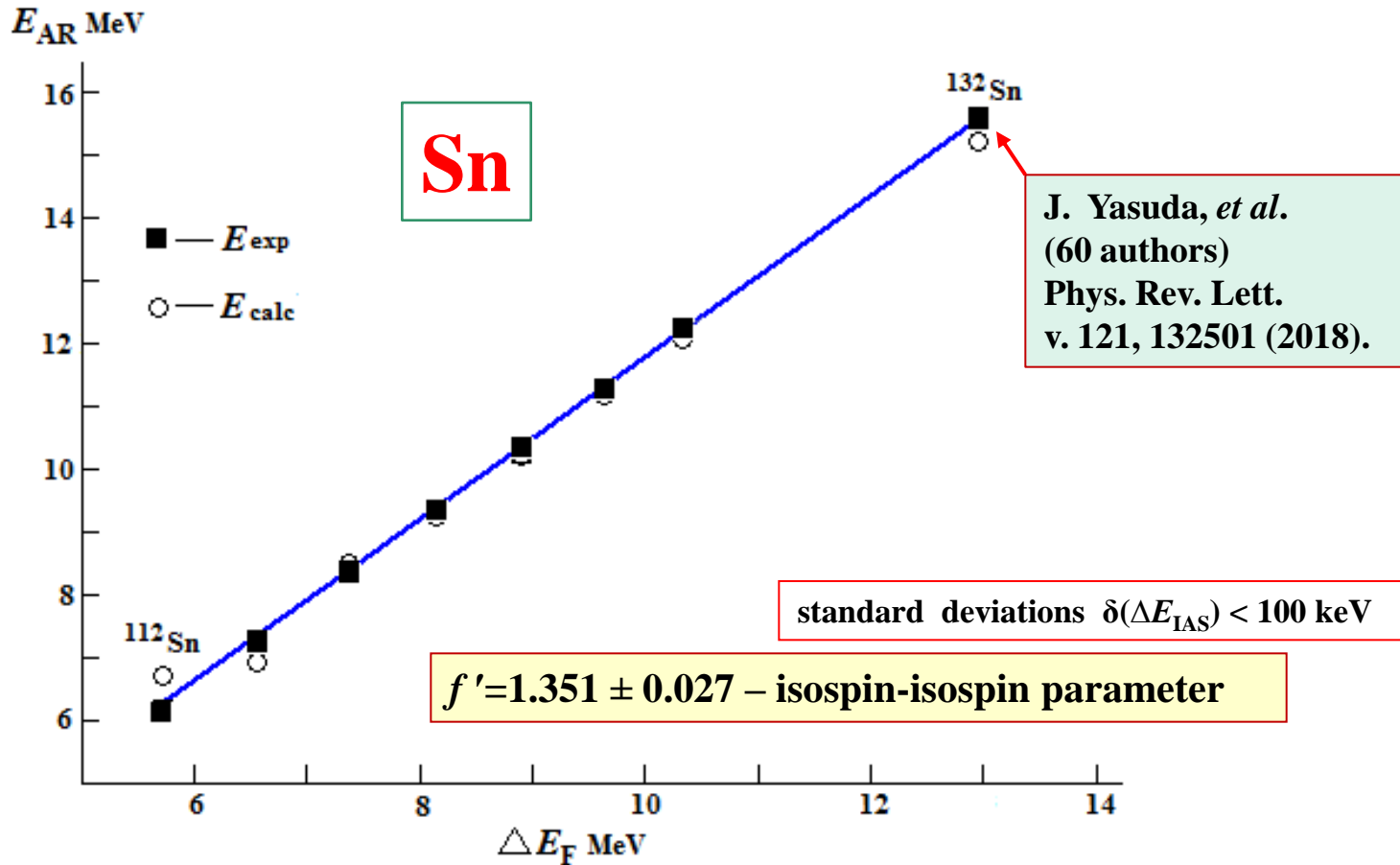
Matrix elements  $M_{GT}$ :  $M_{GT}^2 = \sum_{\lambda_1\lambda_2} \chi_{\lambda_1\lambda_2} A_{\lambda_1\lambda_2} V_{\lambda_1\lambda_2}^\omega$  where  $\chi_{\lambda\nu}$  – mathematical deductions

G -T  $M_i^2$  values are normalized in FFST:  $\sum_i M_i^2 = e_q^2 3(N - Z)$

Effective quasiparticle charge  $e_q^2 = 0.8 - 1.0$  is the “quenching” parameter of the theory.

