Effective Field Theories

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There is a high energy scale M where an effective theory breaks down. Its Lagrangian describes light particles ($m_i \ll M$) and their interactions at $p_i \ll M$ (distances $\gg 1/M$); physics at distances $\lesssim 1/M$ produces local interactions of these light fields.

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The Lagrangian contains all possible operators (allowed by symmetries). Coefficients of operators of dimension $n+4$ contain $1/M^n$. If M is much larger than energies we are interested in, we can retain only renormalizable terms (dimension 4), and, maybe, a power correction or two.

Photonia

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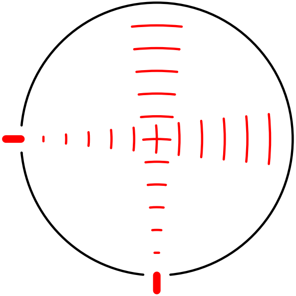
We indignantly refuse to discuss the question "What the experimantalists and their apparata are made of?" as irrelevant.

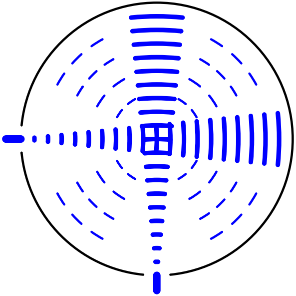
Photonia



Quantum PhotoDynamics (QPD)

$$L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

Photonia



Quantum PhotoDynamics (QPD)

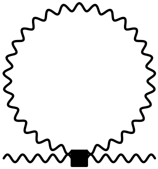
$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + c_1 O_1 + c_2 O_2$$

$$O_1 = (F_{\mu\nu}F^{\mu\nu})^2 \qquad O_2 = F_{\mu\nu}F^{\nu\alpha}F_{\alpha\beta}F^{\beta\mu} \qquad c_{1,2} \sim 1/M^4$$

Photonia

We work at the order $1/M^4$, there can be only 1 4-photon vertex

No corrections to the photon propagator



$= 0$

No renormalization of the photon field

No corrections to the 4-photon vertex
No renormalization of the operators $O_{1,2}$ and the couplings $c_{1,2}$

Qedland

Physicists in the neighboring Qedland are more advanced: in addition to photons, they know electrons and positrons, and investigate their interactions at energies $E \sim M$. They have constructed a nice theory, QED, which describes their experimental results.

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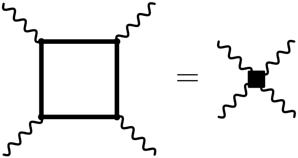
They understand that QPD (constructed in Photonia) is just a low-energy approximation to QED.

Matching

$c_{1,2}$ can be found by matching S-matrix elements



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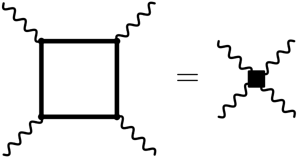

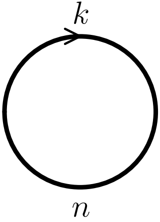

$$= \frac{1}{i\pi^{d/2}} \int \frac{d^d k}{D^n} = M^{d-2n} V(n)$$

$$D = M^2 - k^2 - i0$$

$$V(n) = \frac{\Gamma\left(n - \frac{d}{2}\right)}{\Gamma(n)}$$

Matching

$$T^{\mu_1\mu_2\nu_1\nu_2} = \frac{e_0^4 M^{-4-2\varepsilon}}{(4\pi)^{d/2}} \Gamma(\varepsilon) \frac{(d-4)(d-6)}{2880} \times \left(-5T_1^{\mu_1\mu_2\nu_1\nu_2} + 14T_2^{\mu_1\mu_2\nu_1\nu_2}\right)$$

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Heisengerg–Euler Lagrangian

$$L_1 = \frac{\pi\alpha^2}{180M^4} \left(-5O_1 + 14O_2\right)$$

Wilson line

Physicists in Photonia have some classical (infinitely heavy) charged particles and can manipulate them.

$$S_{\text{int}} = e \int_l dx^\mu A_\mu(x)$$

Feynman path integral: $\exp(iS)$ contains

$$W_l = \exp\left(ie \int_l dx^\mu A_\mu(x) \right)$$

The vacuum-to-vacuum transition amplitude is the vacuum average of the Wilson lines

Potential

Charges e and $-e$ stay at some distance $\vec{r}$ during a large time T: the vacuum amplitude $e^{-iU(\vec{r})T}$

