

Photon-Photon Scattering at the high-intensity frontier

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h2dr

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concept



Outline

1 Scattering Formalism

- Classical Electrodynamics
- Quantum Electrodynamics
 - Optical Photons
 - Hard X-Rays

2 Scientific Studies

- Photon Emission
- Multiphoton Pair Production

Outline

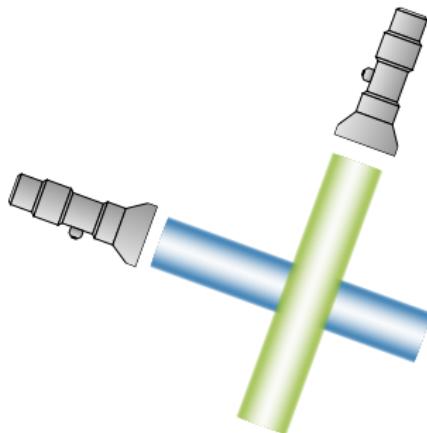
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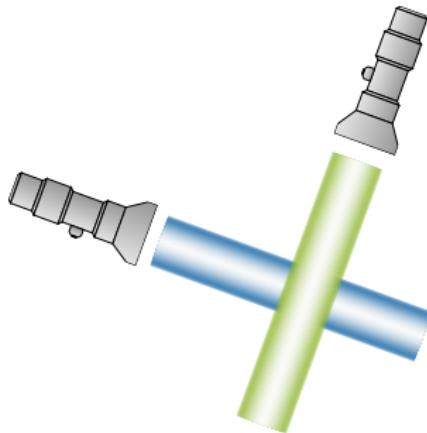
2 Scientific Studies

Vacuum

Classical Light-by-Light Scattering



Classical Light-by-Light Scattering



- Flashlights as light source
- No scattering observed

Classical Lagrangian

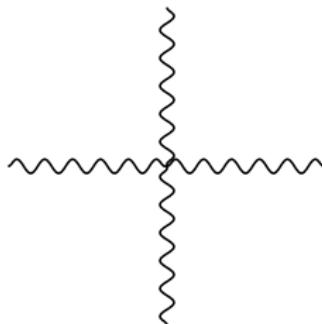
$$\mathcal{L}_{\text{class}} = \mathcal{L}_{\text{int}} + \mathcal{L}_{\text{kin}} = -A_\mu J^\mu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \quad (1)$$

- Field strength tensor
- Background field
- Source term
- Interaction term $-A_\mu J^\mu$
- Kinetic term $-\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$
- Equations of motion → inhomogeneous Maxwell equations

$$F_{\mu\nu}(x) = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad [A_\mu, A_\nu] = 0 \quad (2)$$

- Fields do **not interact directly** with each other!
- Photon-photon **scattering** is **impossible** in classical electrodynamics
- Equations of motion → homogeneous Maxwell equations

Feynman Diagram



- Photons → curly lines
- $[A_\mu, A_\nu] = 0$
- No interaction → no scattering → lines pass each other

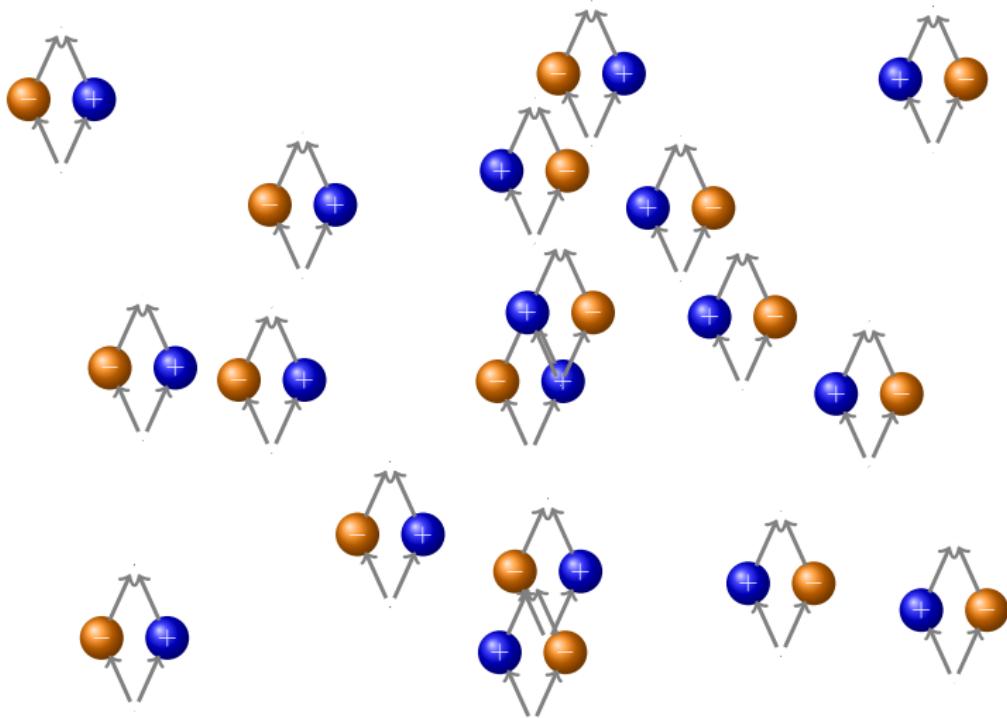
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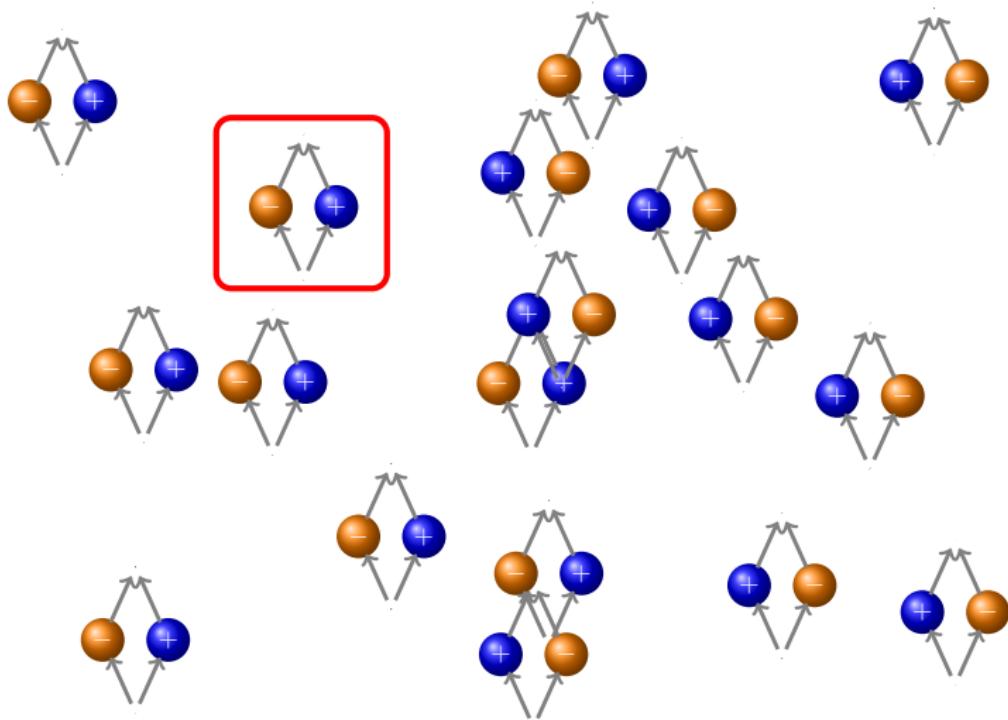
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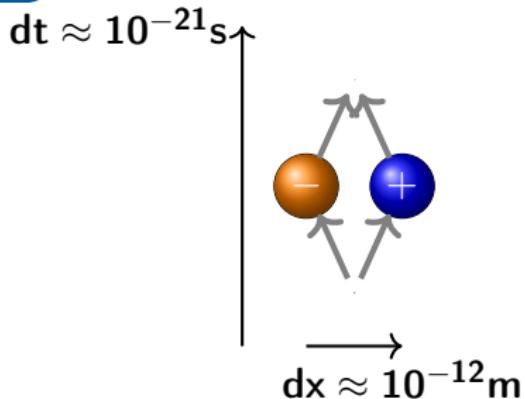
Vacuum in Quantum Field Theory



