Exercises for School: From Strong Fields to Heavy Quarks

Prof. Dr. David Blaschke (University of Wroclaw & JINR Dubna), Email: david.blaschke@gmail.com

1.0 Particle production in heavy-ion collisions may proceed by the decay of color electric flux tubes which are characterized by a linear, stringlike potential between color charges, analogous to the case of a homogeneous electric field considered by Schwinger $|eE| = \sigma$, with $\sigma = 0.19$ GeV^2 being the string tension. The transverse energy spectrum of produced particles according to the Schwinger mechanism would then be

$$\frac{dN_{\text{Schwinger}}}{d^2p_\perp} \sim \exp\left(-\frac{\pi\varepsilon_\perp^2}{\sigma}\right) ,$$

with $\sqrt{m^2+p_\perp^2}$ being the transverse energy, often also denoted as "transverse mass" $m_\perp$. This spectrum of produced particles is nonthermal and thus would contradict the observation of thermal particle spectra in heavy-ion collision experiments

$$\frac{dN_{\text{exp}}}{d^2p_\perp} \sim \exp\left(-\frac{\varepsilon_\perp}{T_{\text{eff}}}\right) ,$$

with an effective temperature $T_{\text{eff}} \sim 160 \ldots 180$ MeV (inverse slope parameter). Thus the question for the thermalization arises.

Show that a thermal particle spectrum would arise when the string tension parameter would fluctuate and have a Poissonian spectrum

$$P(\sigma) = \exp(\sigma/\sigma_0)/\sqrt{\pi\sigma\sigma_0},$$

which is normalized $\int d\sigma P(\sigma) = 1$ and has a mean value $\langle\sigma\rangle = \int d\sigma\, \sigma P(\sigma) = \sigma_0/2$. The effective temperature appears to be the Hawking-Unruh temperature of thermal hadron production,

$$T_{\text{eff}} = \sqrt{\frac{\langle\sigma\rangle}{2\pi}} \sim 173 \text{ MeV} .$$

Hint: Use the integral: $\int_0^\infty dt \exp[-t - k^2/(4t)]/\sqrt{\pi t} = \exp(-k)$.

2.1 The polarization function for gauge bosons in a fermion-antifermion plasma is given by

$$\Pi_{00}(i\omega_l;\mathbf{q}) = N_{\text{dof}}g^2T \sum_{n=-\infty}^{\infty} \sum_{s;s'=\pm} \int \frac{d^3p}{(2\pi)^3} \frac{ss'}{E_pE_{p-q}} \frac{ss'E_pE_{p-q} + \mathbf{p}(\mathbf{p}-\mathbf{q})}{(i\omega_n + sE_p)(i\omega_n - i\omega_l + s'E_{p-q})}. \tag{1}$$

Evaluate the fermionic Matsubara sum and show that the result is

$$\Pi_{00}(i\omega_l;\mathbf{q}) = N_{\text{dof}}g^2 \sum_{ss'=\pm} \int \frac{d^3p}{(2\pi)^3} \frac{ss'}{E_pE_{p-q}} \frac{ss'E_pE_{p-q} + \mathbf{p}(\mathbf{p}-\mathbf{q})}{i\omega_l + sE_p - s'E_{p-q}} [(f(-s'E_p) - f(-sE_{p-q})]. \tag{2}$$

2.2 Perform the static (set $i\omega_l = 0$) and long-wavelength (let $\mathbf{q} \to 0$) limit of the polarization function and derive the result for the Debye mass in a plasma of massless fermions $\Pi_{00}(0;0) = N_{\text{dof}}g^2T^2/3 = m_D^2(T)$.

2.3 Use the Ritz variational principle for the hamiltonian of a heavy quarkonium state in a plasma

$$H = -\frac{\nabla^2}{m_c} - \frac{\alpha}{r}e^{-m_Dr} ,$$

with the trial wave function

$$\psi_\gamma(r) = \sqrt{\frac{\gamma^3}{\pi}} \exp(-\gamma r)$$

to obtain a condition on the critical Debye mass m_D^{Mott} for which the binding energy vanishes. Check that the energy functional is

$$E(\gamma) =< \psi_\gamma|H|\psi_\gamma >= \frac{\gamma^2}{m_c} - \frac{4\alpha\gamma^3}{(m_D+2\gamma)^2}.$$

and derive the result $m_D^{\text{Mott}} = 2\gamma$. Interprete the result in terms of the Bohr radius $a_0 = 2/(\alpha m_Q)$!