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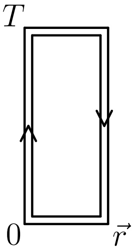
$T \gg r$



$$= e^{-iU(\vec{r})T}$$

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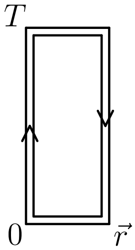


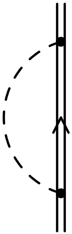
$= e^{-iU(\vec{r})T}$

Coulomb gauge

$$D^{00}(q) = -\frac{1}{\vec{q}^{\,2}}$$

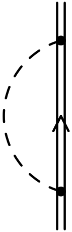
$$D^{ij}(q) = \frac{1}{q^2 + i0}\left(\delta^{ij} - \frac{q^i q^j}{\vec{q}^{\,2}}\right)$$

Wilson line

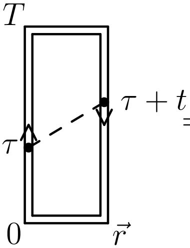


$= 0$

Wilson line



$$= 0$$



$$= -i\, e^2\, T \int D^{00}(t, \vec{r})\, dt$$

$$= -i\, e^2\, T \int \frac{d^{d-1}\vec{q}}{(2\pi)^{d-1}}\, D^{00}(0, \vec{q})\, e^{i\,\vec{q}\cdot\vec{r}}$$

Coulomb potential

$$U(\vec{q}) = e^2 D^{00}(0, \vec{q}) = -\frac{e^2}{\vec{q}^2}$$

$$U(\vec{r}) = -\frac{\alpha}{r}$$

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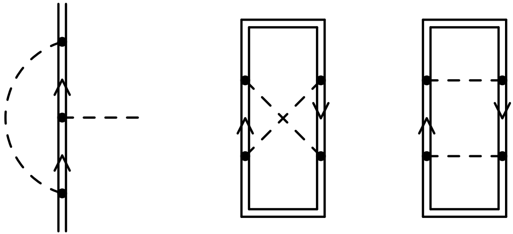

No corrections

Contact interaction

In the presence of sources

$$L_c = c\left(\partial^{\mu} F_{\lambda\mu}\right)\left(\partial_{\nu} F^{\lambda\nu}\right)$$

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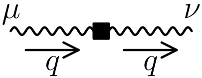

$$= 2icq^2\left(q^2 g_{\mu\nu} - q_\mu q_\nu\right)$$

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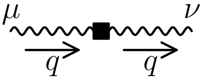

$$= 2icq^2\left(q^2 g_{\mu\nu} - q_\mu q_\nu\right)$$

$$U_c(\vec{r}) = 2c\delta(\vec{r})$$

Qedland

$$D^{00}(\vec{q}) = -\frac{1}{\vec{q}^2}\frac{1}{1-\Pi(-\vec{q}^2)} \qquad U(\vec{q}) = e_0^2 D^{00}(\vec{q})$$

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In macroscopic measurements $\vec{q} \to 0$

$$U(\vec{q}) \to -\frac{e_0^2}{\vec{q}^2}\frac{1}{1-\Pi(0)} = -\frac{e_{\rm os}^2}{\vec{q}^2}$$

On-shell renormalization scheme

$$e_0 = \left[Z_\alpha^{\rm os}\right]^{1/2} e_{\rm os} \qquad A_0 = \left[Z_A^{\rm os}\right]^{1/2} A_{\rm os}$$

$$D^{00}(\vec{q}) = Z_A^{\rm os} D^{00}_{\rm os}(\vec{q}) \qquad D^{00}_{\rm os}(\vec{q}) \to -\frac{1}{\vec{q}^2}$$

$$Z_\alpha^{\rm os} = \left[Z_A^{\rm os}\right]^{-1} = 1-\Pi(0)$$

$\overline{\text{MS}}$ renormalization scheme

Dimensional regularization $d = 4 - 2\varepsilon$

$$e_0 = Z_\alpha^{1/2}(\alpha(\mu)) e(\mu) \qquad A_0 = Z_A^{1/2}(\alpha(\mu)) A(\mu)$$

$$Z_i(\alpha) = 1 + \frac{z_1}{\varepsilon}\frac{\alpha}{4\pi} + \left(\frac{z_{22}}{\varepsilon^2} + \frac{z_{21}}{\varepsilon}\right)\left(\frac{\alpha}{4\pi}\right)^2 + \cdots$$

$$D^{00}(\vec{q}) = Z_A D^{00}(\vec{q};\mu) \qquad D^{00}(\vec{q};\mu) = \text{finite}$$

$$U(\vec{q}) = e^2(\mu) D^{00}(\vec{q};\mu) Z_\alpha Z_A = \text{finite} \qquad Z_\alpha = Z_A^{-1}$$

$$\frac{\alpha(\mu)}{4\pi} = \frac{e^2(\mu)\,\mu^{-2\varepsilon}}{(4\pi)^{d/2}} e^{-\gamma\varepsilon}$$