“Quenching” effect (Losing of sum rule in beta-strength) is the main in heavy nuclei ~ 50%

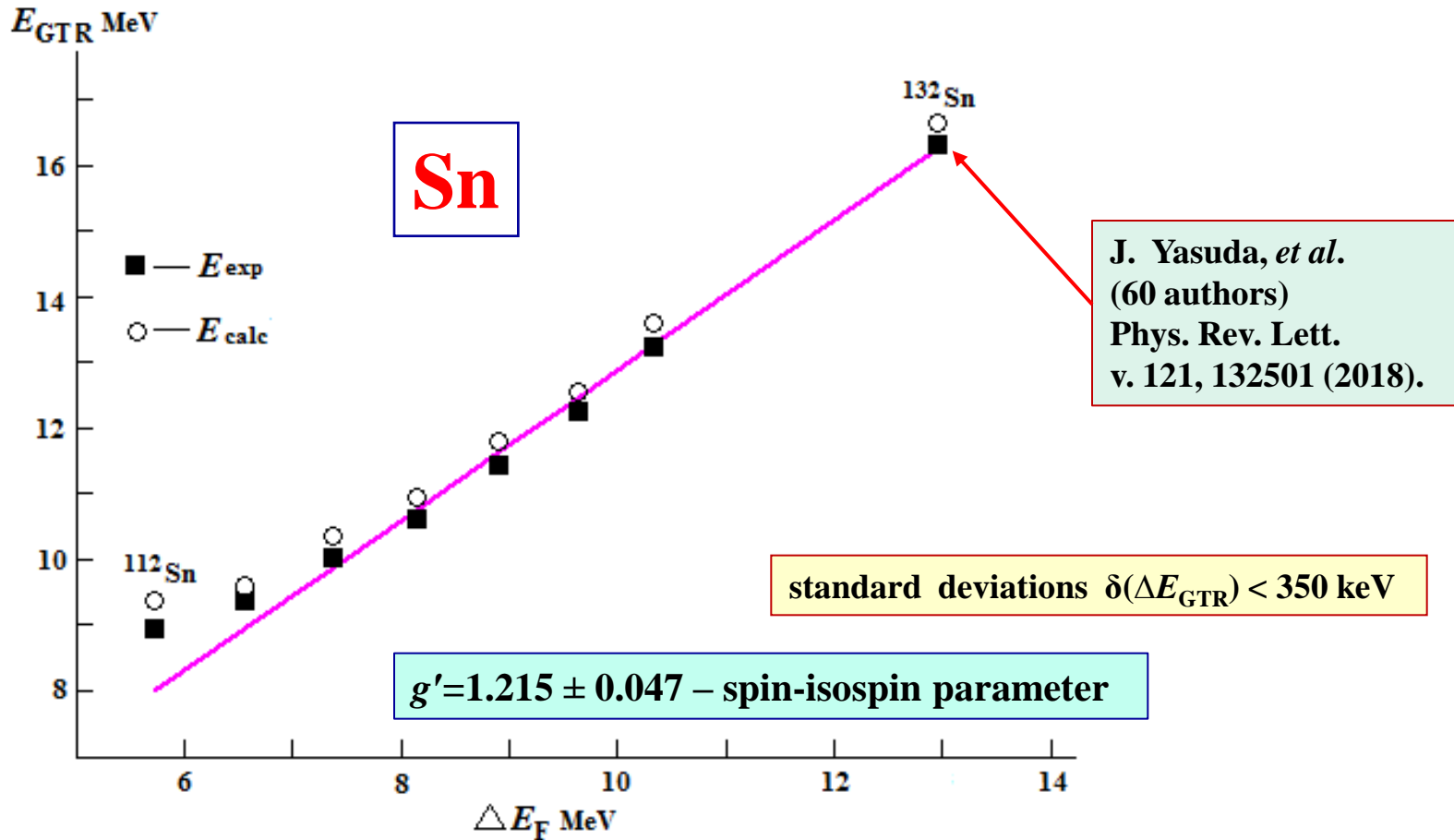
# Sn – Isotopes Analog Resonance – AR = IAS ENERGIES