Vacuum in Quantum Field Theory



Virtual Electron-Positron Pair



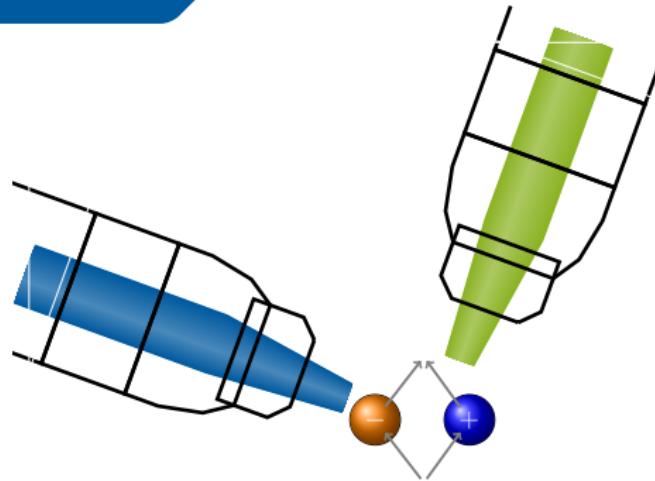
- Vacuum fluctuations
- QED scale: $\varepsilon_{crit} = m_e^2/e = 1.3 \times 10^{18} \text{ V/m}$

F. Sauter: Z. Phys. 69(742), 1931

J. S. Schwinger: Phys. Rev. 82(664), 1951

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High-Intensity Light-by-Light Scattering



- Laser light as source
- Probing quantum vacuum

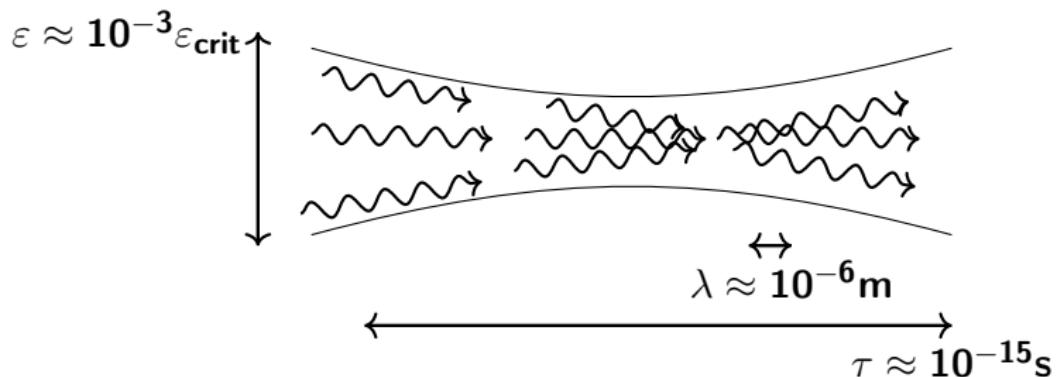
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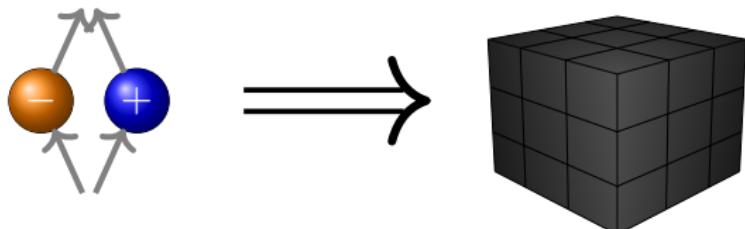
2 Scientific Studies

Low Energy Photons



- All-optical laser system
- Slowly varying background field
- Photons γ_ω with energy $\omega \approx 1 \text{eV}$

Vacuum Fluctuations



- Optical photons cannot resolve quantum fluctuations (different scales)
- Vacuum fluctuations → **effective background field**
- Virtual pair → “**black box**”

QED Lagrangian

$$\mathcal{L}_{\text{QED}} = \mathcal{L}_{\text{int}} + \mathcal{L}_{\text{kin}} = \bar{\psi} (\mathrm{i} \gamma^\mu D_\mu - m) \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \quad (3)$$

- Dirac matrices
- Bispinor fields for spin-1/2 particles
- Covariant derivative $D_\mu \equiv \partial_\mu + \mathrm{i} e A_\mu$
- Interaction term
- Kinetic term
- Coupling constant e , Mass m

$$\mathcal{L}_{\text{int}} (\psi, \bar{\psi}, A_\mu) = \bar{\psi} (i\gamma^\mu (\partial_\mu + ieA_\mu) - m) \psi \quad (4)$$

- “Integrating out” electrons and positrons $\psi, \bar{\psi}$
- Effective Lagrangian $\mathcal{L}_{\text{int}} (\psi, \bar{\psi}, A_\mu) \rightarrow \mathcal{L}_{\text{eff}} (A_\mu)$
- Gauge invariance demands $\mathcal{L}_{\text{eff}} (A_\mu) = \mathcal{L}_{\text{eff}} (F_{\mu\nu})$
- Lowest order non-linear contributions $(F_{\mu\nu} F^{\mu\nu})^2, (F_{\mu\nu} \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta})^2$

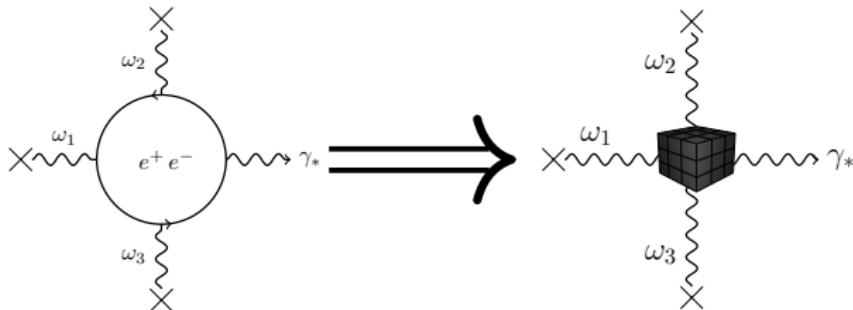
Euler-Heisenberg Lagrangian

$$\mathcal{L}_{\text{EH}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\alpha^2}{90m^4} \left((F_{\mu\nu}F^{\mu\nu})^2 + \frac{7}{4} \left(F_{\mu\nu}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta} \right)^2 \right) \quad (5)$$

- Optical background field $\omega_\gamma \ll m$
- Photon-photon scattering mediated by virtual particles
- Nonlinear dynamics of electromagnetic fields in vacuum
- One-loop \rightarrow fine-structure constant $\alpha \sim e^2$

W. Heisenberg et al.: Z. Phys. 98(714), 1936

Vacuum Emission



- Leading contribution: one loop, four lines
- Effective **nonlinearities**
- Three **couplings** to external field ω
- Single signal **emission** \rightarrow photon γ_* with **new properties**

Field Strength Tensor

$$\overline{F^{\mu\nu}} \rightarrow F^{\mu\nu}(x) + f^{\mu\nu}(x) \quad (6)$$

- Local constant field approximation:
Replace constant field $\overline{F^{\mu\nu}}$ → slowly varying fields
- Background fields $F^{\mu\nu}(x)$ in weak field limit $eF^{\mu\nu} \ll m^2$
- Field strength tensor of signal photons $f^{\mu\nu}(x)$

