1.0 Particle production in heavy-ion collisions may proceed by the decay of color electric flux tubes which are characterized by a linear, stringlike potential between color charges, analogous to the case of a homogeneous electric field considered by Schwinger \(|E| = \sigma\), with \(\sigma = 0.19 \text{ GeV}^2\) being the string tension. The transverse energy spectrum of produced particles according to the Schwinger mechanism would then be
\[
\frac{dN_{\text{Schwinger}}}{d^2p_\perp} \sim \exp \left( -\frac{\pi \varepsilon_\perp^2}{\sigma} \right),
\]
with \(\sqrt{m^2 + \varepsilon_\perp^2}\) being the transverse energy, often also denoted as "transverse mass" \(m_\perp\). This spectrum of produced particles is nonthermal and thus would contradict the observation of thermal particle spectra in heavy-ion collision experiments
\[
\frac{dN_{\text{exp}}}{d^2p_\perp} \sim \exp \left( -\frac{\varepsilon_\perp}{T_{\text{eff}}} \right),
\]
with an effective temperature \(T_{\text{eff}} \sim 160 \ldots 180 \text{ MeV}\) (inverse slope parameter). Thus the question for the thermalization arises.

Show that a thermal particle spectrum would arise when the string tension parameter would fluctuate and have a Poissonian spectrum
\[P(\sigma) = \exp(\sigma/\sigma_0) / \sqrt{\pi \sigma \sigma_0},\]
which is normalized \(\int d\sigma P(\sigma) = 1\) and has a mean value \(\langle \sigma \rangle = \int d\sigma \sigma P(\sigma) = \sigma_0/2\). The effective temperature appears to be the Hawking-Unruh temperature of thermal hadron production,
\[T_{\text{eff}} = \sqrt{\frac{\langle \sigma \rangle}{2\pi}} \sim 173 \text{ MeV} .\]

Hint: Use the integral: \(\int_0^\infty dt \exp[-t - t^2/(4t)]/\sqrt{\pi t} = \exp(-k)\).

2.1 The polarization function for gauge bosons in a fermion-antifermion plasma is given by
\[
\Pi_{00}(i\omega_l; q) = N_{\text{dof}} g^2 T \sum_{n=-\infty}^{\infty} \sum_{s,s'=\pm} \int \frac{d^3p}{(2\pi)^3} \frac{ss'}{E_p E_{p-q}} \frac{s's' E_p E_{p-q} + p(p-q)}{(i\omega_n + s E_p)(i\omega_n - i\omega_l + s'E_{p-q})} , \tag{1}
\]
Evaluate the fermionic Matsubara sum and show that the result is
\[
\Pi_{00}(i\omega_l; q) = N_{\text{dof}} g^2 \sum_{s,s'=\pm} \int \frac{d^3p}{(2\pi)^3} \frac{ss'}{E_p E_{p-q}} \frac{s's' E_p E_{p-q} + p(p-q)}{i\omega_l + s E_p - s'E_{p-q}} [(f(-s' E_p) - f(-s E_{p-q})]. \tag{2}
\]

2.2 Perform the static (set \(i\omega_l = 0\)) and long-wavelength (let \(q \to 0\)) limit of the polarization function and derive the result for the Debye mass in a plasma of massless fermions \(\Pi_{00}(0; 0) = N_{\text{dof}} g^2 T^2 / 3 = m_D^2(T)\).

2.3 Use the Ritz variational principle for the Hamiltonian of a heavy quarkonium state in a plasma
\[H = -\frac{\nabla^2}{m_c} - \frac{\alpha}{r} e^{-m_D r} ,\]
with the trial wave function
\[\psi_\gamma(r) = \sqrt{\frac{2^3}{\pi}} \exp(-\gamma r) \]
to obtain a condition on the critical Debye mass \(m_D^{\text{Mot}}\) for which the binding energy vanishes. Check that the energy functional is
\[E(\gamma) = \langle \psi_\gamma | H | \psi_\gamma \rangle = \frac{\gamma^2}{m_c} - \frac{4\alpha \gamma^3}{(m_D + 2\gamma)^2},\]
and derive the result \(m_D^{\text{Mot}} = 2\gamma\). Interpret the result in terms of the Bohr radius \(a_0 = 2/(\alpha m_Q)\)!