$$E_{AR} = f' \Delta E_F \quad \Delta E_F = E_F(n) - E_F(p) = \frac{4}{3} E_F \frac{N-Z}{A}$$

$$\Gamma^\omega = C_0 (f' + g' \sigma_1 \sigma_2) \tau_1 \tau_2 \delta(r_1 - r_2); \quad \text{Linear dependence } E_{AR} \text{ from } f'$$

# Sn – Isotopes Gamow-Teller Resonance – GTR Energies

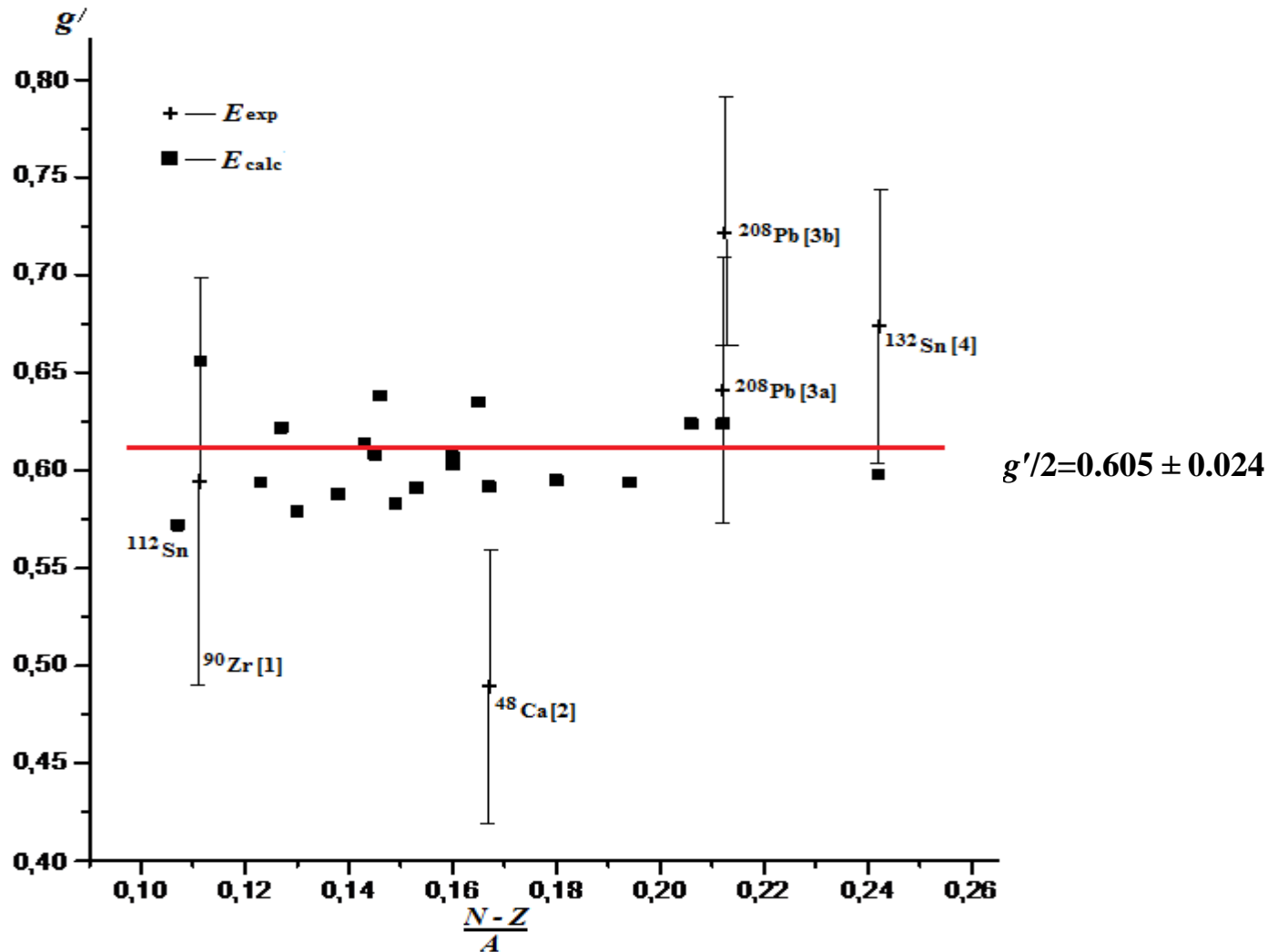


$$E_{GTR} = g'(1+?) \Delta E_F \quad \Delta E_F = E_F(n) - E_F(p) = \frac{4}{3} E_F \frac{N-Z}{A}$$

$$\Gamma^\infty = C_0 (f' + g' \sigma_1 \sigma_2) \tau_1 \tau_2 \delta(r_1 - r_2); \quad \text{No linear dependence } E_{AR} \text{ from } g'$$