Z. Bialynicka-Birula al.: Phys. Rev. D 2 (1970) 2341

Effective Interaction

$$\Gamma = \Gamma^{(1)} + \Gamma^{(2)} + \Gamma^{(3)} + \dots = \int_x f^{\mu\nu}(x) \left. \frac{\partial \mathcal{L}_{\text{HE}}}{\partial \bar{F}^{\mu\nu}} \right|_{\bar{F} \rightarrow F(x)} + \mathcal{O}(m > 1) \quad (7)$$

- Expansion in terms of probe photons m
- $\Gamma^{(1)}$: stimulated vacuum emission
- Expand probe photon field $f^{\mu\nu} \rightarrow$ polarizations states:

$$\hat{f}_{(p)}^{\mu\nu}(k) = k^\mu \epsilon_{(p)}^{*\nu}(k) - k^\nu \epsilon_{(p)}^{*\mu}(k) \quad (8)$$

F. Karbstein et al.: Phys. Rev. D 91 (2015) no.11, 113002

Zero-to-Single Signal Photon Transition Amplitude

$$S_{(p)}(\vec{k}) \sim \epsilon_{(p)}^{*\nu}(\vec{k}) k^\mu \int d^4x e^{ikx} \frac{\partial \mathcal{L}_{\text{HE}}}{\partial \bar{F}^{\mu\nu}} \Big|_{\bar{F} \rightarrow F(x)} \quad (9)$$

- Euler-Heisenberg Lagrangian \mathcal{L}_{HE}
- Electric and magnetic background fields $F^{\mu\nu}(x)$
- Signal photon, polarization p
- Global information
- Momentum spectrum

F. Karbstein et al.: Phys. Rev. D 91 (2015) no.11, 113002

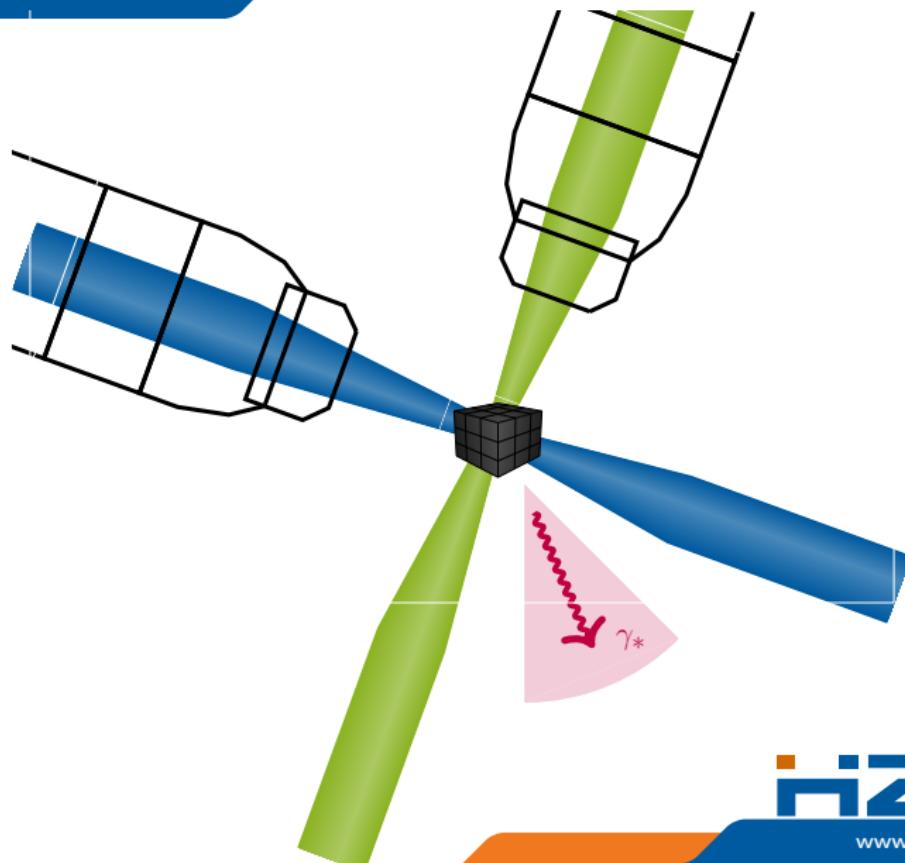
Photon Emission

$$d^3N_{(p)}(\vec{k}) = dk d\phi d\cos\theta \frac{1}{(2\pi)^3} |kS_{(p)}(\vec{k})|^2 \quad (10)$$

- Directional emission characteristics
- Signal photon polarization p
- Spherical coordinates
- Signal photon energies k
- Far-field detection

F. Karbstein et al.: Phys. Rev. D 91 (2015) no.11, 113002

Quantum Vacuum Emission



Résumé: Vacuum Emission Formalism

Positive Aspects

- Electric and magnetic fields as input
- Signal photons as output

Challenges

- Signal to noise ratio
- Signal optimization
- Numerics - Fields varying on different scales

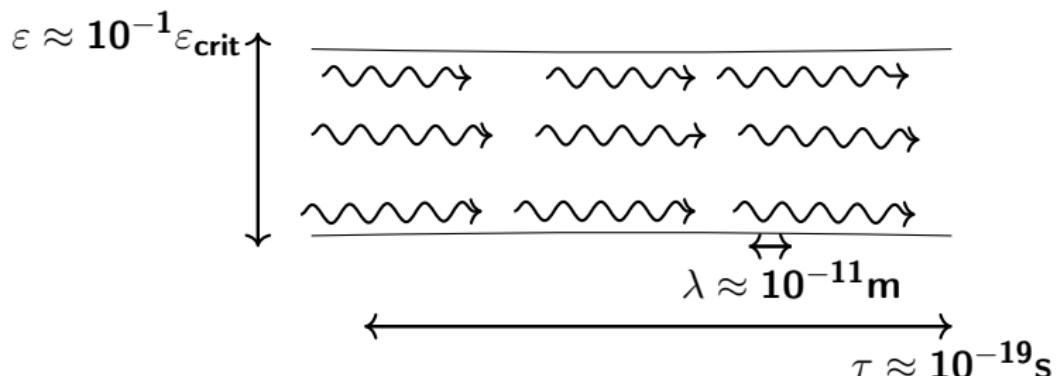
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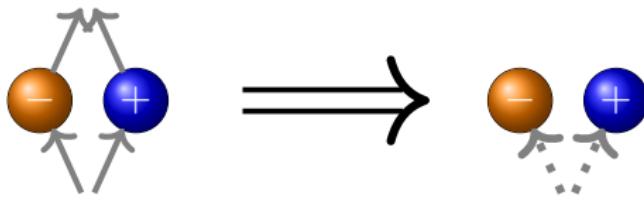
2 Scientific Studies

High Energy Photons



- Rapidly varying background field
- Photon energy $\omega_x \approx 10 \text{keV}$

Multiphoton Pair Production



- Hard X-rays can **probe quantum fluctuations**
- Described by QED Lagrangian
- Transfer of energy, linear & angular momentum
- **Virtual particles** become **real**

E. Brezin et al.: Phys. Rev. D 2 (1970), 1191

Multiphoton Pair Production



- Hard X-rays can **probe quantum fluctuations**
- Described by QED Lagrangian
- Scattering → transfer of energy, linear & angular momentum
- **Virtual particles** can become **real**

E. Brezin et al.: Phys. Rev. D 2 (1970), 1191

QED Lagrangian

$$\mathcal{L}_{\text{QED}} \left(\hat{F}_{\mu\nu} \right) = \bar{\psi} (i\gamma^\mu D_\mu - m) \psi - \frac{1}{4} \hat{F}_{\mu\nu} \hat{F}^{\mu\nu} \quad (11)$$

- Field strength tensor $\hat{F}_{\mu\nu}$
- Dirac matrices
- Bispinor fields for spin-1/2 particles
- Covariant derivative $D_\mu \equiv \partial_\mu + ie\hat{A}_\mu$
- Coupling constant e , Mass m