RG equations

$$\frac{d\log\alpha(\mu)}{d\log\mu} = -2\varepsilon - 2\beta(\alpha(\mu))$$

$$\beta(\alpha(\mu)) = \frac{1}{2}\frac{d\log Z_\alpha(\alpha(\mu))}{d\log\mu} \qquad \beta(\alpha) = \beta_0\frac{\alpha}{4\pi} + \beta_1\left(\frac{\alpha}{4\pi}\right)^2 + \cdots$$

$$\frac{dA(\mu)}{d\log\mu} = -\frac{1}{2}\gamma_A(\alpha(\mu))A(\mu)$$

$$\gamma_A = \frac{d\log Z_A(\alpha(\mu))}{d\log\mu} \qquad \gamma_A(\alpha) = \gamma_{A0}\frac{\alpha}{4\pi} + \gamma_{A1}\left(\frac{\alpha}{4\pi}\right)^2 + \cdots$$

QED $\beta(\alpha) = -\frac{1}{2}\gamma_A(\alpha)$

Charge decoupling

QPD

$$e'_0 = e'_{\mathrm{os}} = e'(\mu)$$

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QPD

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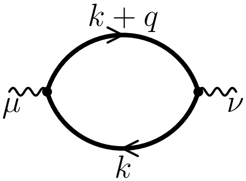
Macroscopically measured charge is the same in QED and QPD

$$e_{\rm os} = e'_{\rm os}$$

$$e_0 = \left[\zeta_\alpha^0\right]^{-1/2} e'_0 \qquad \zeta_\alpha^0 = \left[Z_\alpha^{\rm os}\right]^{-1}$$

$$e(\mu) = \left[\zeta_\alpha(\mu)\right]^{-1/2} e'(\mu) \qquad \zeta_\alpha(\mu) = Z_\alpha \zeta_\alpha^0 = \frac{Z_\alpha}{Z_\alpha^{\rm os}}$$

1 loop



$$= i\left(q^2 g_{\mu\nu} - q_\mu q_\nu\right)\Pi(q^2)$$

$$\Pi(q^2) = -\frac{4}{3}\frac{e_0^2 M_0^{-2\varepsilon}}{(4\pi)^{d/2}}\Gamma(\varepsilon)\left(1 - \frac{d-4}{10}\frac{q^2}{M_0^2} + \cdots\right)$$

1 loop

$$Z_\alpha^{\mathrm{os}} = 1 + \frac{4}{3}\frac{e_0^2 M_0^{-2\varepsilon}}{(4\pi)^{d/2}}\Gamma(\varepsilon) + \cdots$$

$$[\zeta_\alpha(\mu)]^{-1} = \frac{Z_\alpha^{\mathrm{os}}}{Z_\alpha} = \text{finite}$$

$$Z_\alpha = 1 + \frac{4}{3}\frac{\alpha}{4\pi\epsilon} + \cdots \qquad \beta_0 = -\frac{4}{3}$$

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$$[\zeta_\alpha(\mu)]^{-1} = 1 + \frac{4}{3}\left[\left(\frac{\mu}{M(\mu)}\right)^{2\varepsilon} e^{\gamma\epsilon}\Gamma(1+\varepsilon) - 1\right]\frac{\alpha(\mu)}{4\pi\varepsilon} + \cdots$$

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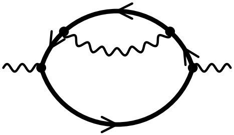
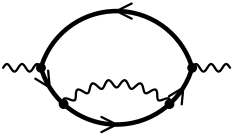
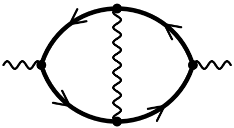
$$\to 1 + \frac{4}{3}\frac{\alpha(\mu)}{4\pi}L \qquad L = 2\log\frac{\mu}{M(\mu)}$$