# 25 – Isotopes: $g'$ – spin-isospin parameter

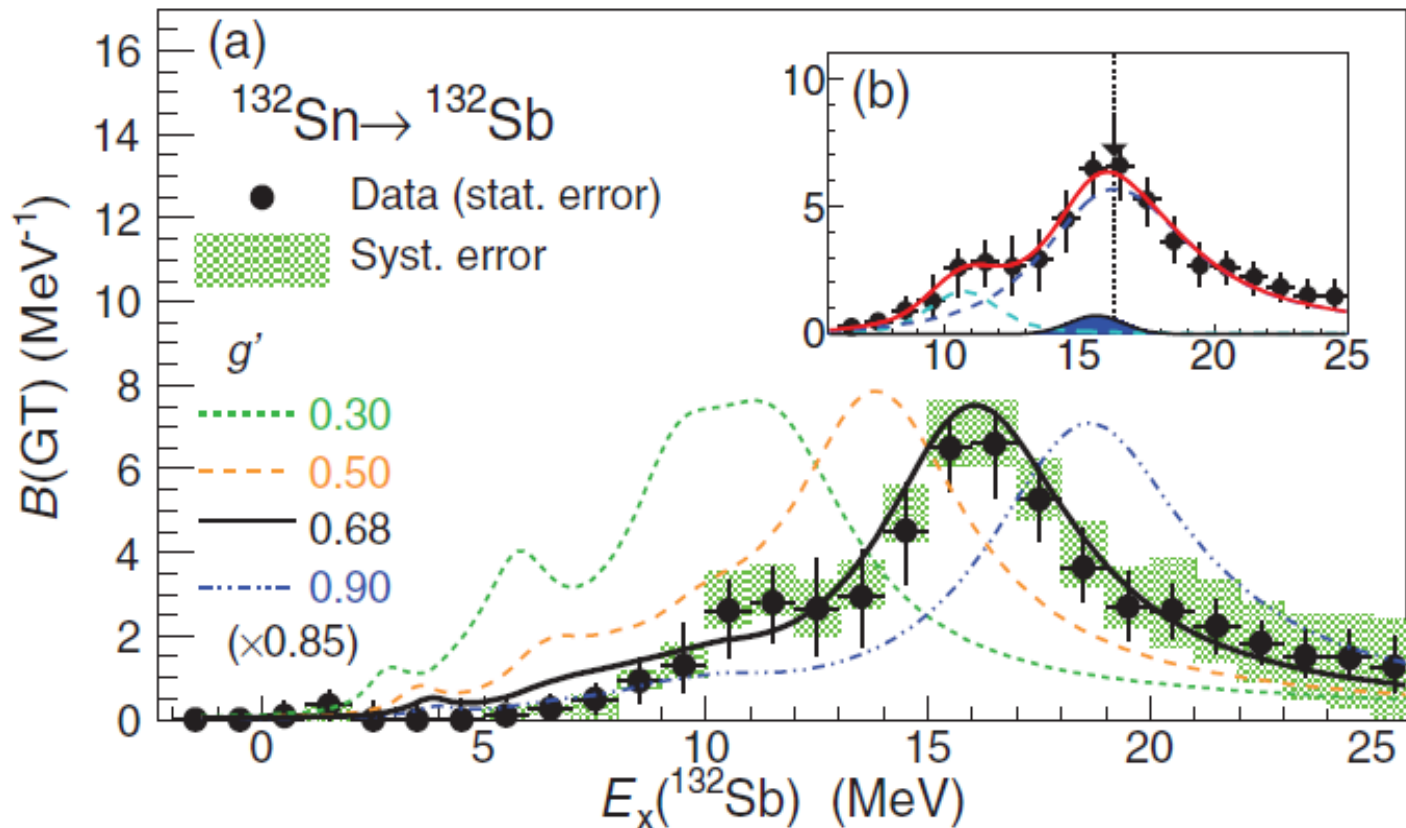


- [1] T. Wakasa, M. Ichimura, and H. Sakai, Phys. Rev. C 72, 067303 (2005).  
 [2] Haozhao Liang, Nguyen Van Giai, and Jie Meng, Phys. Rev. Lett. 101, 122502 (2008).  
 [3a] T. Wakasa *et al.*, Phys. Rev. C 85, 064606 (2012). [3b] T. Suzuki, Nucl. Phys. A379, 110 (1982)  