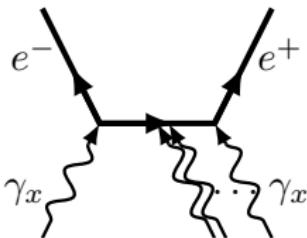
Mean-Field QED Lagrangian

$$\mathcal{L}_{\text{QED}} \left(\hat{F}_{\mu\nu} \right) \rightarrow \mathcal{L}_{\text{QED}} \left(F_{\mu\nu} \right) \quad (12)$$

- Hard X-rays $\omega_x \sim \mathcal{O}(m)$
- Mean-field approximation
 $F_{\mu\nu} \approx \langle \hat{F}_{\mu\nu} \rangle \rightarrow \text{classical background field}$
- Quantum nature of electrons & positrons
- Dynamics of charged particles in electromagnetic background field

D. Vasak et al.: Annals Phys. 173 (1987), 462

Production Process



- Similar to Breit-Wheeler process: $n\gamma_x \rightarrow e^- e^+$
- Emission of electrons and positrons

G. Breit et al.: Phys. Rev. 46 (1934) 1087

D. L. Burke et al.: Phys. Rev. Lett., 79:1626–1629, 1997

Wigner-Weyl Approach

$$\mathbb{W}(x, p) = \frac{1}{2} \int d^4y e^{ipy} U(A_\mu, x, y) \left[\bar{\psi}\left(x - \frac{y}{2}\right), \psi\left(x + \frac{y}{2}\right) \right] \quad (13)$$

- Wigner operator
- Phase-space formalism
- Gauge transporter $U(A_\mu, x, y)$
- $\mathbb{W}(x, p)$ is gauge invariant
- Quasi-probabilities

D. Vasak et al.: Annals of Physics 173(462-492), 1987

Equal-Time Formalism

$$W(x, p, t) = \int \frac{dp_0}{2\pi} \mathbb{W}(x, p) = \frac{1}{4} (\underline{s} + i\gamma_5 \underline{p} + \gamma^\mu \underline{v}_\mu + \gamma^\mu \gamma_5 \underline{a}_\mu + \sigma^{\mu\nu} \underline{t}_{\mu\nu}) \quad (14)$$

- Projection on **equal-time**
- **Initial-value problem**
- Expansion in Dirac bilinears
- **Wigner components:** mass density \underline{s} , charge density \underline{v}_0, \dots

D. Vasak et al.: Annals of Physics 173(462-492), 1987

I. Bialynicki-Birula et al.: Phys. Rev. D 44(6), 1991

Equation of Motion

$$(\mathcal{D}_t + \mathcal{D} + \mathcal{P}) \vec{\mathbf{w}} = \overline{M} \vec{\mathbf{w}} \quad (15)$$

- Matrix \overline{M}
- Wigner components $\vec{\mathbf{w}}$
- Pseudo-differential operators $\mathcal{D}_t(F_{\mu\nu}), \mathcal{D}(F_{\mu\nu}), \mathcal{P}(F_{\mu\nu})$
- Well-defined observables
- Initial-value problem

D. Vasak et al.: Annals of Physics 173(462-492), 1987

I. Bialynicki-Birula et al.: Phys. Rev. D 44(6), 1991

Observables: Particle Density

$$N(t \rightarrow \infty) = \int d^3 p f(\mathbf{p}, t \rightarrow \infty) \quad (16)$$

$$f(\mathbf{p}, t) = \int d^3 x \frac{s(x, \mathbf{p}, t) + \mathbf{p} \cdot v(x, \mathbf{p}, t)}{\omega(\mathbf{p})} \quad (17)$$

- Total production yield $N(t \rightarrow \infty)$
- Particle momentum spectrum $f(\mathbf{p}, t)$
- One-particle energy $\omega(\mathbf{p}) = \sqrt{1 + \mathbf{p}^2}$

Quantum Kinetic Theory

$$\begin{pmatrix} \dot{F} \\ \dot{G} \\ \dot{H} \end{pmatrix} = \begin{pmatrix} 0 & W & 0 \\ -W & 0 & -2\omega \\ 0 & 2\omega & 0 \end{pmatrix} \begin{pmatrix} F \\ G \\ H \end{pmatrix} + \begin{pmatrix} 0 \\ W \\ 0 \end{pmatrix} \quad (18)$$

- Spatially homogeneous electric background field $\mathbf{E}(t) = E(t)\mathbf{e}_z$
- No magnetic field
- Particle density $F(t)$
- Source term W

S. A. Smolyansky et al. hep-ph/9712377 GSI-97-72, 1997

S. Schmidt et al.: Int.J.Mod.Phys. E7 709-722, 1998

J. C. R. Bloch et al.: Phys. Rev. D 60(116011), 1999

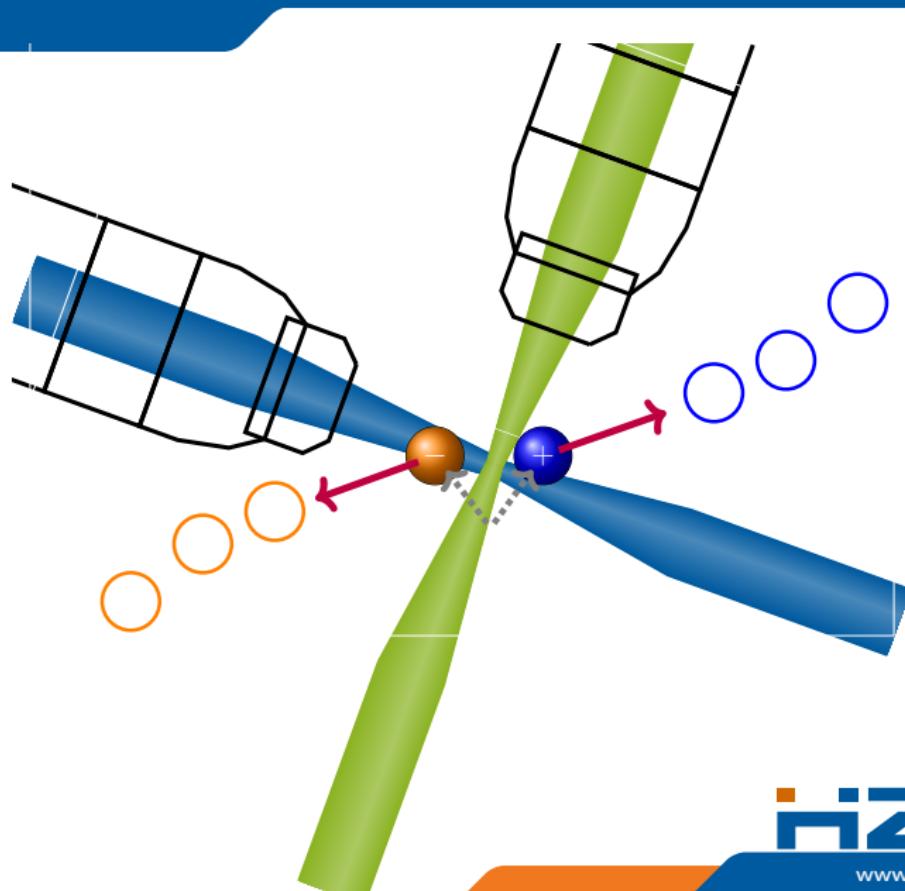
$$\partial_t f^+ + \mathbf{v} \cdot (\nabla_{\mathbf{x}} f^+) + e(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla_{\mathbf{p}} f^+ = 0 \quad (19)$$

$$\partial_t f^- + \mathbf{v} \cdot (\nabla_{\mathbf{x}} f^-) - e(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla_{\mathbf{p}} f^- = 0 \quad (20)$$

- Vlasov equation
- Positron distribution function f^+
- Electron distribution function f^-
- Particle number conservation

G. R. Shin et al. Phys. Rev. A 48:1869–1874, (1993)

High-intensity Light-by-Light Scattering



Résumé: Wigner Formalism

Positive Aspects

- Arbitrary vector potentials as input
- Time evolution
- Particle spectrum

Challenges

- Beyond mean-field approximation
- Back-reaction and particle collisions
- Partial differential equations