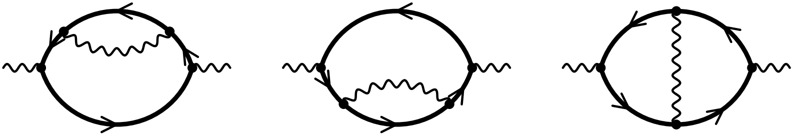
2 loops

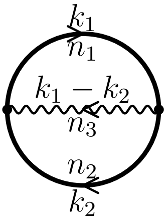






2 loops





$$\frac{\Gamma\left(\frac{d}{2}-n_3\right)\Gamma\left(n_1+n_3-\frac{d}{2}\right)\Gamma\left(n_2+n_3-\frac{d}{2}\right)\Gamma(n_1+n_2+n_3-d)}{\Gamma\left(\frac{d}{2}\right)\Gamma(n_1)\Gamma(n_2)\Gamma(n_1+n_2+2n_3-d)}$$

A. Vladimirov (1980)

2 loops

$$\zeta_A^0 = \left[\zeta_\alpha^0\right]^{-1} = 1 - \Pi(0) = 1 + \frac{4}{3}\frac{e_0^2 M_0^{-2\varepsilon}}{(4\pi)^{d/2}}\Gamma(\varepsilon)$$
$$+ \frac{2}{3}\frac{(d-4)(5d^2-33d+34)}{d(d-5)}\left(\frac{e_0^2 M_0^{-2\varepsilon}}{(4\pi)^{d/2}}\Gamma(\varepsilon)\right)^2$$

2 loops

$$
\begin{aligned}
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+ \frac{2}{3}\frac{(d-4)(5d^2-33d+34)}{d(d-5)}\left(\frac{e_0^2 M_0^{-2\varepsilon}}{(4\pi)^{d/2}}\Gamma(\varepsilon)\right)^2 \\
= 1 + \frac{4}{3}\frac{\alpha(\mu)}{4\pi\varepsilon}e^{L\varepsilon}\left(1 + \frac{\pi^2}{12}\varepsilon^2 + \cdots\right) Z_\alpha(\alpha(\mu)) Z_m^{-2\varepsilon}(\alpha(\mu)) \\
- \varepsilon\left(6 - \frac{13}{3}\varepsilon + \cdots\right)\left(\frac{\alpha(\mu)}{4\pi\varepsilon}\right)^2 e^{2L\varepsilon}
\end{aligned}
$$

2 loops

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\begin{aligned}
\zeta_A^0 = \left[\zeta_\alpha^0\right]^{-1} &= 1 - \Pi(0) = 1 + \frac{4}{3}\frac{e_0^2 M_0^{-2\varepsilon}}{(4\pi)^{d/2}}\Gamma(\varepsilon) \\
&+ \frac{2}{3}\frac{(d-4)(5d^2-33d+34)}{d(d-5)}\left(\frac{e_0^2 M_0^{-2\varepsilon}}{(4\pi)^{d/2}}\Gamma(\varepsilon)\right)^2 \\
&= 1 + \frac{4}{3}\frac{\alpha(\mu)}{4\pi\varepsilon}e^{L\varepsilon}\left(1 + \frac{\pi^2}{12}\varepsilon^2 + \cdots\right) Z_\alpha(\alpha(\mu)) Z_m^{-2\varepsilon}(\alpha(\mu)) \\
&- \varepsilon\left(6 - \frac{13}{3}\varepsilon + \cdots\right)\left(\frac{\alpha(\mu)}{4\pi\varepsilon}\right)^2 e^{2L\varepsilon}
\end{aligned}
$$

$$
Z_\alpha = Z_A^{-1} = 1 + \frac{4}{3}\frac{\alpha(\mu)}{4\pi\varepsilon} + \cdots
$$

Mass renormalization

$$M_0 = Z_m(\alpha(\mu))M(\mu) = Z_m^{\mathrm{os}} M_{\mathrm{os}}$$

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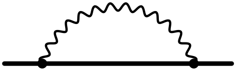
On-shell



$$M(n_1, n_2) = \frac{\Gamma(d - n_1 - 2n_2)\Gamma\left(n_1 + n_2 - \frac{d}{2}\right)}{\Gamma(n_1)\Gamma(d - n_1 - n_2)}$$

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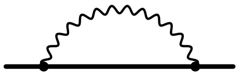
On-shell