 [4] J. Yasuda *et al.* (60 authors) Phys. Rev. Lett. v. 121, 132501 (2018).

# Charge-Exchange Strength Function of Reaction $^{132}\text{Sn}(p,n)^{132}\text{Sb}$

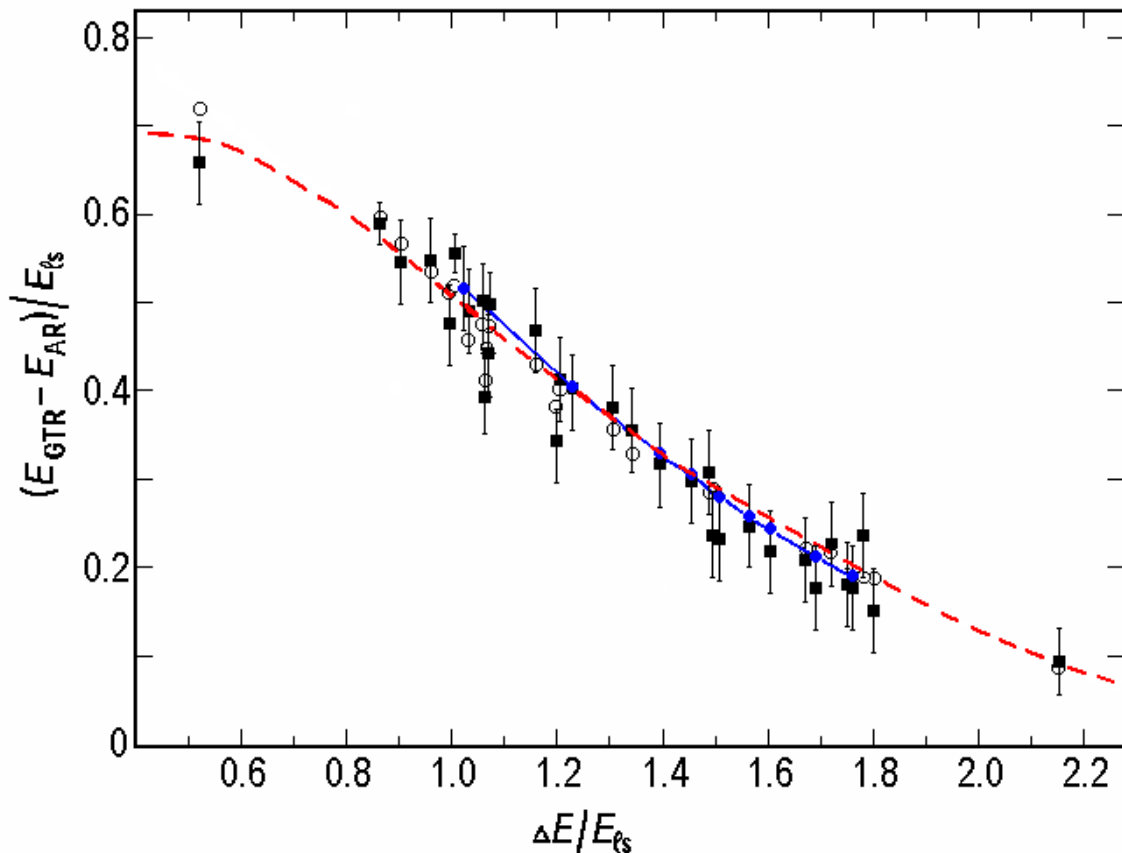
Extraction of the Landau-Migdal Parameter from the Gamow-Teller Giant Resonance in  $^{132}\text{Sn}$