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Background Fields

Generic input

- No constraints on background fields
- Full-scale simulation
- Polarization sensitive results

Paraxial beams

- Good approximation of laser beams
for total number of signal photons
- Highly flexible computational scheme

Generic Background Fields

$$\mathbf{A}(t, \mathbf{x}) = \int d^3k \sum_i a_{0i}(\mathbf{k}) \mathbf{e}_i(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x}} e^{-i\omega t} \quad (21)$$

- Vector potential $\mathbf{A}(t, \mathbf{x})$ given in terms of amplitudes a_{0i}
- Two transverse polarization modes $\mathbf{e}_i(\mathbf{k})$
- Spatial Fourier transform
- Time evolution as phase factor $e^{-i\omega t}$
- Solution to Maxwell equations

A. Blinne et al. Phys. Rev. D (99), (2019) no.1, 016006

Paraxial Approximation

Wave vectors

$$\kappa_{\perp}^2 \ll \kappa^2 \quad (22)$$

Photons in background field **propagate** in same direction κ

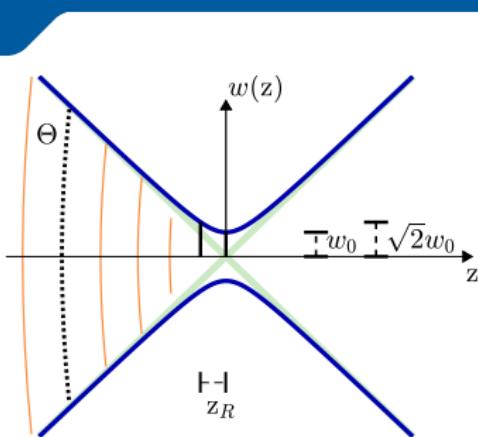
Field components

$$\mathbf{E} = \mathcal{E} \hat{\mathbf{e}}_E, \quad \mathbf{B} = \mathcal{E} \hat{\mathbf{e}}_B \quad (23)$$

$$\hat{\mathbf{e}}_E \cdot \hat{\mathbf{e}}_B = \hat{\mathbf{e}}_E \cdot \hat{\mathbf{e}}_\kappa = \hat{\mathbf{e}}_B \cdot \hat{\mathbf{e}}_\kappa = 0, \quad \hat{\mathbf{e}}_E \times \hat{\mathbf{e}}_B = \hat{\mathbf{e}}_\kappa \quad (24)$$

Fields are characterized by an **overall field amplitude** \mathcal{E}

Paraxial Approximation: Gaussian Beam



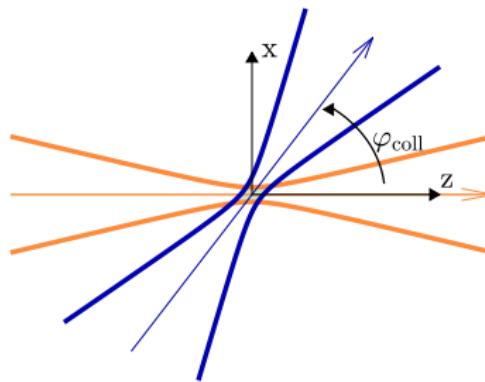
$$\text{Transverse widening } w(z) = w_0 \sqrt{1 + (z/z_R)^2}$$

$$\text{Rayleigh range } z_R = \pi w_0^2 / \lambda$$

$$\text{Beam divergence } \Theta$$

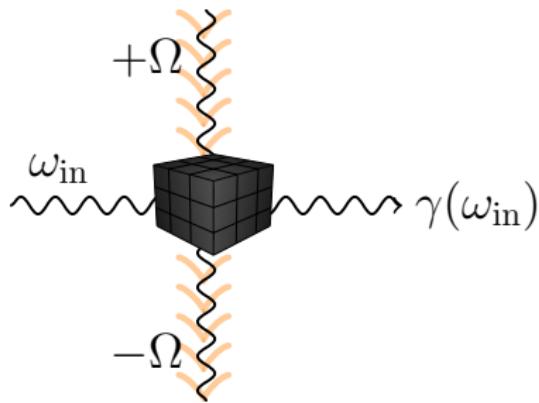
- $\mathcal{E}(x) = \mathcal{E}_0 e^{-\frac{(z\pm t)^2}{(\tau/2)^2}} \frac{w_0}{w(z)} e^{-\frac{r^2}{w^2(z)}} \cos(\omega(z \pm t) + \Phi(x))$
- Frequency $\omega = \frac{2\pi}{\lambda}$ Field strength \mathcal{E}_0 Duration τ

Colliding Beams



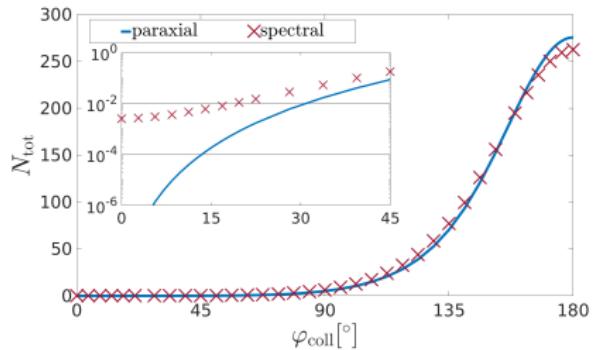
- Collision angle φ_{coll}
- Different angle → change kinematics of scattered photons

Stimulated Photon Emission

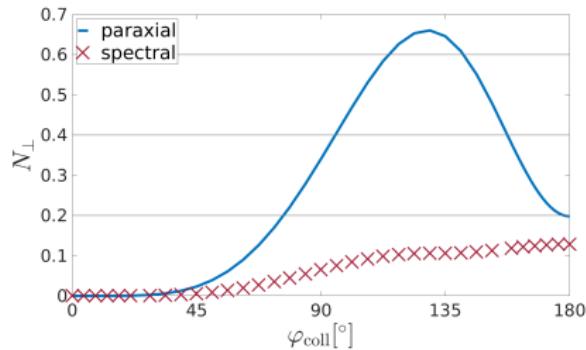


- Photons with energy $\Omega \rightarrow$ **stimulated emission**
- Characteristics of signal photon γ similar to ingoing photon ω_{in}

Signal Photon Rate



Parameters: $W = 25\text{J}$, $\tau = 25\text{fs}$, $\lambda = 800\text{nm}$

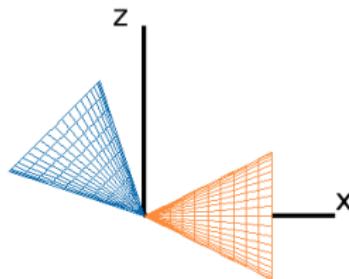


- Collision of two optimally focused laser pulses
- Single-pulse photon emission
- Paraxial approximation cannot resolve signal photon polarization