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$$Z_m^{\mathrm{os}} = 1 - \frac{d-1}{d-3} \frac{e_0^2 M_0^{-2\varepsilon}}{(4\pi)^{d/2}} \Gamma(\varepsilon) + \cdots$$

Mass renormalization

$$M_0 = Z_m(\alpha(\mu))M(\mu) = Z_m^{\rm os} M_{\rm os}$$

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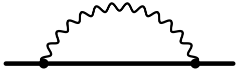

$$M(n_1, n_2) = \frac{\Gamma(d - n_1 - 2n_2)\Gamma\left(n_1 + n_2 - \frac{d}{2}\right)}{\Gamma(n_1)\Gamma(d - n_1 - n_2)}$$

$$Z_m^{\rm os} = 1 - \frac{d-1}{d-3}\frac{e_0^2 M_0^{-2\varepsilon}}{(4\pi)^{d/2}}\Gamma(\varepsilon) + \cdots$$

$\overline{\rm MS}$

Both $M_{\rm os}$ and $M(\mu)$ are finite at $\varepsilon \to 0$

$$Z_m(\alpha) = 1 - 3\frac{\alpha}{4\pi\varepsilon} + \cdots$$

2 loops

$$\zeta_A = Z_A \zeta_A^0 = \text{finite}$$

$$Z_A = Z_\alpha^{-1} = 1 - \frac{4}{3}\frac{\alpha(\mu)}{4\pi\varepsilon} - 2\varepsilon\left(\frac{\alpha(\mu)}{4\pi\varepsilon}\right)^2$$

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$$\zeta_A(\mu) = \zeta_\alpha^{-1}(\mu) = 1 + \frac{4}{3}\left[L + \left(\frac{L^2}{2} + \frac{\pi^2}{12}\right)\varepsilon\right]\frac{\alpha(\mu)}{4\pi} + \left(-4L + \frac{13}{3}\right)\left(\frac{\alpha(\mu)}{4\pi}\right)^2$$

2 loops

$$\zeta_A = Z_A \zeta_A^0 = \text{finite}$$

$$Z_A = Z_\alpha^{-1} = 1 - \frac{4}{3}\frac{\alpha(\mu)}{4\pi\varepsilon} - 2\varepsilon\left(\frac{\alpha(\mu)}{4\pi\varepsilon}\right)^2$$

$$\zeta_A(\mu) = \zeta_\alpha^{-1}(\mu) = 1 + \frac{4}{3}\left[L + \left(\frac{L^2}{2} + \frac{\pi^2}{12}\right)\varepsilon\right]\frac{\alpha(\mu)}{4\pi} + \left(-4L + \frac{13}{3}\right)\left(\frac{\alpha(\mu)}{4\pi}\right)^2$$

Define $M(\bar{M}) = \bar{M}$, then $L = 0$

$$\zeta_A(\bar{M}) = \zeta_\alpha^{-1}(\bar{M}) = 1 + \frac{\pi^2}{9}\varepsilon\frac{\alpha(\bar{M})}{4\pi} + \frac{13}{3}\left(\frac{\alpha(\bar{M})}{4\pi}\right)^2$$

2 loops

Alternatively use M_{os}

$$\frac{M(\mu)}{M_{\mathrm{os}}} = 1 - 6\left(\log\frac{\mu}{M_{\mathrm{os}}} + \frac{2}{3}\right)\frac{\alpha}{4\pi} \qquad L = 8\frac{\alpha}{4\pi}$$

$$\zeta_A(M_{\mathrm{os}}) = \zeta_\alpha^{-1}(M_{\mathrm{os}}) = 1 + \frac{\pi^2}{9}\varepsilon\frac{\alpha(M_{\mathrm{os}})}{4\pi} + 15\left(\frac{\alpha(M_{\mathrm{os}})}{4\pi}\right)^2$$

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For any $\mu = M(1 + \mathcal{O}(\alpha))$, $\zeta_\alpha = 1 + \mathcal{O}(\varepsilon)\alpha + \mathcal{O}(\alpha^2)$

Qedland

Physicists in Qedland suspect that QED is also a low-energy effective theory. They are right: muons exist (let's suppose that pions don't exist). Two ways to search for new physics:

- increase the energy of e^+e^- colliders to produce pairs of new particles
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We were lucky: the scale of new physics in QED is $M \gg m_e$, loops of heavy particles also suppressed by α^n. μ_e agrees with QED without non-renormalizable corrections to a good precision. Physicists expected the same for proton. No luck here.