J. Yasuda *et al.* (60 authors) Phys. Rev. Lett. v. 121, 132501 (2018).



Experimental data on the reaction  $^{132}\text{Sn}(p,n)^{132}\text{Sb}$  were compared with theoretical RPA calculations with different values of the parameter  $g'$  and then fitting of this parameter

# $E_{GTR} - E_{AR}$ MODEL DESCRIPTION - 1



Mat. model developed for the approximate solutions of equations of the FFST theory by the quasi-classical method.

**2 new parameters:**

$$\begin{aligned} \Delta E &= E_F(n) - E_F(p) = \\ &= \frac{4}{3} E_F \frac{N - Z}{A} \end{aligned}$$

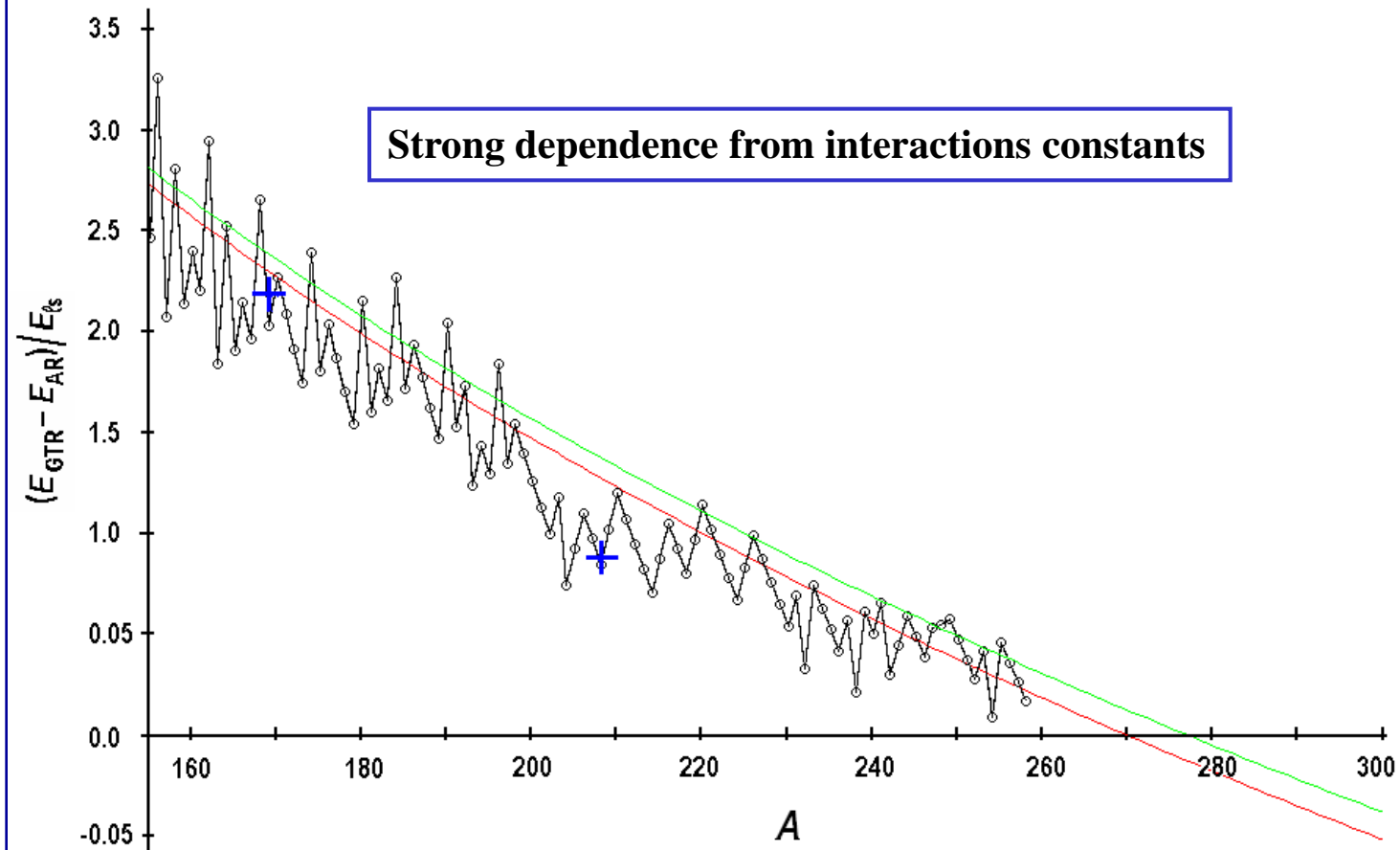
$E_{ls}$  – average energy of the spin-orbit splitting