A. Blinne et al.: Phys. Rev. D 99 (2019) no.1, 016006

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Two-Beam Setup

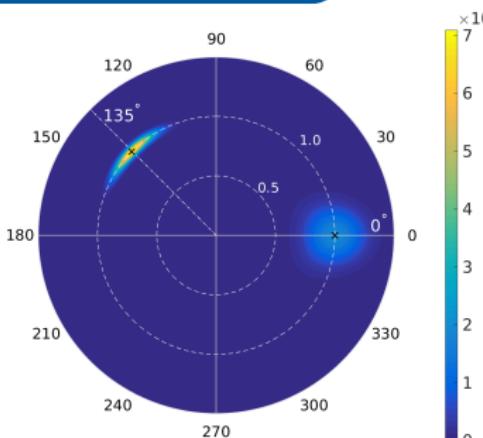


Beam 1: $W = 50\text{J}$, $\tau = 5\text{fs}$, $\lambda = 800\text{nm}$

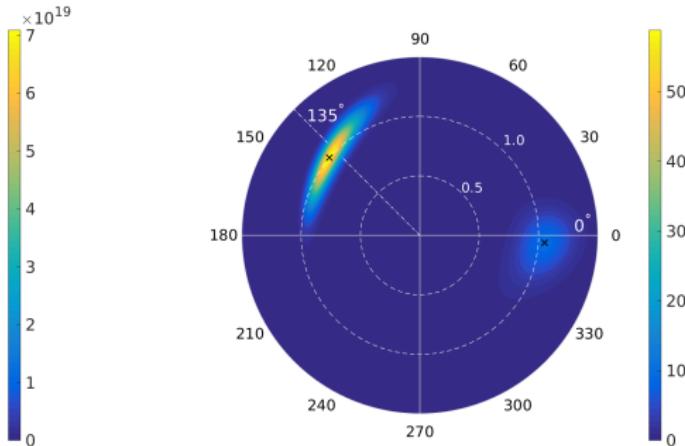
Beam 2: $W = 135\text{J}$, $\tau = 30\text{fs}$, $\lambda = 800\text{nm}$, $\varphi_{\text{coll}} = 135^\circ$

Combine short-pulsed beam with long-pulsed beam

Signal Photon Characteristics



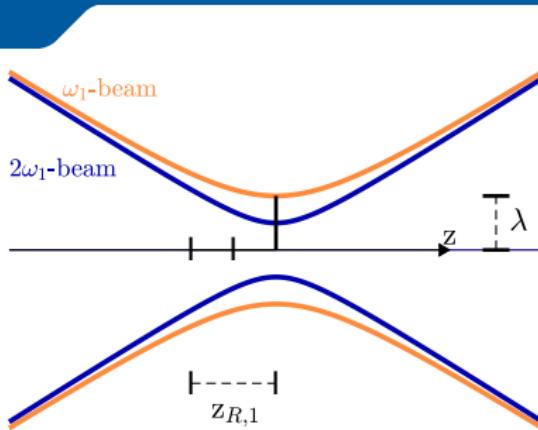
Beam 1: $W = 50\text{J}$, $\tau = 5\text{fs}$, $\lambda = 800\text{nm}$



Beam 2: $W = 135\text{J}$, $\tau = 30\text{fs}$, $\lambda = 800\text{nm}$, $\varphi_{\text{coll}} = 135^\circ$

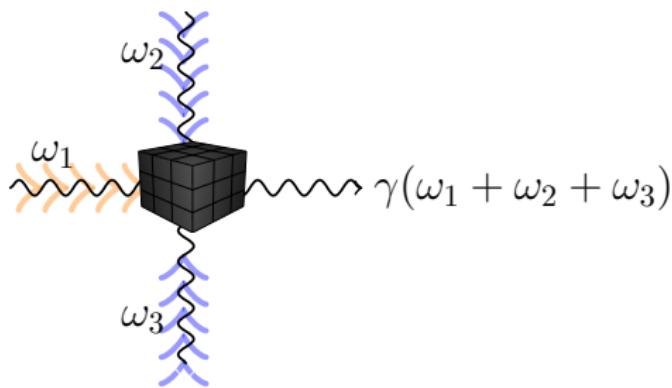
- Differential numbers of **laser** and **signal** photons
- **Maxima** in signal photons **shifted** from laser frequency ω
- Signal photons with wider angular distribution

Frequency Doubling



- Second beam with doubled frequency
- 50 % energy loss
- All beams are “optimally” focused

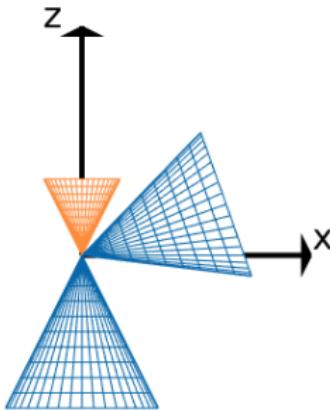
Four-Wave Mixing



- Photon scattering gives rise to **fourth energy**
- **Kinematics** of signal photon $\gamma \rightarrow$ **deviate** from incoming beams

E. Lundstrom et al.: Phys. Rev. Lett. 96 (2006), 083602

Three-Beam Setup



Parameters: $W = 25\text{J}$, $\tau = 25\text{fs}$, $\lambda = 800\text{nm} \rightarrow \omega_1 = 1.55\text{eV}$

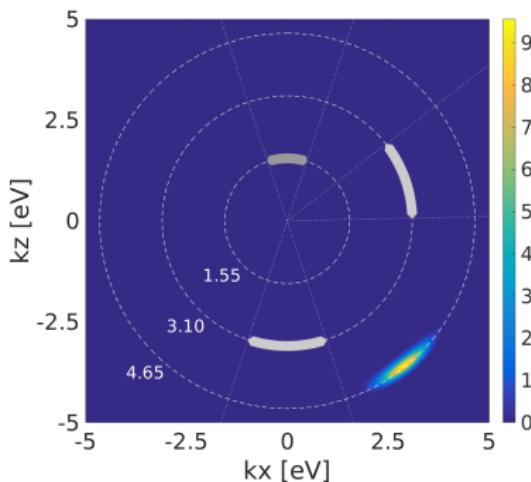
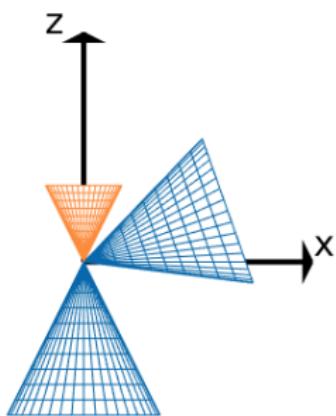
Parameters: $W = 6.25\text{J}$, $\tau = 25\text{fs}$, $\lambda = 400\text{nm} \rightarrow \omega_2 = 3.1\text{eV}$, $\varphi_2 = 70.5^\circ$, $\varphi_3 = 180^\circ$

Combine **high-intensity** beam with two **frequency-doubled** beams

E. Lundstrom et al.: Phys. Rev. Lett. 96 (2006) 083602

N. Seegert: PhD Thesis, 2017

Directional Emission Characteristics



- Frequency-tripled signal
- Outside of background beam foci (grey areas)

Takeaways: Light-by-light Scattering at Low Energies

Summary

- Vacuum emission picture
→ photon-photon scattering **beyond plane-wave** approximation
- **Multi-beam** setups
- **Directional** emission characteristics
- Signatures in signal photon **polarization**

Outlook

- Multi-scale problems
- Higher modes

Outline

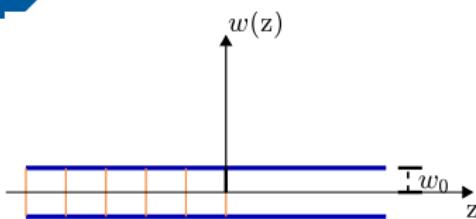
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- Multiphoton Pair Production

Paraxial Approximation: Unfocused Beam

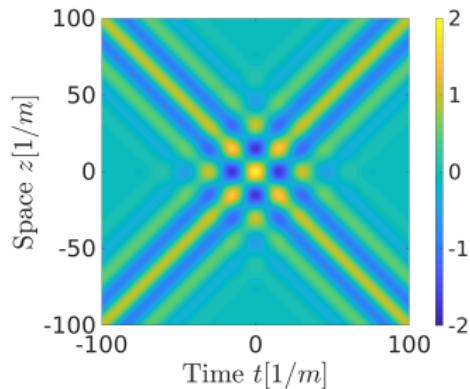


transverse profile $w(z) = w_0$

no beam divergence

- High-energy photon beams → hard to focus
- $\mathcal{E}(t, z) = \varepsilon \varepsilon_{crit} e^{-\frac{(z \pm t)^2}{(\tau/2)^2}} \cos(\omega(z \pm t) + \varphi)$
- Frequency $\omega = \frac{2\pi}{\lambda}$ Field strength ε Duration τ
- Perpendicular, unidirectional, electric & magnetic fields