QCD

- QED: effects of decoupling of muon loops are tiny; pion pairs become important at about the same energies as muon pairs
- QCD: decoupling of heavy flavours is fundamental and omnipresent; everybody using QCD with $n_f < 6$ uses an effective field theory (even if he does not know that he speaks prose)

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Full theory QCD with n_l massless flavours
and 1 flavour of mass M

Effective theory QCD with n_l massless flavours

QCD decoupling

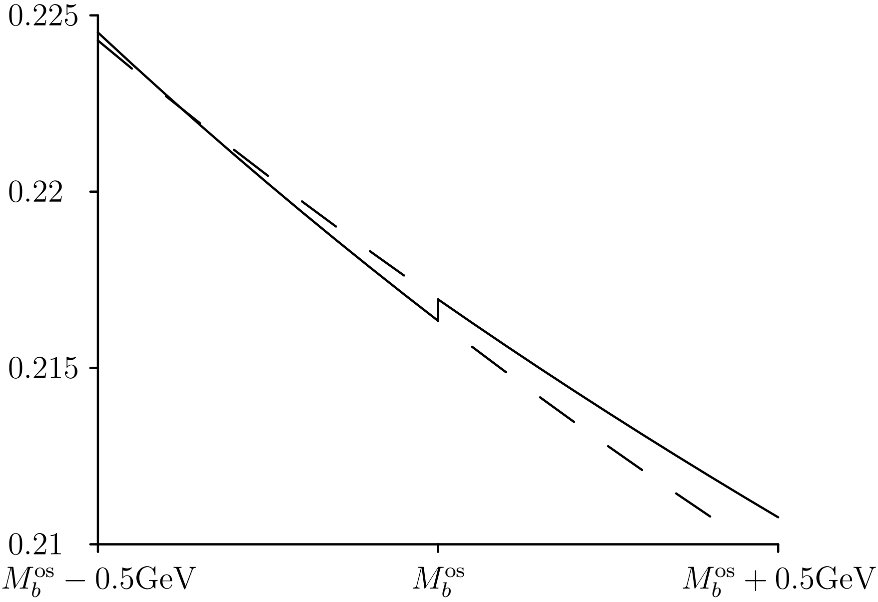
$$\alpha_s^{(n_l+1)}(\mu) = \zeta_\alpha^{-1}(\mu)\alpha_s^{(n_l)}(\mu)$$

$$\zeta_\alpha(\bar{M}) = 1 - \left(\frac{13}{3}C_F - \frac{32}{9}C_A\right) T_F \left(\frac{\alpha_s(\bar{M})}{4\pi}\right)^2 + \cdots$$

RG equation

$$\frac{d\log\zeta_\alpha(\mu)}{d\log\mu} - 2\beta^{(n_l+1)}(\alpha_s^{(n_l+1)}(\mu)) + 2\beta^{(n_l)}(\alpha_s^{(n_l)}(\mu)) = 0$$

QCD



In the past

Only renormalizable theories were considered well-defined: they contain a finite number of parameters, which can be extracted from a finite number of experimental results and used to predict an infinite number of other potential measurements. Non-renormalizable theories were rejected because their renormalization at all orders in non-renormalizable interactions involve infinitely many parameters, so that such a theory has no predictive power. This principle is absolutely correct, if we are impudent enough to pretend that our theory describes the Nature up to arbitrarily high energies (or arbitrarily small distances).

At present

We accept the fact that our theories only describe the Nature at sufficiently low energies (or sufficiently large distances). They are effective low-energy theories. Such theories contain all operators (allowed by the relevant symmetries) in their Lagrangians. They are necessarily non-renormalizable. This does not prevent us from obtaining definite predictions at any fixed order in the expansion in E/M, where E is the characteristic energy and M is the scale of new physics. Only if we are lucky and M is many orders of magnitude larger than the energies we are interested in, we can neglect higher-dimensional operators in the Lagrangian and work with a renormalizable theory.

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For example, if we want to work at the order $1/M^4$, then either a single $1/M^4$ (dimension 8) vertex or two $1/M^2$ ones (dimension 6) can occur in a diagram. UV divergences which appear in diagrams with two dimension 6 vertices are compensated by renormalizing these 2 operators plus dimension 8 counterterms. So, the theory can be renormalized.

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The usual arguments about non-renormalizability are based on considering diagrams with arbitrarily many vertices of nonrenormalizable interactions (operators of dimensions > 4); this leads to infinitely many free parameters in the theory.