**Degeneration of GTR and AR = Wigner's SU(4) super-symmetry restoration in the heavy nuclei**

Calculated TFFS (circles –  $\circ$ ) and experimental ( $\blacksquare$ ) dependencies of the relative energy  $y(x) = \Delta(E_{GTR} - E_{AR})/E_{ls}$  from the dimensionless value  $x = \Delta E/E_{ls}$ . **Blue circles** ( $\bullet$ ) connected by line – calculated values for **Sn** isotopes. **Red line** – calculations with  $E_{ls}(N) = 20N^{-1/3} + 1.25$  (MeV).

$$y = \frac{E_{GTR} - E_{AR}}{E_{ls}} = (g'_0 - f'_0) x + b \frac{1 + b g'_0}{g'_0 x} [1 + c(A)x^2]^{-1}; \quad x = \Delta E/E_{ls}; \quad b = \frac{2}{3} [1 - (2A)^{-1/3}]; \quad c(A) \approx 0.8A^{-1/3}$$

# $E_{GTR} - E_{AR}$ MODEL DESCRIPTION - 2



+ Exp. Data. circles – ○: calc. for Nucl. on Exp. Line of beta-stability up to  $^{258}\text{Fm}$ . **Red line:** calculated values for nuclei on the line of beta-stability with  $E_{ls}(N) = 20N^{-1/3} + 1.25$  (MeV),  $Z_{\beta}(A) = A / (2 + 0.0150A^{2/3})$ , and with  $f_0' = 1.35$ ,  $g_0' = 1.22$  up to  $A_{max} = 270$ . **Green line:** the same with  $f_0' = 1.345$ ,  $g_0' = 1.22$  up to  $A_{max} = 280$ .

## CONCLUSION

- **The are 3 types of the charge-exchange allowed resonances: Giant Gamow–Teller and the analog resonances, and pygmy resonances.**
- **These resonances can be good described using microscopic theory (TFFS) and in its model approximation.**
- **The calculated values of the energies of the Gamow–Teller (GTR), analog (AR) and pygmy resonances, found to be in good agreement with their experimental data. Average deviation is less than 0.40 MeV for 33 nuclei.**
- **The  $E_{\text{GTR}}$  and  $E_{\text{AR}}$  values calculated for heavy and superheavy nuclei on the line of beta-stability up to mass number  $A = 300$ .**
- **The local interaction parameters  $f'$  and  $g'$  of the Landau-Migdal type are found by fitting theoretical and experimental AR and GTR energies.**
  - **Comparison with other results show good agreement.**
- **Degeneration of the GTR and AR resonances ( $\Delta E_{\text{G-A}} \rightarrow 0$ ) in heavy nuclei confirm the SU(4) Wigner super-symmetry restoration.**

**THE END**



**THANK YOU**