Colliding Beams

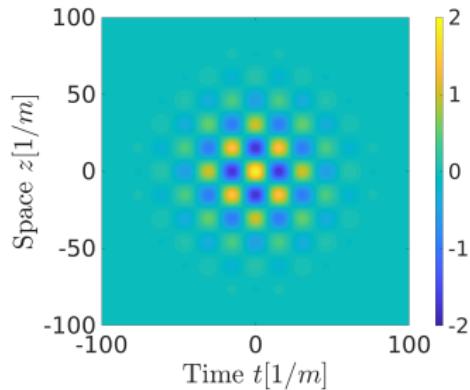


$$\mathcal{E}_{\pm}(z, t) = \varepsilon \varepsilon_{crit} e^{-\frac{(z \pm t)^2}{(\tau/2)^2}} \cos(\omega(z \pm t))$$

Frequency $\omega = \frac{2\pi}{\lambda}$ Field strength ε Duration τ

- Two colliding beams
- Non-vanishing Lorentz invariants in vicinity of collision center
- Incoming and outgoing beams

Standing Wave Approximation

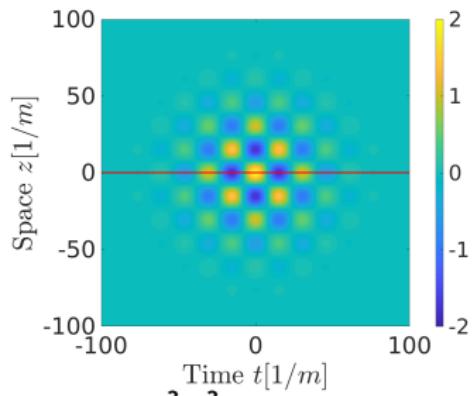


$$\mathcal{E}_{\pm}(z, t) = \textcolor{brown}{\varepsilon} \varepsilon_{crit} e^{-\frac{z^2+t^2}{(\tau/2)^2}} \cos(\omega(z \pm t))$$

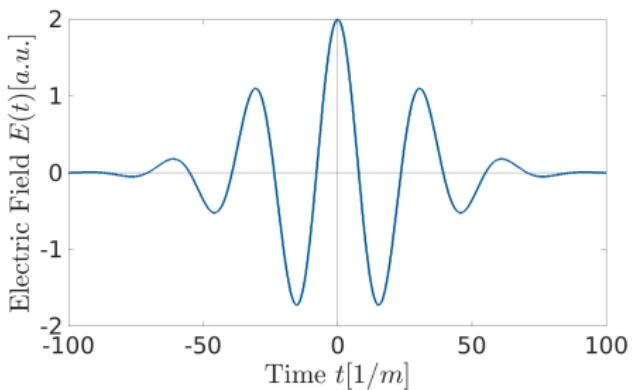
Frequency $\omega = \frac{2\pi}{\lambda}$ Field strength ε Duration τ

- Local standing wave
- Background fields vanish at $t \rightarrow \pm\infty$

Dipole Approximation



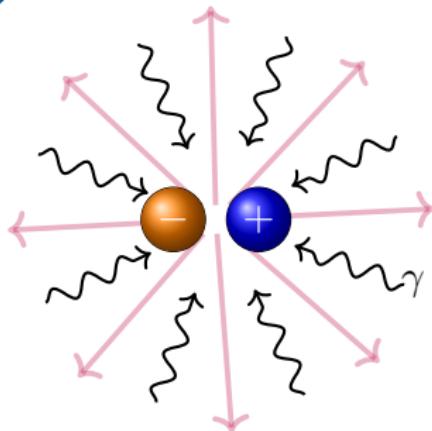
$$\varepsilon_{\pm}(z, t) = \varepsilon_{crit} e^{-\frac{z^2+t^2}{(\tau/2)^2}} \cos(\omega(z \pm t))$$



Frequency $\omega = \frac{2\pi}{\lambda}$ Field strength ε Duration τ

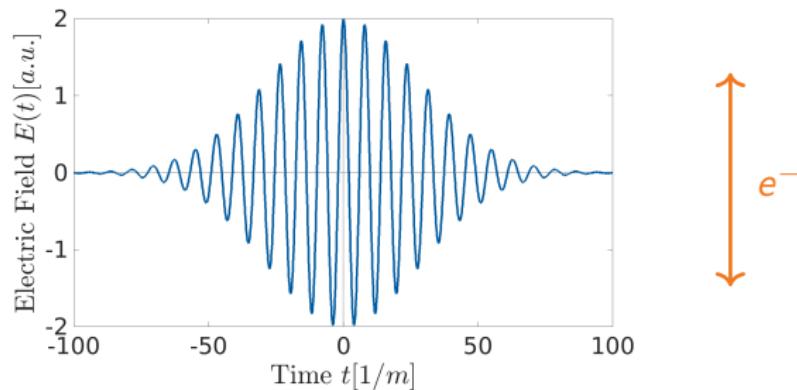
- Single point in space $Z = Z_0$
- Spatially homogeneous electric field
- Magnetic field vanishes automatically

Multiphoton Pair Production: Dipole Approximation



- Photons γ do not carry linear momentum
- Transfer of energy and angular momentum
- One photon can decay into particle pair

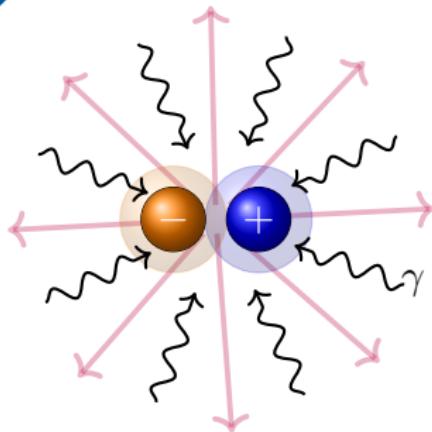
Ponderomotive Energy



Frequency $\omega = \frac{2\pi}{\lambda}$ Field strength ε

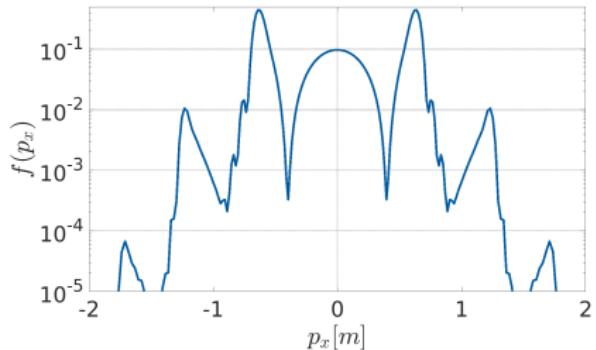
- Created particles **quiver** in oscillating field
- Time-averaged energy $U \propto \varepsilon^2/\omega^2$

Effective Multiphoton Pair Production



- e^-e^+ interact with electric background field
- Particles behave as if they had a higher mass
- Effective mass $m_* = m\sqrt{1 + \varepsilon^2/(2\omega^2)}$

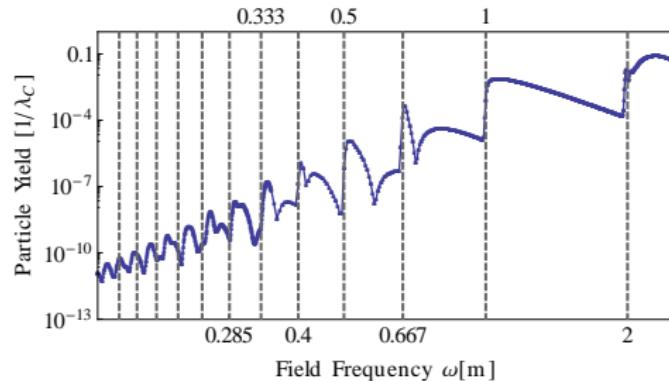
Particle Distribution



Parameters: $\tau = 40 m^{-1}$, $\varepsilon = 0.2$, $\omega = 0.8 m$

- Above-Threshold peaks
- Peak position predictable via effective mass concept
- Energy conservation: $\left(\frac{n\omega}{2}\right)^2 = m_*^2 + p_n^2$

Particle Yield

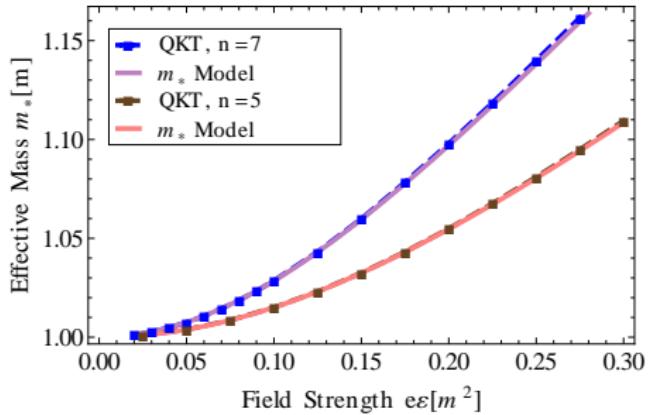


Parameters: $\tau = 100m^{-1}$, $\varepsilon = 0.1$

- Resonant at n-photon frequencies: $\omega_n = 2m_*/n$

C. Kohlfürst et al.: Phys. Rev. Lett. 112 (2014), 050402

Effective Mass

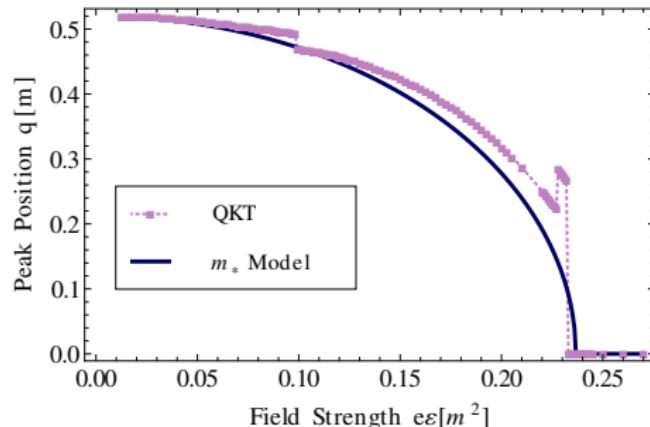


Parameters: $\tau = 100m^{-1}$, $n = 7$

Parameters: $\tau = 100m^{-1}$, $n = 5$

- Comparison: numerical simulation (QKT) - m_* model

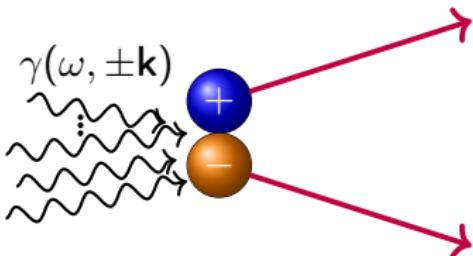
Channel Closing



Parameters: $\tau = 300m^{-1}$, $\omega = 0.322m$

- Above-Threshold **peak position** varies with field strength ε
- Resonance: **Peak at threshold ($q = 0$)**

Multiphoton Pair Production: Standing-Wave Approximation



- Transfer of energy, linear and angular momentum
- Scattering channels distinguishable
- Unique momentum spectrum

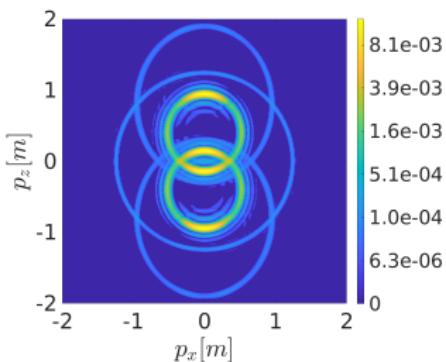
Energy-Momentum Conservation

Energy : $E_{e^+} + E_{e^-} = n_+ \omega + n_- \omega$ (25)

Momentum : $p_{z,e^+} + p_{z,e^-} = n_+ \omega - n_- \omega$ (26)

- Standing wave formed by two laser beams propagating in $\pm z$
- Number of contributing photons per laser beam n_+ and n_-
- Photons with energy ω and momentum $k = \pm \omega$

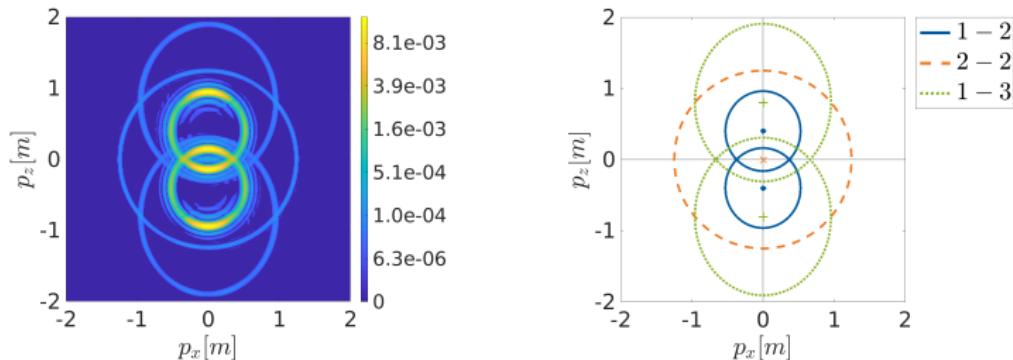
Momentum Spectrum



Parameters: $\tau = 60\text{m}^{-1}$, $\omega = 0.8\text{m}$, $\varepsilon = 0.2$

- Offset in p_z possible
- Above-Threshold pair production

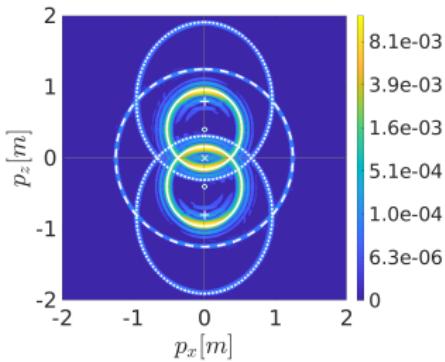
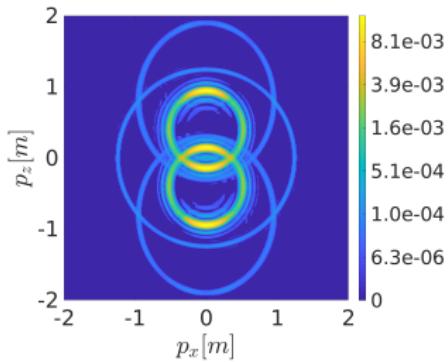
Momentum Spectrum: Channels



Parameters: $\tau = 60 m^{-1}$, $\omega = 0.8 m$, $\varepsilon = 0.2$

- Multiphoton channels: $n_+ - n_-$ and $n_- - n_+$
- Circles: $n_+ = n_-$
- Ellipses: $n_+ \neq n_-$

Momentum Spectrum: Channels



Parameters: $\tau = 60 m^{-1}$, $\omega = 0.8 m$, $\varepsilon = 0.2 E_{\text{cr}}$

- Overlay of analytical and numerical results
- Interference pattern: Angular momentum conservation

Takeaways: Light-by-light Scattering at High Energies

Summary

- Phase-space formalism → pair production processes in the non-perturbative threshold domain
- Background-field dependent threshold

Outlook

- Back-reaction
- Beyond mean-field
- 3D-simulation

Thank you!

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Futher Reading I

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-  C. Kohlfürst, Phys. Rev. D **99** (2019) no.9, 096017 [[arXiv:1812.03130 \[hep-ph\]](https://arxiv.org/abs/1812.03130)].